ANALYSIS OF NONLINEAR PROBLEMS IN HYDRODYNAMICS AND THERMODYNAMICS
REACTION-DIFFUSION, RENSSELAER POLYTECHNIC INSTITUTE, TROY, NY.
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Analysis of Nonlinear Problems in Hydrodynamics and Reaction-Diffusion

ABSTRACT

The research supported by this contract focused on fluid dynamics and hydrodynamic stability and related issues. Other work on this contract included lubrication, work on reaction-diffusion systems, dynamics of biochemical systems and multiphase flows. The problem on the stability and bifurcations of the flow between two rotating cylinders was studied for its simplicity, importance, and its richness in possible flow patterns. The work on lubrication studied the Reynolds equation for two-dimensional and unsteady flows. The work on biological dynamics focused on the stability of motions of cells and chemicals from the point of view of morphogenesis, or the formation of patterns.

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20. ABSTRACT CONTINUED

Reaction-diffusion equations occur in many natural and technological situations. We have been studied extensively under this contract. First, reaction diffusion systems in biology include the release, transport and action of neurotransmitters. The effects of other chemicals that enhance or block the actions of the neurotransmitter ions have been studied. Second, the fluid dynamics of combustion processes have been extensively studied. The stability of flows to inhomogeneities in fuel and temperature have been described.
Final Report

ANALYSIS OF NONLINEAR PROBLEMS IN HYDRODYNAMICS
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Prepared by D. A. Drew
The work supported by this contract is focused on fluid dynamics and hydrodynamic stability and related issues. Other work on this contract included lubrication work, reaction diffusion systems, dynamics of biochemical systems and multiphase flows.

The problem on the stability and bifurcations of the flow between two rotating cylinders was studied for its simplicity, importance, and its richness in possible flow patterns. This flow situation is important because it is a model for the atmosphere (near the equator) and a model for a lubricated journal bearing. It is also a good physical situation for the study of turbulence. The flow is purely azimuthal for sufficiently slow flows, bifurcating to toroidal vortices (Taylor vortices) for sufficiently fast flows, bifurcating again to wavy vortices at still faster flows, eventually leading to turbulence. End conditions due to the finiteness of the apparatus cause the bifurcation to be "gradual," with weak vortices existing for very slow flows.

The analysis of this situation centered around the quasi-linear stability analyses that start from the linear stability problem and assume that the nonlinear solution is an eigensolution of the linear problem, modulated by a slowly varying amplitude. The slowly varying amplitude (or amplitudes, when a second mode occurs, as in the wavy vortices) is governed by a nonlinear ordinary differential equation. The structure of the equations is quite rich, in that many different bifurcation possibilities exist.

The work on lubrication studied the Reynolds equation for two-dimensional and unsteady flows. The Reynolds equation is an equation for the pressure in the lubrication area of a bearing, derived by assuming the fluid film is thin. In two dimensional slider bearings, it is important to describe the leakage out of the sides of the bearing. In order to do so, it is necessary to examine the dynamics near the edges of the bearing. These equations were derived and studied. In addition, some results were obtained on squeeze films, which are unsteady lubrication flows.

The work on biological dynamics focused on the stability of motions of cells and chemicals from the point of view of morphogenesis, or the formation of patterns. It was discovered that spatial pattern formation could be viewed as an instability in chemotaxis, the response of cells to secretion and subsequent decay of chemical attractors by the cells. Work stemming from this early recognition of the possibility has led to similar models for morphogenesis in embryos.

Reaction-diffusion equations occur in many natural and technological situations. Two have been studied extensively under this contract. First, reaction diffusion systems in biology include the release, transport and action of
neurotransmitters. The effects of other chemicals that enhance or block the actions of the neurotransmitter ions have been studied. Second, the fluid dynamics of combustion processes have been extensively studied. The stability of flows to inhomogeneities in fuel and temperature have been described.

Polymerization is the process by which long chain molecules are formed from monomers. The process is dominated by reaction and diffusion. A model for this process has been derived, and results giving the rate of formation of polymers have been obtained.

Multiphase fluid dynamics is also important in many industrial and natural processes. Under the contract, the equations of motion, constitutive equations and predictions from these equations have been studied. Terms to make the equations well posed have been found. The effect of viscosity in shear flows has been quantified. A solution making use of an asymptotic analysis of small and large terms in the combustion of monopropellant particles has been found.

People supported by this contract over its long life include:

R. C. DiPrima, who was a pioneer in the Taylor vortex flow, but passed away before the end of the contract;

L. A. Segel, who was one of the early workers in biomathematics and reaction-diffusion equations in biology;

J. T. Stuart of Imperial College, London, a noted fluid stability researcher;

A. K. Kapila, who did much to couple the fluid dynamics to the combustion processes in deflagrations and detonations;

D. A. Drew, who worked on multiphase flows;

B. Ng, who did numerical work on the stability and bifurcation in Taylor cells;

P. Hall, who did some basic work on the stability of curved flows;

P. Eagles, who did some of the difficult numerical solutions of the Orr Summerfeld equation necessary to quantify the bifurcations;

A. Pridor, who did some numerical work.
The following students received degrees after some support under this contract.

W. Steinmetz (Ph. D.)
J. Schmitt (Ph. D.)
E. F. Pate (Ph. D.)
G. Ganser (Ph. D.)
T. Jackson (Ph. D.)
M. Bentrcia (Ph. D.)

Publications under this contract and related work:


S. I. Rubinow, L. A. Segel and W. Ebel, "A mathematical framework for the study of morphogenetic development in the slime mold." (submitted for publication)


P. Hall, "Centrifugal Instabilities in Finite Boundaries: A Periodic Model", Accepted for publication by the Journal of Fluid Mechanics.


A. K. Kapila, "Response of a Plane Flame to a Normally Incident Acoustic Wave", Accepted for publication in the Proceedings of the International Chemical Reaction Engineering Conference, Pune, India.


D. A. Drew, "Effect of a Wall on the Lift Force", accepted by Chemical Engineering Science


D. A. Drew, "One dimensional burning wave in a bed of monopropellant particles," Combustion Science and Technology, 47, 139 164 (1986)


