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By Robert N. Murtha

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BLAST DESIGN PROCEDURE FOR FLAT SLAB STRUCTURES

ABSTRACT A general step-by-step procedure was developed for designing flat slab structures to resist dynamic blast loads. The procedure is consistent with the Navy's current blast-resistant design manual NAVFAC P-397 and is based on an equivalent single-degree-of-freedom (SDOF) model of a flat slab. The distribution of reinforcement throughout the slab is based on the elastic distribution of design moments outlined by the American Concrete Institute (ACI). The step-by-step procedure is easily adapted to flat slabs of any configuration and considers both flexural and shear behavior.

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INTRODUCTION

Many explosive storage magazines with flat slab roofs, such as the Navy Type IIB magazine, are in use by the Navy, but at the relatively large, nonstandard magazine separation distances required by NAVSEA OP-5 (Ref 1). Since box-type flat slab roof magazines are popular with operations personnel, many new magazines will be of this design. In order to reduce the land requirements, these new magazines will be designed to withstand the larger blast loads associated with the shorter standard magazine separation distances in Reference 1. However, the Navy's current blast-resistant design manual, NAVFAC P-397 (Ref 2), does not contain a design procedure for flat slabs. Therefore, the objective of this report was to develop and document the procedures for designing flat slab structures to resist dynamic blast loads. The work discussed in this report was sponsored by the Department of Defense Explosives Safety Board and is part of the Naval Civil Engineering Laboratory's explosives safety program supporting ordnance logistics to the Fleet.

BACKGROUND

NAVFAC P-397 presents methods of design for protective construction used in facilities for the storage of explosive materials. The primary objectives of this manual were to establish design procedures and construction techniques whereby propagation of explosions or mass detonations would be prevented and protection for personnel and valuable equipment would be provided. The objectives are based upon the results of numerous full- and small-scale structural response and explosive effects tests. Questions about the safety of the older Type IIB design and uncertainties in the design of a new Type A magazine resulted in the tests of 1/2-scale models of these magazines in ESKIMO VI (Ref 3 and 4). The results of ESKIMO VI and tests at U.S. Army Waterways Experiment Station (Ref 5) clearly demonstrated that the existing (pre-1980) design procedure for flat slab roofs provided an excessive margin of safety against failure from design blast loads. A preliminary blast design procedure, using test results and analysis methods, was developed in 1981 by NCEL for safe and efficient flat slab structures (Ref 6). This procedure was expanded by Ammann and Whitney Consulting Engineers (Ref 7) to include requirements on the design of columns, column capitals, drop panels, elastic distribution of reinforcement, and the calculation of stiffness and deflection. The procedure was then used by Ammann and Whitney in the design of a typical flat slab box magazine with two interior circular columns and continuous exterior walls.

SCOPE AND APPROACH

The primary purpose of this study was to develop and document a general design procedure for flat slab structures subjected to blast loads. Eventually this procedure would be implemented into NAVFAC P-397. The structures were to be limited to single-story, box-type configurations with continuous exterior walls. There was to be no limit on the number of continuous spans in any direction, but each panel had to be rectangular and have its ratio of longer to shorter span not greater that 2.0.

The analysis portion of the design would use the same basic theory, most of the same notation, and many of the same equations that were used in NAVFAC P-397. Thus, a working knowledge of P-397 would be very important in understanding the design procedures to be outlined in this report. An equivalent elastic-plastic, single-degree-of-freedom (SDOF) model of the flat slab would form the basis of the design. Its ultimate flexural resistance would be determined from "yield-line" theory using a collapse mechanism similar to that found by tests. Response of the system could then be found using equations and charts in NAVFAC P-397 for idealized impulse or triangular loads, or with numerical integration of the equations of motion for more complicated loading functions. Since the prediction of the response of reinforced-concrete structures to dynamic loads is relatively inexact, simplifying assumptions were to be made, when appropriate, to facilitate the design process. Results of two-way and flat slab tests were to be used to establish flexural failure criteria that would limit the maximum deflection and support rotation of the structure. Sufficient shear capacity must then be provided to preclude premature shear failure and allow development of the flexural capacity of the flat slab.

An important aspect of the design was the use of the ACI (Ref 8) published elastic factored moment distribution for initial selection of the reinforcement throughout the flat slab. Yield-line theory allows freedom in the choice of the reinforcement arrangement; however, an elastic distribution is recommended for several reasons:

- The design is more economical.

- Better service load behavior is obtained in regards to cracking, especially when the design blast loads are relatively low in relation to the service loads.

- Moment distribution required to achieve the design configuration is minimized.

- With the required concentration of the reinforcement in the column strips, the possibility of failure by localized yield patterns is remote.

A detailed discussion of this elastic distribution is contained in Appendix A.

DESCRIPTION OF FLAT SLAB

In reinforced-concrete buildings, slabs are used to provide flat, useful surfaces. A reinforced-concrete slab is a broad, flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so. By definition, a flat slab structure consists of a slab built monolithically with columns and supported directly by these columns without the aid of beams and girders. The flat slab system analyzed in this report has continuous monolithic exterior walls. When the ratio, β, of the long span, L, to the the short span, S, as shown in Figure 1(a), is less than 2, the deflected surface becomes one of double curvature. The roof load is then carried in both directions to the four supporting columns of the panel. The column tends to punch upward through the slab, and the inclined cracking arising from the punching shear must be prevented. Thus, it is common to enlarge the top of the column in the shape of an inverted frustum, known as the column "capital." Further shear (inclined cracking) resistance may be obtained by thickening the slab in the vicinity of the column; this thickened portion is known as the "drop panel" or simply the "drop" (see Figure 1(b)). The columns and column capitals may be either round or square in cross section, but round column capitals are preferred to avoid shear stress concentrations. However, for calculational purposes, the circular capital is sometimes converted to an equivalent square capital via the following equality:

$$\frac{\pi d^2}{4} = c^2$$

where: d = capital diameter (in.)

c = equivalent square length (in.)

Therefore,

$$c = \sqrt{\frac{\pi}{4}}\, d = 0.89\ d \qquad (1)$$

EQUIVALENT SDOF MODEL

As stated earlier, the design method is based on the elasticplastic analysis of a single-degree-of-freedom (SDOF) representation of the flat slab. The following SDOF parameters are required to fully describe the flat slab behavior:

- Ultimate unit flexural resistance, r_u, (psi) of the actual system.

- Equivalent unit stiffness, K_E, (psi/in.) of the actual system.

- Effective unit masses, m_{ef}, (lb-sec²/in.³) of the equivalent SDOF system in the elastic range and in the plastic range.

The effective natural period of vibration of the SDOF system is then given by:

$$T_n = 2\pi\sqrt{\frac{m_{ef}}{K_E}} \tag{2}$$

Structures designed for high pressure loads at short scaled distances, such as storage magazines, will generally be sensitive to impulse loading. The maximum response, X_m, of structural elements that are sensitive to just the impulse loading (area under the pressure-time load history) and that are allowed large deflections (maximum support rotations, θ_m, greater than 5 deg) can be determined from the impulse loading, i_b, the effective unit mass, m_{ef}, in the plastic range, and the ultimate unit resistance, r_u. That is,

$$X_m = \frac{i_b^2}{2\, m_{ef}\, r_u} \tag{3}$$

where: X_m = maximum transient deflection (in.)

At allowable support rotations less than 5 degrees and for pressure-sensitive structures, the elasto-plastic portion of the resistance deflection curve must also be determined and used in the response calculations. Response of the structure can be found using charts in P-397 for idealized impulse or triangular loads, or with numerical integration of the equations of motion for more complicated loading functions.

Ultimate Unit Flexural Resistance

The ultimate unit flexural resistance is the static uniform pressure load, r_u (psi), that the structural element can sustain during plastic yielding of the collapse mechanism. This resistance is assumed to remain essentially constant over a wide range of deflection (θ_m < 12 deg). The r_u value defines the plastic portion of the resistance deflection curve (see Figure 2). A conservative lower bound can be determined using yield-line procedures (Ref 9 and 10).

The ultimate uniform resistance is a function of the amount and distribution of the reinforcement (i.e., moment capacities of the slab strips), the geometry of the slab, and the support conditions. A yield-line analysis can be used to determine r_u in terms of these parameters. Since in-plane compression forces and membrane tensile forces are not considered, the ultimate resistance determined from a yield-line analysis will generally be lower than the actual resistance.

Yield-line analysis is an ultimate load determination method in which a flexural element is assumed to fail along lines that form a valid failure mechanism. The first step is to assume a yield-line pattern consistent with the stated conditions. The pattern will contain one or more unknown dimensions that locate the positions of the yield lines. Sectors between yield lines are assumed to rotate rigidly, and ultimate resisting moments are assumed to develop along the full length of all yield lines. Either equilibrium or energy (virtual work) methods can be used to find the critical collapse mechanism and associated minimum r_u value. Though P-397 uses the equilibrium method, the complexity of reinforcement in most flat slabs makes the energy method a better choice for flat slab design.

Figure 3 shows examples of failure mechanisms found by test and analysis to apply to flat slabs. In order to calculate the ultimate unit resistance, equations for the internal work, E, and external work, W, are written in terms of r_u, the moment capacities of the sections, and the geometry of the structure and failure mechanism. The expression for external work is then set equal to that for internal work, and the resulting equation is solved for the minimum value of r_u and the associated geometry of the failure mechanism.

The external work done by r_u on rotating sector i is:

$$W_i = r_u A_i \Delta_i \tag{4}$$

where: A_i = area of sector i (in.^{2})

Δ_i = deflection of the c.g. of sector i (in.)

The total external work is the sum of the work done on each sector:

$$W = \Sigma W_i = \Sigma r_u A_i \Delta_i \tag{5}$$

For illustration, see Figure 4 which shows a quarter section of the flat slab given in Figure 3(a). The external work on sector B is the sum of the work done on the rectangular portion and the work done on the triangular portion. That is,

$$W_B = r_u \left[\left(S - y - \frac{c}{2}\right) x \frac{\Delta}{2} + x \frac{y}{2} \left(\frac{\Delta}{3}\right)\right] \tag{6}$$

$$= r_u x \Delta \left(\frac{S}{2} - \frac{y}{3} - \frac{c}{4}\right)$$

where: L = long span length (in.)

S = short span length (in.)

x,y = distances to yield lines (in.)

Δ = maximum deflection of the sector (in.)

The internal work, E, done by the actions at the yield lines is due only to the bending moments as the support reactions do not undergo any displacement and the work done by the shear forces is zero when summed over the entire slab. The internal work, E_{ij}, for each yield line is the rotational energy done by moment M_n rotating through θ_n. That is,

$$E_{ij} = M_n \theta_n = m_n \theta_n \ell_n \tag{7}$$

where: M_n = moment capacity along yield line (in.-lb)

m_n = unit moment capacity along yield line (in.-lb/in.)

θ_n = rotation about yield line (radian)

ℓ_n = length of yield line (in.)

The total internal work is the sum of the rotational energies for all yield lines:

$$E = \Sigma E_{ij} = \Sigma\, m_n\, \theta_n\, \ell_n \tag{8}$$

As stated earlier, the flat slab design is based on the ACI elastic distribution of reinforcement. This distribution recognizes three orthogonal bands (i.e., column, middle, exterior), each containing different levels of reinforcement. Thus, it is more convenient to write the internal work in terms of moments (M_x, M_y) and rotations (θ_x, θ_y) in the principal reinforcement directions x and y. That is,

$$E_{ij} = M_x\, \theta_x + M_y\, \theta_y \tag{9}$$

or

$$E_{ij} = m_x\, s_y\, \theta_x + m_y\, s_x\, \theta_y \tag{10}$$

where: m_x, m_y = ultimate unit moment capacities in the x and y directions (in.-lb/in.)

s_y, s_x = lengths of the yield line in the y and x directions over which m_x and m_y apply (in.)

θ_x, θ_y = relative rotations about the yield lines in the x and y directions

As an example, consider the structure in Figure 4 with rotating sectors A through D, areas 1 to 9 of equal moment capacities (in bands of width s defined by dashed lines) and geometry defined by L, S, c, x, and y. The internal work along yield line AB (yield line between sectors A and B) is

$$E_{AB} = \left[m_{9x}\left(s_{ey} - \frac{c}{2}\right)\theta_B + m_{5x}\left(y + \frac{c}{2} - s_{ey}\right)\theta_B\right] + \left[m_{9y}\left(s_{ex} - \frac{c}{2}\right)\theta_D + m_{5y}\left(x + \frac{c}{2} - s_{ex}\right)\theta_D\right] \qquad (11)$$

where: m_{9x} = unit moment capacity in x direction in area 9

m_{5x} = unit moment capacity in x direction in area 5

m_{9y} = unit moment capacity in y direction in area 9

m_{5y} = unit moment capacity in y direction in area 5

Substituting $\theta_B = \Delta/x$ and $\theta_D = \Delta/y$

$$E_{AB} = \frac{\Delta}{x}\left[m_{9x}\left(s_{ey} - \frac{c}{2}\right) + m_{5x}\left(x + \frac{c}{2} - s_{ey}\right)\right] + \frac{\Delta}{y}\left[m_{9y}\left(s_{ex} - \frac{c}{2}\right) + m_{5y}\left(x + \frac{c}{2} - s_{ex}\right)\right] \qquad (12)$$

Likewise, along line BD:

$$E_{BD} = \left[m_{2x}\frac{s_{cy}}{2} + m_{5x}\left(S - \frac{s_{cy}}{2} - \frac{c}{2} - y\right)\right]\left(\theta_A + \theta_B\right) \qquad (13)$$

Substituting $\theta_A = \Delta/(L - x - c)$ and $\theta_B = \Delta/x$

$$E_{BD} = \left[m_{2x}\frac{s_{cy}}{2} + m_{5x}\left(S - \frac{s_{cy}}{2} - \frac{c}{2} - y\right)\right]\left(\frac{\Delta}{L - x - c} + \frac{\Delta}{x}\right) \quad . \; . \; . \; . \qquad (14)$$

The external work on all sectors and internal work on all positive and negative yield lines are determined and summed. An equation for r_u is written from:

$$W = E \qquad (15)$$

$$x\,\Delta\left(\frac{S}{2} - \frac{y}{3} - \frac{c}{4}\right)r_u + \cdots = m_{9x}\left(s_{ey} - \frac{c}{2}\right)\frac{\Delta}{x} + \cdots$$

$$r_u = \frac{m_{9x}\left(s_{ey} - \frac{c}{2}\right) \Big/ x + \cdots}{x\left(\frac{S}{2} - \frac{y}{3} - \frac{c}{4}\right) + \cdots} \tag{16}$$

Variables x and y are varied independently until r_u is minimized. This minimum solution provides the failure mechanism and the value of the ultimate resistance, r_u. A rapid determination of the solution can be obtained using a programmable electronic calculator. A trial and error procedure to solve for the minimum value of the resistance function, r_u, can be accomplished as follows:

- Start with both crack lines located close to the centerline of the middle strip.

- Move one crack line, holding the other constant, in the direction which minimizes the resistance function until r_u begins to increase.

- Hold the first crack line constant, and vary the second crack line in the minimum direction until r_u also begins to increase.

- Once this minimum point is achieved, shift each crack line to either side of the minimum location to check that a further refined shifting of the crack line is not necessary to minimize the resistance function.

It should be noted that if the crack line should shift out of the middle strip, a new resistance function equation must be written and the procedure then repeated. Appendix B contains detailed information on the determination of the ultimate unit flexural resistance for a flat slab.

Equivalent Elastic Unit Stiffness

The elastic deflections for several points of an interior panel of a flat slab are given by the general equation

$$X = C\left[r_u L^4 (1 - \nu^2)\right]/E_c I_a \tag{17}$$

where: C = constant varying with panel aspect ratio L/S, the ratio of support size c to the span length L, and the location within the panel.

E_c = modulus of elasticity of concrete = $w^{1.5}\, 33\sqrt{f'_c}$ (psi)

w = unit weight of concrete (lb/ft^3)

f'_c = static ultimate compressive strength of concrete (psi)

I_a = average of gross and cracked moments of inertia

= $(I_g + I_c)/2$ ($in.^4/in.$)

I_g = moment of inertia of gross concrete section

= $(1/12)(t_{avg})^3$ ($in.^4/in.$)

I_c = moment of inertia of cracked concrete section

= $5.5\, p_{avg}\, (d_{avg})^3$ ($in.^4/in.$)

t_{avg} = average slab thickness within panel (in.)

p_{avg} = average steel reinforcement ratio within panel

d_{avg} = average effective depth within panel (in.)

ν = Poisson's ratio of concrete

Values of the constant C are based on a finite difference method (Ref 11) and are given in Table 1 for the center of the panel and the midpoints of the long and short sides. The deflection for the center of the interior panel is determined by using C_C in the above expression. For the corner, long and short side panels (Figure 1), no simplified solutions for the center deflections are currently available. Generally, the deflections for the side panels will be smaller than the deflection of the interior panel because of the restraining effect of the exterior walls. These deflections can be approximated by using the following expressions:

$$\text{Long Side Panel} \qquad C = C_C - \frac{C_S}{2} \tag{18}$$

$$\text{Short Side Panel} \qquad C = C_C - \frac{C_L}{2} \tag{19}$$

where the values of C_C, C_S, and C_L are those for the interior panel.

When the maximum allowable deflection of the panel is small (allowable support rotation < 1 deg), the dynamic response of the system will be more sensitive to the elastic stiffness, and it will be necessary to obtain a better value of the elastic deflections by using another procedure such as the equivalent frame method of Reference 12.

The equivalent elastic unit stiffness of a flat slab panel is given by:

$$K_E = r_u/X_E = \frac{E_c I_a}{C L^4 (1 - \nu^2)} \tag{20}$$

Because of the complexity of the behavior, the elastic-plastic transition range will be ignored. That is, no method will be given to determine the stiffness within this range. All dynamic response calculations will use the previously given elastic stiffness and deflection relationships rather than an "effective" bilinear resistance function based on the actual non-bilinear function (e.g., fixed-fixed beam representation).

Effective Unit Mass

The mass of an equivalent SDOF system is not the actual mass of the structure since movement of all elements of the mass is not equal. The actual mass of the structure must be replaced by an effective mass, m_{ef}, the mass of the equivalent single-degree-of-freedom system. The value of the effective mass is dependent upon the deflected shape of the structural member, varying with the type of spanning, end conditions, etc., and therefore is different in the elastic, elasto-plastic, and plastic ranges of behavior. The effective unit mass of the equivalent system is related to the unit mass of the actual system by:

$$m_{ef} = K_{LM}\, m \tag{21}$$

where: m_{ef} = effective unit mass (lb-sec²/in.³)

m = actual unit mass (lb-sec²/in.³)

K_{LM} = load-mass factor

For a flat slab without drop panels, the actual unit mass equals:

$$m = m_{slab} + m_{ob} = \rho_{slab}\, t_{slab} + \rho_{ob}\, t_{ob} \qquad (22)$$

where: m_{slab} = unit mass of slab (lb-sec²/in.³)

m_{ob} = unit mass of soil overburden (lb-sec²/in.³)

ρ_{slab} = mass density of slab (lb-sec²/in.⁴)

t_{slab} = thickness of slab (in.)

ρ_{ob} = mass density of soil overburden (lb-sec²/in.⁴)

t_{ob} = thickness of soil overburden (in.)

For a flat slab with drop panels, the actual unit mass must be obtained from this expression:

$$m = M_T/A_T \qquad (23)$$

where: M_T = total mass (slab + soil overburden + drop panel) (lb-sec²/in.)

A_T = total slab area (in.²)

Note that these quantities represent that portion of the structure which rotates (deflects). Therefore, the mass/area inside of the equivalent square capital and outside of the perimeter yield line (wall haunch) are excluded from the calculations.

No data are currently available to determine the load-mass factor, K_{LM}, in the elastic range of behavior. Reference 13 gives a value of 0.64 for a typical interior panel on point supports with L/S = 1. This value is reasonably close to the elastic value (0.61) for a square fixed-ended panel. It is therefore recommended that the following equation for K_{LM} listed in Table 6-1 of Reference 2 for two-way elements with all supports fixed be used for the appropriate L/S ratio for all panels:

$$K_{LM} = 0.61 + 0.16\left(\frac{L}{S} - 1\right) \qquad 1 \leq L/S \leq 2 \tag{24}$$

The load-mass factor in the plastic range is determined using a procedure outlined in Section 6.6 of Reference 2. The procedure uses the equation of angular motion for sections rotating about supports as its basis. This linkage motion results from an assumption of zero moment or curvature changes between plastic hinges under increasing deflection. In Figure 5, a portion of a two-way element bounded by the support and the yield line is shown. The load-mass factor, K_{LM}, for this sector is:

$$K_{LM} = \frac{I_m}{c\, L_1\, M} \tag{25}$$

where: I_m = mass moment of inertia about the axis of rotation AB (lb-in.-sec^2)

c = distance from the resultant applied load F to the axis of rotation AB (in.)

L_1 = total length of sector normal to axis of rotation AB (in.)

M = total mass of sector (lb-sec^2/in.)

When an element (such as a flat slab with drop panels and soil cover) is composed of several sectors, each sector must be considered separately, and the contributions then summed to determine the load-mass factor for the entire element. That is,

$$K_{LM} = \frac{\Sigma(I_m/c\, L_1)}{\Sigma M} \tag{26}$$

For elements of constant depth and therefore of constant unit mass (such as a flat slab without drop panels but with uniform soil cover), the load-mass factor equals:

$$K_{LM} = \frac{\Sigma(I/c\, L_1)}{\Sigma A} \tag{27}$$

where: I = area moment of inertia about the axis of rotation (in.^{4})

A = total area of sector (in.^{2})

The plastic load-mass factors for typical cross sections (rectangle, triangle) are shown in Figure 6. Appendix C contains detailed information on the determination of both the actual unit mass, m, and the plastic load-mass factor, K_{LM}, for flat slabs.

PANEL DEFINITION

The development of the blast design procedure for flat slab structures led to the recognition of the following four panel types:

1. Corner panel, C

2. Long side panel (panel side common to exterior short side), LS

3. Short side panel (panel side common to exterior long side), SS

4. Interior panel, I

These panels are depicted in Figure 1a for a portion of a typical flat slab structure. It is possible to define any arbitrarily configured flat slab structure as a combination of these four panel types. Because of symmetry, the design/analysis can be simplified if the side and interior panels are further divided into subpanels. Figure 7 shows the make-up of five different flat slab configurations. Each of the divided subpanels is equal for the flat slabs shown in Figures 7a through 7d. However, for the 3x4 flat slab (Figure 7e), the distribution of the reinforcement necessitates an A and B subpanel designation for the interior and long side panels. That is, LS/A and LS/B are not identical (different moment distribution and dimensions). By using symmetry in the design procedure, the flat slab can be reduced to a quarter of the total slab. In this report the lower right quadrant is used. Figure 8 shows the symmetric quadrants for the five flat slab configurations of Figure 7.

DESIGN CRITERIA

Ultimate Unit Moment Capacity

The ultimate unit moment capacity of structural sections is based on the ultimate strength design methods of the ACI Building Code (Ref 8) with the strength reduction factor, ϕ, omitted as in Reference 2. For structures that undergo support rotations less than 2 degrees, the static unit moment resistance, m_u, of a Type I cross section (where the cover over the reinforcement on both surfaces remains intact) may be used:

$$m_u = \frac{A_s f_s}{b} (d - a/2) \tag{28}$$

where: A_s = area of tension reinforcement within the width b (in.^{2})

f_s = static design stress for reinforcement (psi)

a = depth of equivalent rectangular stress block (in.)

= $A_s f_s / 0.85\, b\, f'_c$

b = width of compression face (in.)

d = distance from extreme compression fiber to centroid of tension reinforcement (in.)

f'_c = static ultimate compressive strength of concrete (psi)

For structures that undergo rotations greater than 2 degrees, the static unit moment capacity of a Type II or Type III cross section may be used:

$$m_u = \frac{A_s f_s d_c}{b} \qquad A_s \leq A'_s \tag{29}$$

where: A'_s = area of compression reinforcement (in.^{2})

d_c = distance between centroids of the compression and the tension reinforcement (in.)

The static design stress, f_s, can be approximated (as in Ref 2) with a weighted average of the yield strength, f_y, and ultimate strength, f_u, depending on the amount of deflection or rotation of the element (see Table 2).

The dynamic moment capacity of the reinforced-concrete sections is determined from the above equations by substituting the dynamic design stress for the reinforcement, f_{ds}, for f_s, and the dynamic ultimate compressive strength of concrete, f'_{dc}, for f'_c, as applicable, where

$$f'_{dc} = (DIF)\ f'_c \tag{30}$$

$$f_{ds} = (DIF)\ f_s \tag{31}$$

Dynamic increase factors, DIF, for concrete and reinforcing steel are reproduced from Reference 2 in Table 3. It is recommended that no dynamic stress increases be considered when determining shear or bond capacities.

Ultimate Shear Strength

The shear resistance must be sufficient to develop fully the flexural capacity of the slab at large rotations and deflections. The conservative approach in the evaluation of the ultimate resistance (neglecting in-plane compression and membrane tensile forces) requires a conservative evaluation of the shear capacity.

Therefore, the following recommended equations for calculating the shear capacity are significantly lower than those given in Reference 8. In general, shear reinforcement of the slab is to be avoided. The use of a thicker slab is a less costly alternative.

The nominal beam shear strength, v_c, (psi) at a distance d from the face of the wall support is:

$$v_c = 1.9\sqrt{f'_c} + 2{,}500\, p \leq 2.28\sqrt{f'_c} \tag{32}$$

where $p = A_s/b\, d_c$

The above expression for the maximum value of v_c corresponds to a 20 percent increase in the 1.9 factor. This value of 2.28 is used, rather than the 3.5 factor of the ACI Building Code (ACI Section 11.3.2.1), in order to provide a lower bound on the test data used in developing this equation. Beam shears should also be checked at the column capitals and at the drop panels in both the longitudinal and transverse directions.

The shear strength of the slab around the column in two-way action must also be checked. In two-way action, potential diagonal cracking may occur along a truncated cone or pyramid around the column. Thus, the critical section is located so its periphery, b_o, is at a distance equal to one-half of the effective depth through the drop from the periphery of the column capital, and also at a distance equal to one-half of the effective depth outside of the drop from the periphery of the drop. Where no drop is used, of course, there would be only one critical section for two-way action. The nominal punching shear strength at these locations is:

$$v_c = 4.0\sqrt{f'_c} \tag{34}$$

This is identical to the ACI recommended value (ACI Section 11.11.2).

Design of all cross sections subject to shear shall be based on ACI equation 11-1:

$$V_u \leq \phi\, V_c \tag{35}$$

where V_u is the factored shear force at the sections considered, V_c is the nominal shear strength provided by the concrete, and ϕ is the ACI strength reduction factor for shear ($\phi = 0.85$). The ϕ factor is maintained to ensure against a premature failure due to shear which would substantially reduce the overall blast resistant capacity of the slab. Calculation of the factored shear force at any section should be made using the tributary area, A, (in.^{2}) defined by the yield lines and the critical shear section. That is:

$$V_u = r_u A \qquad (36)$$

The nominal shear strength is given by these equations:

$$V_c = v_c b_w d \qquad \text{(Beam shear)} \qquad (37)$$

$$V_c = v_c b_o d \qquad \text{(Punching shear)} \qquad (38)$$

where: b_w = critical section length on which shear stress acts (in.)

b_o = perimeter of critical section for slabs (in.)

d = depth of section (in.)

The critical locations for the shear analysis are shown in Figure 9 for a quarter panel of a flat slab with central column (Figure 3a).

Allowable Deflections

Reference 14 recommended a 12-degree maximum support rotation for laterally unrestrained two-way slabs with L/S ratios less than 2. The static flat slab test in Reference 5 also showed that 12-degree rotations can be attained while maintaining the ultimate resistance. Based on the above, the rotation corresponding to the deflection at incipient failure is 12 degrees. The ultimate deflection is therefore:

$$X_m = L_{min} \tan 12^\circ = 0.2\, L_{min} \qquad (39)$$

where L_{min} is the shortest sector length rotating through 12 degrees. The maximum allowable deflections permitted in the design of a structure vary according to the protection category required. The following maximum values of the support rotation angle, θ, were recommended in Reference 7:

- Personnel shelter θ = 2 degrees
- Equipment shelter θ = 5 degrees
- Explosives magazine θ = 8 degrees

These values are also listed in Table 4.

WALL DESIGN

It was shown in the ESKIMO test series that 12-inch concrete sidewalls and backwalls, reinforced to retain the earth backfill, are adequate to resist blast loads at standard magazine separation distances. Therefore, if these minimums are maintained, these walls need not be checked for blast loads.

COLUMN DESIGN

The columns are designed to resist the axial load and unbalanced moment resulting from the flat slab blast loads and the structure dead load. The columns are designed in accordance with the criteria presented in the ACI Code, Reference 8. Slenderness effects must be included, if applicable, and it is assumed that there is no sidesway since such motion is prevented by the rigidity of the roof slab as a diaphragm and the end shear walls. Fixity at the base of the column is determined from the relative stiffnesses of the column and column footing.

Design of the column shall be based on the ACI equation:

$$P_u \leq \phi P_n \tag{40}$$

where P_u is the factored axial load at given eccentricity, P_n is the nominal axial load strength at given eccentricity, and ϕ is the ACI strength reduction factor for axial compression (ϕ varies from 0.70 to 0.90).

The axial load and moment at the top of the column (the critical section is at the bottom of the capital) are obtained from the flat slab shear forces acting on the perimeter of the column capital plus the load on the tributary area of the equivalent square capital. The dynamic load is essentially applied instantaneously to the column and remains constant for a duration of t_m (time of maximum response for flat slab obtained from the flat slab analysis). The column can then be idealized as an EL-PL SDOF system with an allowable maximum ductility, X_m/X_E, of 3.0.

Useful design procedures and design charts are presented in ACI publication SP-17A (Ref 15). Appendix D contains detailed information on the design of a column.

DETAILING OF REINFORCEMENT

Proper detailing of the reinforcement is required to ensure adequate structural behavior (see Figure 10). A portion of both the top and bottom reinforcement must be continuous across the roof, adequately anchored and spliced, if necessary. Splice locations should be staggered, preferably on opposite sides of the columns and at opposite ends of the slab spans. Splices should always be located in regions of low stress and the number of splices minimized by using the longest rebar possible. All splices of adjacent parallel bars must be staggered to prevent the formation of a local plane of weakness. Added rebar, such as that at the columns, should be discontinued at staggered locations also.

Additional reinforcement at the following two critical locations will also assist in maintaining the integrity of the structure:

- Provide haunches and diagonal bars at the intersection of the exterior wall and roof slab.

- Provide radial diagonal bars enclosed in hoops at the surface of the column capitals.

To ensure proper structural behavior under dynamic loads and also to minimize excessive deformations under conventional loads, the minimum area of flexural reinforcement on each face should be at least equal to that specified in Reference 8 (ACI 7.12.2) for shrinkage and temperature reinforcement. That is,

Minimum A_s = 0.0010 b t each face (Grade 40 or 50)

Minimum A_s = 0.0009 b t each face (Grade 60)

DESIGN PROCEDURE

A summary of the flat slab design procedure is provided here.

Problem: Design a flat slab subjected to a given blast loading

Required Information:

Slab geometry (number of spans, overall dimensions)

Pressure time loading

Material properties

- Concrete
- Soil overburden
- Steel reinforcement

Design criteria

- Minimum static design stress (Table 2)
- Dynamic increase factors (Table 3)
- Deflection failure criteria (Table 4)
- Strength reduction factors

Assumptions:

Capital size

Depth of overburden

Wall/slab thickness ratio

Solution:

Step 1: Distribute the moments in the slab based on elastic response using the direct-design method outlined in ACI 318-77. (See Appendix A for detailed discussion.)

Step 2: Determine yield-line locations and ultimate resistance relationship using energy principles. (See Appendix B for a detailed discussion.)

Step 3: Establish maximum allowable displacement based on failure deflection criteria.

$$X_m = L_{min} \tan \theta_m$$

Step 4: Assume trial slab thickness based on minimum ACI criteria.

$$t_{min} \geq \frac{L}{60}\left(1 + \frac{1}{\beta}\right) \tag{41}$$

Step 5: Perform dynamic analysis on SDOF representation of the slab.

a. Determine SDOF parameters.

Elastic stiffness (Assume $I_c = 0$)

$$K_E = \frac{E_c\ I_a}{c\ L^4\ (1 - \nu^2)}$$

Elastic load-mass factor

$$K_{LM} = 0.61 + 0.16\left(\frac{L}{S} - 1\right)$$

Plastic load-mass factor

$$K_{LM} = \frac{I_M}{c\ L_1\ M}$$

Actual unit mass

$$m = \rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab}$$

Natural time period (elastic range)

$$T_n = 2\pi\sqrt{\frac{K_{LM}\ m}{K_E}}$$

b. Check loading for definition of impulse.

$$\frac{t_d}{T_n} < 0.2 \tag{42}$$

c. Determine required dynamic ultimate unit resistance for impulse sensitive structure using m_{ef} for the plastic range.

$$r_{ud} = \frac{i_b^2}{2\ m_{ef}\ X_m}$$

d. Determine dead load.

$$r_{dl} = g\ m \tag{43}$$

e. Determine the equivalent <u>static</u> required ultimate unit resistance of the slab necessary to flexurally resist both the dead load and the dynamic loading (i.e., must adjust r_{ud}).

$$r_{uf} = \frac{r_{ud}}{DIF} + r_{dl} \tag{44}$$

f. Determine the actual dynamic flexural ultimate unit resistance of the slab. Use Equation 45 to determine the factored shear force for shear design calculations (ACI 11-1).

$$r_{uv} = DIF\ (r_{uf}) \tag{45}$$

g. Determine time to reach maximum response.

$$t_m = \frac{i_b}{r_{ud}} \tag{46}$$

Step 6: Modify slab thickness or steel percentage based upon required ACI minimum steel percentage.

a. Determine required moment capacity and steel percentage at minimum moment section. That is,

$$m_e = f(r_{uf})$$

$$m_{req} = f(m_e)$$

$$(A_s)_{req} = f(m_{req})$$

$$p_{req} = f(A_s)_{req}$$

b. Compare with ACI minimum value, p_{min}, in both L and S directions (ACI 7.12.2).

If $p_{req} \geq p_{min}$ { Increase slab thickness. Return to Step 5.
or
Go to Step 12.

If $p_{req} < p_{min}$ { If possible, decrease slab thickness. (t_{slab} must be equal to or greater than t_{min}.) Return to Step 5.
Otherwise,
Must use p_{min} at all locations where $p_{req} < p_{min}$. Go to Step 7.

Step 7: Revise distribution of moments at all locations where $p_{req} < p_{min}$.

Step 8: Re-determine yield-line locations and value of ultimate resistance for a given slab geometry using energy principles.

Step 9: Re-establish maximum allowable deflection.

Step 10: Perform another dynamic analysis on SDOF representation of slab.

Step 11: Re-check minimum steel percentages.

a. Determine required moment capacity and steel percentage at minimum moment section.

$$m_e = f(r_{uf})$$

$$m_{req} = f(m_e)$$

$$(A_s)_{req} = f(m_{req})$$

$$p_{req} = f(A_s)_{req}$$

b. Compare with ACI minimum value.

If $p_{req} \geq p_{min}$ { Go to Step 12.

If $p_{req} < p_{min}$ { Return to Step 7 if this is the first time through Step 11.
or
Go to Step 12 if values are nearly identical and this is not the first time through Step 11.

Step 12: Check beam shear at walls (within slab).

If $V_u > \phi V_c$ { Increase slab thickness. Return to Step 5.

If $V_u \leq \phi V_c$ { Go to Step 13.

Step 13: Check punching shear at column capital (within slab).

If $V_u > \phi V_c$ { Go to Step 14.

If $V_u \leq \phi V_c$ { Go to Step 17.

Step 14: Design drop panel for punching shear; select dimensions and thickness of drop panel.

If $V_u \leq \phi V_c$ { Go to Step 15.

The following ACI requirements are applicable:

13.4.7.1: Drop panel shall extend in each direction from centerline of support a distance not less that 1/6 the span length measured from center-to-center of supports in that direction. Therefore,

$$L_{dp} \geq \frac{L}{3}$$

$$S_{dp} \geq \frac{S}{3}$$

13.4.7.2: Projection of drop panel below the slab shall be at least 1/4 the slab thickness beyond the drops. Therefore,

$$t_{dp} \geq \frac{t_{slab}}{4}$$

13.4.7.3: In computing required slab reinforcement, the thickness of drop panel below the slab shall not be assumed greater that 1/4 the distance from edge of drop panel to edge of column or column capital. Therefore,

$$t_{dp} < \frac{S_{dp} - d}{8} \qquad \text{where: } d = \text{capital diameter}$$

When determining the steel area required for negative moment in a column strip with a drop panel, the smaller of the actual column strip width or the drop panel width is used. Therefore, it is recommended that the width of the drop panel be set equal to or greater than the column strip width. That is,

$$L_{dp} \geq s_{cx}$$

$$S_{dp} \geq s_{cy}$$

Step 15: Check punching shear at edge of drop panel (within slab).

If $V_u > \phi V_c$ $\left\{\right.$ Increase drop panel dimensions. Return to Step 15.
or
Increase slab thickness. Return to Step 5.

If $V_u \leq \phi V_c$ $\left\{\right.$ Go to Step 16.

Step 16: Check longitudinal and transverse beam shear at drop panel edge (within slab). (Unit width b = 1 ft)

If $V_u > \phi V_c$ — Increase drop panel dimensions. Return to Step 16. or Increase slab thickness. Return to Step 5.

If $V_u \leq \phi V_c$ — Go to Step 17.

Step 17: Check longitudinal and transverse beam shear at column capital (within drop panel). (Unit width b = 1 ft)

If $V_u > \phi V_c$ — Increase drop panel thickness. Return to Step 17.

If $V_u \leq \phi V_c$ — Go to Step 18.

Step 18: Final design of slab.

a. Include drop panel mass in dynamic SDOF analysis *if applicable*.

b. Determine unit moments and steel areas.

c. Check all initial design assumptions (e.g., α_t, α_H).

Step 19: Design column.

Step 20: End.

The flow chart for the above design procedure is depicted in Table 5. Because of the potential number of iterations necessary in the design of the drop panel (size/thickness), the drop panel mass was not included in the SDOF dynamic analysis until the very end (Step 18). However, it is not necessary to redo all of the shear calculations with this new lower ultimate resistance value.

EXAMPLE PROBLEM

<u>Problem</u>: Design a flat slab for an explosives magazine subjected to a blast load

Required Information:

Slab geometry (see Figure 11)

3 x 4 flat slab

$L = 300$ in.

$S = 240$ in.

$H_w = 120$ in.

Loading

Triangular blast load

Peak pressure, $B = 250$ psi

Duration, $t_d = 8$ msec

Impulse, $i_b = 1{,}000$ psi-msec

Material properties

- Concrete (slab, drop panel, capital, column)

  $\rho = 0.000217$ lb-sec²/in.⁴

  $\gamma = 145$ pcf

  $f'_c = 4{,}000$ psi

  $E_c = (145)^{1.5}\ (33)\ \sqrt{4000} = 3.64 \times 10^6$ psi

  $\nu = 0.17$

- Overburden

  $\rho = 0.000150$ lb-sec²/in.⁴

  $\gamma = 100$ pcf

- Reinforcement (Grade 60)

  $f_y = 60{,}000$ psi

  $f_u = 90{,}000$ psi

Design criteria

- Minimum static design stress (see Table 2)

  $f_s = 1/2\ (f_y + f_u) = 75{,}000$ psi

- Dynamic increase factors (see Table 3)

  DIF = 1.20, flexure reinforcement

  DIF = 1.25, concrete compression

  DIF = 1.00, concrete shear

- Allowable support rotation (see Table 4)

  θ_m = 8 degrees (for explosives magazine)

<u>Assumptions</u>:

Capital size

Let $\alpha_{cap} = 0.20$

$d = \alpha_{cap} L = (0.20)(300) = 60$ in. (diameter of capital)

$c = 0.89\ d = (0.89)(60) = 53.4$ in. (equivalent square capital)

Depth of overburden

Let $t_{ob} = 12$ in.

Wall/slab thickness ratio

Let $\alpha_t = 1.0$

where $\alpha_t = t_{wS}/t_{slab}\ ;\ t_{wL}/t_{slab}$

<u>Solution</u>:

Step 1: Distribute moments according to Appendix A. The distribution is shown in Figure 12 (reproduction of Figure A-9). The values of the unit moment coefficients are listed in Table 6 (reproduction of Table A-7).

Step 2: Yield-line analysis according to Appendix B. The yield-line mechanism is shown in Figure 13 (reproduction of Figure B-2). The results of the analysis are listed in Table 7 (reproduction of Table B-11). That is,

$\alpha_{ru} = 10.102$

$x' = 0.40$

$y' = 0.30$

$z' = 0.311$

Now, by definition:

$$(r_u)_{min} = \alpha_{ru} \frac{m_e}{L^2}$$

Therefore,

$$(r_u)_{min} = 10.102 \frac{m_e}{(300)^2} = 0.0001122 \, m_e$$

Step 3: The calculated span lengths for the yield-line mechanism are listed in Table 8. The minimum span length, L_{min}, equals 90.0 in. Therefore,

$$X_m = L_{min} \tan \theta_m = 90.0 \tan 8^o = 12.65 \text{ in.}$$

Step 4: Assume trial slab thickness.

$$t_{min} \geq \frac{L}{60}\left(1 + \frac{1}{\beta}\right) = \frac{300}{60}\left(1 + \frac{1}{1.25}\right) = 9.0 \text{ in.}$$

Step 5: Perform dynamic analysis on SDOF representation of the slab.

a. Determine SDOF parameters.

Elastic stiffness

$$K_E = \frac{E_c \, I_a}{C \, L^4 \, (1 - \nu^2)}$$

where:

$$I_a = \frac{1}{2} I_g = \frac{1}{2}\left[\frac{(9)^3}{12}\right] = 30.4 \text{ in.}^4/\text{in.}$$

(Approximation $I_c = 0$)

$$C = C_C - \frac{C_L}{2} = 0.00189 - \frac{0.00155}{2} = 0.00112$$

(See Table 1)

Because the minimum span length is in the short side panel, use Equation 19. Therefore,

$$K_E = \frac{(3.64 \times 10^6)(30.4)}{(0.00112)(300)^4 \, (1 - 0.17^2)} = 12.6 \text{ psi/in.}$$

Elastic load-mass factor

$$K_{LM} = 0.61 + 0.16\ (1.25 - 1) = 0.65$$

Plastic load-mass factor

$$K_{LM} = 0.689 \quad \text{(from Appendix C)}$$

Actual unit mass

$$m = \rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab}$$

$$= (0.00015)(12) + (0.000217)(9)$$

$$= 0.00375\ \text{lb-sec}^2/\text{in.}^3 \quad \text{or} \quad 3{,}750\ \text{lb-msec}^2/\text{in.}^3$$

Natural period

$$T_n = 2\ \pi \sqrt{\frac{K_{LM}\ m}{K_E}} = 2\ \pi \sqrt{\frac{(0.65)(3{,}750)}{12.6}} = 87.4\ \text{msec}$$

b. Check loading for definition of impulse.

$$\frac{t_d}{T_n} = \frac{8}{87.4} = 0.09 \leq 0.2$$

Therefore, loading is impulsive.

c. Determine required dynamic ultimate unit resistance for impulse sensitive structure.

$$r_{ud} = \frac{i_b^{\ 2}}{2\ m_{ef}\ X_m} = \frac{(1{,}000)^2}{(2)(0.689)(3{,}750)(12.65)} = 15.3\ \text{psi}$$

d. Determine dead load.

$$r_{dl} = g\ m = (386.4)(0.00375) = 1.45\ \text{psi}$$

e. Determine equivalent static required ultimate unit flexural resistance.

$$r_{uf} = \frac{r_{ud}}{DIF} + r_{dl} = \frac{15.30}{1.20} + 1.45 = 14.20\ \text{psi}$$

f. Determine dynamic unit resistance for shear calculations.

$$r_{uv} = (DIF)\ (r_{uf}) = (1.20)\ (14.20) = 17.04\ \text{psi}$$

g. Determine time to reach maximum response.

$$t_m = \frac{i_b}{r_{ud}} = \frac{1,000}{15.30} = 65.4 \text{ msec}$$

Step 6: Check minimum steel percentage.

a. Determine required static moment capacity and steel percentage at minimum moment section in both direction L and S.

From Step 2:

$$(r_u)_{min} = 0.0001122 \, m_e$$

where: $r_{uf} = (r_u)_{min}$

Therefore,

$$m_e = \frac{r_{uf}}{0.0001122} = \frac{14.20}{0.0001122} = 126,560 \text{ in.-lb/in.}$$

From Table 6:

S-direction

$$m_{req} = m_5 = 0.087 \, m_e$$

$$m_5 = (0.087)(126,560) = 11,010 \text{ in.-lb/in.}$$

L-direction

$$m_{req} = m_{14} = 0.179 \, m_e$$

$$m_{14} = (0.179)(126,560) = 22,655 \text{ in.-lb/in.}$$

Determine $(A_s)_{req}$ for these two locations; use f_s:

S-direction

$$m_n = (A_s)_{req} \, f_s \, d_{cS}$$

L-direction

$$m_n = (A_s)_{req} \, f_s \, d_{cL}$$

Since the largest unit moments occur in the L-direction, place the reinforcement for these moments nearest the top/bottom surfaces (see Figure 2b). According to ACI 7.7.1:

Concrete exposed to earth or weather	minimum cover (in.)
#6 - #18 bars	2
#5 bar or smaller	1½

Concrete not exposed to weather or in contact with ground	minimum cover (in.)
#11 bar or smaller	3/4
#14 and #18 bars	1½

Therefore,

top cover = 2 in.
bottom cover = 3/4 in.

$$d_{cL} = t_{slab} - 2 - \frac{3}{4} - d_L = t_{slab} - 2\frac{3}{4} - d_L$$

$$d_{cS} = t_{slab} - 2 - \frac{3}{4} - 2\,d_L - d_S = t_{slab} - 2\frac{3}{4} - 2\,d_L - d_S$$

where: d_L = diameter of long side bar

d_S = diameter of short side bar

For #6 bars

$$d_{cL} = 9 - 2\frac{3}{4} - \frac{6}{8} = 5.5 \text{ in.}$$

$$d_{cS} = 9 - 2\frac{3}{4} - 2\left(\frac{6}{8}\right) - \frac{6}{8} = 4.0 \text{ in.}$$

S-direction (m_5)

$$11{,}010 = (A_s)_{req}\ (75{,}000)\ (4)$$

Therefore,

$$(A_s)_{req} = 0.0367 \text{ in.}^2/\text{in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cS}} = \frac{0.0367}{(1)(4)} = 0.00916$$

L-direction (m_{14})

$$22{,}655 = (A_s)_{req}\ (75{,}000)(5.5)$$

Therefore,

$$(A_s)_{req} = 0.0549 \text{ in.}^2/\text{in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cL}} = \frac{0.0549}{(1)(5.5)} = 0.0100$$

b. Compare with ACI minimum value (p_{min}).

For equal top/bottom reinforced slab:

$$(A_s)_{min} = 0.009\ b\ t_{slab}$$

$$(A_s)_{min} = (0.0009)(1)(9) = 0.0081 \text{ in.}^2/\text{in.}$$

$$p_{min} = \frac{(A_s)_{min}}{b\ d} = \frac{0.0081}{(1)(5.5)} = 0.0015$$

Therefore,

$$p_{req} > p_{min} \left\{ \begin{array}{l} \text{Increase slab thickness.} \\ \text{Return to Step 5.} \\ \text{or} \\ \text{Go to Step 12.} \end{array} \right.$$

Go to Step 12.

Step 12: Check shear at support (see Figure 14).

a. <u>Shear at Location #1 (d_{cL} from wall haunch sector ⟨B⟩):</u>

Critical shear width,

$$b_w = 2\ S - \frac{c}{2} - \frac{d_{cL}}{x}\, y$$

$$= (2)(240) - \frac{53.4}{2} - \frac{5.5}{120}(90)$$

$$= 449.2 \text{ in.}$$

Tributary area,

$$A = \left[\frac{b_w + \left(2\ S - \frac{c}{2} - y\right)}{2}\right](x - d_{cL})$$

$$= \left[\frac{449.2 + (480 - 26.7 - 90)}{2}\right](120 - 5.5)$$

$$= 46{,}516 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (17.04)(46{,}516)$$

$$= 792{,}630 \text{ lb}$$

Nominal shear strength at wall provided by concrete,

$$V_c = v_c b_w d_{cL}$$

where: $v_c = 1.9 \sqrt{f'_c} + 2{,}500\, p \leq 2.28 \sqrt{f'_c}$

$$m_{10} = 0.314\, m_e = (0.314)(126{,}560) = 39{,}740 \text{ in.-lb/in.}$$

$$A_s = \frac{m_{10}}{f_s\, d_{cL}} = \frac{39{,}740}{(75{,}000)(5.5)} = 0.0963 \text{ in.}^2\text{/in.}$$

$$p = \frac{A_s}{b\, d_{cL}} = \frac{0.0963}{(1)(5.5)} = 0.0175$$

$$v_c = 1.9 \sqrt{4{,}000} + 2{,}500(0.0175) \leq 2.28 \sqrt{4{,}000}$$

$$= 163.9 \text{ psi} \leq 144.2 \text{ psi}$$

Therefore, $v_c = 144.2$ psi

$$V_c = v_c b_w d_{cL}$$

$$= (144.2)(449.2)(5.5)$$

$$= 356{,}261 \text{ lb}$$

Therefore,

$$V_u > \phi V_c$$

$$792{,}630 > (0.85)(356{,}261)$$

$$792{,}630 > 302{,}821$$

b. <u>Shear at Location #2 (d_{cS} from wall haunch of Sector ⟨A⟩):</u>

Critical shear width,

$$b_w = 1.5\, L - \frac{c}{2} - \frac{d_{cS}}{y}\, x$$

$$= (1.5)(300) - \frac{53.4}{2} - \frac{4}{90}(120)$$

$$= 418.0 \text{ in.}$$

Tributary area,

$$A = \left[\frac{b_w + \left(1.5\ L - \frac{c}{2} - x\right)}{2}\right](y - d_{cS})$$

$$= \left[\frac{418 + (450 - 26.7 - 120)}{2}\right](90 - 4)$$

$$= 31{,}016 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (17.04)(31{,}016)$$

$$= 528{,}510 \text{ lb}$$

Nominal shear strength at wall provided by concrete,

$$V_c = v_c\ b_w\ d_{cS}$$

where: $m_1 = 0.168\ m_e = (0.168)(126{,}560) = 21{,}260$ in.-lb/in.

$$A_s = \frac{m_1}{f_s\ d_{cS}} = \frac{21{,}260}{(75{,}000)(4)} = 0.0709 \text{ in.}^2\text{/in.}$$

$$p = \frac{A_s}{b\ d_{cS}} = \frac{0.0709}{(1)(4)} = 0.0177$$

$$v_c = 1.9\sqrt{4{,}000} + 2{,}500(0.0177) \leq 2.28\sqrt{4{,}000}$$

$$= 164.4 \text{ psi} \leq 144.2 \text{ psi}$$

Therefore, $v_c = 144.2$ psi

$$V_c = v_c\ b_w\ d_{cS}$$

$$= (144.2)(418)(4)$$

$$= 241{,}102 \text{ lb}$$

Therefore,

$$V_u > \phi V_c$$

$$528{,}510 > (0.85)(241{,}102)$$

$$528{,}510 > 204{,}937$$

Increase slab thickness. Let t_{slab} = 16 in. Return to Step 5.

Step 5: Perform dynamic analysis on SDOF representation of the slab.

a. Determine SDOF parameters.

Elastic stiffness

$$I_a = \frac{1}{2}\left[\frac{(16)^3}{12}\right] = 170.7 \text{ in.}^4/\text{in.}$$

$$C = 0.00112$$

$$K_E = \frac{(3.64 \times 10^6)(170.7)}{(0.00112)(300)^4\ (1 - 0.17^2)}$$

$$= 70.5 \text{ psi/in.}$$

Elastic load-mass factor

$$K_{LM} = 0.65$$

Plastic load-mass factor

$$K_{LM} = 0.689$$

Actual unit mass

$$m = (0.00015)(12) + (0.000217)(16)$$

$$= 0.00527 \text{ lb-sec}^2/\text{in.}^3 \quad \text{or} \quad 5{,}272 \text{ lb-msec}^2/\text{in.}^3$$

Natural period

$$T_n = 2\pi\sqrt{\frac{(0.65)\ (5{,}272)}{70.5}} = 43.8 \text{ msec}$$

b. Check loading for definition of impulse.

$$\frac{t_d}{T_n} = \frac{8}{43.8} = 0.18 < 0.2$$

Therefore, loading is impulsive.

c.

$$r_{ud} = \frac{(1,000)^2}{(2)(0.689)(5,272)(12.65)} = 10.88 \text{ psi}$$

d.

$$r_{dl} = (386.4)(0.00527) = 2.04 \text{ psi}$$

e.

$$r_{uf} = \frac{10.88}{1.20} + 2.04 = 11.11 \text{ psi}$$

f.

$$r_{uv} = (1.20)(11.11) = 13.33 \text{ psi}$$

g.

$$t_m = \frac{1,000}{10.88} = 91.9 \text{ msec}$$

Step 6: Check minimum steel percentage.

a. Determine p_{req}

$$m_e = \frac{r_{uf}}{0.0001122} = \frac{11.11}{0.0001122} = 99,020 \text{ in.-lb/in.}$$

$$d_{cL} = t_{slab} - 2\frac{3}{4} - d_L = 16 - 2\frac{3}{4} - \frac{6}{8}$$

$$= 12.5 \text{ in.}$$

$$d_{cS} = t_{slab} - 2\frac{3}{4} - 2\ d_L - d_S = 16 - 2\frac{3}{4} - 2\left(\frac{6}{8}\right) - \frac{6}{8}$$

$$= 11.0 \text{ in.}$$

S-direction (m_5)

$$m_{req} = m_5 = 0.087\ m_e = (0.087)(99,020)$$

$$= 8,615 \text{ in.-lb/in.}$$

$$(A_s)_{req} = \frac{m_{req}}{f_s\ d_{cS}} = \frac{8,615}{(75,000)(11)} = 0.01044 \text{ in.}^2\text{/in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cS}} = \frac{0.01044}{(1)(11)} = 0.000949$$

L-direction (m_{14})

$$m_{req} = m_{14} = 0.179\ m_e = (0.179)(99{,}020)$$

$$= 17{,}725 \text{ in.-lb/in.}$$

$$(A_s)_{req} = \frac{m_{req}}{f_s\ d_{cL}} = \frac{17{,}725}{(75{,}000)(12.5)} = 0.0189 \text{ in.}^2\text{/in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cL}} = \frac{0.0189}{(1)(12.5)} = 0.001513$$

b. Compare with ACI minimum value.

$$(A_s)_{min} = 0.0009\ b\ t_{slab}$$

$$= (0.0009)(1)(16) = 0.0144 \text{ in.}^2\text{/in.}$$

L-direction

$$p_{min} = \frac{(A_s)_{min}}{b\ d_{cL}} = \frac{0.0144}{(1)(12.5)} = 0.00115$$

S-direction

$$p_{min} = \frac{(A_s)_{min}}{b\ d_{cS}} = \frac{0.0144}{(1)(11)} = 0.00131$$

Therefore,

$p_{req} < p_{min}$ For S-direction only

Go to Step 7.

Step 7: Revise distribution of moments at all locations where $p_{req} < p_{min}$ (for S-direction only) according to Appendix B. That is,

$$(\alpha_{um})_{min} = \frac{p_{min}\ b\ d_{cS}^{\ 2}\ f_s}{m_e} \qquad \text{(Eq. B-14)}$$

$$= \frac{(0.00131)(1)(11)^2\ (75{,}000)}{99{,}020}$$

$$= 0.120$$

Minimum unit moment coefficient in the S-direction equals 0.120. Table 6 shows that α_{um} for m_2, m_3, m_4, and m_5 must all be increased to 0.120.

Step 8: Re-determine yield-line locations and value of ultimate resistance according to Appendix B. The results of the analysis are listed in Table 9 (reproduction of Table B-14). That is,

$$\alpha_{ru} = 10.363$$

$$x' = 0.40$$

$$y' = 0.30$$

$$z' = 0.311$$

By definition:

$$(r_u)_{min} = \alpha_{ru} \frac{m_e}{L^2}$$

$$= 10.363 \frac{m_e}{(300)} = 0.0001151 \ m_e$$

Step 9: Since x', y', and z' did not change, X_m remains the same. Therefore,

$$X_m = 12.65 \text{ in.}$$

Step 10: Perform dynamic analysis. Since X_m did not change, the previous dynamic analysis is still valid. That is,

$$m = 5{,}272 \text{ lb-msec}^2/\text{in.}^3$$

$$m_{ef} = 3{,}632 \text{ lb-msec}^2/\text{in.}^3$$

$$r_{ud} = 10.88 \text{ psi}$$

$$r_{dl} = 2.04 \text{ psi}$$

$$r_{uf} = 11.11 \text{ psi}$$

$$r_{uv} = 13.33 \text{ psi}$$

Step 11: Recheck minimum steel percentages.

a. Determine p_{req}

$$m_e = \frac{r_{uf}}{0.0001151} = \frac{11.11}{0.0001151} = 96{,}525 \text{ in.-lb/in.}$$

S-direction (m_2, m_3, m_4, m_5)

$$m_{req} = 0.120\ m_e = (0.120)\ (96,525) = 11,583 \text{ in.-lb/in.}$$

$$(A_s)_{req} = \frac{m_{req}}{f_s\ d_{cS}} = \frac{11,583}{(75,000)\ (11)} = 0.01404 \text{ in.}^2\text{/in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cS}} = \frac{0.01404}{(1)(11)} = 0.00128$$

L-direction (m_{14})

$$m_{req} = 0.179\ m_e = (0.179)(96,525) = 17,278 \text{ in.-lb/in.}$$

$$(A_s)_{req} = \frac{m_{req}}{f_s\ d_{cL}} = \frac{17,278}{(75,000)(12.5)} = 0.01843 \text{ in.}^2\text{/in.}$$

$$p_{req} = \frac{(A_s)_{req}}{b\ d_{cL}} = \frac{0.01843}{(1)(12.5)} = 0.001474$$

b. Compare with ACI minimum value

L-direction

$$p_{min} = 0.00115$$

S-direction

$$p_{min} = 0.00131$$

Therefore,

$p_{req} < p_{min}$ For S-direction only.

However, these values are close enough. Go to Step 12.

Step 12: Check shear at support.

a. Shear at Location #1 (d_{cL} from wall haunch of Sector ⟨B⟩):

Critical shear width,

$$b_w = 2(240) - \frac{53.4}{2} - \frac{12.5}{120}(90) = 443.9 \text{ in.}$$

Tributary area,

$$A = \left[\frac{443.9 + (480 - 26.7 - 90)}{2}\right](120 - 12.5)$$

$$= 43{,}387 \text{ in.}^2$$

Factored shear force,

$$V_u = (13.33)(43{,}387) = 578{,}349$$

Nominal shear strength at wall provided by concrete,

where:

$$m_{10} = 0.314\ m_e = (0.314)(96{,}525) = 30{,}309 \text{ in.-lb/in.}$$

$$A_s = \frac{m_{10}}{f_s\ d_{cL}} = \frac{30{,}309}{(75{,}000)(12.5)} = 0.0323 \text{ in.}^2\text{/in.}$$

$$p = \frac{A_s}{b\ d_{cL}} = \frac{0.0323}{(1)(12.5)} = 0.00259$$

$$v_c = 1.9\ \sqrt{4{,}000} + 2{,}500(0.00259) \leq 2.28\ \sqrt{4{,}000}$$

$$= 126.6 \text{ psi} \leq 144.2 \text{ psi}$$

Therefore, $v_c = 126.6$ psi

$$V_c = (126.6)(443.9)(12.5) = 702{,}652 \text{ lb}$$

Therefore,

$$V_u < \phi\ V_c$$

$$578{,}349 < (0.85)(702{,}652)$$

$$578{,}349 < 597{,}254$$

b. <u>Shear at Location #2 (d_{cS} from wall haunch of Sector ⟨A⟩):</u>

<u>Critical shear width,</u>

$$b_w = (1.5)(300) - 26.7 - \frac{11}{90}(120) = 408.6 \text{ in.}$$

Tributary area,

$$A = \left[\frac{408.6 + (450 - 26.7 - 120)}{2}\right] (90 - 11)$$

$$= 28{,}120 \text{ in.}^2$$

Factored shear force,

$$V_u = (13.33)(28{,}120) = 374{,}840 \text{ lb}$$

Nominal shear strength at wall provided by concrete,

where:

$$m_1 = 0.168\ m_e = (0.168)(96{,}525) = 16{,}216 \text{ in.-lb/in.}$$

$$A_s = \frac{m_1}{f_s\ d_{cS}} = \frac{16{,}216}{(75{,}000)(11)} = 0.0197 \text{ in.}^2\text{/in.}$$

$$p = \frac{A_s}{b\ d_{cS}} = \frac{0.0197}{(1)(11)} = 0.00179$$

$$v_c = 1.9 \sqrt{4{,}000} + 2{,}500(0.00179) \leq 2.28 \sqrt{4{,}000}$$

$$= 124.6 \text{ psi} \leq 144.2 \text{ psi}$$

Therefore, $v_c = 124.6$ psi

$$V_c = (124.6)(408.6)(11) = 560{,}214 \text{ lb}$$

Therefore,

$$V_u < \phi V_c$$

$$374{,}840 < (0.85)(560{,}214)$$

$$374{,}840 < 476{,}182$$

Go to Step 13.

Step 13: Check punching shear at column capital. The critical column for punching shear is the top column which has the largest tributary area (see Figure 15, Location #1).

Critical shear width,

$$b_o = \pi\left[\frac{d}{2} + \frac{1}{2}(d_c)_{avg}\right]$$

where: $d_{cL} = 12.5$ in.

$d_{cS} = 11.0$ in.

$(d_c)_{avg} = 11.75$ in.

$\frac{1}{2}(d_c)_{avg} = 5.875$ in.

$b_o = \pi(30 + 5.875) = 112.7$ in.

Tributary area,

$$A = \left(\frac{S}{2}\right)\left(1.5L - \frac{c}{2} - x\right) - \frac{\pi}{2}\left[\frac{d}{2} + \frac{1}{2}(d_c)_{avg}\right]^2$$

$$= (120)(303.3) - \frac{\pi}{2}(35.875)^2 = 34{,}375 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (13.33)(34{,}375) = 458{,}220 \text{ lb}$$

Nominal shear strength provided by concrete,

$$V_c = v_c \, b_o \, (d_c)_{avg}$$

where: $v_c = 4\sqrt{f'_c} = 4\sqrt{4{,}000} = 253$ psi

$$V_c = (253)(112.7)(11.75) = 335{,}005 \text{ lb}$$

Therefore,

$$V_u > \phi V_c$$

$$458{,}220 > (0.85)(335{,}005)$$

$$458{,}220 > 284{,}755$$

Go to Step 14.

Step 14: Design drop panel for punching shear. Select trial drop panel dimensions.

$$L_{dp} = s_{cx} = \frac{S}{2} = \frac{240}{2} = 120 \text{ in.}$$

$$S_{dp} = s_{cy} = \frac{S}{2} = \frac{240}{2} = 120 \text{ in.}$$

$$t_{dp} = \frac{t_{slab}}{4} = \frac{16}{4} = 4 \text{ in.}$$

Critical shear width,

$$b_o = \pi\left[\frac{d}{2} + \frac{1}{2}(d_c)_{avg}\right]$$

where: $(d_{cL})_{drop} = d_{cL} + t_{dp} = 12.5 + 4 = 16.5$ in.

$(d_{cS})_{drop} = d_{cS} + t_{dp} = 11.0 + 4 = 15.0$ in.

$(d_c)_{avg} = 15.75$ in.

$\frac{1}{2}(d_c)_{avg} = 7.875$ in.

$$b_o = \pi(30 + 7.875) = 119.0 \text{ in.}$$

Tributary area,

$$A = \left(\frac{S}{2}\right)\left(1.5L - \frac{c}{2} - x\right) - \frac{\pi}{2}\left[\frac{d}{2} + \frac{1}{2}(d_c)_{avg}\right]^2$$

$$= (120)(303.3) - \frac{\pi}{2}(37.875)^2 = 34{,}145 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (13.33)(34{,}145) = 455{,}155 \text{ lb}$$

Nominal shear strength provided by concrete,

$$V_c = v_c\, b_o\, (d_c)_{avg}$$

where: $v_c = 253$ psi

$$V_c = (253)(119.0)(15.75) = 474{,}185 \text{ lb}$$

Therefore,

$$V_u > \phi V_c$$

$$455,155 > (0.85)(747,185)$$

$$455,155 > 403,057$$

Increase t_{dp}. Let t_{dp} = 6 in. and repeat Step 13.

Critical shear width,

where:
$$(d_{cL})_{drop} = 12.5 + 6 = 18.5 \text{ in.}$$

$$(d_{cS})_{drop} = 11.0 + 6 = 17.0 \text{ in.}$$

$$(d_c)_{avg} = 17.75 \text{ in.}$$

$$\frac{1}{2}(d_c)_{avg} = 8.875 \text{ in.}$$

$$b_o = \pi(30 + 8.875) = 122.1 \text{ in.}$$

Tributary area,

$$A = (120)(303.3) - \frac{\pi}{2}(38.875)^2 = 34,020 \text{ in.}^2$$

Factored shear force,

$$V_u = (13.33)(34,020) = 453,485 \text{ lb}$$

Nominal shear strength provided by concrete,

$$V_c = (253)(122.1)(17.75) = 548,320 \text{ lb}$$

Therefore,

$$V_u < \phi V_c$$

$$453,485 < (0.85)(548,320)$$

$$453,485 < 466,070$$

Go to Step 15.

Step 15: Check punching shear at edge of drop panel (see Figure 15, Location #2).

Critical shear width,

$$b_o = L_{dp} + S_{dp} + 2\,(d_c)_{avg}$$

where: $d_{cL} = 12.5$ in.

$d_{cS} = 11.0$ in.

$(d_c)_{avg} = 11.75$ in.

$$b_o = 120 + 120 + 2(11.75) = 263.5$$

Tributary area,

$$A = \left(\frac{S}{2}\right)\left(1.5L - \frac{c}{2} - x\right) - [L_{dp} + (d_c)_{avg}]\left[\frac{S_{dp} + (d_c)_{avg}}{2}\right]$$

$$= (120)(303.3) - (131.75)(65.875) = 27{,}715 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv}\,A$$

$$= (13.33)(27{,}715) = 369{,}440 \text{ lb}$$

Nominal shear strength provided by concrete,

$$V_c = v_c\,b_o\,(d_c)_{avg}$$

where: $v_c = 253$ psi

$$V_c = (253)(263.5)(11.75) = 783{,}319 \text{ lb}$$

Therefore,

$$V_u < \phi\,V_c$$

$$369{,}440 < (0.85)(783{,}319)$$

$$369{,}440 < 665{,}822$$

Go to Step 16. Shear reinforcement is not required.

Step 16: Check longitudinal (L-direction) and transverse (S-direction) beam shear at drop panel edge (see Figure 16). Let b_o = 1 in. (unit width)

a. <u>Location #1 (d_{cL} from edge).</u>

Tributary area,

$$A = b_o \left(L - \frac{c}{2} - x - \frac{L_{dp}}{2} - d_{cL} \right)$$

$$= (1.0)(300 - 26.7 - 120 - 60 - 12.5) = 80.8 \text{ in.}2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (13.33)(80.8) = 1{,}077 \text{ lb}$$

Nominal shear strength at drop panel provided by concrete (see Figure 12 and Table 6),

$$V_c = v_c \, b_o \, d_{cL}$$

$$\text{where: } v_c = 1.9 \sqrt{f'_c} + 2{,}500 \, p \leq 2.28 \sqrt{f'_c}$$

$$m_n = \alpha_{um} \, m_e$$

$$m_{avg} = \alpha_{avg} \, m_e = \frac{\alpha_{15} + \alpha_{11}}{2} m_e$$

$$= \frac{0.349 + 0.233}{2} (96{,}525)$$

$$= 28{,}090 \text{ in.-lb/in.}$$

$$(A_s)_{avg} = \frac{m_{avg}}{f_s \, d_{cL}} = \frac{28{,}090}{(75{,}000)(12.5)}$$

$$= 0.0300 \text{ in.}^2\text{/in.}$$

$$p_{avg} = \frac{(A_s)_{avg}}{b \, d_{cL}} = \frac{0.0300}{(1)(12.5)} = 0.00240$$

$$v_c = 1.9 \sqrt{4{,}000} + 2{,}500(0.00240) \leq 2.28 \sqrt{4{,}000}$$

$$= 126.2 \text{ psi} \leq 144.2 \text{ psi}$$

$$V_c = (126.2)(1)(12.5) = 1{,}578 \text{ lb}$$

Therefore,

$$V_u < \phi V_c$$

$$1{,}077 < (0.85)(1{,}578)$$

$$1{,}077 < 1{,}341$$

b. Location #2 (d_{cS} from edge).

Tributary area,

$$A = b_o \left(\frac{S}{2} - \frac{S_{dp}}{2} - d_{cS} \right)$$

$$= (1.0) \left(\frac{240}{2} - \frac{120}{2} - 11 \right) = 49 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A$$

$$= (13.33)(49) = 653 \text{ lb}$$

Nominal shear strength at drop panel provided by concrete,

$$V_c = v_c\, b_o\, d_{cS}$$

where: $m_{avg} = \alpha_{avg}\, m_e = \dfrac{\alpha_9 + \alpha_5}{2} m_e$

$$= \frac{0.191 + 0.120}{2} (96{,}525)$$

$$= 15{,}010 \text{ in.-lb/in.}$$

$$(A_s)_{avg} = \frac{m_{avg}}{f_s\, d_{cS}} = \frac{(15{,}010)}{(75{,}000(11)} = 0.0182 \text{ in.}^2\text{/in.}$$

$$p_{avg} = \frac{(A_s)_{avg}}{b\, d_{cS}} = \frac{0.0182}{(1)(11)} = 0.00165$$

$$v_c = 1.9\sqrt{4{,}000} + 2{,}500(1.00165) \leq 2.28\sqrt{4{,}000}$$

$$= 124.3 \text{ psi} \leq 144.2 \text{ psi}$$

$$V_c = (124.3)(1)(11) = 1{,}367 \text{ lb}$$

Therefore,

$$V_u < \phi V_c$$

$$653 < (0.85)(1{,}367)$$

$$653 < 1{,}162$$

Because the strips in the transverse direction are shorter, the shear in the longitudinal direction is critical, and there is no need to check shear in the transverse direction.

Go to Step 17.

Step 17: Check longitudinal (L-direction) and transverse (S-direction) beam shear at column capital. (See Figure 17.) Let b_o = 1 in.

a. <u>Location #1 (d_{cL} from equivalent square capital).</u>

Tributary area,

$$A = b_o \left[L - \frac{c}{2} - x - \frac{c}{2} - (d_{cL})_{avg} \right]$$

where: $$(d_{cL})_{avg} = \frac{d_{cL} + (d_{cL})_{drop}}{2} = \frac{12.5 + 18.5}{2} = 15.5 \text{ in.}$$

$$A = (1.0)(300 - 26.7 - 120 - 26.7 - 15.5) = 111.1 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv} A = (13.33)(112.1) = 1{,}481 \text{ lb}$$

Nominal shear strength at column capital provided by concrete,

$$V_c = v_c \, b_o \, (d_{cL})_{avg}$$

where: $$m_{avg} = \alpha_{avg} \, m_e = \frac{\alpha_{16} + \alpha_{12}}{2} m_e = \frac{0.689 + 0.230}{2}(96{,}525) = 44{,}353 \text{ in.-lb/in.}$$

$$(A_s)_{avg} = \frac{m_{avg}}{f_s\ (d_{cL})_{avg}} = \frac{44,353}{(75,000)(15.5)}$$

$$= 0.0382 \text{ in.}^2\text{/in.}$$

$$p_{avg} = \frac{(A_s)_{avg}}{b\ (d_{cL})_{avg}} = \frac{0.0382}{(1)(15.5)} = 0.00246$$

$$v_c = 1.9\sqrt{4,000} + 2,500(0.00246) < 2.28\sqrt{4,000}$$

$$= 126.3 \text{ psi} < 144.2 \text{ psi}$$

Therefore, $v_c = 126.3$ psi

$$V_c = (126.3)(1)(15.5) = 1,958 \text{ lb}$$

Therefore,

$$V_u < \phi\, V_c$$

$$1,481 < (0.85)(1,958)$$

$$1,481 < 1,664$$

b. <u>Location #2 (d_{cS} from equivalent square capital).</u>

Tributary area,

$$A = b_o \left[\frac{S}{2} - \frac{c}{2} - (d_{cS})_{avg}\right]$$

where: $(d_{cS})_{avg} = \dfrac{d_{cS} + (d_{cS})_{drop}}{2}$

$$= \frac{11 + 17}{2} = 14.0 \text{ in.}$$

$$A = (1.0)(120 - 26.7 - 14.0) = 79.3 \text{ in.}^2$$

Factored shear force,

$$V_u = r_{uv}\, A$$

$$= (13.33)(79.3) = 1,057 \text{ lb}$$

Nominal shear strength at column capital provided by concrete,

$$V_c = v_c\, b_o\, (d_{cS})_{avg}$$

$$\text{where: } m_{avg} = \alpha_{avg}\, m_e = \frac{\alpha_7 + \alpha_3}{2} m_e$$

$$= \frac{0.496 + 0.120}{2} (96,525)$$

$$= 29,730 \text{ in.-lb/in.}$$

$$(A_s)_{avg} = \frac{m_{avg}}{f_s\, (d_{cS})_{avg}} = \frac{29,730}{(75,000)(14.0)}$$

$$= 0.0283 \text{ in.}^2\text{/in.}$$

$$p_{avg} = \frac{(A_s)_{avg}}{b\, (d_{cS})_{avg}} = \frac{0.0283}{(1)(14.0)} = 0.00202$$

$$v_c = 1.9 \sqrt{4,000} + 2,500(0.00202) < 2.28 \sqrt{4,000}$$

$$= 125.2 \text{ psi} \leq 144.2 \text{ psi}$$

$$V_c = (125.2)(1)(14.0) = 1,753 \text{ lb}$$

Therefore,

$$V_u < \phi\, V_c$$

$$1,057 < (0.85)(1,753)$$

$$1,057 < 1,490$$

Go to Step 18.

Step 18: Final design of slab.

a. Include drop panel mass in dynamic SDOF analysis.

Plastic load-mass factor (from Appendix C),

$$K_{LM} = 0.689$$

Actual unit mass (from Appendix C),

$$m = \rho_{ob}\, t_{ob} + \rho_{slab}\, t_{slab} + \rho_{dp}\, t_{dp} \left(\frac{A_{dp}}{A_T}\right)$$

$$= (0.00015)(12) + (0.000217)(16)$$

$$+ (0.000217)(6)\,\frac{0.1925\ L^2}{2.0845\ L^2}$$

$$= 0.00539\ \text{lb-sec}^2/\text{in.}^3 \quad \text{or} \quad 5{,}392\ \text{lb-msec}^2/\text{in.}^3$$

(Note: An increase of 120 lb-msec^2/in.^{3})

Dynamic SDOF analysis,

$$r_{ud} = \frac{(1{,}000)^2}{(2)(0.689)(5{,}392)(12.65)} = 10.64\ \text{psi}$$

$$r_{dl} = (386.4)(0.005392) = 2.08\ \text{psi}$$

$$r_{uf} = \frac{10.64}{1.20} + 2.08 = 10.95\ \text{psi}$$

$$r_{uv} = (1.20)(10.95) = 13.14\ \text{psi}$$

$$t_m = \frac{1{,}000}{10.64} = 94.0\ \text{msec}$$

b. Determine unit moments and steel areas.

$$m_e = \frac{r_{uf}}{0.0001151} = \frac{10.95}{0.0001151} = 95{,}135\ \text{in.-lb/in.} \quad \text{(From Step 8)}$$

where: $m_n = \alpha_{um}\, m_e$

$A_s = m_n/f_s\, d_c$

$p = A_s/b\, d_c$

These design quantities are calculated for the entire flat slab and are listed in Table 10. The reinforcement shown in Figures 18 and 19 was then selected to satisfy these quantities. The last three columns of Table 10 reflect these selections.

c. Check initial design assumptions.

The final check of the design involves the validation of Equation 3 using the actual slab properties within the short side panel. This panel has nine unique locations (directly related to the ACI assignment of reinforcement) where the slab properties (i.e., thickness, effective depth, reinforcement ratio) differ. Because these properties also differ in the two directions, there would be a total of 18 values which must be used in determining the average value for each short side panel parameter. The following equations were derived for the 3 x 4 slab shown in Figure 12:

$$t_{avg} = \frac{7}{9} t_{slab} + \frac{2}{9} (t_{slab} + t_{dp})$$

$$d_{avg} = \frac{7}{18} (d_{cS} + d_{cL})_{slab} + \frac{2}{18} (d_{cS} + d_{cL})_{drop}$$

$$p_{avg} = (3p_1 + p_2 + p_3 + 2p_6 + 2p_7 + 2p_{12} + p_{14} + 2p_{16} + p_{18} + 3p_{19})/18$$

Substitution of the values listed in Table 10 yields the following average properties:

$$t_{avg} = \frac{7}{9}(16) + \frac{2}{9}(16 + 6) = 17.3 \text{ in.}$$

$$d_{avg} = \frac{7}{18} (11.0 + 12.5) + \frac{2}{18} (17.0 + 18.5) = 13.1 \text{ in.}$$

$$\begin{aligned} p_{avg} &= [3(0.00176) + 0.00125 + 0.00125 + 2(0.00269) \\ &\quad + 2(0.00218) + 2(0.00186) + 0.00146 \\ &\quad + 2(0.00255) + 0.00207 + 3(0.00125)]/18 \\ &= 0.00187 \end{aligned}$$

Elastic stiffness,

$$I_g = \frac{1}{12} (17.3)^3 = 431.5 \text{ in.}^4/\text{in.}$$

$$I_c = (5.5)(0.00187)(13.1)^3 = 23.1 \text{ in.}^4/\text{in.}$$

$$I_a = \frac{431.5 + 23.1}{2} = 227.3 \text{ in.}^4/\text{in.}$$

Therefore,

$$K_E = \frac{(3.64 \times 10^6)(227.3)}{(0.00112)(300)^4 (1 - 0.17^2)} = 93.9 \text{ psi/in.}$$

Natural period,

$$T_n = 2\pi \sqrt{\frac{(0.65)(5,392)}{93.9}} = 38.4 \text{ msec}$$

$$\frac{t_d}{T_n} = \frac{8}{38.4} = 0.21 \cong 0.20$$

Therefore, loading is impulsive.

For illustrative purposes a dynamic analysis based on elastic-plastic response charts (Ref 16) for triangular loads will be shown.

Given:

$$X_E = \frac{r_{ud}}{K_E} = \frac{10.64}{93.9} = 0.113 \text{ in.}$$

$$\frac{B}{r_{ud}} = \frac{250}{10.64} = 23.5$$

$$\frac{t_d}{T_n} = \frac{8}{38.4} = 0.21$$

Solution:

$$\frac{X_m}{X_E} = 110 \qquad \text{Therefore, } X_m = 12.4 \text{ in.}$$

$$\frac{t_m}{T_n} = 2.4 \qquad \text{Therefore, } t_m = 92.2 \text{ msec}$$

These values compare very favorably with the previously determined values. It should be emphasized that the actual ductility of the flat slab will be less than shown above ($\mu = 100$). This is a result of neglecting the elastic-plastic portion of the resistance function in arriving at the stiffness value.

Step 19: Design Column.

The determination of the design load, P_u, and eccentricity, e, for the lower column is shown in Appendix D. That is,

$$P_u = 918{,}945 \text{ lb}$$

$$e = 1.0 \text{ in.}$$

The design procedure can be obtained from a reinforced concrete design text book. It will not be illustrated in this report.

Step 20: End.

DISCUSSION

A general step-by-step design procedure for flat slab structures subjected to blast loads was presented. This procedure is totally consistent with the philosophy of the Navy's current blast-resistance design manual, NAVFAC P-397. However, because this manual is periodically reviewed and updated, values of the following design parameters may be affected by future P-397 revisions:

- Allowable rotations and deflections, θ_m and X_m
- Design stresses, f_s
- Dynamic increase factors, DIF
- Strength reduction factors, Φ

By dividing a flat slab into four distinct panel types (i.e., corner, interior, exterior short side, and exterior long side), the design procedure is applicable to flat slabs of any configuration (as defined by interior column arrangement and spacing). The establishment of a "step-by-step" process provides efficient execution of the design and also allows designers to more easily understand the complex structural behavior and interaction.

As a result of parameter studies, values for the following design parameters were established:

- Location of positive yield line between interior columns
- Plastic load-mass factor, K_{LM}

The yield-line analyses contained in Appendix B indicate that these positive yield lines can be located midway between the columns. Appendix C shows that K_{LM} is very insensitive to slab configuration and yield-line pattern. In fact, K_{LM} varied between 0.679 and 0.689 for a wide choice of configurations and assumed patterns.

REFERENCES

1. Naval Sea Systems Command, NAVSEA OP-5: Ammunition and Explosives Ashore, vol I, change 12, Washington D.C., Oct 1984.

2. Naval Facilities Engineering Command, NAVFAC P-397, Army TM-5-1300, and Air Force AFM 88-22: Sturctures to resist the effects of accidental explosions, Washington D.C., Jun 1969.

3. Civil Engineering Laboratory. Technical Memorandum 51-80-06: The ESKIMO VI test plan, by J.E. Tancreto and P.E. Tafoya. Port Hueneme, Calif., Apr 1980.

4. Naval Civil Engineering Laboratory. Technical Report R-889: ESKIMO VI test results, by P.E. Tafoya. Port Hueneme, Calif., Nov 1981.

5. U.S. Army Engineer Waterways Experiment Station. Technical Report N-72-10: Design and testing of a blast-resistant reinforced concrete slab system, by M.E. Criswell. Vicksburg, Miss., Nov 1972.

6. Naval Civil Engineering Laboratory. Technical Memorandum M-51-81-15: Preliminary design procedure for flat slab structures subjected to blast loads, by J.E. Tancreto. Port Hueneme, Calif., Sep 1981.

7. Civil Engineering Laboratory. Contract Report No. N62472-76-C-1148: Design of flat slabs subjected to blast loads. Ammann & Whitney Consulting Engineers, New York, N.Y.

8. American Concrete Institute. Building code requirements for reinforced concrete (ACI 318-77), Detroit, Michigan, 1977.

9. L.L. Jones and R.H. Wood. Yield line analysis of slabs. New York, N.Y., American Elsevier Publishing Co. Inc., 1967.

10. R. Park and W.L. Gamble. Reinforced concrete slabs. New York, N.Y., John Wiley and Sons, 1979.

11. M. Vanderbilt, M. Sozen, and C. Seiss. "Deflection of multiple-panel reinforced concrete floor slabs," in Proceedings of the American Society of Civil Engineers, Journal of the Structural Division, Aug 1965.

12. D.E. Branson. Deformation of concrete structures. New York, N.Y., McGraw-Hill International Book Co., 1977.

13. Department of the Army. Technical Manual 5-856-4: Design of structures to resist the effects of atomic weapons. Washington D.C., 1957.

14. Civil Engineering Laboratory. Design criteria for deflection capacity of conventionally reinforced concrete slabs, by J. Tancreto. Port Hueneme, Calif., 1980. (Internal Working Paper)

15. American Concrete Institute, Design Handbook, Volume 2 - Columns, Publication SP-17A, Detroit, Mich., 1978.

16. Naval Civil Engineering Laboratory. Technical Note N-1669: Charts for predicting response of a simple spring-mass system to bilinear blast loads, by J.S. Hopkins. Port Hueneme, Calif., Jun 1983.

17. R.H. Wood. Plastic and elastic design of slabs and plates. New York, N.Y., Ronald Press Co., 1961.

18. Department of the Army. Technical Manual 5-856-3: Design of structures to resist the effects of atomic weapons. Washington, D.C., 1957.

Table 1. Deflection Coefficients for Interior Panels

L/S	Deflection Coefficients for c/L of -			
	0.0	0.1	0.2	0.3*
Center of Interior Panel, C_C				
1.00	0.00581	0.00441	0.00289	0.00200
1.25	0.00420	0.00301	0.00189	0.00120
1.67	0.00327	0.00234	0.00143	0.00080
2.50	0.00284	0.00204	0.00120	0.00065
Midspan of Long Side, C_L				
1.00	0.00435	0.00304	0.00173	0.00100
1.25	0.00378	0.00262	0.00155	0.00085
1.67	0.00321	0.00228	0.00137	0.00075
2.50	0.00284	0.00204	0.00120	0.00065
Midspan of Short Side, C_S				
1.00	0.00435	0.00304	0.00173	0.00100
1.25	0.00230	0.00131	0.00057	0.00020
1.67	0.00099	0.00040	0.00008	0.00005
2.50	0.00031	0.00004	0.00001	---

*Values are extrapolated.

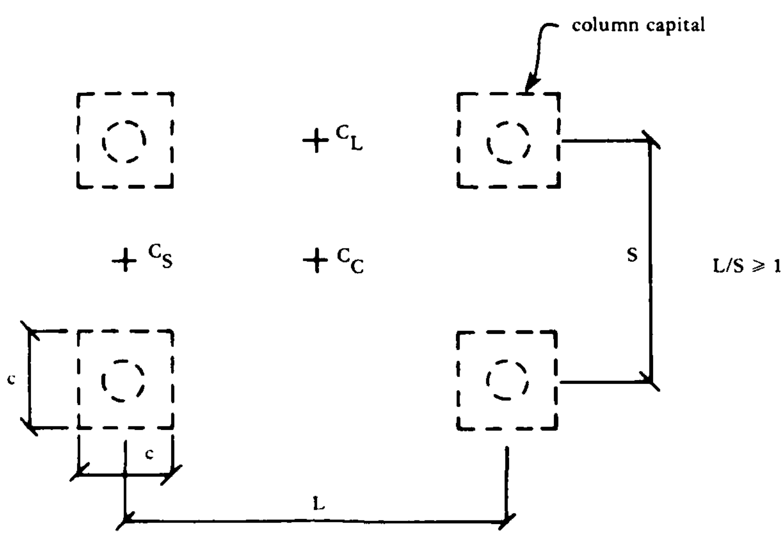


Table 2. Minimum Static Design Stress*

Maximum Support Rotation Angle, θ_m	Static Design Stress, f_s
$0° < \theta_m < 2°$	f_y
$2° < \theta_m < 5°$	$f_y + \dfrac{f_u - f_y}{4}$
$5° < \theta_m < 12°$	$\dfrac{f_y + f_u}{2}$

*From Section 5-6 of Reference 2.

Table 3. Dynamic Increase Factors (DIF)*

Stresses	DIFs for	
	High Pressure Range	Intermediate and Low Pressure Range
Reinforcing Steel		
Bending	1.20	1.10
Shear	1.00	1.00
Concrete		
Compression	1.25	1.25
Diagonal Tension	1.00	1.00
Direct Shear	-	1.10
Bond	-	1.00

*Table 5-3 of Reference 2

Table 4. Allowable Support Rotations

Protection Category	Allowable Support Rotation, θ_m (deg)
Personnel Shelter	2
Equipment Shelter	5
Explosives Magazine	8

Table 5. Flow chart of design procedure for flat slab.

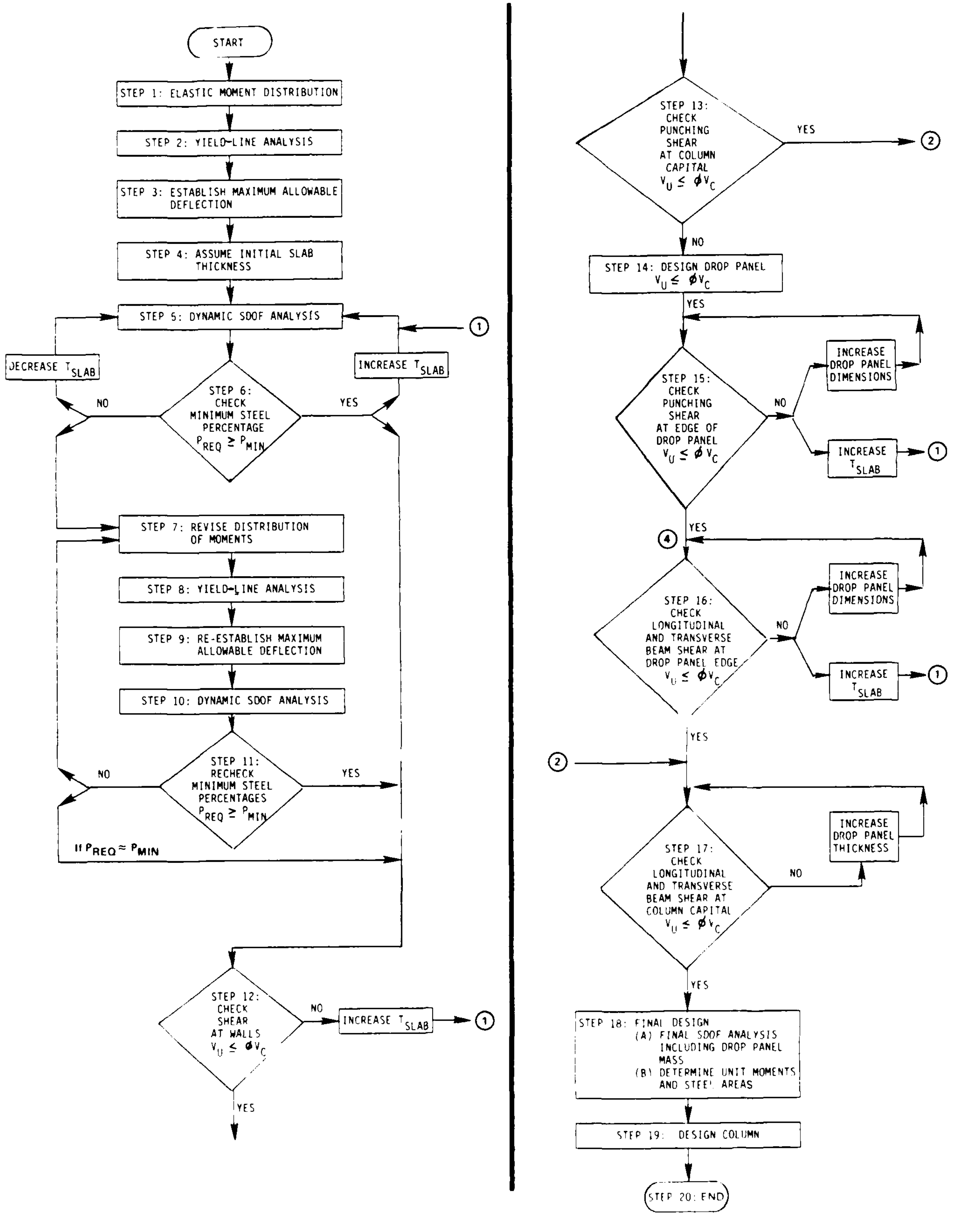


Table 6. Values of Unit Moment Coefficients ($\beta = 1.25$; $\alpha_{cap} = 0.20$; $t_{wS}/t_{slab} = t_{wL}/t_{slab} = 1.00$; $H_w/S = 0.5$)

Unit Moment, m_n	Unit Moment Coefficient, α_{um}	Adjusted Positive Unit Moment Coefficient, α^*_{um}
m_1	0.168	---
m_2	0.114	---
m_3	0.111	---
m_4	0.105	---
m_5	0.090	0.087
m_6	0.257	---
m_7	0.496	---
m_8	0.472	---
m_9	0.203	0.191
m_{10}	0.314	---
m_{11}	0.233	---
m_{12}	0.230	---
m_{13}	0.220	---
m_{14}	0.189	0.179
m_{15}	0.349	---
m_{16}	0.689	---
m_{17}	0.660	---
m_{18}	0.284	0.255
m_{19}	minimum	---

Note: $m_n = \alpha_{um}\ m_e$

$m_e = w\ L^2/8$

Table 7. Ultimate Resistance Calculations

x'	y'	z'	Coefficient of Internal Work,* α_{IW}	Coefficient of External Work,** α_{EW}	Coefficient of Ultimate Resistance,*** α_{ru}
	0.20	0.311	12.457	1.184	10.520
	0.25	0.311	12.047	1.174	10.263
0.35	0.30	0.311	11.836	1.164	10.172
	0.35	0.311	11.781	1.153	10.215
	0.40	0.311	11.898	1.143	10.409
	0.20	0.311	12.291	1.174	10.472
	0.25	0.311	11.871	1.164	10.203
0.40	0.30	0.311	11.649	1.153	10.102
	0.35	0.311	11.582	1.143	10.134
	0.40	0.311	11.683	1.133	10.315
	0.20	0.311	12.260	1.163	10.538
	0.25	0.311	11.829	1.153	10.259
0.45	0.30	0.311	11.595	1.143	10.146
	0.35	0.311	11.514	1.133	10.167
	0.40	0.311	11.599	1.122	10.335

*From Table B-10.

**From Table B-9.

***
$$\alpha_{ru} = \frac{\alpha_{IW}}{\alpha_{EW}} = \frac{r_u}{m_e/L^2}$$

Table 8. Span Lengths

Angle of Support Rotation	Span Direction	Expression	Length (in.)
θ_A	Long	(L - c)/2	123.3
θ_B	Long	L - c - x	126.6
θ_C	Long	x	120.0
θ_D	Short	y	90.0
θ_E	Short	S - c - y	96.6
θ_F	Short	z	93.3
θ_G	Short	S - c - z	93.3

Note: L = 300 in.
S = 240 in.
c = 53.4 in.
x = 120 in.
y = 90 in.
z = 93.3 in.

Table 9. Ultimate Resistance Calculations for

$(\alpha_{um})_{min} = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.120$

x'	y'	z'	Coefficient of Internal Work,* α_{IW}	Coefficient of External Work,** α_{EW}	Coefficient of Ultimate Resistance,*** α_{ru}
	0.20	0.311	12.784	1.184	10.797
	0.25	0.311	12.371	1.174	10.537
0.35	0.30	0.311	12.159	1.164	10.446
	0.35	0.311	12.106	1.153	10.500
	0.40	0.311	12.228	1.143	10.698
	0.20	0.311	12.594	1.174	10.727
	0.25	0.311	12.172	1.164	10.457
0.40	0.30	0.311	11.949	1.153	10.363
	0.35	0.311	11.884	1.143	10.397
	0.40	0.311	11.989	1.133	10.582
	0.20	0.311	12.541	1.163	10.783
	0.25	0.311	12.107	1.153	10.500
0.45	0.30	0.311	11.872	1.143	10.387
	0.35	0.311	11.793	1.133	10.409
	0.40	0.311	11.881	1.122	10.589

*From Table B-13.

**From Table B-9.

$$^{***}\alpha_{ru} = \frac{\alpha_{IW}}{\alpha_{EW}} = \frac{r_u}{m_e/L^2}$$

Table 10. Unit Moments and Reinforcement

[m_e = 95,135 in.-lb/in.; f_s = 75,000 psi]

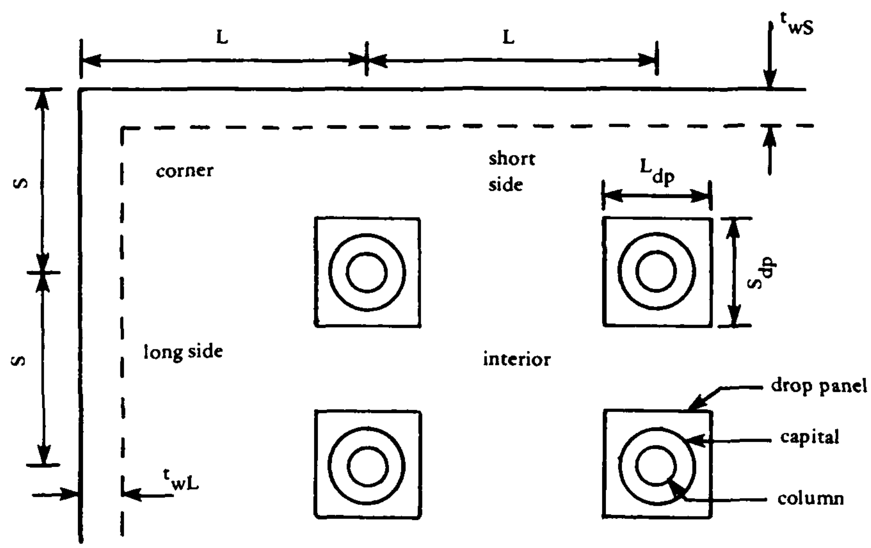
Unit Moment Parameter,† m_n	Unit Moment Coefficient, α_{um}	Unit Moment Value,†† m_n (in.-lb/in.)	Effective Depth, d_c (in.)	Required Reinforcement Area,††† A_s		Reinforcement Percentage,†††† p	Selected Reinforcement		Strip
				(in.^{2}/in.)	(in.^{2}/ft)		Bar Size at Spacing (in.)	A_s (in.^{2}/ft)	
m_1	0.168	15,985	11.0	0.0194	0.233	0.00176	#4 @ 10 #4 @ 14 #4 @ 35	0.240 0.240	column middle wall
m_2^*	0.120	11,415	11.0	0.0138	0.166	0.00125	#4 @ 14	0.171	middle
m_3^*	0.120	11,415	11.0	0.0138	0.166	0.00125	#4 @ 14	0.171	middle
m_4^*	0.120	11,415	11.0	0.0138	0.166	0.00125	#4 @ 14	0.171	middle
m_5^*	0.120	11,415	11.0	0.0138	0.166	0.00125	#4 @ 14	0.171	middle
m_6	0.257	24,450	11.0	0.0296	0.355	0.00269	#4 @ 10 #4 @ 20	0.360	column
m_7	0.496	47,185	17.0	0.0370	0.444	0.00218	#4 @ 5	0.480	column
m_8	0.472	44,905	17.0	0.0352	0.422	0.00207	#4 @ 5	0.480	column
m_9^*	0.191	18,170	11.0	0.0220	0.264	0.00200	#4 @ 10	0.240	column

(Continued)

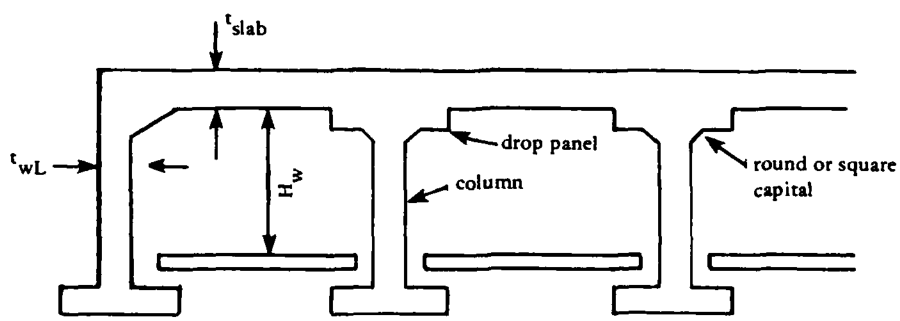
Table 10. Continued

Unit Moment Parameter,† m_n	Unit Moment Coefficient, α_{um}	Unit Moment Value,†† m_n (in.-lb/in.)	Effective Depth, d_c (in.)	Required Reinforcement Area,††† A_s		Reinforcement Percentage,†††† p	Selected Reinforcement		Strip
				(in.^{2}/in.)	(in.^{2}/ft)		Bar Size at Spacing (in.)	A_s (in.^{2}/ft)	
m_{10}	0.314	29,870	12.5	0.0319	0.383	0.00255	#6 @ 14 #4 @ 8 #3 @ 16 #4 @ 7	0.377 0.383 0.343	column middle wall
m_{11}	0.233	22,165	12.5	0.0236	0.283	0.00189	#4 @ 8	0.300	middle
m_{12}	0.230	21,880	12.5	0.0233	0.280	0.00186	#4 @ 8	0.300	middle
m_{13}	0.220	20,930	12.5	0.0223	0.269	0.00178			
m^*_{14}	0.179	17,030	12.5	0.0182	0.218	0.00146	#4 @ 8	0.300	middle
m_{15}	0.349	33,200	12.5	0.0354	0.425	0.00283	#6 @ 14 #4 @ 28	0.463	column
m_{16}	0.689	65,550	18.5	0.0472	0.566	0.00255	#6 @ 14 #4 @ 14	0.548	column
m_{17}	0.660	62,790	18.5	0.0453	0.544	0.00245			
m^*_{18}	0.255	24,260	12.5	0.0259	0.311	0.00207	#6 @ 14	0.377	column
m_{19}	0.120	11,415	11.0	0.0138	0.166	0.00125	#4 @ 14	0.171	wall

†An asterisk denotes adjusted value. ††$m_n = \alpha_{um} m_e$ †††A_s (in.^{2}/in.) $= m_n/f_c d_c$ ††††$p = A_s/b\ d_c$ (b = 1 in.)

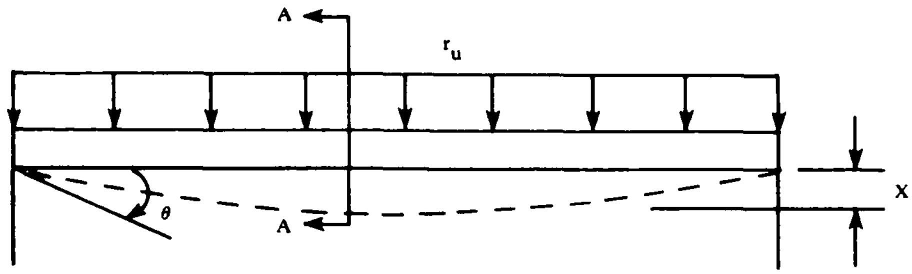


(a) Plan view.

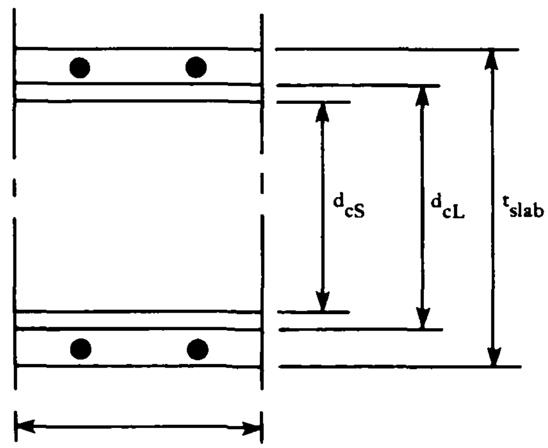


(b) Section View

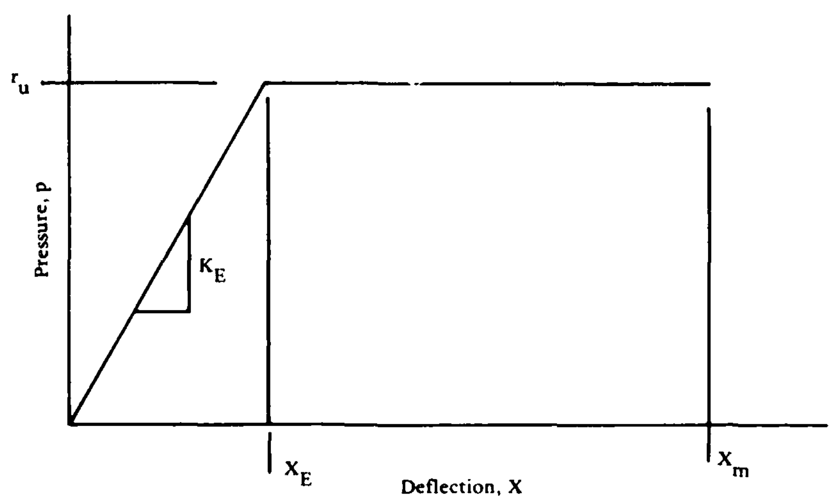
Figure 1. Typical flat slab structure.



(a) SDOF flexural structural element.

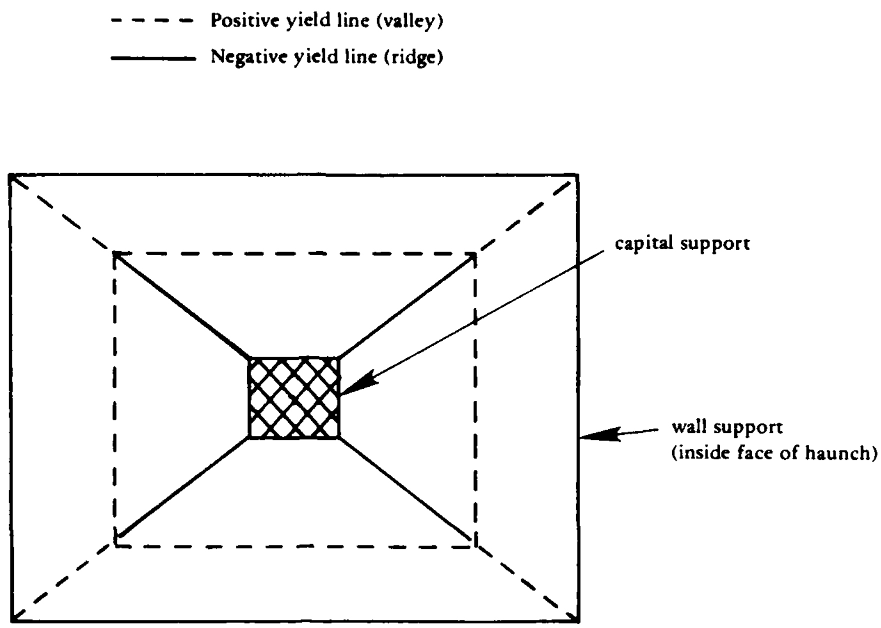


(b) Reinforced-concrete, Section A-A.

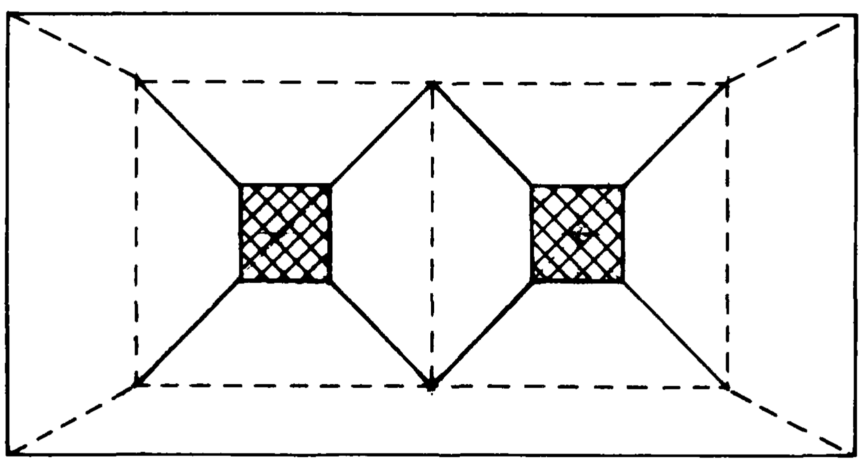


(c) Resistance-deflection function.

Figure 2. SDOF resistance-deflection function.



(a) Flat slab with one column.



(b) Flat slab with two columns.

Figure 3. Possible flat slab yield-line mechanisms.

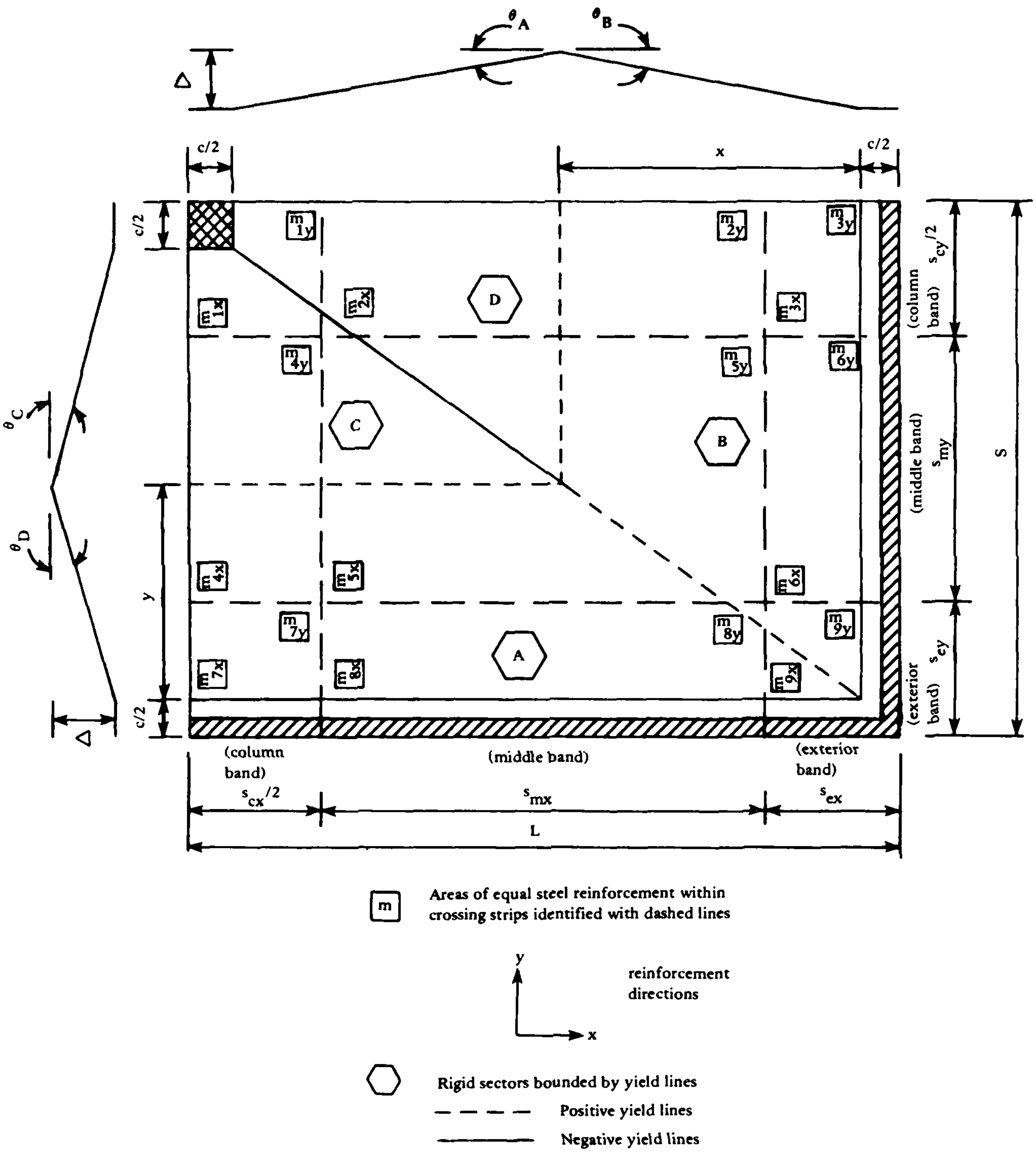
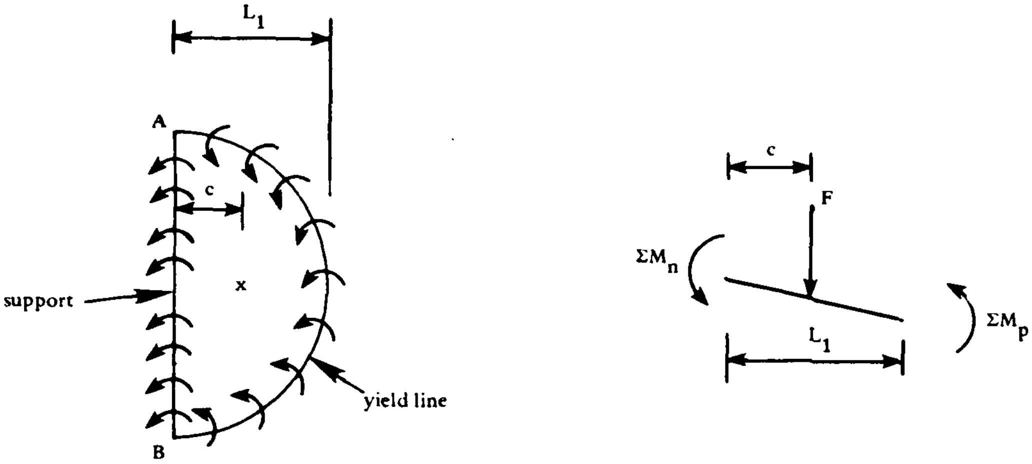
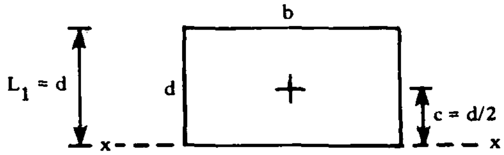


Figure 4. Yield line mechanism for one-quarter panel of flat slab with one central column.



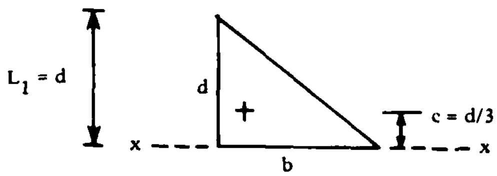
(a) Plan view. (b) Section view.

Figure 5. Determination of load-mass factor in the plastic range.



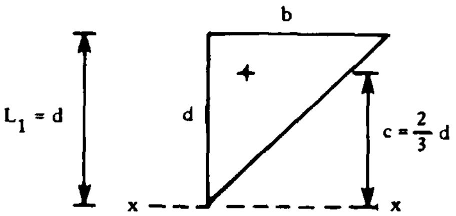
$$\frac{I}{cL_1} = \frac{b\,d^3/3}{\left(\frac{d}{2}\right)(d)} = \frac{2}{3}bd$$

(a) Rectangle.



$$\frac{I}{cL_1} = \frac{b\,d^3/12}{\left(\frac{d}{3}\right)(d)} = \frac{1}{4}b\,d$$

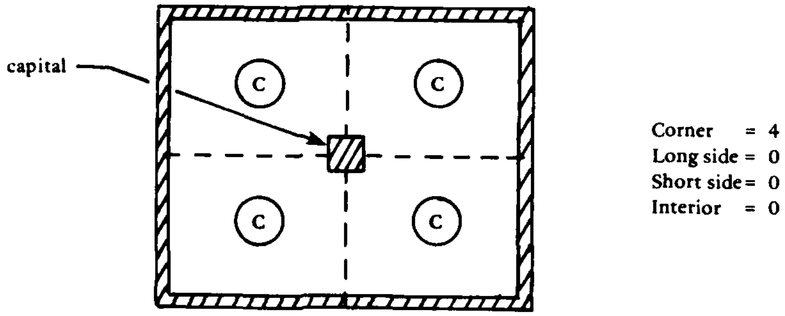
(b) Triangle rotating about side.



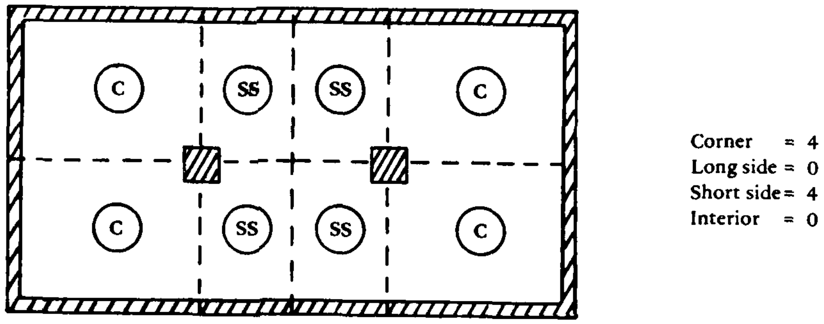
$$\frac{I}{cL_1} = \frac{b\,d^3/4}{\left(\frac{2d}{3}\right)(d)} = \frac{3}{8}b\,d$$

(c) Triangle rotating about corner.

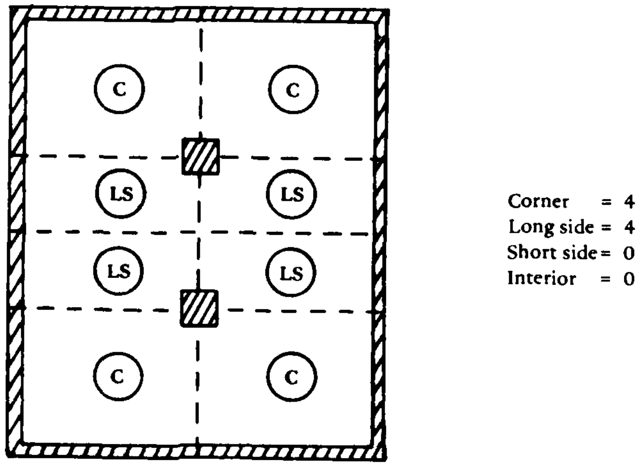
Figure 6. Expressions for I/cL_1 for typical sections.



(a) 2 x 2.

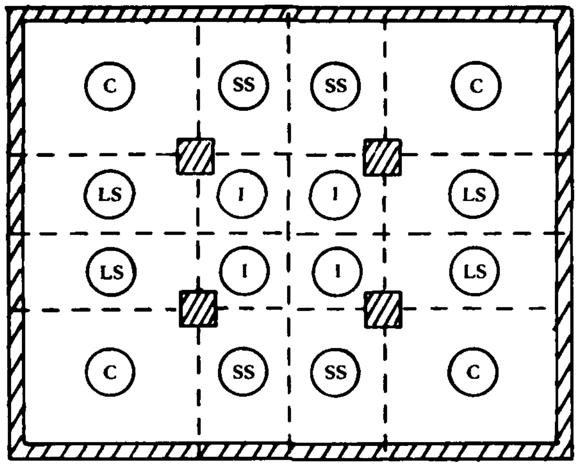


(b) 3 x 2.



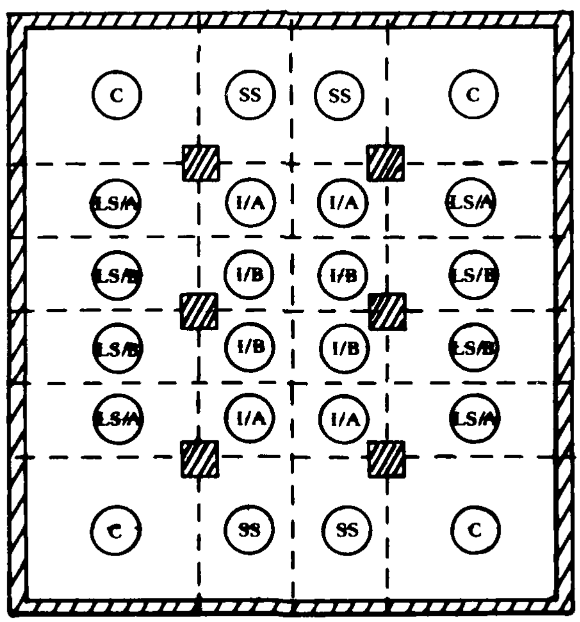
(c) 2 x 3.

Figure 7. Flat slab configurations.



Corner = 4
Long side = 4
Short side = 4
Interior = 4

(d) 3 x 3.



Corner = 4
Long side = 8
Short side = 4
Interior = 8

(e) 3 x 4.

Figure 7. (continued)

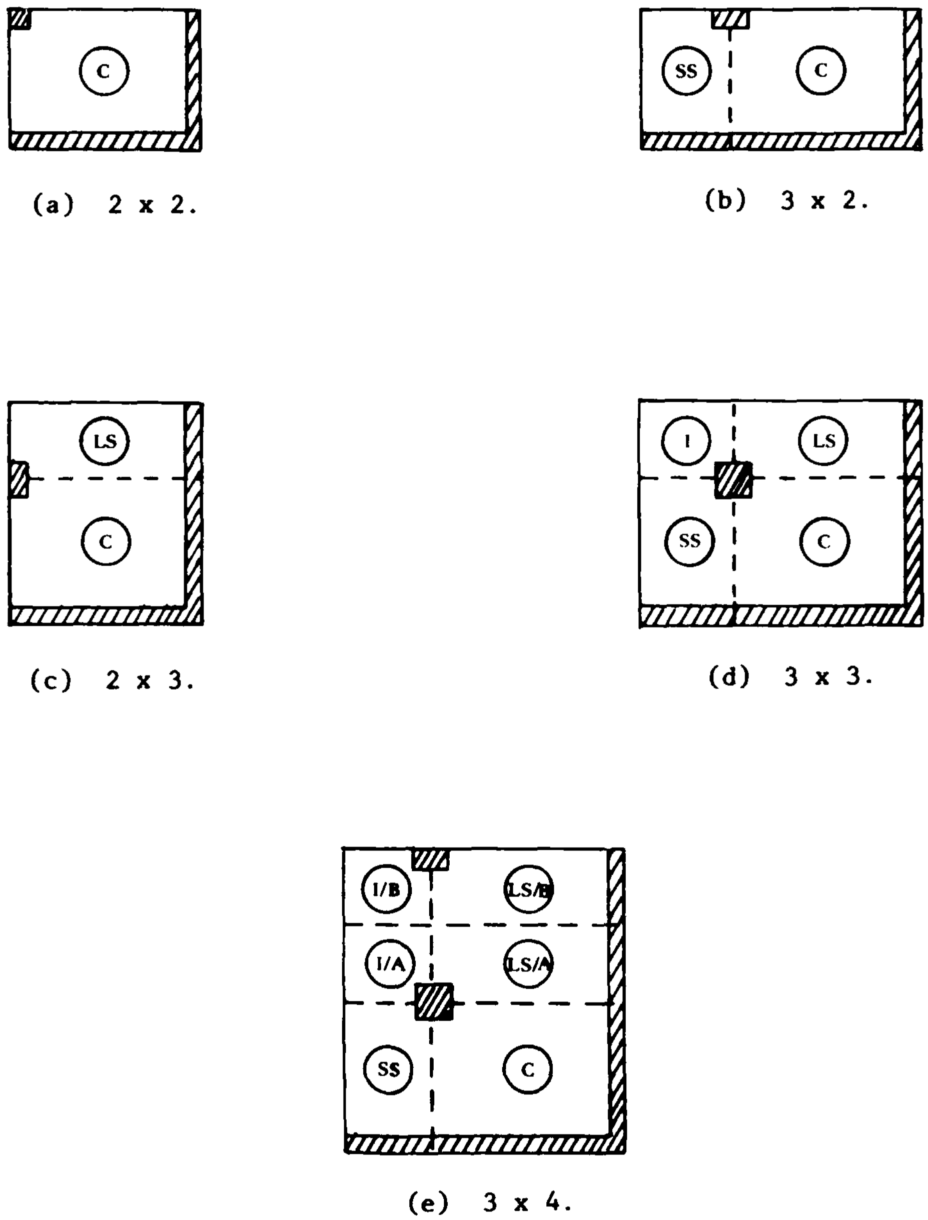
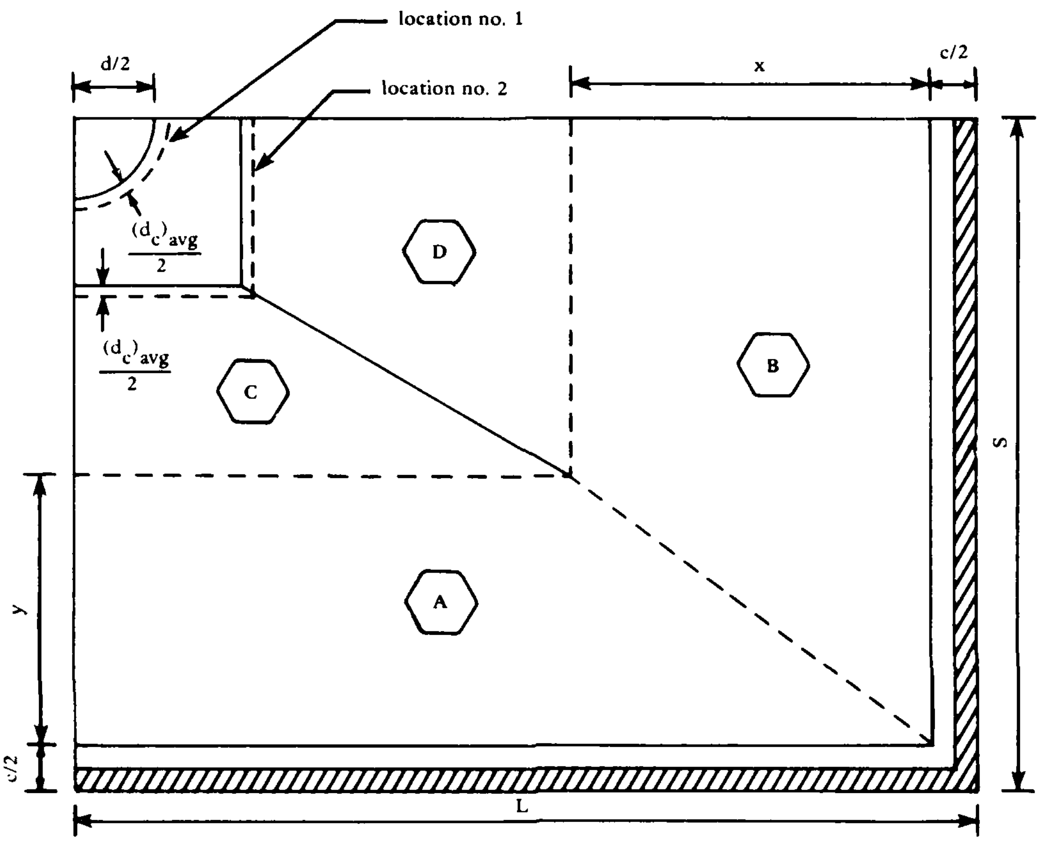


Figure 8. Symmetric flat slab quadrants.

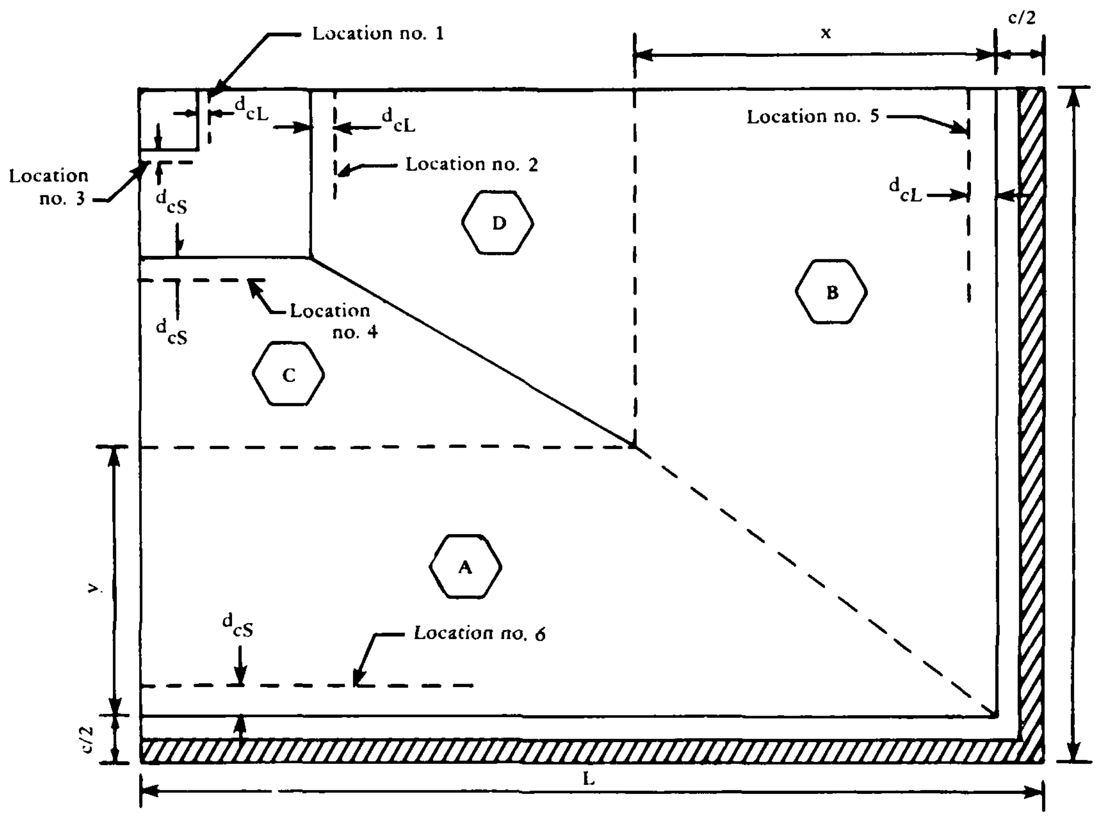


Location no. 1: $\frac{(d_c)_{avg}}{2}$ from circular column capital

Location no. 2: $\frac{(d_c)_{avg}}{2}$ from rectangular drop panel

(a) Punching shear.

Figure 9. Critical shear locations for one-quarter panel of flat slab with central column.



Location no. 1: d_{cL} from column capital (longitudinal)

Location no. 2: d_{cL} from drop panel (longitudinal)

Location no. 3: d_{cS} from column capital (transverse)

Location no. 4: d_{cS} from drop panel (transverse)

Location no. 5: d_{cL} from wall haunch (longitudinal)

Location no. 6: d_{cS} from wall haunch (transverse)

(b) Beam shear.

Figure 9. (continued)

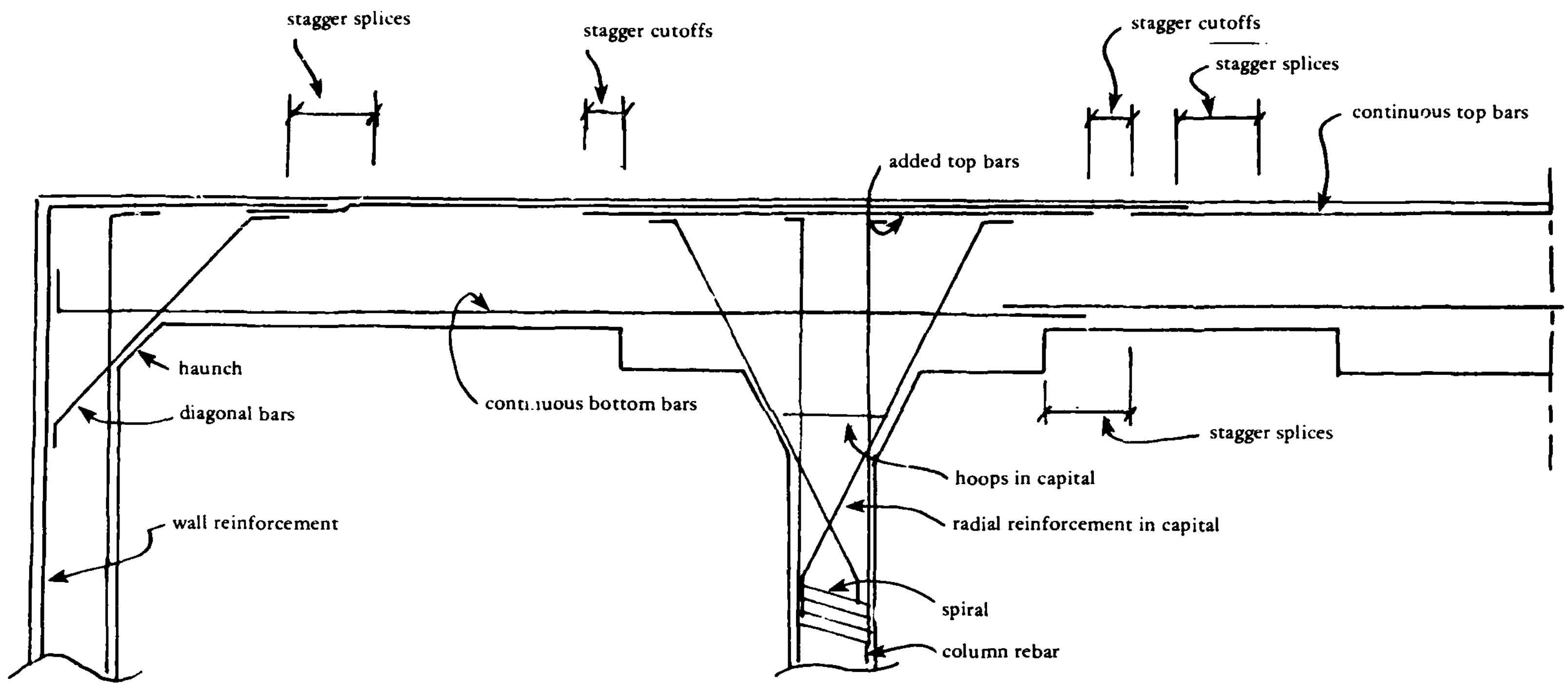


Figure 10. Typical reinforcement details.

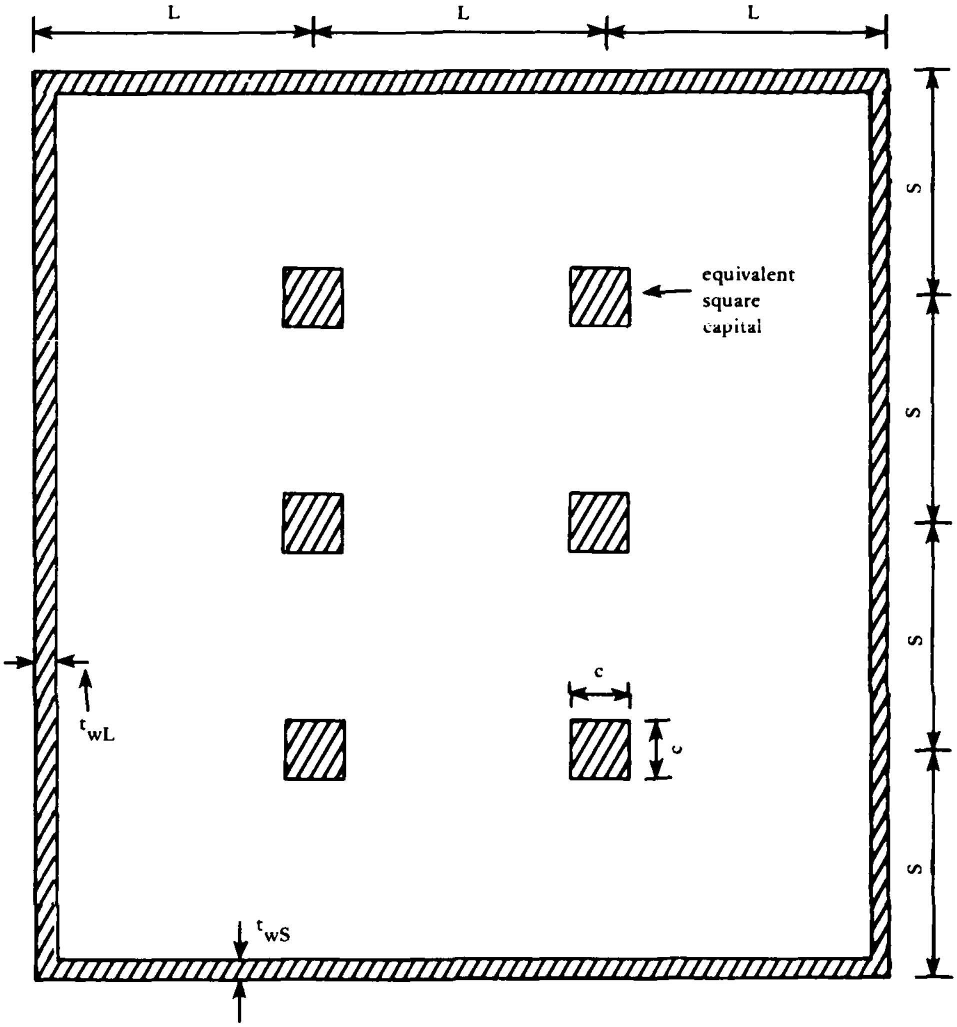


Figure 11. 3 x 4 flat slab configuration.

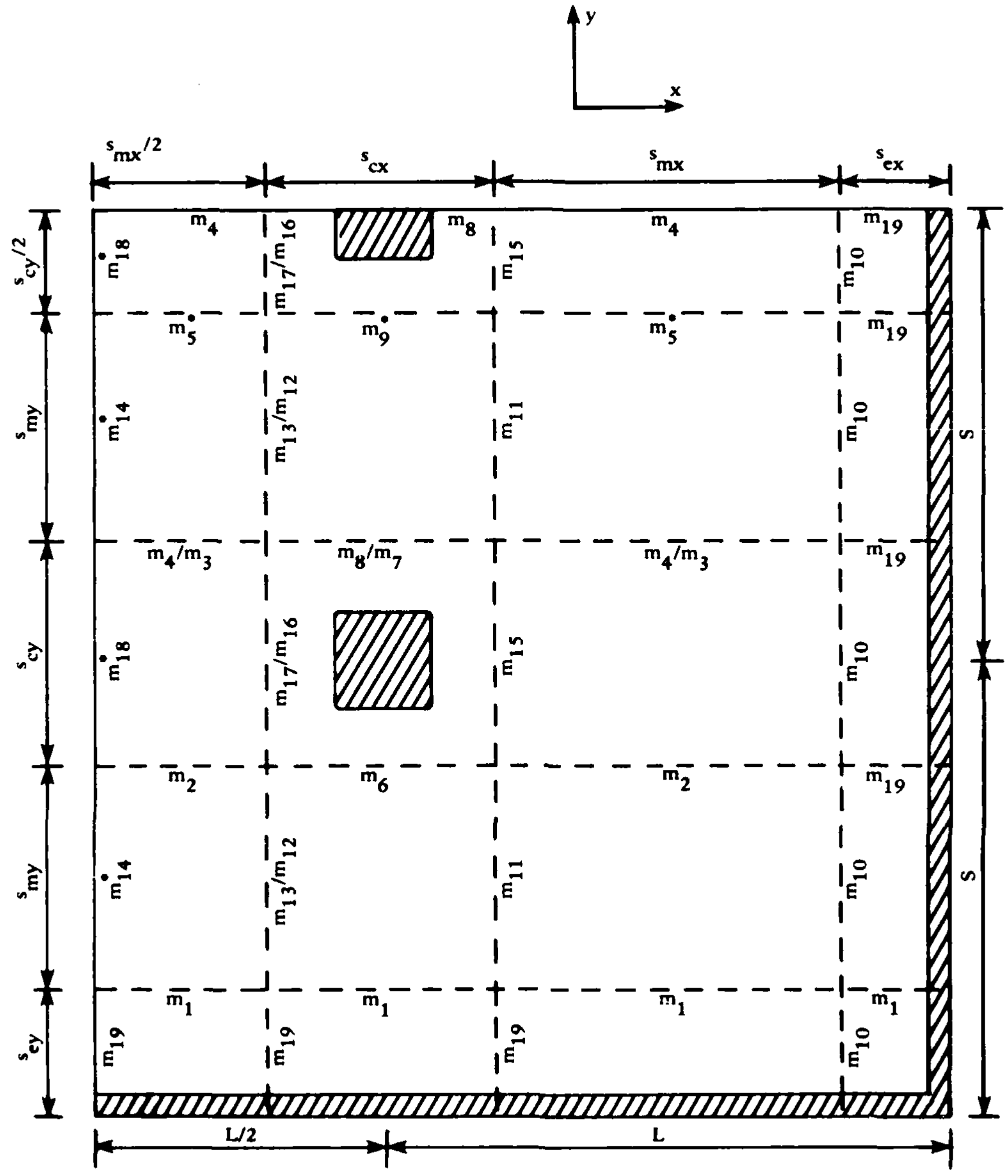


Figure 12. Unit moment distribution for 3 x 4 flat slab.

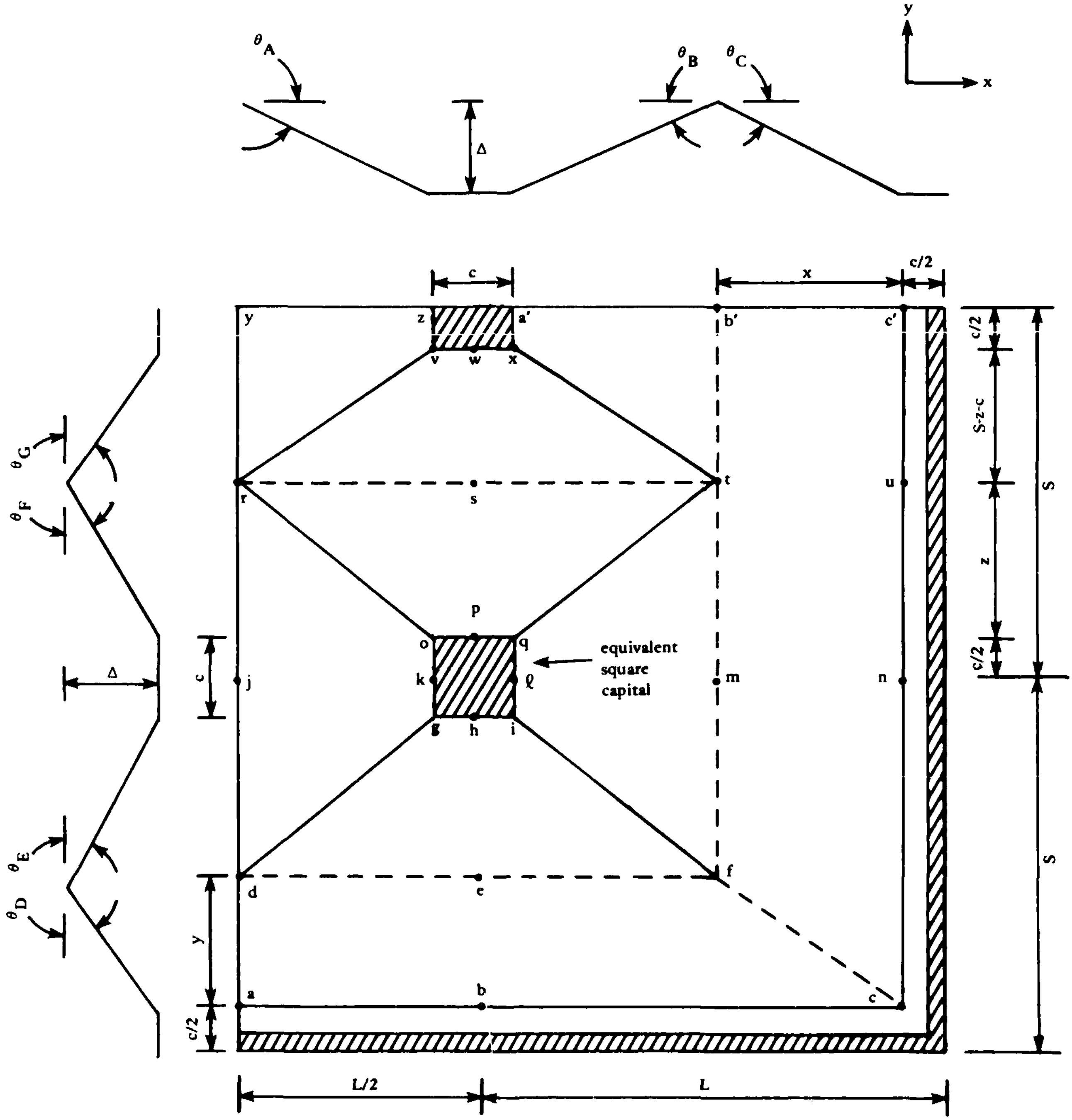


Figure 13. Yield-line mechanism for a 3 x 4 flat slab.

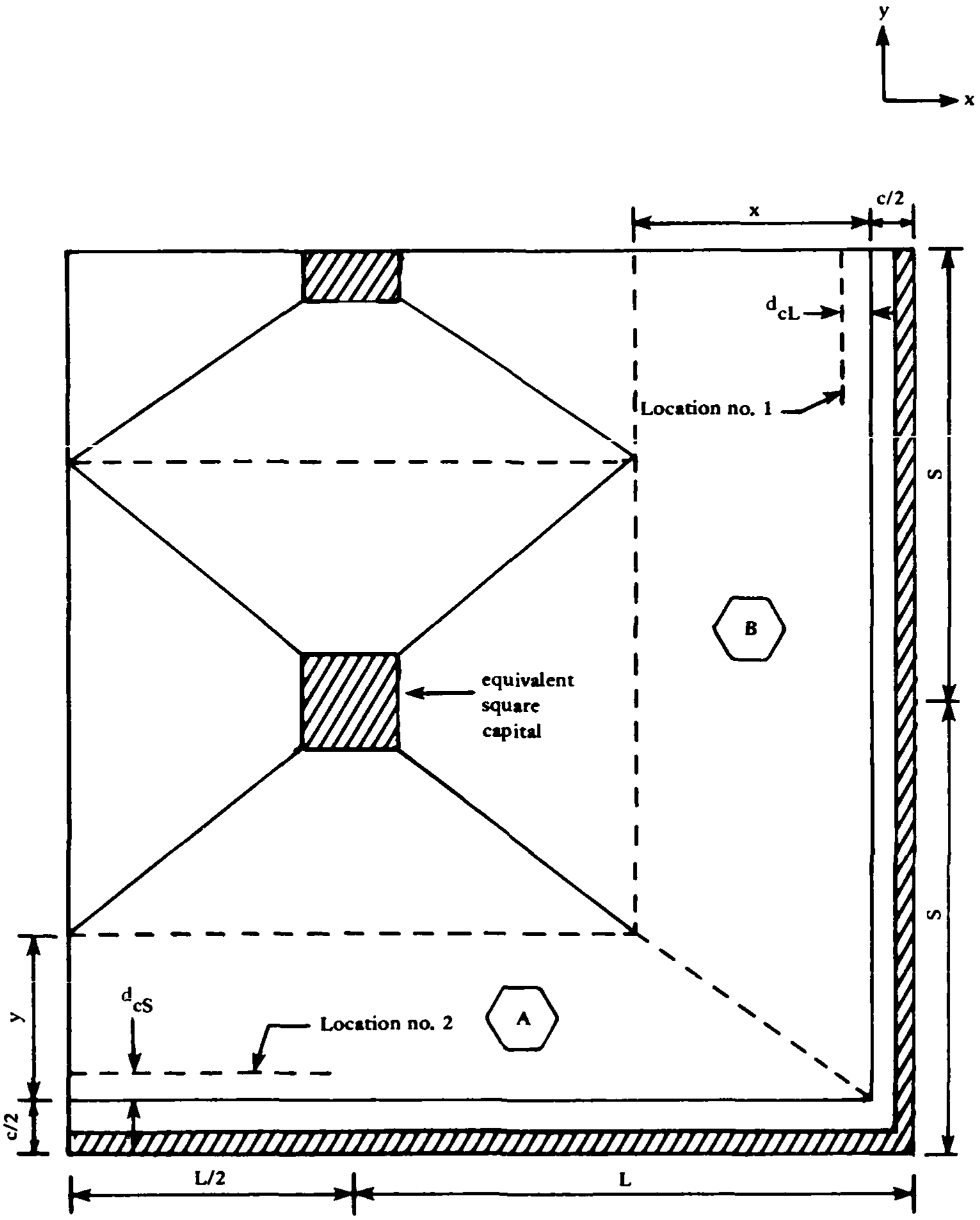


Figure 14. Critical locations for beam shear at support for 3 x 4 flat slab.

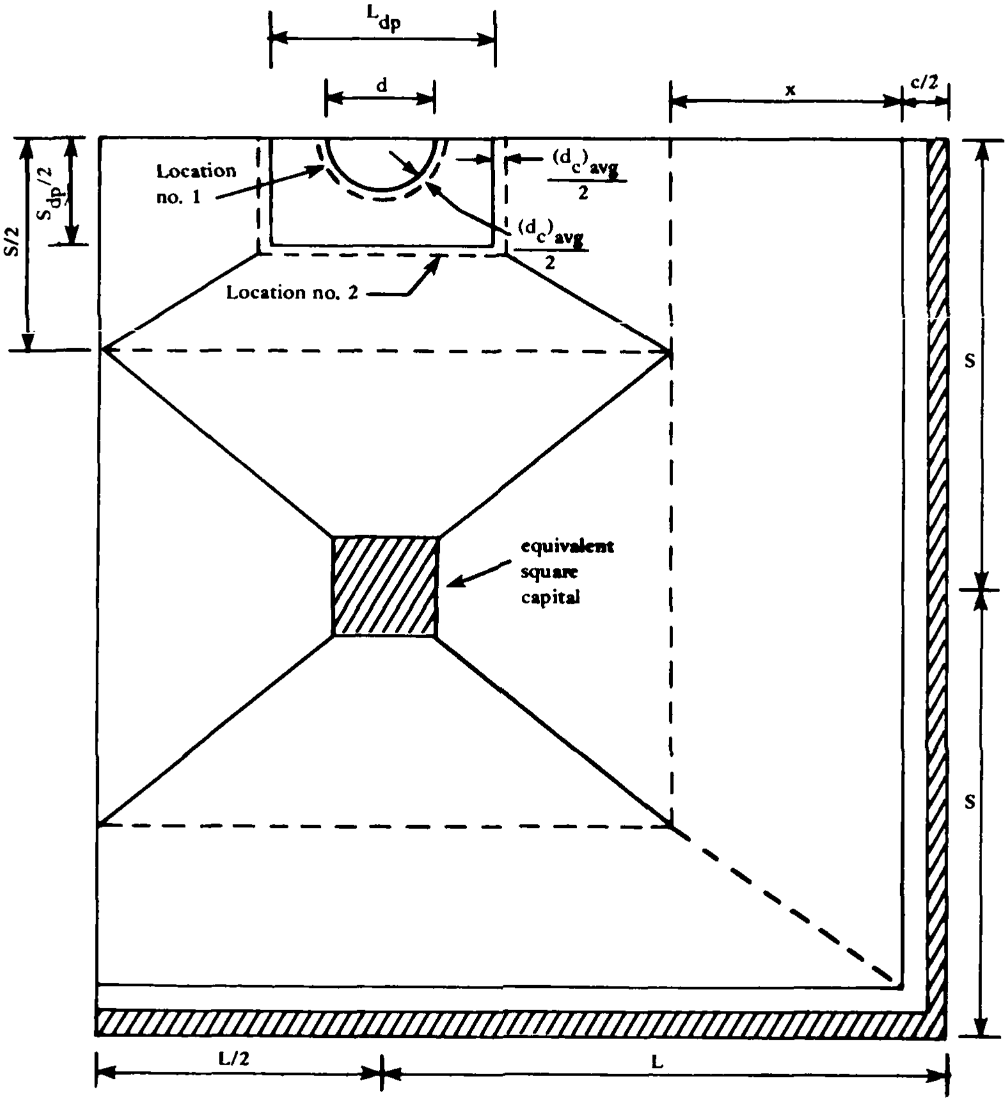


Figure 15. Critical locations for punching slear for a 3 x 4 flat slab.

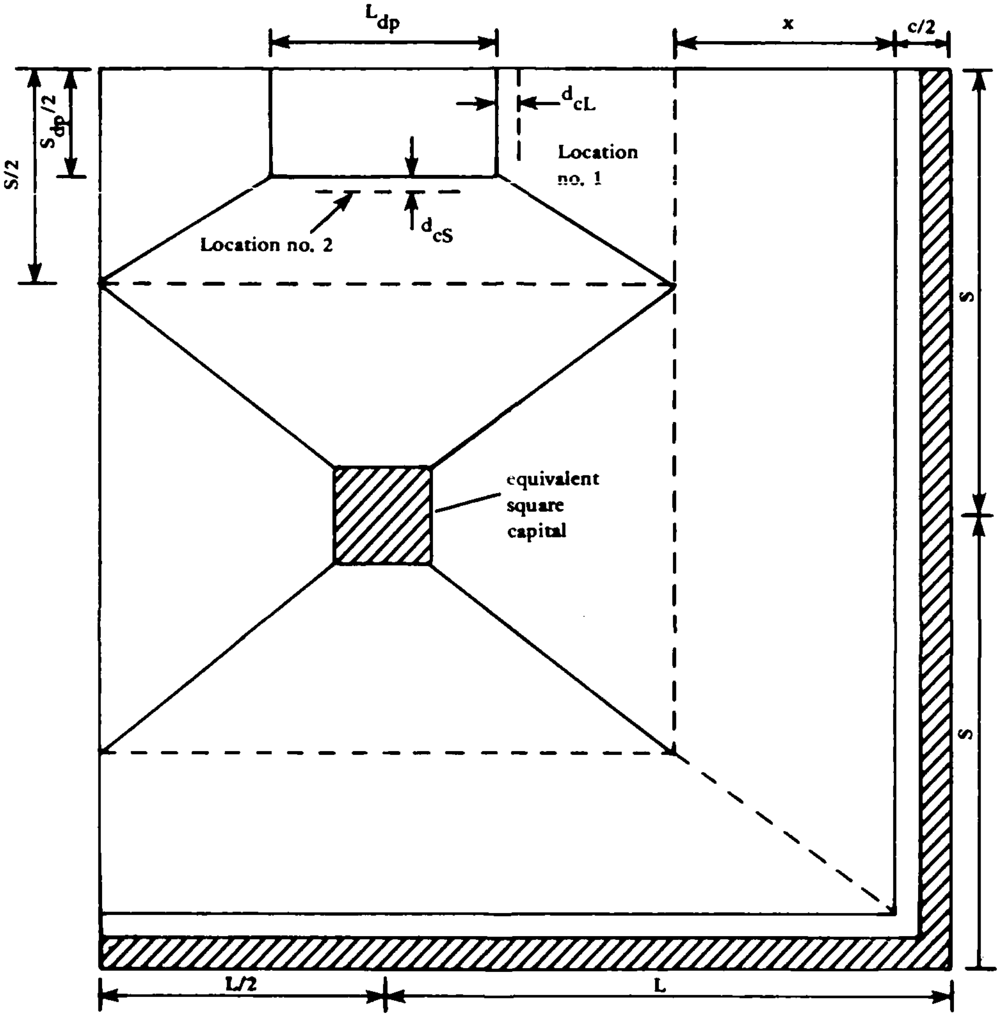


Figure 16. Critical locations for beam shear at drop panel edge for a 3 x 4 flat slab.

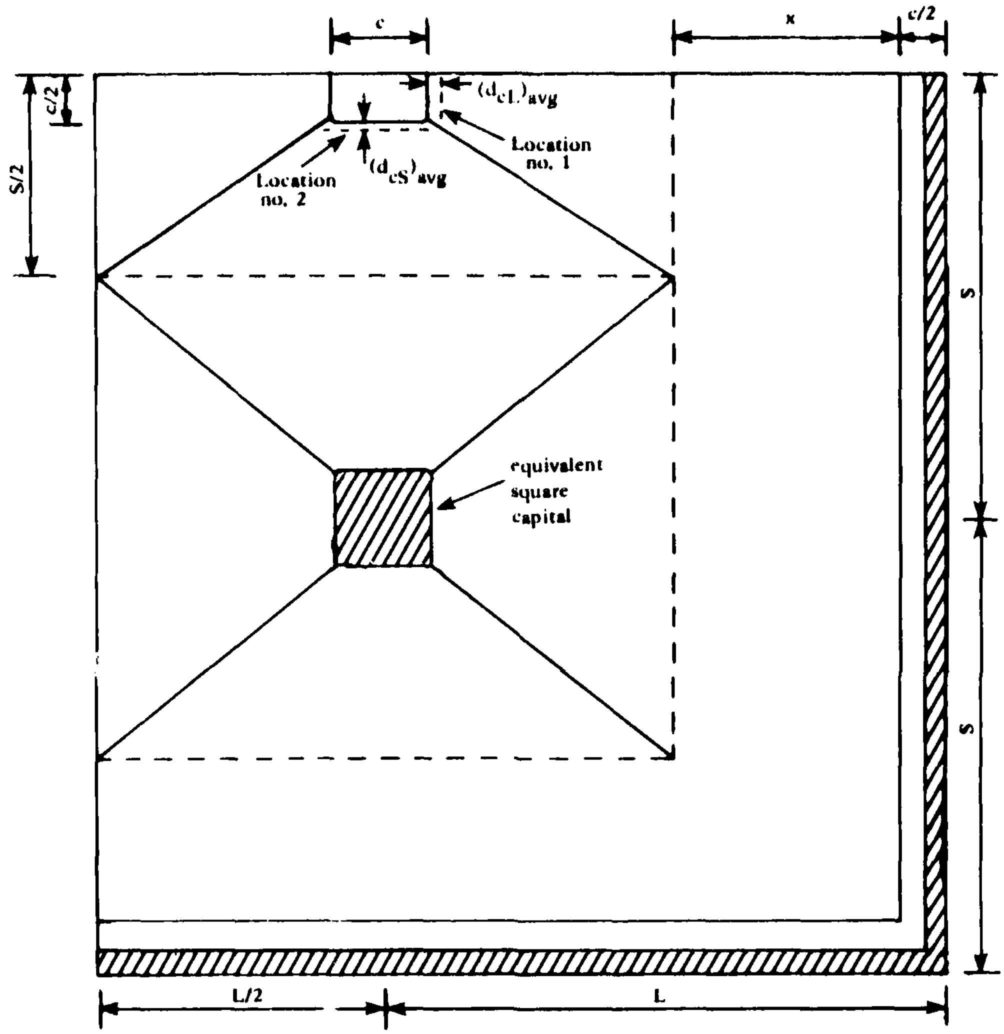
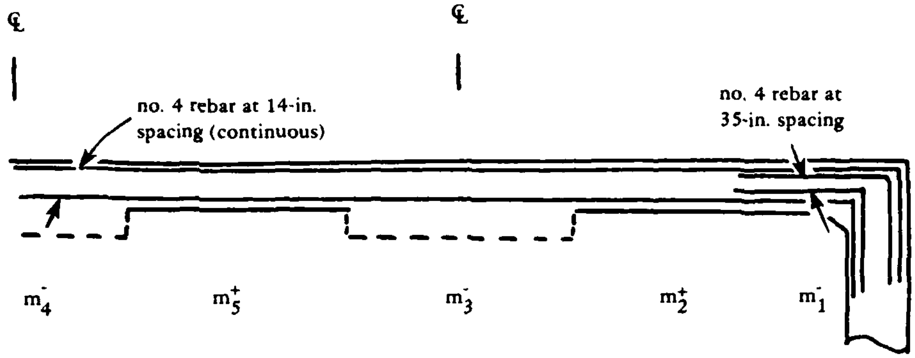
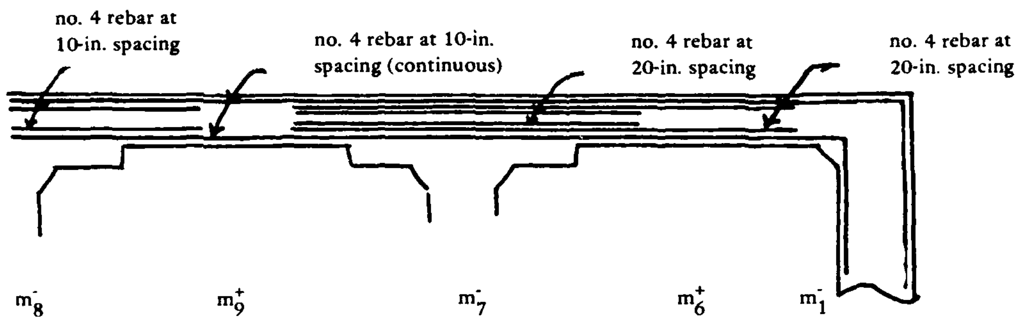


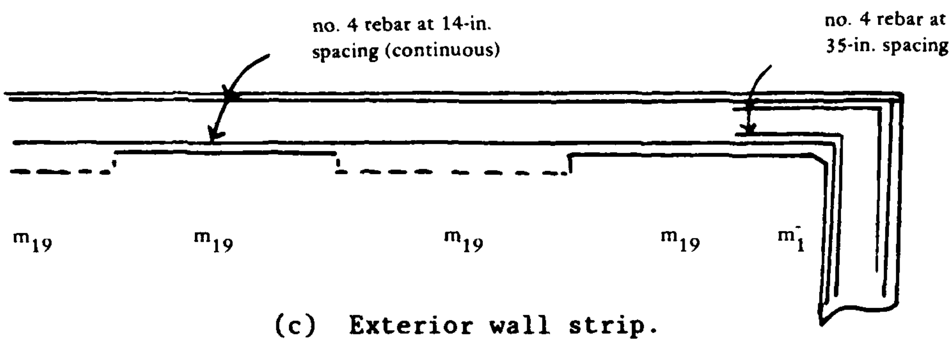
Figure 17. Critical locations for beam shear at column capital for a 3 x 4 flat slab.



(a) Middle strip.

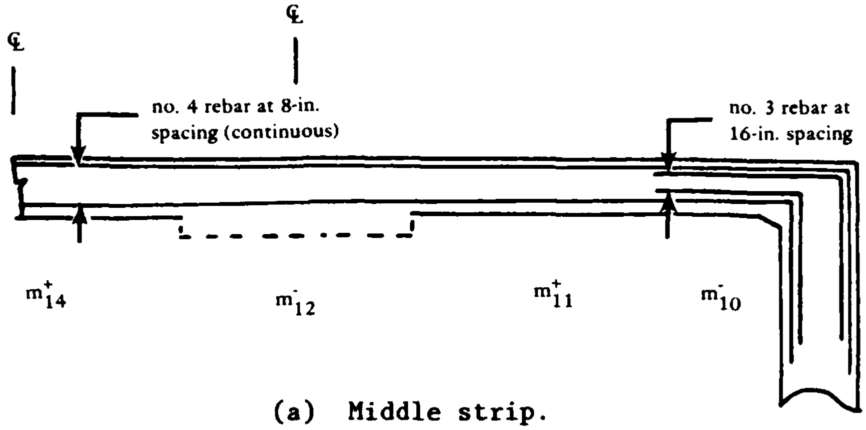


(b) Column strip.

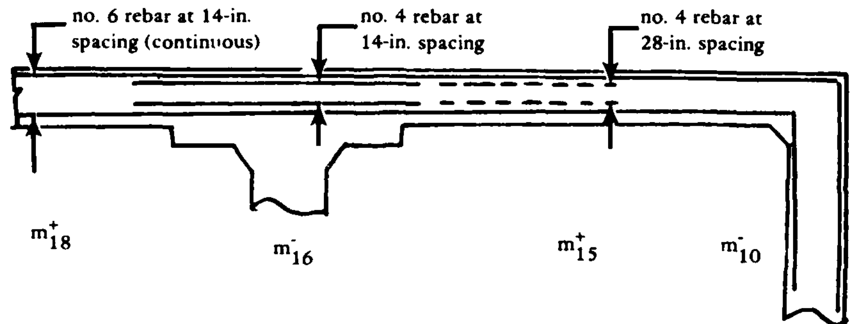


(c) Exterior wall strip.

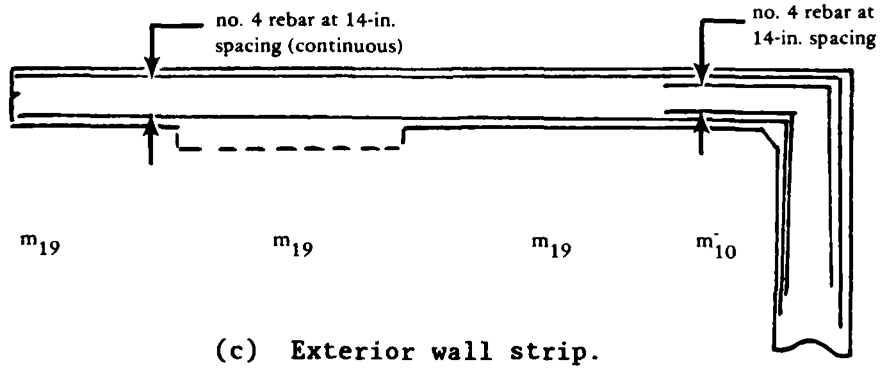
Figure 18. Reinforcement for 3 x 4 flat slab in S-direction.



(a) Middle strip.



(b) Column strip.



(c) Exterior wall strip.

Figure 19. Reinforcement for flat slab in L-direction.

Appendix A

ACI ELASTIC DISTRIBUTION OF REINFORCEMENT

INTRODUCTION

In general, yield-line theory allows freedom in the choice of reinforcement arrangements. For the design of flat slabs, however, it is recommended that an elastic distribution of the reinforcement be used. Several reasons may be cited (Ref 10 and 17):

- The design is more economical.
- Better service load behavior is obtained in regards to cracking, especially when the design blast loads are relatively low in relation to the service loads.
- Moment distribution required to achieve the design configuration is minimized.
- With the required concentration of reinforcement in the column strips, the possibility of failure by localized yield patterns is remote.

DIRECT DESIGN METHOD

The determination of the elastic distribution of moments follows the ACI procedures outlined in Chapter 13 of Reference 8. The ACI Code recommends two methods for the design of two-way slab systems; they are the direct-design method and the equivalent-frame method. In this study the direct-design method will be adopted. In the direct-design method, the distribution (between positive and negative moment zones) of the total static design moment in each direction may be made according to a set of coefficients prescribed in the ACI Code. As applied to flat slabs, this method may be used under the following limitations on continuity, dimensions, and live-load-to-dead-load ratios:

- There must be a minimum of three continuous spans in each direction.
- The panels must be rectangular, each having the ratio of longer to shorter spans not greater than 2.0.
- The successive span lengths in each direction must not differ by more than one-third of the longer span.
- Columns may not be offset more than 10% of the span in the direction of the offset.
- The live load must not exceed three times the dead load.

The basic ACI approach used in the design of flat slabs involves the consideration of rigid frames taken separately in the longitudinal and the transverse directions. When a typical horizontal span in a rigid frame is subjected to a total design load of wL_2 per foot, as shown in Figure A-1(a), equilibrium requires that the sum of the absolute average value of the negative moments at the center of supports and the positive moment at midspan be equal to $wL_2L_1^2/8$ where L_1 is the span length between centerlines of supports; thus

$$M_{pos} + \frac{1}{2}\left(M_{ni} + M_{nj}\right) = \frac{1}{8}\, w\, L_2\, L_1^2 \qquad \text{(A-1)}$$

While the maximum envelope value at midspan can be used directly in design, the value at the centerline of supports can be used only as a basis for obtaining the reduced value at the face of the supports, at which location the slab thickness is investigated and reinforcement designed. In the direct-design method, the reacting shears are assumed to act on the clear span at the face of the supports, as shown in Figure A-1(b). Thus, Equation A-1 becomes:

$$M_{pos} + \frac{1}{2}\left(M_{ni} + M_{nj}\right) = \frac{1}{8} w \, L_2 \, L_n^2 = M_0 \qquad \text{(A-2)}$$

where L_n is the clear span, and M_{ni} and M_{nj} are the negative moments at the face of the supports. Thus, the ACI Code uses the total static design moment, M_0, which is then distributed using coefficients between M_{pos} at midspan and M_{ni} and M_{nj} at the face of the supports. The total static design moment, M_0, is further differentiated in this report as the moments M_{0L} and M_{0S} acting in the long and short directions, respectively.

In the direct-design method, design moment curves in the direction of the span length are nominally defined for regular situations. Referring to Figure A-2(a), L_1 and L_2 are the centerline spans in the longitudinal and transverse directions, while L_n is the clear span in the longitudinal direction. The total static moment in the longitudinal direction has been defined by Equation A-2. In the direct-design method, the design curves, as shown in Figure A-2(c), may be "directly used for design" for the exterior and interior spans. By incorporating these design curves into the blast design methodology, the L_n values (for a given direction) for the interior and exterior spans must then be equal.

These moment values are for the entire width (sum of two half panel widths in the transverse direction, for an interior column line) of the equivalent rigid frame. Each of these moments is to be divided between a column strip and two half middle strips as defined in Figure A-3. For the typical flat slab with continuous exterior walls and L/S $\geq$1, the column strips are S/2 in width in each direction with the middle strips forming the remaining portion of each panel. For flat slabs without beams, the quantity $\alpha_1(L_2/L_1)$ equals zero. According to the ACI Code, the negative moment at the interior supports is distributed 75% to the column strip and 25% to the middle strip (ACI-13.6.4.1), while the positive moment is distributed 60% to the column strip and 40% to the middle strip (ACI-13.6.4.4). When the exterior support consists of a wall extending for a distance greater than three-fourths of the transverse width, the exterior negative moment is to be uniformly distributed over the transverse width (ACI-13.6.4.3).

The rules governing the elastic distribution of moments throughout a flat slab have now been completely defined. All these rules are contained in Table A-1, which shows the calculations employed in determining the entire set of possible unit moments, m_n. Figure A-4 shows the location of these unit moments for three flat slab configurations. These configurations were selected because they illustrate the full spectrum of ACI unit moment distribution. Distributions for other configurations are easily obtained from these figures. All the distributions as shown are symmetric in both directions. The proposed design methodology is limited to this condition.

In Figure A-4(a) and (b) along the first rows of interior columns (in either direction), two values of the negative unit moment acting in the other direction are shown. One value represents a distribution from the interior span, while the other value represents a distribution from the end span. In all cases the value printed nearest the exterior wall is associated with the end span distribution. According to ACI convention, the larger of the two negative factored moments shall be used. However, ACI allows a reduction in the adjacent positive moment so as to maintain the total panel moment (ACI 13.6.7). For example, if m_7 is greater than m_8, then m_7 is used as the negative moment capacity at the column, and m_9 is decreased. In Figure A-4(a) and (b), this reduction is $(m_7 - m_8)/2$ and $m_7 - m_8$, respectively. Note that in Figure A-4(c) there is no interior span and, therefore, all the unit moments are based on the end span distribution.

ABSOLUTE VALUES OF UNIT MOMENTS

Expressions for the unit moments listed in Table A-1 are of a general nature and involve the following parameters.

L = long span measured center-to-center of supporting columns*

S = short span measured center-to-center of supporting columns*

α'_{ecS} = coefficient of flexural stiffness of exterior wall and slab in the short span direction

α'_{ecL} = coefficient of flexural stiffness of exterior wall and slab in the long span direction

M_{OS} = total factored static moment in the short direction

M_{OL} = total factored static moment in the long direction

To define these unit moments further, it is necessary to introduce these two design parameters:

β = span ratio (L/S)

α_{cap} = capital ratio (d/L)

where d = diameter of capital. By definition,

$$M_0 = \frac{w L_2 L_n^2}{8} \tag{A-3}$$

For the short side moment, M_{OS}:

$$L_2 = L$$

$$L_n = S - c = S - 0.89\ d = \frac{L}{\beta} - 0.89\ \alpha_{cap} L = L\left(\frac{1}{\beta} - 0.89\ \alpha_{cap}\right)$$

*For exterior spans these quantities have no direct physical correspondence. That is, it is the clear span, L_n, in the interior and exterior spans that are equal.

where c = equivalent square capital length = 0.89 d. For the long side moment, M_{OL}:

$$L_2 = S = \frac{L}{\beta}$$

$$L_n = L - c = L - 0.89\ d = L - 0.89\ \alpha_{cap}\ L = L\left(1 - 0.89\ \alpha_{cap}\right)$$

Substitution into Equation A-3:

$$M_{OS} = \frac{w\ L\left[L\ (\frac{1}{\beta} - 0.89\ \alpha_{cap})\right]^2}{8} = \frac{w\ L^3\ (\frac{1}{\beta} - 0.89\ \alpha_{cap})^2}{8} \qquad \text{(A-4)}$$

$$M_{OL} = \frac{w\ \frac{L}{\beta}\left[L\ (1 - 0.89\ \alpha_{cap})\right]^2}{8} = \frac{w\ L^3\ (1 - 0.89\ \alpha_{cap})^2}{8\ \beta} \qquad \text{(A-5)}$$

To simplify the design process, NCEL recommends the following substitution:

$$m_e = \frac{w\ L^2}{8} \qquad \text{(A-6)}$$

Therefore,

$$M_{OS} = (\frac{1}{\beta} - 0.89\ \alpha_{cap})^2\ L\ m_e = \alpha_{OS}\ L\ m_e \qquad \text{(A-7)}$$

$$M_{OL} = \frac{(1 - 0.89\ \alpha_{cap})^2}{\beta}\ L\ m_e = \alpha_{OL}\ L\ m_e \qquad \text{(A-8)}$$

where:

$$\alpha_{OS} = (\frac{1}{\beta} - 0.89\ \alpha_{cap})^2 \qquad \text{(A-9)}$$

$$\alpha_{OL} = \frac{(1 - 0.89\ \alpha_{cap})^2}{\beta} \qquad \text{(A-10)}$$

Through direct substitution of the expressions in Equations A-6, A-7, and A-8, it is now possible to reduce the unit moment expressions found in Table A-1 to functions solely of α'_{ecS}, α'_{ecL}, α_{OS}, α_{OL}, β, and m_e. Let:

$$m_n = \alpha_{um} m_e \tag{A-11}$$

where α_{um} = unit moment coefficients. The unit moment coefficients for the flat slab are listed in Table A-2. Values of the panel moment coefficients (α_{OS} and α_{OL}) are listed in Tables A-3 and A-4 for typical values of β and α_{cap}. These values are also plotted in Figures A-5 and A-6.

Two additional design parameters are required to fully describe α'_{ecS} and α'_{ecL}. These parameters are:

α_t = wall thickness ratio (t_{wL}/t_{slab} or t_{wS}/t_{slab})

α_H = wall height ratio (H_w/S)

where: t_{slab} = slab thickness
t_{wL} = long side wall thickness*
t_{wS} = short side wall thickness*
H_w = wall height (interior clear height from floor to ceiling)

By definition,

$$\alpha'_{ecS} = \frac{1}{1 + \frac{1}{\alpha_{ecS}}} \tag{A-12}$$

$$\alpha'_{ecL} = \frac{1}{1 + \frac{1}{\alpha_{ecL}}} \tag{A-13}$$

*In this symmetric analysis, the sidewall thicknesses are considered equal as are the backwall and headwall thicknesses. However, for typical box-shaped ammunition storage magazines, the headwall will be thicker than the backwall. For these cases NCEL recommends maintaining symmetry by using the backwall thickness in all calculations involving the moment distribution. This results in lower negative moments calculated at the headwall, but greater positive moments between the headwall and the column line.

where:

$$\alpha_{ecS} = \frac{t_{wS}^3\ S}{t_{slab}^3\ H_w} \qquad \text{(A-14)}$$

$$\alpha_{ecL} = \frac{t_{wL}^3\ L}{t_{slab}^3\ H_w} \qquad \text{(A-15)}$$

Therefore,

$$\alpha'_{ecS} = \frac{1}{1 + \dfrac{t_{slab}^3\ H_w}{t_{wS}^3\ S}} \qquad \text{(A-16)}$$

$$\alpha'_{ecL} = \frac{1}{1 + \dfrac{t_{slab}^3\ H_w}{t_{wL}^3\ \beta\ S}} \qquad \text{(A-17)}$$

Tables A-5 and A-6 list values of α'_{ecS} and α'_{ecL} for typical values of β, t_{wS}/t_{slab}, t_{wL}/t_{slab}, H_w/S, and H_w/L. These values are also plotted in Figures A-7 and A-8.

EXAMPLE PROBLEM

It is now possible to determine the unit moments for the 3 x 4 flat slab configuration shown in Figure 11. Let,

$$\beta = 1.25$$

$$\alpha_{cap} = 0.20$$

$$\frac{t_{wS}}{t_{slab}} = \frac{t_{wL}}{t_{slab}} = 1.00$$

$$\frac{H_w}{S} = 0.50$$

The following coefficients are obtained from Tables A-3, A-4, A-5, and A-6:

$$\alpha_{OS} = 0.387$$

$$\alpha_{OL} = 0.541$$

$$\alpha'_{ecS} = 0.666$$

$$\alpha'_{ecL} = 0.714$$

Substitution of these quantities into the expressions found in Table A-2 yields the unit moment coefficients, α_{um}, listed in Table A-7. Their locations are shown in Figure A-9. The allowable ACI adjusted unit moment coefficients in the positive interior moment regions are also listed. For example,

$$\alpha_9^* = \alpha_9 - \frac{\alpha_7 - \alpha_8}{2} = 0.203 - \frac{0.496 - 0.472}{2} = 0.191$$

Table A-1. ACI Unit Moment Definitions

Unit Moment Parameter, m_n	Span Direction	Strip	Negative or Positive	Strip Width	Longitudinal Distribution Factor	Transverse Distribution Factor	Total Positive/Negative Moment	Unit Moment Expression, m_n
m_1	Short	mid/col/wall	Neg	L	$0.65\ \alpha'_{ecS}$	1.00	$0.65\ \alpha'_{ecS}\ M_{OS}$	$0.65\ \alpha'_{ecS} M_{OS}/L$
m_2	Short	mid	Pos	L-(S/2)	$0.63-0.28\ \alpha'_{ecS}$	0.40	$(0.252-0.112\ \alpha'_{ecS})M_{OS}$	$(0.252-0.112\ \alpha'_{ecS})M_{OS}/[L-(S/2)]$
m_3	Short	mid	Neg	L-(S/2)	$0.75-0.10\ \alpha'_{ecS}$	0.25	$(0.188-0.025\ \alpha'_{ecS})M_{OS}$	$(0.188-0.025\ \alpha'_{ecS})M_{OS}/[L-(S/2)]$
m_4	Short	mid	Neg	L-(S/2)	0.65	0.25	$0.163\ M_{OS}$	$0.163\ M_{OS}/[L-(S/2)]$
m_5	Short	mid	Pos	L-(S/2)	0.35	0.40	$0.14\ M_{OS}$	$0.14\ M_{OS}/[L-(S/2)]$
m_6	Short	col	Pos	S/2	$0.63-0.28\ \alpha'_{ecS}$	0.60	$(0.378-0.168\ \alpha'_{ecS})M_{OS}$	$(0.378-0.168\ \alpha'_{ecS})M_{OS}/(S/2)$
m_7	Short	col	Neg	S/2	$0.75-0.10\ \alpha'_{ecS}$	0.75	$(0.563-0.075\ \alpha'_{ecS})M_{OS}$	$(0.563-0.075\ \alpha'_{ecS})M_{OS}/(S/2)$
m_8	Short	col	Neg	S/2	0.65	0.75	$0.488\ M_{OS}$	$0.488\ M_{OS}/(S/2)$
m_9	Short	col	Pos	S/2	0.35	0.60	$0.21\ M_{OS}$	$0.21\ M_{OS}/(S/2)$
m_{10}	Long	mid/col/wall	Neg	S	$0.65\ \alpha'_{ecL}$	1.00	$0.65\ \alpha'_{ecL}\ M_{OL}$	$0.65\ \alpha'_{ecL}\ M_{OL}/S$
m_{11}	Long	mid	Pos	S/2	$0.63-0.28\ \alpha'_{ecL}$	0.40	$(0.252-0.112\ \alpha'_{ecL})M_{OL}$	$(0.252-0.112\ \alpha'_{ecL})M_{OL}/(S/2)$
m_{12}	Long	mid	Neg	S/2	$0.75-0.10\ \alpha'_{ecL}$	0.25	$(0.188-0.025\ \alpha'_{ecL})M_{OL}$	$(0.188-0.025\ \alpha'_{ecL})M_{OL}/(S/2)$
m_{13}	Long	mid	Neg	S/2	0.65	0.25	$0.163\ M_{OL}$	$0.163\ M_{OL}/(S/2)$
m_{14}	Long	mid	Pos	S/2	0.35	0.40	$0.14\ M_{OL}$	$0.14\ M_{OL}/(S/2)$
m_{15}	Long	col	Pos	S/2	$0.63-0.28\ \alpha'_{ecL}$	0.60	$(0.378-0.168\ \alpha'_{ecL})M_{OL}$	$(0.378-0.168\ \alpha'_{ecL})M_{OL}/(S/2)$
m_{16}	Long	col	Neg	S/2	$0.75-0.10\ \alpha'_{ecL}$	0.75	$(0.563-0.075\ \alpha'_{ecL})M_{OL}$	$(0.563-0.075\ \alpha'_{ecL})M_{OL}/(S/2)$
m_{17}	Long	col	Neg	S/2	0.65	0.75	$0.488\ M_{OL}$	$0.488\ M_{OL}/(S/2)$
m_{18}	Long	col	Pos	S/2	0.35	0.60	$0.21\ M_{OL}$	$0.21\ M_{OL}/(S/2)$
m_{19}	Short/Long	wall	Pos/Neg	---	minimum	minimum	----	----

Note: $\alpha'_{ecS} = 1/[1 + (1/\alpha_{ecS})]$ $\qquad \alpha_{ecS} = t^3_{wS}\ S/t^3_{slab}\ H_w$

$\alpha'_{ecL} = 1/[1 + (1/\alpha_{ecL})]$ $\qquad \alpha_{ecL} = t^3_{wL}\ L/t^3_{slab}\ H_w$

Table A-2. Unit Moment Coefficients

Unit Moment, m_n	Unit Moment Coefficient, α_{um}
m_1	$0.65\ \alpha'_{ecS}\ \alpha_{OS}$
m_2	$(0.504-0.224\ \alpha'_{ecS})\alpha_{OS}\ \beta/(2\beta-1)$
m_3	$(0.376-0.05\ \alpha'_{ecS})\alpha_{OS}\ \beta/(2\beta-1)$
m_4	$0.326\ \alpha_{OS}\ \beta/(2\beta-1)$
m_5	$0.28\ \alpha_{OS}\ \beta/(2\beta-1)$
m_6	$(0.756-0.336\ \alpha'_{ecS})\alpha_{OS}\ \beta$
m_7	$(1.126-0.15\ \alpha'_{ecS})\alpha_{OS}\ \beta$
m_8	$0.976\ \alpha_{OS}\ \beta$
m_9	$0.42\ \alpha_{OS}\ \beta$
m_{10}	$0.65\ \alpha'_{ecL}\ \alpha_{OL}\ \beta$
m_{11}	$(0.504-0.224\ \alpha'_{ecL})\alpha_{OL}\ \beta$
m_{12}	$(0.376-0.05\ \alpha'_{ecL})\alpha_{OL}\ \beta$
m_{13}	$0.326\ \alpha_{OL}\ \beta$
m_{14}	$0.28\ \alpha_{OL}\ \beta$
m_{15}	$(0.756-0.336\ \alpha'_{ecL})\alpha_{OL}\ \beta$
m_{16}	$(1.126-0.15\ \alpha'_{ecL})\alpha_{OL}\ \beta$
m_{17}	$0.976\ \alpha_{OL}\ \beta$
m_{18}	$0.42\ \alpha_{OL}\ \beta$
m_{19}	minimum

Note: $\alpha_{um} = m_n/m_e$

Table A-3. Values of α_{OS}

Case No.	β	α_{cap}	α_{OS}
1	1.0	0.15 0.20 0.25	0.751 0.676 0.605
2	1.25	0.15 0.20 0.25	0.444 0.387 0.334
3	1.50	0.15 0.20 0.25	0.284 0.239 0.197
4	1.75	0.15 0.20 0.25	0.192 0.155 0.122
5	2.00	0.15 0.20 0.25	0.134 0.104 0.077

Note: $\alpha_{OS} = [(1/B) - 0.89\ \alpha_{cap}]^2$

Table A-4. Values of α_{OL}

Case No.	β	α_{cap}	α_{OL}
1	1.0	0.15 0.20 0.25	0.751 0.676 0.605
2	1.25	0.15 0.20 0.25	0.601 0.541 0.484
3	1.50	0.15 0.20 0.25	0.501 0.451 0.403
4	1.75	0.15 0.20 0.25	0.429 0.386 0.345
5	2.00	0.15 0.20 0.25	0.375 0.338 0.302

Note: $\alpha_{OL} = (1 - 0.89\ \alpha_{cap})^2/\beta$

Table A-5. Values of α'_{ecS}

H_w/S	α'_{ecS} for t_{wS}/t_{slab} =				
	0.75	1.00	1.25	1.50	1.75
0.4	0.513	0.714	0.830	0.894	0.931
0.5	0.458	0.666	0.796	0.871	0.915
0.6	0.413	0.625	0.765	0.849	0.899

Note: $$\alpha'_{ecS} = \frac{1}{1 + \dfrac{t_{slab}^3 H_w}{t_{wS}^3 S}}$$

Table A-6. Values of α'_{ecL}

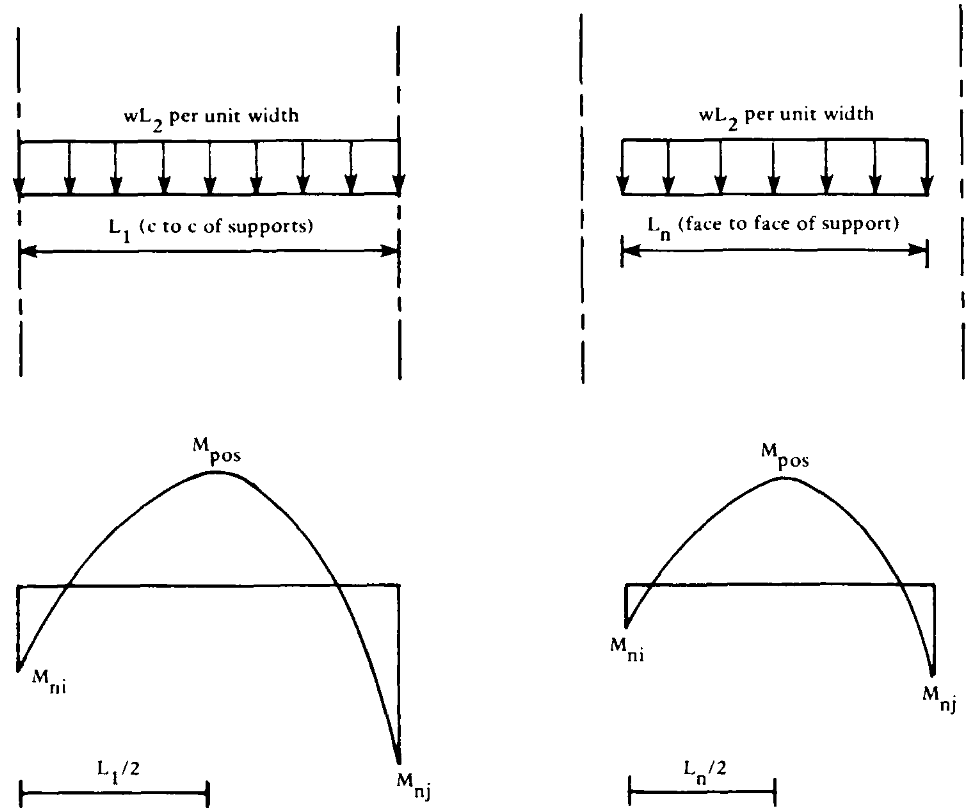
H_w/L	α'_{ecL} for t_{wL}/t_{slab} =				
	0.75	1.00	1.25	1.50	1.75
0.20	0.678	0.833	0.907	0.944	0.964
0.25	0.628	0.800	0.887	0.931	0.955
0.30	0.584	0.769	0.867	0.918	0.947
0.35	0.547	0.741	0.848	0.906	0.939
0.40	0.513	0.714	0.830	0.894	0.931
0.45	0.484	0.690	0.813	0.882	0.923
0.50	0.458	0.667	0.796	0.871	0.915
0.55	0.434	0.645	0.780	0.860	0.907
0.60	0.413	0.625	0.765	0.849	0.899

Note: $$\alpha'_{ecL} = \frac{1}{1 + \dfrac{t_{slab}^3 H_w}{t_{wL}^3 L}}$$

Table A-7. Values of Unit Moment Coefficients (β = 1.25; α_{cap} = 0.20; t_{wS}/t_{slab} = t_{wL}/t_{slab} = 1.00; H_w/S = 0.5)

Unit Moment, m_n	Unit Moment Coefficient, α_{um}	Adjusted Positive Unit Moment Coefficient, α^*_{um}
m_1	0.168	-----
m_2	0.114	-----
m_3	0.111	-----
m_4	0.105	-----
m_5	0.090	0.087
m_6	0.257	-----
m_7	0.496	-----
m_8	0.472	-----
m_9	0.203	0.191
m_{10}	0.314	-----
m_{11}	0.233	-----
m_{12}	0.230	-----
m_{13}	0.220	-----
m_{14}	0.189	0.179
m_{15}	0.349	-----
m_{16}	0.689	-----
m_{17}	0.660	-----
m_{18}	0.284	0.255
m_{19}	miniumum	-----

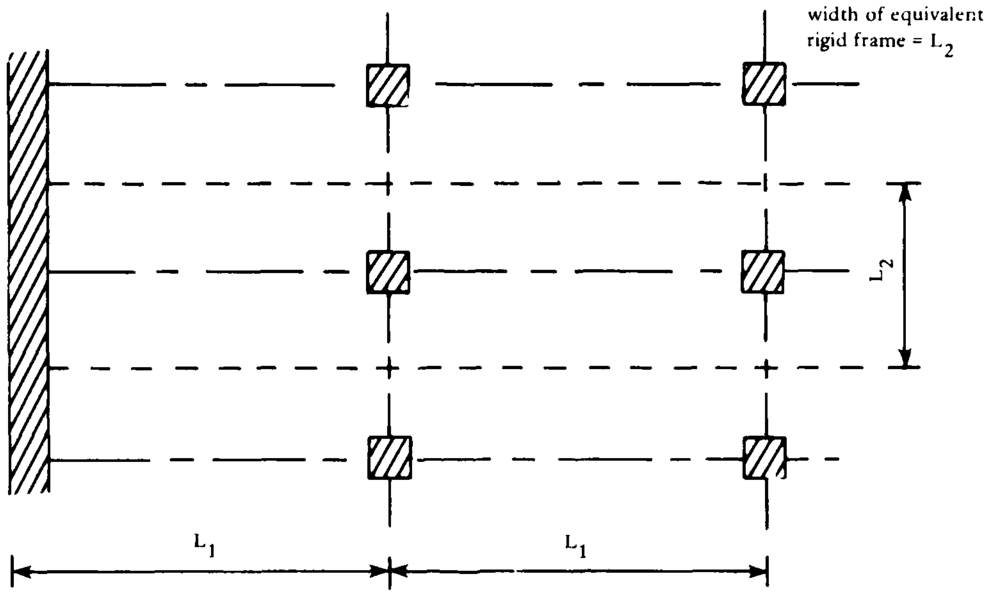
Note: $m_n = \alpha_{um} m_e$, $m_e = w L^2/8$



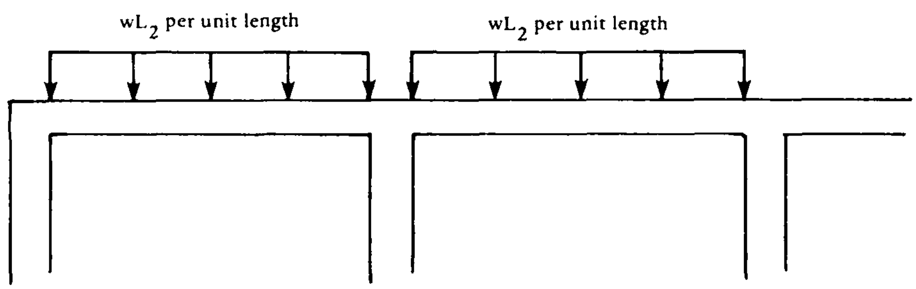
(a) Moment diagram referring to centerline of supports.

(b) Moment diagram referring to faces of support.

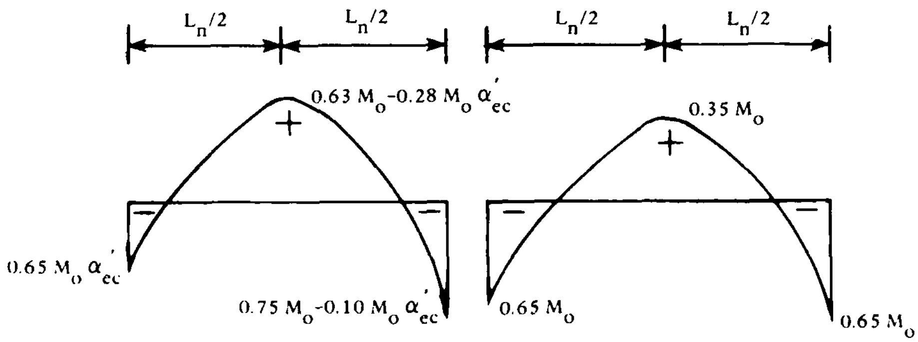
Figure A-1. Typical moment diagram of a horizontal span.



(a) Plan view.



(b) Equivalent rigid frame.



(c) Longitudinal design moment curve.

Figure A-2. Direct-design method: longitudinal distribution of moments.

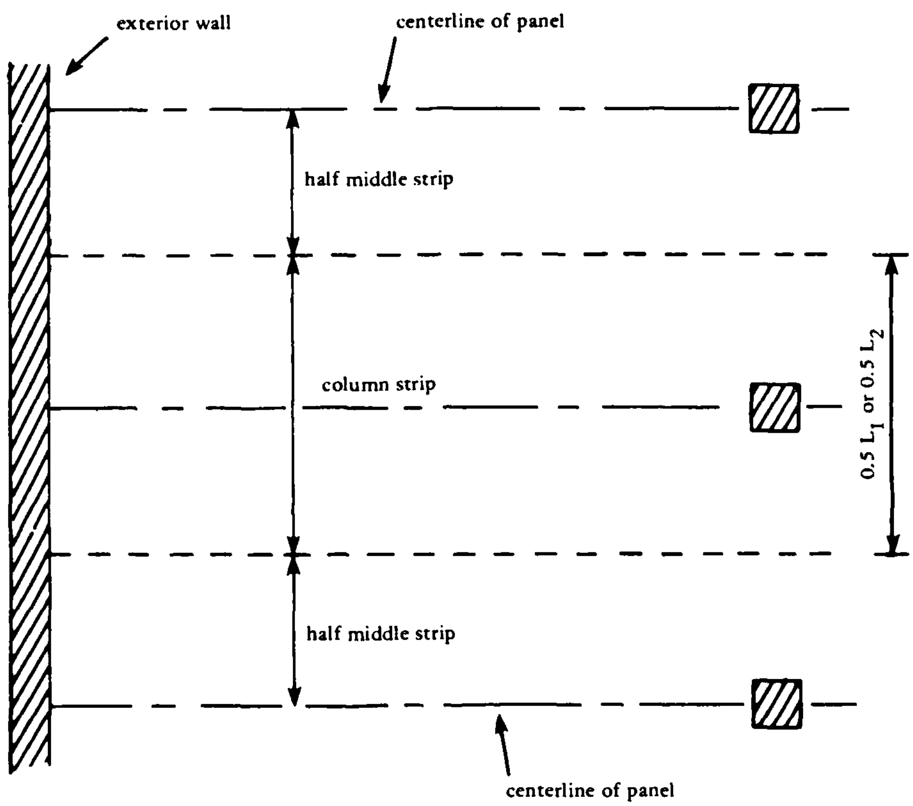


Figure A-3. Definition of column and middle strips.

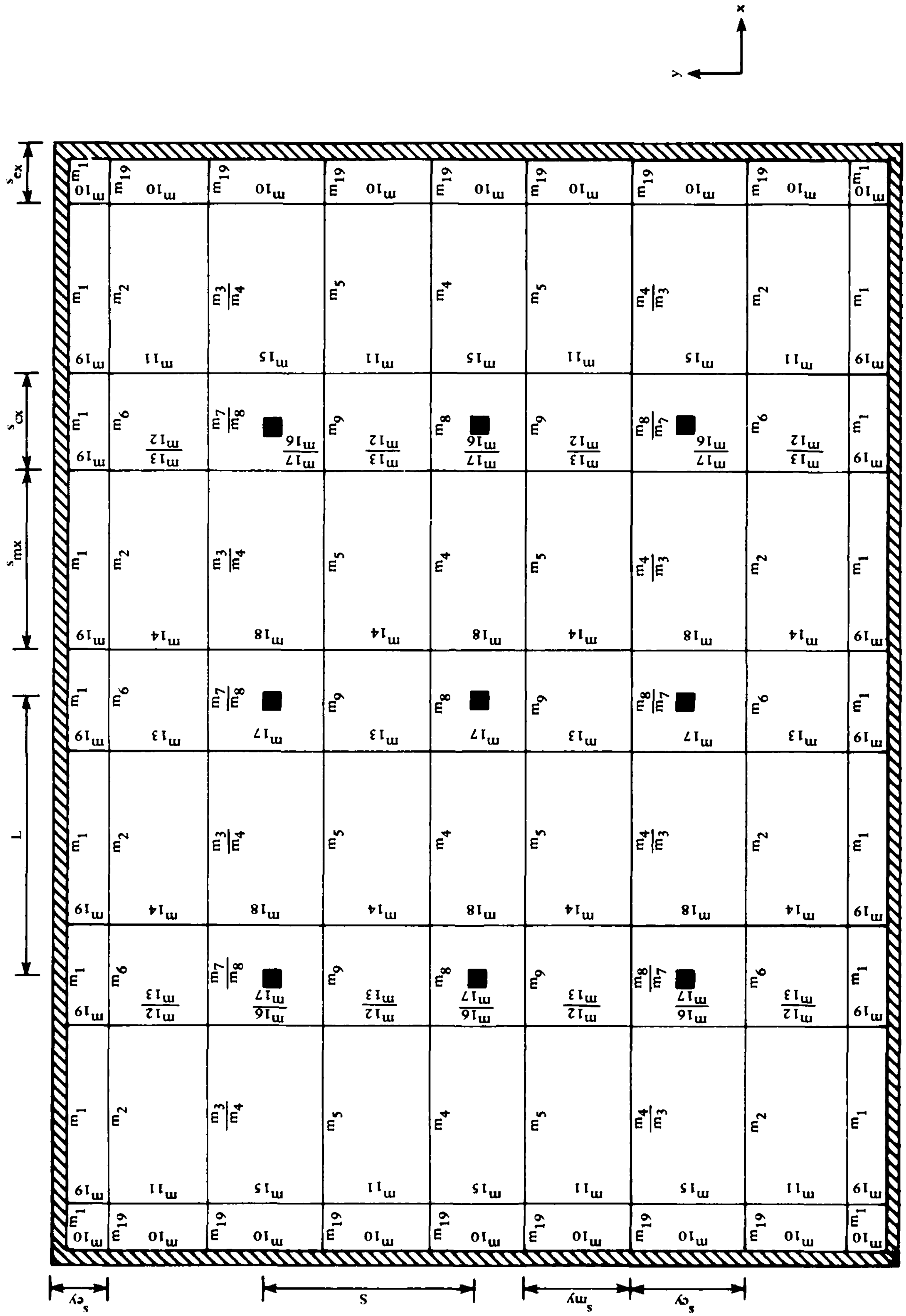


Figure A-4(a). Flat slab: 4 x 4.

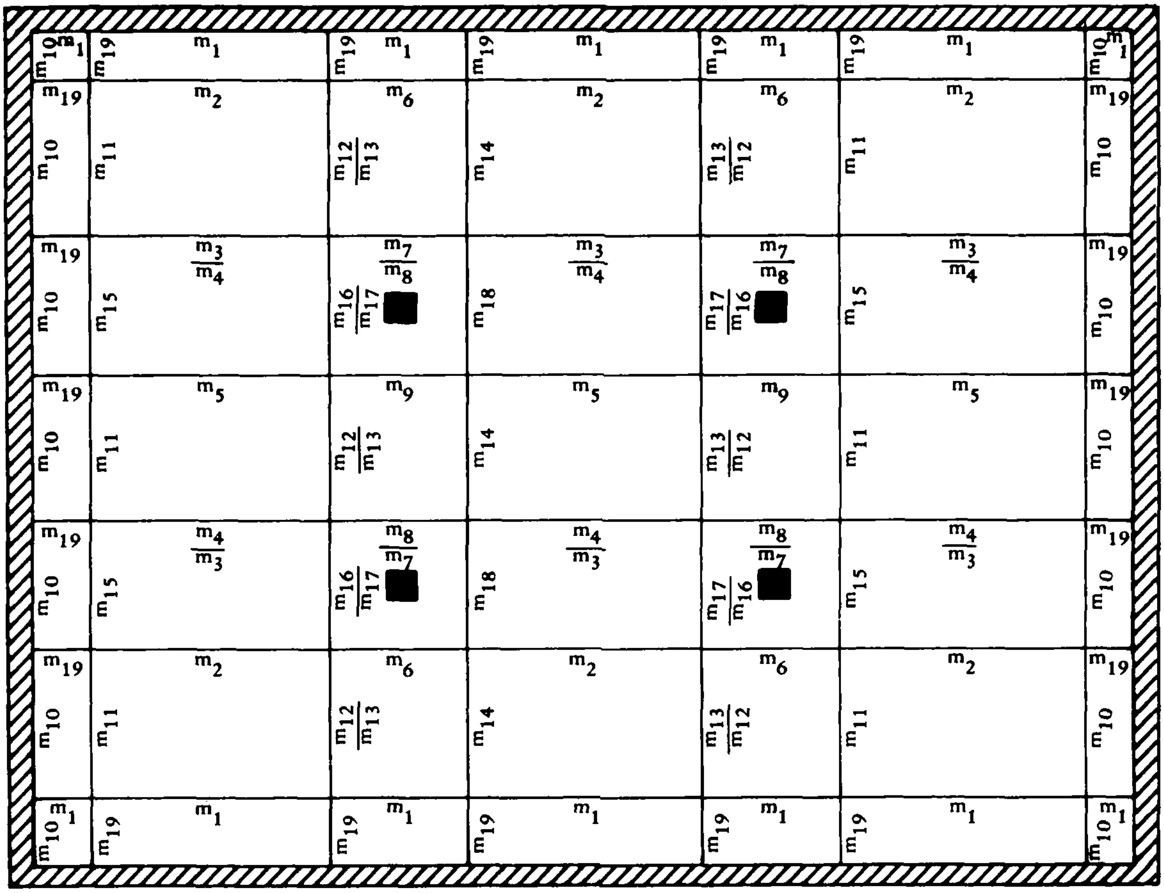


Figure A-4(b). Flat slab: 3 x 3.

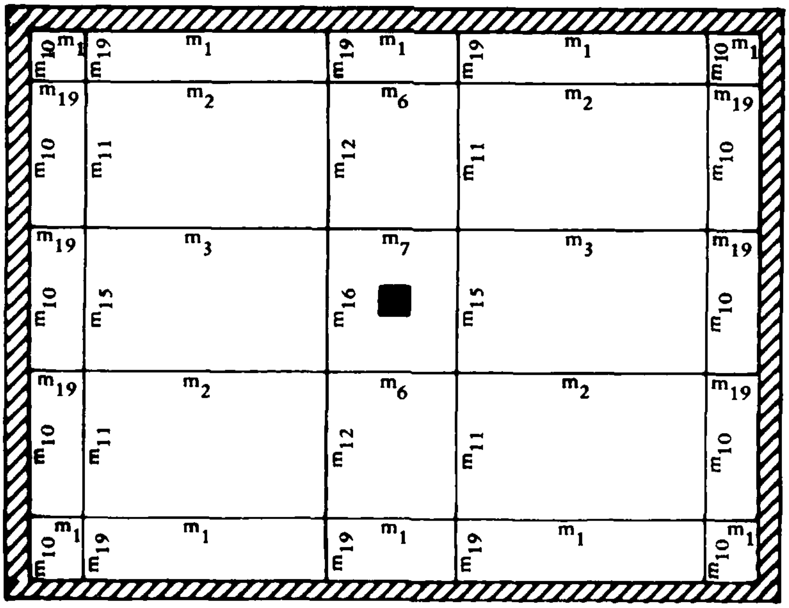


Figure A-4(c). Flat slab: 2 x 2.

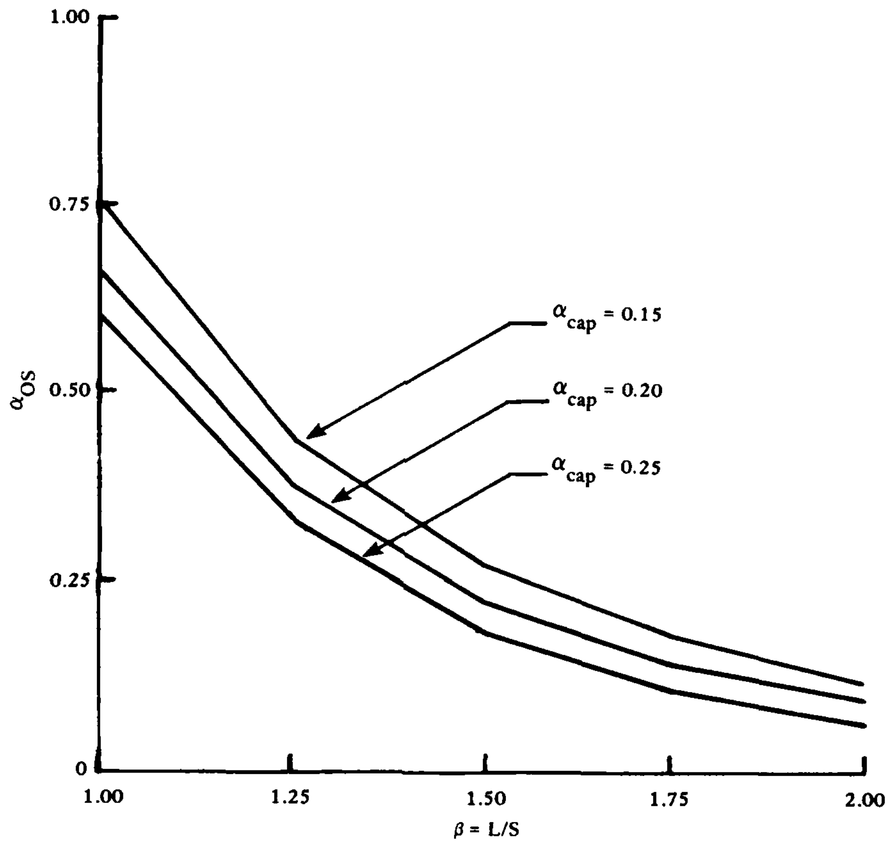


Figure A-5. Values of α_{OS}.

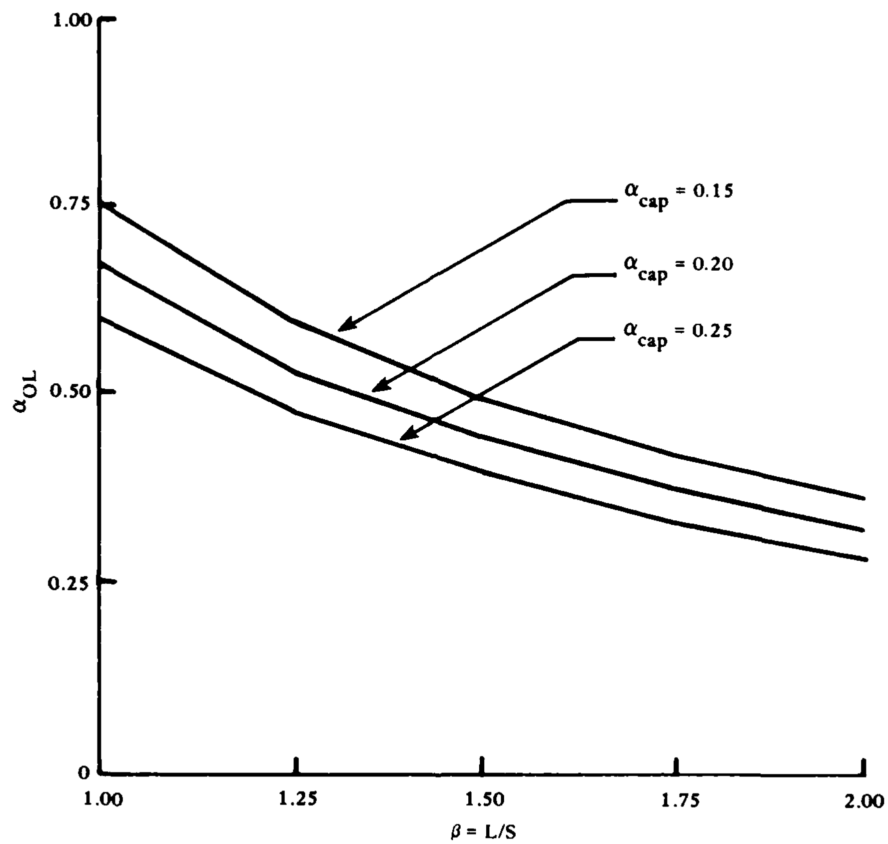


Figure A-6. Values of α_{OL}.

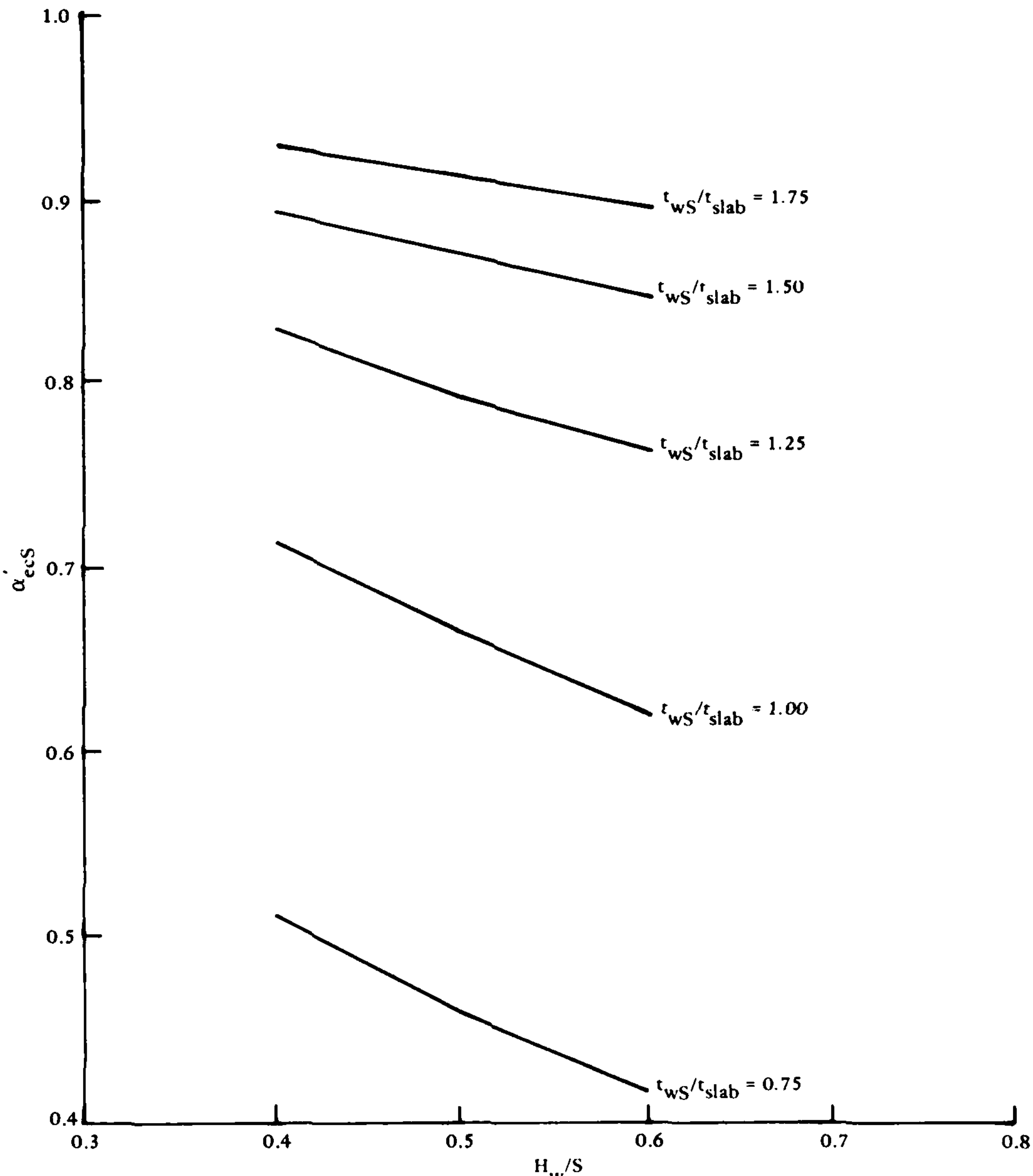


Figure A-7. Values of α'_{ecS}.

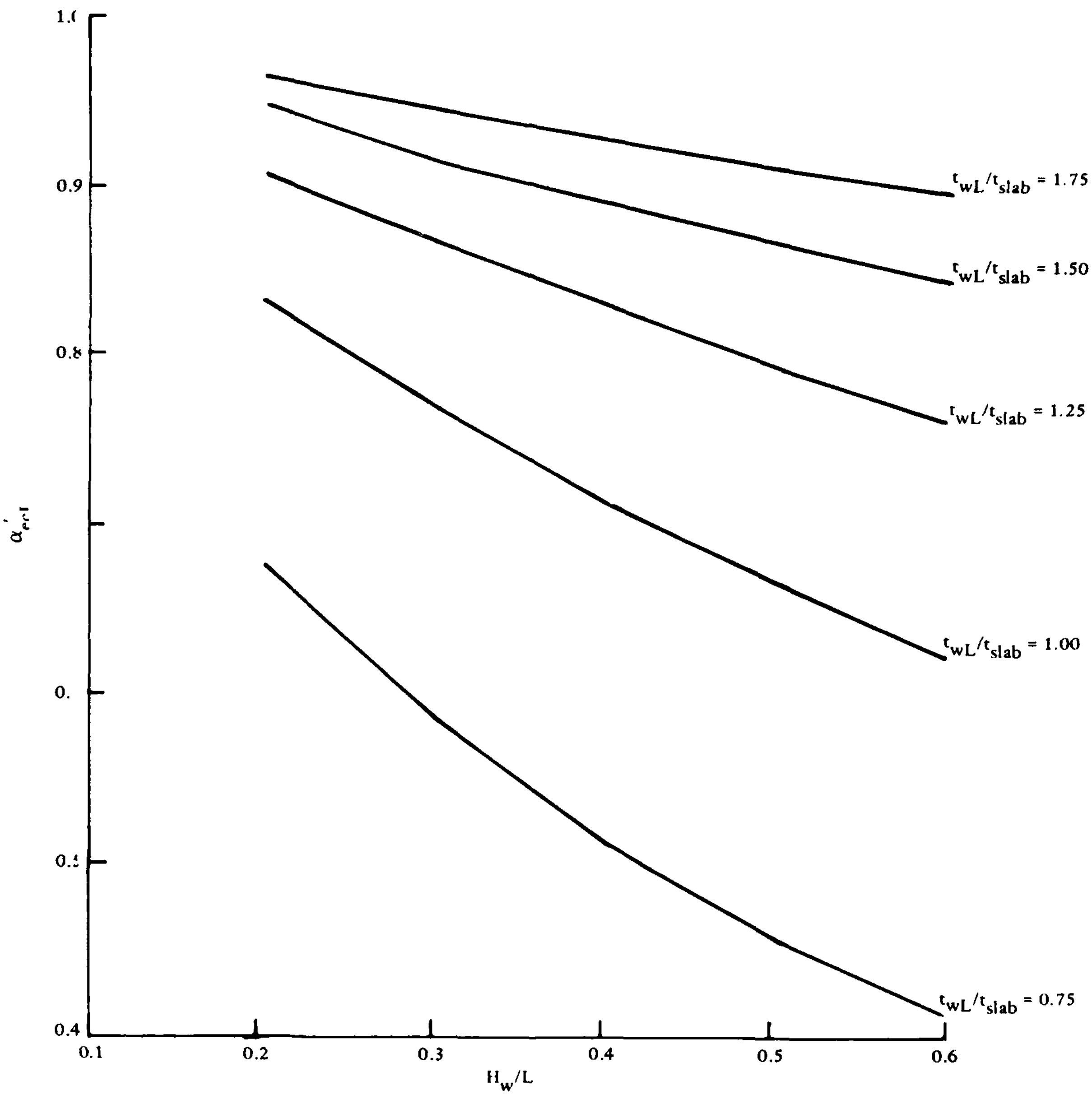


Figure A-8. Values of α'_{ecL}.

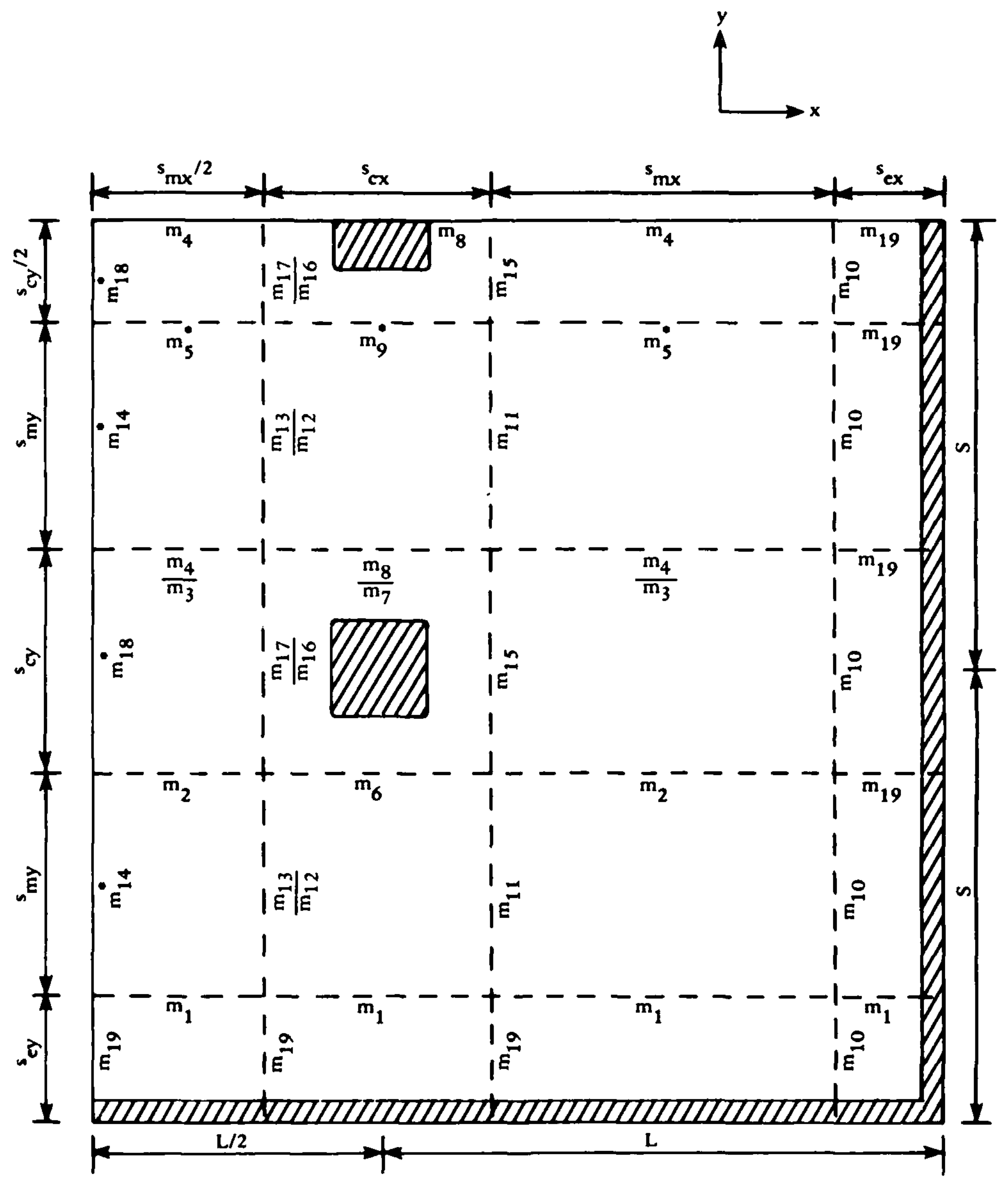


Figure A-9. Unit moment distribution for 3 x 4 flat slab.

Appendix B

ULTIMATE UNIT FLEXURAL RESISTANCE FOR FLAT SLAB

INTRODUCTION

The ultimate unit flexural resistance is the static uniform pressure load, r_u (psi), that a structural element can sustain during plastic yielding of its collapse mechanism. The ultimate uniform resistance is a function of the amount and distribution of the reinforcement (i.e., moment capacity of the slab strips), the geometry of the slab, and the support conditions. A yield-line analysis is used to determine r_u in terms of these parameters.

Because of the complexity and wide choice of parameter values for flat slab structures, it becomes imperative to develop a general procedure for determining the ultimate resistance of any flat slab configuration. The key to this or any general procedure is in the development of a simple, but efficient, method that can be used for any flat slab. Basically one needs to set up the methodology of calculating both the iuternal and external work for any flat slab configuration (i.e., $1.0 < \beta < 2.0$; various number of spans in either direction; various wall/slab thickness ratios). In this report the calculations were programmed on an HP-41CV.

The procedure is illustrated for 3 x 4 flat slab structures (Figure 11) for which the ACI elastic unit moment distribution has been determined previously in Appendix A. The location and values of the unit moments are shown in Figure B-1 and Table B-1 (duplication of Figure A-9 and Table A-7). The assumed yield-line mechanism is shown in Figure B-2. The unknown distance quantities for this pattern are x, y, and z. The negative yield moment along the walls is assumed to occur at a distance of c/2 from the outer face of the wall (i.e., $c/2 \approx$ wall thickness plus haunch width). This assumption satisfies the clear span equality criterion for interior and exterior spans mentioned in Appendix A (i.e., $L_n = L - c$ or $L_n = S - c$).* The division of the symmetric one-quarter slab into panel types (i.e., interior, I; corner, C; long side, LS; and short side, SS) is shown in Figure B-3. These panels are further divided into rotating sectors (rectangles and quadralaterals) about supports for use in the calculation of external work (see Figure B-4).

EXTERNAL WORK

The external work done by r_u on rotating sector i is:

$$W_i = r_u A_i \Delta_i \quad \text{(B-1)}$$

where: A_i = area of sector i

Δ_i = deflection of the center of gravity (c.g.) of sector i

Δ = maximum deflection of the sector

Each quadralateral sector is further divided into rectangular and triangular sub-sectors. The values of A_i and Δ_i for all sub-sectors are listed in Table B-2. The total external work is the sum of the work done on each sector. That is,

$$W = \sum W_i = \sum r_u A_i \Delta_i \quad \text{(B-2)}$$

The external work is determined separately for each panel type after making the following substitutions:

$x = x' L$

$y = y' L$

$z = z' L$

$S = L/\beta$

$c = 0.89 \alpha_{cap} L$

*If the wall thickness plus haunch width does not equal c/2, one must adjust the exterior span length accordingly. Thus, all the design assumptions remain unaffected.

Computer programs (Table B-3) were written for an HP-31CV to calculate the external work ($r_u \Delta L^2$ is factored out). The parameters stored in the registers are shown in Table B-4. The five dimensionless input parameters for a given yield line analysis are stored in registers 00 through 04. The parameters stored in registers 05 through 20 are obtained from the initialization program INIT (Table B-5). The unit moment coefficients are stored in registers 21 through 39. The sum of the output from the external work programs is designated the coefficient of external work, α_{EW}. That is,

$$W = \alpha_{EW} r_u \Delta L^2 \tag{B-3}$$

INTERNAL WORK

The internal work, E_{ij}, for each yield line is the rotational energy done by moment M_n rotating through θ_n. That is:

$$E_{ij} = M_n \theta_n = m_n \theta_n \ell_n = m_x s_y \theta_x + m_y s_x \theta_y \tag{B-4}$$

where: m_x, m_y = ultimate unit moment capacities in the x and y directions

s_y, s_x = lengths of the yield line in the y and x directions over which m_x and m_y apply

θ_x, θ_y = relative rotations about the yield lines in the x and y directions

Equations for the internal work are developed separately for each panel type. The equations are in general terms such that they can be employed in the design process of any flat slab configuration. Table B-6 was developed at NCEL from Figures B-1 and B-2 to show the parameters involved in the internal work calculations for all the yield lines occurring in each panel type. The absolute and dimensionless values of the lengths and rotational angles are listed in Table B-7. Computer programs were written for an HP-31CV to calculate the internal work ($m_e \Delta$ is factored out) for each panel type. The necessary parameters stored in the calculator registers (obtained from program INIT) are listed in Table B-4. The internal work program listings are shown in Table B-8. The sum of the output from these programs is designated the coefficient of internal work, α_{IW}. That is,

$$E = \Sigma E_{ij} = \Sigma m_n \theta_n \ell_n$$

Eventually

$$E = m_e \Delta \alpha_{IW} \tag{B-5}$$

SOLUTION OF ENERGY EQUATION

The total external work for all panels is set equal to the total internal work:

$$W = E \tag{B-6}$$

or

$$r_u L^2 \Delta \alpha_{EW} = m_e \Delta \alpha_{IW} \tag{B-7}$$

Therefore,

$$r_u = \frac{m_e \Delta \alpha_{IW}}{L^2 \Delta \alpha_{EW}} = \frac{m_e}{L^2} \frac{\alpha_{IW}}{\alpha_{EW}} = \alpha_{ru} \frac{m_e}{L^2} \tag{B-8}$$

Usually, α_{cap} and β are given, and the solution involves varying x', y', and z' independently until $r_u/(m_e/L^2)$ is minimized. This minimum solution provides both the failure mechanism and the value of the ultimate resistance. To simplify and shorten this iterative procedure, the positive yield line (rst) between columns is initially located at the mid-point. That is, let:

$$z = \frac{1}{2}(S - c) \tag{B-9}$$

Substituting yields:

$$z = \frac{1}{2}\left(\frac{L}{\beta} - 0.89\ \alpha_{cap}\ L\right) \tag{B-10}$$

or

$$z' = \frac{1}{2}\left(\frac{1}{\beta} - 0.89\ \alpha_{cap}\right) \tag{B-11}$$

The iterative minimization solution process is then employed to determine the appropriate values of x' and y'. The process is then repeated for other values of z' until the $r_u/(m_e/L^2)$ expression is minimized. According to yield-line theory, the positive yield line (rst) will move towards the column with the smallest negative y-moment (m_y) capacity (either line vwx or opq).

EXAMPLE PROBLEM

Calculations were made in this section to determine the yield-line locations and the ultimate resistance expression for the 3 x 4 flat slab. Equation B-11 was used to initially establish a value for z'. That is,

$$z' = \frac{1}{2}\left(\frac{1}{\beta} - 0.89\ \alpha_{cap}\right) = \frac{1}{2}\left[\frac{1}{1.25} - (0.89)(0.20)\right] = 0.311$$

A number of calculations were then made for various values of x' and y'. These results are listed in Tables B-9, B-10, and B-11 and plotted in Figure B-5. The minimum solution is:

$$\alpha_{ru} = 10.102$$

$$x' = 0.40$$

$$y' = 0.30$$

Substituting into Equation B-8:

$$(r_u)_{min} = 10.102 \left(\frac{m_e}{L_2}\right) \tag{B-12}$$

To obtain the actual minimum resistance, z' was then varied while keeping x' and y' fixed at 0.4 and 0.3, respectively. The external work coefficient, α_{EW}, is unaffected by a change in the z' value. Therefore, it remained fixed at 1.1531 (see Table B-9). The calculations for the internal work coefficient are listed in Table B-12. It was necessary to carry out the calculations to six significant figures in order to detect a change in the z' value associated with the minimum resistance. Therefore, for most flat slabs, a satisfactory value for z' can be obtained directly from Equation B-11. That is, it is not necessary to employ the iterization procedure for other values of z'.

This completes Step 2 of the Design Procedure (determination of ultimate resistance relationship). The actual required absolute value of r_u is not obtained until Step 5 (dynamic SDOF analysis). Equation B-12 is then used to calculate an absolute value of m_e. That is,

$$m_e = \frac{r_u L^2}{\alpha_{ru}} \tag{B-13}$$

The individual values of all the unit moments are then determined using Equation A-11 (i.e., $m_n = \alpha_{um} m_e$) in conjunction with the value of the unit moment coefficients (α_{um}) listed in Table B-1.

UNIT MOMENT READJUSTMENT

In Steps 6 and 11 of the Design Procedure, a check on the minimum steel percentage is made. In some cases (usually in middle bands), the required steel percentages are less than the specified ACI minimum (p_{min}), and these steel percentages must be increased. If so, then the original unit moment coefficients for these sections must also be increased. The following expression is the minimum unit moment coefficient that can occur:

$$(\alpha_{um})_{min} = \frac{p_{min}\ b\ d^2\ f_s}{m_e} \qquad \text{(B-14)}$$

This new value is then used in another yield-line analysis. Note that the values of the external work coefficients (see Table B-10) remain unchanged. Only the internal work calculations are affected.

As an example, suppose that m_2, m_3, m_4, and m_5 were too low and had to be increased so that $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.120$. If these new unit moment coefficients were introduced into the internal work calculations, the values in Tables B-14 and B-15 would result. As can be seen, the minimum resistance still occurs at x' = 0.4 and y' = 0.3. However, α_{ru} increases from 10.102 to 10.363. Since the absolute value of the required r_u remains unchanged, the required m_e value (calculated from Equation B-12) decreases from $0.0990\ r_u\ L^2$ to $0.0965\ r_u\ L^2$. This results in an overall 2.5 percent decrease in the absolute values of the unadjusted unit moments (i.e., m_1, m_6 through m_{18}). Engineering intuition would have predicted this effect.

Table B-1. Values of Unit Moment Coefficients (β = 1.25; α_{cap} = 0.20; t_{wS}/t_{slab} = t_{wL}/t_{slab} = 1.00; H_w/S = 0.5)

Unit Moment, m_n	Unit Moment Coefficient, α_{um}	Adjusted Positive Unit Moment Coefficient, $(\alpha_{um})_{min}$
m_1	0.168	-----
m_2	0.114	-----
m_3	0.111	-----
m_4	0.105	-----
m_5	0.090	0.087
m_6	0.257	-----
m_7	0.496	-----
m_8	0.472	-----
m_9	0.203	0.191
m_{10}	0.314	-----
m_{11}	0.233	-----
m_{12}	0.230	-----
m_{13}	0.220	-----
m_{14}	0.189	0.179
m_{15}	0.349	-----
m_{16}	0.689	-----
m_{17}	0.660	-----
m_{18}	0.284	0.255
m_{19}	miniumum	-----

Note: $m_n = \alpha_{um} m_e$, $m_e = w L^2/8$

Table B-2. External Work Parameters

Panel Type	Sector	Sub-Sector			
		Rectangular		Triangular	
		Area, A_i	Deflection, Δ_i	Area, A_i	Deflection, Δ_i
Corner (C)	C-1	(c/2) (L - c - x)	(1/2) Δ	(1/2) (S - c - y) (L - c - x)	(2/3) Δ
	C-2	[S - (c/2) - y] (x)	(1/2) Δ	(1/2) (y) (x)	(1/3) Δ
	C-3	(c/2) (S - c - y)	(1/2) Δ	(1/2) (L - c - x) (S - c - y)	(2/3) Δ
	C-4	[L - (c/2) - x] (y)	(1/2) Δ	(1/2) (x) (y)	(1/3) Δ
Interior (I/A)	I/A-1	(c/2) (z)	(1/2) Δ	(1/2) [(L/2) - (c/2)] (z)	(2/3) Δ
	I/A-2	(c/2) [(L/2) - (c/2)]	(1/2) Δ	(1/2) (z) [(L/2) - (c/2)]	(2/3) Δ
Interior (I/B)	I/B-1	(c/2) (S - c - z)	(1/2) Δ	(1/2) [(L/2) - (c/2)] (S - c - z)	(2/3) Δ
	I/B-2	(c/2) [(L/2) - (c/2)]	(1/2) Δ	(1/2) (S - c - z) [(L/2) - (c/2)]	(2/3) Δ
Short Side (SS)	SS-1	(c/2) [(L/2) - (c/2)]	(1/2) Δ	(1/2) (S - c - y) [(L/2) - (c/2)]	(2/3) Δ
	SS-2	(c/2) (S - c - y)	(1/2) Δ	(1/2) [(L/2) - (c/2)] (S - c - y)	(2/3) Δ
	SS-3	(L/2) (y)	(1/2) Δ	---	---
Long Side (LS/A)	LS/A-1	(c/2) (z)	(1/2) Δ	(1/2) (L - c - x) (z)	(2/3) Δ
	LS/A-2	(c/2) (L - c - x)	(1/2) Δ	(1/2) (z) (L - c - x)	(2/3) Δ
	LS/A-3	[z + (c/2)] (x)	(1/2) Δ	---	---
Long Side (LS/B)	LS/B-1	(c/2) (S - c - z)	(1/2) Δ	(1/2) (L - c - x) (S - c - z)	(2/3) Δ
	LS/B-2	(c/2) (L - c - x)	(1/2) Δ	(1/2) (S - c - z) (L - c - x)	(2/3) Δ
	LS/B-3	[S - (c/2) - z] (x)	(1/2) Δ	---	---

Table B-3. External Work Program Listings

a. Corner (C)

```
01*LBL "EC"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 10
07 *
08 3
09 1/X
10 RCL 12
11 *
12 RCL 10
13 *
14 +
15 2
16 1/X
17 RCL 13
18 *
19 RCL 00
20 *
21 +
22 6
23 1/X
24 RCL 01
25 *
26 RCL 00
27 *
28 +
29 2
30 1/X
31 RCL 07
32 *
33 RCL 12
34 *
35 +
36 3
37 1/X
38 RCL 10
39 *
40 RCL 12
41 *
42 +
43 2
44 1/X
45 RCL 11
46 *
47 RCL 01
48 *
49 +
50 6
51 1/X
52 RCL 00
53 *
54 RCL 01
55 *
56 +
57 END
```

b. Interior (I/A)

```
01*LBL "EIA"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 02
07 *
08 2
09 ENTER↑
10 3
11 /
12 RCL 20
13 *
14 RCL 02
15 *
16 +
17 2
18 1/X
19 RCL 07
20 *
21 RCL 20
22 *
23 +
24 END
```

c. Interior (I/B)

```
01*LBL "EIB"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 14
07 *
08 2
09 ENTER↑
10 3
11 /
12 RCL 20
13 *
14 RCL 14
15 *
16 +
17 2
18 1/X
19 RCL 07
20 *
21 RCL 20
22 *
23 +
24 END
```

(continued)

Table B-3. Continued

d. Short Side (SS)

```
01*LBL "ESS"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 20
07 *
08 2
09 ENTER↑
10 3
11 /
12 RCL 12
13 *
14 RCL 20
15 *
16 +
17 2
18 1/X
19 RCL 07
20 *
21 RCL 12
22 *
23 +
24 2
25 1/X
26 2
27 /
28 RCL 01
29 *
30 +
31 END
```

e. Long Side (LS/A)

```
01*LBL "ELSA"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 20
07 *
08 2
09 ENTER↑
10 3
11 /
12 RCL 10
13 *
14 RCL 02
15 *
16 +
17 2
18 1/X
19 RCL 07
20 *
21 RCL 10
22 *
23 +
24 2
25 1/X
26 RCL 18
27 *
28 RCL 00
29 *
30 +
31 END
```

f. Long Side (LS/B)

```
01*LBL "ELSB"
02 2
03 1/X
04 RCL 07
05 *
06 RCL 14
07 *
08 2
09 ENTER↑
10 3
11 /
12 RCL 10
13 *
14 RCL 14
15 *
16 +
17 2
18 1/X
19 RCL 07
20 *
21 RCL 10
22 *
23 +
24 2
25 1/X
26 RCL 15
27 *
28 RCL 00
29 *
30 +
31 END
```

Table B-4. HP-31CV Storage for Flat Slab Yield-Line Analysis

a. Input Parameters

Absolute Value	Dimensionless Value	Register
x	x'	00
y	y'	01
z	z'	02
1/S	β	03
d*	α_{cap}	04

*c = 0.89 d

(continued)

Table B-4. Continued

b. Calculated Parameters

Absolute Value	Dimensionless Value	Register
S	$1/\beta$	05
c	$0.89\ \alpha_{cap}$	06
c/2	$0.445\ \alpha_{cap}$	07
L-c	$1 - 0.89\ \alpha_{cap}$	08
S-c	$(1/\beta) - 0.89\ \alpha_{cap}$	09
L - c - x	$1 - 0.89\ \alpha_{cap} - x'$	10
L - (c/2) - x	$1 - 0.445\ \alpha_{cap} - x'$	11
S - c - y	$(1/\beta) - 0.89\ \alpha_{cap} - y'$	12
S - (c/2) - y	$(1/\beta) - 0.445\ \alpha_{cap} - y'$	13
S - c - z	$(1/\beta) - 0.89\ \alpha_{cap} - z'$	14
S - (c/2) - z	$(1/\beta) - 0.445\ \alpha_{cap} - z'$	15
x + (c/2)	$x' + 0.445\ \alpha_{cap}$	16
y + (c/2)	$y' + 0.445\ \alpha_{cap}$	17
z + (c/2)	$z' + 0.445\ \alpha_{cap}$	18
L - (S/2)	$1 - (1/2\beta)$	19
(L - c)/2	$(1 - 0.89\ \alpha_{cap})/2$	20

(continued)

Table B-4. Continued

c. Unit Moment Coefficients, α_{um}

Unit Moment, m_n	Register Number	Unit Moment, m_n	Register Number
m_1	21	m_{11}	31
m_2	22	m_{12}	32
m_3	23	m_{13}	33
m_4	24	m_{14}	34
m_5	25	m_{15}	35
m_6	26	m_{16}	36
m_7	27	m_{17}	37
m_8	28	m_{18}	38
m_9	29	m_{19}	39
m_{10}	30	---	--

Note: $m_n = \alpha_{um} m_e$

Table B-5. Program to Initialize HP-31CV Storage Registers

```
01*LBL "INIT"
02 RCL 03
03 1/X
04 STO 05
05 RCL 04
06 .89
07 *
08 STO 06
09 RCL 06
10 2
11 /
12 STO 07
13 1
14 ENTER↑
15 RCL 06
16 -
17 STO 08
18 RCL 05
19 RCL 06
20 -
21 STO 09
22 RCL 08
23 RCL 00
24 -
25 STO 10
26 RCL 07
27 CHS
28 1
29 +
30 RCL 00
31 -
32 STO 11
33 RCL 09
34 RCL 01
35 -
36 STO 12
37 RCL 07
38 CHS
39 RCL 05
40 +
41 RCL 01
42 -
43 STO 13
44 RCL 09
45 RCL 02
46 -
47 STO 14
48 RCL 05
49 RCL 07
50 -
51 RCL 02
52 -
53 STO 15
54 RCL 00
55 RCL 07
56 +
57 STO 16
58 RCL 01
59 RCL 07
60 +
61 STO 17
62 RCL 02
63 RCL 07
64 +
65 STO 18
66 1
67 ENTER↑
68 RCL 05
69 2
70 /
71 -
72 STO 19
73 RCL 08
74 2
75 /
76 STO 20
77 END
```

Table B-6a. Internal Work Parameters, Corner

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	if	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_B	m_{11}	$S - y - (s_{cy}/2) - (c/2)$	θ_B	-	-	-
	fc	-	-	-	m_{11}	$y + (c/2) - s_{ey}$	θ_C	m_{10}	$s_{ey} - (c/2)$	θ_C
	fm	m_{15}	$s_{cy}/2$	$\theta_B + \theta_C$	m_{11}	$S - y - (s_{cy}/2) - (c/2)$	$\theta_B + \theta_C$	-	-	-
	cn	m_{10}	$s_{cy}/2$	θ_C	m_{10}	s_{my}	θ_C	m_{10}	$s_{ey} - (c/2)$	θ_C
	iℓ	m_{17}/m_{16}	$c/2$	θ_B	-	-	-	-	-	-
y	bc	m_1	$s_{cx}/2$	θ_D	m_1	s_{mx}	θ_D	m_1	$s_{ex} - (c/2)$	θ_D
	ef	m_6	$s_{cx}/2$	$\theta_D + \theta_E$	m_2	$L - x - (s_{cx}/2) - (c/2)$	$\theta_D + \theta_E$	-	-	-
	ni	m_8/m_7	$c/2$	θ_E	-	-	-	-	-	-
	if	m_8/m_7	$(s_{cx}/2) - (c/2)$	θ_E	m_2	$L - x - (s_{cx}/2) - (c/2)$	θ_E	-	-	-
	fc	-	-	-	m_2	$x + (c/2) - s_{ex}$	θ_D	m_1	$s_{ex} - (c/2)$	θ_D

NOTE: The m_n which is circled is the largest, m_n^* must be adjusted (decreased) accordingly.

Table B-6b. Internal Work Parameters, Interior (I/A)

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	ko	m_{17}/m_{16}	$c/2$	θ_A	-	-	-	-	-	-
	jr	m^*_{18}	$s_{cy}/2$	θ_A	m^*_{14}	$z + (c/2) - (s_{cy}/2)$	θ_A	-	-	-
	ro	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_A	m^*_{14}	$z + (c/2) - (s_{cy}/2)$	θ_A	-	-	-
y	op	m_8/m_7	$c/2$	θ_F	-	-	-	-	-	-
	rs	m^*_9	$s_{cx}/2$	θ_F	m^*_5	$s_{mx}/2$	θ_F	-	-	-
	ro	m_8/m_7	$(s_{cx}/2) - (c/2)$	θ_F	m^*_5	$s_{mx}/2$	θ_F	-	-	-

Table B-6c. Internal Work Parameters, Interior (I/B)

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	zv	m_{17}/m_{16}	$c/2$	θ_A	-	-	-	-	-	-
	yr	m^*_{18}	$s_{cy}/2$	θ_A	m^*_{14}	$S - z - (c/2) - (s_{cy}/2)$	θ_A	-	-	-
	rv	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_A	m^*_{14}	$S - z - (c/2) - (s_{cy}/2)$	θ_A	-	-	-
y	vw	m_8	$c/2$	θ_G	-	-	-	-	-	-
	rs	m^*_9	$s_{cx}/2$	θ_G	m^*_5	$s_{mx}/2$	θ_G	-	-	-
	rv	m_8	$(s_{cx}/2) - (c/2)$	θ_G	m^*_5	$s_{mx}/2$	θ_G	-	-	-

Table B-6d. Internal Work Parameters, Short Side (SS)

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	gk	m_{17}/m_{16}	$c/2$	θ_A	-	-	-	-	-	-
	dg	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_A	m^*_{14}	$S - y - (s_{cy}/2) - (c/2)$	θ_A	-	-	-
	dj	m^*_{18}	$s_{cy}/2$	θ_A	m^*_{14}	$S - y - (s_{cy}/2) - (c/2)$	θ_A	-	-	-
y	gh	m_8/m_7	$c/2$	θ_E	-	-	-	-	-	-
	de	m_6	$s_{cx}/2$	$\theta_D + \theta_E$	m_2	$s_{mx}/2$	$\theta_D + \theta_E$	-	-	-
	ab	m_1	$s_{cx}/2$	θ_D	m_1	$s_{mx}/2$	θ_D	-	-	-
	dg	m_8/m_7	$(s_{cx}/2) - (c/2)$	θ_E	m_2	$s_{mx}/2$	θ_E	-	-	-

Table B-6e. Internal Work Parameters, Long Side (LS/A)

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	ℓq	m_{17}/m_{16}	$c/2$	θ_B	-	-	-	-	-	-
	mt	m_{15}	$s_{cy}/2$	$\theta_B + \theta_C$	m_{11}	$z + (c/2) - (s_{cy}/2)$	$\theta_B + \theta_C$	-	-	-
	nu	m_{10}	$s_{cy}/2$	θ_C	m_{10}	$z + (c/2) - (s_{cy}/2)$	θ_C	-	-	-
	qt	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_B	m_{11}	$z + (c/2) - (s_{cy}/2)$	θ_B	-	-	-
y	pq	m_8/m_7	$c/2$	θ_F	-	-	-	-	-	-
	st	m_9^*	$s_{cx}/2$	θ_F	m_5^*	$L - x - (s_{cx}/2) - (c/2)$	θ_F	-	-	-
	qt	m_8/m_7	$(s_{cx}/2) - (c/2)$	θ_F	m_5^*	$L - x - (s_{cx}/2) - (c/2)$	θ_F	-	-	-

Table B-6f. Internal Work Parameters, Long Side (LS/B)

Moment Direction	Yield Line	Column Band			Middle Band			Exterior Wall Band		
		Unit Moment	Length	Rotation	Unit Moment	Length	Rotation	Unit Moment	Length	Rotation
x	xa'	m_{17}/m_{16}	$c/2$	θ_B	-	-	-	-	-	-
	b't	m_{15}	$s_{cy}/2$	$\theta_B + \theta_C$	m_{11}	$S - z - (c/2) - (s_{cy}/2)$	$\theta_B + \theta_C$	-	-	-
	c'u	m_{10}	$s_{cy}/2$	θ_C	m_{10}	$S - z - (c/2) - (s_{cy}/2)$	θ_C	-	-	-
	xt	m_{17}/m_{16}	$(s_{cy}/2) - (c/2)$	θ_B	m_{11}	$S - z + (c/2) - (s_{cy}/2)$	θ_B	-	-	-
y	wx	m_8	$c/2$	θ_G	-	-	-	-	-	-
	st	m_9^*	$s_{cx}/2$	θ_G	m_5^*	$L - x - (s_{cx}/2) - (c/2)$	θ_G	-	-	-
	xt	m_8	$(s_{cx}/2) - (c/2)$	θ_G	m_5^*	$L - x - (s_{cx}/2) - (c/2)$	θ_G	-	-	-

Table B-7. Internal Work Parameters

Expression	Absolute Value	Dimensionless Value	Storage Registers[a]
θ_A	$2\Delta/(L - c)$	$2/(1 - 0.89\ \alpha_{cap})$	1/ 20
θ_B	$\Delta/(L - c - x)$	$1/(1 - 0.89\ \alpha_{cap} - x')$	1/ 10
θ_C	Δ/x	$1/x'$	1/ 00
θ_D	Δ/y	$1/y'$	1/ 01
θ_E	$\Delta/(S - c - y)$	$1/[(1/\beta) - 0.89\ \alpha_{cap} - y']$	1/ 12
θ_F	Δ/z	$1/z'$	1/ 02
θ_G	$\Delta/(S - c - z)$	$1/[(1/\beta) - 0.89\ \alpha_{cap} - z']$	1/ 14
$c/2$	$c/2$	$0.445\ \alpha_{cap}$	07
$s_{cy}/2$	$S/4$	$1/4\beta$	05 /4
$(s_{cy}/2) - (c/2)$	$(S/4) - (c/2)$	$(1/4\beta) - 0.445\ \alpha_{cap}$	(05 /4) - 07
s_{my}	$S/2$	$1/2\beta$	05 /2
$s_{my}/2$	$S/4$	$1/4\beta$	05 /4
$s_{ey} - (c/2)$	$(S/4) - (c/2)$	$(1/4\beta) - 0.445\ \alpha_{cap}$	(05 /4) - 07
$S - (c/2) - y - (s_{cy}/2)$	$S - (c/2) - y - (S/4)$	$(1/\beta) - 0.445\ \alpha_{cap} - y' - (1/4\beta)$	13 - (05 /4)
$y + (c/2) - s_{ey}$	$y + (c/2) - (S/4)$	$y' + 0.445\ \alpha_{cap} - (1/4\beta)$	17 - (05 /4)

(continued)

Table B-7. Continued

Expression	Absolute Value	Dimensionless Value	Storage Registers[a]
$s_{cx}/2$	$S/4$	$1/4\beta$	$05/4$
$(s_{cx}/2) - (c/2)$	$(S/4) - (c/2)$	$(1/4\beta) - 0.445\ \alpha_{cap}$	$(05/4) - 07$
s_{mx}	$L - (S/2)$	$1 - (1/2\beta)$	19
$s_{mx}/2$	$[L - (S/2)]/2$	$[1 - (1/2\beta)]/2$	$19/2$
$s_{ex} - (c/2)$	$(S/4) - (c/2)$	$(1/4\beta) - 0.445\ \alpha_{cap}$	$(05/4) - 07$
$L - (c/2) - x - (s_{cx}/2)$	$L - (c/2) - x - (S/4)$	$1 - 0.445\ \alpha_{cap} - x' - (1/4\beta)$	$11 - (05/4)$
$x + (c/2) - s_{ex}$	$x + (c/2) - (S/4)$	$x' + 0.445\ \alpha_{cap} - (1/4\beta)$	$16 - (05/4)$
$z + (c/2) - (s_{cy}/2)$	$z + (c/2) - (S/4)$	$z' + 0.445\ \alpha_{cap} - (1/4\beta)$	$18 - (05/4)$
$S - (c/2) - z - (s_{cy}/2)$	$S - (c/2) - z - (S/4)$	$(1/\beta) - 0.445\ \alpha_{cap} - z' - (1/4\beta)$	$15 - (05/4)$

[a] Circled numbers are storage registers as identified on Table B-4.

Table B-8. Internal Work Program Listings

a. Corner (C)

```
01*LBL "IC"
02 RCL 05
03 4
04 /
05 RCL 19
06 +
07 RCL 05
08 2
09 /
10 +
11 RCL 07
12 2
13 *
14 -
15 RCL 01
16 /
17 RCL 21
18 *
19 RCL 11
20 RCL 05
21 4
22 /
23 -
24 RCL 01
25 /
26 RCL 22
27 *
28 +
29 RCL 11
30 RCL 05
31 4
32 /
33 -
34 RCL 12
35 /
36 2
37 *
38 RCL 22
39 *
40 +
41 RCL 16
42 RCL 05
43 4
44 /
45 -
46 RCL 01
47 /
48 RCL 22
49 *
50 +
51 RCL 01
52 1/X
53 RCL 12
54 1/X
55 +
56 RCL 05
57 *
58 4
59 /
60 RCL 26
61 *
62 +
63 RCL 05
64 4
65 /
66 RCL 12
67 /
68 RCL 27
69 *
70 +
71 RCL 05
72 4
73 /
74 RCL 00
75 /
76 RCL 30
77 *
78 +
79 RCL 05
80 2
81 /
82 RCL 00
83 /
84 RCL 30
85 *
86 +
87 RCL 05
88 4
89 /
90 RCL 07
91 -
92 RCL 00
93 /
94 2
95 *
96 RCL 30
97 *
98 +
99 RCL 13
100 RCL 05
101 4
102 /
103 -
104 RCL 10
105 /
106 2
107 *
108 RCL 31
109 *
110 +
111 RCL 13
112 RCL 05
113 4
114 /
115 -
116 RCL 00
117 /
118 RCL 31
119 *
120 +
121 RCL 17
122 RCL 05
123 4
124 /
125 -
126 RCL 00
127 /
128 RCL 31
129 *
130 +
131 RCL 10
132 1/X
133 RCL 00
134 1/X
135 +
136 RCL 05
137 *
138 4
139 /
140 RCL 35
141 *
142 +
143 RCL 05
144 4
145 /
146 RCL 10
147 /
148 RCL 36
149 *
150 +
151 END
```

(continued)

Table B-8. Continued

b. Interior (I/A)

```
01*LBL "IIA"
02 RCL 19
03 RCL 02
04 /
05 RCL 25
06 *
07 RCL 05
08 4
09 /
10 RCL 02
11 /
12 RCL 27
13 *
14 +
15 RCL 05
16 4
17 /
18 RCL 02
19 /
20 RCL 29
21 *
22 +
23 RCL 18
24 RCL 05
25 4
26 /
27 -
28 RCL 20
29 /
30 2
31 *
32 RCL 34
33 *
34 +
35 RCL 05
36 4
37 /
38 RCL 20
39 /
40 RCL 36
41 *
42 +
43 RCL 05
44 4
45 /
46 RCL 20
47 /
48 RCL 38
49 *
50 +
51 END
```

c. Interior (I/B)

```
01*LBL "IIB"
02 RCL 19
03 RCL 14
04 /
05 RCL 25
06 *
07 RCL 05
08 4
09 /
10 RCL 14
11 /
12 RCL 28
13 *
14 +
15 RCL 05
16 4
17 /
18 RCL 14
19 /
20 RCL 29
21 *
22 +
23 RCL 15
24 RCL 05
25 4
26 /
27 -
28 RCL 20
29 /
30 2
31 *
32 RCL 34
33 *
34 +
35 RCL 05
36 4
37 /
38 RCL 20
39 /
40 RCL 36
41 *
42 +
43 RCL 05
44 4
45 /
46 RCL 20
47 /
48 RCL 38
49 *
50 +
51 END
```

(continued)

Table B-8. Continued

d. Short Side (SS)

```
01*LBL "ISS"
02 .5
03 ENTER↑
04 RCL 01
05 /
06 RCL 21
07 *
08 RCL 01
09 1/X
10 RCL 12
11 1/X
12 2
13 *
14 +
15 RCL 19
16 *
17 2
18 /
19 RCL 22
20 *
21 +
22 RCL 01
23 1/X
24 RCL 12
25 1/X
26 +
27 RCL 05
28 *
29 4
30 /
31 RCL 26
32 *
33 +
34 RCL 05
35 4
36 /
37 RCL 12
38 /
39 RCL 27
40 *
41 +
42 RCL 13
43 RCL 05
44 4
45 /
46 -
47 RCL 20
48 /
49 2
50 *
51 RCL 34
52 *
53 +
54 RCL 05
55 4
56 /
57 RCL 20
58 /
59 RCL 36
60 *
61 +
62 RCL 05
63 4
64 /
65 RCL 20
66 /
67 RCL 38
68 *
69 +
70 END
```

(continued)

Table B-8. Continued

e. Long Side (LS/A)

```
01*LBL "ILSA"
02 RCL 11
03 RCL 05
04 4
05 /
06 -
07 RCL 02
08 /
09 2
10 *
11 RCL 25
12 *
13 RCL 05
14 4
15 /
16 RCL 02
17 /
18 RCL 27
19 *
20 +
21 RCL 05
22 4
23 /
24 RCL 02
25 /
26 RCL 29
27 *
28 +
29 RCL 05
30 4
31 /
32 RCL 00
33 /
34 RCL 30
35 *
36 +
37 RCL 18
38 RCL 05
39 4
40 /
41 -
42 RCL 00
43 /
44 RCL 30
45 *
46 +
47 RCL 18
48 RCL 05
49 4
50 /
51 -
52 RCL 10
53 /
54 2
55 *
56 RCL 31
57 *
58 +
59 RCL 18
60 RCL 05
61 4
62 /
63 -
64 RCL 00
65 /
66 RCL 31
67 *
68 +
69 RCL 10
70 1/X
71 RCL 00
72 1/X
73 +
74 RCL 05
75 *
76 4
77 /
78 RCL 35
79 *
80 +
81 RCL 05
82 4
83 /
84 RCL 10
85 /
86 RCL 36
87 *
88 +
89 END
```

f. Long Side (LS/B)

```
01*LBL "ILSB"
02 RCL 11
03 RCL 05
04 4
05 /
06 -
07 RCL 14
08 /
09 2
10 *
11 RCL 25
12 *
13 RCL 05
14 4
15 /
16 RCL 14
17 /
18 RCL 28
19 *
20 +
21 RCL 05
22 4
23 /
24 RCL 14
25 /
26 RCL 29
27 *
28 +
29 RCL 05
30 4
31 /
32 RCL 00
33 /
34 RCL 30
35 *
36 +
37 RCL 18
38 RCL 05
39 4
40 /
41 -
42 RCL 00
43 /
44 RCL 30
45 *
46 +
47 RCL 18
48 RCL 05
49 4
50 /
51 -
52 RCL 10
53 /
54 2
55 *
56 RCL 31
57 *
58 +
59 RCL 18
60 RCL 05
61 4
62 /
63 -
64 RCL 00
65 /
66 RCL 31
67 *
68 +
69 RCL 10
70 1/X
71 RCL 00
72 1/X
73 +
74 RCL 05
75 *
76 4
77 /
78 RCL 35
79 *
80 +
81 RCL 05
82 4
83 /
84 RCL 10
85 /
86 RCL 36
87 *
88 +
89 END
```

Table B-9. External Work Calculations for 3 x 4 Flat Slab

x'	y'	z'	Panel Types						Total* External Work Coefficient, α_{EW}
			Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)	
0.35	0.20	0.311	0.3414	0.1173	0.1173	0.2027	0.2027	0.2027	1.1841
	0.25	0.311	0.3346	0.1173	0.1173	0.1993	0.2027	0.2027	1.1739
	0.30	0.311	0.3277	0.1173	0.1173	0.1958	0.2027	0.2027	1.1635
	0.35	0.311	0.3209	0.1173	0.1173	0.1924	0.2027	0.2027	1.1533
	0.40	0.311	0.3140	0.1173	0.1173	0.1890	0.2027	0.2027	1.1430
0.40	0.20	0.311	0.3362	0.1173	0.1173	0.2027	0.2001	0.2001	1.1737
	0.25	0.311	0.3294	0.1173	0.1173	0.1993	0.2001	0.2001	1.1635
	0.30	0.311	0.3225	0.1173	0.1173	0.1958	0.2001	0.2001	1.1531
	0.35	0.311	0.3157	0.1173	0.1173	0.1924	0.2001	0.2001	1.1429
	0.40	0.311	0.3088	0.1173	0.1173	0.1890	0.2001	0.2001	1.1326
0.45	0.20	0.311	0.3311	0.1173	0.1173	0.2027	0.1975	0.1975	1.1634
	0.25	0.311	0.3242	0.1173	0.1173	0.1993	0.1975	0.1975	1.1531
	0.30	0.311	0.3174	0.1173	0.1173	0.1958	0.1975	0.1975	1.1428
	0.35	0.311	0.3105	0.1173	0.1173	0.1924	0.1975	0.1975	1.1325
	0.40	0.311	0.3037	0.1173	0.1173	0.1890	0.1975	0.1975	1.1223

$$^{*}\alpha_{EW} = \frac{W}{r_u L^2 \Delta}$$

Table B-10. Internal Work Calculations for 3 x 4 Flat Slab

x'	y'	z'	Panel Types						Total* Internal Work Coefficient, α_{IW}
			Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)	
0.35	0.20	0.311	3.9594	1.2432	1.2278	2.0972	1.9725	1.9571	12.4572
	0.25	0.311	3.6928	1.2432	1.2278	1.9538	1.9725	1.9571	12.0472
	0.30	0.311	3.5463	1.2432	1.2278	1.8886	1.9725	1.9571	11.8355
	0.35	0.311	3.4911	1.2432	1.2278	1.8893	1.9725	1.9571	11.7810
	0.40	0.311	3.5305	1.2432	1.2278	1.9665	1.9725	1.9571	11.8976
0.40	0.20	0.311	3.8705	1.2432	1.2278	2.0972	1.9336	1.9182	12.2905
	0.25	0.311	3.5944	1.2432	1.2278	1.9538	1.9336	1.9182	11.8710
	0.30	0.311	3.4373	1.2432	1.2278	1.8886	1.9336	1.9182	11.6487
	0.35	0.311	3.3697	1.2432	1.2278	1.8893	1.9336	1.9182	11.5818
	0.40	0.311	3.3938	1.2432	1.2278	1.9665	1.9336	1.9182	11.6831
0.45	0.20	0.311	3.8388	1.2432	1.2278	2.0972	1.9342	1.9188	12.2600
	0.25	0.311	3.5516	1.2432	1.2278	1.9538	1.9342	1.9188	11.8294
	0.30	0.311	3.3823	1.2432	1.2278	1.8886	1.9342	1.9188	11.5949
	0.35	0.311	3.3008	1.2432	1.2278	1.8893	1.9342	1.9188	11.5141
	0.40	0.311	3.3081	1.2432	1.2278	1.9665	1.9342	1.9188	11.5986

$^*\alpha_{IW} = \frac{E}{m_e \Delta}$

Table B-11. Ultimate Resistance Calculations

x'	y'	z'	α_{IW} *	α_{EW} **	α_{ru} ***
0.35	0.20	0.311	12.457	1.184	10.520
	0.25	0.311	12.047	1.174	10.263
	0.30	0.311	11.836	1.164	10.172
	0.35	0.311	11.781	1.153	10.215
	0.40	0.311	11.898	1.143	10.409
0.40	0.20	0.311	12.291	1.174	10.472
	0.25	0.311	11.871	1.164	10.203
	0.30	0.311	11.649	1.153	10.102
	0.35	0.311	11.582	1.143	10.134
	0.40	0.311	11.683	1.133	10.315
0.45	0.20	0.311	12.260	1.163	10.538
	0.25	0.311	11.829	1.153	10.259
	0.30	0.311	11.595	1.143	10.146
	0.35	0.311	11.514	1.133	10.167
	0.40	0.311	11.599	1.122	10.335

*From Table B-10.

**From Table B-9.

*** $\alpha_{ru} = \dfrac{\alpha_{IW}}{\alpha_{EW}} = \dfrac{r_u}{m_e/L^2}$

Table B-12. Internal Work Calculations for 3 x 4 Flat Slab for x' = 0.40 and y' = 0.30

z'	Panel Types						Total Internal Work Coefficient,* α_{IW}
	Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)	
0.187	3.4373	1.5395	1.1664	1.8886	2.0354	2.0535	12.1207
0.249	3.4373	1.3410	1.1830	1.8886	1.9337	1.9716	11.7552
0.280	3.4373	1.2837	1.2009	1.8886	1.9252	1.9404	11.6761
0.311	3.4373	1.2432	1.2278	1.8886	1.9336	1.9182	11.6487
0.312	3.4373	1.2421	1.2288	1.8886	1.9341	1.9176	11.6485
0.313	3.4373	1.2411	1.2299	1.8886	1.9346	1.9171	11.6486
0.342	3.4373	1.2150	1.2666	1.8886	1.9544	1.9080	11.6699
0.373	3.4373	1.1959	1.3217	1.8886	1.9845	1.9144	11.7424
0.435	3.4373	1.1774	1.5138	1.8886	2.0646	2.0098	12.0915

$$* \ \alpha_{IW} = \frac{E}{m_e \Delta}$$

Table B-13. Internal Work Calculations for 3 x 4 Flat Slab for $(\alpha_{um})_{min} = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.120$

x'	y'	z'	Panel Types						Total Internal Work Coefficient,* α_{IW}
			Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)	
0.35	0.20	0.311	3.9877	1.3069	1.2915	2.1148	2.0491	2.0337	12.7837
	0.25	0.311	3.7188	1.3069	1.2915	1.9707	2.0491	2.0337	12.3707
	0.30	0.311	3.5717	1.3069	1.2915	1.9058	2.0491	2.0337	12.1587
	0.35	0.311	3.5173	1.3069	1.2915	1.9077	2.0491	2.0337	12.1062
	0.40	0.311	3.5590	1.3069	1.2915	1.9873	2.0491	2.0337	12.2275
0.40	0.20	0.311	3.8974	1.3069	1.2915	2.1148	1.9996	1.9842	12.5944
	0.25	0.311	3.6188	1.3069	1.2915	1.9707	1.9996	1.9842	12.1717
	0.30	0.311	3.4609	1.3069	1.2915	1.9058	1.9996	1.9842	11.9489
	0.35	0.311	3.3937	1.3069	1.2915	1.9077	1.9996	1.9842	11.8836
	0.40	0.311	3.4196	1.3069	1.2915	1.9873	1.9996	1.9842	11.9891
0.45	0.20	0.311	3.8642	1.3069	1.2915	2.1148	1.9896	1.9742	12.5412
	0.25	0.311	3.5744	1.3069	1.2915	1.9707	1.9896	1.9742	12.1073
	0.30	0.311	3.4041	1.3069	1.2915	1.9058	1.9896	1.9742	11.8721
	0.35	0.311	3.3226	1.3069	1.2915	1.9077	1.9896	1.9742	11.7925
	0.40	0.311	3.3312	1.3069	1.2915	1.9873	1.9896	1.9742	11.8807

* $\alpha_{IW} = \frac{E}{m_e \Delta}$

Table B-14. Ultimate Resistance Calculations for $(\alpha_{um})_{min} = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0.120$

x'	y'	z'	Coefficient of Internal Work,* α_{IW}	Coefficient of External Work,** α_{EW}	Coefficient of Ultimate Resistance,*** α_{ru}
0.35	0.20	0.311	12.784	1.184	10.797
	0.25	0.311	12.371	1.174	10.537
	0.30	0.311	12.159	1.164	10.446
	0.35	0.311	12.106	1.153	10.500
	0.40	0.311	12.228	1.143	10.698
0.40	0.20	0.311	12.594	1.174	10.727
	0.25	0.311	12.172	1.164	10.457
	0.30	0.311	11.949	1.153	10.363
	0.35	0.311	11.884	1.143	10.397
	0.40	0.311	11.989	1.133	10.582
0.45	0.20	0.311	12.541	1.163	10.783
	0.25	0.311	12.107	1.153	10.500
	0.30	0.311	11.872	1.143	10.387
	0.35	0.311	11.793	1.133	10.409
	0.40	0.311	11.881	1.122	10.589

*From Table B-13.

**From Table B-9.

*** $\alpha_{ru} = \dfrac{\alpha_{IW}}{\alpha_{EW}} = \dfrac{r_u}{m_e/L^2}$

$s_{ex} = \frac{S}{4}$ $\qquad$ $s_{ey} = \frac{S}{4}$

$s_{mx} = L - \frac{S}{2}$ $\qquad$ $s_{my} = \frac{S}{2}$

$s_{cx} = \frac{S}{2}$ $\qquad$ $s_{cy} = \frac{S}{2}$

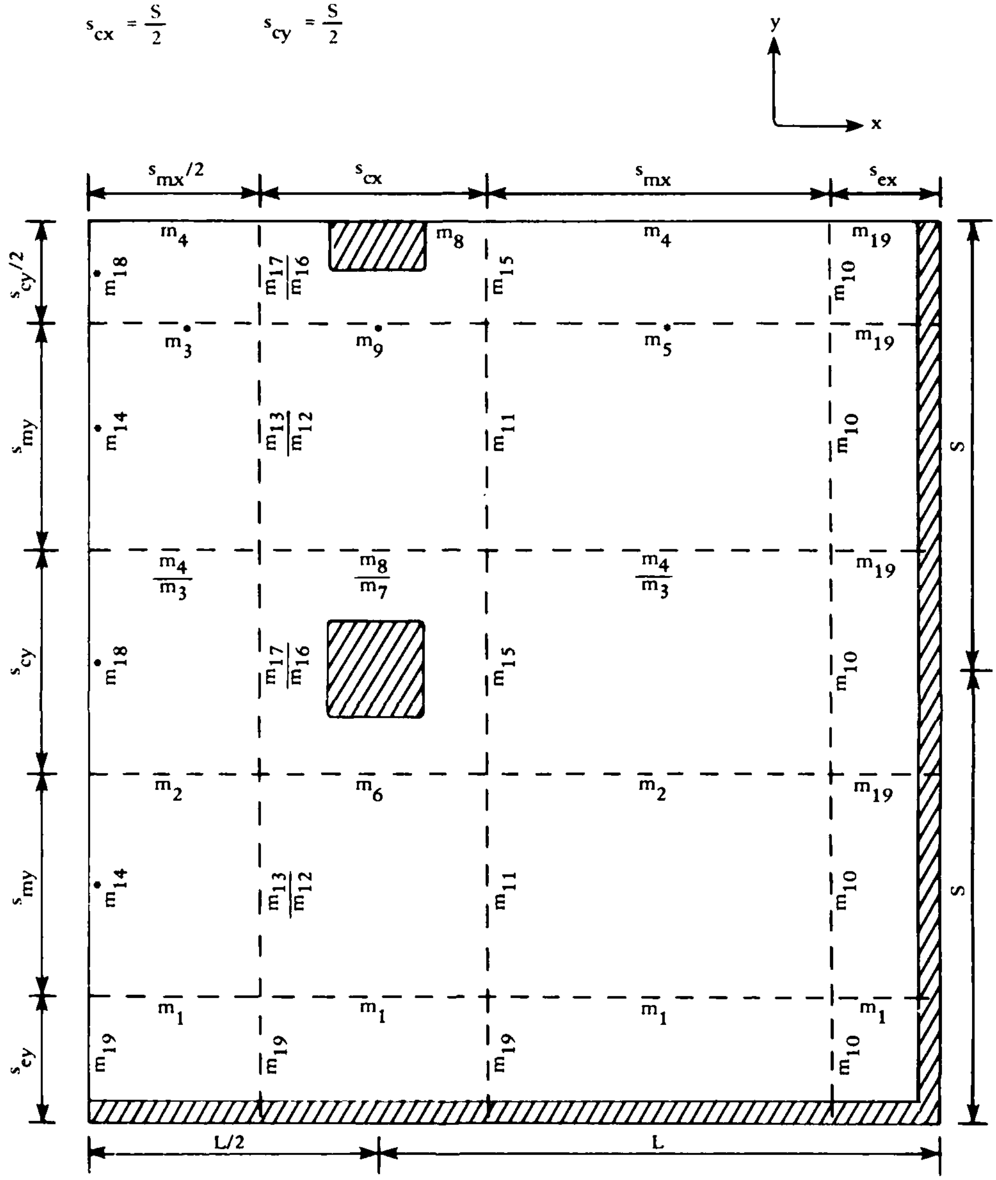


Figure B-1. Unit moment distribution for 3 x 4 flat slab.

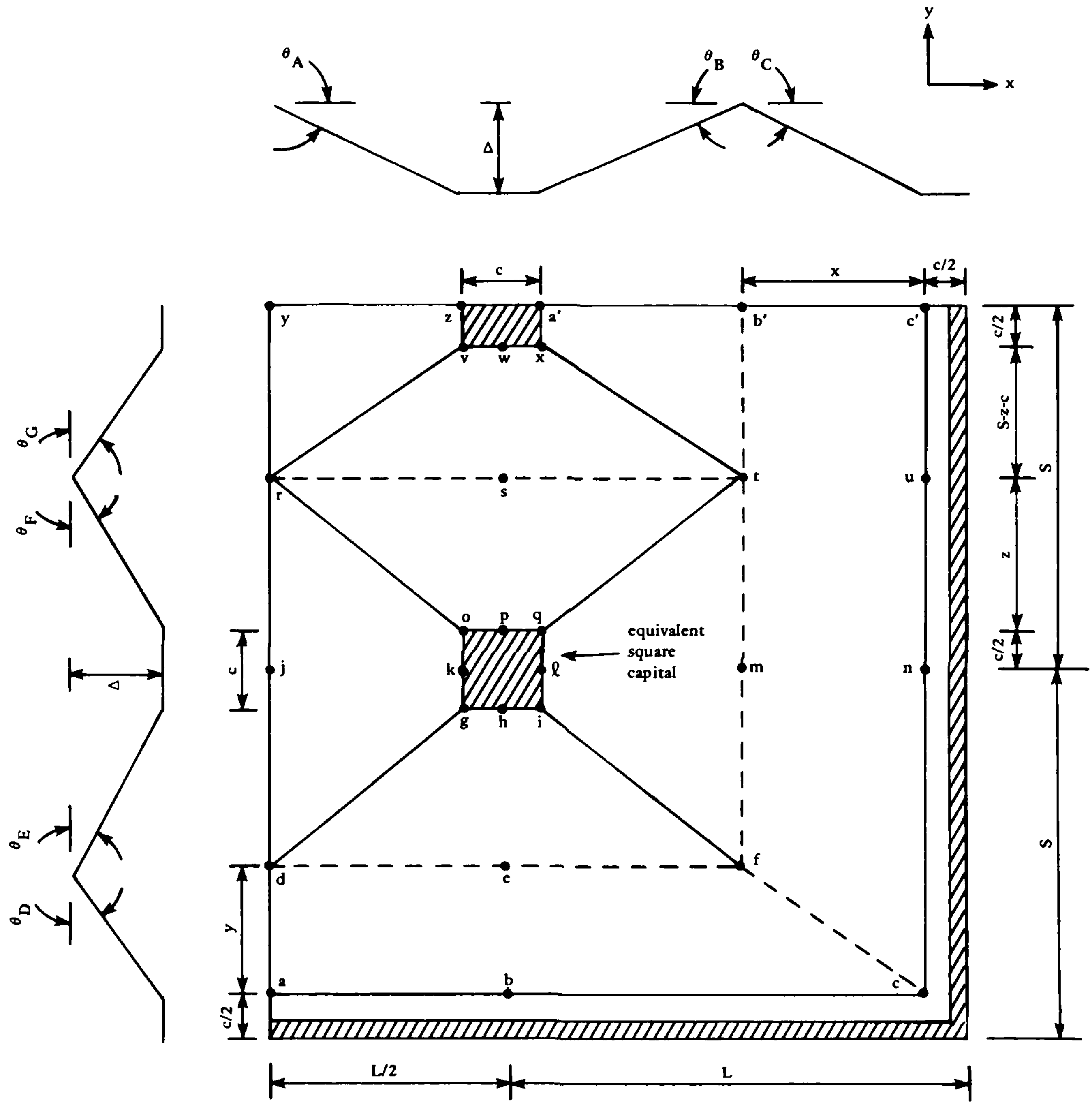


Figure B-2. Yield-line mechanism for a 3 x 4 flat slab.

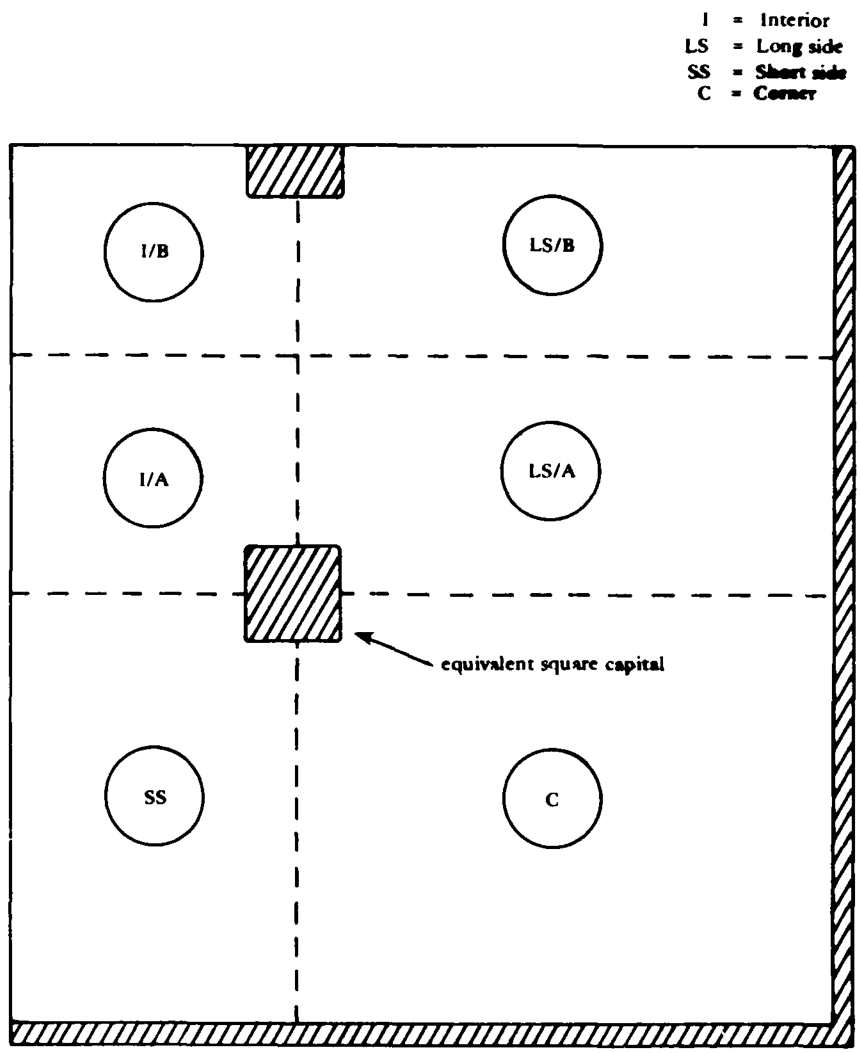


Figure B-3. Panel designations for 3 x 4 flat slab.

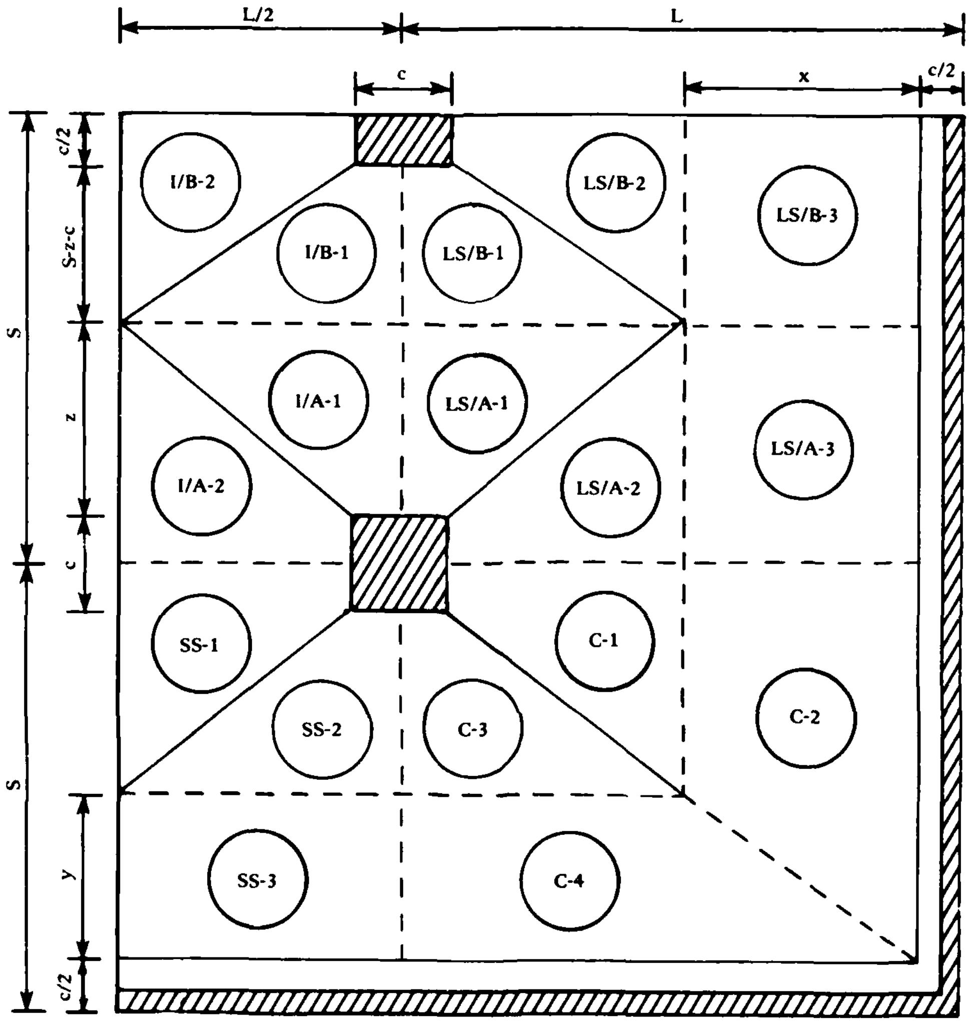


Figure B-4. Rotating sectors for internal work calculations for 3 x 4 flat slab.

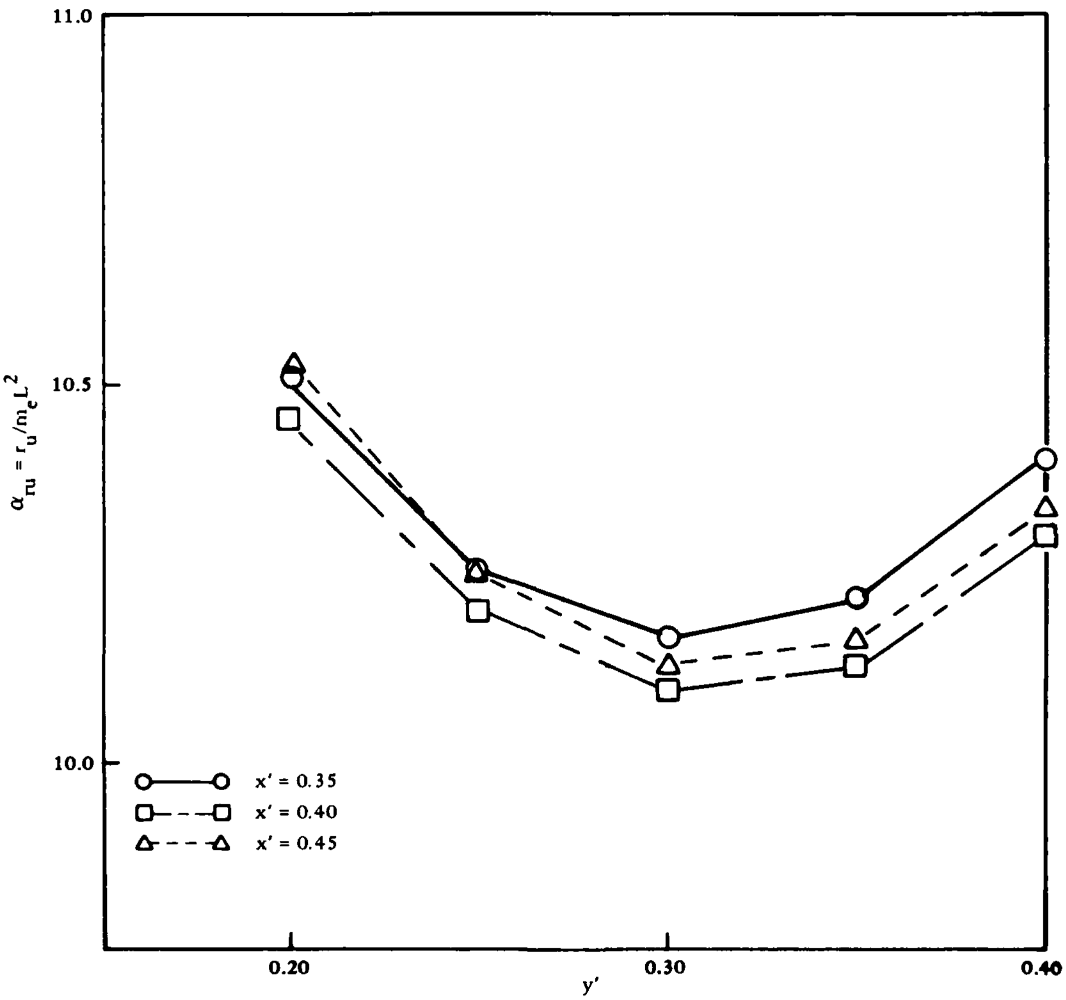


Figure B-5. Minimum r_u curves (z' = 0.311).

Appendix C

EFFECTIVE UNIT MASS FOR FLAT SLAB

INTRODUCTION

The effective unit mass of a flat slab is given by:

$$m_{ef} = K_{LM}\, m \qquad \text{(C-1)}$$

where: m = actual unit mass (lb-sec^2/in.^{3})

m_{ef} = effective unit mass (lb-sec^2/in.^{3})

K_{LM} = load-mass factor

Because of the complexity and wide choice of parameter values for flat slab structures, it becomes imperative to develop a general procedure for determining the effective plastic unit mass for any flat slab configuration. Also, adding to the complexity is the possible presence of drop panels and soil cover. Both of these conditions are addressed in this Appendix. The key to this or any general procedure is in the development of a simple, but efficient, method which can be used for any flat slab. This Appendix contains the recommended procedure, which has been programmed on an HP-41CV.

The procedure is illustrated for the 3 x 4 flat slab structure (Figure 11) previously analyzed in Appendixes A and B. Parameter studies are also made for other configurations.

ACTUAL UNIT MASS

For a flat slab without drop panels, the actual unit mass equals:

$$m = m_{slab} + m_{ob} = \rho_{slab}\, t_{slab} + \rho_{ob}\, t_{ob} \qquad \text{(C-2)}$$

where: m_{slab} = actual unit mass of slab

m_{ob} = actual unit mass of soil overburden

ρ_{slab} = mass density of slab

t_{slab} = thickness of slab

ρ_{ob} = mass density of soil overburden

t_{ob} = thickness of soil overburden

For a flat slab with drop panels, the actual unit mass is obtained from this expression:

$$m = \frac{M_T}{A_T} \tag{C-3}$$

where: M_T = total mass (slab + soil overburden + drop panel)

A_T = total slab area

Note that these quantities represent that portion of the structure which rotates (deflects). Therefore, the mass/area inside of the equivalent square capital and outside of the perimeter yield line (haunch) is excluded in the calculations. Now,

$$M_T = \left(m_{ob} + m_{slab}\right) A_T + m_{dp} A_{dp} \tag{C-4}$$

or

$$M_T = \left(\rho_{ob} t_{ob} + \rho_{slab} t_{slab}\right) A_T + \rho_{dp} t_{dp} A_{dp} \tag{C-5}$$

where: m_{dp} = actual unit mass of drop panel

A_{dp} = area of drop panel (minus area of capital)

ρ_{dp} = mass density of drop panel

t_{dp} = thickness of drop panel

Substituting Equation C-5 into Equation C-3 yields:

$$m = \rho_{ob} t_{ob} + \rho_{slab} t_{slab} + \rho_{dp} t_{dp} \frac{A_{dp}}{A_T} \tag{C-6}$$

PLASTIC LOAD-MASS FACTOR

As shown in the main text, the plastic load-mass factor for flat slabs without drop panels and with or without a uniform soil overburden equals

$$K_{LM} = \frac{\sum \frac{I}{cL_1}}{\Sigma A} \tag{C-7}$$

where, for each sector:

I = area moment of inertia about the axis of rotation

c = distance from the resultant applied load to the axis of rotation

L_1 = total length of sector normal to axis of rotation

A = total area of sector

For a flat slab with drop panels and with or without soil overburden the load-mass factor equals:

$$K_{LM} = \frac{\sum \frac{I_m}{cL_1}}{\Sigma M} \tag{C-8}$$

where: I_m = mass moment of inertia about the axis of rotation

M = total mass of sector

To determine the numerators of Equations C-7 and C-8, the flat slab is divided into the same rotating sectors used for the external work calculations. Figure C-1 is a reproduction of Figure B-4, which is for the 3 x 4 flat slab example. For a flat slab with drop panels, the additional rotating sectors are shown in Figure C-2 as the dotted areas. The numerator of Equation C-7 then becomes:

$$\sum \frac{I}{cL_1}\begin{pmatrix}\text{flat}\\ \text{slab}\end{pmatrix} = \frac{I}{cL_1}\ (\text{corner}) + \frac{I}{cL_1}(\text{interior}) + \frac{I}{cL_1}\begin{pmatrix}\text{short}\\ \text{side}\end{pmatrix} + \frac{I}{cL_1}\begin{pmatrix}\text{long}\\ \text{side}\end{pmatrix} \qquad \text{(C-9)}$$

The numerator of Equation C-8 becomes:

$$\sum \frac{I_m}{cL_1}\begin{pmatrix}\text{flat}\\ \text{slab}\end{pmatrix} = \frac{I_m}{cL_1}(\text{corner}) + \frac{I_m}{cL_1}\ (\text{interior}) + \frac{I_m}{cL_1}\begin{pmatrix}\text{short}\\ \text{side}\end{pmatrix} + \frac{I_m}{cL_1}\begin{pmatrix}\text{long}\\ \text{side}\end{pmatrix} \qquad \text{(C-10)}$$

Each of the four terms on the RHS of Equation C-10 contains mass contributions from the slab, soil overburden, and drop panel. If the drop panel contributions are separated, then Equation C-10 can be rewritten as:

$$\frac{I_m}{cL_1}\begin{pmatrix}\text{flat}\\ \text{slab}\end{pmatrix} = \left(\rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab}\right)\sum \frac{I}{cL_1}\begin{pmatrix}\text{flat}\\ \text{slab}\end{pmatrix} + \rho_{dp}\ t_{dp}\sum \frac{I}{cL_1}\begin{pmatrix}\text{drop}\\ \text{panel}\end{pmatrix} \qquad \text{(C-11)}$$

Since $\Sigma I/cL_1$ (flat slab) was already calculated in Equation C-9, only the additional effects of the drop panel have to be considered. Therefore, the amount of calculations has been reduced.

To determine the denominators of Equations C-7 and C-8, it is not necessary to individually determine the area or mass of each rotating sector. Since the total area or mass is independent of the yield-line locations, it is easier to determine the total area or mass by using the overall slab dimension and then subtracting the area of the nonrotating column capitals and wall haunches. The total mass for a flat slab with drop panels is given by Equation C-5. That is,

$$M = (\rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab})\ A_T + \rho_{dp}\ t_{dp}\ A_{dp}$$

Finally, after substituting Equations C-11 and C-5 into Equation C-8:

$$K_{LM} = \frac{(\rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab}) \sum \frac{I}{cL_1} \binom{\text{flat}}{\text{slab}} + \rho_{dp}\ t_{dp} \sum \frac{I}{cL_1} \binom{\text{drop}}{\text{panel}}}{(\rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab})\ A_T + \rho_{dp}\ t_{dp}\ A_{dp}} \quad \text{. . . . . (C-12)}$$

Figure C-3 shows the I/cL_1 values for triangular and rectangular rotating sectors. Each value consists of the quantity b d times coefficient α_{LM}, where b is the length of the sector side parallel to the axis of rotation, and d is the length of the sector side perpendicular to the axis of rotation. Therefore, the I/cL_1 values for any rotating quadralateral sector equals:

$$\frac{I}{cL_1} = d \left[\left(\alpha_{LM}\ b\right)_{\text{rectangle}} + \left(\alpha_{LM}\ b\right)_{\text{triangle}} \right] \quad \text{(C-13)}$$

EXAMPLE PROBLEM

Calculations were made in this section to determine the effective plastic unit mass for the 3 x 4 flat slab structure (Figure 11). Both configurations with and without drop panels are considered.

The load-mass parameters for the flat slab without drop panels are listed in Table C-1. The I/cL_1 expressions are then listed in Table C-2. Programs were written for an HP-31CV to calculate these values for each panel type (corner, short side, interior, long side); these program listings are shown in Table C-3. These programs make use of some of the storage registers assigned earlier in the yield-line analysis; they are reproduced in Table C-4. The result of the yield-line analysis of Appendix B is as follows:

x' = 0.40

y' = 0.30

z' = 0.311

That is, according to Table B-11, the minimum ultimate resistance of the flat slab occurs for the above yield-line locations. Therefore, it is only necessary to calculate K_{LM} for this yield-line pattern. The denominator (ΣA) of Equation C-7 is calculated as follows:

$$\Sigma A = (1.5L)(2S) - c^2 - \frac{1}{2}c^2 - \frac{c}{2}(2S) - \left(\frac{c}{2}\right)(1.5L) + \left(\frac{c}{2}\right)^2 \qquad \text{(C-14)}$$

where: $$S = \frac{L}{\beta} = \frac{L}{1.25} \qquad \text{(C-15)}$$

$$c = 0.89\,\alpha_{cap}\,L = (0.89)(0.20)\,L = 0.178\,L \qquad \text{(C-16)}$$

Substitution yields,

$$\Sigma A = 2.0845\,L^2 \qquad \text{(C-17)}$$

The calculated I/cL_1 values for the six panel types are listed in Table C-5. The total $\Sigma I/cL_1$ (flat slab) value equals $1.4352\,L^2$. Therefore, substituting into Equation C-7 yields:

$$K_{LM} = \frac{1.4352\,L^2}{2.0845\,L^2} = 0.689$$

The actual unit mass is given by Equation C-2. Assume that the following conditions exist:

$$t_{slab} = 16 \text{ in.}$$

$$t_{ob} = 12 \text{ in.} = 0.75\,t_{slab}$$

$$\rho_{slab} = 0.000217 \text{ lb-sec}^2/\text{in.}^4$$

$$\rho_{ob} = 0.000150 \text{ lb-sec}^2/\text{in.}^4 = 0.69\,\rho_{slab}$$

Substitution into Equation (C-2) yields:

$$m = (\rho_{slab})(t_{slab}) + (0.69\,\rho_{slab})(0.75\,t_{slab})$$

$$= 1.518\,\rho_{slab}\,t_{slab}$$

$$= 0.00527 \text{ lb-sec}^2/\text{in.}^3$$

For the flat slab with drop panels (assume $L_{dp} = 0.4$ L and $S_{dp} = 0.4$ L), Equation C-12 must be used to determine K_{LM}. The first terms of the numerator and denominator which represent the slab and overburden, only, are already known. That is,

$$\rho_{ob}\, t_{ob} + \rho_{slab}\, t_{slab} = 1.518\, \rho_{slab}\, t_{slab}$$

$$A_T = \Sigma A = 2.0845\ L^2$$

$$\frac{I}{cL_1}\begin{pmatrix}\text{flat}\\ \text{slab}\end{pmatrix} = 1.4352\ L^2$$

Table C-6 lists the parameters (d, b, α_{LM}) needed to determine the second terms (drop panel contribution). Because the drop panel is square (i.e., $L_{dp} = S_{dp}$), all 12 sectors are identical. Therefore,

$$\frac{I}{cL_1}\begin{pmatrix}\text{drop}\\ \text{panel}\end{pmatrix} = 12\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right)\left(0.2\ L - \frac{c}{2}\right) + \frac{3}{8}\left(0.2\ L - \frac{c}{2}\right)\left(0.2\ L - \frac{c}{2}\right)\right] \tag{C-18}$$

$$= 12\left[\left(\frac{2}{3}\right)\left(\frac{0.178\ L}{2}\right)\left(0.2\ L - \frac{0.178\ L}{2}\right) + \frac{3}{8}\left(0.2\ L - \frac{0.178\ L}{2}\right)\left(0.2\ L - \frac{0.178\ L}{2}\right)\right]$$

$$= 0.1346\ L^2$$

The drop panel area, A_{dp}, equals:

$$A_{dp} = 1.5\ [(0.4\ L)(0.4\ L) - c^2] = 0.1925\ L^2 \tag{C-19}$$

Substitution into Equation (C-12) yields:

$$K_{LM} = \frac{(1.518\ \rho_{slab}\ t_{slab})(1.4352\ L^2) + \rho_{dp}\ t_{dp}(0.1346\ L^2)}{(1.518\ \rho_{slab}\ t_{slab})(2.0845\ L^2) + \rho_{dp}\ t_{dp}(0.1925\ L^2)} \quad \ldots \text{(C-20)}$$

Assume that the following conditions exist:

$t_{slab} = 16$ in.

$t_{dp} = 6$ in. $= 0.375\ t_{slab}$

$\rho_{slab} = \rho_{dp} = 0.000217$ lb-sec^2/in.^{4}

Substituting into Equation C-20 yields:

$$K_{LM} = \frac{[(1.518\ \rho_{slab}\ t_{slab})(1.4352\ L^2) + (\rho_{slab})(0.375\ t_{slab})(0.1346\ L^2)]}{[1.518\ \rho_{slab}\ t_{slab})(2.0845\ L^2) + (\rho_{slab})(0.375\ t_{slab})(0.1925\ L^2)]}$$

$$= \frac{2.229\ \rho_{slab}\ t_{slab}\ L^2}{3.236\ \rho_{slab}\ t_{slab}\ L^2} = 0.689$$

Thus, there is no change in the K_{LM} value. The actual mass is given by Equation C-6. That is,

$$m = \rho_{ob}\ t_{ob} + \rho_{slab}\ t_{slab} + \rho_{dp}\ t_{dp} \frac{A_{dp}}{A_T}$$

Substituting yields:

$$m = 1.518\ \rho_{slab}\ t_{slab} + (1.0\ \rho_{slab})(0.375\ t_{slab})\left(\frac{0.1925\ L^2}{2.0845\ L^2}\right)$$

$$= 1.553\ \rho_{slab}\ t_{slab}$$

$$= 0.00539 \text{ lb-sec}^2/\text{in.}^3$$

Therefore, there is only a 2.3% increase in the unit mass because of the drop panels. For practical purposes this can be neglected during the design process.

PARAMETER STUDIES

For comparitive purposes, K_{LM} was calculated for the 15 iterative values of x' and y' employed in the yield-line analysis of Appendix B. These results, listed in Table C-5, indicate the relative insensitivity of K_{LM} to yield-line locations. That is, K_{LM} varies slightly from 0.679 to 0.698.

Another comparative sensitivity study was made for different values of β (i.e., $1.00 \leq \beta \leq 2.00$). The location of the yield lines were assumed as follows:

$$x' = 0.4$$

$$y' = 0.375/\beta$$

$$z' = \left(\frac{1}{\beta} - 0.89\ \alpha_{cap}\right)/2$$

The calculated values of K_{LM} for five β values are listed in Table C-7. These results again indicate the relative insensitivity of K_{LM}. That is, K_{LM} varies from 0.682 to 0.691.

CONCLUSION

This Appendix outlines a procedure for determining the effective unit mass for flat slabs with and without drop panels. The plastic load-mass factor, K_{LM}, was shown to be unaffected by the introduction of normal sized drop panels. Therefore, Equation C-7 can be used for flat slabs with or without drop panels. The actual unit mass was shown to increase only slightly (2.3%) upon the introduction of normal sized drop panels. Therefore, Equation C-2 can be used to determine the actual unit mass for flat slabs with or without drop panels.

Table C-1. Load-Mass Parameters

Panel	Sector	d	Rectangle*	Triangle	
			b	b	α_{LM}
Corner (C)	C-1	L - c - x	c/2	S - c - y	3/8
	C-2	x	S - (c/2) - y	y	1/4
	C-3	S - c - y	c/2	L - c - x	3/8
	C-4	y	L - (c/2) - x	x	1/4
Interior (I/A;I/B)	I/A - 1	z	c/2	(L - c)/2	3/8
	I/A - 2	(L - c)/2	c/2	z	3/8
	I/B - 1	S - c - z	c/2	(L - c)/2	3/8
	I/B - 2	(L - c)/2	c/2	S - z - c	3/8
Short Side (SS)	SS - 1	(L - c)/2	c/2	S - c - y	3/8
	SS - 2	S - c - y	c/2	(L - c)/2	3/8
	SS - 3	y	L/2	-	-
Long Side (LS/A;LS/B)	LS/A - 1	z	c/2	L - c - x	3/8
	LS/A - 2	L - c - x	c/2	z	3/8
	LS/A - 3	x	z + (c/2)	-	-
	LS/B - 1	S - c - z	c/2	L - c - x	3/8
	LS/B - 2	L - c - x	c/2	S - z - c	3/8
	LS/B - 3	x	S - z - (c/2)	-	-

*For rectangular sectors: $\alpha_{LM} = 2/3$.

Table C-2. Expressions for $\frac{I}{cL_1}$

Panel Type	$\frac{I}{cL_1} = \sum d[(\alpha_{LM}\, b)_{rectangle} + (\alpha_{LM}\, b)_{triangle}]$
Corner (C)	$(L-c-x)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(S-c-y)\right] + x\left[\left(\frac{2}{3}\right)\left(S-\frac{c}{2}-y\right) + \left(\frac{1}{4}\right)(y)\right] + (S-c-y)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(L-c-x)\right] + y\left[\left(\frac{2}{3}\right)\left(L-\frac{c}{2}-x\right) + \left(\frac{1}{4}\right)(x)\right]$
Interior (I/A)	$z\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)\left(\frac{L-c}{2}\right)\right] + \left(\frac{L-c}{2}\right)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(z)\right]$
Interior (I/B)	$(S-c-z)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)\left(\frac{L-c}{2}\right)\right] + \left(\frac{L-c}{2}\right)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(S-z-c)\right]$
Short Side (SS)	$\left(\frac{L-c}{2}\right)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(S-c-y)\right] + (S-c-y)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)\left(\frac{L-c}{2}\right) + y\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right]$
Long Side (LS/A)	$z\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(L-c-x)\right] + (L-c-x)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(z)\right] + x\left[\left(\frac{2}{3}\right)\left(z+\frac{c}{2}\right)\right]$
Long Side (LS/B)	$(S-c-z)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(L-c-x)\right] + (L-c-x)\left[\left(\frac{2}{3}\right)\left(\frac{c}{2}\right) + \left(\frac{3}{8}\right)(S-z-c)\right] + x\left[\left(\frac{2}{3}\right)\left(S-z-\frac{c}{2}\right)\right]$

Table C-3. Program Listings for I/cL_1 Calculations

a. Corner (C)

```
01*LBL "CC"
02 RCL 06
03 3
04 /
05 RCL 12
06 3
07 *
08 8
09 /
10 +
11 RCL 10
12 *
13 RCL 13
14 2
15 *
16 3
17 /
18 RCL 01
19 4
20 /
21 +
22 RCL 00
23 *
24 +
25 RCL 06
26 3
27 /
28 RCL 10
29 3
30 *
31 8
32 /
33 +
34 RCL 12
35 *
36 +
37 RCL 11
38 2
39 *
40 3
41 /
42 RCL 00
43 4
44 /
45 +
46 RCL 01
47 *
48 +
49 END
```

b. Interior (I/A)

```
01*LBL "IA"
02 RCL 06
03 3
04 /
05 RCL 20
06 3
07 *
08 8
09 /
10 +
11 RCL 02
12 *
13 RCL 06
14 3
15 /
16 RCL 02
17 3
18 *
19 8
20 /
21 +
22 RCL 20
23 *
24 +
25 END
```

c. Interior (I/B)

```
01*LBL "IB"
02 RCL 06
03 3
04 /
05 RCL 20
06 3
07 *
08 8
09 /
10 +
11 RCL 14
12 *
13 RCL 06
14 3
15 /
16 RCL 14
17 3
18 *
19 8
20 /
21 +
22 RCL 20
23 *
24 +
25 END
```

(continued)

Table C-3. Continued

d. Short Side (SS)	e. Long Side (LS/A)	f. Long Side (LS/B)
01*LBL "SS"	01*LBL "LSA"	01*LBL "LSB"
02 RCL 06	02 RCL 06	02 RCL 06
03 3	03 3	03 3
04 /	04 /	04 /
05 RCL 12	05 RCL 10	05 RCL 10
06 3	06 3	06 3
07 *	07 *	07 *
08 8	08 8	08 8
09 /	09 /	09 /
10 +	10 +	10 +
11 RCL 20	11 RCL 02	11 RCL 14
12 *	12 *	12 *
13 RCL 06	13 RCL 06	13 RCL 06
14 3	14 3	14 3
15 /	15 /	15 /
16 RCL 20	16 RCL 02	16 RCL 14
17 3	17 3	17 3
18 *	18 *	18 *
19 8	19 8	19 8
20 /	20 /	20 /
21 +	21 +	21 +
22 RCL 12	22 RCL 10	22 RCL 10
23 *	23 *	23 *
24 +	24 +	24 +
25 RCL 01	25 RCL 18	25 RCL 15
26 3	26 2	26 2
27 /	27 *	27 *
28 +	28 3	28 3
29 END	29 /	29 /
	30 RCL 00	30 RCL 00
	31 *	31 *
	32 +	32 +
	33 END	33 END

Table C-4. HP-31CV Storage for Flat Slab Yield-Line Analysis

a. Input Parameters

Absolute Value	Dimensionless Value	Register
x	x'	00
y	y'	01
z	z'	02
1/S	β	03
d*	α_{cap}	04

*c = 0.89 d

(continued)

Table C-4. Continued

b. Calculated Parameters

Absolute Value	Dimensionless Value	Register
S	$1/\beta$	05
c	$0.89\ \alpha_{cap}$	06
c/2	$0.445\ \alpha_{cap}$	07
L-c	$1 - 0.89\ \alpha_{cap}$	08
S-c	$(1/\beta) - 0.89\ \alpha_{cap}$	09
L - c - x	$1 - 0.89\ \alpha_{cap} - x'$	10
L - (c/2) - x	$1 - 0.445\ \alpha_{cap} - x'$	11
S - c - y	$(1/\beta) - 0.89\ \alpha_{cap} - y'$	12
S - (c/2) - y	$(1/\beta) - 0.445\ \alpha_{cap} - y'$	13
S - c - z	$(1/\beta) - 0.89\ \alpha_{cap} - z'$	14
S - (c/2) - z	$(1/\beta) - 0.445\ \alpha_{cap} - z'$	15
x + (c/2)	$x' + 0.445\ \alpha_{cap}$	16
y + (c/2)	$y' + 0.445\ \alpha_{cap}$	17
z + (c/2)	$z' + 0.445\ \alpha_{cap}$	18
L - (S/2)	$1 - (1/2\beta)$	19
(L - c)/2	$(1 - 0.89\ \alpha_{cap})/2$	20

Table C-5. Plastic Load-Mass Factor Calculations for 3 x 4 Flat Slab

[$\beta = 1.25$, $\alpha_{cap} = 0.20$, $\Sigma A = 2.0845 L^2$]

x'	y'	z'	Panel Types						Total $\frac{I}{cL_1}$	K_{LM}*
			Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)		
0.35	0.20	0.311	0.4315	0.1387	0.1387	0.2462	0.2499	0.2499	1.4549	0.698
	0.25	0.311	0.4266	0.1387	0.1387	0.2445	0.2499	0.2499	1.4483	0.695
	0.30	0.311	0.4217	0.1387	0.1387	0.2427	0.2499	0.2499	1.4416	0.692
	0.35	0.311	0.4168	0.1387	0.1387	0.2410	0.2499	0.2499	1.4350	0.688
	0.40	0.311	0.4119	0.1387	0.1387	0.2393	0.2499	0.2499	1.4284	0.685
0.40	0.20	0.311	0.4280	0.1387	0.1387	0.2462	0.2486	0.2486	1.4488	0.695
	0.25	0.311	0.4229	0.1387	0.1387	0.2445	0.2486	0.2486	1.4420	0.692
	0.30	0.311	0.4179	0.1387	0.1387	0.2427	0.2486	0.2486	1.4352	0.689
	0.35	0.311	0.4128	0.1387	0.1387	0.2410	0.2486	0.2486	1.4284	0.685
	0.40	0.311	0.4077	0.1387	0.1387	0.2393	0.2486	0.2486	1.4216	0.682
0.45	0.20	0.311	0.4246	0.1387	0.1387	0.2462	0.2473	0.2473	1.4428	0.692
	0.25	0.311	0.4193	0.1387	0.1387	0.2445	0.2473	0.2473	1.4358	0.689
	0.30	0.311	0.4140	0.1387	0.1387	0.2427	0.2473	0.2473	1.4287	0.685
	0.35	0.311	0.4087	0.1387	0.1387	0.2410	0.2473	0.2473	1.4217	0.682
	0.40	0.311	0.4034	0.1387	0.1387	0.2393	0.2473	0.2473	1.4147	0.679

$$* \; K_{LM} = \frac{\sum \frac{I}{cL_1}}{\Sigma A}$$

Table C-6. Load-Mass Parameters for Drop Panel

Panel	Sector	d	Rectangle*	Triangle	
			b	b	α_{LM}
Corner (C)	C - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	C - 3	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	I/A - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
Interior (I/A; I/B)	I/A - 2	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	I/B - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	I/B - 2	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
Short Side (SS)	SS - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	SS - 2	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
Long Side (LS/A; LS/B)	LS/A - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	LS/A - 2	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	LS/B - 1	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8
	LS/B - 2	0.2L - (c/2)	c/2	0.2L - (c/2)	3/8

*For rectangle: $\alpha_{LM} = 2/3$

Table C-7. Plastic Load-Mass Calculations for 3 x 4 Flat Slab

[$1.0 \leq \beta \leq 2.0$]

β	x'	y'	z'	Total Area, A_T	Panel Types						Total $\frac{I}{cL_1}$	K_{LM}^*
					Corner (C)	Interior (I/A)	Interior (I/B)	Short Side (SS)	Long Side (LS/A)	Long Side (LS/B)		
1.00	0.4	0.375	0.411	2.6489	0.5387	0.1755	0.1755	0.3137	0.3128	0.3128	1.8290	0.6905
1.25	0.4	0.300	0.311	2.0845	0.4179	0.1387	0.1387	0.2427	0.2486	0.2486	1.4352	0.6885
1.50	0.4	0.250	0.244	1.7080	0.3373	0.1142	0.1142	0.1954	0.2057	0.2057	1.1725	0.6865
1.75	0.4	0.214	0.197	1.4395	0.2798	0.0968	0.0968	0.1617	0.1753	0.1753	0.9857	0.6848
2.00	0.4	0.188	0.161	1.2379	0.2365	0.0836	0.0836	0.1363	0.1522	0.1522	0.8444	0.6821

$$* K_{LM} = \frac{\sum \frac{I}{c\,L_1}}{A_T}$$

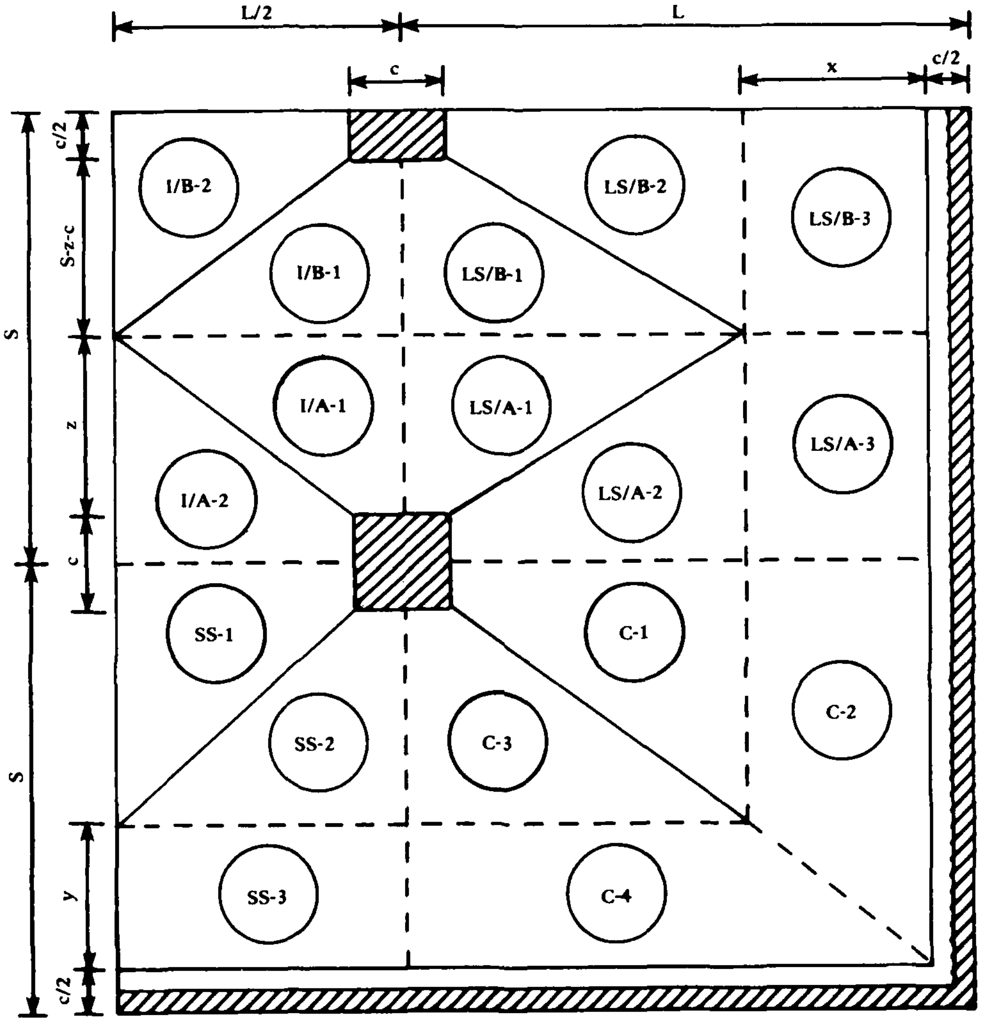


Figure C-1. Rotating sectors for load-mass factor calculations for 3 x 4 flat slab.

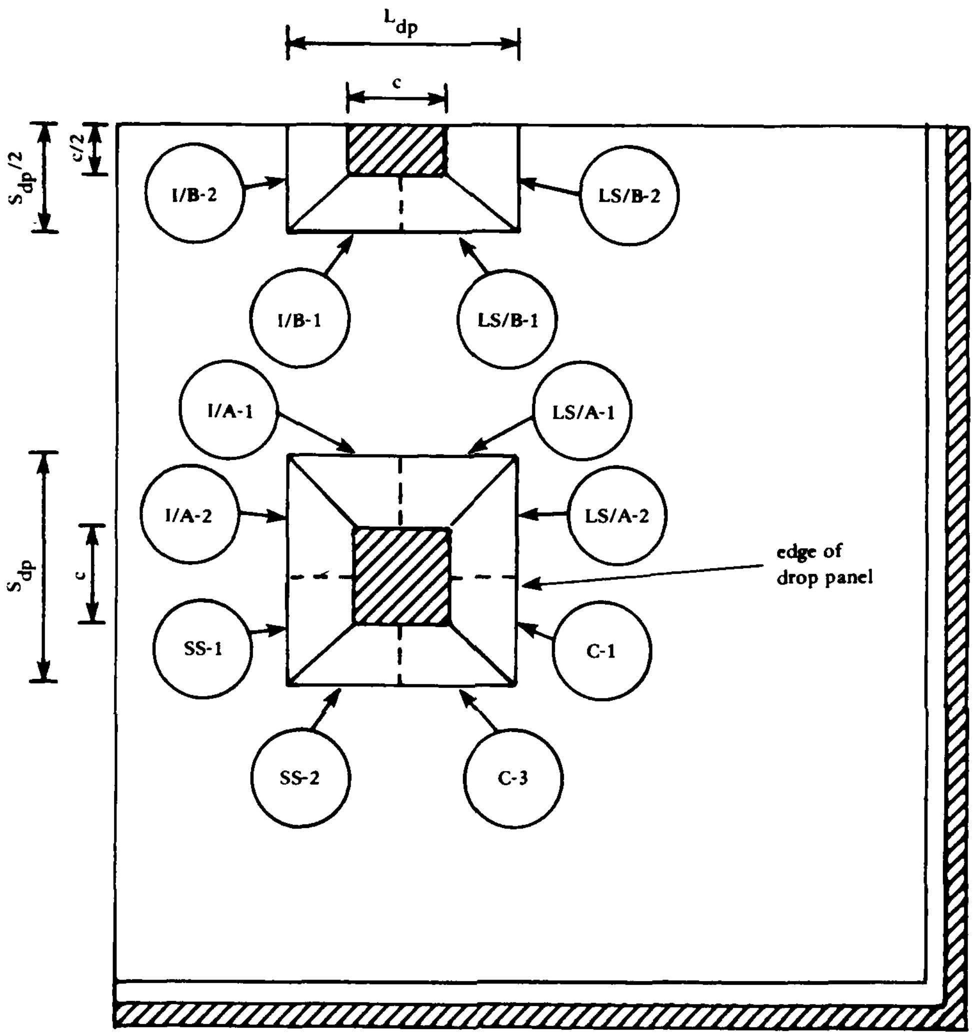
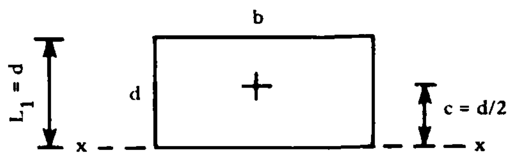
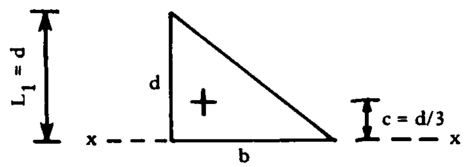


Figure C-2. Rotating sectors for load-mass factor calculations for 3 x 4 flat slab with drop panels.



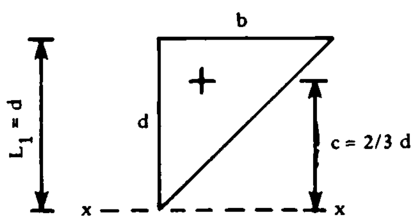
$$\frac{I}{cL_1} = \frac{b\,d^3/3}{\left(\frac{d}{2}\right)(d)} = \frac{2}{3}bd$$

(a) Rectangle.



$$\frac{I}{cL_1} = \frac{b\,d^3/12}{\left(\frac{d}{3}\right)(d)} = \frac{1}{4}bd$$

(b) Triangle rotating about side.



$$\frac{I}{cL_1} = \frac{b\,d^3/4}{\left(\frac{2d}{3}\right)(d)} = \frac{3}{8}b\,d$$

(c) Triangle rotating about corner.

Figure C-3. Expressions for I/cL_1 for typical sections.

Appendix D

COLUMN DESIGN

INTRODUCTION

The columns are designed in accordance with the criteria presented in the ACI Code. They must be designed to resist the axial load and unbalanced moment resulting from the flat slab blast load and the structure dead load, P_{dl}. The axial load and moment at the top of the column (the critical section is at the bottom of the capital) are obtained from the flat slab shear forces, P_i, acting on the perimeter of the column capital plus the load, P_{cap}, on the equivalent square capital (Figure D-1).

The determination of these design loads are illustrated for the lower right-hand column of the 3 x 4 flat slab structure (Figure 11). The actual column design is not shown, but it can be obtained from any reinforced concrete design textbook.

CALCULATE TOTAL DYNAMIC COLUMN LOADS

The previously determined yield-line pattern for the flat slab is shown in Figure D-2 along with the loading parameters for the lower column. The required dynamic ultimate unit resistance, r_{ud}, equals 10.72 psi. (See Step 18 of main text.) The column loads are determined as follows:

<u>Capital</u>: $P_{cap} = (53.3)(53.3)(10.72) = 30{,}450$ lb

<u>Area 1</u>: $P_1' = \frac{1}{2}(93.3)(123.3)(10.72) = 61{,}660$ lb

$P_1'' = (93.3)(53.4)(10.72) = 53{,}410$ lb

$P_1''' = \frac{1}{2}(93.3)(126.6)(10.72) = 63{,}310$ lb

$P_1 = \Sigma P_1 = 178{,}380$ lb

$$e'_{1x} = -26.7 - \frac{123.3}{3} = -67.8 \text{ in.}$$

$$e''_{1x} = 0.0 \text{ in.}$$

$$e'''_{1x} = 26.7 + \frac{126.6}{3} = 68.9 \text{ in.}$$

$$e_{1x} = \frac{\Sigma P_1 e_{1x}}{\Sigma P_1} = [(61{,}660)\ (-67.8) + (53{,}410)\ (0) + (63{,}310)\ (68.9)]/178{,}380$$

$$= +1.02 \text{ in.}$$

$$e_{1y} = +26.7 \text{ in.}$$

<u>Area 2</u>:

$$P'_2 = \frac{1}{2}(123.3)(93.3)(10.72) = 61{,}660 \text{ lb}$$

$$P''_2 = (123.3)(53.4)(10.72) = 70{,}580 \text{ lb}$$

$$P'''_2 = \frac{1}{2}(123.3)(96.6)(10.72) = 63{,}840 \text{ lb}$$

$$P_2 = \Sigma P_2 = 196{,}080 \text{ lb}$$

$$e'_{2y} = 26.7 + \frac{93.3}{3} = 57.8 \text{ in.}$$

$$e''_{2y} = 0.0 \text{ in.}$$

$$e'''_{2y} = -26.7 - \frac{96.6}{3} = -58.9 \text{ in.}$$

$$e_{2y} = \frac{\Sigma P_2 e_{2y}}{\Sigma P_2} = [(61{,}660)\ (57.8) + (70{,}580)\ (0) + (63{,}840)\ (-58.9)]/196{,}080$$

$$= -1.00 \text{ in.}$$

$$e_{2x} = -26.7 \text{ in.}$$

<u>Area 3</u>:

$$P'_3 = \frac{1}{2}(126.6)(93.3)(10.72) = 63{,}310 \text{ lb}$$

$$P''_3 = (126.6)(53.4)(10.72) = 72{,}470 \text{ lb}$$

$$P_3''' = \frac{1}{2}(126.6)(96.6)(10.72) = 65{,}550 \text{ lb}$$

$$P_3 = \Sigma P_3 = 201{,}330 \text{ lb}$$

$$e_{3y}' = 57.8 \text{ in.}$$

$$e_{3y}'' = 0.0 \text{ in.}$$

$$e_{3y}''' = -58.9 \text{ in.}$$

$$e_{3y} = \frac{\Sigma P_3 e_{3y}}{\Sigma P_3} = [(63{,}310)(57.8) + (72{,}470)(0) + (65{,}550)(-58.9)]/201{,}330$$

$$= -1.00 \text{ in.}$$

$$e_{3x} = 26.7 \text{ in.}$$

Area 4:

$$P_4' = \frac{1}{2}(96.6)(123.3)(10.72) = 63{,}840 \text{ lb}$$

$$P_4'' = (96.6)(53.4)(10.72) = 55{,}290 \text{ lb}$$

$$P_4''' = \frac{1}{2}(96.6)(126.6)(10.72) = 65{,}550 \text{ lb}$$

$$P_4 = \Sigma P_4 = 184{,}690 \text{ lb}$$

$$e_{4x}' = -67.8 \text{ in.}$$

$$e_{4x}'' = 0.0 \text{ in.}$$

$$e_{4x}''' = 68.9 \text{ in.}$$

$$e_{4x} = \frac{\Sigma P_4 e_{4x}}{\Sigma P_4} = [(63{,}840)(-67.8) + (55{,}290)(0) + (65{,}550)(68.9)]/184{,}690$$

$$= 1.02 \text{ in.}$$

$$e_{4y} = -26.7 \text{ in.}$$

Design Loads:

$$M_x = \Sigma P_i e_{iy} = (178{,}380)(26.7) + (196{,}080)(-1.00) \quad \text{(D-1)}$$

$$+ (201{,}330)(-1.00) + (184{,}690)(-26.7)$$

$$= -565{,}890 \text{ in.-lb}$$

$$M_y = \Sigma P_i e_{ix} = (178{,}380)(1.02) + (196{,}080)(-26.7) \quad \text{(D-2)}$$

$$+ (201{,}330)(26.7) + (184{,}690)(1.02)$$

$$= 510{,}510 \text{ in.-lb}$$

$$M = \sqrt{M_x^2 + M_y^2} = \sqrt{(-565{,}890)^2 + (510{,}510)^2} = 762{,}130 \text{ in.-lb} \quad \ldots \text{(D-3)}$$

$$P = \Sigma P_i + P_{cap} = 790{,}930 \text{ lb} \quad \text{(D-4)}$$

$$e = \frac{M}{P} = \frac{762{,}130}{790{,}930} = 1.0 \text{ in.} \quad \text{(D-5)}$$

This load is assumed to be a suddenly applied constant load with a limited duration equal to the time (t_m = 93.3 msec) calculated for the slab to reach its maximum response (from Step 18).

CALCULATE NATURAL PERIOD

Column Mass

The mass of the column includes the column, capital, drop panel, and that portion of the roof slab and soil overburden within the boundaries of the drop panel; see Figure D-3. Therefore,

$$\text{Column: } M = \left(\frac{\pi}{4}\right)(30)^2\,(119)(0.000217) = 18.25 \text{ lb-sec}^2/\text{in.}$$

$$\text{Capital: } M = \frac{1}{3}(15)\left[\left(\frac{\pi}{4}\right)(30)^2 + \left(\frac{\pi}{4}\right)(60)^2 + \left(\frac{\pi}{4}\right)(30)(60)\right](0.000217)$$

$$= 5.37 \text{ lb-sec}^2/\text{in.}$$

Drop Panel: $M = (120)^2 (6)(0.000217) = 18.75$ lb-sec^2/in.

Roof Slab: $M = (120)^2 (16)(0.000217) = 50.00$ lb-sec^2/in.

Overburden: $M = (120)^2 (12)(0.000150) = 25.92$ lb-sec^2/in.

$M_{Total} = 118.29$ lb-sec^2/in.

Column Stiffness (Axial)

$$K = \frac{E_c A}{L} = \frac{(3.64 \times 10^6) \left(\frac{\pi}{4}\right) (30)^2}{119} = 21.62 \times 10^6 \text{ lb/in.} \qquad \text{(D-6)}$$

Natural Period

$$T_n = 2\pi \sqrt{\frac{K_{LM} M}{K}} \qquad \text{(D-7)}$$

where $K_{LM} = 1.0$. Therefore,

$$T_n = 2\pi \sqrt{\frac{(1.0)\ (118.29)}{21.62 \times 10^6}} = 0.0147 \text{ sec}$$

ELASTIC-PLASTIC SDOF RESPONSE

The required dynamic strength of the column, P_d, (considering only the blast loading) is obtained from the SDOF maximum response chart, Figure 5.25, in Reference 18. That is, for an allowable design ductility, X_m/X_E, of 3.0:

$$\frac{t_d}{T_n} = \frac{93.3}{14.7} = 6.35 \longrightarrow C_R = 1.20$$

$$P_d = C_R P = (1.20)\ (790{,}930) = 949{,}120 \text{ lb} \qquad \text{(D-8)}$$

This value is converted to an equivalent static column strength by dividing by DIF. That is,

$$P_s = \frac{P_d}{DIF} \tag{D-9}$$

For concrete compression, DIF equals 1.25. Therefore,

$$P_s = \frac{949,120}{1.25} = 759,300 \text{ lb}$$

TOTAL COLUMN LOAD

The total factored axial load, P_u, consists of the previously determined equivalent static load, P_s, plus the structure dead load, P_{dl}, within the boundaries of the yield lines shown in Figure D-1. The dead load calculations are as follows:

$$\text{Capital:} \quad P = \frac{1}{3}(15)\left[\left(\frac{\pi}{4}\right)(30)^2 + \left(\frac{\pi}{4}\right)(60)^2 + \frac{\pi}{4}(30)(60)\right]\left(\frac{145}{1,728}\right) = 2,076 \text{ lb}$$

$$\text{Drop Panel:} \quad P = (120)^2 (6)\left(\frac{145}{1,728}\right) = 7,250 \text{ lb}$$

$$\text{Roof Slab:} \quad P = (303.3)(243.3)(16)\left(\frac{145}{1,728}\right) = 99,074 \text{ lb}$$

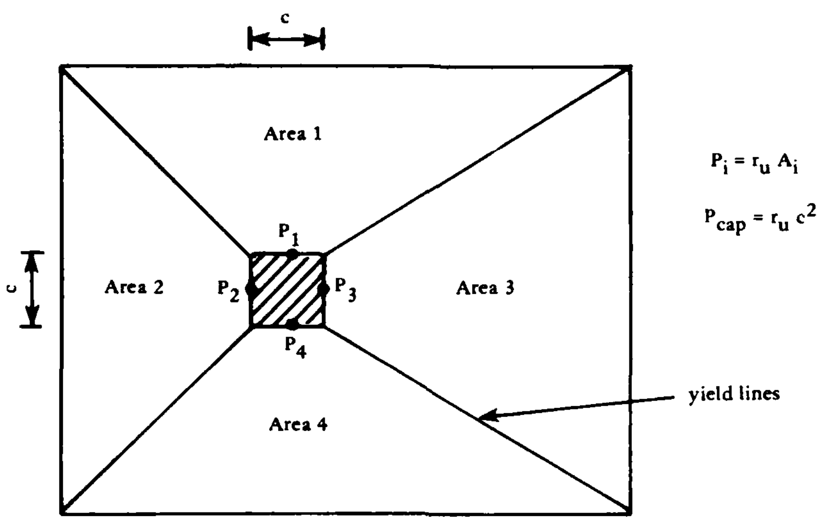
$$\text{Soil Overburden:} \quad P = (303.3)(243.3)(12)\left(\frac{100}{1,728}\right) = 51,245 \text{ lb}$$

$$P_{dl} = \Sigma P = 159,645 \text{ lb}$$

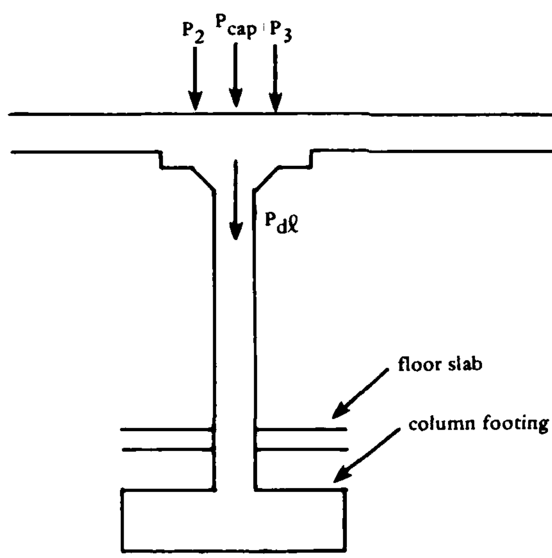
Therefore, the total factored axial load equals:

$$P_u = P_s + P_{dl} = 759,300 + 159,645 = 918,945 \text{ lb} \tag{D-10}$$

The design eccentricity remains at 1.0 in.



(a) Roof plan.



(b) Column elevation.

Figure D-1. Typical column loads.

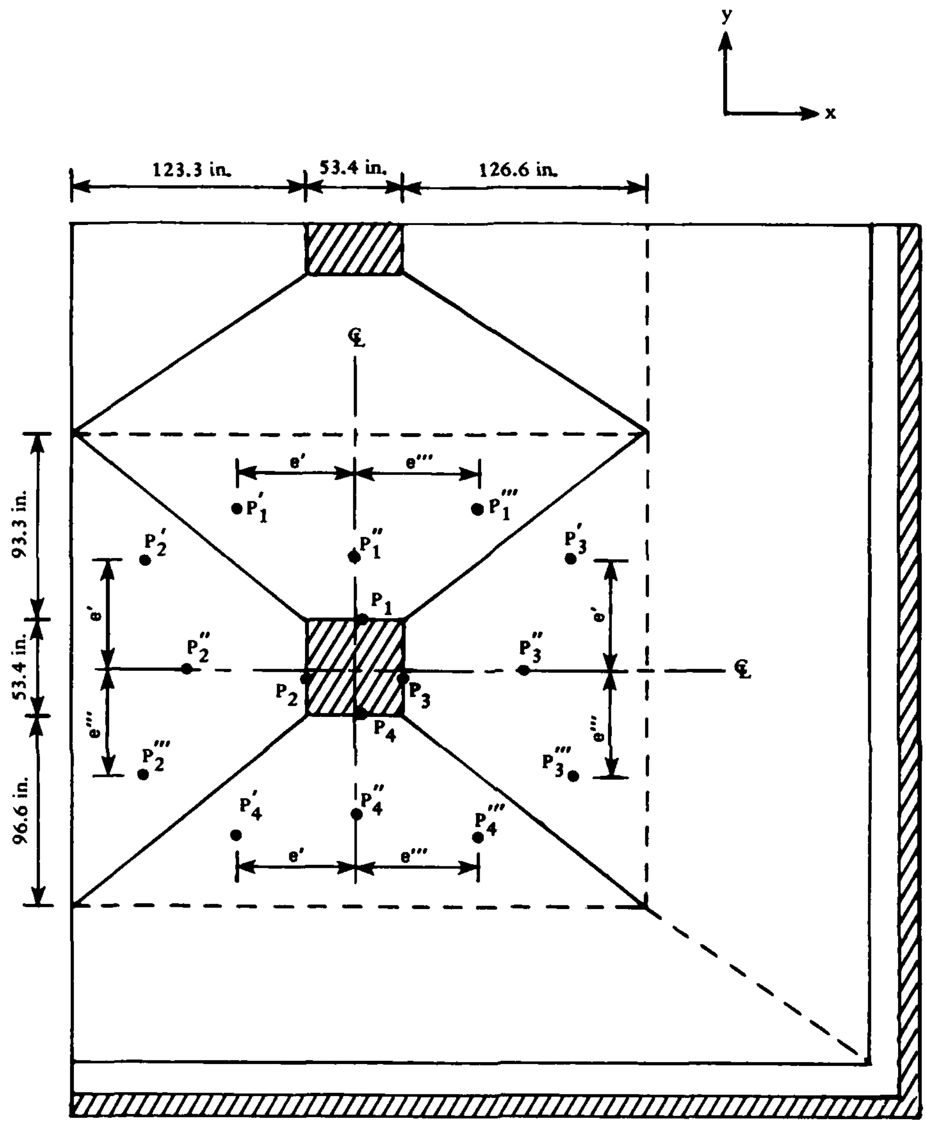


Figure D-2. Column loads for 3 x 4 flat slab.

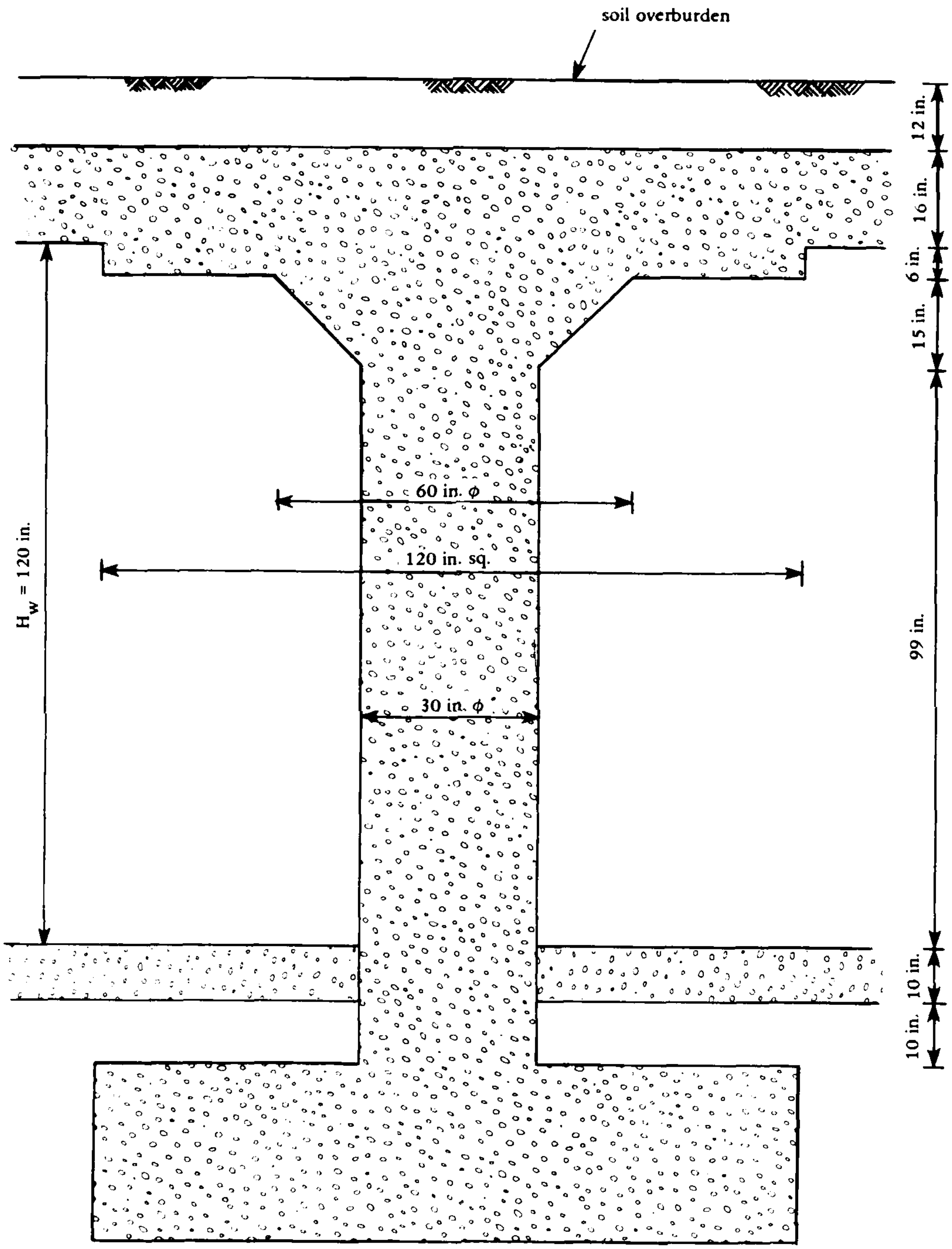


Figure D-3. Assumed column.

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