Research in nonlinear acoustics was carried out during the period
1 April 1975 - 31 August 1986. The general goal was to understand the behavior of sound waves of finite amplitude. Progress toward this goal was made by attacking specific problems in the following areas:

1) Theoretical work on periodic waves, including Burgers' equation and sound beams.

N waves diffraction nonlinear acoustics nonlinear ray acoustics
focusing sound beams Burgers' equation nonlinear porous materials
schlieren optoacoustics B/A for sea water interaction of tone and noise
waveguides shock rise time parametric receiver suppression of sound by sound
weak shocks outdoor sound parametric transmitter water-to-air transmission
20. (Cont'd)

(2) Tones, noise, and interaction—mainly experimental, including
outdoor propagation of tones and noise, suppression of sound by
sound, interaction of noise with a tone, and noncollinear inter-
action of two tones in a rectangular waveguide.

(3) N waves and other transients, including development of a schlieren
system, diffraction by a slit, propagation in a cylindrical tube,
shock rise time, and propagation of a weak shock having a tail of
arbitrary waveform.

(4) Propagation in inhomogeneous fluids, including values of B/A for
sea water and nonlinear geometrical acoustics with applications
to long range underwater propagation.

(5) Nonlinear effects in air-filled porous materials.

(6) Parametric receiving array.

(7) Focused beam parametric transmitter and

(8) Miscellaneous topics.

Work was also done on selected topics in linear acoustics, such as thermo-
acoustics and water-to-air sound transmission. Most of the research was
carried out as graduate student thesis projects. Students supported under
the contract earned nine B.S. degrees, ten M.S. degrees, and three Ph.D.
degrees. Three foreign scientists also contributed substantially to the
research.
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I. INTRODUCTION

Basic research on nonlinear acoustics was carried out under Contract N00014-75-C-0867 during the period 1 April 1975 - 31 August 1984. Predecessor contracts were Contract N00014-70-A-0166, Task 0021, for the period 1 April 1974 - 31 March 1975 (73-1)* and Contract N00014-70-A-0166, Task 0004, for the period 1 January 1971 - 31 December 1972 (75-3). Some of the work described in this report was started during the earlier contracts.

The basic objective of the research was to investigate the behavior of sound waves of sufficiently high intensity that linear theory no longer suffices to describe the phenomena. Nonlinearity affects propagation, absorption, diffraction, focusing, and other acoustic phenomena. The study of nonlinear wave behavior was carried out by performing the investigations described in Section III.

Although nonlinear acoustics was the central theme of the research, a few investigations on topics in linear physical acoustics, notably opto-acoustics and transmission across a water-air interface, were included. They are also described in Section III.

*Numbers in parenthesis stand for items in the Chronological Bibliography. For example, (73-1) means Ref. 1 in the list for 1973.
II. TASK O. NONLINEAR ACOUSTICS (GENERAL)

A. Theoretical Work on Periodic Waves

1. Burgers' Equation

One of the early breakthroughs in the modern era of nonlinear acoustics (post World War II) was the application of Burgers' equation to the problem of propagation of plane waves of finite amplitude in a thermoviscous fluid. Burgers' equation is

\[ u_x - \left( \frac{\beta}{c_0^2} \right) uu_t = Au_t' t' \quad , \]

where \( u \) is particle velocity, \( x \) is distance, \( t' = t - x/c_0 \) is retarded time (\( t \) is actual time), \( c_0 \) is the small-signal sound speed, \( \beta \) is the coefficient of nonlinearity, and \( A \) is a constant proportional to the coefficients of viscosity and heat conduction. The small-signal attenuation coefficient \( \alpha \) in such a medium is \( \alpha = A \omega^2 \), where \( \omega \) is the angular frequency. The attraction of Burgers' equation is twofold. First, it is an accurate model for the physical problem (finite-amplitude distortion in combat with thermoviscous absorption). Second, it has a known exact solution. Unfortunately, investigators have had trouble extending the model to other, more practical problems involving (1) other wave types, e.g., spherical waves, cylindrical waves, and so on, and (2) other media, such as relaxing fluids. The following two projects dealt with the problem of generalizing the approach based on Burgers' equation.

a. Asymptotic Solution of Burgers' Equation for Spherical, Time Harmonic Waves

The Burgers equation for spherical waves is

\[ u_r + u/r - \left( \frac{\beta}{c_0^2} \right) uu_t = Au_t' t' \quad , \]

where $r$ is radial distance, the delay time is $t' = t - \frac{(r - r_0)}{c_0}$, and $r_0$ is source radius. For single frequency sources, the boundary condition is $u(r_0, t) = u_0 \sin \omega t$. No exact solution of this problem is known. Solutions valid in various regions have been found, but difficulty has been encountered in obtaining the asymptotic solution valid for great distances. We obtained a solution (78-10) in the form of a Fourier series,

$$u = \sum_{n=1}^{\infty} u_n(r) \sin n\omega t' ,$$

(3)

in which the components $u_n$ have the following form at great distance ($kr >> 1$):

Weak waves ($\Gamma \leq 1$): $u_n = u_0 (\Gamma/4)^{n-1} (r_0/r)^n e^{-n\alpha (r-r_0)}$,  

(4)

Strong waves ($\Gamma >> 1$): $u_n = (4\Gamma c_0/\beta k) K^n (r_0/r)^n e^{-n\alpha (r-r_0)}$  

(5)

In these expressions $\Gamma = \beta \varepsilon k/\omega$ is the Gol'dberg number, $\varepsilon = u_0/c_0$ is the peak particle velocity Mach number at the source, $k = \omega/c_0$ is the wave number for the fundamental frequency component, and $K$ is an undetermined constant. Although our inability to determine a value for $K$ (other than the limiting value $K \rightarrow 1$ as $r_0 \rightarrow \infty$) is regrettable, Eqs. 4 and 5 do show some interesting results. Small-signal theory would lead one to expect that a spherically spreading signal of angular frequency $n\omega$ (i.e., one of the higher harmonics) would decay as $(r_0/r)^n e^{-n\alpha (r-r_0)}$. This expectation is clearly not fulfilled. The explanation is that at great distances the received harmonic signals come predominantly from virtual sources near the receiver, not virtual sources near the source. Judged by the harmonic components ($n \geq 2$), therefore, the old age regime of the wave is not one in which linear theory holds. This result, which is also pointed out in Refs. 78-3, 78-7, and 79-7, has altered our view of the character of the old age region: linear theory holds for the fundamental but not for the higher harmonics.

---


b. A Burgers Equation for Arbitrary Media

To obtain a Burgers equation for plane waves in fluids with arbitrary attenuation and dispersion properties, we replaced Eq. 1 with

\[ u_x - \left( \frac{1}{c_0^2} \right) uu_t + L(u) = 0 \]

(6)

where the operator \( L(u) \) describes the attenuation and dispersion properties, even if known only empirically (81-6, 85-3). The following excerpt from the abstract of Ref. 85-3 summarizes the work:

"Specific forms of \( L(u) \) are given for thermoviscous fluids, relaxing fluids, and fluids for which viscous and thermal boundary layers are important. A method for specifying \( L(u) \) when the attenuation and dispersion properties are known only empirically is described. A perturbation solution of the generalized Burgers equation is carried out to third order. An example is discussed for the case \( a_2^2 = 2a_1^2 \), where \( a_1 \) and \( a_2 \) are the small-signal attenuation coefficients at the fundamental and second-harmonic frequencies, respectively. The growth/decay curve of the second harmonic component is given both with and without the inclusion of dispersion. Dispersion causes a small reduction of the component. The extension of the generalized Burgers equation to cover nonplanar one-dimensional waves is given."

2. Nonlinear Effects in Sound Beams, Including the Parametric Array

After making great progress during the 1960s with the propagation of simple plane, spherical, and cylindrical waves, investigators turned their attention to intense sound beams. Their interest was stimulated (1) by the development of the parametric array, and (2) by the fact that most practical acoustical sources are directional. To analyze propagation in beams, one must find some way of accounting for diffraction, which shapes the beam. In some early work, \(^4\) considerable success was achieved by essentially superposing nonlinear propagation effects on an already formed beam, that is, by ignoring the inherent interaction between diffraction and nonlinear distortion. \(^4\) However, the need to include the

inherent interaction, particularly for the nearfield, led to interest in
the now famous Zabolotskaya–Khokhlov equation, later generalized by
Kuznetsov to include the effect of thermoviscous dissipation.

The work on intense sound beams under Contract N00014-75-C-0867
was carried out largely by S. and J. N. Tjøtta during the period 1978-80
and by M. F. Hamilton during the period 1981-83. The Tjøttas were at
Applied Research Laboratories, The University of Texas at Austin (ARL:UT)
on leave from the University of Bergen, Bergen, Norway. Hamilton,
a doctoral candidate at Pennsylvania State University, joined our group to
complete his doctoral research (in absentia) on parametric arrays, namely
collinear and noncollinear interaction in dispersive fluids.

a. Intense Sound Beams and Parametric Arrays (Tjøttas)

A highlight during the two-year period 1978-80 was the
association of Sigve and Jacqueline N. Tjøtta with our research group.
Having spent the first year at The University of Texas at Austin as Visiting
Scholars (supported by the Norwegian government), the Tjøttas continued
for a second year with support from ONR under Contract N00014-75-C-0867 and
Naval Coastal Systems Center. Supported by ONR, Sigve Tjøtta was principally
responsible for work on nonlinear effects in sound beams, while Jacqueline
N. Tjøtta, supported by NCSC, was principally responsible for analyzing
the interaction of beams (both parametrically and linearly generated) with
a water-sediment interface. In fact, however, each helped the other, and
all their publications bear both their names. Only the ONR associated work
is reported here.

During their first year (September 1978 - August 1979), the
Tjøttas investigated nonlinear effects in sound beams, mostly with appli-
cations to the parametric array in mind. They wished to account not only
for nonlinear interaction, which is what produces the parametric array,
but also for absorption and diffraction, which determine many of the
characteristics of the parametric array. In this instance diffraction

---

refers to that caused by the spatial extent of the virtual sources in the interaction region, not nearfield effects in the primary beams themselves. Thus in their first analysis (79-2, 80-1), they assumed spherically spreading beams; no account was taken of the primary nearfields.

Subsequently, however, they began to examine the role played by the primary nearfields, in particular the phase and amplitude variations therein. Reference 79-3 includes a brief report of a new analytical approximation of the small-signal nearfield and farfield of a circular piston. Using this approximation, they analyzed the effect of phase and amplitude oscillations in the carrier nearfields on the performance of the parametric array. The oscillations generally cause a reduction in parametric array amplitude but may sharpen the directivity somewhat. Also included in the paper are solutions, for a two-frequency source, of the plane-wave Burgers equation for both weak and strong waves. The weak-wave solution shows that taper functions used by some authors in modeling the virtual source distribution are in error. For the case of strong waves, an asymptotic expression (resembling the Fay solution) is given for the difference frequency wave and all its harmonics.

References 79-5 and 80-3 contain a detailed account of their work on the field of a circular piston (linear theory, including absorption). Both rigid and pressure release baffles are considered.

In Refs. 79-4 and 80-6, the Tjøttaas consider several different carrier field properties that affect the parametric array. Some of the topics described briefly in Ref. 79-3 are treated in detail and extended. Highly collimated uniform beams (the appropriate model when the zone of interaction ends prior to $R_0/2\pi$, where $R_0$ is the Rayleigh distance for the primary beams) and Gaussian beams, for which the amplitude variation across the beam is Gaussian, are considered. A generalized aperture factor is derived (the Tjøttaas had published their discovery of the aperture effect 15 years earlier). A more complete treatment of the effect of carrier nearfield oscillations is also included. The problem of extra attenuation (caused by nonlinearity) on the primary beams is taken up in Ref. 80-2. Reference 80-5 is a review paper about parametric arrays.
The Tjødtas' final article under the contract, "Nonlinear equations of acoustics, with applications to parametric acoustic arrays" (81-5) is a fitting work with which to conclude their effort during the two-year period. It is a tour de force. Starting from first principles, the authors derive a nonlinear wave equation that contains the most important terms relating to nonlinearity, dissipation (for a thermoviscous fluid), and diffraction. Certain well known equations derived by other investigators, e.g., those of Westervelt, Zabolotskaya and Khokhlov, and Kuznetsov, turn out to be special cases. A number of specific phenomena, such as nonlinear attenuation and several aspects of parametric array behavior, are then analyzed. Besides including new results, the paper brings a remarkable unification to a wide range of investigations.

b. Effect of Dispersion and Noncollinear Interaction on Parametric Arrays (Hamilton)

M. F. Hamilton, a graduate student at The Pennsylvania State University, was about halfway through his doctoral research when his supervisor, F. H. Fenlon, died in June 1981. Hamilton was invited to come to ARL:UT to continue the research program that had been planned by Fenlon. The arrangement was agreed upon mutually by Hamilton and by Drs. Tichy, Maynard, and Piggot of Penn State, Blackstock of ARL:UT, and Hargrove of ONR. The very successful conclusion of the research is a tribute to the foresight of Fenlon, the dedication of Hamilton, and the good will of all persons concerned. Although the direct support for the work continued to come from the Penn State contract (N00014-79-C-0624, J. Maynard, Principal Investigator), the indirect support of the ARL:UT contract, which provided the research group and atmosphere in which Hamilton completed his work, was substantial.

Kuznetsov's paraxial wave equation was used as the starting point for the analysis, and the parametric array was assumed to be formed by Gaussian primary beams. The effect of inherent dispersion (dispersion due to the properties of the medium) on parametric generation was first carefully analyzed for primary beams that are collinear (82-7, 84-4).

Noncollinear primary beams were then considered (83-5, 84-5) with the hope that geometric dispersion (dispersion due to noncollinearity) might be used to compensate for inherent dispersion. It was found that although compensation may be realized under certain very restricted conditions, it is not a practical approach because a penalty must be paid in overall efficiency. The investigation is thus of high scientific value but of practical value only in the negative sense. Because of the excellence and thoroughness of his work (83-1), Hamilton was awarded the F. V. Hunt Postdoctoral Fellowship by the Acoustical Society of America for 1983-84. He took the Fellowship at the University of Bergen, Bergen, Norway, with the Tjøttas.

B. Tones, Noise, and Interaction--Mainly Experimental

In the 1960s investigators in nonlinear acoustics devoted their attention primarily to the following two topics: (1) one-dimensional radiation from single frequency sources, and (2) parametric arrays. Although work on the two topics continued into the 1970s, interest in new, related problems developed. Propagation of finite-amplitude noise was a problem begging to be treated because of its application to intense sound fields generated by jet aircraft. A study of plane waves of noise, carried out with partial support of the first of the two predecessor contracts (73-1), had led to the discovery of important phenomena. The extension to the more practical problem of spherically spreading noise in the atmosphere was one of the first projects of Contract N00014-75-C-0867. At about the same time, we began work on the suppression of sound by sound, a topic that had grown out of work done elsewhere on parametric arrays. These projects and their descendants are described in this section.


1. Propagation of Intense Sound in the Atmosphere

The primary goal of this work was to measure the propagation of intense noise in the atmosphere. We wished to determine whether the effects discovered in the earlier, laboratory study\(^8\) have application to aircraft noise. An outdoor propagation path was set up parallel to an 85 m tall radio tower. By choosing a vertical path, we were able to examine the effects of spherical spreading and (random) inhomogeneity of the medium without including the additional complication of ground reflection or bending of the rays due to refraction. The source, either an array of conventional horns or a siren, was located on the ground. After passing through a relatively short nearfield, the waves spread spherically over the remainder of the propagation path. The receiving microphone was mounted on a boom carried by the tower elevator. Two sets of experiments, called Phase I and Phase II, were carried out. In Phase I, done primarily to test the system and study the effect of inhomogeneity of the medium, the source emitted a pure tone. The distortion of the propagating tone was measured under various conditions over a path as long as 76 m. Measurements in which the source emitted noise constituted Phase II.

a. Phase I. Outdoor Propagation of Intense Tones

In Phase I the source radiated intense periodic waves. The experiments were done by M. A. Theobald and D. A. Webster. The source frequency was generally in the 6-8 kHz band, and the source level SPL\(_{1m}\) (farfield sound pressure level extrapolated back to 1 m) was in the range 140-150 dB. The experiments were done during the summer and early fall of 1976, and the various results were reported in oral presentations (76-4, 77-2, 77-4, 77-5, 77-6) and in detailed technical reports (77-1, 78-3). The major results of Phase I are as follows.

(1) Substantial nonlinear effects were observed, including generation of higher harmonics, formation of shocks, and extra attenuation (up to about 10 dB) of the fundamental.

(2) Random inhomogeneity of the medium (temperature and wind fluctuations) caused short term variations in the time waveform of the propagating wave and in the levels of the various harmonic components. Long-term
averages of the harmonic levels, however, agree well with predictions based on theories of finite-amplitude distortion in a homogeneous medium.

(3) Interesting new effects attributed to the interaction between (near-field) diffraction and finite-amplitude distortion were observed (76-4, 77-1).

(4) A graphical display was developed that has proved to be very useful in the design of nonlinear acoustics experiments. Called a source-frequency level (SFL) chart, the display allows one to analyze known acoustical sources to determine whether their fields are expected to be affected by nonlinear propagation distortion (76-4, 77-1, 78-3).

b. **Phase II. Outdoor Propagation of Intense Noise**

The noise experiments were carried out during the summer of 1977 by D. A. Webster, with the assistance of D. E. Alexander, whose support came from other sources. For the noise experiments we had originally planned to use a very powerful modulated air source, a Ling EPT 200 capable of delivering 10,000 acoustic watts that had been provided GFE by NASA. Unfortunately, however, an air supply sufficient to run the unit could not be found. We had to rely instead on horn arrays, driven by conventional electroacoustic horn drivers, of the sort used in Phase I. Although much less powerful, the arrays produced noise intense enough to be subject to important nonlinear effects. Ten propagation experiments with noise were completed, and some auxiliary experiments on fluctuations and tones were done.

Typical results are shown in Fig. 1, which gives a comparison of spectra at various distances for a noise of relatively low intensity (first and third columns) and for a high intensity noise (second and fourth columns). The distances given for each spectrum are with respect to the mouth of the horn array; negative distance means the microphone was inside the horn. The electronic input signal was an octave band of noise centered at 4 kHz (see Input Current Spectrum). The acoustical spectra show the following. Even the low level noise was somewhat affected by nonlinearity: note the steady growth of the second-harmonic noise band, centered at 8 kHz. In the case of the high level noise, however, the growth of the
FIGURE 1
Change in noise spectrum with distance caused by nonlinear propagation distortion. First and third columns: OASPL_{1m} = 123 dB (low level).
entire high frequency end of the spectrum is spectacular. For example, at the horn mouth (distance = 0.0 m) the level of the signal at 20 kHz is at least 55 dB below the level of the 4 kHz band. At 70 m the difference in levels is only 25 dB. An enormous spectral distortion has occurred. Put another way, the 20 kHz signal has an absolute band level (SPL in a 100 Hz band) of about 67 dB at the horn mouth; yet it has nearly the same level at a distance of 70 m. The powerful attenuative effects of both spherical spreading and atmospheric absorption have been counteracted by the addition of energy pumped into the band as a result of nonlinear distortion.

To see the application of our results to jet noise, consider Fig. 2. Here spectrum levels of a KC-135A aircraft at 250 ft (ground runup, single engine only, measurement angle 30° off the jet axis) are compared with spectrum levels measured at the same distance in two of our highest intensity experiments. The KC-135A noise is basically much lower in frequency than the noise in our experiments. To compensate, we have scaled our spectra in order that they may be compared directly with the jet spectra. The scaling is done in such a way as to preserve the effect nonlinearity has on the noise. It will be seen that the KC-135A noise is roughly 10 dB higher, in the mid and high frequency ranges, than our noise. Since our noise was very definitely affected by nonlinearity, one must conclude that even stronger nonlinear effects were at work during the propagation of the KC-135A noise. Moreover, although the KC-135A is one of the noisiest aircraft, many other current aircraft produce noise that is within the 10 dB range below the noise of the KC-135A. Nonlinear effects must therefore be common in jet noise. This is a very important conclusion.

The results were reported in papers at meetings (77-10, 78-7), in an extensive technical report (78-3), and in a journal article (79-7).

c. Associated Work on Distortion of Random Signals

This is a collection of smaller tasks that grew out of questions or problems that arose during our work on finite-amplitude noise and interaction of noise with a finite-amplitude tone (see Section 2.a(4) below).
FIGURE 2

Comparison of actual jet noise spectra with scaled spectra (scaled down by a factor of 10 in frequency, scaled up 20 dB in level) from experiments AEM3 and JBL9. From Ref. 78-3.
(1) **Amplitude Density of a Finite-Amplitude Random Signal**

We found that as long as the waveform does not contain shocks, propagation distortion does **not** change the amplitude density of the wave (78-3, 79-1).

(2) **Effect of Finite-Amplitude Distortion on the Gaussian Character of a Noise**

Does a Gaussian signal remain Gaussian despite non-linear propagation? The answer is no. Although the amplitude density of a signal, if Gaussian to begin with, remains Gaussian as the wave distorts (see the previous task), the amplitude density of the derivative of the signal does not. The Gaussian character of the noise is therefore not preserved under propagation distortion.

(3) **Construction of a Noise Waveform Given only the Spectrum of the Noise (78-4, 86-3)**

In order to use Pestorius's algorithm to make calculations of distortion in the time domain, it is necessary to know the time waveform of the source noise. We have constructed the input noise waveform by starting with a measured spectrum of the noise (picked up by a monitor microphone as close to the source as possible) and assuming that the phase of the spectral components is a random variable with uniform probability density over the range $-\pi$ to $\pi$ (78-1). Having the amplitude and phase of all the spectral components, one obtains the time waveform by a simple FFT operation. We call this the random phase method. The method worked successfully in our research on tone-noise interaction (78-1). In that case the tone was intense but the noise was a small signal. The method failed to work, however, in our research on outdoor noise propagation (78-3). In this case the noise was more intense and some distortion had already taken place by the time the monitor microphone picked up the signal. To try to determine why the method works in some cases but not in others, we carried out some noise distortion measurements in our plane wave tube. It was found that the random phase method works well when the monitor microphone is in a region where either (1) very little, or (2) a great deal of distortion has occurred. In the early stages of propagation distortion,
the distortion apparently increases the phase coherence of the noise. The coherence reaches a maximum and then decreases as more distortion takes place. For the random phase assumption to work, therefore, the monitor microphone must be placed either very close to the source or far enough away that considerable propagation distortion has already occurred.

(4) **Spectral Shape of the Difference Frequency Noise When the Bandwidth of the Source Noise is One Octave or Less**

Each pair of spectral components in the source band interacts to produce a difference frequency component. Adding energies of the various contributions at each difference frequency $f_d$, we find that the pressure amplitude $P_d$ at that difference frequency is, for plane waves,

$$P_d \propto f_d \sqrt{BW - f_d},$$

where $BW$ is the bandwidth of the source noise. For a directional source of spherical waves the farfield result is

$$P_d \propto f_d^2 \sqrt{BW - f_d}.$$

The spectrum indicated by Eq. 7 has a peak at $f_d = (2/3)BW$; (for Eq. 8 the peak is at $(4/5)BW$). Measurements in our plane wave tube (see the previous task) roughly confirm the predicted position of the peak.

2. **Interaction of Sound with Sound**

Although the best known example of the interaction of one sound with another is the parametric array, other interaction phenomena have been studied. We performed the following experiments on collinear interaction of plane waves: suppression of sound by sound, interaction of noise with a finite-amplitude tone, and degenerate parametric amplification. Later some experiments with tones in a waveguide were done in order to study noncollinear interaction.

a. **Collinear Interaction**

Our first work was done on the suppression of sound by sound. The use of one sound to suppress another was discovered in the early 1970s by several investigators in the United States$^{9-11}$ and in the

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Soviet Union.\textsuperscript{12} It is interesting to note that solutions for the interaction of two sounds had been obtained earlier by Mrass and Brinkmann\textsuperscript{13} (76-1) and by Fenlon.\textsuperscript{14} Motivated by other applications, they apparently did not recognize suppression as one of their predictions. Our study of suppression, which was stimulated by Moffett’s report,\textsuperscript{9} began under the immediate predecessor contract, Contract N00014-70-A-0166, Task 0021 (75-3).

(1) Suppression of Sound by Sound

Suppression is caused by the modulation of one sound wave by another. The wave being suppressed is called the weak wave (frequency $f_w$) because it is usually a small signal. The wave doing the suppressing (frequency $f_p$) is a finite-amplitude signal called the pump. The modulation causes sideband signals to be produced. One hopes that the growth of the sidebands is at the expense of the weak signal. If this hope is realized, the amplitude $u_w$ (particle velocity) at frequency $f_w$ will be reduced.

For the region prior to shock formation ($x < x_p$, where $x_p$ is the shock formation distance for the pump), the formula for $u_w$ is

$$u_w = u_{w0} \left| J_0 \left( \frac{f_p}{f_w} \sigma_p \right) \right|$$

where $u_{w0}$ is the initial weak wave amplitude (before the pump is introduced) and $\sigma_p = x/x_p$ (Eq. 9 is valid for $\sigma \leq 1$). Since the Bessel function $J_0$ has a zero when its argument is 2.4, a complete null of the weak wave is in principle possible at the distance

$$x_{null} = 2.4 \left( \frac{f_p}{f_w} \right) x_p$$

Two cases may now be recognized: Case I in which the pump is the lower frequency wave, and Case II in which the pump is the higher frequency wave. It is clear that in Case I a complete null is possible (provided $f_p < f_w/2.4$) before the pump forms a shock. No null is possible, at least in the shock free region, in Case II; indeed the potential for suppression is much less.


\textsuperscript{13}H. Mrass and K. Brinkmann, Acustica 14, 205-211 (1964) (in German).

Case I suppression was studied by Schaffer (75-2, 75-4) and found to be very effective. Our 30 m long, 5 cm i.d. progressive plane wave tube (air-filled) was used for the measurements. The properties of the signals at the source were as follows: $f_w = 2.8$ to 10.5 kHz, $SPL_w < 100$ dB, $f_p = 0.769$ to 1 kHz, $SPL_p = 130$ to 150 dB. As much as 40 dB of suppression was measured in one experiment.

The main disadvantage of Case I suppression is that sidebands about the weak signal (the modulation components produced have frequencies $f_w + f_p$, $f_w + 2f_p$, $f_w + 3f_p$, and so on) and some of the harmonics of the pump signal pollute the frequency region in the neighborhood of $f_w$. Willshire (77-3) therefore undertook a thorough study of Case II suppression, which was known to offer much less suppression in the preshock region but also to cause much less pollution of the spectrum in the neighborhood of the weak signal frequency. It was hoped that by extending the theory and measurements into the postshock region, we would find increased suppression. This hope was not realized, however. On the other hand, the study did greatly increase our understanding of the nonlinear interaction of two periodic signals. A good summary of Willshire's research is given by the abstract of his thesis (77-3):

"The purpose of this research was to study the suppression of one sound by another sound of higher frequency. More specifically, the planar propagation in air of a finite amplitude tone (the pump) and a smaller amplitude tone (the weak signal) of lower frequency was investigated both theoretically and experimentally. . . . The amount of suppression depends qualitatively on whether the pump frequency is lower than the weak signal frequency (Case I) or vice versa (Case II). Case I has been previously studied and large suppression of the weak signal has been obtained. Even though preliminary analytical work indicated that little suppression is possible in Case II, this research was undertaken to determine the amount of suppression possible.

"The theoretical investigation was concerned with both Case I and Case II; a general notation was adopted so that the analysis of a single solution was applicable to both cases. Various exact and approximate
preshock solutions were reviewed in the time (Earnshaw solution) and frequency (Fenlon solution) domains. These solutions were then compared with each other and ranked with regard to their exactness. Because general solutions valid in the postshock region were not available, postshock solutions were developed based on weak shock theory. Weak shock theory was implemented in a computer algorithm with corrections for attenuation and dispersion. Predictions obtained from the use of the program were compared to experimental results. The experimental work was restricted to Case II. For the weak signal the experiments (performed in a 30 m progressive wave tube) covered a range of source SPL of 104 to 121 dB and a frequency range of 0.6 to 1 kHz. The corresponding ranges for the pump were 125 to 158 dB and 1.5 to 6.6 kHz, respectively. The measurement distances ranged from 3.9 m to 27 m. The agreement between experimental results and theoretical predictions was excellent. In particular the experiments confirmed the theoretical prediction that little suppression of the weak signal is obtained for Case II either before or after shock formation. The sidebands in both cases form around the high frequency signal and its harmonics and the energy for the sideband formation comes primarily from the high frequency signal (including its harmonics). Thus another general conclusion is that the low frequency signal modulates the high frequency signal, regardless of their respective amplitudes."

(2) Interaction of Noise with a Finite-Amplitude Tone

The next interaction study, effect of an intense tone on small-signal noise, was qualitatively different from the two studies of suppression. The results of a typical experiment, also done in our progressive plane wave tube, are shown in Fig. 3 (77-2, 77-6, 78-1). The top row shows spectra of the intense tone and its harmonics (no noise present) at three different distances: \( x=0 \) (source), \( x = \bar{x} = 7.38 \text{ m} \) (shock formation distance), and \( x = \hat{x} = 22.1 \text{ m} \) (well-formed sawtooth distance). For the spectra in the second row, the tone is turned off and only the noise is on. Finally, the third row shows the spectra when both tone and noise are turned on. The noise portion of the combination spectrum may be inferred by mentally filtering out the tone and its harmonics. It is not surprising
FIGURE 3
Interaction of tone with noise.
From Refs. 77-2, 77-6, and 78-1.
that the noise spectrum broadens as the combined signal propagates. What is surprising is the extent and character of the broadening. The noise spectrum is tremendously enhanced, decaying at only 2 or 3 dB/octave. In fact, when the spectrum analyzer bandwidth was extended to 50 kHz (its maximum), the slow rolloff property was observed to extend at least that far.

In the journal article (78-1) in which this investigation was reported, we described computations done to support the observations. A computer program based on classical nonlinear acoustics theory, with corrections added for ordinary attenuation, yielded predictions that agree with the experimental data. Unfortunately, the computational work provided little physical understanding of the phenomenon. Qualitatively, we attributed the spectral effects to modulation of the tone by the noise. Since modulation may be expected to produce upper and lower noise sidebands about each tonal harmonic, we thought the observed broadband noise might be simply a composite of all the various sidebands. Recall Willshire's study of Case II suppression, i.e., the interaction of a weak low frequency tone with an intense high frequency pump. Spectra measured by Willshire are shown in Fig. 4. The sidebands, which in this case occur at frequencies $f_p \pm f_w$, are clearly evident in Fig. 2(c). It can be seen that if the weak signal were a band of noise rather than a tone, the sidebands would tend to overlap. The amount of overlap would depend on the noise bandwidth and the proximity of the noise band to the pump frequency.

![FIGURE 4](image-url)

Tone-tone interaction for a 1 kHz weak wave and a 4.5 kHz pump. Spectra measured 4.2 m from the source. From Ref. 77-3.
After the publication of Ref. 78-1, a brief analysis was done that quantitatively supports the notion that the broadband noise produced in the tone-noise experiments is indeed sideband noise. Since this analysis has not been published, it is presented here in some detail. For our analytical tool we used results from Fenlon's solution for a two-frequency source.¹⁴ This solution, which is valid prior to shock formation, had been adapted by Willshire for the case he studied (Ref. 77-3). For plane waves in a lossless medium, the particle velocity amplitude for the primary sideband at frequency $nf ± f_w$ is

$$u_{1,n} = u_{wo} \left\{ \begin{array}{l} \frac{nf_p ± f_w}{f_p} \left( \frac{f_p}{2} \sigma_p \right) \\ \end{array} \right\}, \quad (11)$$

where the symbols are the same as those used in Eq. 9. The + sign signifies the upper sideband, the - sign the lower sideband. To account approximately for attenuation, we modify $u_{wo}$ and $\sigma_p$ in Eq. 11 as follows:

$$u_{wo} \rightarrow u_{wo} e^{-\alpha_w x},$$

$$\sigma_p \rightarrow \frac{1 - e^{-\alpha_p x}}{\alpha_p x},$$

(the second correction is discussed in Ref. 75-4). Here $\alpha_w$ and $\alpha_p$ are the ordinary attenuation coefficients at the weak wave and pump frequencies, respectively. We made use of Eq. 11 to compute the expected spectrum level at the particular frequency $f = 8114$ Hz (a point in the broadband noise) at the $x = 7.38$ m measurement distance. Eight spectral components in the original noise band were identified that combine with appropriate pump harmonics to satisfy the frequency relation $nf_p ± f_w = 8114$ Hz. When Eq. 1 (corrected for attenuation as described) was used to compute the contributions of the eight pairs of signals, only four contributions were found to be important. They were combined on an intensity basis. The resulting composite signal was found to have a (100 Hz) band level of 102.5 dB. This level compares very favorably with the measured level, 103 dB, which may be
read from Fig. 3 at 8114 Hz. Although the calculation was done for only
one frequency in the broadband spectrum, the excellent agreement is strong
evidence in favor of the sideband thesis. Moreover, further analysis of
Eq. 11 showed that a very slow high frequency rolloff is to be expected.

(3) Degenerate Parametric Amplification

A very small effort was spent on this topic. Let the
two primary waves be tones of frequencies $f_w$ and $f_p$, ordered as in Case II
suppression. If the pump frequency is chosen so that $f_p = 2f_w$, then the
frequency of the difference tone (the first lower sideband; see Fig. 4(c)) is
equal to $f_w$. One may therefore use the interaction to amplify the weak
wave, a possibility that was considered many years ago. Calculations at
that time showed that the relative phase $\phi$ of the two primary signals is
very important. If $\phi = \pi/2$, the difference frequency wave adds to the weak
wave; if $\phi = 0$, subtraction occurs. In the present work, Fenlon's solution was
used to obtain the most general formula for the gain $G$ at the weak wave
frequency:

$$G = \sqrt{\frac{2}{3} - 2J_0^2 J_1^2 \cos 2\phi + J_1^2},$$

(12)

where the argument of each of the two Bessel functions is $\sigma_p/2$. The largest
gain, which occurs when $\phi = \pi/2, \sigma = 1$, is only 1.2 dB. The least gain, which
occurs when $\phi = 0, \sigma_p = 1$, is -3.1 dB. Because of the small effect, we did
not pursue the matter further. However, measurements made in Japan, with
theory improved to include absorption (a version of Pestorius's algorithm was used),
confirm the important role played by the phase (82-5).

(4) Review Paper

Finally, a review of work on collinear, tone-tone inter-
actions (research of others as well as our own) was given at an interna-
tional symposium in 1982 and published in 1983 (82-5).

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15 During 1964-65, in work supported by AFOSR under Contract AF49 (638)-1320.
b. **Noncollinear Interaction in a Rectangular Waveguide**

A rectangular (air-filled) waveguide was used in an experiment done by J. A. TenCate to study the interaction of two noncollinear wave fields. The first field was that of an intense low frequency pump signal propagating down the guide in the \((0,0)\), or principal, mode. The second wave, a weak high frequency signal, traveled down the guide in the \((0,1)\), or first bouncing, mode. The arrangement is shown in Fig. 5. As indicated in the figure, the angle \(\theta\) between the two fields is given by

\[
\sin \theta = \frac{c_o}{2a\omega_w},
\]

where \(a\) is the separation distance between the two walls that support the \((0,1)\) mode, and \(\omega_w\) is the (weak wave) drive frequency. One may thus vary the angle by varying the drive frequency.

![Propagation of \((0,0)\) and \((1,0)\) modes in a waveguide.](image)

**FIGURE 5**

Propagation of \((0,0)\) and \((1,0)\) modes in a waveguide. From Ref. 83-7.
This work was originally motivated by a desire to study the dependence of the coefficient of nonlinearity $\beta$ on the interaction angle $\theta$. For collinear interaction ($\theta=0$) $\beta$ is known to be given by

$$\beta = 1 + (\gamma-1)/2 \quad \text{(usually written } (\gamma+1)/2) \quad (14a)$$

for gases, where $\gamma$ is the ratio of specific heats. For liquids the expression takes the form

$$\beta = 1 + B/2A \quad , \quad (14b)$$

where $B/2A$ is the dimensionless coefficient of the quadratic term in the isentropic relation between pressure and density. The factor "1" in Eqs. 14a,b is associated with convection (a particle set in motion by the passage of a wave contributes its own velocity to the total propagation speed of the wave), whereas $(\gamma-1)/2$ in Eq. 14a, or $B/2A$ in Eq. 14b, may be traced to nonlinearity of the pressure-density relation. When the two fields interact at an angle $\theta$, one expects the contribution due to nonlinearity of the pressure-density relation to be unchanged, since that relation is a scalar. Convection is, however, a vector effect. Only the component of the pump particle velocity in the direction of the weak signal propagation would be expected to affect the weak signal. Accordingly, the "1" in Eqs. 14a,b should be replaced by $\cos \theta$. We designate the value of $\beta$ so obtained, $\beta_{\text{eff}}$:

$$\beta_{\text{eff}} = \cos \theta + (\gamma-1)/2 \quad \text{ (for gases) } \quad (15a)$$

$$\beta_{\text{eff}} = \cos \theta + B/2A \quad \text{ (for liquids) } \quad (15b)$$

This result, derived heuristically here, first appeared in the literature in a paper by Westervelt. How should the validity of the result be tested? Crossed beam experiments had been suggested, but they are notori-
ously affected by diffraction. We decided to use avoid diffraction by using a waveguide to contain the "beams" (in a waveguide beams became modes). Because the role of convection in determining $\beta_{\text{eff}}$ is much greater for gases than for liquids (for air $(\gamma-1)/2=0.2$, for water $B/2A=2.5$), an air-filled waveguide was used in our experiments.

To our chagrin, use of a waveguide did not eliminate all the "extraneous" effects inherent in noncollinear interaction. It turned out that geometric dispersion—the failure of the two modes to have the same group velocity—is much more important than the $\theta$ dependence of $\beta_{\text{eff}}$. The distortion of the weak wave by the pump was dominated by geometrical dispersion. Because of this, the experimental test of Eq. 15a was not easy to carry out. Great care and precision had to be exercised. At the same time, however, the intrusion of geometric dispersion opened up an entirely new set of phenomena to be explored.

(1) Noncollinear Modulation of Sound by Sound in a Rectangular Waveguide

TenCate (82-2, 83-7) used the suppression of sound by sound as a means of measuring $\beta_{\text{eff}}$ (Eq. 15a). Consider, for example, Eq. 10, which applies to collinear modulation. Since $x_p = 1/\beta c_p k_p$, where $c_p = u_p/c$, $u_p$ is the pump amplitude at the source, and $k_p = 2\pi f_p/c$ is the pump wave number, Eq. 10 may be solved for $\beta$,

$$\beta = \frac{2.4c_0}{2\pi f_w \epsilon x_{\text{null}}} .$$

This expression shows that a measurement of the distance $x_{\text{null}}$ could be used to determine $\beta$. In the case of noncollinear interaction Eq. 9 is replaced by

$$u_w = u_{w0} \left| J_0 \left( \mu \frac{\sin \psi}{\psi} \right) \right| ,$$

where $\mu = \beta_{\text{eff}} c_{p w} k_p r$, $\psi = k_r r (1-\cos \theta)$, and $r = x/c \cos \theta$. The value of $\beta_{\text{eff}}$ could in principle be determined by finding the null distance, that is, the distance at which $\mu (\sin \psi)/\psi = 2.4$. However, under practical conditions, suppression in a noncollinear interaction experiment is generally limited...
to a few dB; complete nulls are not a viable option. TenCate therefore settled for a given amount of suppression, namely 3 dB. For a given distance x and pump frequency $f_p$, he varied pump SPL$_p$ (a convenient measure of $\xi_p$) and weak wave frequency $f_w$ (which in turn varied the interaction angle $\theta$; see Eq. 13) until a suppression of 3 dB was achieved. The data for SPL$_p$ versus $f_w$ were then compared with the theoretical values based on Eqs. 14a (no dependence of $\beta$ on $\theta$) and 15a. A typical graph is given in Fig. 6. Since in most cases the two theoretical curves were separated by only about 1.5 dB, great care was required in the measurements. The appearance of unwanted modes (especially the (0,1) and (2,0) modes—the dip in the data in Fig. 6 is associated with the cuton frequency of the (0,1) mode), combined with a ceiling of about 156 dB for SPL$_p$, limited the frequency $f_w$ to a relatively narrow range, approximately 4.1-4.9 kHz. Three different pump frequencies—400 Hz, 450 Hz, and 500 Hz—were used. The measurements tend to confirm the validity of Eq. 15a.

![Graph of observed data compared to theoretical predictions for SPL$_p$ vs. $f_w$](image)

**FIGURE 6**

Comparison of observed data with predicted SPL$_p$ for 3 dB weak wave suppression ($f_p = 500$ Hz). From Ref. 83-7.
An unexpected bonus was the quantification of the very strong role played by geometric dispersion. Consider the first sideband signals, i.e., the sum and difference frequency tones. They are found to alternately grow and diminish with distance down the waveguide. The pattern of scallops, which has a large dynamic range (as much as 25 dB in one set of measurements), is caused by the continually varying phase of the virtual sources that produce the sum and difference tones. Following methods he had developed earlier (83-1), Hamilton obtained theoretical curves that provide an excellent fit to the data. The early work on this topic (83-7) has since been refined and expanded (86-1) and is continuing under Contract N00014-84-K-0574.

(2) **Finite-Amplitude Distortion of a Single Wave in the (1,0) Mode of a Rectangular Waveguide**

This project, a joint effort by TenCate and Hamilton, began during the last few months of the contract and is being continued under Contract N00014-84-K-0574. The following abstract (85-2) describes the results:

"Finite amplitude propagation of higher-order modes in a rectangular waveguide is analyzed by decomposing the modes into plane waves. Two types of nonlinear interactions may then be considered. The self interaction of an individual plane wave generates harmonics that propagate in the same direction. Such interactions are unaffected by dispersion because each harmonic propagates at the same speed, although in a different mode. The second type includes noncollinear interactions between plane waves. In this case geometric dispersion prevents efficient transfer of energy between the interacting components. A single transverse mode excited at a frequency not far from cutoff is composed of two plane waves propagating in very different directions. The noncollinear interactions are then so highly dispersive that as a first approximation they may be ignored. The remaining, nondispersive interactions were modeled using a modified Burgers equation that accounts for tube wall absorption of each mode. Theoretical results for this case agree very well with experiment. If the fundamental frequency is well above cutoff, dispersive interactions can no longer be
ignored. Experimental waveforms then resemble those observed within finite amplitude sound beams."

C. **N Waves and Other Transients**

N waves, so called because of the resemblance of their waveform to the letter N, are common in nature and technology. The sonic boom is perhaps the most famous N wave. For many years we have used N waves produced by electric sparks to study nonlinear (and linear) acoustical phenomena. Some of the earlier work is reviewed in Refs. 73-1 and 75-3 and elsewhere.19

A typical sequence of microphone measurements of an N wave at increasing distance r from the spark is shown in Fig. 7. Nonlinear behavior is

<table>
<thead>
<tr>
<th>Distance from Spark</th>
<th>Oscilloscope Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 cm</td>
<td>0.2 V/cm</td>
</tr>
<tr>
<td>120 cm</td>
<td>0.1 V/cm</td>
</tr>
<tr>
<td>210 cm</td>
<td>0.1 V/cm</td>
</tr>
</tbody>
</table>

**FIGURE 7**

Microphone recordings of N waves at various distances from a spark. First wave has a peak pressure of about 1070 Pa and a (transient) sound pressure level of about 150 dB (re 20 μPa).

immediately evident: The N wave stretches out as it propagates. Stretching occurs because the propagation speed is not the same over the entire waveform. For example, the head shock propagates supersonically, the tail shock subsonically. In fact only the zero at the middle of the N travels at the small-signal sound speed. Amplitude measurements (discussed elsewhere\textsuperscript{19}) reveal further nonlinear behavior: The wave decays faster than would be expected on the basis of spherical spreading and ordinary absorption.

The N waves we measure generally have rise times as short as 0.5 μs, durations in the range 10-80 μs (corresponding to spatial extents of 0.34-2.7 cm), and amplitudes in the range 10-10\textsuperscript{4} Pa. Since no commercially available microphone is capable of faithfully recording such short, fast rising pulses, we construct our own (condenser) microphones.\textsuperscript{19} A very wide bandwidth is required. For example, to record a rise time as short as 0.5 μs, the passband of the microphone must be at least 700 kHz. Nevertheless, the basic simplicity of the equipment--only a high voltage generator and spark gap, microphone, and oscilloscope are needed--makes possible a wide variety of measurements. To complement the time waveform measurements, we added a schlieren apparatus to visualize the spatial field.

1. Schlieren System

D. R. Kleeman began developing the schlieren system (from an old wind tunnel system) under the previous contract (75-3) and completed the work under the present contract (76-5). To demonstrate the usefulness of the apparatus, Kleeman used it to measure several wave phenomena well known in acoustics: reflection of a spherical wave from a flat wall, reflection and diffraction by a barrier, pulse propagation in a parallel plate duct, and focusing by a spherical mirror.

The schlieren pictures in Fig. 8 record the processes of reflection and diffraction. Figure 8(a) shows a spherical N wave traveling from left to right toward a rigid flat plate (heavy black vertical line). The thin bright curved line is the head shock. The slightly dark section behind the bright line, about 1 cm in extent, is the expansion phase of the N wave (the roughly linear middle section of the N; see Fig. 7). The tail shock does not show up as a second bright line because it is not a well
FIGURE 8

Schlieren photographs showing reflection and diffraction of a spherical N wave. From Ref. 76-5.
formed shock, i.e., it is a gradual, not abrupt, compression (see Fig. 7). In Fig. 8(b) the wave has hit the flat plate and is being reflected. Both the incident wave (upper trace) and reflected wave (lower trace) are in view. Figures 8(c),(d) show a spherical N wave being diffracted and reflected by a thin barrier (another flat plate). Some penetration into the shadow zone can be seen.

2. Diffraction of an N Wave by a Slit

We have long been interested in the interaction of nonlinearity with diffraction. In our previous experimental attacks on the problem, we were limited by our complete dependence on microphone measurements. Using the schlieren apparatus (which he greatly improved), in combination with microphone measurements, W. N. Cobb was able to show in some detail how nonlinearity causes a substantial change in the wave field in a classical diffraction problem (77-8).

The problem studied by Cobb, diffraction of a plane N wave by a slit, was selected because of the simplicity of the geometry and the relative clarity of the schlieren pictures that could be obtained. (The plane N wave was produced by placing an electric spark at the focus of a parabolic dish.) A similar problem, diffraction of a spherical shock wave when it emerges from the mouth of a conical horn, had been studied earlier by Coombs and Thornhill. The linear theory prediction is that the shock wave is transmitted as a simple beam, a parallel beam in the slit problem, a conical one in the conical horn problem. As Coombs and Thornhill showed, however, nonlinearity causes the diffracted wave (scattered from the edges of the slit in our problem, from the horn mouth in the spherical wave problem) to propagate faster than the transmitted shock and thus to gradually "eat" into the transmitted shock beam. If the initial wave is strong enough, the diffracted wave may "eat" all the way to the center of the transmitted beam. When this happens, a strong decay of the axial pressure disturbance occurs.

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A set of Cobb's schlieren measurements is shown in Fig. 9. The diagrams at the left represent the traditional description based on linear diffraction theory. The wavefronts shown are for the shock at the head of the N wave. As the plane wave comes through the slit, the two edges scatter cylindrical waves, which are headed by the circular arcs marked DE. The plane wave segment that passes through the slit, i.e., the transmitted beam, is headed by the front marked U. In linear theory the width of the beam is a constant equal to the width of the slit. As noted above, however, when nonlinear effects are taken into account, it is found that the scattered cylindrical waves travel faster than the plane wave segment. The width of the beam is therefore predicted to decrease with distance from the slit. The schlieren measurements, along with complementary microphone measurements, confirmed this prediction (77-8, 77-9).

3. **Propagation of an N Wave through a Cylindrical Tube**

This project was actually a precursor to the project on non-collinear interaction in a rectangular waveguide (see Item 2.b above). Our initial intent was to use a cylindrical tube as the waveguide, a small-signal N wave from a low energy spark located on the tube axis as the weak wave, and an intense plane periodic wave (sinusoidal at the source and traveling in the (0,0) mode) as the pump. The receiver, a microphone located on the tube axis downstream, would pick up a train of pulses: first the direct arrival, second a pulse that had been reflected once from the tube wall, next a pulse that had been reflected twice from the tube wall, and so on. Each pulse would have intersected the pump at a different angle \( \theta \). The effect of angle on modulation of the weak wave by the pump could therefore have been determined.

As it turned out, the problem of the N wave field by itself in the tube proved sufficiently challenging that it became R. D. Essert's thesis project (79-6, 80-8, 80-10). The work was both theoretical and experimental. The main investigation was done with small-signal N waves. However, some measurements of finite-amplitude N waves were made and interpreted. The abstract of Essert's thesis (80-10) provides a good summary of the work.
Schlieren measurements of the diffraction of a plane N wave by a slit; X represents the distance of the shock from the slit. From Ref. 77-8.
"An experimental and theoretical study of the propagation of a spherical N wave in a rigid cylindrical tube is presented. Both source, an electric spark, and receiver, a very wideband condenser microphone, were located on the tube axis. Spark energies of between 0.04 J and 2.4 J were used. The received signal is a series of pulses. The first pulse is the direct N wave, which travels straight down the axis. Subsequent pulses represent waves reflected from the tube wall; their waveforms vary (they are not N shaped). There is, however, a shape repetition every fourth reflected wave. A theoretical analysis of the problem was based on a solution of the linear wave equation for a dissipative medium. The solution, expressed as a ray expansion, indicates that the variation in pulse shape for low energy sparks is due primarily to focusing of the reflected rays each time they cross the tube axis: Each focus is accompanied by a 90° phase shift of the components making up the pulse. It was also found that atmospheric absorption and microphone directivity have important effects on the shapes of the received pulses. Both analytical and numerical methods were used to compute individual pulse waveforms from the theoretical results. For low spark energies (<0.1 J) computed and measured waveforms are in good agreement. For higher spark energies (>0.1 J) the shapes of the measured waveforms are altered by finite amplitude effects, and the simple linear theory is no longer sufficient to explain the results. A numerical propagation algorithm, which includes nonlinear propagation distortion, has been proposed. A qualitative rendering of the algorithm accounts reasonably well for the observed changes in wave shape."

Reported in the appendix of Ref. 80-10 are measurements of the decay of small-signal N waves in open air. The results are used as a high frequency test of the (then new) ANSI and ASA standard on atmospheric absorption.21

Figure 10 shows a sequence of measured pulse trains (oscillograms) as spark energy increases. Oscillogram (a) (very weak spark) illustrates the behavior for a small-signal N wave. The direct arrival (marked D) is

FIGURE 10

Family of measured pressure pulse trains produced by a spark on the axis of a cylindrical tube. Receiver is also on the axis. Parameter is spark energy in joules. From Ref. 80-10.
exceedingly weak because it is carried by a single ray. The reflected signals (marked 1, 2, 3, ..., according to the number of reflections) are much stronger because each represents the confluence of a bundle of rays. The $90^\circ$ phase shift from pulse to pulse is characteristic of cylindrical focusing. Note that the pulse shape inverts every second pulse ($180^\circ$ total phase shift); see, e.g., pulses 2, 4, 6, and 8. Moreover, the shape is restored after every fourth pulse ($360^\circ$ total phase shift); see, e.g., pulses 2 and 6, 3 and 7, 4 and 8. Next examine the change in pulse train as spark energy increases. The total phase shift at a given point in the pulse train seems to be a function of N wave amplitude. Notice, for example, the change in shape of pulse 8 as spark energy increases. By oscillogram (d) (moderately strong spark), pulse 8 no longer resembles pulse 8 in oscillogram (a) but rather pulse 7. Essert reasoned that the wavelet time shift caused by nonlinear distortion produced changes in the pulse shape that can be interpreted as phase shift. He proposed a computer algorithm to test this hypothesis. Although he did not implement the algorithm, he produced rough sketches that qualitatively confirm his arguments.

4. Rise Time of N Waves Produced by Electric Sparks

The purposes of this study, which was carried out by L. B. Orenstein (82-1, 82-4), were to measure the rise time of the head shock of a spark-produced N wave, determine the effect of N wave amplitude and half-duration on the rise time, and compare the results with various theoretical predictions. The impetus for the study was the still unsolved mystery about sonic boom rise times: Why are they so large? The long rise times have been variously attributed to inhomogeneity of the atmosphere, large-scale turbulence, small-scale turbulence, and relaxation effects. In our laboratory study we intended to examine the role played by relaxation.

Earlier laboratory investigations of N wave rise time had been reported. Wright (83-2) measured N waves having short rise times, 0.5 to 2.5 $\mu$s, while Holst-Jensen, 22 who used exploding wires as well as sparks, concentrated on N waves having rise times an order of magnitude larger. A

very brief study had also been done several years earlier by Bourgeoisie, a member of our group. Orenstein's measurements filled the gap between those of Wright and Holst-Jensen.

The theoretical work was initially limited to a review of existing predictions of shock rise time in a thermoviscous medium and in a relaxing medium. It became apparent, however, that these analyses, which are for a steady shock (a plane step wave), were not applicable to the N waves in our study. Our shocks were unsteady: they were affected by the expansion following the shock. In particular, the rise time was found to depend on the N wave's half-duration as well as on its amplitude. Since little was found in the literature on rise time of actual N waves, Orenstein adapted a computer algorithm originally developed by Anderson that permits one to account simultaneously for waveform distortion, spherical spreading, and atmospheric attenuation of an N wave. Orenstein also added the small effect of dispersion, which is caused by relaxation, to the algorithm.

The experimental measurements required that a new microphone be designed, constructed, and calibrated (the general construction technique is described by Anderson). A new, much improved preamplifier was also designed and assembled. The sensitivity of the new microphone-preamplifier combination was measured by a special technique and found to be $-107$ dB re $1 \text{ V}/\mu\text{bar}$. Nagging questions about the sensitivity later forced the use of an independent calibration based on a comparison measurement at low frequencies in a tube. The difference between the sensitivity measured by the two techniques was found to be less than $1$ dB. A digital oscilloscope was used to record the N wave signatures picked up by the microphone.

The N waves used, which came from sparks having energies in the range 0.06-26.5 J, generally had pressure amplitudes in the range 15-1500 Pa, half-durations 7.5-41.5 $\mu$s, and rise times 0.65-7 $\mu$s. Measurements were made at distances of 0.1 to 5.5 m from the spark. The data were taken in two ways. First, the spark energy was held constant and N waves were recorded at various distances. Since in this method both

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amplitude and half duration of the N wave vary, the dependence of rise time on each could not be isolated. In the second method of taking data, spark energy and distance were both varied, but in such a way that either amplitude or half duration remained constant. Finally a few measurements were made with very weak (essentially small-signal) N waves.

The computed waveforms were found to be in good agreement—in amplitude, half duration, and rise time—with the measured waveforms. Further computations showed that the measured rise times could not be attributed to either relaxation (including dispersion) or thermoviscous effects alone. Both mechanisms were important for the N waves used in our study.

5. Reviews

Two general reviews of our work on N waves up through 1980 were presented (80-4, 80-9). The following abstract (80-9) gives the scope of the coverage:


"This paper is a review of experiments done in our laboratory on airborne N waves generated by electric sparks. The purpose of the experiments has been to investigate the effects of finite amplitude on a variety of classical wave phenomena. The N waves used have duration 10-70 μsec, amplitude 0.1-100 mbar, and rise time frequently less than 1 μsec. Experiments have been done on simple propagation in an open medium; refraction and diffraction by an acoustic lens (a gas-filled soap bubble); diffraction by a circular aperture, by a disk, and by a slit; focusing by a spherical mirror; and propagation in waveguides (parallel plates and circular tube). All nonlinear effects observed may be traced to modification of the propagation speed by the acoustic field itself. The modification causes elongation and extra decay of simple propagating N waves, unusual interference effects in diffraction, and self-refraction. Self-refraction, which is bending of rays due to a variation of particle velocity (and therefore propagation speed) from ray to ray, sometime prevents converging rays from actually focusing."

39
W. M. Wright also published a review of mainly experimental propagation of work he had done on N waves (82-2), primarily earlier under Contract Nonr-3932(00)* but also partly as a consultant under the present contract and its predecessors (73-1, 75-3). Subjects covered include extra attenuation and lengthening of the N wave, N wave rise time, microphones, and experimental technique.

6. Weak Shock Followed by a Tail of Arbitrary Waveform

Interest in this problem goes back to 1969. While working on a history of nonlinear acoustics,24 Blackstock became interested in a remark of Earnshaw25 that a distant observer, listening to a cannon shot, heard the report of the gun first and then the vocal command "Fire". To determine whether this sequence of acoustic arrivals could be explained by the supersonic speed of the shock wave from the gun, Blackstock modeled the gunshot signal as a weak shock followed by an exponential tail. Expressions for the diminution of the shock amplitude and the increase in decay time of the tail were obtained, for both spherical and plane waves. Although the results were reported to the sponsor19 (AFOSR) they were not published. Subsequently P. H. Rogers published an independent solution of the problem.26

Work was done during the present contract to extend the previous analysis to cover shocks with tails of arbitrary waveform. The exponential tail is an unrealistic model for explosion sounds because it has no negative phase (undershoot). Work on more realistic waveforms was carried out and reported, as given by the following abstract (78-9):

"Weak-shock theory is used to find the decay of a shock that is propagating into a quiet fluid. The tail, or flow field behind the shock, is assumed to be smooth but otherwise arbitrary. Let the time waveform of the


pressure at the initial measurement point be given by \( p = P_0 g(t) \), where \( P_0 \) is the initial pressure amplitude and \( g(t) \) is the arbitrary tail function normalized so that \( g(0) = 1 \). For plane waves the dependence of the shock amplitude \( P \) on distance \( x \) is found from the following coupled equations: 
\[
Ax = \int_0^{\phi_b} g(\lambda) d\lambda \quad \text{and} \quad P = P_0 g(\phi_b) ,
\]
where \( A \) is a constant and \( \phi_b \) is the time-like parameter (for the shock) from the Earnshaw solution. For example, if \( G = \exp(-at) \) (an exponential tail), the coupled equations yield 
\[
P = (P_0/Ax) [(2Ax+1)^{-1/2} + 1] ,
\]
in agreement with P. H. Rogers' result [J. Acoust. Soc. Am. 62, 1412-1419 (1977)]. Solutions have also been obtained for the model tail functions described by J. W. Reed [J. Acoust. Soc. Am. 61, 39-49 (1977)]. For spherical waves replace \( x \) by \( r_0 \ln(r/r_0) \) where \( r \) is radial distance and \( r_0 \) is the initial measurement distance."

Another facet of the problem was stressed in two later papers (83-4, 84-6), namely Earnshaw's example of the apparent reversal of arrival of the sounds of the cannon shot and the command "Fire." The example was based on an actual event recorded during some speed of sound measurements in the Arctic in the 1820s. The reversed arrival of the two sounds, which was indeed observed, was in Earnshaw's time (35-40 years later) taken as evidence of the correctness of Earnshaw's theory of finite-amplitude sound waves. By applying weak shock theory, however, we showed that the magnitude of the reported reversal (about 3/8 s) is far too large to be explained as a finite-amplitude effect. Moreover, overtaking of the speech signal by the shock wave would have destroyed the identity of the speech signal. The true explanation for the apparent reversal is still unknown.

D. Other Topics in Nonlinear Acoustics

This section is a collection of miscellaneous topics on finite-amplitude waves: propagation in inhomogeneous fluids, propagation through porous materials, and diffraction of a periodic wave by a circular opening.

1. Inhomogeneous Fluids

Our work on waves of finite amplitude in inhomogeneous fluids was done at two widely separated times. The early work was a follow-up of
research done under the predecessor contract (75-3). Under that contract we had investigated vertical propagation of plane waves through a gravity-stratified fluid, such as the atmosphere or the ocean (74-1, 74-2). We had also developed for sea water an empirical relation for the dependence of the parameter of nonlinearity $B/2A$ on salinity, temperature, and hydrostatic pressure (or depth). During the first year of the present contract, the empirical relation was revised (76-3). The formulas for $\beta(=1+B/2A)$ are as follows.

In terms of $S$, $T$, and $P'$

$$\beta = 3.3685 + 9.874E-07 \, S^3 + 7.799E-06 \, T^3 + 1.027E-03 \, P'$$
$$- 2.276E-10 \, P'^3 + 5.429E-04 \, ST - 8.641E-07 \, STP'$$
$$- 1.542E-05 \, ST^2 + 1.620E-08 \, T^3P'$$

where $S$ is salinity in °/oo, $T$ is temperature in °C, and $P'$ is gauge pressure in kgf/cm$^2$ ($1$ kgf/cm$^2$ = 0.980665 bar).

In terms of $S$, $T$, and $D$

$$\beta = 3.3683 + 9.908E-07 \, S^3 + 7.785E-06 \, T^3$$
$$+ 1.063E-04 \, D - 2.447E-13 \, D^3 + 5.407E-04 \, ST$$
$$- 8.849E-08 \, STD - 1.532E-05 \, ST^2$$
$$+ 1.601E-09 \, T^3D$$

where $D$ is depth in m.

Near the end of the contract, but with primary support from Contract N00014-82-K-0805, we tackled the more general problem of nonlinear geometrical acoustics, that is, ray theory for finite-amplitude waves. The intent was to determine the extent to which nonlinear effects are important in long range underwater propagation. Despite the fact that support for most of work at this time came from Contract N00014-82-K-0805, it is necessary to describe briefly the entire project in order to appreciate the role played by the present contract. The project was divided into the following three tasks.

Task I. Shock pulse propagation in a homogeneous ocean
Task II. Nonlinear propagation in a depth-dependent ocean
Task III. Nonlinear propagation in a caustic region

Under Task I we developed a computer algorithm in which weak-shock propagation, real-ocean attenuation, and spherical spreading were taken into
account. The effect of inhomogeneity, which was omitted for simplicity in Task I, was taken up in Task II. To keep Task II simple we assumed that although distortion occurred, no shocks were present. The plan was ultimately to merge Tasks I and II and to obtain thereby a general treatment of nonlinear geometrical acoustics—not including effects that take place at caustics. The caustics problem would be solved separately under Task III.

Tasks I and II were largely completed (separately) by C. L. Morfey with little help from the present contract. The job of merging the results of the two tasks fell, however, to the present contract, and its successor (Contract N00014-84-K-0574). The work was carried out by F. D. Cotaras.

Cotaras derived the equations of nonlinear geometrical acoustics from first principles and modified the propagation algorithm (Task I) to include the effects of inhomogeneity (Task II). He also computed a variety of interesting examples in order to delineate and compare the roles of the various competing mechanisms in long range underwater propagation. References 84-3 and 85-4 include the work done under the present contract. The abstract of Cotaras's thesis (85-4) provides a good summary of his contribution and is as follows.

"In this report the propagation of finite amplitude acoustic signals through an inhomogeneous ocean is investigated both analytically and numerically. The effects of reflections and focusing are not considered. From simplified versions of the lossless hydrodynamics equations the theories of linear and nonlinear geometrical acoustics are developed. Losses are accounted for directly in the numerical routine. The eikonal equation, from which an equation for the ray paths is derived, is assumed to be the same for both small-signal and finite amplitude waves. The transport equation is found to be different, however. The transport equation leads to a standard first-order progressive wave equation, linear for small-signal waves, but nonlinear for finite amplitude waves. All the analysis is carried out in the time domain and is for a fully inhomogeneous ocean.

"In the numerical study the ocean is assumed to be stratified. The effects of inhomogeneity, ordinary attenuation and dispersion, and nonlinear propagation are investigated using a numerical implementation of nonlinear geometrical acoustics. Two explosion waveforms are considered: a weak shock with an exponentially decaying tail and a more realistic waveform that includes the first bubble pulse. Numerical propagation of the simpler wave along a 58.1 km path starting at a depth of 300 m leads to the following conclusions: (1) The effect of inhomogeneity on nonlinear distortion is small. (2) Dispersion plays an important role in determining the arrival time of the pulse. (3) Neither nonlinearity nor ordinary attenuation (and dispersion) are paramount; both need to be included. For the more realistic wave the propagation is along a 23 km ray path starting from a depth of 4300 m. Two charge weights, 0.818 kg and 22.7 kg TNT, are assumed. In each case the energy spectrum of the signal obtained by considering finite amplitude effects for the entire 23 km path is compared with spectra obtained by neglecting finite amplitude effects (1) entirely, (2) after the first 150 m, and (3) after the first 1100 m. Finite amplitude effects are found to be of small consequence in the case of the 0.818 kg TNT explosion for frequencies below 6 kHz at distances beyond 1100 m. For the 22.7 kg explosion the corresponding quantities are 4 kHz and 1100 m."

2. Propagation through (Air-Filled) Porous Materials

This project also had another sponsor for its primary support, NASA Lewis Research Center (Grant NSG 3198). The research was carried out by H. L. Kuntz (Ph.D. student) during 1978-82 and by D. A. Nelson (M.S. student) during 1982-85. Their direct support came from the NASA grant. In 1980 the present contract began participating by furnishing support for undergraduate assistants (at various times J. E. Estes, J. S. McLean, and E. A. Tschoepe), use of equipment and supplies, and computing.

The project goal was to determine how high intensity sound propagates in (air-filled) porous materials, such as Kevlar. Kevlar is a very tough fibrous material that has been proposed for use as a sound absorbing liner in jet engine inlets. The results of Kuntz's investigation (81-2, 81-4, 82-3, 83-6, 86-4) are given in the abstract of his dissertation.
"An investigation of the interaction of high-intensity sound with bulk porous materials is reported. The work is mainly experimental but some theoretical results are obtained. Previous studies of high-intensity sound in porous materials have been limited to porous sheets. Most tests in the present study were done on Kevlar® 29, a fibrous plastic material, for the porosity range between 0.985 and 0.809. The nonlinear behavior of the materials was first described by dc flow resistivity tests. Then acoustic propagation and reflection were measured. Small signal (100 dB re 20 \mu Pa spectrum level), broadband (frequency range from 0.1 to 10 kHz) measurements of phase speed and attenuation were carried out. High-intensity tests (120-172 dB) were made with 1, 2, and 3 kHz tone bursts to measure harmonic generation and extra attenuation of the fundamental. Small signal (100 dB), standing wave tests were used to measure impedance between 0.1 and 3.5 kHz. High level tests (120-165 dB) with single cycle tone bursts at 1 to 4 kHz show that impedance increases with intensity. A theoretical analysis is presented for high-porosity, rigid-frame, isothermal materials (the isothermal assumption is justified by a separate analysis of heat transfer effects). One-dimensional equations of motion are derived and solved by perturbation. The measured data is not, however, well explained by the perturbation results. The experiments show that there is excess attenuation of the fundamental component and in some cases a close approach to saturation. A separate theoretical model is developed to explain the excess attenuation. This model yields predictions that are in good agreement with the measurements. Impedance and attenuation at high intensities are modeled by substituting the nonlinear flow resistivity relation into the linear impedance expressions. The model is useful in predicting the effects of a porous material on sinusoidal signals."

We concluded from Kuntz's study that ordinary "hydrodynamic nonlinearity," embodied in the nonlinear terms in the conservation equations for an ordinary fluid, is not important for porous materials. Sound absorption in porous materials is so high that classical cumulative distortion (which dominates the behavior in an ordinary fluid) is insignificant. The important nonlinearity is that associated with the flow resistance of the porous medium.
The tendency toward saturation of the fundamental component deserves special comment. We had earlier investigated saturation as a phenomenon that occurs in ordinary fluids, in particular water\textsuperscript{28} and air.\textsuperscript{29} It is interesting to note that despite the vast difference in nonlinear mechanisms, porous materials and ordinary fluids have quite similar saturation behaviors. In fact, their saturation properties may be described by the same model equation (86-4).

One of Nelson's most important contributions was the recognition that the flow resistance should be modeled with a cubic, rather than a quadratic, nonlinearity. Using this model, he was able to explain much of Kuntz's data. Nelson's results (83-9, 84-7, 85-1) are aptly summarized by the abstract of his thesis (84-7).

"The subject of this investigation is the propagation of high intensity sound waves through an air-filled porous material. The material is assumed (1) to be rigid, incompressible, and homogeneous, and (2) to be adequately described by two properties: resistivity $r$ and porosity $\Omega$. The resistivity was measured as a function of velocity for static flows and found to follow the empirical relation $r = r_1 + r_2 \, v \, \text{sgn}(v)$. This relation is assumed to apply for acoustic signals as well. Ordinary hydrodynamic nonlinearity (which leads to shock formation) is neglected because of the very high attenuation in the porous material. The resulting wave equation is still nonlinear, however, because of the $v \, \text{sgn}(v)$ term in the resistivity. The equation is solved in the frequency domain as an infinite set of coupled inhomogeneous Helmholtz equations, one for each harmonic. An approximate first integral formulation of these equations gives relations for progressive waves. The source wave considered is a slightly distorted intense tone, that is, a finite-amplitude fundamental accompanied by weak higher harmonics. An approximate but analytical solution leads to predictions of excess attenuation, saturation, and phase speed reduction for the fundamental component. A more general numerical solution is used to calculate the propagation curves for the higher harmonics. The $v \, \text{sgn}(v)$ nonlinearity produces a cubic distortion pattern; when the input signal is a


pure tone, only odd harmonic distortion products are generated. Quantitative experiments were performed on batted Kevlar® 29 having porosities in the range Ω = 0.94 to 0.98. Measurements were taken in the 400 to 6200 Hz frequency range for small-signal waves, 500-1500 Hz for finite-amplitude waves. Source levels for the finite-amplitude waves extended to 160 dB. Qualitative confirmation of predictions has been obtained in all cases, and quantitative confirmation in most cases. The theory is then applied to the problem of propagation and attenuation in lined ducts. A vector version of the wave equation is derived along with the corresponding inhomogeneous Helmholtz equations. These are used as the basis of a perturbation solution for reflection from a porous half-space and propagation in lined ducts. The solution becomes extremely complicated, however, when used for high intensity waves in lined ducts. An ad hoc model is therefore presented for the purpose of illustrating the gross nonlinear effects on absorption."

3. **Diffraction of a Periodic Wave Emerging from a Circular Duct**

Early in the course of Phase I of the outdoor propagation project (Section B.1.a above) we noticed some rather peculiar wave shapes in the radiation of fairly intense periodic waves from a horn. A set of waveforms measured along the axis from the horn mouth is shown in Fig. 11.

**FIGURE 11**

On-axis waveforms of intense sound radiated by an exponential horn. Distance is measured from the mouth of the horn. Frequency 7.36 kHz. From Refs. 76-4, 77-1.
The scalloped wave shape (bottom waveform), which would otherwise be a stage in the classical approach to a sawtooth, is explained by the interaction of diffraction with finite-amplitude distortion. The wave suffers the usual finite-amplitude distortion as it travels through the horn, but that form of distortion slows down considerably when the wave weakens as it emerges from the mouth of the horn. Ordinary small-signal diffraction effects then become paramount, at least out to and somewhat beyond the Rayleigh distance. Model the horn mouth as a vibrating piston. A well known result from piston radiation theory is that the farfield pressure signal on the axis of the piston is proportional to the time derivative of the piston velocity (i.e., its acceleration), not the velocity itself. This behavior is caused by the addition of the diffracted wave (scattered from the mouth of the horn) to the directly transmitted wave. By inspection (and suitable phase shift) one can see that the waveform at 3.05 m does indeed resemble the derivative of the waveform at 0 m (mouth waveform).

If our explanation of the scalloped wave shape is correct, we should expect even more fanciful waveforms if the wave is more highly distorted when it reaches the mouth of the horn. If it is a full-fledged sawtooth, for example, the axial pressure signal in the farfield should be a train of impulses (the derivative of a sawtooth). To investigate this prediction, J. R. Kuhn carried out some measurements of the radiation from the open end of a pipe. The results of the study are summarized by the following abstract (78-5).

"The radiation of sawtooth waves from the open end of a circular pipe is considered. Time domain piston radiation theory is used to compute the waveform for all regions of the field. For example, the waveform on the axis in the farfield is predicted to be a series of impulses, one for each of the shocks in the sawtooth at the mouth of the pipe. Experiments done with a 3.04-m long, 0.0467-m i.d. pipe generally confirm the predictions. The sawtooth wave was achieved by generating intense sinusoidal waves (8-kHz tone bursts, SPL 138 dB re 20 μPa) at the other end of the pipe. Nonlinear propagation distortion transformed the sinusoidal wave into a sawtooth as it traveled through the pipe. (Use of linear theory to compute the radiation into the open medium is justified because the wave rapidly
diminished in amplitude when it was no longer confined to the pipe.) The work has application to horn radiation, both in air and underwater, and to radiation of core noise from jet engines."

E. Optoacoustics and Thermoacoustics

During the later stages of the contract a project on optoacoustics and thermoacoustics was initiated. The work was carried out by W. M. Wright and his students at Kalamazoo College, Kalamazoo, Michigan.

One of the devices used to measure air pollution is the so-called optoacoustic spectrometer. A modulated laser beam is used to generate sound in an acoustical cavity. See Figs. 12 and 13. A gas or aerosol with optical absorption at the wavelength of the laser is contained in the cavity, or cell. Optical absorption generates heat along the path of the beam. Modulation of the laser beam by chopping produces periodic heating, which generates acoustic waves at the modulation frequency. The sound is detected by a microphone mounted in the cell. The system sensitivity is enhanced by choosing the modulation frequency to be one of the resonance frequencies of the cavity. In the cell used by Wright, the cavity is designed to be excited in its lowest order azimuthal mode. Since the sound produced depends on optical absorption by the gas, which is strongly dependent on the optical wavelength, the system is an optoacoustic spectrometer. It can be used to detect the presence of a small amount of a given gas.
The research under the contract began in 1981 and was directed toward improving the understanding of how the cell operates. The motivation was to learn how to design more efficient and sensitive cells for future optoacoustic experiments. In a substantial part of the work, a wire carrying an ac electric current was used to simulate the heating by the laser beam. Although the work was done to help optimize the acoustic and electronic systems for the cell, the use of a heating wire as a thermal sound source became of interest in itself. When the wire was biased with a dc current to prevent frequency doubling, the acoustic signal had the same frequency as the ac electric current. A frequency response of the cell was then easily taken. The resonance at the lowest order azimuthal mode frequency was found to have a Q of about 100. A paper reporting this work was given at the Fall 1981 Meeting of the Acoustical Society of America (81-10).

The work also led to a senior thesis by M. T. Houk (81-12). About one-third of Houk's project was devoted to the heating wire simulator. The rest was a study of sound produced by heating the surface of a solid with a chopped light beam. Beams from both an incandescent light bulb and a laser were used. The modulation frequency was low (245 Hz) because theoretical analysis shows that acoustic output decreases with increasing frequency. A nonresonant chamber was used, and the solid used to absorb the light was black foam rubber. The experiments were essentially a test.
of theoretical predictions given in the literature.  

Wright later wrote a journal article on cavity excitation by a heated wire source (86-2).

F. Miscellaneous

Activities of a general nature are reported in this section.

A paper on the history of physical acoustics in North America (76-2) was given in the special session "History of American Acoustics" organized for the Washington, D.C., Meeting of the Acoustical Society of America during the Bicentennial Year 1976. Particular emphasis was given in the paper to the relatively unknown work of Joseph Henry (1799-1878) in atmospheric acoustics. Driven by the need to develop better fog signaling apparatus, Henry carried out remarkable research on sound outdoors—propagation, reflection, refraction (primarily that due to wind gradients), and diffraction—from 1865 until his death 13 years later. Also reviewed in the paper were the first ever atmospheric sound absorption measurements, which were made by A. W. Duff (1898 and 1900), and some of the researches of G. W. Pierce.

Other miscellaneous activities include reviews of the 9th and 10th International Congresses on Acoustics, which were held in Madrid in 1977 and in Sydney in 1980, respectively (78-2, 81-1), a tutorial review of nonlinear acoustics (81-7), a tutorial review of the limit on sound transmission imposed by finite-amplitude saturation (81-8), and a review of acoustics education at The University of Texas at Austin (80-7).

III. OTHER TASKS

A. TASK 1. Parametric Receiving Array

Work on this task was directed by J. J. Truchard and grew out of research he had conducted earlier on the parametric receiving array. Among the several problems with which he was concerned, two are notable here.

(1) The proper value for the parameter of nonlinearity $\beta$ as applied to the parametric receiver, in particular, is $\beta$ equal to $1+B/2A$ or $\cos \theta + B/2A$?

(2) The use of signal processing based on the phase characteristics of the sideband signals (sum and difference tones) in order to improve the performance of the parametric receiver.

The question posed in Item (1) has already been discussed in connection with our work on noncollinear interaction in a waveguide; see Eqs. 14 and 15. Truchard's measurements did not shed much light on the matter, a result that is not surprising. First, his measurements were made in water. Since for water the factor $B/2A$ is by far the larger of the two components of $\beta$, distinguishing between 1 and $\cos \theta$ amounts to finding a small difference between large numbers. (Truchard himself recognized that air would be a much better medium for the test.) Second, the sideband signals are strong only in the vicinity of $\theta=0$, where $\cos \theta \approx 1$. At angles for which $\cos \theta$ differs much from unity, the signals are very weak and hard to measure.

Results pertaining to Item (2) represent the main contribution of the task (76-6, 78-8, 79-8). Truchard's analysis showed that combining the carrier (pump) and sideband signals leads to a carrier signal that has both phase modulated and amplitude modulated components. Use of the phase modulation for detection upgrades the performance of the receiver. In Fig. 14 beam patterns based on both phase modulation and amplitude modulation data are shown. When the pump transducer is properly aligned with the hydrophone receiver (Fig. 14(a)), the patterns are the same. When the pump

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is slightly misaligned (Fig. 14(b)), however, the performance of the ampli-
tude based system is badly degraded whereas the phase based measurement is
hardly affected.

![Diagram of beam patterns](image)

**FIGURE 14**

Parametric receiving array beam patterns for a 5 kHz signal. Both phase detector output and amplitude detector output were used to obtain the patterns.
(a) Pump transducer aligned with receiving hydrophone.
(b) Pump transducer misaligned (rotated to its 3 dB down point).

Finally, Truchard completed some earlier work on the relation of the parametric receiving array to the general problem of scattering of sound by sound (78-6).

B. TASK 2. Storage and Retrieval of Energy in Nonlinear Acoustical Radiations

This study, the use of a focusing source for the parametric transmitting array, was directed by T. G. Muir. One of the reasons for the low efficiency of the parametric array is so called finite-amplitude loss. As the two primary waves lose energy to their higher harmonics, they have less with which to build up the difference frequency signal. After shock formation occurs, finite-amplitude losses became particularly severe. It
was long ago reasoned that if a way could be found to phase invert the primary signals near the shock formation point, post-inversion distortion would, at least for some distance, pump energy out of the harmonics back into the primaries. The effective zone of interaction (the parametric array), where the difference frequency sound is generated, could thus be lengthened. Reflection from a pressure release surface produces the desired phase inversion of the primary waves, but the accompanying difference frequency wave also suffers phase inversion. It then interferes destructively with the difference frequency sound produced after the inversion. Muir reasoned that focusing might produce the desired $180^\circ$ phase shift of the primary waves without causing the same phase shift in the difference frequency wave. Because of the much longer wavelength of the difference frequency sound, diffraction might prevent the geometrical convergence necessary for inversion. As it turned out, this hope was at least partially realized.*

The research was accomplished in two stages. First theory and experiments were carried out for a focusing source by B. G. Lucas. Studies were done both for two-frequency excitation (the parametric array problem) and for single frequency excitation (the ordinary harmonic distortion problem). The work is reported in Lucas's dissertation (81-11) and elsewhere (82-6, 82-8, 83-3, 83-8). Small-signal focusing (applicable to each primary wave) is the subject of Ref. 82-6. By using the parabolic approximation of the Helmholtz equation, Lucas was able to register a theoretical advance. He obtained a solution for the field at any point that required only one numerical integration. Experiments generally confirmed the predictions.

Focusing of finite-amplitude waves is taken up in Ref. 83-3 for a two-frequency source and in Ref. 83-8 for a single-frequency source. Figure 15 shows computed and measured pressure amplitude along the acoustic axis for the fundamental, second harmonic, and difference frequency components. The fundamental frequency is 403 kHz, the source radius 0.2 m, and the focal length 0.85 m (for (c) the second primary frequency is 373 kHz).

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*Although the difference frequency wave was found to undergo focusing, subsequent research done under another contract showed that the phase shift is not simply $180^\circ$. 

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FIGURE 15
Propagation of sound along the acoustic axis of an F/2 lens. From Refs. 81-11, 82-6, 82-8, and 82-3.
Note the sharper focus at the higher frequencies. The following abstract (83-3) summarizes the results for the two-frequency (parametric) source:

"An analytical description for the field of a parametric focusing source is derived. It is valid for spherically concave sources with small aperture angle and high ka, under conditions of quasilinear interaction (strong shocks precluded). The solution furnishes computations on the phase and amplitude of difference frequency sound along the axis and in the focal plane, as well as on the width of the radiation lobe in the focal region. Underwater experiments conducted with an f/2 lens coupled to a dual, interleaved primary array are discussed. The results support the utility and validity of the analytical model for describing the distribution of sound along the acoustic axis and across the focal plane. The difference frequency radiation was found to be effectively focused, in that the width of the beam became quite narrow in the focal plane."

The second stage of the research was less extensive and consisted of analytical modeling of the focusing source by J. Saito. Saito found that for weak focusing the primary radiation could be modeled as a succession of three simple fields: a spherically converging field in the prefocus region, a plane wave field in the focal region, and a spherically diverging field in the postfocal region (83-10, 84-2). The analytical computations associated with this model are much simpler than those required for the parabolic approximation of the Helmholtz equation. At the same time, the sacrifice in accuracy appears to be small. Saito also analyzed the second harmonic field in some detail (84-1).

C. TASK 3. Transmission of Sound from Water to Air

This work, which was carried out by E. E. Mikeska and J. A. Shooter, was motivated by the need to cope with multipath interference whenever underwater measurements must be made near the water surface. For example, the presence of the air-water interface makes it difficult to calibrate acoustical sources, such as sonar transmitters and explosive charges. A novel approach is to eliminate the multipath interference by taking the receiver across the air-water boundary and making the measurement in air.
The task had three objectives. The primary objective was to test the practicality, precision, and accuracy with which measurements can be made with the source in water and the receiver in air. The second objective was to demonstrate the measurement-in-air technique by recording beam patterns for an ordinary cw source. The last objective was to extend the measurements to include high-energy sources, such as underwater sparks or explosives.

Theory pertinent to the experiments reported here may be found in a paper by Young\(^{34}\) and references therein.

All of the experimental work was carried out in an indoor tank facility. The tank was 18.3 m long, 4.6 m wide, and 3.65 m deep. For single frequency measurements the projector was an 11 in. diam circular piston, which was operated at frequencies in the range 15-38 kHz. The projector was first mounted near the bottom with its axis pointing straight upward. From this position precise measurements were made of the transmission loss across the air-water interface. For measuring beam patterns, the source was mounted on a rotating column that allowed scanning in azimuth and grazing angle. Receivers were a USRL H-23 hydrophone for underwater measurements and a B&K 4133 half-inch condenser microphone for measurements in air. The hydrophone was used to independently obtain source level and beam patterns (the underwater measurements were done with pulses in order to avoid multipath interference). The microphone was used to measure sound pressure level (SPL) along the main acoustical axis as a function of height above the water surface. It was also used to map out the acoustic field in planes parallel to the surface.

The primary objective was achieved. Measurements in air were made with sufficient accuracy to determine the source level in water to within 0.5 dB. This level of accuracy requires not only an even greater accuracy for the SPL measurement in air but also accurate environmental data. In particular, temperature in the air and the water must be known to ±3°C, and barometric pressure must be known to ±10 mm Hg. Moreover, salinity must be known if the medium is the ocean. Relative humidity is unimportant unless the air path is long enough that atmospheric attenuation is significant. At the frequencies used in our tests the air-water surface must be smooth.

---

As an example, in one measurement the transmission loss across the air-water interface was found to be 65.4 dB ± 0.5 dB. This is the expected value for the temperatures and barometric pressure that existed when the measurement was made (81-3).

Figure 16 demonstrates that the second objective, to measure beam patterns, was also met (81-9). The practical requirements were found to be that (1) the underwater source level must be high enough that a good signal-to-noise ratio is achieved in the air, and (2) the grazing angle (angle made by the acoustical axis with the surface) should be 25° or less in order that the nulls and sidelobes in the directivity patterns occur at their proper angles. Of course the length of the sound path underwater must exceed the farfield distance if one wishes to measure the farfield directivity.

![Diagram showing directivity patterns in water and air. Grazing angle 20°, frequency 38 kHz. From Ref. 81-9.](image-url)

FIGURE 16

Directivity patterns measured in water and in air. Grazing angle 20°, frequency 38 kHz. From Ref. 81-9.
The last objective, to extend the measurements to high energy sources, was never reached. Some preliminary experiments with a spark source were carried out, but the measurements obtained had poor repeatability. Future work in this area will require much more sophistication than was available with the apparatus at hand.
IV. VITAL STATISTICS

A. Tasks

For administrative purposes the effort was organized by tasks as follows:

Task 0. Nonlinear Acoustics (General)
Responsible technical person: D. T. Blackstock
Period: 1 April 1975 to 31 August 1984

Task 1. Parametric Receiving Array
Responsible technical person: J. J. Truchard
Period: 15 June 1976 to 30 June 1977

Task 2. Storage and Retrieval of Energy in Nonlinear Acoustical Radiations
Responsible technical person: T. G. Muir
Period: 1 July 1980 to 31 August 1984

Task 3. Transmission of Sound from Water to Air
Responsible technical persons: J. A. Shooter and E. E. Mikeska
Period: 1 July 1980 to 30 June 1982

B. Other Sponsors (Task 0 only)
Most of the research done under Task 0 was cosponsored by other government agencies. The agencies and time periods are listed below. Approximately 30% of the total funding for the projects carried out under Task 0 came from the cosponsors.

<table>
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<tr>
<th>Cosponsor</th>
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<td>AFOSR</td>
<td>1 April 75 to 31 October 78</td>
</tr>
<tr>
<td>NOAA</td>
<td>1 September 74 to 28 February 78</td>
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<tr>
<td>NASA (Langley Research Center)</td>
<td>15 September 75 to 31 May 78</td>
</tr>
<tr>
<td>NASA (Lewis Research Center)</td>
<td>1 September 78 to 31 August 84</td>
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</table>
C. Personnel

For Task 0 the research group was composed mainly of graduate and undergraduate students, and the work was organized as M.S. and Ph.D. thesis research topics. Undergraduates normally assisted the graduate students although occasionally an undergraduate had his own project. Supervision was provided by the principal investigator (D. T. Blackstock) except for a few instances in which a consultant (W. M. Wright) was the supervisor or cosupervisor. A few projects were not student oriented; they were carried out by senior staff or a visiting foreign scientist. Task 2 operated much in the same way as Task 0. For Tasks 1 and 3, however, the personnel were entirely senior staff. In the lists below the person worked on Task 0 unless otherwise indicated.

<table>
<thead>
<tr>
<th>Student</th>
<th>Period on Contract</th>
<th>Degree</th>
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<tbody>
<tr>
<td>B. J. Bourgeois</td>
<td>October - November 1976</td>
<td>B.S./Phys., December 1976</td>
</tr>
<tr>
<td>A. J. Gregorcyk</td>
<td>May 1975 - April 1976</td>
<td>B.S./Phys., December 1977</td>
</tr>
<tr>
<td>M. T. Houk</td>
<td>September - December 1981</td>
<td>B.S./Phys., June 1982*</td>
</tr>
<tr>
<td>J. D. Jessup</td>
<td>May - August 1976</td>
<td>B.S./Phys., June 1977**</td>
</tr>
<tr>
<td>J. R. Kuhn</td>
<td>March - June 1976</td>
<td>B.S./Phys., June 1977*</td>
</tr>
<tr>
<td>G. D. Landry</td>
<td>June - August 1984</td>
<td>not yet graduated</td>
</tr>
<tr>
<td>F. Schellenburg</td>
<td>March - June 1981</td>
<td>B.S./Phys., June 1981*</td>
</tr>
<tr>
<td>E. A. Tschoeppe</td>
<td>June - August 1984</td>
<td>B.S./Phys., December 1984</td>
</tr>
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</table>

* Degree from Kalamazoo College, Kalamazoo, Michigan

**Degree from Hendrix College, Conway, Arkansas
2. Graduate Students, M.S.

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<tr>
<td>T. W. Carlton</td>
<td>August - September 1975</td>
<td>M.B.A., Univ. Chicago, date unknown</td>
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<tr>
<td>F. D. Cotaras</td>
<td>August 1983 - August 1984</td>
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<td>D. R. Kleeman</td>
<td>April 1975 - September 1976</td>
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3. Graduate Students, Ph.D.

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<tr>
<td>D. A. Webster</td>
<td>January 1977 - October 1978</td>
<td>degree not completed (E.E.)</td>
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4. Foreign Scientists

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<tr>
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<tr>
<td>S. Saito (Task 2)</td>
<td>May 1982 - August 1983</td>
<td>Tokai University, Japan</td>
</tr>
<tr>
<td>J. N. Tjøtta</td>
<td>September 1979 - August 1980</td>
<td>University of Bergen, Norway</td>
</tr>
<tr>
<td>S. Tjøtta</td>
<td>September 1979 - August 1980</td>
<td>University of Bergen, Norway</td>
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5. Staff Members

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<tr>
<td>D. T. Blackstock</td>
<td>April 75 - August 84</td>
</tr>
<tr>
<td>J. M. Estes</td>
<td>January 80 - October 83</td>
</tr>
<tr>
<td>E. E. Mikeska (Task 3)</td>
<td>July 80 - June 82</td>
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<tr>
<td>T. G. Muir (Task 2)</td>
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<td>J. A. Shooter (Task 3)</td>
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<td>J. J. Truchard (Task 1)</td>
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6. **Consultant**

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<th>Name</th>
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<tr>
<td>W. M. Wright</td>
<td>August 77, July 80 - June 84</td>
<td>Kalamazoo College, \nKalamazoo, Michigan</td>
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D. **ONR Scientific Officers**

For the first year of the contract, the Scientific Officer was W. M. Madigosky. In April 1976 L. E. Hargrove became the Scientific Officer and continued throughout the remainder of the contract.
CHRONOLOGICAL BIBLIOGRAPHY

Code
AR = administrative report O = oral presentation
B = chapter in a book P = paper in a proceedings
J = journal publication T = thesis or dissertation
JS = submitted for journal publication Tr = translation
TR = technical report

Code 1973

Code 1974


Code 1975


*Attributed to predecessor contract, Contract N00014-70-A-0166, Task 0004.

**Attributed to predecessor contract, Contract N00014-70-A-0166, Task 0021.


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*Primary support for this work came from the Norwegian Government, which sponsored the Tjøttas during their first year at Applied Research Laboratories, The University of Texas at Austin (September 1978 - August 1979).


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**Primary support for this work came from NASA Grant NSG 3198.

H. L. Kuntz, D. T. Blackstock, and N. D. Perreira, "Reflection and absorption of high-intensity sound at the surface of a bulk porous material," Noise-Con 81 Meeting, Raleigh, 8-10 June 1981, Proceedings, pp. 73-76.


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*Primary support for this work came from ONR Contract N00014-79-C-0624 with Pennsylvania State University.

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*Primary support for this work came from ONR Contract N00014-79-C-0624 with Pennsylvania State University.

**Primary support for this work came from NASA Grant NSG 3198.
Code  1984


*Primary support for this work came from ONR Contract N00014-79-C-0624 with The Pennsylvania State University.

**Primary support for this work came from NASA Grant NSG 3198.

***Primary support for this work came from ONR Contract N00014-82-K-0805.


Code 1986


JS 2. W. M. Wright, "Generation of sound within a closed cell by an alternating current in a straight wire," to be submitted to J. Acoust. Soc. Am.


**Primary support for this work came from NASA Grant NSG 3198.
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<td>University of California at Los Angeles Physics Department Los Angeles, CA 90024 Attn: I. Rudnick</td>
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<td>Georgetown University Physics Department Washington, DC 20057 Attn: W. G. Mayer</td>
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<td>Georgia Institute of Technology School of Mechanical Engineering Atlanta, GA 30332 Attn: Y. H. Berthelot</td>
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<td>Kalamazoo College Department of Physics Kalamazoo, MI 49007 Attn: W. M. Wright</td>
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<td>58</td>
<td>University of Mississippi Physics Department University, MS 38677 Attn: H. E. Bass</td>
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<td>Rutgers University Mechanical and Aerospace Engineering Department New Brunswick, NJ 08903 Attn: S. Temkin</td>
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<td>Jet Propulsion Laboratory 4800 Oak Grove Pasadena, CA 91103 Attn: T. G. Wang</td>
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<td>67</td>
<td>National Research Council Division of Physics Montreal Road Ottawa, Ontario CANADA K1A 0S1 Attn: T. F. W. Embleton</td>
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<td></td>
<td>Technical University of Denmark Industrial Acoustics Laboratory Building 352 DK-2800 Lyngby DENMARK Attn: L. Bjørnø</td>
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<td>70</td>
<td>University of Bath School of Physics Claverton Down Bath BA2 7AY UNITED KINGDOM Attn: H. O. Berktoy</td>
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<td></td>
<td>Cambridge University Applied Math. &amp; Theoretical Physics Dept. Silver Street Cambridge CB3 9EW ENGLAND Attn: D. G. Crittton</td>
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<td>71</td>
<td>Institute of Sound and Vibration Research University of Southampton Southampton S09 5NH UNITED KINGDOM Attn: G. L. Morfey</td>
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<td>Osaka University The Institute of Scientific and Industrial Research B-1, Mihogaoka, Ibaraki Osaka 567 JAPAN Attn: A. Nakamura</td>
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<td>73</td>
<td>University of Bergen Mathematics Department Allegaten 59-55 Bergen 5000 NORWAY Attn: J. M. Tjøtta</td>
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<td>75</td>
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<td>76</td>
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<td>78</td>
<td>Acoustic Systems F. O. Box 8610 Austin, Texas 78764 Attn: D. A. Nelson</td>
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82 Lori B. Orenstein  
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