THE CALCULATION AND ANALYSIS OF THE VERTICAL DISTRIBUTION OF TURBULENT PARAMETERS IN THE NEUTRAL PBL [Planetary boundary layer]

by

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THE CALCULATION AND ANALYSIS OF THE VERTICAL DISTRIBUTION OF TURBULENT PARAMETERS IN THE NEUTRAL PBL [Planetary boundary layer]

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By referencing the Lettman Method\cite{1}, the vertical distribution of turbulent parameters - momentum transfer coefficient $k_{m}$, friction velocity $u_{*}$, turbulent shear stress $\overline{\tau}$ and mixing length $l$, were calculated from the observed data of average wind profiles in the neutral planetary boundary layer (PBL). The results were compared with research results from abroad and they were basically in fair agreement. Some of the parameters should be modified.

I. PREFACE

The basic characteristic of the planetary boundary layer is its turbulency in movement. Turbulent motion is resulted from the combined effects of "vortex" groups, which span quite wide in the time and space scales, under complicated boundary conditions. To directly measure the turbulent flow field requires high-tech devices and is generally very difficult. Therefore, the development and application of turbulent statistical theories has been limited. Moreover, using the K theory to solve the closed equations is also not very easy. Another method is to use the observed average wind and temperature profiles to calculate the distribution of turbulent characteristics under assumed conditions. This is valuable.

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for both theoretical and experimental studies.

This paper used the field wind data measured by the bivane anemometer at Yizheng of Jiangsu Province on April 29 and August 12, 1982 to calculate the vertical distribution of turbulent transfer coefficient $k_m$, mixing length $l$, friction velocity $u_*$ and turbulent shear stress $\tau$ in the neutral planetary full boundary layer. It provided a base for the research of turbulent structures in the planetary boundary layer.

II. CALCULATION METHOD

We referenced the Lettau Method. Assuming the atmospheric layer structure is neutral and with positive pressure, and the horizontal flow field is steady with uniform linear motion, then the motion equations can be simplified to:

$$\begin{align*}
\frac{d\tau_x}{dz} &= -\rho f(u - u_x) \\
\frac{d\tau_y}{dz} &= \rho (u - u_y)
\end{align*}$$

(1)

where $\tau_x$ and $\tau_y$ are the horizontal components of turbulent shear stress respectively. $u$ and $v$ are the horizontal components of the average wind velocity field. $u_g$ and $v_g$ the horizontal components of ground rotation wind. $f$ is the Clarke parameter and $\rho$ is the density of air. Based on semi-experience theory:

$$\tau = \nu k_n \frac{d\nu}{dz},$$

that is,

$$\tau_x = \rho k_n \frac{du}{dz}, \quad \tau_y = \rho k_n \frac{dv}{dz}$$

(2)

Substituting Equation (2) into Equation (1) and not considering the vertical changes of density, then the integration with respect to $x$ along the ground rotation wind direction gives:

$$\begin{align*}
\tau_x &= \tau_{x0} - \rho f \int_u^{u_x} dz - \rho k_n \frac{du}{dz} \\
\tau_y &= \tau_{y0} + \rho f \int (u - u_x) dz - \rho k_n \frac{dv}{dz}
\end{align*}$$

(3)
Take two special layers at altitudes \( z_1 \) and \( z_2 \), which are the altitudes where \( u \) and \( v \) reach their maximum values, then the components of ground shear stress can be written as:

\[
\begin{align*}
\tau_{x,z} &= \rho \int_{z_1}^{z_2} v \, d\zeta \\
\tau_{y,z} &= -\rho \int_{z_1}^{z_2} (u - u_g) \, d\zeta 
\end{align*}
\]  

(4)

How is \( u_g \) determined? If the angle \( \alpha_0 \) between ground rotational wind and ground wind is given, and assuming the direction of shear stress very close the ground is the same as that of the ground wind, then

\[
\tau_{x,z} = \tau_{y,z} \tan \alpha_0
\]

(5)

Substituting Equation (5) into the second equation of (4), we get

\[
u_x = \frac{1}{z_2} \left[ \int_{z_1}^{z_2} u \, d\zeta + \tan \alpha_0 \int_{z_1}^{z_2} \nu \, d\zeta \right]
\]

(6)

Now the main problem is the determination of \( \alpha_0 \). Trial and error method can be used. Assuming turbulent transfer coefficient is a standard quantity, i.e.

\[
k_x = k_y = \tan^{-1} \left[ \left( \frac{\nu}{\delta z} \right) + \left( \frac{\nu}{\delta z} \right) \right]^{1/2}
\]

(7)

where \( l \) is the mixing length. An initial \( \alpha_0 \) value is first given. The observed wind velocity can be divided into \( u(z) \) and \( v(z) \), and \( z_1 \) and \( z_2 \) are located. \( u_g \) is calculated from Equation (6). \( \tau_{x0} \) can be obtained by substituting \( u_g \) into Equation (4), and \( \tau_{y0} \) is obtained from Equation (5). \( \tau_x(z) \) and \( \tau_y(z) \) can then be obtained from Equation (3), and finally \( k_x \) and \( k_y \) are calculated from Equation (2). \( \alpha_0 \) value is continuously changed until \( (k_x - k_y) \), \( M \rightarrow 0 \) (the subscript \( M \) represents average value with respect to height). Thus accurate \( \alpha_0 \) is obtained. Meanwhile, the turbulent shear stress \( \tau_x, \tau_y \) at various altitudes is obtained, and so is the turbulent transfer coefficient \( k_m \). Then mixing length can be solved from

\[
1(z) = \left\{ k_m(z) \right\} \left[ \left( \frac{\nu}{\delta z} \right) + \left( \frac{\nu}{\delta z} \right) \right]^{1/2}
\]

(7')
The friction velocity can be calculated according to its definition:

\[ u_* = \left( \frac{\tau}{\rho} \right)^{1/3} \]  

(8)

where

\[ \tau = \left[ \tau(x) + \tau_r \right] \]

For details of the calculations, refer to Reference [1].

We employed the digital calculation method, and the Simpson formula is used for integration

\[ \int f(x) \, dx = \frac{h}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + f(x_m) \right] \]  

(9)

where

\[ h = \frac{b-a}{m} \]

In this paper, \( h = 10 \text{ m} \).

Differential calculation: The second-order polynomial approximate fitting method is used. The coefficients of said polynomial are determined by the least mean square method using the observed data of five points. At a straighter portion of the profile, simple average difference formula is used, i.e.

\[ \frac{dY_i}{dx} = \frac{Y_{i+1} - Y_{i-1}}{2h} \]  

(10)

We improved the method for determining ground wind direction. Instead of extrapolating from wind data measured by the small balloon, accurate anemometer for direct measurement of instantaneous wind velocity and wind direction was used. The above calculations were conducted in a PC-1500 computer.

III. OBSERVATION BACKGROUND AND MACROSCOPIC METEOROLOGICAL CONDITIONS

The paper primarily used the wind velocity profile data measured by the bivane anemometer at Yizheng of Jiangsu Province on August 12, 1982. The terrain surrounding the influence by the Chang Jiang, data leaning toward north wind were selected.

From 8:30-17:30 on August 12, there were 12 groups of wind velocity profiles. It could be seen from the meteorological
diagram that the isobars on that day in said area were quite straight and that the isotherms were basically parallel. There were no changes in the meteorological system, and the flow field was rather stable. At 14:00 the low altitude air observations gave: average vertical temperature decreasing rate was \(-0.61^\circ C/100m\) under 1 kilometer, average temperature gradient was \(0.43^\circ C/100m\). Thus the atmospheric structure was neutral. The observation field and macroscopic meteorology matched the aforementioned assumed conditions.

IV. ANALYSIS OF CALCULATED RESULTS

First of all, the trial and error method was used to determine \(\alpha_0\) value. On August 12, ground barometric pressure \(P_0=976.2\) milibar, temperature \(T_0=26.5^\circ C\); calculated density \(\rho =1.24 \times 10^{-3} g/cm^3\); latitude \(\phi =32^\circ\); the Clarke parameter \(f=0.773 \times 10^{-4}\) sec\(^{-1}\). Table 1 gives calculated \(\nu_g\) results for different \(\alpha_0\) values.

<table>
<thead>
<tr>
<th>(\alpha_0 (\circ))</th>
<th>21.1</th>
<th>23.7</th>
<th>24.4</th>
<th>25.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z_1 (\circ))</td>
<td>19.0</td>
<td>21.6</td>
<td>24.1</td>
<td>26.7</td>
</tr>
<tr>
<td>(z_2 (\circ))</td>
<td>21.0</td>
<td>23.0</td>
<td>24.9</td>
<td>27.0</td>
</tr>
<tr>
<td>(\nu_g (\text{cm/s}))</td>
<td>3.54</td>
<td>3.14</td>
<td>3.30</td>
<td>3.45</td>
</tr>
<tr>
<td>(\nu_g (\text{cm/s}))</td>
<td>0.25</td>
<td>1.23</td>
<td>2.72</td>
<td>3.99</td>
</tr>
<tr>
<td>((\nu_x - \nu_y)_M)</td>
<td>0.5</td>
<td>-1.4</td>
<td>-3.3</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

Key: (a) degree; (b) meter; (c) dynes/cm\(^2\).

\((k_x-k_y)_M\) in the table represents the average transfer coefficient under the altitude of 400 meters. It can be seen from the above table that the angle \(\alpha_0\) between ground rotational wind and ground wind should be selected as \(24.4^\circ\) with a ground rotational wind velocity of \(14.71\text{m/sec}\).
The August 12 spirals diagram of the average wind velocity profiles is as shown in Figure 1. The profile of the wind velocity component is as shown in Figure 2. In Figure 2, the Y-axis is the net height. Compared with Lepzig data, the trend of the two distributions is similar. Figure 3 shows the spirals of turbulent stress calculated from the average wind velocity profile and $\alpha_0$, and they are in fair agreement with those of Lettau\(^1\).

Using the aforementioned method, the vertical distribution of friction velocity $u_*$, turbulent transfer coefficient $k_m$ and mixing length $\alpha$ were calculated.

![Figure 1](image1.jpg)

**Fig. 1.** Spiral of the observed wind velocity on August 12, 1982, with $v_g$ direction as X-axis direction.

![Figure 2](image2.jpg)

**Fig. 2.** Observed wind velocity component comparing with Lepzig data

Dotted line: Lepzig
Solid line: observed data of this paper.
The \( k_m \) values at various altitudes from the ground to 1 kilometer are listed in Table 2 and are as shown in Fig. 4. The figure gives the \( k_m \) profiles of August 12 and April 29 and those calculated from the Lepzig data. The comparison shows that the trend of their distribution is in fair agreement (especially obvious above 400 meters). On the average, the \( k_m \) values of this paper are greater than those of Lepzig's. The difference mainly occurs at a lower altitude. The Lepzig \( k_m \) curve increases very quickly under 100 meters with the maximum occurring at 200 meters. The \( k_m \) curve of our measuring point increases slowly under 100 meters, then increases sharply reaching its maximum at around 250 meters. The curves for April 29 and August 12 match well under 200 meters (the discrepancy at a high altitude was due to the flow field of April 29 not meeting the assumed conditions fully, and effects of thermal wind were present at a higher altitude). Moreover, the Lepzig profiles were the average profiles of 28 observed data in one day, yet the profiles of this paper for August 12 were the average of 12 observed data on that day, making the curve not as smooth. The general trend, however, is in fair agreement. Therefore, the Lettau method is creditable. It can be seen from comparing Fig. 2 and Fig. 4 that the distributions of \( k_m \) and \( \nu \) are basically similar. Just as explained by Lettau [1], the energy of turbulent momentum transfer is obtained from the "potential energy" of horizontal barometric pressure through the effects of \( \nu \).
Table 2. Vertical distribution of wind velocity component and \( k_m \)

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( 0 )</th>
<th>( 50 )</th>
<th>( 100 )</th>
<th>( 150 )</th>
<th>( 200 )</th>
<th>( 250 )</th>
<th>( 300 )</th>
<th>( 350 )</th>
<th>( 400 )</th>
<th>( 450 )</th>
<th>( 500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{u}{Z} )</td>
<td>( \frac{v}{Z} )</td>
<td>( \frac{w}{Z} )</td>
<td>( \frac{u}{Z} )</td>
<td>( \frac{v}{Z} )</td>
<td>( \frac{w}{Z} )</td>
<td>( \frac{u}{Z} )</td>
<td>( \frac{v}{Z} )</td>
<td>( \frac{w}{Z} )</td>
<td>( \frac{u}{Z} )</td>
<td>( \frac{v}{Z} )</td>
<td>( \frac{w}{Z} )</td>
</tr>
<tr>
<td>( 0.00 )</td>
<td>( 5.65 )</td>
<td>( 7.44 )</td>
<td>( 10.34 )</td>
<td>( 11.21 )</td>
<td>( 11.64 )</td>
<td>( 11.92 )</td>
<td>( 12.36 )</td>
<td>( 12.82 )</td>
<td>( 13.25 )</td>
<td>( 13.72 )</td>
<td></td>
</tr>
<tr>
<td>( 0.00 )</td>
<td>( -2.44 )</td>
<td>( -3.34 )</td>
<td>( -5.15 )</td>
<td>( -6.36 )</td>
<td>( -4.27 )</td>
<td>( -4.16 )</td>
<td>( -4.04 )</td>
<td>( -3.96 )</td>
<td>( -2.85 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 1.00 )</td>
<td>( 5.65 )</td>
<td>( 7.44 )</td>
<td>( 10.34 )</td>
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<td>( -4.04 )</td>
<td>( -3.96 )</td>
<td>( -2.85 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The vertical distribution of the calculated mixing length \( l \) values is as shown in Fig. 5. From the ground up, \( l \) increases with altitude. At the altitude between 260-300m, it reaches the maximum, then starts to decrease slightly. This is consistent with the observation of Rossby\(^2\). Figure 5 also gives the calculated values based on the Blackader\(^3\) and Lettau\(^3\) experience models.

![Fig. 4. Vertical distribution of \( k_m \) --- August 12 --- April 29 --- Leipzig.](image)

![Fig. 5. Vertical distribution of \( l \) --- August 12 --- Lettau --- Blackader.](image)
It can be seen from Fig. 5 that there is a difference between our calculated $l$ values and those of the above two formulae. The main discrepancy occurs in the upper portion of the boundary layer where the trend of $l$ distribution is quite different. Formula (11) gives $l$ values which increase quickly from the ground up to about 300 m, then the increase slows down as altitude increases, formula (12) gives $l$ values which increases sharply up to 400m, and from then on the values stay essentially unchanged. The $l$ values of this paper which were calculated from the observed data increase from the ground up and the maximum value occurs at around 300m. From then on, they slowly decrease. If formula (12) is modified as

\[ l = \frac{xz}{1 + (z/z_m)^{0.3}} \]  

and its $x$ is set as 0.35 which was proposed by Bussinger (1971), then the calculated values of formula (13) are in better agreement with those of the observed data as shown by the solid line in Fig. 5.

The calculated results of friction velocity $u_*$ are as shown in Fig. 6. From the ground up, $u_*$ decreases linearly with respect to altitude. The experience formula for $u_*$ in the neutral tower layer proposed by Clarke (1970) is:

\[ u_* = u_* - 6fz \]  

Based on our calculated results, at altitude under 200m, the following formula

\[ u_* = u_* - 5fz \]
will fit better. As for the basic trend of linear decrease with respect to altitude for $u_*$, both formulae are in total agreement.

Fig. 6. Vertical distribution of friction velocity $u_*$ - results of this paper - Clarke.

V. CONCLUSIONS

(1) Our calculated results have shown: by using the Lettau method, it is feasible to calculate turbulent parameters of the neutral, horizontally uniform and steady flow field from average wind velocity profiles. More satisfactory results can be obtained as long as observed data meeting the assumed conditions are available.

(2) The primary results of the vertical distribution of the neutral boundary layer turbulent parameters calculated from observed wind velocity data are as follows:

The $k_m$ values, from the ground up, increase linearly with respect to altitude. The maximum values occur around 250m.
decrease gradually.

The $u_*$ values, from the ground up, increase linearly to the range of 250-300m where the maximum occurs, then decrease gradually.

The $u_*$ values decrease linearly from the ground up. The rate of direct decrease is 5f.

(3) Our calculation results are compared with the research results from abroad, and the distribution profiles of $\tau$, $u_*$, $k_m$ and $l$ in the entire boundary layer are in fair agreement. But there are discrepancies in specific distribution profiles, which are shown when comparing formulae (11) and (12) with formula (13); formula (14) with formula (15); and $k_m$ profile in Fig. - 4. As stated before, the discrepancies are caused by difference in ground roughness. In addition, our observed data for August 12 were obtained from 12 groups of profiles for the period between 08:30 and 17:30. There was a little change in the temperature structure during this period. By analyzing the synchronized data from low altitude air observations: 

$$\frac{\Delta T}{\Delta z} = 0.4 - 0.8 \degree C/100m; \quad \frac{\Delta \theta}{\Delta z} = 0.45 \degree C/100m.$$ 

They did not satisfy the neutral conditions completely.

(4) There are given limitations in the use of wind data measured by the bivane anemometer to calculate turbulent parameters:

(a) It can not accurately calculate the average wind velocity at different altitudes for the same time instant from the trace of the small balloon which rises at a constant speed. First of all, the timing of measurement for different altitudes can not be strictly "synchronized". For a small balloon with a rising speed of 100m/min, the time lag can reach 15 minutes between the first measurement and that at an altitude of 1.5km. Secondly, the time intervals of the average values are not very
representative. If more measuring points are added, then the time interval for taking readings must be shortened (i.e. the average time interval). Our observation took 4 readings in one minute, i.e. every wind velocity value represents the average wind velocity within a 25m altitude range. Obviously this time interval is quite small and can not include turbulent vortexes of a wider scale. Therefore, the profiles are not smooth enough and must conduct numerous repeat observations to obtain the average. Yet it is difficult to maintain stable temperature structure and macroscopic meteorological conditions during the repeat observation period, thus causing the application of said method to have given limitations.

(b) The baseline selected for bivane anemometer observations must be perpendicular to wind direction. Wind direction close to the ground and the boundary layer usually would shift 20°-30° or more as altitude increases. Therefore, the same baseline, which is proper for use on the ground, becomes not perpendicular to wind direction as the balloon rises to the top portion of the boundary layer and the wind direction shifts. Even a difference of just a few degrees in the two bivane anemometer direction readings will cause great calculation errors, and the vertical projection method or vector method must be used. If the directional angles differ little and so do the attack angles, then the above two methods will do little good. We imagine, if two baselines are used simultaneously along with a trivane anemometer to conduct observations, the aforementioned defects can expect to be alleviated, thereby raising the accuracy of wind velocity profile measurements in the boundary layer.

The observed wind velocity profile data used in this paper were provided by the "Yizheng Chemical Fibers Industry United Company Environmental Effects Evaluation" cooperative group (atmospheric). Those who organized and participated in the field
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LITERATURE

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