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INTRODUCTION TO HOLOGRAPHIC INTERFEROMETRY
APPLIED TO STRAIN DETERMINATION

by
S.J. RUMBLE

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INTRODUCTION TO HOLOGRAPHIC INTERFEROMETRY APPLIED TO STRAIN DETERMINATION

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S.J. RUMBLE

SUMMARY

The application of holographic interferometry to displacement and strain determination has been reviewed. A brief introduction to the theoretical basis of holography and holographic interferometry, and a summary of recent developments is given. It is concluded that with continued development, holographic interferometry has the potential to become a routine tool for whole field displacement and strain measurement, and non-destructive testing of aircraft structures.
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1. INTRODUCTION

An accurate knowledge of the stresses and strains in aircraft structures, in all stages of their lives, is vital for safe operation of aircraft. In the design and evaluation stages, individual components may need to be tested for ultimate strength and fatigue life. Large substructures eg. frames, spars, wings, require assessment for possible strength inadequacies or fatigue critical areas, which can be aided by identification of regions of high stress. Full-scale fatigue tests on complete structures require measurement of flight loads at critical areas, and subsequent verification that representative loads are applied during testing. Later in the life of aircraft it is desirable to know the changes in stresses and strains caused by modifications, cracks or other forms of damage, and also subsequent repair schemes.

At present the standard technique for strain measurement is the application of resistive strain gauges on the surface at the point of interest. However, this results in point-by-point measurement and strain gauges can be difficult to apply in regions of interest eg. strongly curved surfaces. The technique also requires an initial judgement of potential problem areas. Thus there has been continuing research into techniques which enable strain measurement over a large surface area. These are collectively called whole-field techniques, and include holographic and speckle interferometry, shadow moire, in-plane moire, thermoelectricity and microscopic measurement of deformation of very fine grids. Ultrasonics offer promise for residual and applied stress measurements within the volume of a specimen. All the above techniques are currently under development or consideration at ARL.

This paper reviews the application of holographic interferometry to whole-field surface displacement and strain measurements. It contains a brief introduction to the theory of holography and holographic interferometry (HI). This is followed by discussion of the interpretation of the HI data, experimental aspects of HI and recent areas of development.
2. BACKGROUND

The technique of HI has been used for quite some time (1-5) and there have been numerous examples of its application to specific problems (6-8). However, the acceptance of the technique by the general engineering community appears to have been limited by a number of factors. These include, 1) experimental difficulties due to vibration, laser problems and skill required by operator; 2) difficulties in interpreting data; and 3) time consuming and costly operation. In recent years these limitations have gradually been reduced. A number of commercially produced HI systems are available, including Newport Corporation HC-1000 and HC-2000 holographic cameras; Apollo Lasers Impart 1 Holocamera; Laser Technology, Inc Holomatic recording systems (9); and Rottenkolber HRC-110 Holo-recorder (10). These systems considerably reduce the skill and time required to record holograms. The accuracy and speed of interpretation of data is also improving, due to the use of photothermoplastic holographic cameras, digital imaging cameras, more reliable lasers and digital computer analysis. These improvements to the HI technique, along with the desire for a detailed knowledge of stresses and strains in aircraft structures, point to this field as an ideal area for the application of HI as a routine tool.

3. HOLOGRAPHY

Holography is a technique for storing the information required to recreate a portion of an optical field, and the subsequent recreation of that field. The region stored is that portion which can interact with the recording medium. The stored information and recording medium is said to be a hologram. A discussion of the theoretical basis of the technique is given in Appendix I. Until recently the most common recording medium has been high resolution (greater than 1000 line/mm) photographic plates. However, plates are gradually being replaced, for more routine applications, by systems utilizing photothermoplastic materials.

Holography differs significantly from ordinary photography. In photography the information recorded will only recreate the time-averaged intensity of the optical field (the square of the field's amplitude) at
the recording surface. By comparison, in holography, information is stored that enables the recreation of the amplitude and relative phase of the optical field over the recording aperture. Thus, the same three-dimensional optical field will be formed as that which occurred when the hologram was recorded (see Appendix I). This ability to store and recreate optical fields means that it is possible for the optical fields (of an object) to interfere even if they occurred at different times, and with the object in slightly different states. The next section shows that this interference can be used to determine the surface displacements between the two object states.

4. HOLOGRAPHIC INTERFEROMETRY

Holographic interferometry involves the comparison of the optical fields of an object in two or more slightly different states, and with at least one of the optical fields recreated with a hologram. The comparison results in the formation of a pattern of interference fringes. The fringe pattern can be related to the surface displacements of the object over the whole field of view. The main advantages of HI compared to other whole-field techniques, such as moire and gridding, are that the technique is non-contact, the object can have a complex three-dimensional shape and be diffusely reflecting. In addition the displacements can be measured down to a fraction of a wavelength of light if desired.

There are three main types of HI. The different types are distinguished by the number of separate object fields that are stored in the same recording plate. In the most common type, termed double exposure HI, the interference is between two optical fields stored on the same plate, each optical field having been recorded at separate times, with the object in slightly different states. The other main types of holographic interferometry are time-averaged HI and real time HI. In real time HI only an initial state of the object is recorded. The interference is between the reconstructed field and the actual object field. Hence the fringes alter in real time as the object is stressed or altered slightly. In time-averaged HI, the optical fields of a vibrating surface are recorded over a period which is long compared to the period of vibration. The resultant recording can be considered as the superposition
of a very large number of object states, each at a slightly displaced position. Because the object is stationary at the extremes of motion and also is close to the extremes for the majority of the period of vibration, the resultant fringe pattern is essentially equivalent to that which would be observed if a double exposure hologram had been recorded between those extremes.

The theoretical basis for the relationship between the fringe pattern and the surface displacements is most easily explained for the double exposure case. The other types can be explained as extensions to the double exposure case. An outline of the theoretical basis of HI is given in Appendix II; therefore only the final results will be shown here. In Appendix II it is shown that the fringe pattern is related to the phase differences between the two optical fields. The phase differences are introduced by the displacements of the object's surface between the two object states. For a small displacement \( \mathbf{d} \) of a general point \( Q \), shown in Figure 1, the resultant phase difference between the optical fields can be written as

\[
\delta \phi = \mathbf{K} \cdot \mathbf{d}
\]  

(1)

where \( \mathbf{K} = \mathbf{k}_2 - \mathbf{k}_1 \) is the sensitivity vector, with \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \) the propagation vectors of the incident and scattered light respectively. Using equation (1), the vector \( \mathbf{d} \) can be fully specified by utilizing different directions of \( \mathbf{K} \). This can be achieved by using different illumination directions and/or different observation directions.

There are a number of other techniques for determining displacement information developed by earlier workers in the HI field. These involve the concepts of fringe localization and parallax mentioned in Appendix II. The work by Briers (11) includes an historical review and summary of these interpretation techniques.

In a review of HI displacement measurement techniques using the concept of sensitivity vectors, Pryputniewicz and Bowley (12) found that the technique of Dhir and Sikora (5) was most reliable. In their method, multiple observation directions on the one hologram were used and a least-
squares analysis was applied to the data. In more recent work, Hariharan et al (13) have recorded multiple holograms with different illumination directions to enable determination of the vector displacements.

5. PHASE DETERMINATION

All the HI techniques that utilize the sensitivity vector concept require a measurement of the phase difference between optical fields. The main phase measurement techniques are photographic, optical heterodyning, and phase shifting. The photographic techniques rely on the following relationship for the intensity in an interference pattern (for simplicity given in one dimension only and following Katzir et al (14)).

\[ I(x) = I_o(x) \left[ 1 + V(x) \cos \phi(x) \right] \]  

(2)

where \( I_o(x) \) is the average intensity over a small region centred on \( x \), \( V(x) \) is the visibility and \( \phi(x) \) is the desired phase difference. The visibility \( V(x) \) is related to the local maximum and minimum intensities nearby to \( x \), \( I_{\text{max}} \) and \( I_{\text{min}} \) respectively, by the following expression.

\[ V(x) = \frac{I_{\text{max}}(x) - I_{\text{min}}(x)}{I_{\text{max}}(x) + I_{\text{min}}(x)} \]  

(3)

In general \( I_o(x) \) and \( V(x) \) are not constant because of variations in object reflectivity and object illumination, and localization of fringes. However, if they are approximately constant, then at the extremes of \( I(x) \), \( \phi(x) = n\pi \) (\( n \) integer). Hariharan (15) has noted that it is often difficult to locate the maxima of fringes to better than 0.1 of their spacing, and hence the accuracy of this technique is limited. Also when the maxima are unevenly and widely spaced, interpolation becomes uncertain. However, Katzir et al (14) have used this technique in conjunction with a 1024 pixel, linear solid-state light detector and computer processing, to demonstrate the capability of the technique for routine non-destructive testing.

In the heterodyne technique developed by Dandliker (16-18) each exposure of a double-exposure hologram is taken with a different, angularly separated reference beam of the same frequency. On reconstruction both reference beams are used. However a small frequency
shift is introduced to one of the beams using a rotating grating or acousto-optic modulator. It can be shown (15) that if the frequency shift is $\nu$, then the detected intensity from a point on the object will be modulated at the frequency $\nu$ and the phase will be equal to the optical phase difference. Currently this method is the most accurate of all phase measurement techniques. It has a phase accuracy of better than 0.3 degrees which corresponds to a displacement accuracy of $\lambda/2400$ for the most sensitive optical arrangement, where $\lambda$ is the wave length of the illumination. However its use is limited as it involves a slow, point by point measurement.

A good compromise between accuracy, and the simplicity and speed of data acquisition, is the technique involving phase shifting of the reference beam (19,20). In this technique, the real-time image is formed on a digital two-dimensional detector array and the outputs of the individual pixels are recorded by a computer as the phase of the reference beam is shifted relative to the object beam. The phase of the reference beam is changed, under computer control, by means of a mirror mounted on a piezoelectric translator. In the initial work by Hariharan et al (20), the intensities were recorded for phase shifts of $0, \frac{2\pi}{3}$ and $\frac{\pi}{3}$. The three intensities can be simply related to the optical phase difference between the two optical fields. However, as attaining exact phase shifts requires precise calibration of the piezoelectric translator, Creath (21) developed a system in which the actual phase shift need not be known. In this system four intensities are recorded for phase shifts of $3\alpha, \alpha, -\alpha$, $-3\alpha$, where $\alpha$ need not be known. A further development by Chang et al (22) involved the integration of the intensities into four regions as the phase is shifted. Again, there is a simple mathematical relationship between the intensities and the optical phase difference. The reported accuracies of the phase shifting techniques are $2^\circ$ and $1^\circ$ by Hariharan et al (2) and Chang et al (22) respectively. These phase accuracies correspond to displacement accuracies of $\lambda/360$ and $\lambda/720$ respectively for the most sensitive optical arrangement.
6. STRAIN DETERMINATION

While whole-field displacement information can be quite useful, for example in holographic non-destructive testing (23, 24, 25), it is often necessary to determine the local strains. The quantitative determination of strains using HI has been approached in a number of different ways. These have included, holographic moire (26-29), graphical interpolation and differentiation (30), fringe-vector theory (31-33), numerical interpolation and differentiation (34), and optoelectronic heterodyning (17, 35). More recently Sciammarella (36) has discussed a solution to the problem of complete determination of the components of the strain tensor by HI. The solution proposed, uses four illumination beam directions and a contouring technique, coupled with hologram analysis using electronic data processing techniques. The most suitable technique for a particular problem depends on the phase measurement method used, and the complexity of the strain field to be determined.

7. EXPERIMENTAL ASPECTS

The main elements in HI systems are a laser, optics, recording medium and vibration isolation table. A vibration isolation table is usually only necessary if a continuous wave (CW) laser is used. The most common CW lasers are He/Ne and argon ion lasers which have typical single line powers of 5 to 15mW and 2 to 5W respectively. However, whilst the argon ion laser has greater power than the He/Ne laser, it is significantly more expensive to buy and maintain. Even more expensive are the current types of pulsed lasers. The pulsed lasers need to have very short pulse widths (10 to 30nsec) and to be able to generate double or multiple pulses with a well defined time interval (1µsec up to 500µsec) between pulses and with energies from 10mJ to 3J per pulse. Currently, the pulsed lasers that can meet these requirements are Q-switched ruby lasers and Q-switched frequency-doubled neodymium:YAG lasers.

Requirements of all laser systems are that they operate in TEM$_{00}$ mode and have good temporal coherence. For practical work, the coherence length needs to be at least 10cm and preferably a few metres. To achieve the temporal coherence requirements, intracavity Fabry-Perot etalons can
be installed. This has the effect of reducing the number of longitudinal modes involved in the laser action of the cavity and consequently reducing the frequency uncertainty of the light output. Attainment of TEM\textsubscript{00} mode usually requires adjustment of intracavity apertures. In the case of pulsed lasers this leads to the system design of an oscillator cavity followed by one or more amplifier stages to increase the energy per pulse.

The typical optical components required for a HI system are beam steering mirrors, beam splitters, beam expanders with spatial filters, power-meters, plate holder, exposure control shutters, light baffles and possibly piezoelectric translators and digital camera system. The type of optical components required depends on whether a CW or pulsed laser is used. The high power (up to several hundred MW) of pulsed lasers, necessitates careful selection of mirror coatings, beam splitting system and spatial filtering.

There are numerous recording mediums available that are capable of the resolution required for holography. Hariharan (15) has summarized the materials and included a brief discussion of the principles of operation of each material. The most common photographic emulsions are Agfa-Gevaert (holotest) 10E75, 10E56, 8E5cHD, 8E75HD and Kodak 649F. These can all be processed in a variety of ways to create phase or amplitude holograms. Over recent years advances in photothermoplastic materials and techniques, have led to commercial availability and to subsequent widespread use.

During recording of a hologram, the relative motions of the object, optical components and plate must be less than \( \lambda/4 \). If this requirement is not met, then the interference fringes, formed between the scattered light and the reference beam, will be washed out, and a hologram will not be created. As exposure times of a number of seconds are common, when using CW lasers, it is often necessary to mount all components and object on the same rigid surface, which is vibration isolated from the floor. When vibration isolation is not possible, or the object is moving, pulsed lasers need to be used. In this case, a hologram can still be recorded with object velocities up to 2.9 m/sec (7) if the pulse width is less than 50nsec.
8. RECENT DEVELOPMENTS AND APPLICATIONS

There has been quite a diverse range of applications of HI reported in recent publications. In particular, the work of Sharnoff (37) on microdifferential holography and of Rastogi (38-40) on comparative holographic moire interferometry, represent significant advances on past applications. Sharnoff introduced a technique enabling discrimination between amplitude and phase changes by an object in double exposure holograms. This has special application to the use of HI on objects viewed at a microscopic level. The comparative holographic moire interferometry technique, developed by Rastogi, enables comparison of a fringe pattern response of a master specimen to a test specimen, subjected to the same stress levels. This has important applications as a nondestructive inspection tool.

More conventional applications of HI have been reported by Snell and Marchant (41, 42), Mader (43), Kabunai et al (44), and Hsu and Lewak (45). Snell and Marchant have used HI to determine the flexural stiffness of anisotropic carbon composite plates. Mader applied HI to investigate the deformation of pipes, cylinders and tubes, when they were subjected to various loads and thermal expansions. The work of Kabunai et al, looked at the detection of micromechanical phenomena using the high accuracy available with heterodyne HI. In particular, a very accurate measurement was made of the apparent Young's modulus during the bending of a cantilever beam. This was compared with the results from couple-stress theory. Hsu and Lewak have attempted to address the problem of using HI when large distortions occur. In these circumstances, the fringe density becomes so large, that the accuracy rapidly falls off. A holographic cinematographic technique was developed in which a moving shutter allowed high speed multi-exposure HI to take place. The system was designed to verify experimentally a theoretical model of the deformation of nuclear reactor fuel cladding. Rapid deformations can occur when large power transients are generated during a loss of coolant accident. Initial experience with the camera system revealed that significant errors arose due to the high sensitivity of the data analysis to the position of the viewing points. Subsequent tests, with modifications to the camera, agreed more closely with the theoretical results.
Other developments of HI have included work by Stetson and Brohinsky (46) on the application of electro-optic holography to HI and improvements by Wagner (47) of the heterodyne HI technique to include holographic recording of transient phenomena using a triple exposure dual-reference beam arrangement. Also of interest is the recent review by Reynolds (48) of nondestructive testing of fibre-reinforced composite materials. Reynolds gives a brief report of the application of HI to nondestructive testing, by a number of aircraft manufacturers.

A related technique to HI, worthy of mention and which has been the subject of much development, is the technique of speckle-pattern interferometry. The technique, which was primarily developed by Butters (49, 50), has lately been given added impetus by the commercial production of the Ealing Electro-optics VIDISPEC system. This system uses a video camera and digital frame store to produce fringe patterns containing information about the deformation of a surface. Recent work involving speckle interferometry has included theoretical development (51 - 53), application to vibration observations (54), and deformation and strain measurements (56 - 58).

9. CONCLUDING REMARKS

Holographic interferometry has the potential to determine surface strains, produced in response to applied loads or thermal stress, over complex three-dimensional shapes. The technique is non-contact and usually requires no surface preparation. HI utilizes the ability of holography to store and recreate optical fields of an object. This enables interferometric comparisons of optical fields of an object in slightly different states. This comparison can be interpreted so as to give the displacements and strains of each point on the surface of the object. With continued development, HI can become a routine tool for whole field displacement and strain measurement, and nondestructive testing of aircraft structures. This would lead to a more accurate and detailed knowledge of the stresses and strains in aircraft structures and to potentially safer and higher performance aircraft.
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APPENDIX I  Brief theory of holography

If an object is illuminated with an ordinary incandescent lamp, the electric field at any given point in the resultant optical field, will be undergoing rapid, random variations in magnitude and direction. The variation will have frequency components in the range $4-7 \times 10^{14}$ Hz and there will be poor correlation between different points in the field. However, if the object is illuminated with monochromatic, coherent light, the resultant electric field $E$ at any given point will be varying sinusodially at a single frequency and with constant amplitude and phase. It can also be correlated with other points in the optical field. This variation can be expressed as follows:

$$E(x,y,z,t) = E_0(x,y,z) \cos(2\pi vt - \phi(x,y,z))$$

where $E_0$ is the amplitude, $v$ the frequency and $\phi$ is the phase. The single frequency nature of the light enables a complex amplitude notation to be used for the electric field. This has the form

$$E(x,y,z) = E_0(x,y,z) \exp(-i\phi(x,y,z)).$$

This notation will be used in the following discussion as it simplifies the presentation of the mathematical representation of holography.

If a large opaque screen, with an aperture, is placed in the optical field, the electric field on the opposite side to the object is determined by the values of the electric field over the aperture. As the aperture defines a planar surface, the electric field at this surface can be written as

$$E(x,y) = E_0(x,y) \exp(-i\phi(x,y)).$$

The significance of this, is that if this two dimensional variation of electric field can be reproduced by some other means, then the field in the half space beyond the plane will be identical to the original field. Thus only two-dimensional information is required to create a three-dimensional electric field variation that can be interpreted by the eye or
other optical system as a three-dimensional image. A two-dimensional variation of electric field can be produced using holography. In this case the aperture is the photographic plate. Detailed explanations of holography have been given in many texts (7, 15, 59, 60). A brief explanation following Hariharan (15) is as follows.

Consider an object illuminated by coherent, monochromatic light. Let the variation of electric field at the photographic plate due to light scattered from the object be as follows:

\[ o(x,y) = |o(x,y)| \exp[-i\phi(x,y)]. \]

If in addition to the object field, a plane wave beam is allowed to fall at an angle \( \theta \) to the photographic plate (see Figure 2), then the variation of electric field due to this beam is

\[ r(x,y) = r \exp[i2\pi \xi] \]

where \( \xi = \sin(\theta)/\lambda \). This beam is called the reference beam. The photographic plate responds to the intensity of the combination of these two waves. The resultant intensity is

\[
I(x,y) = |r(x,y) + o(x,y)|^2
= |r(x,y)|^2 + |o(x,y)|^2
+ r|o(x,y)| \exp[-i\phi(x,y)] \exp[-i2\pi \xi]
+ r|o(x,y)| \exp[i\phi(x,y)] \exp[i2\pi \xi]
= r^2 + |o(x,y)|^2 + 2r|o(x,y)| \cos(2\pi \xi \cos(\phi(x,y)))
\]

If the photographic plate is processed so that its amplitude transmittance is linearly related to intensity, then the transmittance can be written as

\[
t(x,y) = t_0 + \kappa \left( |o(x,y)|^2 + r|o(x,y)| \exp[-i\phi(x,y)] \exp[-i2\pi \xi]
+ r|o(x,y)| \exp[i\phi(x,y)] \exp[i2\pi \xi] \right)
\]
where \( t_0 \) is the constant transmittance from \( r^2 \) and background, and \( k \) is related to exposure and development characteristics, and exposure time.

To reconstruct the original object field at the plate, the hologram is illuminated with the same reference beam used to record it (see Figure 3). The complex amplitude of the transmitted wave is then

\[
u(x,y) = r(x,y) t(x,y)
\]

\[
= t_0 r \exp \left(-i2\pi r x \right)
\]

\[
+ kr |o(x,y)|^2 \exp \left[i2\pi x \right]
\]

\[
+ kr^2 |o(x,y)| \exp \left[-i\phi(x,y) \right] \exp \left[-i2\pi x \right] \exp \left[+i2\pi x \right]
\]

\[
+ kr^2 |o(x,y)| \exp \left[+i\phi(x,y) \right] \exp \left[+i2\pi x \right] \exp \left[-i2\pi x \right]
\]

\[
= t_0 r \exp \left[i2\pi x \right]
\]

\[
+ kr |o(x,y)|^2 \exp \left[i2\pi x \right]
\]

\[
+ kr^2 o(x,y)
\]

\[
+ kr^2 o^*(x,y) \left[+i4\pi x \right]
\]

The third term in the above expression for \( u(x,y) \) is the required original field, except for a constant term. The other terms have the following explanations. Term one is an attenuated reference beam around which a halo is created by term two. Term four is the conjugate object field modified by the \( \exp[14\pi x] \) term. This exponential term indicates that the propagation direction is altered compared to the original object wave. The conjugate object wave is deflected off the reference beam axis by an angle approximately equal to the angle between the reference beam and the normal to the hologram.
APPENDIX II  Brief theory of holographic interferometry.

The theoretical basis of HI has been developed and discussed in numerous papers and texts (11, 15, 60-66). The following discussion has been drawn from a number of the above works. It is well established that if two optical fields of a diffusely reflecting object, in slightly different states, are arranged to interfere (using holographic techniques), then there will be a modulation of the intensity in the reconstructed image. This modulation of intensity is often referred to as the fringe pattern. However, the fringe pattern is in reality a separate entity to the image, as the region in space at which the viewing optics must be focussed to give maximum visibility is not, in general, in the region of the image. This region in space where the fringes do form is termed the region of localization. The position of the region of localization depends on the type of displacement undergone by the object. Simultaneous viewing of the fringes and image thus requires a lens system with an adequate depth of field. Whilst in early work the position of localization of fringes was used to calculate displacements, it was found that this information is not necessary.

The relevant information is the phase differences between the two optical fields at the point of interest on the object, the phase difference being due to the path difference caused by the displacement of the point. The measurement of these phase differences requires the formation of an image by an optical system. The finite resolution of this system will result in light from a small area on the object impinging on each point in the image. Because the two fields are coherent there will be interference between the two fields. There will also be random phase variations in each object field due to the diffusely reflecting nature of the surface. The random phase variations in each optical field will only be correlated with each other if the light came from the same region on the object. Correspondingly, the random phase variations of the light from different regions of the object, in the two object states will be uncorrelated. Thus at a point in the image, the observed phase difference will be due to the correlated component which arises from the same point on the object in its two states. Therefore there is a one-to-one correspondence between the phase difference measured at a point in the image and the displacement of the corresponding region on the object.
the following section the mathematical relationship between the phase differences and the displacements is derived.

Consider the interference corresponding to a displacement \( d \) of a point \( Q \) on an object illuminated by a point source, as shown in Figure 4. The point source at \( O \) has been arbitrarily chosen as the origin. The position vectors of the point \( Q \) before and after the displacements are \( r \) and \( r' \) respectively. The object and fringes (assumed to be localized at the object) are viewed from \( S \) which has the position vector \( R \). A discussion of the more general case when the fringes are not localized at the object has been given by Dubas and Schumann (57). The light propagation vector for the light illuminating \( Q \), before and after displacement are \( k_1 \) and \( k_2 \) respectively. The propagation vectors have a magnitude of \( 2\pi / \lambda \) where \( \lambda \) is the wavelength of the light used. The phase of the light travelling along paths \( k_1 \), \( k_2 \) and \( k_3 \), \( k_4 \) can be written as

\[
\phi_1 = \vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot (\vec{R} - \vec{r}_1) - 3\pi \quad (1)
\]

\[
\phi_2 = \vec{k}_3 \cdot \vec{r}_3 + \vec{k}_4 \cdot (\vec{R} - \vec{r}_3) - 3\pi \quad (2)
\]

where \( 3\pi \) is a constant phase change on interaction with the surface at \( Q \). The propagation vectors \( k_3 \) and \( k_4 \) can be written as

\[
k_3 = \vec{k}_1 + \Delta k_1 \quad (3)
\]

\[
k_4 = \vec{k}_2 + \Delta k_2 \quad (4)
\]

The phase difference \( \delta \phi = \phi_1 - \phi_2 \) can then be written as

\[
\delta \phi = \phi_1 - \phi_2 = (\vec{k}_1 - \vec{k}_2) \cdot (\vec{r}_1 - \vec{r}_3) + \Delta \vec{k}_1 \cdot \vec{r}_3 + \Delta \vec{k}_2 \cdot (\vec{R} - \vec{r}_3) \quad (5)
\]

As \( |d| = |r_3 - r_1| \) is typically only several microns and \( |r_1| \) and \( |r_3| \) are typically approximately equal, then \( |r_1| \) and \( |r_3| \) are very nearly equal and \( \Delta k_1 \) and \( \Delta k_2 \) are perpendicular to \( r_3 \) and \( (\vec{R} - \vec{r}_3) \) respectively. Therefore the last two terms in equation (5) vanish and the phase difference reduces to
\[ \delta \phi = (k_2 - k_1) \cdot (r_1 - r_3) \]
\[ = K \cdot d \]

where \( K = k_2 - k_1 \) and \( d = r_3 - r_1 \). This result is fundamental to determining displacements using HI. The vector \( K \), called the sensitivity vector, is shown in Figure 1. It bisects the angle \( 2\psi \) between the illumination and viewing directions, and has a magnitude of

\[ 2k_o \cos (\psi), \text{ where } k_o = 2\pi/\lambda. \]
\[ k = k_2 - k_1 \]

sensitivity vector

\[ |k_i| = \frac{2\pi}{\lambda} \]

\( i = 1, 2 \)
FIG. 2. OPTICAL ARRANGEMENT DURING RECORDING OF A HOLOGRAM.
FIG. 3 Optical arrangement during reconstruction.
FIG. 4 OPTICAL ARRANGEMENT FOR CALCULATION OF PHASE DIFFERENCE
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### INTRODUCTION TO HOLOGRAPHIC INTERFEROMETRY APPLIED TO STRAIN DETERMINATION

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**Abstract**
The application of holographic interferometry to displacement and strain determination has been reviewed. A brief introduction to the theoretical basis of holography and holographic interferometry, and a summary of recent developments is given. It is concluded that with continued development, holographic interferometry has the potential to become a routine tool for whole field displacement and strain measurement, and non-destructive testing of aircraft structures.
This paper is to be used to record information which is required by the Establishment for its own use but which will not be added to the DISTIS data base unless specifically requested.

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