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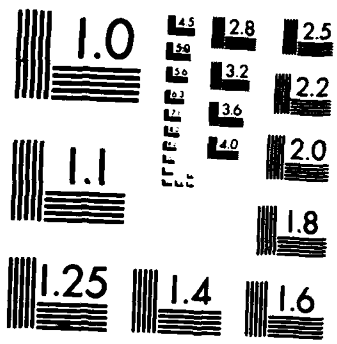

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APPLICATION OF ADAPTIVE CONTROL TO AIRCRAFT SYSTEMS

Chen Zongji and Zhang Hongyue

(Beijing Institute of Aeronautics and Astronautics)

Abstract

This paper reviews the competition of the adaptive technique with other techniques. The inherent drawback of the adaptive technique in application to aircraft systems is considered in the light of the analysis of the features of flight control systems. The potentials and perspectives of the adaptive technique are also suggested.

The development of advanced aircraft demands enhanced performances which go beyond the capacity of airdata scheduling technique. Therefore, the adaptive control schemes are expected to be applied to these flight control problems. In order to make full use of the advantages, such as saving sensor hardware, requiring less priori knowledge of aircraft and providing higher flexibility, it is necessary to carry out robustness research of existing adaptive control schemes and to develop new more robust adaptive control schemes.

APPLICATION OF ADAPTIVE CONTROL TO AIRCRAFT SYSTEMS

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Submitted 20 June 1984

In 1979, Stein[1], Kreisselmeier[2] and Rynaski[3] presented separate papers on the application of adaptive control to aircraft systems in the symposium for the Application of Adaptive Control. This paper will, through a general review of these papers, establish a clearer knowledge on the application status of adaptive aircraft control, the problems it faces and its possible application direction and perspectives.

I. Preface

For twenty or more years, many theoreticians and practical engineering technicians have expended much effort trying to apply the adaptive control to aircraft systems. However, very few results have been obtained up to now despite the efforts.

Theoretically, the aircraft control problems which have the characteristics of parameter uncertainty and slow time-variance should be a perfect domain for the application of the adaptive control. Yet, why has the application of adaptive control to aircraft systems been so limited? This is a question that has puzzled many adaptive control theoreticians and aeronautical engineering technicians who are interested in adaptive control. In 1979, in the Symposium for the Application of Adaptive Control held by Yale University, Stein, Kreisselmeier and Rynaski presented papers on the application of adaptive control to aircraft systems. These were more encompassing, more in-depth papers with substances prepared by people who tried to answer the above question in recent years. This paper will, on the basis of introducing these papers and combining with the development of adaptive control in recent years, conduct a general review on the application status of adaptive control to aircraft systems, existing special problems, its possible application direction and perspectives.

II. Features of an Aircraft and Its Control Systems

The rigid aircraft is a moving object of the complex six degrees of freedom and multiple maneuvering surfaces. Its demands for high speed, high mobility and high reliability make the control problem of the aircraft more complicated. In addition, the aircraft control systems have the following two special problems, i.e., the uncertainty of the controlling object's parameter and model, and the man-aircraft union.

1. Uncertainty

The uncertainties of aircraft are expressed in two areas, i.e., parameter uncertainty and model uncertainty.

The parameter uncertainties of aircraft are caused by the flight conditions with a large range of change and the change in aircraft structure. The change in flight conditions mainly means the change in flight altitude, flight velocity and flight posture. The change in aircraft structure mainly means the change in fuel, loads and geometric dimensions during the flight process. These changes cause the parameters in the aircraft's dynamics and kinematics equations to change. During the flight process, the changes are actually unpredictable. However, research has shown that numerous parameter changes can be grouped eventually into a few parameter changes of slow time-variance. Take the vertical control problem, for example: the dominant parameters are the pitching moment M_δ generated by the stabilizer and the pitching moment M_α generated by the attack angle[4]. It must be pointed out that the flight enveloping lines of modern aircraft have a tendency to increase to a large attack angle and post-stall ability. However, we still lack sufficient knowledge on the aircraft dynamic responses under this kind of flight condition. Moreover, the change of these parameters are no longer of slow time-variance.

The uncertainty of the model means the unknown high frequency dynamic responses or those that are not considered during the process of model building. They include high frequency structural harmonic vibration, unstable air distribution dynamic responses, hydraulic mechanism, transfer delay of aerodynamic responses, etc. In essence, aircraft is a dynamic system of infinite dimension, but low order systems are generally used to approximate in designing. For example, the low frequency input-output responses of the aircraft in vertical motion can be approximated by a third-order rigid model or an elastic model with a little higher order, but there is great difference between its high frequency end and the third-order model. Figure 1 gives the Bode diagram of the third-order transfer function. Meanwhile, it also gives the enveloping lines of the actual aircraft frequency response. The frequency response of the aircraft high frequency end can be any continuous, smooth curve in the shaded area of the figure.

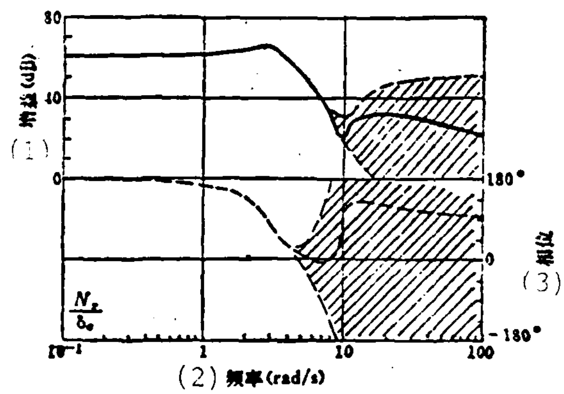

Fig. 1. [illegible] frequency response and high frequency uncertainties
Key: (1) Gain; (2) Frequency; (3) Phase.

2. Man-Aircraft System and Flight Quality

The purpose of any flight control system is to improve or increase the flight quality of aircraft, and the flight quality defines the coordinating relations of the man-aircraft system. The flight quality of the vertical short cycle motion is related to the three parameters of the aircraft dynamic response, and they are the zero point, inherent frequency and damping ratio of the pitching speed with respect to the stabilizer transfer function.

According to the U.S. military aircraft flight quality specification MIL-F-8785B(ASG)[5], for first class flight quality standard, the allowable change for the damping ratio is about 4 times and the allowable change for the inherent frequency is about 10 times. Accordingly, the pilot has strong adaptability to the large range change in aircraft inherent frequency, i.e., the man-aircraft system actually includes an "adaptive controller"-pilot.

Yet, research has shown that the pilot's adaptability to time delay is weaken. When the pilot gives an instruction to the aircraft, he expects the aircraft to have immediate response to the instruction. If there is a 100 ms delay caused by the computation time of the computer and other factors, this will seriously affet the pilot. In addition, the pilot's adaptability to the response of the nonminimal phase is also very poor. The nonminimal phase will cause its initial response to be in the opposite direction of the demand of the instruction.

From the standpoint of flight quality demand of the man-aircraft system, the dynamic response of the aircraft is allowed to vary within a larger range, but the higher order dynamic delay, transfer delay of the numerical computer and the nonminimal phase should be avoided if possible.

III. Adaptive Control and Its Control Scheme in the Competition History of Aircraft Systems

In the history of aeronautical development, the competition among various control schemes has usually been fierce. People generally think that the adaptive technique performs better than the regular feedback system in systems with uncertain situations. Therefore, from the stand point of uncertainty of the aircraft dynamic response, the adaptive technique, in principle, should be a more ideal control scheme. However, whether the aircraft manufacturing companies and customers are willing to adopt this scheme still requires consideration of aircraft performance, cost effectiveness, reliability, etc. provided by said scheme. In the early 60's and mid 70's, there had been two rounds of competition between the adaptive technique and other control schemes, and each time was ended with the defeat of the adaptive control scheme.

The primary competitor of the adaptive control scheme is the airdata scheduling control laws. It can be seen from section II that the change of aircraft parameters can be grouped into the change of certain dominant parameters, such as M_δ and M_α, and these dominant parameters are closely related to airdata kinetic pressure, Mach number or attack angle. Thus, the control laws expressed in M_δ and M_α can be approximated by the control laws expressed in kinetic pressure, Mach number and attack angle.

As early as the 60's, people had higher requirements for flight enveloping lines and performance of military aircraft. But the manufacture and maintenance of an airdata sensor with high accuracy and good reliability at the time could not satisfy the requirements for the airdata scheduling control; thus, the adaptive control scheme was once a very popular scheme and was involved in the test flights of the X-15, F-111, F-101, and F-4 test aircraft. The test flight results were not good, and especially the crash of the X-15 test aircraft had a bad psychological influence on the application of adaptive control to aircraft systems. At the same time, with this, the accuracy and maintainability of the airdata sensor had been improved, and its cost had dropped. Consequently, airdata scheduling control laws became the primary scheme for various aircraft control systems thereafter. Therefore, some people think that the control engineer had lost to the sensor manufacturer in the first round of competition between the adaptive control scheme and the airdata scheduling control scheme.

However, the competion between the two schemes did not end. In the mid 70's, another generation of advocates for adaptive control started the second round of competion, and their contentions are:

1. There have been new developments in the theory of adaptive control, and they are primarily displayed in the areas of: development in the identification theory (e.g., parameter estimation using on-line maximum approximation method); development in the stability analysis and verification of adaptive control (the use of [Liyapnov's] stability and superstability theories to design); further perfection of the theory and application of the self-tuning regulator.

2. The rapid development in computer software and hardware technologies make some of the complex adaptive control laws easy to be implemented. For example, the F-8 on-board computer AP-101 needs only 5 ms in every sampling period to complete the solution[4] of on-line parameter estimation and control laws using the maximum approximation method when the sampling frequency is 50 Hz.

3. The adoption of a fly-by-wire control system must require the use of the redundancy technique. Consequently, it is inevitable that the circuitry of the airdata sensor and measurement will multiply manyfold in the scheduling control laws. This kind of increase not only directly raises the cost, but also causes difficulties as to finding proper places to install these sensors.

4. some of the control laws cannot be simply expressed by kinetic pressure, Mach number or attack angle (e.g., the active control and direct force control modes of oscillation/tremor).

The above contentions received much attention from the aircraft control industry. NASA had arranged adaptive control testing in the digital fly-by-wire control system experimental project of F-8C. But the testing results showed that the performance of aircraft employing the adaptive control scheme was not as good as expected; it did not even exceed the performance of the airdata scheduling control scheme. The adaptive scheme used in the F-8C and based on the on-line identification of M_δ , M_α and air velocity may perform satisfactorily only under the effects of sustained and excited experimental signals. Therefore, the adaptive control scheme represented by the self tuning adaptive scheme had lost again in the second round competition with the airdata scheduling control scheme.

IV. Existing Problems in the Application of the Adaptive Technique to Aircraft Systems

This section will, through the analysis of the existing problems in the application of adaptive control to aircraft systems, attempt to understand why the adaptive control scheme had failed in past competitions.

The two primary schemes of the adaptive control are: direct adaptive control and indirect adaptive control. The direct adaptive control is also known as the model reference adaptive. This scheme does not need to conduct identification of the system's parameters; it adjusts the controller parameters according to a certain stability standard, making the output of the controlled object to follow asymptotically the output of the reference model. Contrary to this, the indirect adaptive control, represented by the self tuning regulator, first needs to conduct identification of the system's parameters, then, based on these parameters and according to a certain standard, the appropriate control laws are generalized. We will analyze below, respectively, what problems will occur when the two schemes are applied to aircraft systems.

1. Model Reference Adaptive Control

The structural principle of the model reference adaptive control system is as shown in Fig. 2. Basically, it uses the feedback compensator with adjustable parameters to eliminate the object zero point and assigned model extreme points; the model zero point is introduced using the [feedforward] compensator, making the combined system of the controlled object, [feedforward] and feedback compensators and the reference model of equal value. In this process, since the zero point of the object is eliminated, the model reference adaptive scheme cannot be applied to the nonminimal phase system.

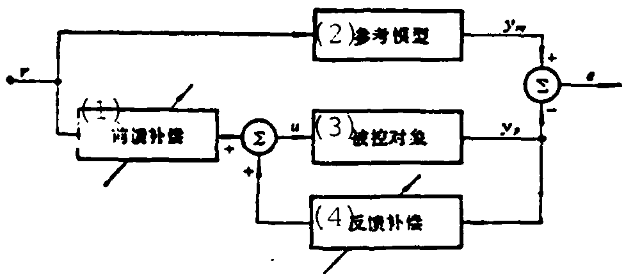

Fig. 2. Model reference adaptive control system
Key: (1) [Feedforward] compensator; (2) Reference model; (3) Controlled object; (4) Feedback compensator.

We also noticed that the parameters of these compensators are regulated based on the errors between the object and model output, thus, such control laws cause serious nonlinearity. This will affect the pilot's judgment and maneuvering.

The verification of the asymptotic stability of the model reference adaptive system is conducted under a series of assumptions. These assumptions are: no external interference, the highest order and the relative order (the order difference of the denominator and numerator) of the object are known, minimalphase system and the sign of gain of the object is known. However, since these conditions are not all present in our actual design problem, we still lack clear knowledge on the stability of the entire system when the adaptive control scheme is applied to the actual aircraft control systems.

From the standpoint of the multi-purposes of an aircraft, a single reference model is not capable of handling it. For example, the dynamic effects of an aircraft during landing are impossible to be as fast as those at high flying speed. Consequently, the reference model will be adjusted according to flight duty, and this will increase the complexity in design.

The advantages of the model reference adaptive are that it does not require sustained and excited signals, nor does it require that the adjustable parameters of the controller converge uniformly, and the object output is guaranteed to follow asymptotically the model output.

It can be seen from the above analyses that before the robustness of the model reference adaptive system for external interference and model error is verified, the aircraft manufacturer and the customers cannot accept this scheme psychologically. Meanwhile, the introduction of serious nonlinearity and the nonapllicability to the non-minimal phase system also bring about certain difficulties in the application of the said scheme.

2. Self Tuning Regulator

The structural principle of the self tuning regulator is shown in Fig. 3. It conducts on-line identification of the system parameters first, then the obtained parameter estimation is treated as the true value of the system parameter and the control laws are combined according to different design standards. These design standards can be model tracing, extreme points assignment, minimum direction difference, etc. Therefore, the self tuning regulator is more flexible.

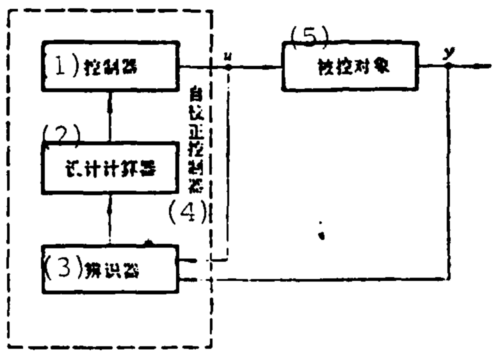

Fig. 3. Self tuning regulator
Key: (1) Controller; (2) Design calculator; (3) Identifier; (4) Self tuning controller; (5) Controlled object.

Kreisselmeier completed the verification[7] of full stability under sustained and excited conditions for one kind of adaptive observer and condition feedback control laws. Yet, since the regulation law of the current self tuning regulator is not built upon a stability theory, the verification of general full stability has yet to be completed.

It should also be pointed out that the on-line parameter estimation up to now is still not a mature technique. Even the off-line parameter estimation, according to experience, is a method whose accuracy is difficult to guarantee. It is determined by the dynamic features of the object accuracy of the sensor, motion features of the aircraft and the identification method adopted. Since the success or failure of this kind of scheme is determined by the accuracy of the parameter estimation, this requires that the object be subject to sustained and excited input signals to assure that the parameter estimation value converges at the true parameter value. Kreisselmeier believes that, in the flight evnironment of interference caused by scattered wind, the additional excited signals are no longer needed. Stein, however, points out that the test flight of F-8C showed that the excited signals were the key to obtaining satisfactory performance.

The only information required before designing the self tuning regulator is the order of the system, unlike the model reference adaptive which requires more prior knowledge. Also, the self tuning regulator designed according to extreme point assignment and generalized standards for minimum direction difference can be applied to the nonminimal phase system.

According to the above analyses, it is believed that the self tuning adaptive control scheme is more robust is the apllication to aircraft control systems. But the self tuning adaptive still lacks clear conclusions in the area of full stability and requires sustained and excited signals. This also hampers the smooth application of this scheme.

V. The Possible Application Direction and Perspectives of the Adaptive Control in the Aeronautical Industry

Since the adaptive control cannot demonstrate its superiority in situations where the airdata scheduling control laws also apply, we should search for the applications of the adaptive contorl is situations where the scheduling control laws do not work.

For example, in the high speed flight during re-entry, the airdata cannot be timely and accurately measured; for the control of a small guided missile, it is not only ineconomical to install a highly sensitive sensor and corresponding measurement circuitry for scheduling, but also will be limited by its structure; in the active control duties, such as direct force control and oscillation/tremor, etc., the interrelation between the airdata and the parameters of the controlled object is very poor. The scheduling control laws are not suitable for these situations. We will introduce in more detail below why the airdata scheduling control does not work in direct force control and oscillation/tremor suppression.

1. Direct Force Control

The primary trend of improvement of modern fighter performance is to increase the so-called performance of direct force cotnrol. The multiple control surfaces of this kind of aircraft can use the coordinated maneuvers of a certain coupling form to generate direct lift or lateral force, but not the pitching or deviation moment. This has greatly increased the performance of aircraft. In the desigr of this kind of control system, we must know the aerodynamic response parameters of each control surface. Yet, the interrelation between these response parameters and the airdata is rather poor, making it impossible to adjust the controller parameters according to airdata.

2. Oscillation/Tremor Suppression

For light structure or high speed aircraft, the interplay between the structural dynamic response and air flow becomes more distinctive. When a light damping vibration is generated by these interplays, it is called oscillation; when these vibrations are unstable, they are called tremor. These phenomena all occur during high speed flight. In order to prevent aircraft structural fatigue and crash, the speed of aircraft is somewhat limited. In recent years, the expansion of such speed limit has gained extensive attention. However, due to the complexity of the oscillation/tremor model, the research in this kind of control problem is progressing quite slowly. The conlexity of the oscillation/tremor model is displayed in that it includes numerous structural vibration modes that interplay with air flow in the $10\sim200$ Hz high frequency end, and the uncertainty of these modes as the flight conditions change. The active control duty of oscillation/tremor is to measure the vibration mode parameters that require suppression at appropriate locations using the appropriate sensor, and then increase the damping of these modes by designing an appropriate compensator. When it is necessary to design the oscillation/tremor controller in full flight enveloping lines, the location of the sensor and the parameter of the compensator must all be adjusted as the mode changes. Since the airdata are only partially related to these modes, they still do not possess this kind of adjustment means up to now. The adaptive control will be one of the methods that could possible solve this type of problem.

Stein, Kreisselmeier and Rynaski also believe that:

(1) The self tuning adaptive scheme is more robust in the application to aircraft systems than the model reference adaptive, since the model building error of the model reference adaptive will introduce higher order dynamic influences.

(2) Unless the speed of the numerical computer is very high to keep the computation delay less than 50 ms, it is still better to use a simulation controller.

(3) Measures must be taken to increase the accuracy of the online parameter identification, since identification error can also introduce high order dynamic delay.

It must be pointed out here that their conclusions were based on the results in the theory and application of adaptive control before 1979.

In recent years, the theoreticians of adaptive control have started to pay attention to the research of robustness of the model reference and model error. Kreisselmeier and Narendra[8] verified that under bounded disturbances all the signals of the model reference adaptive system are bounded. Kokotovic[9] analyzed that when the high speed mode has weak influences on the system, or the high speed mode with strong influences is subject to low general filtering, the model reference adaptive system has robustness with respect to model error. The authors of this paper[10, 11] analyzed, using the stability theory, that the model reference adaptive system possesses robustness with respect to the external interference and model building error. There has been progress in how to design a model reference adaptive system with strong robustness. Peterson and Narendra[12] have established a nonlinear adaptive controller to suppress the influences on system by external interference. At the same time with this development, the verification of the full stability of the self tuning adaptive has not obtained a true breakthrough, and it seems that the requirement for sustained and excited signals is inevitable. From the above analyses, the point that the model reference adaptive will be more robust in applications from now on is not unfounded.

Recently, the theoretical study of constant parameter robust control and the research in its application to aircraft control systems have gained substantial progress[13, 14]. In order to develop purposefully the study of applications of various techniques, we must clearly understand under what conditions can the constant parameter robust controller satisfy the performance requirements; under what conditions will the airdata scheduling controller be a better control scheme; and under what conditions is the adaptive controller a must. The research in this area requires concerted efforts by the theoreticians and the flight control engineers.

As of now, the existing problems with the airdata scheduling control laws not only still remain unsolved, but will also become more noticeable as the flight enveloping lines of modern aircraft further enlarge and its mobility further increases. At the same time with this, the further perfection of adaptive control theory and the rapid development of computer software and hardware technologies will make the application of adaptive control to flight control systems more and more mature. All these indicate that, in the development of aircraft control systems, the third round competition between the adaptive control and the airdata scheduling control schemes could be inevitable.

VI. Conclusions

Due to the special features and inherent defects of the aircraft control systems, the application of adaptive control to flight control systems is not successful up to now. However, as the performance requirements of modern aircraft are raised further, and with the further perfection of adaptive control theory and the development of computer software and hardware technologies, the adaptive scheme could expect to be used in situations where the airdata scheduling scheme does not work. Accordingly, the advantages of the adaptive control scheme such as saving sensor and measurement circuitry, requiring less prior knowledge, saving the expenditure and time of wind test, shortening the design cycle, providing higher flexibility, etc. will be fully utilized.

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APPLICATION OF ROTOR INDUCED VELOCITY FIELD OF THE VORTEX THEORY TO THE STUDY OF STABILITY AND CONTROLLABILITY OF HELICOPTER WITH HINGELESS ROTOR

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Abstract

With the aim of developing a new rotor system, a method is presented for calculation of stability and controllability of helicopter with hingeless rotor. The effects of nonuniformity of rotor induced velocity distribution and blade flapping elasticity are taken into account and investigated in detail.

It is the first time that the first harmonic term of rotor induced velocity distribution derived from the generalized vortex theory of rotor is employed and a close form of equation for the induced velocity and circulation is established. Finally, analytic formulae suitable to engineering application are given, which consider the effects of motion parameter, control parameter, blade flapping elasticity and so on. Also, the effect of the second flapping mode on stability and controllability of helicopter with hingeless rotor is discussed in an all-round way. The calculations show that only the first flapping bending mode should be taken into account in stability and controllability calculation of helicopter with hingeless rotor in the range of $\mu < 0.3$.

Finally, a typical helicopter Bo-105 is taken as a numerical example, and it is found that the results are in good agreement with its flight-test data[8].

APPLICATION OF ROTOR INDUCED VELOCITY FIELD OF THE VORTEX THEORY TO THE STUDY OF STABILITY AND CONTROLLABILITY OF A HELICOPTER WITH HINGELESS ROTOR

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Submitted 26 April 1984

This paper presents the computation method of the stability and controllability of a helicopter with hingeless rotor. The effects of the first harmonic term of rotor induced velocity distribution and the first and second flapping bending modes of a rotor blade are taken into consideration and investigated. Especially, the application of rotor induced velocity field derived from Wang Shicun's generalized vortex theory to the study of stability and controllability of a helicopter with hingeless rotor is described in detail. The derivation method and numerical derivative method for μ_x, μ_y and μ_z are employed. Finally, the Bo-105 helicopter is taken as a numerical example, and the computed results are in good agreement with its flight-test data.

Nomenclature

a_{0N}, a_{1N}, b_{2N} Flapping coefficients corresponding to the N^{th} flapping mode of the rotor blade.

$(A)_0$, $(A)_{1C}$, $(A)_{1S}$, $(A)^{o}{}_{1C}$, $(A)^{o}{}_{1S}$ Correction coefficients of the monuniformity of rotor induced velocity distribution.

M_{XH}, M_{ZH}, m_{XH}, m_{ZH} Rolling and pitching rotor hub moments and corresponding coefficients

$S_N(X_1)$ N^{th} flapping bending vibration model function of a rotor blade

k Number of rotor blade

T, H, S, C_T, C_H, C_S Rotor tension, vertical force, side force and corresponding coefficients

V_1 Rotor composite velocity (nondimensional $\bar{V}_1=V_1/\Omega R$)

α_1 Rotor attack angle with respect to V_1

v_1 Rotor induced velocity (nondimensional $\bar{v}_1=v_1/\Omega R$)

Ω Rotory rpm

ω_x, ω_z Rolling and pitching angular velocity of helicopter ($\hat{\omega}_x=\omega_x/\Omega$, $\hat{\omega}_z=\omega_z/\Omega$)

μ, μ_x, μ_y, μ_z Characteristic coefficients of flight condition and components along rotor axis system

λ_N Frequency ratio corresponding to N^{th} flapping mode

ζ_N Generalized coordinates of N^{th} flapping mode

φ_7 Total distance angle

$\Delta\varphi$ Negative torque of rotor blade

θ_1 Horizontal periodic distance change of rotor

θ_2 Vertical periodic distance change of rotor

θ_{7TR} Total distance angle of tail rotor

$\bar{\Gamma}$ Nonfactor circulation ($\bar{\Gamma} = \Gamma / \Omega R^2$)

I. Preface

The rotor induced velocity field distribution of a helicopter is an important and extremely complicated problem. It is particularly important to a helicopter with hingeless rotor, especially the first harmonic term[1]. In the past, the helicopter balance, stability and controllability computations all employed the assumption of uniform distribution, and then modified methods were introduced[1~3], yet they were all based on the classical momentum theory. These assumptions are too simplified such that not only the effects of lateral disturbance to the rotor induced velocity field are not considered, but also cannot reflect the effects of various structures, motion and control parameter of a helicopter. The development of the vortex theory is becoming more and more mature, and using it to calculate rotor induced velocity field distribution is more accurate and reasonable. It has been extensively utilized in the rotor blade airloads and rotor aerodynamics. However, due to the complexity of vortex theory, it is still rarely used in the study of helicopter controllability. This paper adopts Wang Shicun's generalized vortex theory and employs the close form of equations for the rotor induced velocity and circulation to give the rotor induced velocity field distribution and the relational equations of parameters such as motion, structure and control for helicopter with hingeless rotor are given. Moreover, the rotor induced velocity field of the vortex theory is introduced into all the hingeless rotor force, moment and flapping motion, and finally analytic formulae suitable to engineering application are obtained.

The effects of rotor blade elasticity mode number to the features of controllability and stability for a helicopter with hingeless rotor are not only a important theoretical problem, but also a practical problem in engineering application. This paper has made detailed study on them.

II. Basic Methods

1. Load and Velocity at the Rotor Blade Element

The hingeless rotor blade can be treated as a suspended beam. Also, assume the blade is infinitely rigid in twisting and chordwise bending directions, and only flexible in a flapping direction. The blade flapping bending differential equation is:

$$\frac{\partial^2}{\partial r^2}\left\{EI\frac{\partial^2 Y}{\partial r^2}\right\}-\frac{\partial}{\partial r}\left\{N\frac{\partial Y}{\partial r}\right\}+m\frac{\partial^2 Y}{\partial t^2}=\frac{\partial F}{\partial r} \qquad (1)$$

where $N=\int_r^R m\Omega^2 r dr$ is the centrifugal tension, $\partial F/\partial r$ is the airloads and Coriolis inertia loads;

$$\frac{\partial E}{\partial r}=\frac{\partial F_{\ell}}{\partial r}+\frac{\partial F_{i}}{\partial r}.$$

Here the effects of reverse flow region and compressibility are not considered.

$$\frac{\partial F_4}{\partial r}=\frac{1}{2}\rho a_{\infty} b(\Omega R)^2(\bar{u}_T^2\phi_* + \bar{u}_T\bar{u}_p)$$

where

$$\phi_*=\theta_r+\Delta\varphi x+\theta_1\cos\psi+\theta_2\sin\psi,\quad \theta_r=\varphi_7-0.7\Delta\varphi$$

$$\frac{\partial F_{\ell}}{\partial r}=2m\Omega^2 R[\hat{\omega}_x x\cos\psi-\hat{\omega}_z x\sin\psi]$$

The velocity at rotor blade element:

$$\left.\begin{aligned}\bar{u}_T &= x + \mu_x \sin\psi + \mu_z \cos\psi \\ \bar{u}_p &= -\mu_y - \bar{v}_0 - \bar{v}_{1c}\cos\psi - \bar{v}_{1s}\sin\psi + (\hat{\omega}_x \sin\psi + \hat{\omega}_z \cos\psi)\, x \\ &\quad -(\mu_z\cos\psi - \mu_x \sin\psi)\frac{\partial y}{\partial x} - \dot{y}/\Omega \end{aligned}\right\} \quad (2)$$

where $x = r/R,\ y = Y/R,\ y = \sum_{N=1}^{2} S_N(x)\zeta_N(t)$

The first harmonic distribution of the rotor induced velocity is considered.

2. Rotor Induced Velocity Field

According to the blade element theory and Rukowski formula, and adopting the regular assumption that $u \approx u_x$, $C_y \approx a_\infty \alpha$, the circulation relational formula is obtained as:

$$\bar{\Gamma} = \frac{1}{2} a_\infty \bar{b}(\bar{u}_T \phi_* + \bar{u}_p) = \bar{\Gamma}_0 + \bar{\Gamma}_{1c}\cos\psi + \bar{\Gamma}_{1s}\sin\psi + \cdots\cdots$$

Substituting $\bar{u}_T$, $\bar{u}_p$ and ϕ_* and after

$$\left.\begin{aligned}\bar{\Gamma}_0 &= \frac{1}{2} a_\infty \bar{b}\Big[\theta_* x + \Delta\varphi x^2 + \frac{1}{2}(\mu_x\theta_2 + \mu_z\theta_1) - \mu_y - \bar{v}_0 \\ &\quad + \frac{1}{2}\mu_x \Sigma a_{1N}\frac{ds_N}{dx} - \frac{1}{2}\mu_z \Sigma b_{1N}\frac{ds_N}{dx}\Big] \\ \bar{\Gamma}_{1c} &= \frac{1}{2} a_\infty \bar{b}\Big[\theta_1 x + \mu_z\theta_* + \mu_z \Delta\varphi x - \bar{v}_{1c} + \hat{\omega}_z x + \Sigma s_N b_{1N} \\ &\quad - \mu_x \Sigma a_{0N}\frac{ds_N}{dx}\Big] \\ \bar{\Gamma}_{1s} &= \frac{1}{2} a_\infty \bar{b}\Big[\theta_2 x + \mu_x\theta_* + \mu_x \Delta\varphi x - \bar{v}_{1s} + \hat{\omega}_x x - \Sigma s_N a_{1N} \\ &\quad + \mu_z \Sigma a_{0N}\frac{ds_N}{dx}\Big] \end{aligned}\right\} \quad (3)$$

According to the stationary vortex theory of [illegible] Shicun[4], and only limited to the first harmonic term, the relational formula between induced velocity and circulation is:

$$\left.\begin{aligned}\bar{v}_0 &= \frac{k}{4\pi \bar{V}_1}\bar{\Gamma}_0 \\ \bar{v}_{1c} &= \frac{k}{4\pi \bar{V}_1}\Big[\frac{2\cos\alpha_1}{1+\sin\alpha_1}\Big(-\frac{2}{3} + x + \frac{1}{2}x^2\Big)\bar{\Gamma}_0 + \frac{2\sin\alpha_1}{1+\sin\alpha_1}\bar{\Gamma}_{1c}\Big] \\ \bar{v}_{1s} &= \frac{k}{4\pi \bar{V}_1}\Big[\frac{2\cos\alpha_1}{1+\sin\alpha_1}\bar{V}_1(3 - 3x + x^2)\bar{\Gamma}_0 + \frac{2}{1+\sin\alpha_1}\bar{\Gamma}_{1s}\Big] \end{aligned}\right\} \quad (4)$$

Solving (3) and (4) simulataneously and eventually the induced velocities are obtained: $\bar{v}_1 = \bar{v}_0 + \bar{v}_{1c}\cos\psi + \bar{v}_{1s}\sin\psi + \cdots\cdots$

$$\left.\begin{aligned}
\bar{v}_0 &= (A)_0\left[\theta_r x + \Delta\varphi x^2 - \mu_y + \frac{1}{2}\mu_x\theta_2 + \frac{1}{2}\mu_x\theta_1 + \frac{1}{2}\Sigma(\mu_x a_{1N} - \mu_x b_{1N})\frac{ds_N}{dx}\right] \\
\bar{v}_{1c} &= (A)_{1c}\left[\theta_1 x + \mu_x\theta_r + \mu_x\Delta\varphi x + \hat{\omega}_x x + \Sigma\left(s_N b_{1N} - \mu_x a_{0N}\frac{ds_N}{dx}\right)\right] \\
&\quad + (A)^0_{1c}\left(-\frac{2}{3} + x + \frac{1}{2}x^2\right)\left[\theta_r x + \Delta\varphi x^2 - \mu_y + \frac{1}{2}\mu_x\theta_2 + \frac{1}{2}\mu_x\theta_1\right. \\
&\quad \left. + \frac{1}{2}\Sigma(\mu_x a_{1N} - \mu_x b_{1N})\frac{ds_N}{dx}\right] \\
\bar{v}_{1s} &= (A)_{1s}\left[\theta_2 x + \mu_x\theta_r + \mu_x\Delta\varphi x + \hat{\omega}_x x - \Sigma\left(s_N a_{1N} - \mu_x a_{0N}\frac{ds_N}{dx}\right)\right] \\
&\quad (A)^0_{1s}(3 - 3x + x^2)\left[\theta_r x + \Delta\varphi x^2 - \mu_y + \frac{1}{2}\mu_x\theta_2 + \frac{1}{2}\mu_x\theta_1 + \frac{1}{2}\Sigma(\mu_x a_{1N}\right. \\
&\quad \left. - \mu_x b_{1N})\frac{ds_N}{dx}\right]
\end{aligned}\right\} \quad (5)$$

where the definitions of $(A)_0$, $(A)_{1C}$, $(A)^0_{1C}$, $(A)_{1S}$ and $(A)^0_{1S}$ are as follows:

$$A = \frac{1}{4\pi}a_\infty \bar{b}k_1 \quad A^0_0 = A/2\bar{V}_1; \quad (A)_0 = \frac{A^0_0}{1 + A^0_0}$$

$$A^0_{1c} = A^0_0\frac{2\cos\alpha_1}{1 + \sin\alpha_1}; \quad A^{1c}_{1c} = A^0_0\frac{2\sin\alpha_1}{1 + \sin\alpha_1}; \quad (A)_{1c} = \frac{A^{1c}_{1c}}{1 + A^{1c}_{1c}}$$

$$(A)^0_{1c} = \frac{1}{1 + A^{1c}_{1c}}\,\frac{A^0_{1c}}{1 + A^0_0}; \quad A^{1s}_{1s} = A^0_0\frac{2}{1 + \sin\alpha_1}; \quad A^0_{1s} = A^0_{1c}$$

$$(A)_{1s} = \frac{A^{1s}_{1s}}{1 + A^{1s}_{1s}}; \quad (A)^0_{1s} = \frac{\bar{V}_1}{1 + A^{1s}_{1s}}\,\frac{A^0_{1s}}{1 + A^0_0}$$

3. Flapping Motion

Using the rotor blade vibration model equation and the model's normal feature, then from the blade flapping bending equation the rotor flapping motion equation can be obtained:

$$\frac{d^2\zeta_N(\psi)}{d\psi^2} + \lambda_N^2\zeta_N(\psi) = \frac{\int_0^1 \frac{\partial F}{\partial x}S_N(x)dx}{\Omega^2R^2\int_0^1 mS_N^2(x)dx} \qquad N = 1,\ 2\cdots\cdots \qquad (6)$$

Generally speaking, it is difficult to accurately solve the vibration model equation. This paper adopts the "asymptotic method" to obtain the solution. It is already sufficient to select the first harmonic term. $\zeta_N(\psi) = a_{0N} - a_{1N}\cos\psi - b_{1N}\sin\psi\cdots\cdots$ then the above equation can be changed to:

$$\lambda_N^2 a_{0N} - (\lambda_N^2 - 1) a_{1N} \cos\psi - (\lambda_N^2 - 1) b_{1N} \sin\psi = \frac{\int_0^1 \frac{\partial F}{\partial x} S_N(x)\, dx}{\Omega^2 R^2 \int_0^1 m S_N^2(x)\, dx} \quad (7)$$

In essence, Equation (6) and (7) describe the forced effects of each order vibration model $S_N(x)$ under external loads. As a centrally hinged type rigid blade, $\lambda_1=1$, $S_1(x)=x$, and Equation (7) is simplified to the hinged type rotor flapping equation $a_0 \Omega^2 I_* = \int_0^R \frac{\partial F}{\partial r} r dr$, with which we are familiar. Obviously, the hinged type rotor blade is a special case of hingeless rotor. According to the aforementioned external loads and induced velocity distribution, and eliminating harmonic terms above the second order and those cross-coupling terms between each order mode, two groups of linear algebraic equations can be eventually obtained:

$$[A][\zeta_1]=[B]$$

$$[C][\zeta_2]=[D] \quad (8)$$

corresponding first order mode: $[\zeta_1]=[a_{01} a_{11} b_{11}]^T$

corresponding second order mode: $[\zeta_2]=[a_{02} a_{12} b_{12}]^T$

When flight conditions are given, [A], [B], [C] and [D] are all determined. The [ζ_1] and [ζ_2] can be solved and the corresponding derivatives can be obtained.

4. Hingeless Rotor Force and Rotor Hub Moments

The rotor hub moments are particularly important to the hingeless rotor. Take the integral of moment generated by the aerodynamic force on the blade, cetrifugal force and the flapping and Coriolis inertia forces with respect to the center of rotor:

$$M_H(0, t) = \int_0^R \left(m\ddot{Y} + m\Omega^2 Y - \frac{\partial F}{\partial r} \right) r dr$$

If only the first order flapping coefficient is selected, the rotor hub moments generated by the centrifugal force and the flapping inertia force cancel each other exactly. Therefore,

$$M_{xH} = -\frac{kR}{2\pi} \int_0^{2\pi} \int_0^1 \frac{\partial F}{\partial x} x \sin\psi dx d\psi$$

$$M_{zH} = -\frac{kR}{2\pi} \int_0^{2\pi} \int_0^1 \frac{\partial F}{\partial x} x \cos\psi dx d\psi$$

Take rotor blade elasticity first and second flapping modes and first harmonic term of induced velocity into account, then the following can be obtained after integration and nondimensionalization:

$$m_{xH} = m_{xH_0} + \Delta m_{xH}; \quad m_{zH} = m_{zH_0} + \Delta m_{zH}$$

Similar to a hinged rotor, the elementary forces of a hingeless rotor are:

$$dT \cong dL = \frac{1}{2} \rho a_\infty b (u_T^2 \phi_* + u_T u_p) dr$$

$$dH \cong (dD - dL\beta_*) \sin\psi - dL \frac{dy}{dx} \cos\psi$$

$$dS \cong -(dD - dL\beta_*) \cos\psi - dL \frac{dy}{dx} \sin\psi$$

After the same manifuplation, the following can be obtained:

$$C_T = C_{T0} + \Delta C_T; \quad C_H = C_{H0} + \Delta C_H; \quad C_s = C_{s0} + \Delta C_s$$

C_{T0}, C_{H0}, C_{S0}, m_{xH0} and m_{zH0} are the basic terms; ΔC_T, ΔC_H, ΔC_S, Δm_{xH} and Δm_{zH} are the increments derived from induced velocity harmonic terms in which this paper is very interested:

$$\Delta C_T = \frac{1}{2}\mu_x \int_0^1 \bar{v}_{1s}\,dx - \frac{1}{2}\mu_x \int_0^1 \bar{v}_{1c}dx \tag{9}$$

$$\begin{aligned}
\Delta C_H = {} & \frac{1}{2}\theta_r \int_0^1 x\bar{v}_{1s}\,dx + \frac{1}{2}\Delta\varphi \int_0^1 x^2\bar{v}_{1s}\,dx + \frac{1}{8}\theta_1\left(\mu_x \int_0^1 \bar{v}_{1c}dx + \mu_x \int_0^1 \bar{v}_{1s}dx\right) \\
& + \frac{1}{8}\theta_2\left(4\mu_x \int_0^1 \bar{v}_{1c}dx + 3\mu_x \int_0^1 \bar{v}_{1s}\,dx\right) - \mu_y \int_0^1 \bar{v}_{1s}\,dx - \int_0^1 \bar{v}_0\bar{v}_{1s}dx \\
& + \frac{1}{8}\mu_x\left[a_{11}\int_0^1 \bar{v}_{1s}\frac{ds_1}{dx}\,dx + a_{12}\int_0^1 \bar{v}_{1s}\frac{ds_2}{dx}dx\right] + \frac{1}{8}\mu_x\left[b_{11}\int_0^1 \bar{v}_{1c}\frac{ds_1}{dx}dx\right. \\
& \left. + b_{12}\int_0^1 \bar{v}_{1s}\frac{ds_2}{dx}dx\right] - \frac{5}{8}\mu_x\left[a_{11}\int_0^1 \bar{v}_{1c}\frac{ds_1}{dx}dx + a_{12}\int_0^1 \bar{v}_{1c}\frac{ds_2}{dx}dx\right] \\
& - \frac{7}{8}\mu_x\left[b_{11}\int_0^1 \bar{v}_{1s}\frac{ds_1}{dx}\,dx + b_{12}\int_0^1 \bar{v}_{1s}\frac{ds_2}{dx}dx\right] \\
& + \frac{1}{2}\left[a_{01}\int_0^1 \bar{v}_{1c}x\frac{ds_1}{dx}\,dx + a_{02}\int_0^1 \bar{v}_{1c}x\frac{ds_2}{dx}dx\right]
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Delta C_Y = {} & -\frac{1}{2}\theta_r \int_0^1 x\bar{v}_{1c}\,dx - \frac{1}{2}\Delta\varphi \int_0^1 x^2\bar{v}_{1c}dx - \frac{1}{8}\theta_1\left(\mu_x \int_0^1 \bar{v}_{1s}dx + 3\mu_x \int_0^1 \bar{v}_{1c}dx\right) \\
& - \frac{1}{8}\theta_2\left(\mu_x \int_0^1 \bar{v}_{1c}dx + \mu_x \int_0^1 \bar{v}_{1s}dx\right) + \mu_y \int_0^1 \bar{v}_{1c}\,dx + \int_0^1 \bar{v}_0\bar{v}_{1c}dx \\
& - \frac{7}{8}\mu_x\left[a_{11}\int_0^1 \bar{v}_{1c}\frac{ds_1}{dx}\,dx + a_{12}\int_0^1 \bar{v}_{1c}\frac{ds_2}{dx}\,dx\right] - \frac{5}{8}\mu_x\left[b_{11}\int_0^1 \bar{v}_{1s}\frac{ds_1}{dx}dx\right. \\
& \left. + b_{12}\int_0^1 \bar{v}_{1s}\frac{ds_2}{dx}dx\right] + \frac{1}{8}\mu_x\left[a_{11}\int_0^1 \bar{v}_{1c}\frac{ds_1}{dx}\,dx + a_{12}\int_0^1 \bar{v}_{1s}\frac{ds_2}{dx}dx\right] \\
& + \frac{1}{8}\mu_x\left[b_{11}\int_0^1 \bar{v}_{1c}\frac{ds_1}{dx}\,dx + b_{12}\int_0^1 \bar{v}_{1c}\frac{ds_2}{dx}dx\right] \\
& + \frac{1}{2}\left[a_{01}\int_0^1 \bar{v}_{1s}x\frac{ds_1}{dx}\,dx + a_{02}\int_0^1 \bar{v}_{1s}x\frac{ds_2}{dx}dx\right]
\end{aligned} \tag{11}$$

$$\Delta m_{xH} = \frac{1}{2}\int_0^1 x^2\bar{v}_{1s}dx \tag{12}$$

$$\Delta m_{yH} = \frac{1}{2}\int_0^1 x^2\bar{v}_{1c}dx \tag{13}$$

According to the induced velocity distribution of the vortex theory, the hingeless rotor force and rotor hub moment which take the induced velocity harmonic components and blade elasticity into account can be eventually obtained:

$$\frac{1}{a_\infty\sigma}\{F\} = [R_0]\{E\} + [\Delta R]\{E\} \tag{14}$$

where

$$[F]=[C_T\ C_H\ C_y\ m_{xH}\ m_{zH}]^T$$

$$\{E\}=\{\theta_7\ \ \Delta\varphi\ \ \mu_y\ \ \theta_1\ \ \theta_2\ \ \hat{\omega}_x\ \ \hat{\omega}_z\ \ \mu_z\ \ 1\}^T$$

$[R_0]$ and $[\Delta R]$ are functions of μ_x and $(A)_0$, $(A)_{1C}$, $(A)^o_{1C}$, $(A)_{1S}$, $(A)^o_{1S}$. $\{E\}$ is the flight conditions and control parameter matrix. Similarly, when $\lambda_1=1$, $S_1(x)=x$, $\lambda_2=0$, $S_2(x)=0$, the ordinary hinged rotor forces and rotor hub moments can be obtained.

5. Disturbance Motion Equations and Control Derivatives

The linear small disturbance motion differential equation groups are divided into longitudinal and lateral coupling and noncoupling for consideration. The disturbance motion equaiton is:

$$[W]\{H\}=[U]\{G\} \tag{15}$$

where [W] is the stability derivative matrix; [U] is the control derivative matrix; $\{H\}$ is the motion parameter matrix; and $\{G\}$ is the control parameter matrix.

The coefficients of the equation, i.e., all the stability derivatives in matrix [W] and the control derivatives in matrix [U], are obtained by computer using the triple-point formula numerical derivative method with step increment H=0.005. The computations have shown that, for stability and controllability, the numerical derivative method is feasible and conducive to the rotor aerodynamic and dynamic model's being more truthful and perfect.

III. Examples

In order to explain the method of this paper, a Bo-105 helicopter with hingeless rotor is selected as a numerical example and the fuselage data is substituted by test results of similar fuselage (longitudinal data are taken from Reference [5], lateral data are taken from H-5). See Reference [6] for original data and the balance data are taken from Reference [7]. The "asymptotic method" is adopted for the blade frequency vibration model to obtain $\lambda_1=1.124$, $\lambda_2=2.86$. Four kinds of flight conditions at $\mu=0.1$, 0.2, 0.3 for hovering and cruise are calculated under three situations of considering both the induced velocity harmonic component and second flapping mode and with either one eliminated. In order for convenient comparison, Fig. 1. shows $a^*_{11}=a_{11}-\theta_2$; $b^*_{11}=b_{11}\div\theta_1$. It can be seen that the induced velocity harmonic term primarily affects the lateral flapping coefficients. There is a peak at $\mu\approx 0.1$ and this phenomenon has been observed in many experiments. This result matches completely with those of Reference [3] and [4]. It can be seen from Fig. 2 that, within the range of $\mu<0.3$, the effects of the second flapping do not excee. 10% of the total value. When $\mu>0.3$, the effects of the second mode must be considered. The analysis of this paper is basically consistent with the conclusions of Reference [1] and [8]. The results have shown that the vortex theory induced velocity distribution introduced by this paper is reasonable and correct.

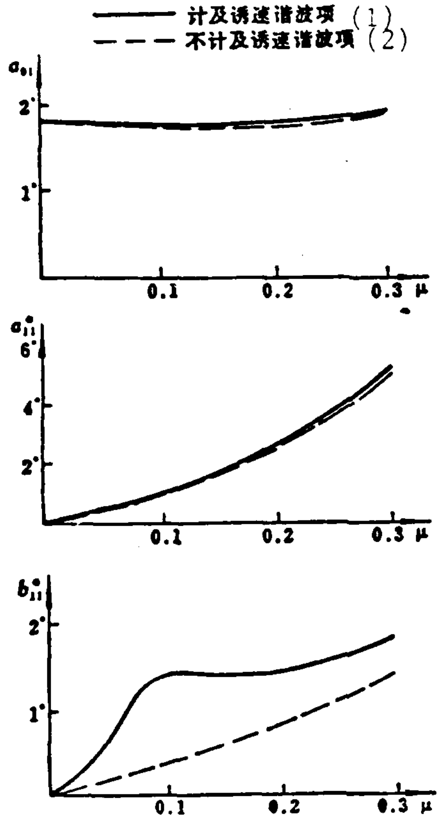
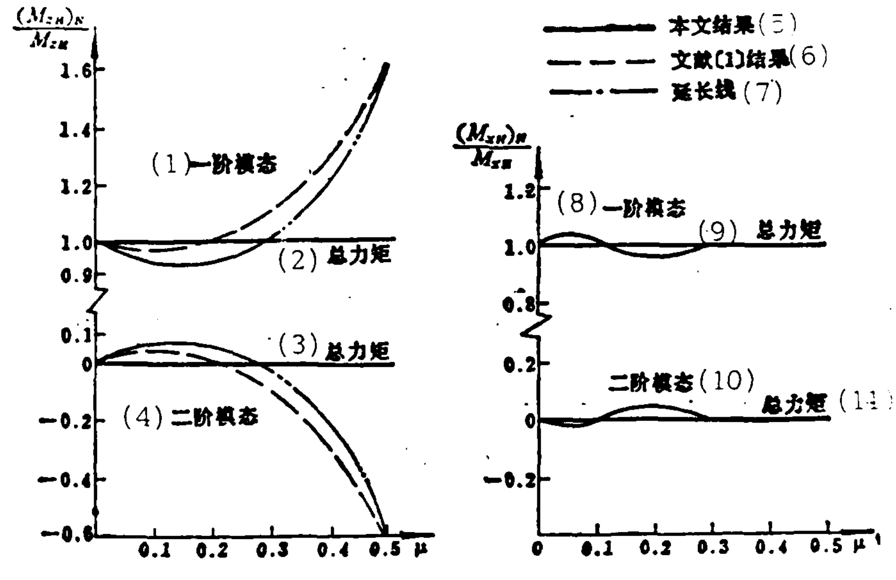


Fig. 1. Eff[illegible] of nonuniformity of induced velocity distribution on flapping [illegible]ficient.
Key: (1 With induced velocity harmonic term accounted for; (2 With induced velocity harmonic term not accounted for.



F[illegible]. Effect of flapping bending mode of blade on t[illegible] hub m[illegible].
Key: (1) first mode; (2) total moment; (3) total moment; (4) second mode; (5) results of this paper; (6) results of Reference [1];

(7) extended line; (8) first mode; (9) total moment; (10) second mode; (11) total moment.

The stability and controllability features under various conditions are computed. Figure 3 shows the comparison of computed results with the theoretical values and flight-test data. Also, the primary stability and control derivatives for typical flight conditions are selected to compare with the theoretical values and flight results given b·· Reference [9], and they are in better agreement.

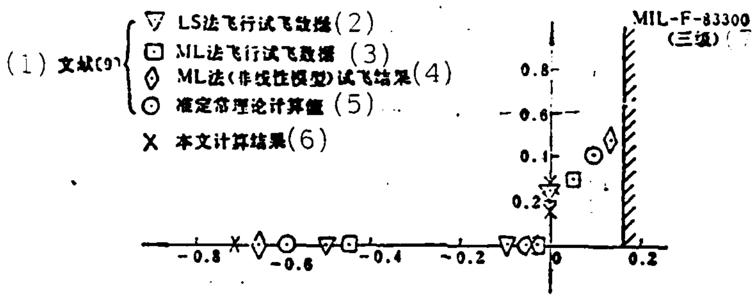


Fig. ·. ˜mparison of characteristic root resul·.·.
Key: (1) Reference [9]; (2) flight-test data of ·· method; (3) flight-test data of ML method; (4) flight-test data of ML method (nonlinear model); (5) quasi-steady theory computed values; (6) computed results of this paper; (7) third class.

Table 1

(1) 导数			(4) 本文计算结果1750 kg, 43.6m/s			(6) 文献[9]数据2100 kg, 38.1m/s		
[9]	本文 (2)	单位 (3)	全考虑 (5)	$S_2=0$	$\bar{v}_{1c}=\bar{v}_{1s}=0$ (7)	准定常理论计算值 (8)	(9) 试飞数据 最小二乘法 (10)	最大似然法 (11)
X_u	X^{V_x}	1/s	-0.027	-0.027	-0.0278	-0.0346	-0.043	-0.0138
Y_v	Z^{V_z}	1/s	-0.1825	-0.182	-0.184	-0.1252	-0.229	-0.110
Z_w	Y^{V_y}	1/s	-0.98	-0.98	-0.98	-0.9818	-0.439	-0.720
L_v	$M_X^{V_z}$	1/m·s	-0.20	-0.20	-0.214	-0.291	-0.370	-0.057
L_p	$M_X^{\omega_x}$	1/s	-9.78	-9.8	-10.14	-10.493	-4.26	-1.89
L_r	$M_X^{\omega_z}$	1/s	2.73	2.76	1.19	1.672	4.026	3.946
M_u	$M_Z^{V_x}$	1/m·s	0.009	0.011	0.034	0.047	0.018	0.0066
M_w	$-M_Z^{V_y}$	1/m·s	0.027	0.032	-0.0118	0.055	0.010	0.0136
M_p	$M_Z^{\omega_x}$	1/s	-1.08	-1.095	-0.50	-1.37	-0.70	-1.648
M_q	$M_Z^{\omega_z}$	1/s	-5.3	-5.3	-4.2	-4.434	-2.49	-2.02
M_{θ_c}	$M_Z^{\theta_c}$	1/s²·deg	0.73	0.73	0.76	0.97	—	0.333
N_v	$-M_Y^{V_z}$	1/m·s	0.112	0.113	0.115	0.208	0.075	0.091
N_r	$M_Y^{\omega_y}$	1/s	-1.313	-1.32	-1.32	-1.389	-0.91	-1.94

Table 1.
Key: (1) derivative; (2) This paper; (3) unit; (4) Computed results of this paper 1750 kg, 43.6 m/sec; (5) All taken into account; (6) data of Reference [9] 2100 kg, 38.1 m/sec; (7) quasi-steady theory; (8) computed values; (9) flight-test data; (10) minimum second multiplication method; (11) maximum approximation method.

IV. Conclusions

This paper establishes for the first time the stability and controllability computations of a helicopter with hingeless rotor while taking the rotor induced velocity first harmonic distribution of vortex theory into account. It presents analytic formulae suitable to engineering applications. Within the range of $\mu < 0.3$, the calculation only needs to consider the first flapping mode.

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A SECOND-ORDER THEORY FOR THREE-DIMENSIONAL UNSTEADY FLOWS AND ITS APPLICATION TO THIN WINGS WITH ANGLE OF ATTACK AT SUPERSONIC AND HYPE-RSONIC SPEEDS

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Abstract

According to the assumption of high Mach numbers the secondorder equation for unsteady flow reduces to a form analogous to that for steady flow. As an approximation, a particular solution is derived from the assumption of local two-dimensionality instead of the approximate three-dimensional particular solution.

In treatment of the unsteady problem for delta wings with low aspect ratio and supersonic leading edges, the non-linear pressure relation and the modified reference flow parameters are adopted to take the effect of mean angles of attack into account. Analytical expressions are obtained for the first order expansion of the unsteady velocity potential. They conduce to simplifying the calculation of stability derivatives to a great extent.

Stability derivatives for flat plate, double-wedge or biconvex delta wings oscillating at low frequencies are given respectively. Since the corresponding experimental data are unavailable, the obtained results are merely compared with those of Hui's flat plate[4], and both results are in good agreement.

A SECOND-ORDER THEORY FOR THREE-DIMENSIONAL UNSTEADY FLOWS AND ITS APPLICATION TO THIN WINGS WITH ANGLE OF ATTACK AT SUPERSONIC AND HYPERSONIC SPEEDS

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I. Preface

The dynamic derivatives are indispensable parameters in the dynamic quality analysis of aircraft and the design of its control systems; they directly affect the dynamic characteristics of aircraft. Therefore, since the 70's, there have been many studies made either in wind tunnel tests, flight tests' or analyses', computation, etc., and more progress has been made[1].

The computation of dynamic derivatives are essentially unsteady aerodynamic computation problems. The existing linear theories cannot satisfy the requirements, and the demand for taking nonlinear effects into account in the dynamic derivative computation has grown more urgent by the day. The second-order theory is one kind of method to compute nonlinear effects. It is established by second iteration or the second-order expansion of the object surface deviation angle, and the primary nonlinear effects are computed from the second-order equation. Van Dyke[2] and Qian Fuxing have worked in this area. Whereas when subtraction is performed between the pressure coefficients, which have undergone the second-order theory series expansion, of the upper and lower wing surfaces, the unsteady effects of the angle of attack are cancelled out; therefore, Reference [3] only computed the pitching damping features of the compression surface of a three-dimensional thin wing. In recent years, Hui et al.[4] have proposed a unified supersonic and hypersonic theory which can give better results for low frequency flat plate wing with high Mach number and angle of attack, but it is not suitable to wings with a small aspect ratio under low supersonic conditions, and it cannot handle thickness effects.

This paper, on the basis of a supersonic unsteady three-dimensional linear solution, utilizes the approximate local two-dimensional results of the second-order theory at a high Mach number to take the mean angle of attack and nonlinear effects of thickness into account and specifically solves the unsteady problem of supersonic delta wings with angle of attack at leading edges. In order to compute the nonlinear effects of the angle of attack, this paper conducts computations using the nonlinear isothermal pressure relation and the reference incident flow parameters, and compares with the results of Reference [4]. They are in good agreement.

The basic equation of the unsteady second-order theory and its solution are omitted here. See Reference [3] for details. It needs to be explained that, when solving the unsteady second-order equation, our method adopts the assumption of a high Mach number approximation and local two-dimensionality of the flow on a thin wing surface.

II. Nonlinear Isothermal Pressure Relation (Method 1)

A three-dimensional thin wing with a small aspect ratio in a uniform supersonic flow is subject to small amplitude simple harmonic pitching oscillation near the mean angle of attack α_0. If second-order is considered, the flow is isothermal. Rectangular coordinates oxyz are selected with the origin at the tip of the wing's leading edge, x-axis along the flow direction, y-axis pointing to the right when facing the flow and z-axis being vertical with up as positive. t represents time. Assume $\Phi(x, y, z, t)$ is the disturbance velocity potential, then the formula for nonlinear pressure coefficient is:

$$C_p\big|_{z=H} = \frac{2}{\gamma Ma_\infty^2}\left\{\left[1-\frac{\gamma-1}{2}Ma_\infty^2(2\Phi_x+\Phi_x^2+\Phi_y^2+\Phi_z^2+2\Phi_t/V_\infty)\right]^{\frac{\gamma}{\gamma-1}}-1\right\} \qquad (2.1)$$

where V_∞, Ma_∞, γ and H are free flow velocity, Mach number, adiabatic index and height of unsteady object surface, respectively; the subscript respresents a deviation derivative such as $\Phi_x = \partial\Phi/\partial x$. Φ can be represented, using the sum of steady, unsteady first and second disturbance velocity potentials, as[3]

$$\Phi = \varphi_0(x, y, z) + \varphi_1(x, y, z) + \theta_0 e^{i(\omega t - \bar{\omega} x)}[\psi_0(x, y, z) + \psi_1(x, y, z)] \qquad (2.2)$$

By expanding Equation (2.1) and omitting a small quantity of higher order, the pressure coefficient formula of the second-order theory is obtained

$$\begin{aligned} C_p\big|_{z=0} = & -2\theta_0 e^{i(\omega t - \bar{\omega} x)}(\psi_{0x}^0 + i\bar{K}\psi_0^0 - i\bar{\omega}\psi_0^0) \\ & -2\theta_0 e^{i(\omega t - \bar{K} x)}[\psi_{1x} + (1 - Ma_\infty^2)\varphi_{0x}\psi_{0x} - i\bar{K}\varphi_{0x}\psi_0 \\ & + \varphi_{y0}\psi_{0y} + \varphi_{0z}\psi_{0z} + \psi_{0xz}(h + \alpha_0 x) - e^{i\bar{K}x}(x - b)\varphi_{0xz}] \end{aligned} \qquad (2.3)$$

where $i=\sqrt{-1}$, $\bar{K}=\omega/V_\infty$, $\bar{\omega}=\bar{K}Ma_\infty^2/(Ma_\infty^2-1)$, ω and θ_0 are the Angular frequency and angular amplitude of the simple harmonic oscillation; h and b are the height of steady object surface and x coordinate of the vibration center. φ_0, ψ_0 and ψ_1 are the local two-dimensional solutions, ψ_0^* is the unsteady three-dimensional linear solution without the high Mach number approximation to take the three-dimensional effects into account and raise the computation accuracy under low supersonic conditions.

When computing the dynamic derivatives, the pressure difference $C_{pu}-C_{pl}$ between the upper and lower surface will be integrated along the object surface. If the upper and lower wing surfaces are symmetrical, the difference between the solutions of upper and lower surfaces is only the sign change of α_0 and θ_0. It can be seen from Equation (2.3) that, in the second-order theory, there is no $\theta_0\alpha_0$ effect included in $C_{pu}-C_{pl}$. Therefore, Equation (2.3) cannot be used to obtain the unsteady effect $\theta_0\alpha_0$ of the angle of attack using the velocity potential of the second-order theory; the nonlinear Equation (2.1) of the pressure coefficient should be used. First, convert Equation (2.1) to obtain solution at z=0, then solve for derivative with respect to θ, and take limit $\theta_0 \rightarrow 0$ and omit a small quantity of higher orders, thus

$$\lim_{\theta_0\to 0}\frac{\partial}{\partial\theta}C_p\Big|_{z=0}=\left[1-(\gamma-1)Ma_\infty^2\varphi_{0x}\right]^{\frac{1}{\gamma-1}}C_p^* \qquad (2.4)$$

where $\theta=\theta_0 e^{i\omega t}$; the representation of C_p^* is similar to Equation (2.3), i.e., Equation (2.3) divided by θ and then adding $-2e^{-i\bar{k}x2}$ $-Ma_\infty^2\varphi_{0x}\psi_{0x}$.

The unsteady pitching moment coefficient on the compression surface or expansion surface is

$$\lim_{\theta_0 \to 0} \frac{\partial C_m}{\partial \theta} = \frac{1}{Sc_0} \iint_s (x-b)[1-(\gamma-1)Ma_\infty^2 \varphi_{0x}]^{\frac{1}{\gamma-1}} C_p^* dxdy = [C_{m\theta} + ikC_{m\dot{\theta}}]_{\cdot m} \quad (2.5)$$

where $C_{m\dot{\theta}}$ and $C_{m\theta}$ are the pitching damping derivative and static derivative; $\dot{\theta}=(d\theta/dt)\cdot(c_0/V_\infty)=ik\theta_0 e^{i\omega t}$, $k=\omega c_0/V_\infty$ is the folding frequency; c_0 is the chord length at wing root; S is the surface area of wing. For low frequency, ψ_0^* is calculated to k^3 term using the frequency series expansion method. Here "one surface" means either the compression surface or the expansion surface, and the total stability derivative is the algebraic sum of upper and lower surfaces. The sign of α_0 must be changed on the upper and lower surfaces of the wing. In actual computation, the wing crosssection considered is near-jet similar with the object surface equations as:

$$h(x,y) = f \cdot Z(\bar{x}/f)$$

double-wedge: $Z(\bar{x}/f) = \varepsilon[c_0 - |c_0 - 2\bar{x}/f|]/2$

double-arc: $Z(\bar{x}/f) = 2\varepsilon(\bar{x}/f)\cdot(1-\bar{x}/fc_0)$

where $\bar{x}=x-ytgX_s$; $f=1-ytgX_s/c_0$; X_s is the swept angle of the wing's leading edges; flat plate corresponds to the situation of thickness ratio $\varepsilon=0$.

III. Reference Incident Flow Parameter (Method 2)

Based on the second-order theory of isothermal flow, the change in enthalpy after the flow crosses the shocks wave is completely ignored. Thus, under the conditions of a high Mach number and larger angle of attack, the error in the second-order theory increases as the shock wave strengthens. The reference incident flow parameter method is exactly a method proposed to solve the problem of computing steady effects of the angle of attack and thickness effects.

When a two-dimensional steady supersonic flow V_∞ flows over a flat plate at angle of attack α_0, the undisturbed free flow parameters of the streamlines of a three-dimensional thin wing are replaced by flow parameters of the object surface behind the shock wave, then the upper and lower surfaces of the wing are computed respectively using the second-order theory. The reference incident flow parameters can be completely determined by the shock wave-expansion wave method and is denoted by a subscript 1. It should be pointed out that in Method 2 the mean angle of attack α_0 does not appear directly and its effects are included in the reference incident flow parameters. The formula for pressure coefficient is

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{p - p_1}{\frac{1}{2}\rho_\infty V_\infty^2} + \frac{p_1 - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \qquad (3.1)$$

The second term to the right of the equality sign is the steady pressure. It is independent of the stability derivatives $C_{m\dot{\theta}}$ and $C_{m\theta}$ and thus is not considered.

Let

$$\bar{C}_p = \frac{p - p_1}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\rho_1 V_1^2}{\rho_\infty V_\infty^2} \cdot \frac{p - p_1}{\frac{1}{2}\rho_1 V_1^2} = G(Ma_\infty,\ \alpha_0) C_p' \qquad (3.2)$$

$$G(Ma_\infty,\ \alpha_0) = \rho_1 V_1^2 / \rho_\infty V_\infty^2 \qquad (3.3)$$

where p, ρ and V are pressure, density and velocity, respectively. Using Ma_∞ and α_0, the G of the compression surface and expansion surface can be obtained. The form of C_p' is identical to Equation (2.3), except that the undisturbed flow parameters are replaced by the reference incident flow coefficients. Take the integral of $\bar{C}_p$ on the wing, then the unsteady aerodynamic forces and the desired stability derivatives are obtained.

IV. Results and Discussions

It can be observed from Figs. 1 and 2 that, for a flat plate delta wing, the results of this method and the theoretical results of Reference [4] are in good agreement both in the trend and order of magnitude.

For a delta wing with double-wedge crosssection, the results of the two methods in this paper have shown that, when $Ma_\infty \alpha_0 < 1.3$, the consistency of the two methods are better. Under high Mach number condition, Method 2 is more reliable. However, at $Ma_\infty < 2.5$ and larger α_0, the reference incident flow Mach number Ma_1 on the compression surface is smaller and the nearsonic or subsonic leading edges will occur. At such time, Method 2 becomes useless and Method 1 can be used. The two methods are complementary to each other. They can, within the range of Mach number $2 \sim 8$ and folding frequency $0 \sim 1.0$, more accurately estimate the thickness effects, especially the unsteady effects of angle of attack.

This method is both analytic and semi-analytic. It is convenient to use and its operation fast. It can combine the unsteady second-order theory results of a single wing and the theoretical interference factors of a long thin object to conduct unsteady aerodynamic computations for the assembled body of wings, fuselage and tail wings.

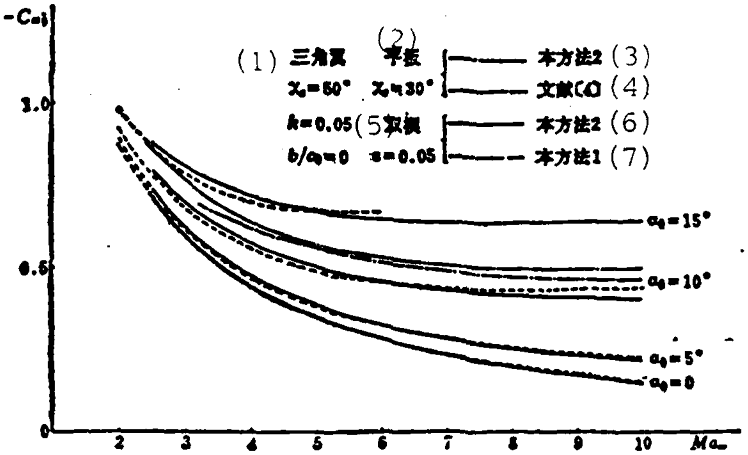

Fig. 1. Variation of $C_{m\dot{\theta}}$ with Mach number Ma_∞

Key: (1) delta wing; (2) flat plate; (3) Method 2 of this paper; (4) Reference [4]; (5) double-wedge; (6) Method 2 of this paper; (7) Method 1 of this paper.

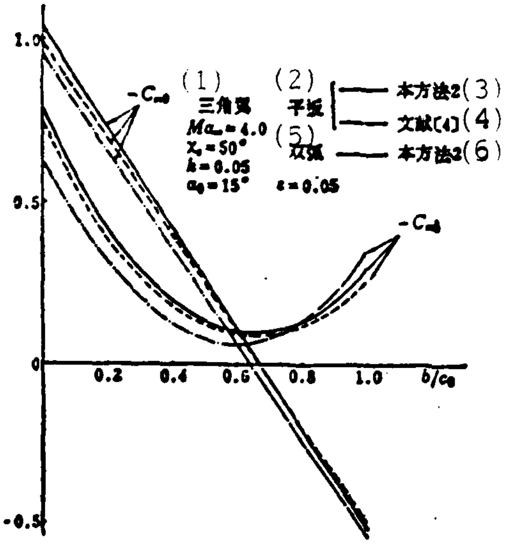

Fig. 2. Variation of $C_{m\dot{\theta}}$, $C_{m\theta}$ with center of oscillation b/c_0

Key: (1) delta wing; (2) flat plate; (3) Method 2 of this paper; (4) Reference [4]; (5) double-arc; (6) Method 2 of this paper.

At a low supersonic speed such as $Ma_\infty = 2$, the accuracy of the second-order solution is not high. However, since one uses the unsteady three-dimensional solution ψ_0^*, which does not have as high Mach number approximation, the overall computation accuracy can be assured, and the linear results are improved.

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