MODELING OF RIGID-BODY AND ELASTIC AIRCRAFT DYNAMICS FOR FLIGHT CONTROL DEVELOPMENT

THESIS

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Wright-Patterson Air Force Base, Ohio
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Thesis Chairman: Dr. Robert A. Calico
Professor of Aeronautical Engineering
Abstract

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FOR FLIGHT CONTROL DEVELOPMENT

THESIS

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of the Air Force Institute of Technology
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Preface

The interaction between the flight control system, structural dynamics, and aerodynamics of aircraft has become a major concern to aircraft designers. There has been considerable effort to develop an accurate method of modelling such interactions. This effort develops a simple method to create an accurate model of this interaction. This model can then be used for stability and control analysis including the effects of structural dynamics.

I wish to thank my advisor, Dr. Robert A. Calico for his invaluable guidance and assistance during this research effort. I would also like to thank my thesis committee members, Dr. Peter J. Torvik and Maj. Lanson Hudson for their helpful comments and thorough editing of this document. In addition, I would like to thank my sponsor, Dr. Thomas E. Noll for his guidance on applying aeroelastic concepts and for the use of his mathematical model of the YF-17. My thanks also go to Mr. Maxwell Blair who helped me use ADAM, and 1Lt. William Blake who helped run Digital DATCOM. Finally, I wish to thank my wife Colleen for her understanding and support during this work.
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Nomenclature

Scalars

\( a_i \) - components of a skew symmetric matrix

\( b \) - reference semi-chord length

\( c \) - reference chord length

\( D_i \) - denominator coefficient of the Pade polynomial

\( F_i \) - \( i \)th generalized force

\( F_x, F_y, F_z \) - thrust forces in the \( x, y, \) or \( z \) direction

\( g \) - gravity

\( h_i \) - normalized magnitude of the \( i \)th mode shape

\( h_0 \) - magnitude of the plunge mode shape

\( I_{xx}, I_{xy}, I_{xz}, I_{yy}, I_{yz}, I_{zz} \) - mass moment or product of inertia

\( k \) - reduced frequency \((k=\omega/V)\)

\( L \) - Lagrangian

\( L \) - aerodynamic moment about the \( x \) axis

\( M \) - aerodynamic moment about the \( y \) axis

\( M \) - mass

\( M_\theta \) - \( \partial M/\partial \omega \)

\( m_j \) - generalized mass of the \( j \)th control surface

\( N \) - aerodynamic moment about the \( z \) axis

\( N_i \) - numerator coefficient of the Pade polynomial

\( p \) - roll rate

\( Q_{ij} \) - force on the \( j \)th mode due to the \( i \)th motion

\( q \) - pitch rate
\( q \) - dynamic pressure
\( r \) - yaw rate
\( S \) - reference area
\( S \) - surface area
\( s \) - Laplace variable
\( T \) - kinetic energy
\( V \) - potential energy
\( V \) - velocity
\( V \) - volume
\( \alpha \) - angle of attack
\( \Delta P_j \) - pressure on the jth mode
\( \delta_j \) - jth control surface
\( \delta_{jo} \) - magnitude of the jth control mode
\( \phi \) - bank angle
\( \psi_i \) - mode shape of the ith mode of the elastic motion
\( \gamma \) - twist position of wing
\( \eta_i \) - ith generalized coordinate
\( \mu_i \) - ith generalized mass
\( \psi \) - yaw angle
\( \rho \) - density
\( \omega \) - frequency
\( \omega_i \) - frequency of vibration of the ith mode
\( \xi_i \) - motion of the ith mode of the elastic motion
\( \xi_{io} \) - magnitude of the ith mode shape
\( \theta \) - pitch angle
\( \theta_0 \) - magnitude of the pitch mode shape
Vectors

\( a, b, c \) - general vectors

\( F \) - vector of generalized forces producing elastic motion

\( F_R \) - vector of forces producing rigid-body translation

\( H \) - angular momentum

\( M_0 \) - vector of moments producing rigid-body rotation

\( P \) - linear momentum

\( q \) - vector of generalized force

\( \mathbf{g} \) - generalized coordinates

\( \mathbf{R}_0 \) - position vector of the origin of the body axis with respect to the inertial frame

\( \dot{\mathbf{R}}_0^I \) - velocity of the body axis origin with respect to the inertial frame as seen by an observer in the inertial frame

\( \mathbf{R}_p \) - position vector of point \( P \) on the body with respect to the inertial frame

\( \dot{\mathbf{R}}_p^I \) - velocity of point \( P \) with respect to the inertial frame as seen by an observer in the inertial frame

\( \mathbf{L}_0 \) - position vector of the undeformed point \( P \) with respect to the body axis

\( \mathbf{u} \) - m vector of inputs to the aircraft

\( \mathbf{V}_0 \) - velocity vector of the body axis origin with respect to the inertial frame

\( \mathbf{x} \) - m vector of states of the aircraft

\( \mathbf{y} \) - p vector of the outputs of the aircraft

\( \delta \) - elastic deformation of point \( P \) with respect to the undeformed position

\( \mathbf{q} \) - vector of generalized coordinates

\( \Omega^B/I \) - angular velocity vector of the body axis with respect to the inertial frame
Matrices

\([A]\) - \(n\) by \(n\) plant matrix

\([\tilde{A}]\) - skew symmetric matrix

\([a_k]\) - matrix coefficients of Padé fit equations of motion

\([B]\) - \(n\) by \(m\) input matrix

\([C]\) - \(p\) by \(n\) output matrix

\([\dot{C}]\) - generalized damping matrix

\([I]\) - identity matrix

\([I_C]\) - inertial coupling matrix

\([I_E]\) - elastic inertial matrix

\([I_R]\) - rigid inertial matrix

\([K]\) - generalized stiffness matrix

\([M]\) - generalized mass matrix

\([Q(k)]\) - generalized aerodynamic force matrix

\([X]\) - aircraft mass matrix (including aerodynamics)

\([Y]\) - aircraft damping matrix (including aerodynamics)

\([Z]\) - aircraft stiffness matrix (including aerodynamics)
Abstract

The purpose of this effort was to provide a method of developing a linear model of an elastic aircraft. The model provides the capability to analyze the coupling between the rigid and elastic motion of the aircraft. The method developed in this effort obtains stability derivatives directly from unsteady aerodynamic forces. This results in a state-space model whose states are just the normal aircraft states and rates, the structural coordinates and rates, and the control surface positions and rates. Using a representation of the YF-17 wind tunnel flutter model, it was demonstrated that the methodology developed predicted the required dynamics to make this a viable method of modelling rigid-body and flutter behavior of the model. Flutter control laws were designed for motion about an equilibrium condition represented by a velocity 20% above the flutter velocity. Both classical and modern techniques yielded acceptable control laws. The control laws were also analyzed at off design conditions to check robustness.
I. Introduction

With the advent of high gain control systems on high performance, structurally efficient aircraft, the interaction between aerodynamics, structural dynamics and the flight control system, has become a major concern to aircraft designers. This interaction has been termed aeroservoelasticity (ASE). Virtually all major U.S. fighter aircraft in use today have encountered this phenomenon. The F-15 has encountered several ASE problems that have influenced both ground and flight tests (18:Vol I, 8-22). The F/A-18 also had many ASE encounters, primarily in ground tests (18:Vol II, 205-211). The YF-16 and YF-17 also had ASE encounters (7:482). Even the experimental X-29 had ASE problems, although not in flight. The ASE interactions are not limited to new aircraft. The ASE phenomenon has been encountered on the F-4 (18:Vol I, 3-7; 7:482) and many aircraft that were in test phases (18:Vol II, 226-231; 21:10). This interaction, if severe enough, can limit the
performance capabilities of an aircraft. Avoidance of such problems calls for the development of an accurate aircraft modelling technique, which can model the interactions correctly.

**Background**

Typically, the three phenomenon involved in ASE, have been analyzed using separate models for each. The aerodynamic models are usually based on the assumption that the aircraft is a rigid-body. An accurate representation of the aerodynamic shape of the aircraft is required to obtain accurate aerodynamic parameters. The flight control models are typically linearized rigid-body models, using stability derivatives developed for a rigid aircraft. These linear flight control models are then represented by state-space models or through transfer functions. The usual way to account for flexibility is to add an elastic correction factor to the stability derivatives, which does not account for the dynamics of the structure. The structural dynamics model, on the other hand, is concerned mainly in a correct representation of the elastic structure to predict such things as flutter and divergence. Typically, each of the three models leads to an analysis which is never communicated to the other two. This lack of a unified model
resulted in the problems that were noted previously.

Correcting the unwanted ASE interactions after the aircraft is built and tested is the most common procedure for dealing with ASE problems. The typical "quick fixes" usually involve the control system. Such fixes have included, adding filters, reducing control system gains, relocating sensors (18; 21:8), introducing flutter placards, and in the case of the X-29, limiting the flight envelope until a better control system could be designed. The process of separate modelling, and then fixing the problems after the aircraft is built limits the potential of modern aircraft design. A unified approach to aircraft modelling can be accomplished, with all three of the phenomenon involved in ASE being accounted for.

The history of developing a unified model for aircraft design goes back well over twenty years. Bisplinghoff and Ashley (1:450-486) and Milne (13) were some of the first to develop the equations of motion for an elastic aircraft. The implementation of these equations was beyond the capabilities of computers of that day. In 1974, the first computer program that addressed the ASE phenomenon was FLEXSTAB (5). FLEXSTAB, developed by Boeing, in cooperation with NASA and the Flight Dynamics Laboratory (FDL), combined aerodynamic, elastic structure and control system models into one computer program for analysis and design purposes.
FLEXSTAB was a large and cumbersome program, and as new design and analysis techniques were developed, they could not be easily incorporated into FLEXSTAB. The desire to develop a unified model was continued throughout the 70's. Etkin (6), Schwanz (22; 23), Taylor and Woodcock (26), Warren (29), Rodden (19), NASA engineers (18), and many others (4; 11; 25) all proposed approaches to develop ASE models. However, it became apparent that even by 1984 there was no consensus on the best way to develop a unified model (18).

Recently, the Flight Dynamics Laboratory has developed an in-house computer program called ADAM (Analog and Digital Aeroservoelastic Method). It has the capability to combine unsteady aerodynamics, multi-input multi-output control systems and a structural dynamics model into one analysis package (14:1).

ADAM does have some limitations. The first limitation is that typical stability derivatives that flight controls engineers use are not directly available (14:8). A second difficulty is that ADAM does not include control surface states and thus cannot predict control surface instabilities (2: Vol 1, 12). In addition, ADAM has numerical difficulties inherent in the complexity of the modelling process (14:8). A method is needed to solve these problems in ADAM to make it a true ASE analysis tool.
Purpose

The purpose of this thesis is to develop a method for use in concert with ADAM, in the analysis of ASE problems. This method includes prediction of stability derivatives, addition control surface dynamics, and reducing model complexity.

Scope

This effort will be limited to the development of a linear time-invariant state-space model of an elastic aircraft. This effort will examine only longitudinal motion, although the theory can be easily extended to lateral directional motion. The work is also limited to continuous time models and control systems.
Approach

This thesis will develop the equations of motion of an elastic aircraft, using Lagrange's equations. The resulting equations will then be linearized about an equilibrium point, and the associated non-linear aerodynamics will also be linearized. The kinematic coupling between the rigid-body and elastic motion will be eliminated. Thus the only coupling between the rigid-body and elastic motion will be through the aerodynamics. The resulting second order equations will then be transformed into a first order state-space model for the elastic aircraft. A computer program will then be developed which will automate this procedure. The methodology will be demonstrated with a representation of the YF-17 wind tunnel flutter model.
II. Theoretical Development

It is desired to develop the equations of motion of an elastic aircraft, in the form of Eqs 1 and 2

\[ \dot{x} = [A]x + [B]u \]

\[ y = [C]x \]

where

- \( x \) - n vector of states of the aircraft
- \( u \) - m vector of inputs to the aircraft
- \( y \) - p vector of outputs of the aircraft
- \([A]\) - n by n plant matrix
- \([B]\) - n by m input matrix
- \([C]\) - p by n output matrix

There are three general ways that the equations of motion for an elastic aircraft can be determined. The first way, and probably the most common way is to use classical Newtonian dynamics. Etkin (6:122-145) developed the equations of motion for a rigid aircraft using Newtonian dynamics. Milne (13:4), Taylor (26:22), and Bisplinghoff and Ashley (1:450-468) developed the equations of motion for an elastic aircraft using this method. The second method is to use Hamilton's equations to derive the equations of motion (3:1684-1687). The final method of deriving the equations of motion uses Lagrange's equations, as Schwanz had done (22). This is the method used in this development, since it is the most direct and easy to use. Lagrange's
equations contain all the information needed to derive the equations of motion.

**Lagrange's Equations**

Lagrange's equations are based on energy principles. If the aircraft's total potential and kinetic energy can be described, in reference to an inertial frame, then the equations of motion of that aircraft can be derived. Stated in a general form, Lagrange's equations for a holonomic system are

\[
\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) - \frac{\partial L}{\partial \eta} = Q
\]

\[L = T - V\]

where

- \(T\) - total kinetic energy,
- \(V\) - total potential energy,
- \(\eta\) - generalized coordinates,
- \(Q\) - generalized forces, and
- \(\frac{\partial L}{\partial \eta} = \left(\frac{\partial L}{\partial \eta_1} \ldots \frac{\partial L}{\partial \eta_n}\right)^T\)

Therefore in order to use Lagrange's equations, the kinetic and potential energy of the elastic aircraft must be found.

Consider an elastic body, as shown in Figure 1. The inertial axis origin, \(O_1\), is a fixed point attached to the earth. It will be assumed that the earth is flat and non-rotating. In general the flat non-rotating earth
Figure 1. Coordinates for a General Elastic Body
assumption does not have to be made, however, it is reasonable to do so for aircraft motion in subsonic and low supersonic flight (6:148). This assumption will allow the notation to be kept simple, and simplifies the dynamics.

The position vector from this inertial origin to any arbitrary point \( P \) on the body, \( \mathbf{R} \) is given by:

\[
\mathbf{R}_p = \mathbf{R}_0 + \mathbf{r}_0 + \mathbf{\delta}
\]

(5)

where

\[ \mathbf{R}_0 \] - position vector of the body frame origin to the inertial frame origin,
\[ \mathbf{r}_0 \] - position vector of the undeformed position of \( P \) to the body axis origin, and
\[ \mathbf{\delta} \] - elastic deformation position vector of point \( P \) from the undeformed position.

From this the kinetic and potential energies can be obtained.

**Kinetic Energy.** The kinetic energy of the body is defined as

\[
T = \frac{1}{2} \rho \mathbf{R}_p^I \cdot \mathbf{\dot{R}}_p^I \, dV
\]

(6)

where

\[ \rho \] - density
\[ V \] - volume,
\[ \mathbf{\dot{R}}_p^I \] - velocity of point \( P \) with respect to \( O_I \)

as seen by an observer in the inertial reference frame.

\[ \mathbf{\dot{R}}_p^I \] can be represented by
\[ \dot{\mathbf{x}}_p^I = \dot{\mathbf{x}}_0^I + \mathbf{d}^I/dt(\tau_0 + \dot{\phi}) \]  

(7)

which can be expanded

\[ \dot{\mathbf{x}}_p^I = \dot{\mathbf{x}}_0^I + \mathbf{d}^B + \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \]  

(8)

where

\[ \dot{\omega}^B/I - \text{angular velocity of the body axis with respect to the inertial axis} \]

\[ \mathbf{d}^B - \text{velocity of the deformed point P with respect to the body axis frame} \]

Substituting this back into Eq 6 yields

\[ T = \frac{1}{2} \int \rho \{ \mathbf{v}_0 + \mathbf{d}^B + \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \cdot \{ \mathbf{v}_0 + \mathbf{d}^B + \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \, dV \]  

(9)

where, \( \dot{\mathbf{x}}_0^I = \mathbf{v}_0 \). Expanding Eq 9 term by term gives

\[ T = \frac{1}{2} \int \rho \mathbf{v}_0 \cdot \mathbf{v}_0 \, dV + \frac{1}{2} \int \rho \mathbf{d}^B \cdot \mathbf{d}^B \, dV + \frac{1}{2} \int \rho \mathbf{v}_0 \cdot (\dot{\omega}^B/Ix(\tau_0 + \dot{\phi})) \cdot (\dot{\omega}^B/Ix(\tau_0 + \dot{\phi})) \, dV \]

\[ + \frac{1}{2} \int \rho \mathbf{d}^B \cdot (\dot{\omega}^B/Ix(\tau_0 + \dot{\phi})) \cdot (\dot{\omega}^B/Ix(\tau_0 + \dot{\phi})) \, dV \]

\[ + \frac{1}{2} \int \rho \{ \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \cdot \mathbf{v}_0 \cdot dV + \frac{1}{2} \int \rho \{ \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \cdot \mathbf{d}^B \, dV \]

\[ + \frac{1}{2} \int \rho \{ \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \cdot \{ \dot{\omega}^B/Ix(\tau_0 + \dot{\phi}) \} \, dV \]  

(10)

The total mass of the aircraft is defined as
Combining term in Eq 10, and expanding the remaining products yields

\[ T = \frac{1}{2} M_{V_0} \cdot V_0 + \int \rho \dot{V}_0 \cdot \dot{V}_0 \, dV + \int \rho \ddot{V}_0 \cdot (\ddot{\Omega}^B/Ix_{\ddot{\Omega}}) \, dV + \int \rho \dddot{V}_0 \cdot (\dddot{\Omega}^B/Ix_{\dddot{\Omega}}) \, dV + \frac{1}{2} \int \rho (\dddot{\Omega}^B/Ix_{\dddot{\Omega}}) \cdot (\dddot{\Omega}^B/Ix_{\dddot{\Omega}}) \, dV \]

Since \( V_0 \) and \( \Omega^B/I \) do not depend on the position within the volume, they can be pulled outside the integrals. Now the following assumptions are made to help simplify Eq 12. First, it is assumed that the body axis origin is at the center of mass of the equilibrium configuration of the aircraft. This implies that

\[ 0 = \int \rho \rho \, dV \]

The second major assumption is that the aircraft body axis system is the mean axis system, which Milne states as the axis "... at every point, the linear and angular momentum of the relative motion with respect to the body axis is
identically zero" (13:5). This results in the following

\[ 0 = \int \rho \dot{\mathbf{v}} \mathbf{d}V = \int \rho \mathbf{p} \mathbf{d}V = \int \rho \mathbf{p} \mathbf{B} \mathbf{d}V = \int \rho \mathbf{p} \mathbf{B} \mathbf{d}V \]

(14)

This reduces the coupling between the overall motion and the elastic deformation (13:27), thus reducing the kinetic energy equation (Eq 12) to

\[
T = \frac{1}{2} M \mathbf{v}_0 \cdot \mathbf{v}_0 + \int \int \rho \dot{\mathbf{v}} \mathbf{B} \mathbf{d}V \cdot \int \int \rho \mathbf{p} \mathbf{B} \mathbf{d}V \cdot \mathbf{v}_0 \cdot \mathbf{B} \mathbf{d}V
\]

+ \int \int \rho (\mathbf{B} / I_x) \cdot \mathbf{v}_0 \cdot \mathbf{B} \mathbf{d}V \cdot \mathbf{v}_0 \cdot \mathbf{B} \mathbf{d}V

+ \int \int \rho (\mathbf{B} / I_x) \cdot \mathbf{v}_0 \cdot \mathbf{B} \mathbf{d}V

(15)

Noting that the components of the cross product of two vectors expressed in the same orthogonal reference frame, say \( \mathbf{a} \times \mathbf{b} = \mathbf{c} \), may be obtained from

\[
[\mathbf{a}] \mathbf{b} = \mathbf{c}
\]

(16)

where \([\mathbf{a}]\) is a skew symmetric matrix, defined by

\[
[\mathbf{a}] = \begin{bmatrix}
0 & -a_3 & a_2 \\
-a_3 & 0 & -a_1 \\
-a_2 & a_1 & 0
\end{bmatrix}
\]

Eq 15 now becomes
\[
T = \frac{1}{2} MV_o \cdot V_o + \frac{1}{2} \int \rho \, \dot{x}^B \cdot \dot{x}^B \, dV + \frac{1}{2} Q^B/I \cdot \dot{Q}^B/I
\]

\[
+ \int \rho Q^B/I \cdot (\dot{x}^B \cdot \dot{x}^B) \, dW + Q^B/I \cdot \dot{Q}^B/I
\]

\[
+ \frac{1}{2} Q^B/I \cdot [I_E] \cdot \dot{Q}^B/I
\]

(17)

where

- \([I_R]\) - rigid body inertia matrix
- \([I_C]\) - inertial coupling matrix between rigid and elastic motion
- \([I_E]\) - elastic motion inertia matrix

It is at this point that knowledge about the equilibrium conditions is necessary to simplify Eq 17. Since it is desired to develop a linear model, perturbation equations about an equilibrium condition will be developed. The equilibrium condition in this development will be straight and level flight of an undeformed airframe, thus the equilibrium body rotation rates \((p, q, r)\) are zero. Thus, \(Q^B/I\) is now just a vector of the perturbation rotation rates. Both the coupling and elastic inertial terms of Eq 17 are functions of \(\delta\), a perturbation motion. Thus the last three terms of Eq 17 are third, third, and fourth order respectively of the perturbation motion, and thus will be ignored. Thus, Eq 17 becomes

\[
T = \frac{1}{2} MV_o \cdot V_o + \frac{1}{2} \int \rho \, \dot{x}^B \cdot \dot{x}^B \, dV + \frac{1}{2} Q^B/I \cdot \dot{Q}^B/I
\]

(18)

As can be seen, this expression for kinetic energy is
much simpler than the original equation (Eq 10). The only term that involves the elastic motion is the middle term in Eq 18. It is at this point that some assumptions about the elastic deformations must be made. In general, the elastic deformations of a body can be described by a set of \( n \) coupled second order ordinary differential equations, representing the free vibration of the body. These coupled equations can be decoupled by use of a linear transformation (12:143-144). Therefore, in this development, it will be assumed that a set of uncoupled orthogonal modes are available. This assumption will allow diagonal mass and stiffness matrices to be developed for the equations of motion, thus allowing easier decoupling of the elastic and rigid body equations of motion. The elastic deformations are defined as

\[
\mathbf{d} = \sum_{i=1}^{n} \varphi_i(x, y, z) \xi_i(t)
\]

(19)

where,

- \( \varphi_i(x, y, z) \) - mode shape of the \( i \)th mode, and
- \( \xi_i(t) \) - time dependent motion of the mode

Since the modes are orthogonal, \( \int \varphi_i \varphi_j = 0 \) for \( i \neq j \) (12: 143). Now defining the generalized mass as
\[ u_i = \int \rho \phi_i^2 dV \]  

(20)

and substituting Eqs 19 and 20 into Eq 18 yields

\[
\Sigma = \frac{1}{2} \sum_{i=1}^{n} W_i \phi_i^2 + \frac{1}{2} \sum_{i=1}^{n} \omega_i^2 \phi_i^2
\]

(21)

This is the equation that is required for the kinetic energy.

**Potential Energy.** The potential energy of an elastic structure can be represented by

\[
\Sigma = -\frac{1}{2} \sum_{i=1}^{n} \omega_i^2 \phi_i^2
\]

(22)

where

\[ \omega_i^2 \] - the squared natural frequencies of the various modes.

The same assumption about orthogonality that were made earlier is also made here (12:168). The potential energy due to the Earth's gravitation will be dealt with as an external force acting on the aircraft.

Eqs 21 and 22 can now be substituted into Eq 4, the equation for \( L \), and Lagrange's equations formed, and noting that the rigid-body terms do not appear, gives
This can be represented in matrix notation as

\[
\begin{bmatrix}
\mu_1 & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \cdots & \mu_n
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\vdots \\
\xi_n
\end{bmatrix}
+
\begin{bmatrix}
\omega_1^2 \mu_1 \\
\vdots \\
\omega_n^2 \mu_n
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\vdots \\
\xi_n
\end{bmatrix}
=
\begin{bmatrix}
F_1 \\
\vdots \\
F_n
\end{bmatrix}
\]

(24)

where

\begin{align*}
\mu_i & \quad \text{ith generalized mass} \\
\omega_i & \quad \text{ith modal frequency} \\
\xi_i & \quad \text{ith generalized modal coordinate} \\
F_i & \quad \text{ith generalized force}
\end{align*}

In this formulation, the control surface motions will be treated as extra degrees of freedom in the elastic motion. In order to correctly develop the remaining equations for rigid body motion, the definitions of linear and angular momentum must be used. The linear momentum may be found from

\[
P = \partial T / \partial \dot{V}_0
\]

(25)

Then the equations of motion for rigid translation become

\[
F_R = dI/dt(P) = dI/dt(\partial T / \partial V_0)
\]

\[
= dB/dt(\partial T / \partial V_0) + B/1x(\partial T / \partial V_0)
\]

(26)

Using the typical formulation, the aircraft equations of motion become
\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix}
+ M
\begin{bmatrix}
0 & -r & p \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
= \begin{bmatrix}
X + F_x - Mg\sin\theta \\
Y + F_y - Mg\sin\phi\cos\theta \\
Z + F_z - Mg\cos\phi\cos\theta
\end{bmatrix}
\] (27)

where

- \( M \) - mass
- \( u \) - velocity in the x direction
- \( v \) - velocity in the y direction
- \( w \) - velocity in the z direction
- \( p \) - roll rate
- \( q \) - pitch rate
- \( r \) - yaw rate
- \( X \) - aerodynamic force in the x direction
- \( Y \) - aerodynamic force in the y direction
- \( Z \) - aerodynamic force in the z direction
- \( F_x \) - thrust in the x direction
- \( F_y \) - thrust in the y direction
- \( F_z \) - thrust in the z direction
- \( g \) - acceleration of gravity
- \( \theta \) - pitch angle
- \( \phi \) - bank angle

with the following equations for kinematics and trajectory

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & -\sin\theta \\
0 & \cos\phi\sin\theta & \cos\phi \\
0 & -\sin\phi & \cos\phi
\end{bmatrix}
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix}
\] (28)

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix}
= \begin{bmatrix}
c\cos\psi & c\cos\psi & -s\theta \\
s\sin\psi - c\sin\theta \cos\phi & s\sin\psi + c\sin\theta \cos\phi & c\cos\theta \\
c\sin\psi - c\sin\theta \cos\phi & s\sin\psi + c\sin\theta \cos\phi & -s\cos\phi
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\] (29)

where

- \( c \) - cosine
- \( s \) - sine
- \( \psi \) - yaw angle

A similar method can be used to develop the equations of motion for the attitude motion. The angular momentum is given by
The equations for rigid body rotation become

\[ M_0 = d^2H/dt^2 = 4dH/dt \frac{\partial H}{\partial \Omega^B} + \Omega^B \times (\frac{\partial H}{\partial \Omega^B}) \]

(31)

These equations (Eqs 26 and 31) are sometimes termed Lagrange’s equations in quasi-coordinates. Eq 31 in matrix notation becomes

\[
\begin{bmatrix}
I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

\[ + \begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
\begin{bmatrix}
-I_{xx} & -I_{xy} & -I_{xz} \\
-I_{xy} & I_{yy} & -I_{yz} \\
-I_{xz} & -I_{yz} & I_{zz}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

(32)

where

- \(I_{xx}, I_{xy}, I_{xz}, I_{yy}, I_{yz}, I_{zz}\) - mass moment or product of inertia
- \(L\) - aerodynamic moment about the \(x\) axis
- \(M\) - aerodynamic moment about the \(y\) axis
- \(N\) - aerodynamic moment about the \(z\) axis

Eqs 24, 27, 28, 29, and 32 comprise the desired equations of motion. The forces in Eqs 24, 27, and 32 are now function of both the rigid and elastic motion.

**Longitudinal Equations of Motion**

The desired equations of motion, Eqs 1 and 2, are
linear in nature. The above equations (Eqs 27, 28, 29, and 32) still contain non-linear terms which need to be further linearized. In this development, however, only the longitudinal equations will be developed. Therefore, only those equations appropriate to the longitudinal motion will be addressed. The equilibrium condition is again straight and level flight, but now using longitudinal motion, equations reduce to the following scalar equations

\[
\begin{align*}
\dot{M}w + Mqw &= -X + F_x - Mgsin\theta \\
\dot{M}w - Mqu &= -Z + F_z - Mgcose \\
I_{yy}q &= H \\
q &= \dot{\theta} \\
u &= xcos\theta - zsin\theta \\
w &= xsin\theta + zcos\theta
\end{align*}
\]

(33) \hspace{2cm} (34) \hspace{2cm} (35) \hspace{2cm} (36) \hspace{2cm} (37) \hspace{2cm} (38)

The attitude equations are uncoupled from the trajectory equations (Eqs 37 and 38), and thus the trajectory equations will be disregarded in this development. Now it will be assumed that each variable will be the sum of its' equilibrium value and perturbed value (i.e. \(u = u_e + u'\)), and the derivatives of the equilibrium values are zero. It will also be assumed that the mass and inertia remain essentially constant. If the thrust is to be used as an input it can be used as a control input. Substituting these into Eqs 33-38 yields

- 20 -
\[ M\dot{w}' + N(q')(w_e + w') = -(X_e + X') + F_x - Mgsin(\theta_e + \theta') \] (39)

\[ M\dot{w}' - N(q')(u_e + u') = -(Z_e + Z') + F_z - Mgcos(\theta_e + \theta') \] (40)

\[ I_{yy}\dot{q}' = N_e + M' \] (41)

\[ (q') = \dot{\theta} \] (42)

\[ u' = x'cos(\theta_e + \theta') - z'sin(\theta_e + \theta') \] (43)

\[ v' = x'sin(\theta_e + \theta') + z'cos(\theta_e + \theta') \] (44)

Now taking all the terms that strictly deal with equilibrium conditions yields Eqs 45-47 while the remaining terms result in the perturbation equations, Eqs 48-53.

\[ 0 = -X_e + F_x - Mgsin\theta_e \] (45)

\[ 0 = -Z_e + F_z - Mgcos\theta_e \] (46)

\[ M_e = 0 \] (47)

\[ M\dot{w} + N(w_e q' + w' q') = -X' - Mgsin\theta' \] (48)

\[ M\dot{w} + N(u_e q' + u' q') = -Z' \] (49)

\[ I_{yy}\dot{q}' = M' \] (50)

\[ q' = \dot{\theta}' \] (51)

\[ u' = \dot{x}' \] (52)

\[ w' = \dot{z}' \] (53)

The equilibrium equations and the perturbation equations are identical to those developed in Taylor (26:39-44). Assuming that the terms where products of perturbation quantities exists are small results in the linear set of equations in...
Eq 54, where the primed superscript is dropped.

\[
\begin{bmatrix}
  M & 0 & 0 \\
  0 & M & 0 \\
  0 & 0 & I_{yy}
\end{bmatrix}
\begin{bmatrix}
  \dot{u} \\
  \dot{w} \\
  \dot{q}
\end{bmatrix}
+ \begin{bmatrix}
  0 & 0 & M_{ee} \\
  0 & 0 & M_{ee} \\
  0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  u \\
  w \\
  q
\end{bmatrix}
= \begin{bmatrix}
  -X - Mg \theta \\
  -Z \\
  M
\end{bmatrix}
\]

The equations can be combined with the elastic equations of motion to yield the following matrix equation:

\[
\begin{bmatrix}
  M_{00} & 0 & 0 \\
  0 & M_{00} & 0 \\
  0 & 0 & I_{yy}
\end{bmatrix}
\begin{bmatrix}
  \dot{X} \\
  \dot{Z} \\
  \dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
  0 & 0 & M_{ee} \\
  0 & 0 & M_{ee} \\
  0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  X \\
  Z \\
  \theta
\end{bmatrix}
= \begin{bmatrix}
  -X \\
  -Z \\
  M
\end{bmatrix}
\]

These equations are identical to those of Schwan (23:54). The linear aerodynamics which drive these equations must now be determined to complete the model of the flexible aircraft. From this point on, only the short period motion will be considered. Thus the first equation in Eq 55 will not be considered and there will not be an x state.
Linear Aerodynamics

Since the equations of motion of the aircraft have been linearized, the aerodynamics associated with this model must also be linearized. Etkin (6:159) and Taylor (26:45) have both shown that the aerodynamics can be approximated linearly. Ignoring symmetric and antisymmetric motion interactions, and neglecting most higher order aerodynamic derivatives, the longitudinal aerodynamic forces are of the following form

\[ M = M_u u' + M_q q' + M_a \alpha + M_d \dot{\alpha} + \sum_{j=1}^{m} (M_{d1j} \dot{\delta}_j + M_{d3j} \dot{\delta}_j + M_{d2j} \ddot{\delta}_j) \]

\[ + \sum_{i=1}^{n} (M_{e1i} \epsilon_i + M_{e3i} \dot{\epsilon}_i + M_{e2i} \ddot{\epsilon}_i) \quad (56) \]

where,

\[ M_u = \frac{\partial M}{\partial u}, \text{ etc.} \]

In general these derivatives are not directly available from the unsteady aerodynamic methods such as the Doublet Lattice (8), which is used in flutter prediction work. Rodden and Giesing (20) have shown however, that stability derivatives can be obtained from such methods. In developing unsteady aerodynamic forces and moments, it is assumed that the motion is purely oscillatory and of the form \( e^{i \omega t} \). The resulting generalized forces therefore
contain real and imaginary parts. The aerodynamics are
determined at a constant altitude and Mach number for
various reduced frequencies \((K=\omega b/V, \text{ where } b \text{ is a}
reference semi-chord and } V \text{ is the velocity})\). This results
in a set of forces for each degree of freedom for each
reduced frequency.

One way to approximate the frequency dependent is to
use a Pade fit as a function of reduced frequency (14:3-4).
In general the equations of motion can be represented by the
second order equations

\[
[M]\ddot{q} + [C]\dot{q} + [K]q = -\left(\rho V^2 S/2\right) [Q(K)] g
\]  

(57)

where

\[
[M] \quad \text{generalized mass matrix} \\
[C] \quad \text{generalized damping matrix} \\
[K] \quad \text{generalized stiffness matrix} \\
[Q(K)] \quad \text{generalized aerodynamic matrix} \\
g \quad \text{generalized coordinate vector} \\
V \quad \text{velocity} \\
S \quad \text{reference area}
\]

The \([Q(K)]\) matrix is a matrix of complex coefficients based
on simple harmonic motion. In order to solve the flutter
problem in a classical way, many different values of \(k\) and
the associated \([Q(K)]\) matrices are required. An alternate
technique is to curve fit each coefficient in the \([Q(K)]\)
matrix with respect to \(k\), using Pade polynomials. The
polynomials are of the form
\[ Q_{ij}(k) = \frac{N_0 + N_1(ik) + N_2(ik)^2 + N_3(ik)^3 + N_4(ik)^4}{1 + N_0 + D_1(ik) + D_2(ik)^2} \]  
(58)

for a fourth order over second order fit. Then substituting \( bs/V = ik \) into Eq 58 yields

\[ Q_{ij}(s) = \frac{N_0 + N_1(bs/V) + N_2(bs/V)^2 + N_3(bs/V)^3 + N_4(bs/V)^4}{1 + N_0 + D_1(bs/V) + D_2(bs/V)^2} \]  
(59)

Taking the Laplace transform of the right side of Eq 57, and equating terms of like power in \( s \), results in the following equation

\[ ([I]s^4 + [a_3]s^3 + [a_2]s^2 + [a_1]s + [a_0])g(s) = 0 \]  
(60)

As can be seen, this method, increases the order of the system by the multiple associated with the order of the denominator of the Pade fit. The system order is therefore altered unless the Pade denominator is a constant. This makes it extremely difficult to form a state-space model of the aircraft in which all the states relate directly to physical quantities. Another method, suggested by Rodden and Giesing (20) allows the state variables to be explicitly stated, and may also reduce the order of the model. This is the method that will be used in this study.
In order to derive the equations necessary to find the derivatives some assumptions must be made. In the analysis of the unsteady aerodynamic forces, the rigid-body motions are assumed to be pitch \( q \) and plunge \( h \). In aircraft dynamics, plunge is typically replaced with angle of attack \( \alpha \). Plunge is pure vertical motion, and is related directly to \( w \) by \( \dot{h} = w \) if measured from the center of mass. If \( \alpha \) is small, which can be assumed since perturbation motion is being discussed, then; \( \alpha = w/V = \dot{h}/V \). And since the equilibrium condition assumes level flight then \( \alpha = 0 \) for the equilibrium conditions.

The method of Rodden and Giesing was developed to calculate dynamic stability derivatives from unsteady aerodynamics (20). This methodology is summarized here, using the moment equation as an example. First, it is assumed that stability derivatives can be represented by a Maclaurian series

\[
M = M_\alpha + M_\alpha \alpha + M_\alpha \dot{\alpha} + M_\alpha \ddot{\alpha} + M_\alpha q + \ldots
\]

(61)

Oscillatory theory cannot predict the \( M_\alpha \) term, and therefore it will be omitted. Next it is assumed that the pitching and plunging motion are oscillatory and are of the form \( \alpha = \theta_0 e^{i\omega t} \) and \( h = h_0 e^{i\omega t} \), and thus for pitching motion
\[ M = \text{Real}(M e^{i\omega t}) \]  \hfill (62)

Hence the complex pitching amplitude \( M \) due to pitching motion can be represented by the series expansion to first order

\[ R = \theta_0 [M_\alpha + i\omega(M_\alpha + M_q)] \]  \hfill (63a)

Similarly for plunging motion with \( \alpha = \dot{h}/V \)

\[ R = h_0/2E(i\omega M_\alpha - \omega^2 M_\alpha) \]  \hfill (63b)

The terms of Eqs 63 can now be directly related to the appropriate terms of the matrix of complex frequency dependent aerodynamic coefficients from the Doublet Lattice method. According to Rodden and Giesing, the steady dynamic stability derivatives are defined by the limiting values as \( k \) approaches 0, so in general, a small value of \( k \) should be used. For greater detail on the procedure used by Rodden and Giesing, the reader is referred to their paper (20).

The application of the procedure outlined above in this effort is as follows. The unsteady aerodynamic forces were obtained from FASTOP (30) which uses Doublet Lattice aerodynamics. The generalized forces for a reduced frequency are
\[ Q_{ij} = \left( \frac{1}{s^2} \right) \int_S \left( \frac{\Delta P_j}{\bar{q}} \right) \left( \frac{h_i}{s} \right) dS \]  

(64)

where

\( Q_{ij} \) - the force on the \( i \)th mode due to the \( j \)th modal deflection
\( \Delta P_j \) - pressure on the \( j \)th mode
\( h_i \) - magnitude of the \( i \)th mode
\( s \) - reference length
\( S \) - surface area
\( q \) - dynamic pressure

In order to obtain stability derivatives the \( Q_{ij} \) terms must be normalized by both the \( i \)th and \( j \)th mode shape magnitudes as was demonstrated by Noll with lateral-directional derivatives (15). To develop dimensional stability derivatives the \( Q_{ij} \) terms must be multiplied by the dynamic pressure \( (q) \), and the reference area \( (S) \). Also, any \( Q_{ij} \) terms which are related to moments must also be multiplied by a reference length \( (\bar{c} - \text{reference chord}) \).

Thus the terms in Eqs 65-73 are now dimensional quantities. Eqs 65 are the rigid-body results of Rodden and Giesing in dimensional form

**Pitch and Plunge Influence on "Rigid Forces"**

\[ Q_{ZZ} = (1 + \omega Z_a - \omega^2 Z_a^2) \left( \frac{h_0}{V} \right) \left( \frac{h_0}{h_0} \right) \exp(\omega t) \]  

(65a)

\[ Q_{MZ} = (1 + \omega M_a - \omega^2 M_a^2) \left( \frac{h_0}{V} \right) \left( \theta_0 \right) \exp(\omega t) \]  

(65b)

\[ Q_{ZM} = (Z_a + i \omega (Z_a + Z_q)) \left( \theta_0 \right) \left( \frac{h_0}{h_0} \right) \exp(\omega t) \]  

(65c)

\[ Q_{MM} = (M_a + i \omega (M_a + M_q)) \left( \theta_0 \right) \left( \theta_0 \right) \exp(\omega t) \]  

(65d)
Rodden and Giesing's method for rigid-body motion can easily be extended further to obtain the elastic and control surface influences on the rigid body forces and visa versa, along with the interactions among themselves. These are shown in Eqs 66-73.

**Elastic Influences on "Rigid Forces"**

\[
Q_{Z\xi_i} = (Z\xi_i + i\omega Z\xi_i - \omega^2 Z\xi_i)(\xi_i)(\theta_0)e^{i\omega t} \\
Q_{M\xi_i} = (M\xi_i + i\omega M\xi_i - \omega^2 M\xi_i)(\xi_i)(\theta_0)e^{i\omega t}
\]  
(66a)
(66b)

**Pitch and Plunge Influence on "Elastic Forces"**

\[
Q_{\xi_i Z} = (i\omega F_{\xi_i} \alpha - \omega^2 F_{\xi_i} \alpha)(\theta_0/V)(\xi_i) e^{i\omega t} \\
Q_{\xi_i M} = [F_{\xi_i} \alpha + i\omega (F_{\xi_i} \alpha + F_{\xi_i} q)](\theta_0)(\xi_i) e^{i\omega t}
\]  
(67a)
(67b)

**Elastic Influence on "Elastic Forces"**

\[
Q_{\xi_j \xi_i} = (F_{\xi_j} \xi_i + i\omega F_{\xi_j} \xi_i - \omega^2 F_{\xi_j} \xi_i)(\xi_i)(\xi_j) e^{i\omega t}
\]  
(68)

**Control Surface Influence on "Rigid Forces"**

\[
Q_{Z\delta_i} = (Z\delta_i + i\omega Z\delta_i - \omega^2 Z\delta_i)(\delta_i)(\theta_0)e^{i\omega t} \\
Q_{M\delta_i} = (M\delta_i + i\omega M\delta_i - \omega^2 M\delta_i)(\delta_i)(\theta_0)e^{i\omega t}
\]  
(69a)
(69b)

**Pitch and Plunge Influence on "Control Surface Forces"**

\[
Q_{\delta_i Z} = (i\omega F_{\delta_i} \alpha - \omega^2 F_{\delta_i} \alpha)(\theta_0/V)(\delta_i) e^{i\omega t} \\
Q_{\delta_i M} = [F_{\delta_i} \alpha + i\omega (F_{\delta_i} \alpha + F_{\delta_i} q)](\theta_0)(\delta_i) e^{i\omega t}
\]  
(70a)
(70b)
Control Surface Influence on "Elastic Forces"

\[ \frac{Q_j}{Q_j} = (F_{\xi_j} \delta_j + \omega F_{\xi_j} \delta_j - \omega^2 F_{\xi_j} \delta_j) (\delta_j) \left( \varepsilon_{\xi_j} \right) e^{i\omega t} \]  \hspace{1cm} (71)

Elastic Influence on "Control Surface Forces"

\[ \frac{Q_j}{Q_j} = (F_{\delta_j} \varepsilon_1 + \omega F_{\delta_j} \varepsilon_1 - \omega^2 F_{\delta_j} \varepsilon_1) (\varepsilon_1) (\delta_j) e^{i\omega t} \]  \hspace{1cm} (72)

Control Surface Influence on "Control Surface Forces"

\[ \frac{Q_j}{Q_j} = (F_{\delta_j} \delta_1 + \omega F_{\delta_j} \delta_1 - \omega^2 F_{\delta_j} \delta_1) (\delta_1) (\delta_j) e^{i\omega t} \]  \hspace{1cm} (73)

In FASTOP, to find the steady derivatives, \( k \) is set to zero, thus only the real terms of the above equations appear. The remaining derivatives are found at a small value of \( k \). The expansion was kept to second order which results in no increase in system order. If higher order dynamics are needed, they can be easily added by adding higher order stability derivatives. This will, of course increase the order of the system.

Resulting Equations

The stability derivatives from Eqs 65-73 can now be moved to the left hand side of Eq 55, thus filling the matrices with terms. The only terms left on the right hand side of Eq 55 are the generalized forces due to the torques applied to the control surface hinges from the control inputs, as shown in Eq 74.
\[ [\vec{X}] \ddot{\vec{X}} + [Y] \dot{\vec{X}} + [Z] \vec{X} = Q \] (74)

It is shown Kane (9:81-83) that the generalized forces, \( Q \), due to the control torques can be represented by

\[ Q_i = \sum_{j=1}^{m} \frac{\partial Q}{\partial \dot{\eta}_i} \cdot T_j \] (75)

where

\( T_j \) - torques applied to the control surface hinges

This can be represented by

\[ Q = \left[ \frac{\partial Q}{\partial \dot{\eta}_1} \right] [T_1 \ldots T_m]^T \] (76)

The torque can be related to the input signal to the surface by

\[ T_j = m_j \delta_j \] (77)

where

\( m_j \) - the generalized mass of the \( j \)th control surface

\( \delta_j \) - is the command signal to the \( j \)th control surface

Now letting \( \delta = [\delta_1 \ldots \delta_m] \) Eq 75 becomes

\[ Q = [E]u \] (78)

where
Thus Eq 74 becomes

\([X] \ddot{x} + [Y] \dot{x} + [Z] x = [E] y\) \hspace{1cm} (79)

If the matrix \([X]\) can be inverted, then Eq 79 can be rearranged into the typical state-space representation

\[
\begin{bmatrix}
\dot{x} \\
x
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-x^{-1}Z & -x^{-1}Y
\end{bmatrix}
\begin{bmatrix}
x \\
x
\end{bmatrix} +
\begin{bmatrix}
0 \\
x^{-1}E
\end{bmatrix} y
\] \hspace{1cm} (80)

Eq 80 is the desired equation in the form of Eq 1. Eq 80 and the procedure for obtaining the dimensional stability derivatives from the generalized forces (Eqs 65-73) were programmed into a computer procedure called MAC (Methodology for Aeroelasticity and Controls). Appendix B is the User's Manual, and Appendix C are the subroutines that comprise MAC.
III. YF-17 Model Development

In order to demonstrate the methodology developed in the previous section, a simple model that exhibited flutter characteristics was needed. A mathematical representation of the YF-17 wind tunnel flutter model was used. This model had the required structural properties needed to validate the methodology. Second, only the first two elastic modes were necessary to adequately predict flutter. This results in a simple representation of the model dynamics.

The model used is shown in Figure 2. It is represented by six modes; the rigid body pitch and plunge modes, two structural modes (first wing bending and first wing torsion modes), and two control surface modes (the trailing edge flap and an all moveable horizontal tail). This model will allow full testing of the influences desired. Table I contains the generalized masses and structural frequencies for the model. The model developed includes an elastic wing, connected by a rigid extension to a rigid tail. The mode shapes for the elastic modes were obtained from Noll’s work (16). The geometric and structural data were entered into FASTOP, a flutter prediction program (30). FASTOP was used to determine the generalized forces for reduced
### TABLE I

**YF-17 Flutter Model Structural Data**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Generalized Mass</th>
<th>Vibration Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.061</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.177</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>0.3115</td>
<td>4.628</td>
</tr>
<tr>
<td>4</td>
<td>0.1019</td>
<td>7.186</td>
</tr>
<tr>
<td>5</td>
<td>0.1027</td>
<td>50.0</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>60.0</td>
</tr>
</tbody>
</table>

![Figure 2. YF-17 Model Planform](image)
frequencies of 0.0 and 0.02. The flutter prediction capability of FASTOP, which uses the P-K method of flutter prediction, was also used to check the validity of this prediction method. The flutter prediction capability of ADAM (2:Vol 1, 24-25), which uses a 'velocity root locus' technique to predict flutter, was also used as a check.

Two sensors were chosen near the trailing edge flap, at 40.154 inches along the span, as shown in Figure 2. Each sensor measures translational motion at that wing location. The two outputs are then differenced and divided by the difference between the two sensors, to obtain the twist angle at that span station. The resulting output matrix, \([C]\) is then a combination of the two structural mode shapes at that span station. Appendix A contains the output matrix used. The sensor and actuator dynamics are not modelled in this development.
IV. Results and Discussion

The resulting equations (Eq 80) are linearized approximations to the actual aircraft dynamics. Where the aerodynamics are linear with respect to the reduced frequencies, this model should successfully capture the essential dynamics and thus predict the low frequency flutter, with the added benefit of predicting rigid body stability derivatives. The resulting state space model will also be of lower order than that obtained using aerodynamics modelled using the Padé approximant method, since it will not contain the often unnecessary higher order aerodynamics associated with curve-fitting the aerodynamics in the frequency domain.

In order to prove that this method is a useful way to model elastic aircraft dynamics for flight control use, three tasks were analyzed. First, the ability of this method to predict stability derivatives, and thus, the ability to predict rigid body dynamics, was investigated. Next, the methodology's ability to predict flutter and associated structural instabilities was investigated. Finally, the model was used to develop a workable flutter control law.
TABLE II

STABILITY DERIVATIVE COMPARISON FOR THE YF-17 MODEL

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Datcom</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{L\alpha}$</td>
<td>4.305</td>
<td>4.142</td>
</tr>
<tr>
<td>$C_{M\alpha}$</td>
<td>-0.6501</td>
<td>-0.5038</td>
</tr>
<tr>
<td>$C_{L\delta}$</td>
<td>2.623</td>
<td>2.276</td>
</tr>
<tr>
<td>$C_{M\delta}$</td>
<td>-4.699</td>
<td>-7.312</td>
</tr>
<tr>
<td>$C_{Lq}$</td>
<td>6.03</td>
<td>8.145</td>
</tr>
<tr>
<td>$C_{Mq}$</td>
<td>-6.799</td>
<td>-7.474</td>
</tr>
<tr>
<td>$C_{L\delta_e}$</td>
<td>1.054</td>
<td>1.153</td>
</tr>
<tr>
<td>$C_{M\delta_e}$</td>
<td>-1.886</td>
<td>-1.909</td>
</tr>
<tr>
<td>$C_{L\delta_f}$</td>
<td>0.1432</td>
<td>0.233</td>
</tr>
<tr>
<td>$C_{M\delta_f}$</td>
<td>-0.0751</td>
<td>-0.4366</td>
</tr>
</tbody>
</table>

**Stability Derivative Prediction**

The capability of MAC to predict rigid body stability derivatives was validated using the YF-17 model. The stability derivatives from MAC were compared to the analytical methods of Digital Datcom (28). As can be seen in Table II, MAC does an acceptable job in predicting most stability derivatives, with the exception of the flap derivatives. If there are better sources for stability derivatives, MAC has the capability to incorporate them into the analysis.
Flutter Prediction

The capability of MAC to predict flutter was also evaluated with the YF-17 model. The results from MAC were compared to two different flutter prediction methods. The first is FASTOP, which uses the P-K method of predicting flutter speeds and frequencies (30). The second is ADAM, which uses a 'velocity root locus' technique to predict flutter speeds and frequencies (2:Vol 1, 24-25). Figure 3 is a plot of ADAM's root locus. MAC uses a similar technique using Eq 80. Figure 4 shows the plot of the velocity root locus of MAC for various speeds. As can be seen by Figures 3 and 4, both methods are in very close agreement in predicting the dynamics of the structural modes. All three methods agree very well in predicting the flutter speed and frequency (Table III). Using FASTOP as a basis, it can be seen that both ADAM and MAC slightly underpredict the speed of flutter while slightly overpredicting the frequency. All the predictions are within 5% of one another.

In comparing the eigenvalues of MAC with ADAM (Table IV) it is seen that the higher order aerodynamic roots associated with the Pade fit method of ADAM do not seem to be important to the dynamics involved. This is similar to the result that Pasquini reported (17:31-32).
### TABLE III

**COMPARISON OF FLUTTER PREDICTION METHODS FOR THE YF-17 MODEL**

<table>
<thead>
<tr>
<th>Flutter Prediction Method</th>
<th>Flutter Speed (ft/sec)</th>
<th>Flutter Frequency (Hertz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FASTOP</td>
<td>387</td>
<td>6.10</td>
</tr>
<tr>
<td>ADAM</td>
<td>371</td>
<td>6.25</td>
</tr>
<tr>
<td>MAC</td>
<td>382</td>
<td>6.18</td>
</tr>
</tbody>
</table>

### TABLE IV

**EIGENVALUE COMPARISON OF MAC WITH ADAM FOR THE 458 FT/SEC CASE**

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>MAC</th>
<th>ADAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid Body</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.000</td>
<td>1.838+3.3141</td>
<td>1.838-3.3141</td>
</tr>
<tr>
<td>0.000</td>
<td>-4.215+6.0841</td>
<td>-4.215-6.0841</td>
</tr>
<tr>
<td>-2.412+5.5861</td>
<td>-5.901+34.261</td>
<td>-5.901-34.261</td>
</tr>
<tr>
<td>Elastic</td>
<td>-0.518+314.41</td>
<td>-</td>
</tr>
<tr>
<td>-0.518-314.41</td>
<td>3.485+36.6661</td>
<td>3.485-36.6661</td>
</tr>
<tr>
<td>-12.93+373.31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-12.93-373.31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Control Surface</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-93.92+184.71</td>
<td>-93.92-184.71</td>
<td>-93.92+184.71</td>
</tr>
<tr>
<td>-94.01+184.01</td>
<td>-94.01-184.01</td>
<td>-94.01-184.01</td>
</tr>
<tr>
<td>-94.34+184.21</td>
<td>-94.34-184.21</td>
<td>-94.34-184.21</td>
</tr>
<tr>
<td>-94.23+184.31</td>
<td>-94.23-184.21</td>
<td>-94.23-184.21</td>
</tr>
<tr>
<td>Aerodynamic</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Real Component (1/sec)

Figure 3. Velocity Root Locus for the Unaugmented YF-17 Model Using ADAM

Real Component (1/sec)

Figure 4. Velocity Root Locus for the Unaugmented YF-17 Model Using MAC
Control Design

From the previous two sections it can be concluded that Eq 80 accurately captures the essential dynamics of the YF-17 model. The model from MAC was then used to develop a flutter control system, using both classical and modern control techniques. The benefit of using this method of developing the aircraft equations of motion is that the state-space model (Eq 80) is formed from the second order form of the equations of motion (Eq 79). The state vector then, is comprised of the generalized coordinates and their rates. In contrast, if the Pade approximation method is used, the state-space model must be formed from the transfer function equation (Eq 60). Typically, this introduces additional states, and yields states which are not the generalized coordinates and their rates.

Both control designs were based on the YF-17 model at 1.2 times the flutter speed (458 ft/sec). All the eigenvalues of this condition are in Table V.

Classical Design. Transfer functions were determined from the state-space model for the above condition using MATRIXx (10). The transfer function from the sensor to the flap \((y/\delta_f)\) was then used to develop a feedback control
TABLE V

EIGENVALUES OF THE UNAUGMENTED YF-17 AT 458 FT/SEC

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>-2.412+5.5861</td>
<td>0.889</td>
</tr>
<tr>
<td>-2.412-5.5861</td>
<td>5.464</td>
</tr>
<tr>
<td>-5.890+34.331</td>
<td>5.836</td>
</tr>
<tr>
<td>-5.890-34.331</td>
<td></td>
</tr>
<tr>
<td>3.371+36.671</td>
<td>50.04</td>
</tr>
<tr>
<td>3.371-36.671</td>
<td>59.42</td>
</tr>
<tr>
<td>-0.518+314.41</td>
<td></td>
</tr>
<tr>
<td>-0.518-314.41</td>
<td></td>
</tr>
<tr>
<td>-12.93+373.31</td>
<td></td>
</tr>
<tr>
<td>-12.93-373.31</td>
<td></td>
</tr>
</tbody>
</table>

law. Using root locus techniques in TOTAL (27), the gain was varied until the unstable structural mode became stable while keeping the remaining structural mode and the short period roots stable. The gain chosen was -50. Figure 5 is the root locus for the $\gamma/\delta_e$ transfer function, with twist position feedback. As can be seen, the unstable structural mode is made stable, however, the very lightly damped control surface mode goes slightly unstable. This is not of much concern since the control surface actually has damping due to frictional forces, which are not accounted for in this model. The resulting roots of this condition with the control system are in Table VI.

This simple control system was then tested at off design conditions. As can be seen from the 'velocity root
Figure 5. Root Locus of $\gamma/\theta_f$ of the YF-17 Model at 458 ft/sec

x-poles
o-zeros
$\Delta$-root locus values for a gain of -50
TABLE VI

EIGENVALUES OF THE YF-17 MODEL AT 458 FT/SEC

USING TWIST POSITION FEEDBACK

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.251</td>
</tr>
<tr>
<td>-2.826+1.5791</td>
<td>5.556</td>
</tr>
<tr>
<td>-2.826-1.5791</td>
<td>5.959</td>
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<tr>
<td>-3.003+35.541</td>
<td>49.67</td>
</tr>
<tr>
<td>-3.003-35.541</td>
<td>59.42</td>
</tr>
</tbody>
</table>

locus' in Figure 6, the control system keeps the structural modes and the short period mode stable for all conditions above 250 ft/sec up to the design condition, thus yielding an acceptable control system, if it is implemented after the model reaches 250 ft/sec.

Modern Design. Again using the 1.2 flutter speed condition as the design point, a state-space feedback control law was developed. Eigenvalue assignment was used to assign the eigenvalues as shown in Table VII. The resulting feedback matrix was found using MATRIXx (10).
Figure 5. Velocity Root Locus for the YF-17 Model with Twist Position Feedback

TABLE VII

EIGENVALUES OF THE YF-17 MODEL AT 458 FT/SEC USING STATE-SPACE FEEDBACK

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.890</td>
</tr>
<tr>
<td>-2.412+5.5941</td>
<td>5.464</td>
</tr>
<tr>
<td>-2.412-5.5941</td>
<td>5.836</td>
</tr>
<tr>
<td>-5.889+34.331</td>
<td>50.04</td>
</tr>
<tr>
<td>-5.889-34.331</td>
<td>59.42</td>
</tr>
<tr>
<td>-3.375+36.671</td>
<td>59.42</td>
</tr>
<tr>
<td>-3.375-36.671</td>
<td>50.04</td>
</tr>
<tr>
<td>-10.00+314.41</td>
<td>50.04</td>
</tr>
<tr>
<td>-10.00-314.41</td>
<td>59.42</td>
</tr>
<tr>
<td>-12.93+373.31</td>
<td></td>
</tr>
<tr>
<td>-12.93-373.31</td>
<td></td>
</tr>
</tbody>
</table>
The feedback matrix was then used at the off design conditions. This state-space feedback system kept the structural and short period modes stable for all the conditions above 100 ft/sec, up to the design conditions, and even beyond (Figure 7).

Quite often the states are not readily measurable. However, the modelling technique as outlined in this effort can be used in the design of an estimator or observer. This model can be usefully employed in a modern control design.

![Velocity Root Locus for the YF-17 Model Using State-space Feedback](image)

**Figure 7.** Velocity Root Locus for the YF-17 Model Using State-space Feedback
V. Conclusions and Recommendations

The equations of motion of an elastic aircraft, have been developed in this thesis, including the effects of control surface dynamics. As was shown, the model developed is useful for stability and control analysis, including the effects of structural dynamics. The model can be used as a basis for either classical or modern control methods. This model also has merit as a flutter prediction method, at least where the aerodynamics are well behaved, and can be linearly approximated at low reduced frequencies.

There are several advantages to using this method. First, stability derivatives can be predicted, but if a better source of stability derivatives is known, they can be directly incorporated into the model. A second advantage is that the states are the normal aircraft states and rates, plus those associated with the structural generalized coordinates and rates, and those associated with the control surface rotations and rates. This makes this method directly applicable to classical and modern control design procedures. A third advantage is that the second order approximation used in this thesis results in a lower order model than the methodology used in ADAM. Even if higher order dynamics are needed, they can be easily added to the model as pointed out by Rodden (20). Finally, this method
can allow the prediction of control surface instabilities, and unlike ADAM can predict rigid body motion.

This project is a useful design tool and it should continue to be expanded. The use of other dynamic models should be used to further validate this method, including models in which the rigid modes couple with the structural modes. Second, the ability to extend this model to beyond second order, and to extend it to lateral-directional motion. Finally, the ability to account for sensor and actuator dynamics, along with control system dynamics needs to be addressed, to provide a better dynamic model.
A. **YF-17 State-Space Model at 1.2 $V_f$**

This appendix contains the A, B, and C matrices for the YF-17 model at 20 percent above the flutter speed (458 ft/sec). The states of the system are in Table A1. The A matrix is contained in Table A2. The B matrix is shown in Table A3, while the C matrix is in Table A4.

**TABLE A1**

THE STATES OF THE YF-17 STATE-SPACE MODEL

<table>
<thead>
<tr>
<th>$Z$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_1$ - Structural mode</td>
<td>$\xi_2$ - Structural mode</td>
</tr>
<tr>
<td>$\delta_1$ - Control surface mode</td>
<td>$\delta_2$ - Control surface mode</td>
</tr>
<tr>
<td>$dZ/dt$</td>
<td>$d\theta/dt$</td>
</tr>
<tr>
<td>$d\xi_1/dt$</td>
<td>$d\xi_2/dt$</td>
</tr>
<tr>
<td>$d\delta_1/dt$</td>
<td>$d\delta_2/dt$</td>
</tr>
</tbody>
</table>
## Table A2

**The A Matrix of the YF-17 Model at 458 ft/sec**

<table>
<thead>
<tr>
<th>ROW 1</th>
<th>0.0000E+00</th>
<th>0.0000E+00</th>
<th>0.0000E+00</th>
<th>0.0000E+00</th>
<th>0.0000E+00</th>
<th>0.0000E+00</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW 2</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 3</td>
<td>-0.1972E+01</td>
<td>0.4510E+03</td>
<td>0.3167E+03</td>
<td>0.1976E+02</td>
<td>0.2958E+00</td>
<td>0.2355E+00</td>
</tr>
<tr>
<td>ROW 4</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 5</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 6</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 7</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 8</td>
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<td>0.0000E+00</td>
<td>0.0000E+00</td>
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<td>0.0000E+00</td>
<td>0.0000E+00</td>
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<tr>
<td>ROW 9</td>
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<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 10</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 11</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
<tr>
<td>ROW 12</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
<td>0.0000E+00</td>
</tr>
</tbody>
</table>
### TABLE A3

**THE B MATRIX OF THE YF-17 MODEL AT 458 FT/SEC**

\[
\begin{bmatrix}
0.0000E+00 & 0.0000E+00 \\
0.0000E+00 & 0.0000E+00 \\
0.0000E+00 & 0.0000E+00 \\
0.0000E+00 & 0.0000E+00 \\
0.0000E+00 & 0.0000E+00 \\
-0.2679E+03 & -0.9205E+03 \\
0.1689E+03 & 0.3064E+03 \\
-0.1233E+04 & -0.4638E+04 \\
-0.2295E+04 & -0.6524E+04 \\
0.9885E+05 & -0.3273E+02 \\
0.1011E+03 & 0.1396E+06
\end{bmatrix}
\]

### TABLE A4

**THE C MATRIX OF THE YF-17 MODEL AT 458 FT/SEC USING TWIST POSITION SENSOR**

\[
\begin{bmatrix}
0 & 0 & 0 & 0.008089 & -0.01584 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
B. **MAC User's Manual**

The Methodology for Aeroelasticity and Controls (MAC) program was developed from the theory presented in the main body of this thesis. MAC integrates the results of unsteady aerodynamics codes, structural dynamics, and controls into a useful tool for both structural dynamics and flight control engineers. MAC was developed under AFIT thesis number AFIT/GAE/AA/86J-02 for the Flight Dynamics Laboratory. The program resides on the Flight Dynamics Laboratory Vax 11/785 computer with a VMS operating system. MAC accesses the IMSL library, the PLOT10 plotting library and the DI3000 plotting library. The example input data in this appendix is the YF-17 model data used in this thesis.

The majority of input to MAC can either be input from the terminal or automatically input from files. There are three exceptions to this. The aerodynamic forces and the print input data must be input from files, while the feedback matrices must be input from the terminal. The following logical file units are assigned to the following data files.
<table>
<thead>
<tr>
<th>Logical Unit</th>
<th>Data File Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Structural Data</td>
</tr>
<tr>
<td>2</td>
<td>Aerodynamic Data</td>
</tr>
<tr>
<td>3</td>
<td>Initial Conditions</td>
</tr>
<tr>
<td>4</td>
<td>Output File</td>
</tr>
<tr>
<td>5</td>
<td>Terminal Input</td>
</tr>
<tr>
<td>6</td>
<td>Terminal Output</td>
</tr>
<tr>
<td>7</td>
<td>Root Locus Input</td>
</tr>
<tr>
<td>9</td>
<td>Output Files for Use in MATRIXx</td>
</tr>
<tr>
<td>10</td>
<td>Print Data</td>
</tr>
<tr>
<td>11</td>
<td>Forcing Function</td>
</tr>
<tr>
<td></td>
<td>Matrix</td>
</tr>
<tr>
<td>12</td>
<td>C Matrix</td>
</tr>
<tr>
<td>90</td>
<td>Data for Plotting</td>
</tr>
<tr>
<td></td>
<td>Root Locus</td>
</tr>
<tr>
<td>98</td>
<td>Scratch File</td>
</tr>
<tr>
<td>99</td>
<td>Scratch File</td>
</tr>
</tbody>
</table>

The first thing MAC does when started is to read the system print parameters from a file. Then the user will be prompted by the following menu.
MAC ready for input

Enter (#):

1. Automatic input of items (2)-(7)
2. Read in structural data
3. Read in aerodynamic data
4. Read in initial conditions
5. Read in root locus values
6. Read in forcing function matrix
7. Read in the C matrix
8. Completion of input data

If the user chooses option 1, MAC will automatically read in all the data from items 2-7 from files. If the user wishes to input the data separately, the options 2-8 must be accomplished in order. Once the data is input, the remainder of the program will prompt the user for input.

The terminal input is in free field format. The remaining portion of this manual describes the format of the input data on files.

Print Data. The system constants for the determination of what is to be output is in this file. The file is in namelist format, as shown in Table B2. Below is an explanation of each variable and it's default value.

NAERO(0) - set to 1 to print the aerodynamic matrices onto unit 4, the output file

NMAT(0) - set to 1 to print the X, Y, and Z matrices onto unit 4
NINV(0) - Set to 1 to print the $X^{-1}$ matrix onto unit 4

NAMAT(0) - set to 1 if the A and B matrices are to be set to files that are compatible with MATRIXx

NEIG(1) - set to 0 if the eigenvalues are not to be sent to the plot file

NSTAB(0) - set to 1 if the rigid body non-dimensional stability derivatives are to be modified

### Table B2

**PRINT DATA EXAMPLE**

$PRNT NAERO=0, NMAT=1, NINV=0, NAMAT=1, NEIG=1, NSTAB=0$ $END$

**Structural Data.** The structural data is input the diagonal elements of mass matrix, and the associated frequency (in Hz) of vibration. Table B3 contains an example. The data is input in the following format.

**Line 1** - N=total # of modes (limited to 20), NRB=# of rigid modes (limited to 2), NS=# of structural modes (limited to 13), NC=# of control surface modes (limited to 5)
Format (4I5)

**Line 2** - Generalized masses in the order of rigid modes, structural modes, and control surface modes.
Line 2 is repeated as often as necessary to account for all the modes
Format (4E15.7)

**Line 3** - Frequencies of the modes of vibration in the same order as above. Line 3 is also repeated as necessary
Format (4E15.7)
TABLE B3

EXAMPLE STRUCTURAL DATA

6  2  2  2
0.7061300E+01  0.7776900E+02  0.3115000E+00  0.1019000E+00
0.1027000E+00  0.1000000E+00
0.0000000E+00  0.0000000E+00  0.4628000E+01  0.7186000E+01
0.5000000E+02  0.6000000E+00

Aerodynamic Data. The generalized aerodynamic forces (Q_{ij}s), from unsteady aerodynamic codes is input as complex pairs with j increasing before i. Table B4 is an example of this data for the YF-17. The format of the variables is shown below Table B4. This is all the data that MAC will use from this file. It will ignore any other aerodynamic data. K1 should be 0.0, and K2 should be a small value of reduced frequency.

Initial Condition Data. This data includes the density speed of sound, reference areas and lengths, and the mode shape magnitudes. Table B5 is an example of this data for the YF-17. The variable format is explained below.

Line 1 - RHOO=density used in slugs/ft^3, A0=speed of sound at that density altitude in ft/sec, SREF=reference wing area in ft^2, CBAR=reference chord (Mean Aerodynamic Chord) in ft, BR=reference semi-chord in ft Format (F10.7,F10.2,3F10.5)

Line 2 - ALPHA0 and HO, the magnitudes of the pitch and
<table>
<thead>
<tr>
<th>YF-17 MODEL &amp; MODES</th>
<th>EXAMPLE AERODYNAMIC DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.600000E+00</td>
<td>0.600000E+00</td>
</tr>
<tr>
<td>0.992680E+02</td>
<td>0.8800120E+03</td>
</tr>
<tr>
<td>0.793070E+03</td>
<td>0.6905413E+03</td>
</tr>
<tr>
<td>0.143720E+01</td>
<td>0.614278E+03</td>
</tr>
<tr>
<td>0.716600E+02</td>
<td>0.3244382E+01</td>
</tr>
<tr>
<td>0.227110E+02</td>
<td>0.769980E+02</td>
</tr>
<tr>
<td>0.167600E+02</td>
<td>0.786688E-09</td>
</tr>
<tr>
<td>0.167601E+02</td>
<td>0.3337670E+01</td>
</tr>
<tr>
<td>0.460642E+02</td>
<td>0.3106519E+01</td>
</tr>
<tr>
<td>0.259841E+01</td>
<td>0.658868E+02</td>
</tr>
<tr>
<td>0.780980E+02</td>
<td>0.314400E+01</td>
</tr>
<tr>
<td>0.143847E+01</td>
<td>0.992680E+03</td>
</tr>
<tr>
<td>0.145692E+02</td>
<td>0.333800E+01</td>
</tr>
<tr>
<td>0.209675E+02</td>
<td>0.513978E-09</td>
</tr>
</tbody>
</table>

Line 1 - TITLE=title of the aerodynamics, Format(80A)
Line 2 N=number of modes, XMACh=mach number of the aerodynamics, Format (I5,5X,E15.7)
Block 1
Line 1 - KL=first reduced frequency (0), Format (E15.7)
Line 2 - GF1(I,J)=first generalized aerodynamic matrix, input in complex pairs, with J coming before I. Line 2 is repeated as often as necessary.
Format (4(2E15.7))
Block 2
Line 1 - K2=second reduced frequency (small), Format (E15.7)
Line 2 - GF2(I,J)=second generalized aerodynamic matrix, input in complex pairs, with J coming before I. Line 2 is repeated as often as necessary.
Format (4(2E15.7))
plunge mode shapes
Format (2F10.5)

Line 3 - ZETA(I): the magnitudes of the structural mode shapes. This line is repeated as often as necessary to account for all the elastic modes
Format (6F10.5)

Line 4 - DELTA(I): the magnitudes of the control surface mode shapes
Format (6F10.5)

TABLE B5
EXAMPLE INITIAL CONDITION DATA

 Root Locus Data. The root locus data contains the velocities that are to be used to compute the eigenvalues for plotting. The maximum number is 20. Table B6 shows an example.

Line 1 - NVEL: number of velocities to be used
Format (15)

Line 2 - V(I): each velocity in ft/sec, repeated NVEL times
Format (F10.3)
TABLE B6
EXAMPLE ROOT LOCUS DATA

15
  0.000
100.000
200.000
250.000
300.000
350.000
381.000
400.000
420.000
440.000
458.000
470.000
480.000
490.000
500.000

Forcing Matrix. The forcing matrix is used to create the B matrix. It is defined in the main text of the thesis. Table B7 is the example used for the YF-17.

Line 1 - HINP=#of inputs the system Format (I5)

Line 2 - QMAT(I,J)=the forcing matrix (NxNINP) input row first, j increases before i. Line 2 is repeated as often as necessary. Format (6E15.7)
TABLE B7

EXAMPLE FORCING MATRIX

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>-0.000000E-02</td>
<td>-0.158300E-01</td>
<td>0.000000E+00</td>
</tr>
<tr>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td></td>
</tr>
</tbody>
</table>

Measurement Matrix. The C matrix is input from this file. Table B8 is the example used for the YF-17.

Line 1 - NOUT= # of outputs into the system
Format (15)

Line 2 - CMAT(I, J)= the measurement matrix (NOUT x N)
input row first, j increases before i. Line 2 is repeated as often as necessary.
Format (6E15.7)

TABLE B8

EXAMPLE MEASUREMENT MATRIX

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.100000E-01</td>
<td>0.100000E-01</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
</tr>
<tr>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.100000E-01</td>
<td>0.000000E+00</td>
<td>0.000000E+00</td>
<td>0.100000E-01</td>
<td></td>
</tr>
</tbody>
</table>

The remaining inputs are from the terminal, as requested by the program prompts.
C. **MAC Program**

This Appendix contains the program listings of all the programs that comprise the MAC program. MAC uses the IMSL subroutine library for matrix inversion (LINV1F), matrix multiplication (VMULFF), and eigenvalue computation (EIGRF). MAC also uses the PLOT10 plot routine library in the velocity root locus plotting program to clear the screen before plotting. Finally, MAC uses the DI3000 plot routine library to create both screen plots and the hard copy plots. The main calling program (MAC) is listed first. The remaining subroutine libraries are in alphabetical order. All the routines are listed below as they appear in this appendix.

<table>
<thead>
<tr>
<th>TABLE C1</th>
<th>MAC PROGRAM LISTING ORDER</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAC</td>
<td>INPUT</td>
</tr>
<tr>
<td>AMAT</td>
<td>MATSAV</td>
</tr>
<tr>
<td>AUGMENT</td>
<td>MINV</td>
</tr>
<tr>
<td>BMAT</td>
<td>MMULT</td>
</tr>
<tr>
<td>EVAL</td>
<td>RL PLOT</td>
</tr>
<tr>
<td>FASTCHG</td>
<td>STABDER</td>
</tr>
</tbody>
</table>

The subroutines are fairly well commented so that a person familiar with the theory developed in the main text and with FORTRAN should be able to interpret the program.
PROGRAM MAC

This program was developed under AFIT Thesis AFIT/GAE/AA/86J-2 by John J. Cerra II.

This program uses generalized mass and stiffness matrices, along with generalized forces from an external source (FASTOP) and combines them into an appropriate state space formulation for use by any control analysis and design package. The state space model is the minimum order attainable, 2N, where N is the number of modes used (including rigid body and control surface modes).

Subroutine INPUT reads in the generalized mass and stiffness matrices. It also reads in the generalized forces from FASTOP at two reduced frequencies (usually k=0 and a small k). The initial conditions are also input here, along with the root locus data. Finally it also inputs the forcing function matrix, Q, and the C matrix if needed.

Subroutine STABDER computes stability derivatives from the FASTOP unsteady aerodynamics. There is also an option to modify the derivatives if derivatives from a better source are known.

Subroutine AMAT uses the above data to compute the state space A matrix used for stability analysis. The order of A is 2N. A root locus can be performed as in a typical flutter solution.

Subroutine BMAT computes the B matrix from the forcing function matrix Q. B is a function of dynamic pressure.

Subroutine MINV, called by AMAT, computes the inverse of the X matrix using an IMSL routine LINVIF.

Subroutine MMULT, called by AMAT, and BMAT multiplies matrices. This is used to form the A and B matrices.

Subroutine EVAL computes the eigenvalues of the A matrix. The eigenvalues for various velocities can be accomplished and then can be plotted on the real and imaginary plane to produce a velocity root locus.

Subroutine AUGMENT augments the A matrix with the appropriate feedback, to form the augmented (closed loop) A matrix.

Subroutine FASTCHG creates non-dimensional coefficients out of the unsteady aerodynamics of FASTOP.

Subroutine RLPILOT plots the eigenvalues of the A matrix with respect to velocity on the complex plane, creating a 'velocity root locus'.
Subroutine MATSAV saves matrices to a file in a compatible form for MATRIXx

CHARACTER*1 QAUG, QLPT
CHARACTER*80 TITLE
INTEGER NVEL
REAL VE, V(20)
COMMON/RL/NVEL, V

Open output file
OPEN(4, STATUS='NEW')
CALL INPUT
OPEN(98, STATUS='SCRATCH')
WRITE(98, *) NVEL

Velocity root locus loop for the unaugmented aircraft
IF (NVEL.GT.1) THEN
   DO 1 I = 1, NVEL
      CALL STABDER(NVEL, V(I))
      CALL AMAT
      CALL BMAT(V(I))
      CALL EVAL
   1 CONTINUE
ELSE
   NVEL = 0
   VE = 0.001
   CALL STABDER(NVEL, VE)
   CALL AMAT
   CALL BMAT(VE)
   CALL EVAL
ENDIF

Option to plot the open loop aircraft roots
WRITE(*, '(A)') ' Do you want to plot the open loop roots?'
READ(*, 100) QPLT
IF (QPLT.EQ. 'Y') THEN
   WRITE(*, '(A)') ' What title do you want for the plot?'
   READ(*, 100) TITLE
   CALL RLPLT(0.0, 8.0, -6.0, 6.0, TITLE)
   CLOSE(90)
ENDIF
WRITE(*, '(A)') ' Do you want to augment the A matrix?'
READ(*, 100) QAUG
IF (QAUG.EQ. 'Y') THEN
Call to closed loop augmentation program
CALL AUGMENT
ENDIF
IDAT=99
CLOSE(1)
CLOSE(2)
CLOSE(3)
CLOSE(4)
CLOSE(7)
CLOSE(90)
CLOSE(9)
CLOSE(10)
CLOSE(11)
CLOSE(12)
CLOSE(98)
WRITE(*','(A)') ' This program has ended.'
100 FORMAT(A)
END
SUBROUTINE AMAT

This subroutine uses the stability derivatives above for use in developing the open loop reduced order A matrix for stability analysis, and control work. It is of much smaller order than that resulting from using Padé fit aerodynamics.

CHARACTER*1 QASAV
CHARACTER*20 ANAM

INTEGER N, NRB, NS, NC, NV
COMMON/INDEX/N, NRB, NS, NC
INTEGER NAERO, NINV, NMAAT, NEIG
COMMON/PRTR/NAERO, NINV, NMAAT, NEIG
REAL ZALPHA, MALPHA, ZALPHDT, MALPHDT, ZQ, MQ,
QFZALPHA, FZALPDTN, FZETQ, ZZETA, MZETA,
QZETADT, MZETADT, ZZETDDT, MZETDDT,
QFZETZT, FZETZDT, FZEZDDT,
QFZETDEL, FZETDDT, FZEZDDT,
QFDELZET, FDELZDT, FDEZDDT,
QZDELTA, MDELTA, FDALPHA, FDALPDT, FDETQ,
QZDELDT, MDELDT, ZDELDT, MDELDT,
QFDELDL, FDELDL, FDEDDDT, VEL

DIMENSION FZALPHA(15), FZALPDT(15), FZETQ(15), ZZETA(15), MZETA(15),
QZETADT(15), MZETADT(15), ZZETDDT(15), MZETDDT(15),
QFZETZT(15, 15), FZETZDT(15, 15), FZEZDDT(15, 15),
QFZETDEL(15, 5), FZETDDT(15, 5), FZEZDDT(15, 5),
QFDELZET(5, 15), FDELZDT(5, 15), FDEZDDT(5, 15),
QZDELTA(5), MDELTA(5), FDALPHA(5), FDALPDT(5), FDETQ(5),
QZDELDT(5), MDELDT(5), ZDELDT(5), MDELDT(5),
QFDELDL(5, 5), FDELDL(5, 5), FDEDDDT(5, 5)
COMMON/DERIV/ZALPHA, MALPHA, ZALPHDT, MALPHDT, ZQ, MQ, FZALPHA, FZALPDT,
QFZETQ, ZZETA, MZETA, ZZETADT, MZETADT, ZZETDDT, MZETDDT, FZETZT,
QFZETZT, FZETZDT, FZETDEL, FZETDDT, FZEZDDT, FDELZET, FDELZDT, FDEZDDT,
QZDELTA, MDELTA, FDALPHA, FDALPDT, FDETQ,
QZDELDT, MDELDT, ZDELDT, MDELDT, FDELDL, FDELDL, FDEDDDT, VEL

REAL GM, GK
DIMENSION GM(20), GK(20)
COMMON/STRUCT/GM, GK
REAL K1, K2
COMPLEX GF1, GF2
DIMENSION GF1(20, 20), GF2(20, 20)
COMMON/AERO/ K1, K2, GF1, GF2, PI
REAL X, Y, Z, AMATRIX, XINV, XINVY, XINVZ
DIMENSION X(20, 20), Y(20, 20), Z(20, 20), AMATRIX(40, 40), XINV(40),
QXINVY(20, 20), XINVZ(20, 20)
COMMON/INV/X, XINV
COMMON/MMUL/Y, Z, XINVY, XINVZ
COMMON/EIG/AMATRIX
REAL XMOM(5)
COMMON/MOMENT/XMOM
COMPLEX E(20)
DIMENSION WKX(20)
REAL*8 A(40,40),DUMMY
N2=N*2

C Initialization of the matrices of: Xx+Yx+Zx=0.
C
DO 30 I=1,N
   DO 30 J=1,N
      X(I,J)=0.0
      Y(I,J)=0.0
      Z(I,J)=0.0
30 CONTINUE

C Input of the structural information into the matrices.
C
DO 1 I=1,N
   X(I,I)=GM(I)
   Z(I,I)=GK(I)
1 CONTINUE

C Input of the aerodynamics into the matrices.
C
IF(NRB.EQ.0) GOTO 24

C Rigid aero
C
   X(1,1)=X(1,1)-ZALPHDT/VEL
   X(2,1)=-MALPHDT/VEL
   Y(1,1)=-ZALPHA/VEL
   Y(2,1)=-MALPHA/VEL
   Y(1,2)=-ZQ-GM(1)*VEL
   Y(2,2)=-MQ
24 CONTINUE

IF (NS.EQ.0) GOTO 21

C Elastic/Rigid and Rigid/Elastic aero
C
DO 2 I=1,NS
   J=I+NRB
   IF (NRB.EQ.0) GOTO 20
   X(I,J)=-FZALPDT(I)/VEL
   X(1,J)=-ZZETDDT(I)
   X(2,J)=-MZETDDT(I)
   Y(J,1)=FZALPHA(I)/VEL
   Y(J,2)=FZETQ(I)
   Y(1,J)=ZZETADT(I)
   Y(2,J)=MZETADT(I)
   Z(1,J)=ZZETA(I)
   Z(2,J)=MZETA(I)
20 CONTINUE
IF (NS.EQ.0) GOTO 4

C Elastic/Elastic aero
C
DO 4 K=1,NS
   L=K+NRB
   X(J,L)=-FZEZDDT(I,K)+X(J,L)
   Y(J,L)=-FZETZDT(I,K)
   Z(J,L)=-FZETZET(I,K)+Z(J,L)
4 CONTINUE

IF (NC.EQ.0) GOTO 21

C Elastic/Control and Control/Elastic aero
C
DO 5 K=1,NC
   L=K+NS+2
   X(L,J)=-FZEZDDT(K,I)
   X(J,L)=-FZEDDDT(I,K)
   Y(L,J)=-FDELZDT(K,I)
   Y(J,L)=-FZETDDT(I,K)
   Z(L,J)=-FDELZET(K,I)
   Z(J,L)=-FZETDEL(I,K)
5 CONTINUE
21 CONTINUE

C Control/Rigid, Rigid/Control aero
C
DO 3 I=1,NC
   J=I+NRB+NS
   X(J,1)=-FDALPDT(I)/VEL
   X(1,J)=-ZDELDDT(I)
   X(2,J)=-MDELDDT(I)
   Y(J,1)=-FDALPHA(I)/VEL
   Y(1,J)=-ZDELTDT(I)
   Y(2,J)=-MDELTDT(I)
   Z(1,J)=-ZDELTA(I)
   Z(2,J)=-MDELTA(I)
23 CONTINUE

C Control/Control aero
C
DO 6 K=1,NC
   L=K+NS+NRB
   X(J,L)=-FDEDDDT(I,K)+X(J,L)
   Y(J,L)=-FDELDDT(I,K)
   Z(J,L)=-FDELDEL(I,K)+Z(J,L)

6 CONTINUE
DO 32 J=1,NC
   L=J+NS+NRB
   XMOM(J)=Z(L,L)
32 CONTINUE

CONTINUE

IF (NMAT.EQ.0) GOTO 14
WRITE(4,'(/A)') ' The X matrix is'
DO 10 I=1,N
   WRITE(4,300) I
   DO 11 J=1,N,6
      WRITE(4,301) (X(I,K),K=J,J+5)
11 CONTINUE
10 CONTINUE
WRITE(4,'(/A)') ' The Y matrix is'
DO 12 I=1,N
   WRITE(4,300) I
   DO 13 J=1,N,6
      WRITE(4,301) (Y(I,K),K=J,J+5)
13 CONTINUE
12 CONTINUE
WRITE(4,'(/A)') ' The Z matrix is'
DO 14 I=1,N
   WRITE(4,300) I
   DO 15 J=1,N,6
      WRITE(4,301) (Z(I,K),K=J,J+5)
15 CONTINUE
14 CONTINUE

C Initialization of the A matrix.

DO 31 I=1,N2
   DO 31 J=1,N2
      AMATRIX(I,J)=0.0
31 CONTINUE

C Call to invert X and to multiply XINV*Y and XINV*Z

CALL MINV
CALL MMULT
DO 8 I=1,N
   DO 8 J=1,N
      K=I+N
      L=J+N
      AMATRIX(K,L)=-XINVY(I,J)
      AMATRIX(I,K)=1.0
      AMATRIX(K,J)=-XINVZ(I,J)
8 CONTINUE

IF (NAMAT.EQ.1) THEN
   OPEN(99,STATUS='SCRATCH')
NV=NINT(VEL)
WRITE(99,310) NV
310 FORMAT('[.MATRX]A',I4.4)
RESE 99
READ(99,302) ANAM
CLOSE(99)
DO 40 I=1,N2
DO 40 J=1,N2
AIJ=AMATRIX(I,J)
A(I,J)=DBLE(AIJ)
40 CONTINUE
OPEN (9,FILE=ANAM,STATUS='NEW')
C
C Call to the routine to put the A matrix in MATRIXx format
C
CALL MATSAV(9,ANAM,40,N2,N2,0,A,DUMMY,'(1P8E1S.7)')
CLOSE(9)
ENDIF
WRITE(4,'(/A)') ' The A matrix of dx/dt=A*x is'
DO 16 I=1,N2
WRITE(4,300) I
DO 17 J=1,N2,6
WRITE(4,301) (AMATRIX(I,K) ,K=J,J+5)
17 CONTINUE
16 CONTINUE
WRITE(98,*) ((AMATRIX(I,J),J=1,N2),I=1,N2)
IF (NRB.NE.0) WRITE(4,304)
IF (NS.NE.0) WRITE(4,305) (I,I=1,NS)
IF (NC.NE.0) WRITE(4,306) (I,I=1,NC)
IF (NRB.NE.0) WRITE(4,307)
IF (NS.NE.0) WRITE(4,308) (I,I=1,NS)
IF (NC.NE.0) WRITE(4,309) (I,I=1,NC)
300 FORMAT(9,9,9)
301 FORMAT(1X,6E11.4)
302 FORMAT(A)
304 FORMAT(/, ' The states of the A matrix are:',/, 

Q' Z',/, 
Q' THETA')
305 FORMAT( ' ZETA(',I2,')-Structural mode')
306 FORMAT(' DELTA(',I2,')-Control surface mode')
307 FORMAT(' dZ/dt',/, ' dTHETA/dt')
308 FORMAT(' dZETA(',I2,')/dt')
309 FORMAT(' dDELTA(',I2,')/dt')
303 FORMAT(6E15.7)
RETURN
END
SUBROUTINE AUGMENT

This subroutine will augment the state space equations with the appropriate feedback (output or state) to form the closed loop state equations.

REAL AUG(40,40),ACL(40,40),TEMP(40,40),CMAT(40,40),
OBMATRIX(40,40),FMAT(5,40),V(20)
INTEGER N,N2,NRB,NS,NC,IFDBK,NP,NOUT,IER,NA,NB
CHARACTER*1 QPLT,QCHG
CHARACTER*80 TITLE
COMMON/EIG/ACL
COMMON/INDEX/N,NRB,NS,NC
COMMON/CMATRX/NOUT,CMAT
COMMON/BMATRIX/NP,BMATRIX
COMMON/RL/NVEL,V
N2=2+N
3 CONTINUE
DO 8 I=1,5
DO 8 J=1,40
FMAT(I,J)=0.0
8 CONTINUE

Find out whether output or state feedback is to be used

WRITE(*,700)
READ(*,701) IFDBK
IF (IFDBK.GT.2) GOTO 3
IF (IFDBK.EQ.0) GOTO 7
IF (IFDBK.EQ.1) THEN

output feedback augmentation loop, creates BKC=AUG

WRITE(*,702) NP,NOUT
NA=NP
NB=NOUT
DO 1 I=1,NP
WRITE(*,703) I
READ(*,*) (FMAT(I,J),J=1,NOUT)
1 CONTINUE
GOTO 13
14 CONTINUE

Call to IMSL routine to multiply BK, and then BK*C

CALL VMULFF(BMATRIX,FMAT,N2,NP,NOUT,40,5,TEMP,40,IER)
CALL VMULFF(TEMP,CMAT,N2,NOUT,N2,40,40,AUG,40,IER)
WRITE(4,'(A)') 'For OUTPUT feedback'
WRITE(4,'(A)') 'With a feedback matrix of'
DO 16 I=1,NP
WRITE(4,703) I
   DO 17 J=1,NOUT,6
       WRITE(4,706) (FMAT(I,K),K=J,J+5)
17    CONTINUE
16    CONTINUE
ELSE
   C state feedback augmentation loop, creates BK=AUG
   WRITE(*,702) NP,N2
   NA=NP
   NB=N2
   DO 2 I=1,NP
       WRITE(*,703) I
       READ(*,*) (FMAT(I,J),J=1,N2)
2    CONTINUE
   GOTO 13
15    CONTINUE
   C Call to IMSL routine to multiply B*K
   CALL VMULFF(BMATRIX,FMAT,N2,NP,N2,40,5,AUG,40,IER)
   WRITE(4,'(A)') 'For STATE feedback'
   WRITE(4,'(A)') 'With a feedback matrix of'
   DO 18 I=1,NP
       WRITE(4,703) I
       DO 19 J=1,N2,6
           WRITE(4,706) (FMAT(I,K),K=J,J+5)
19      CONTINUE
18      CONTINUE
   ENDIF
   DO 4 J=1,N2
       DO 4 I=1,N2
           TEMP(I,J)=0.0
4    CONTINUE
C Unit 98 contains the unaugmented A matrices created during
C the first root locus. This section now creates another
C root locus loop, computing eigenvalues and using the
C closed loop A matrix Acl=A-AUG
C
   REWIND (98)
   READ(98,*) NVEL
   DO 5 K=1,NVEL
       READ(98,*) ((TEMP(I,J),J=1,N2),I=1,N2)
       DO 6 I=1,N2
           DO 6 J=1,N2
               ACL(I,J)=TEMP(I,J)-AUG(I,J)
6      CONTINUE
   WRITE(*, 704) V(K)
   WRITE(4,704) V(K)
CALL EVAL
5 CONTINUE

C Option to plot the root locus points
C
WRITE(*,'(A)') ' Do you want to plot the closed loop roots?'
READ(*,705) QPLT
IF(QPLT.EQ.'Y') THEN
  WRITE(*,'(A)') ' What title do you want for the plot?'
  READ(*,705) TITLE
  CALL RLPLT (0.0,8.0,-6.0,6.0,TITLE)
  CLOSE(90)
ENDIF
GOTO 3
C
This section gives the user the option to change the values
C of any portion of the feedback matrix.
C
13 CONTINUE
  DO 10 I=1,NA
    WRITE(*,703) I
    DO 11 J=1,NB,6
      WRITE(*,706) (FMAT(I,K),K=J,J+5)
  11 CONTINUE
  10 CONTINUE
  WRITE(*,'(A)') ' Do you want to change any values?'
  READ(*,705) QCHG
  IF (QCHG.EQ.'N') GOTO 12
  9 CONTINUE
  WRITE(*,'(A)') ' Please input the row, column, and new value.'
  READ(*,*) I,J,FMAT(I,J)
  WRITE(*,'(A)') ' Do you want to change another value?'
  READ(*,705) QCHG
  IF (QCHG.EQ.'Y') GOTO 9
  12 CONTINUE
  IF (IFDBK.EQ.1) GOTO 14
  IF (IFDBK.EQ.2) GOTO 15
  700 FORMAT(' What type of feedback are you using?',//,
              0'  0. Return to main program',//,
              0'  1. Output',//,
              0'  2. State',//)
  701 FORMAT(I5)
  702 FORMAT(' The feedback matrix will have',I3,' rows by',
              QI3,' columns.',//,' Please input the feedback matrix by rows.',//)
  703 FORMAT(' Row',I3)
  704 FORMAT(' The closed loop eigenvalues for ',F10.3,' ft/sec')
  705 FORMAT(A)
  706 FORMAT(1X,6E11.4)
  7 CONTINUE
RETURN
END
SUBROUTINE BMAT(VEL)

This subroutine takes the forcing function matrix and converts it into the B matrix of \( \frac{dx}{dt} = -Ax + Bu \) where \( u \) is the control input vector.

REAL X(20,20), XINV(20,20), QMAT(20,5), BMATRIX(40,5), BMAT1(20,5)
INTEGER N, NRB, NS, NC, NINP, NAERO, NMAT, NINV, NAMAT, NEIG, N2, NV
REAL*8 B(40,40), DUMMY
CHARACTER*20 BNAM
COMMON/PRT/NAERO, NMAT, NINV, NAMAT, NEIG, NV
COMMON/INDEX/N, NRB, NS, NC
COMMON/BMATRIX/NINP, QMAT
COMMON/INV/X, XINV
COMMON/BMATRX/NINP, BMATRIX
REAL XMOM(5)
COMMON/MOMENT/XMOM

Call to IMSL routine to multiply \( XINV \times QMAT \)

CALL VMULFF(XINV, QMAT, N, N, NINP, 20, 20, BMAT1, 20, IER)
NP=NINP

Creates the actual B matrix

DO 2 I=1,N
  K=N+I
  DO 2 J=1,NINP
    BMATRIX(K, J) = BMAT1(I, J) * XMOM(J)
  2 CONTINUE

WRITE (4,603)
N2=2*N
DO 1 I=1,N2
  WRITE(4,600) I
  WRITE(4,601) (BMATRIX(I,J), J=1,NINP)
  1 CONTINUE

IF (NAMAT.EQ.1) THEN
  OPEN (99, STATUS='SCRATCH')
  NV=NINT(VEL)
  WRITE(99,604) NV
  REWIND 99
  READ(99,605) BNAM
  604 FORMAT(' [.MATRXX]B', I4,4)
  605 FORMAT(A)
  CLOSE(99)
  DO 10 I=1,N2
    DO 10 J=1,NINP
      BIJ=BMATRIX(I,J)
      B(I,J)=DBLE(BIJ)
    10 CONTINUE
OPEN(9,FILE=BNAM,STATUS='NEW')
C Call to routine that puts the B matrix into MATRIXx format
C
CALL MATSAV(9,BNAM,40,N2,NINP,O,B,DUMMY,'(1P8E15.7)')
CLOSE(9)
ENDIF
600 FORMAT(' Row',I3)
601 FORMAT(1X,6E12.4)
602 FORMAT(6E15.7)
603 FORMAT(//' The B matrix of dx/dt=Ax+Bu is')
RETURN
END
SUBROUTINE EVAL

C
C This subroutine computes the eigenvalues of the A matrix
C using the IMSL routine EIGRF.

C
REAL AMATRIX,WK
DIMENSION AMATRIX(40,40),WK(40)
COMPLEX/EIG/AMATRIX
COMPLEX EIGVAL(40)
INTEGER N,NRB,NS,NC,IER,N2
COMMON/INDEX/N,NRB,NS,NC
INTEGER NAERO,NMAT,NINV,NAMAT,NEIG
COMMON/PRT/NAERO,NMAT,NINV,NAMAT,NEIG
CHARACTER*64 QEIFLE
CHARACTER*14 QEVAFL
REAL*8 XXXY,XZ
N2=N*2
PI=3.14159

C Call to IMSL eigenvalue solver routine
C
CALL EIGRF(AMATRIX,N2,40,0,EIGVAL,Z,IZ,WK,IER)
IF (IER.NE.0) THEN
  WRITE(*,600) IER
  STOP
ENDIF
IF (NEIG.EQ.1) OPEN(90,STATUS='SCRATCH',FORM='UNFORMATTED')
WRITE(*,')' 'The eigenvalues are:
WRITE(4,')' 'I Eigenvalue Freq(Hz)
DO 2 I=1,N2
  FREQ=AIMAG(EIGVAL(I))/(2.*PI)
  XY=DBLE(FREQ)
  RV=REAL(EIGVAL(I))
  XX=DBLE(RV)
  XIM=AIMAG(EIGVAL(I))
  XZ=DBLE(XIM)
  WRITE(*,602) I,REAL(EIGVAL(I)),AIMAG(EIGVAL(I)),FREQ
  WRITE(4,602) I,REAL(EIGVAL(I)),AIMAG(EIGVAL(I)),FREQ
  IF (NEIG.EQ.1) WRITE(90) XX,XY,XZ
2 CONTINUE
1 CONTINUE
WRITE(4,')' '1
600 FORMAT(/,**** There was an error in the eigenvalue',
0' computation, # ',I3,' ****'//)
601 FORMAT(A)
602 FORMAT(I3,2E11.4,'i',E11.4)
RETURN
END
SUBROUTINE FASTCHG

C This subroutine changes FASTOP generalized forces into
C non-dimensional form by multiplying by s**2 and by dividing
C by SREF for forces and by SREF*CBAR for moments
C also the difference in the coordinate systems is taken into
C account. FASTOP is a left-hand rule axis system with positive
C Z up and positive moment down. Normal stability in body axes
C positive Z is downward and positive moment is up.
C
COMPLEX GF1,GF2
REAL S,SREF,CBAR,PI,SR,CB,K1,K2,VO,RHOO,ALPHAO,HO,ZETAO, 
ODELTAO,BR
INTEGER N,NRB,NS,NC
DIMENSION GF1(20,20),GF2(20,20),ZETAO(15),DELT AO(5)
COMMON/AERO/K1,K2,GF1,GF2,PI 
COMMON/IC/V0,RHOO,SREF,CBAR,ALPHAO,HO,ZETAO,DELT AO,BR 
COMMON/INDEX/N,NRB,NS,NC 
NRB1=NRB+1 
NSNRB=NS+NRB 
NSNRB1=NSNRB+1 
NCNSNRB=NSNRB+NC 
S=12.0 
SR=SREF*144.0 
CB=CBAR*12.0 
IF (NRB.EQ.0) GOTO 1
DO 1 J=1,N 
GF1(1,J)=-GF1(1,J)*S**2/SR 
GF2(1,J)=-GF2(1,J)*S**2/SR 
GF1(2,J)=-GF1(2,J)*S**3/(SR*CB) 
GF2(2,J)=-GF2(2,J)*S**3/(SR*CB) 
1 CONTINUE 
IF (NS.EQ.0) GOTO 2
DO 2 I=NRB1,NSNRB 
DO 2 J=1,N
GF1(I,J)=-GF1(I,J)*S**2/SR 
GF2(I,J)=-GF2(I,J)*S**2/SR 
2 CONTINUE 
IF (NC.EQ.0) GOTO 3
DO 3 I=NSNRB1,NCNSNRB 
DO 3 J=1,N
GF1(I,J)=-GF1(I,J)*S**3/(SR*CB) 
GF2(I,J)=-GF2(I,J)*S**3/(SR*CB)
3 CONTINUE
RETURN
END
SUBROUTINE INPUT

C This subroutine contains all the input necessary to run the program.
C The total dimensions are 20 total modes, 2 rigid body modes, 5 control
C surface modes and 15 elastic modes maximum.
C
INTEGER N,NRB,NS,NC,NSQ,NVEL,INPT,NINP,N2,NOUT
INTEGER NAERO,NMAT,NINV,NAMAT,NEIG,NSTAB
CHARACTER*64 STRUCT,AERO,INCOND,RTLCSF,INPTM,TITLE,OUTPM
CHARACTER*1 Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9,Q10,Q11
REAL K1,K2,VO,RHOO,SREF,CBAR,ALPHAO,H0,XMACH,AO,BR
REAL GM,GK,ZETAO,DELTAO,OMEGA,V,F(20,20),G(20,20)
REAL QMAT(20,5),CMAT(40,40)
DIMENSION GM(20),GK(20),ZETAO(15),DELTAO(5),OMEGA(20),V(20)
COMPLEX GF1,GF2
DIMENSION GF1(20,20),GF2(20,20)
COMMON/STRUCT/GM,GK
COMMON/AERO/K1,K2,GF1,GF2,PI
COMMON/INDEX/N, NR, NS, NC
COMMON/IC/VO,RHOO,SREF,CBAR,ALPHAO,H0,ZETAO,DELTAO,BR
COMMON/RL/NVEL,V
COMMON/PRT/NAERO,NMAT,NINV,NAMAT,NEIG
COMMON/BMATRX/NINP,QMAT
COMMON/CMATRX/NOUT,CMAT
NAMELIST/PRNT/NAERO,NMAT,NINV,NAMAT,NEIG,NSTAB

C Default values for the namelist variables
C
NAERO=0
NMAT=0
NINV=0
NAMAT=0
NEIG=1
NSTAB=0
OPEN(10,STATUS='OLD')
READ(10,PRNT)
PI=3.14159

C Question to find out whether the data is to be automatically
C input, or whether it will be done manually.
C
24 CONTINUE
WRITE(*,120)
READ(*,121) INPT
IF (INPT.EQ.1) THEN

C Auto input of Structural info
C
OPEN(1,STATUS='OLD')
READ(1,116) N, NR, NS, NC
\[ N2 = 2 \times N \]
\[ \text{READ}(1,117) \ (GM(I), I=1,N) \]
\[ \text{READ}(1,117) \ (OMEGA(I), I=1,N) \]

C

Auto input of Aero

C

OPEN(2,STATUS='OLD')
READ(2,111) TITLE
READ(2,128) N,XMACH
READ(2,127) K1
READ(2,126) (((F(I,J),G(I,J)),I=1,N),J=1,N)
DO 7 I=1,N
   DO 7 J=1,N
      GF1(I,J)=CMPLX(F(I,J),G(I,J))
   CONTINUE
READ(2,127) K2
READ(2,126) (((F(I,J),G(I,J)),I=1,N),J=1,N)
DO 11 I=1,N
   DO 11 J=1,N
      GF2(I,J)=-CMPLX(F(I,J),G(I,J))
   CONTINUE

C Auto input of initial conditions

C

OPEN(3,STATUS='OLD')
READ(3,118) RHOO,AO,SREF,CBAR,BR
IF (NRB.NE.0) READ(3,119) ALPHAO,HO
IF (NS.NE.0) READ(3,119) (ZETAO(I),I=1,NS)
IF (NC.NE.0) READ(3,119) (DELTAO(I),I=1,NC)

C Auto input of root locus info

C

OPEN(7,STATUS='OLD')
READ(7,121) NVEL
DO 15 I=1,NVEL
   READ(7,122) V(I)
CONTINUE

C Auto input of forcing function matrix

C

OPEN(11,STATUS='OLD')
READ(11,121) NINP
READ(11,123) ((QMAT(I,J),J=1,NINP),I=1,N)

C Auto input of C matrix

C

OPEN(12,STATUS='OLD')
READ(12,121) NOUT
READ(12,123) ((CMAT(I,J),J=1,N2),I=1,NOUT)
GOTO 23
ENDIF
IF (INPT.LE.0) GOTO 24
IF (INPT.EQ.2) GOTO 20
IF (INPT.EQ.3) GOTO 21
IF (INPT.EQ.4) GOTO 22
IF (INPT.EQ.5) GOTO 26
IF (INPT.EQ.6) GOTO 25
IF (INPT.EQ.7) GOTO 23
IF (INPT.EQ.8) GOTO 24
20 CONTINUE

C Input of the structural information. N=# of modes, NRB=# of rigid
C body modes (set at two for now), NS=# of structural modes,
C NC=# of control surface modes. OMEGA-modal frequencies
C GM-generalized mass, GK-generalized stiffness.

WRITE(*,'(A)') ' Is the structural information in a file?'
READ(*,111) Q1
IF (Q1.EQ.'Y') THEN
  WRITE(*,'(A)') ' Please input the name of the file.'
  READ(*,111) STRUCT
  OPEN(1,FILE=STRUCT,STATUS='OLD')
  READ(1,116) N,NRB,NS,NC
  N2=2*N
  READ(1,117) (GM(I),I=1,N)
  READ(1,117) (OMEGA(I),I=1,N)
ELSE
  WRITE(*,'(A)') ' Do you wish to save the data on a file for'
  WRITE(*,'(A)') ' future use?'
  READ(*,111) Q2
  IF (Q2.EQ.'Y') THEN
    WRITE(*,'(A)') ' What is the name of the new file?'
    READ(*,111) STRUCT
    OPEN(1,FILE=STRUCT,STATUS='NEW')
  ENDFI
WRITE(*,'(A)') ' Input total # of modes used (max 20).'
READ(*,*) N
N2=2*N
WRITE(*,'(A)') ' Input # of rigid modes used (2).'
READ(*,*) NRB
WRITE(*,'(A)') ' Input # of structural modes used (max 15).'
READ(*,*) NS
WRITE(*,'(A)') ' Input # of control surface modes used (max 5).'
READ(*,*) NC
WRITE(*,'(A)') ' Input the generalized masses in the order of'
WRITE(*,'(A)') ' rigid modes (plunge then pitch), structural,'
WRITE(*,'(A)') ' modes, and control surface modes.'
READ(*,*) (GM(I),I=1,N)
WRITE(*,'(A)') ' Input the modal frequencies (in Hz) in the order'
WRITE(*,'(A)') ' of rigid modes (plunge then pitch), structural,'
READ(*,*) (OMEGA(I), I=1,N)
IF (Q2.EQ.'Y') THEN
WRITE(1,116) N, NRB, NS, NC
WRITE(1,117) (GM(I), I=1,N)
WRITE(1,117) (OMEGA(I), I=1,N)
ENDIF
ENDIF
GOTO 24
21 CONTINUE
C Input of the generalized aerodynamic forces. It is in the format
C of the output of FASTOP as modified by Max Blair, where the two
C reduced frequencies are combined into one file.
C K1 is the first reduced frequency, K2 is the second reduced frequency.
C GF1 are the generalized forces related to the first reduced frequency,
C GF2 are the generalized forces related to K2. K1 is usually 0.0 and
C K2 is a small reduced frequency (0.001).
C WRITE(*,'(A)') ' What is the name of the file for the generalized'
WRITE(*,'(A)') ' forces (aerodynamics)'
READ(*,111) AERO
OPEN(2,FILE=AERO,STATUS='OLD')
READ(2,111) TITLE
READ(2,128) N, XMACH
READ(2,127) K1
READ(2,126) (((F(I,J), G(I,J)), I=1,N), J=1,N)
DO 17 I=1,N
  DO 17 J=1,N
    GF1(I,J)=CMPLX(F(I,J), G(I,J))
17 CONTINUE
READ(2,127) K2
READ(2,126) (((F(I,J), G(I,J)), I=1,N), J=1,N)
DO 18 I=1,N
  DO 18 J=1,N
    GF2(I,J)=CMPLX(F(I,J), G(I,J))
18 CONTINUE
8 CONTINUE
GOTO 24
22 CONTINUE
C Input of initial conditions, velocity, density, reference area,
C reference chord (should be the actual A/C area & chord).
C Also the appropriate magnitudes of the mode shapes are input.
C WRITE(*,'(A)') ' Are the initial conditions in a file?'
READ(*,111) Q3
IF (Q3.EQ.'Y') THEN
  WRITE(*,'(A)') ' Please input the name of the file.'
  READ(*,111) INCOND
  OPEN(3,FILE=INCOND,STATUS='OLD')
  READ(3,118) RHOO, AO, SREF, CBAR, BR
IF (NRB.NE.0) READ(3,119) ALPHAO,H0
IF (NS.NE.0) READ(3,119) (ZETAO(I),I=1,NS)
IF (NC.NE.0) READ(3,119) (DELTAO(I),I=1,NC)
ELSE
   WRITE(*,'(A)') ' Do you wish to save the initial conditions'
   WRITE(*,'(A)') ' on a file for future use?
   READ(*,111) Q4
   IF (Q4.EQ.'Y') THEN
      WRITE(*,'(A)') ' What is the name of the new file?'
      READ(*,111) INCOND
      OPEN(3,FILE=INCOND,STATUS='NEW')
   ENDIF
   WRITE(*,'(A)') ' Please input the following initial conditions.'
   WRITE(*,'(A)') ' Density (slugs/ft**3):'
   READ(*,*) RHOO
   WRITE(*,'(A)') ' Speed of sound (ft/sec) at density altitude:'
   READ(*,*) AO
   WRITE(*,'(A)') ' Reference Area (in ft**2):'
   READ(*,*) SREF
   WRITE(*,'(A)') ' Reference Chord (MAC) (in ft):'
   READ(*,*) CBAR
   WRITE(*,'(A)') ' What is the reference semichord used in the'
   WRITE(*,'(A)') ' aerodynamics routine (from kbw/V) in ft.'
   READ(*,*) BR
   IF (NRB.EQ.0) GOTO 19
   WRITE(*,'(A)') ' Pitch mode shape magnitude:'
   READ(*,*) ALPHAO
   WRITE(*,'(A)') ' Plunge mode shape magnitude:'
   READ(*,*) H0
19 CONTINUE
   IF (NS.EQ.0) GOTO 5
   DO 5 I=1,NS
      WRITE(*,108)I
      READ(*,*) ZETAO(I)
   CONTINUE
5 CONTINUE
   IF (NC.EQ.0) GOTO 6
   DO 6 I=1,NC
      WRITE(*,109)I
      READ(*,*) DELTAO(I)
   CONTINUE
6 CONTINUE
   IF(Q4.EQ.'Y') THEN
      WRITE(3,118) RHOO,AO,SREF,CBAR,BR
      IF (NRB.NE.0) WRITE(3,119) ALPHAO,H0
      IF (NS.NE.0) WRITE(3,119) (ZETAO(I),I=1,NS)
      IF (NC.NE.0) WRITE(3,119) (DELTAO(I),I=1,NC)
   ENDIF
ENDIF
GOTO 24
26 CONTINUE
C
C Manual input of velocity root locus data
WRITE(*,'(A)') 'Do you wish to perform a velocity root locus?'
READ(*,100) Q5
IF(Q5.EQ.'Y') THEN
WRITE(*,'(A)') 'Are the values on file?'
READ(*,100) Q6
IF (Q6.EQ.'Y') THEN
WRITE(*,'(A)') 'What is the name of the file?'
READ(*,100) RTLCSF
OPEN(7,FILE=RTLCSF,STATUS='OLD')
READ(7,121) NVEL
DO 16 I=1,NVEL
   READ(7,122) V(I)
16 CONTINUE
ELSE
WRITE(*,'(A)') 'Do you want to save this data for future use?'
READ(*,111) Q7
IF(Q7.EQ.'Y') THEN
WRITE(*,'(A)') 'What is the name of the new file?'
READ(*,111) RTLCSF
OPEN(7,FILE=RTLCSF,STATUS='NEW')
ENDIF
WRITE(*,'(A)') 'How many velocities to use (max 20)?'
READ(*,*) NVEL
IF (Q7.EQ.'Y') WRITE(7,101) NVEL
WRITE(*,'(A)') 'Please input the velocities (in ft/sec).'
READ(*,*) (V(I),I=1,NVEL)
IF (Q7.EQ.'Y') WRITE(7,102) (V(I),I=1,NVEL)
ENDIF
ENDIF
GOTO 24
25 CONTINUE
C Manual input of the forcing function matrix
C
WRITE(*,'(A)') 'Is the forcing function matrix on file?'
READ(*,111) Q8
IF(Q8.EQ.'Y') THEN
WRITE(*,'(A)') 'Please input the name of the file.'
READ(*,111) INPTM
OPEN(11,FILE=INPTM,STATUS='OLD')
READ(11,121) NINP
READ(11,123) ((QMAT(I,J),J=1,NINP),I=1,N)
ELSE
WRITE(*,'(A)') 'Do you wish to save the data to file?'
READ(*,111) Q9
IF (Q9.EQ.'Y') THEN
WRITE(*,'(A)') 'What is the name of the new file?'
READ(*,111) INPTM
OPEN(11,FILE=INPTM,STATUS='NEW')
ENDIF
WRITE(*,'(A)')  ' How many inputs are there?'  
READ(*,*')  NINP  
WRITE(*,124)  N,NINP  
DO 30  I=1,N  
   WRITE(*,125)  I  
   READ(*,*')  (QMAT(I,J),J=1,NINP)  
30  CONTINUE  
IF (Q9.EQ.'Y')  THEN  
   WRITE(11,121)  NINP  
   WRITE(11,123)  ((QMAT(I,J),J=1,NINP),I=1,N)  
ENDIF  
ENDIF  
GOTO 24  
27  CONTINUE  
C  
C Manual input of the C matrix  
C  
WRITE(*,'(A)')  ' Is the C matrix on file?'  
READ(*,111)  Q10  
IF (Q10.EQ.'Y')  THEN  
   WRITE(*,'(A)')  ' Please input the name of the file.'  
   READ(*,111)  OUTPM  
   OPEN(12,FILE=OUTPM,STATUS='OLD')  
   READ(12,121)  NOUT  
   READ(12,123)  ((CMAT(I,J),J=1,N2),I=1,NOUT)  
ELSE  
   WRITE(*,'(A)')  ' Do you wish to save the data to file?'  
   READ(*,111)  Q11  
   IF (Q11.EQ.'Y')  THEN  
      WRITE(*,'(A)')  ' What is the name of the new file?'  
      READ(*,111)  OUTPM  
      OPEN(12,FILE=OUTPM,STATUS='NEW')  
   ENDIF  
   WRITE(*,'(A)')  ' How many outputs are there?'  
   READ(*,*')  NOUT  
   WRITE(*,129)  NOUT,N2  
   DO 34  I=1,NOUT  
      WRITE(*,125)  I  
      READ(*,*')  (CMAT(I,J),J=1,N2)  
34  CONTINUE  
IF (Q11.EQ.'Y')  THEN  
   WRITE(12,121)  NOUT  
   WRITE(12,123)  ((CMAT(I,J),J=1,N2),I=1,NOUT)  
ENDIF  
ENDIF  
GOTO 24  
23  CONTINUE  
DO 9  I=1,N  
   GK(I)=GM(I)*((OMEGA(I)*2.*PI)**2)  
9  CONTINUE  
VO=XMACH*AO
WRITE(4,100) N
WRITE(4,101) NRB
WRITE(4,102) NS
WRITE(4,103) NC
WRITE(4,104)
DO 1 I=1,N
  WRITE(4,105) I,I,GM(I)
1 CONTINUE
WRITE(4,106)
DO 2 I=1,N
  WRITE(4,105) I,I,CK(I)
2 CONTINUE

C Call to the routine that non-dimensionalizes FASTOP aero forces

CALL FASTCHG
IF (NAERO.EQ.0) GOTO 4
WRITE(4,114) XMACH
WRITE(4,107) K1
DO 3 I=1,N
  WRITE(4,112) ((REAL(GF1(I,J)),AIMAG(GF1(I,J))),J=1,N)
3 CONTINUE
WRITE(4,107) K2
DO 4 I=1,N
  WRITE(4,112) ((REAL(GF2(I,J)),AIMAG(GF2(I,J))),J=1,N)
4 CONTINUE
WRITE(4,115) RHOO,SREF,CBAR,BR
WRITE(4,116) '1'

100 FORMAT(/,' The total number of modes are:',I3)
101 FORMAT( ' The total number of rigid body modes are:',I3)
102 FORMAT( ' The total number of elastic modes are:',I3)
103 FORMAT( ' The total number of control surface modes are:',I3)
104 FORMAT(/,' The generalized mass matrix is',/,' RowCol Gen Mass')
105 FORMAT(1X,2I3,E12.4)
106 FORMAT(/,' The generalized stiffness matrix is',
  0/,' RowCol Gen Stiff')
107 FORMAT(/,' The generalized force matrix for reduced frequency of',
  0F7.4,' is')
108 FORMAT( ' Structural mode',I2,' magnitude:')
109 FORMAT( ' Control surface mode',I2,' magnitude:')
110 FORMAT(1X,4I4,2E15.7)
111 FORMAT(A)
112 FORMAT(3(1X,2E12.4,'i'))
113 FORMAT(15X,E15.7)
114 FORMAT(/,' For a Mach number of',F6.3)
115 FORMAT( ' Density=',F11.5,'Slugs/ft**3, Reference Area=',
  0E11.4,' ft**2',/,
  0' Chord=',E11.4,' feet, and an aerodynamic reference',)
semi-chord=',E11.4,' feet.')
116 FORMAT(4I5)
117 FORMAT(4E15.7)
118 FORMAT(F10.7,F10.2,3F10.5)
119 FORMAT(6F10.5)
120 FORMAT(' MAC ready for input',/,
0' Enter ($): ',//,
0' 1. Automatic input of items (2)-(7)',/
0' 2. Read in structural data',/
0' 3. Read in aerodynamic data',/
0' 4. Read in initial conditions',/
0' 5. Read in root locus values',/
0' 6. Read in forcing function matrix',/
0' 7. Read in the C matrix',/
0' 8. Completion of input data',//)
121 FORMAT(I5)
122 FORMAT(F10.3)
123 FORMAT(6E15.7)
124 FORMAT(' For n modes, there are n modes with m controls',/,
0' The modes will be set up as follows:',//,
0' Rigid modes',/,
0' Structural modes',/,
0' Control surface modes',/,
0' The matrix will have',I3,' rows and ',I3,' columns',/,
0' Please input the matrix by row.'//)
125 FORMAT(' Row',I3)
126 FORMAT(8E15.7)
127 FORMAT(E15.7)
128 FORMAT(I5,5X,E15.7)
129 FORMAT(' The matrix will have ',I3,' rows and ',I3,' columns',/,
0' Please input the matrix by row.'//)
RETURN
END
SUBROUTINE MATSAV ( LUNIT, NAME, NR, M, N, IMG, $ XREAL, XIMAG, FORMT )

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MATSAV writes a matrix to a file in a format suitable for the MATRIXx LOAD operation.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Type</th>
<th>On input-</th>
<th>On output-</th>
</tr>
</thead>
<tbody>
<tr>
<td>LUNIT</td>
<td>INTEGER</td>
<td>unchanged.</td>
<td>unchanged.</td>
</tr>
<tr>
<td>NAME</td>
<td>CHARACTER*(* maximum length 10)</td>
<td>unchanged.</td>
<td>unchanged.</td>
</tr>
<tr>
<td>NR</td>
<td>INTEGER</td>
<td>Row-dimension in the defining dimension or type statement in the calling program. NR must be greater than or equal to M.</td>
<td>unchanged.</td>
</tr>
<tr>
<td>M</td>
<td>INTEGER</td>
<td>Number of rows of the matrix</td>
<td>unchanged.</td>
</tr>
<tr>
<td>N</td>
<td>INTEGER</td>
<td>Number of columns of the matrix</td>
<td>unchanged.</td>
</tr>
<tr>
<td>IMG</td>
<td>INTEGER</td>
<td>If IMG = 0, the imaginary part (XIMAG) is assumed to be zero and is not saved.</td>
<td>unchanged.</td>
</tr>
<tr>
<td>XREAL</td>
<td>DOUBLE</td>
<td>Real part of the matrix to</td>
<td>unchanged.</td>
</tr>
</tbody>
</table>
Example: The following Fortran program generates an elementary matrix in X and writes it to Fortran unit 1. Assume that unit 1 has been preallocated as file (data set) TEST.

```
DOUBLE PRECISION X(20,3), DUMMY
DO 200 J=1,3
   DO 100 I=1,10
      X(I,J)=0.000
   100 CONTINUE
   X(J,J)=1.000
  200 CONTINUE
CALL MATSAV( 1, 'AMATRIX', 20, 10, 3, 0, X, DUMMY, '(1P2E24.15)')
STOP
END
```

After this program runs, invoke MATRIXx and type:

```
<> LOAD 'TEST'
```

This will put X on the stack as stack-variable-name AMATRIX.
 | write real-part of the matrix. |
\[
\text{WRITE}(\text{LUNIT}, \text{FORM}) \ (\text{XREAL}(I,J), I=1,M), J=1,N)
\]

| write imaginary-part if nonzero. |
\[
\text{IF}(\text{IMG}. \neq 0) \text{ WRITE}(\text{LUNIT}, \text{FORM}) \ (\text{XIMAG}(I,J), I=1,M), J=1,N)
\]
RETURN
END
SUBROUTINE MINV

C This subroutine inverts the X matrix using the IMSL routine LINV1F.

C REAL X,XINV,WKAREA
DIMENSION X(20,20),XINV(20,20),WKAREA(20)
COMMON/INV/X,XINV
INTEGER N,NRB,NS,NC,IER
COMMON/INDEX/N,NRB,NS,NC
INTEGER NAERO,NMAT,NINV,NAMAT,NEIG
COMMON/PRT/NAERO,NMAT,NINV,NAMAT,NEIG

C Call to IMSL matrix inversion routine

CALL LINV1F(X,N,20,XINV,4,WKAREA,IER)
IF (IER.EQ.34) THEN
  WRITE(*,403)
  GOTO 5
ENDIF
IF (IER.NE.0) THEN
  WRITE(*,400) IER
  STOP
ENDIF
5 CONTINUE
IF(NINV.EQ.0) GOTO 2
WRITE(4,'(/A)') 'X inverse is'
DO 2 I=1,N
  WRITE(4,401) I
  DO 3 J=1,N,6
    WRITE(4,402) (XINV(I,K),K=J,J+5)
  3 CONTINUE
2 CONTINUE
400 FORMAT(///,' **** There was an error in inverting X, # ',I3,
         0' ****',///)
401 FORMAT(' ROW',I3)
402 FORMAT(6E12.4)
403 FORMAT(///,' The inversion did not meet error criterion',///)
1 CONTINUE
RETURN
END
SUBROUTINE MMULT

C This subroutine multiplies Xinv*Y and Xinv*Z using the
C IMSL routine VMULFF
C
REAL Y,Z,XINV,XINVY,XINVZ
DIMENSION X(20,20),Y(20,20),Z(20,20),XINV(20,20),
XINVY(20,20),XINVZ(20,20)
COMMON/MMUL/Y,Z,XINVY,XINVZ
COMMON/INV/X,XINV
INTEGER N,NRB,NS,NC,IER
COMMON/INDEX/N,NRB,NS,NC
INTEGER NAERO,NMAT,NINV,NAMAT,NEIG
COMMON/PRT/NAERO,NMAT,NINV,NAMAT,NEIG

C Call to IMSL matrix multiplication routine to multiply
C XINV*Y

CALL VMULFF(XINV,Y,N,N,N,20,20,XINVY,20,IER)
IF (IER.NE.0) THEN
  WRITE(*,500) IER
  STOP
ENDIF
IF(NINV.EQ.0) GOTO 2
WRITE(4,'(/A)') 'XINV*Y is'
DO 2 I=1,N
  WRITE(4,501) I
  DO 3 J=1,N,6
    WRITE(4,502) (XINVY(I,K),K=J,J+5)
  3 CONTINUE
2 CONTINUE
IER=0

C Call to IMSL matrix multiplication routine to multiply
C XINV*Z

CALL VMULFF(XINV,Z,N,N,N,20,20,XINVZ,20,IER)
IF (IER.NE.0) THEN
  WRITE(*,500) IER
  GOTO 1
ENDIF
IF(NINV.EQ.0) GOTO 4
WRITE(4,'(/A)') 'XINV*Z is'
DO 4 I=1,N
  WRITE(4,501) I
  DO 5 J=1,N,6
    WRITE(4,502) (XINVZ(I,K),K=J,J+5)
  5 CONTINUE
4 CONTINUE

500 FORMAT(/,****There was an error in the matrix multiply, #')
0, I3, ' *** ***' ///)
501 FORMAT(' B O W', I3)
502 FORMAT(6E12.4)
1 CONTINUE
   RETURN
   END
SUBROUTINE RLPLOT(FL, FH, RL, RH, TITLE)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*4 XPOS, YPOS
CHARACTER*4 NUM
CHARACTER*23 DT
CHARACTER*40 TITLE
LOGICAL PLOT
REAL VAL(13)/.01, .1, .2, .5, 1., 2., 5., 10., 20., 50., 100., 200., 500./
C
RECORD 90
ZERO=0.DO
ONE=1.DO
WRITE(*,1010)
READ(*,*)
CALL JBEGIIN
C CLEAR SCREEN WITH PLOT10 COMMANDS
CALL INITT(960)
CALL FINITI(0,2800)
C
C SETUP GRAPHICS
C
PLOT=.FALSE.
30 IF(.NOT. PLOT) THEN
CALL JDINIT(1)
CALL JDEVON(1)
ELSE
CALL JDINIT(2)
CALL JDEVON(2)
END IF
C
CALL JWINDO(-4., 12., -3., 15.)
CALL JOPEN
CALL JSIZE(.27, .35)
IF(PLOT) CALL JFONT(5)
IF(.NOT. PLOT) CALL JFONT(1)
C
DRAW BOX AND SET UP AXES
C
CALL JMOVE(0., 0.)
CALL JDRAW(10., 0.)
CALL JDRAW(10., 10.)
CALL JDRAW(0., 10.)
CALL JDRAW(O., O.)
C
XRANGE=RH-RL
XRAT=XRANGE/10.
YRANGE=FH-FL
YRAT=YRANGE/10.
C
DO 50 I=1, 13
IF (XRANGE .GE. VAL(I)) XSTEP = VAL(I)/5.
50 IF (YRANGE .GE. VAL(I)) YSTEP = VAL(I)/5.
C
XPOS = SNGL (-RL/XRAT)
CALL JMOVE (XPOS, 0.)
CALL JDRAW (XPOS, 10.)
C
C DRAW TIC MARKS AND NUMBERS
C
X = RL
60 XPOS = SNGL (X/XRAT - RL/XRAT)
IF (XPOS .GT. 10.) GOTO 65
CALL JMOVE (XPOS, 0.)
CALL JRDRAW (0., -.3)
C
ENCODE (4, 1020, NUM) X
CALL JJUST (2, 3)
CALL JMOVE (XPOS, -.5)
CALL JHSTRG (NUM)
C
X = X + XSTEP
GOTO 60
C
C
Y = FL
70 YPOS = SNGL (Y/YRAT - FL/YRAT)
IF (YPOS .GT. 10.) GOTO 75
ENCODE (4, 1020, NUM) Y
CALL JJUST (3, 2)
CALL JMOVE (0., YPOS)
CALL JRDRAW (-3, 0.)
CALL JMOVE (-5, YPOS)
CALL JHSTRG (NUM)
Y = Y + YSTEP
GOTO 70
C
75 CALL JSIZE (.4, .45)
CALL JJUST (2, 3)
CALL JBASE (1., 0., 0.)
CALL JMOVE (.5, -.2.)
CALL JHTEXT (42, 42HR [BLC] EAL [ELC] C [BLC] OMponent (1/SEC) [ELC])
CALL JBASE (0., 1., 0.)
CALL JJUST (2, 1)
CALL JMOVE (-2., 5.)
CALL JHTEXT (44, 44HI [BLC] MAGINARY [ELC] C [BLC] OMponent (Hz) [ELC])
C
C PRINT TITLE AND DATE AT TOP OF PLOT
C
C
CALL JBASE (1., 0., 0.)
CALL JJUST (1, 1)
CALL JMOVE (-3., 12.)
CALL JHSTRG(TITLE)
C CALL LIB$DATE_TIME(DT)
C CALL JMVE(-.3.,11.)
C CALL JHSTRG(DT)
C
80 READ(90,END=999) XH,YH,RF
IF(XH.GT.RH.OR.XH.LT.RL) GOTO 80
IF(YH.GT.FH.OR.YH.LT.FL) GOTO 80
XPOS=SNGL(XH/XRAT-RL/XRAT)
YPOS=SNGL(YH/YRAT-FL/YRAT)
CALL JMVE(XPOS,YPOS)
C DRAW A LITTLE SQUARE
C CALL JRDRW(.05,.05)
CALL JRDRW(-.1,0.)
CALL JRDRW(0.,-.1)
CALL JRDRW(.1,0.)
CALL JRDRW(0.,.1)
GOTO 80
C SHUT EVERYTHING OFF
C 999 CALL JCLOSE
   IF(.NOT.PLOT) THEN
      CALL JPAUSE(1)
      CALL JDEVOF(1)
      CALL JDEND(1)
   ELSE
      CALL JPAUSE(2)
      CALL JDEVOF(2)
      CALL JDEND(2)
   END IF
C SEND QMS FILE TO THE PRINTER
C IF(PLOT) THEN
   WRITE(*,1040)
   ISPAWN=LIB$SPAWN('Q[MAKASE.ADMAD.D3SPAWN]MAXRL.COM')
   IF(.NOT.ISPAWN) CALL LIB$SIGNAL(%VAL(ISPAWN))
   PLOT=.FALSE.
C END IF
C MENU
C CLEAR SCREEN WITH PLOT10 COMMANDS
210 CALL INITT(960)
     CALL FINITT(0,2800)
     WRITE(*,1030)
     READ(*,*) IGOTO
     IF(IGOTO.LT.1.OR.IGOTO.GT.3) GOTO 210
     GOTO (300,400,500), IGOTO
C RETURN TO ADAM
300 CALL JEND
    WRITE(*,1050)
    READ(*,*)
    RETURN
C C CHANGE PLOT LIMITS
C
400 WRITE(*,1060) RL,RH,FL,FH
    WRITE(*,1070)
    READ(*,1065) RL
    WRITE(*,1080)
    READ(*,1065) RH
    WRITE(*,1090)
    READ(*,1065) FL
    WRITE(*,1100)
    READ(*,1065) FH
    REWIND 90
    GOTO 30
C C MAKE HARDCOPY OF CURRENT PLOT
500 PLOT=.TRUE.
    REWIND 90
    GOTO 30
C
1010 FORMAT(' SWITCH TO GRAPHICS MODE AND HIT RETURN')
1020 FORMAT(F4.0)
1030 FORMAT(' PLOT ROUTINE MENU'//', 1-RETURN TO MAC',//', 2-CHANGE
        PLOT LIMITS',//', 3-MAKE A HARDCOPY OF CURRENT PLOT')
1040 FORMAT(' SUBPROCES SPAWNED. FILE ON ITS WAY TO PLOTTER')
1050 FORMAT(' RETURN TERMINAL TO STANDARD MODE AND HIT RETURN KEY')
1060 FORMAT(' XMIN=',F8.4//' XMAX=',F8.4//' YMIN=',F8.4//' YMAX=',F8.4)
1065 FORMAT(F9.5)
1070 FORMAT(' NEW XMIN ?')
1080 FORMAT(' NEW XMAX ?')
1090 FORMAT(' NEW YMIN ?')
1100 FORMAT(' NEW YMAX ?')
1110 FORMAT(' NEW ROW ?')
1120 FORMAT(' NEW COL ?')
1130 FORMAT(I3)
END
SUBROUTINE STABDER(NVEL,V)

C This subroutine computes the dimensional, and some non-dimensional
C stability derivatives for use in developing the reduced order model.
C
CHARACTER*1 QCHANGE
INTEGER N,NRB,NS,NC
COMMON/INDEX/N, NRB, NS, NC
REAL K1,K2
COMPLEX GF1,GF2
DIMENSION GF1(20,20),GF2(20,20)
COMMON/AERO/K1,K2,GF1,GF2,PI
REAL VO,RHOO,SREF,CBAR,ALPHA0,H0,ZETA0,DELTA0,BR
DIMENSION ZETA0(15),DELTA0(5)
COMMON/IC/VO,RHOO,SREF,CBAR,ALPHA0,H0,ZETA0,DELTA0,BR
REAL ZALPHA,MALPHA,ZALPHDT,MALPHDT,ZQ,MQ,
QFZALPHA,FZALPDT,FZETQ,ZZETA,MZETA,
QZZETADT,MZETADT,ZZETDDT,MZETDDT,
QFZETZET,FZETZDT,FZETZDDT,
QFZETDEL,FZETDDT,FZETDDDT,
QFDELZET,FDELZDT,FDELZDDT,
QZDELTA,MDELTA,MDELTA,FDELDEL,FDELDEL,FDELDEL,
QDELDEL,FDELDEL,FDELDEL,VEL
DIMENSION FZALPHA(15),FZALPDT(15),FZETQ(15),ZZETA(15),MZETA(15),
QZZETADT(15),MZETADT(15),ZZETDDT(15),MZETDDT(15),
QFZETZET(15,15),FZETZDT(15,15),FZETZDDT(15,15),
QFZETDEL(15,5),FZETDDT(15,5),FZETDDDT(15,5),
QFDELZET(5,15),FDELZDT(5,15),FDELZDDT(5,15),
QZDELTA(5),MDELTA(5),FDELDEL(5),FDALPDT(5),FDALPDT(5),FDALPDT(5),
QDELDEL(5),MDELDEL(5),ZDELDEL(5),MDELDEL(5),
QDELDEL(5,5),FDELDDL(5,5),FDELDDL(5,5),FDELDDL(5,5),FDELDDL(5,5),FDELDDL(5,5),
COMMON/DERIV/ZALPHA,MALPHA,ZALPHDT,MALPHDT,ZQ,MQ,FZALPHA,FZALPDT,
QFZETQ,ZZETA,MZETA,ZZETADT,MZETADT,ZZETDDT,MZETDDT,FZETZET,
QFZETZET,FZETZDT,FZETDEL,FZETDDT,FZETDDDT,FDELZET,FDELZDT,FDELZDDT,
QZDELTA,MDELTA,MDELTA,FDELDEL,FDELDEL,FDELDEL,FDELDEL,VEL
REAL CMALPHA,CMALPHA,CMALPDT,CMALPDT,CMQ,CZQ,CMZDELTA,
QDELDEL,MODE
DIMENSION MODE(20),CMDELTA(5),CZDELTA(5)

C If a velocity root locus is desired, then the velocity takes on
C the velocities desired, otherwise the initial velocity is used.
C
IF (NVEL.EQ.0) THEN
VEL=VO
ELSE
VEL=V
ENDIF
WRITE(*,203) VEL,RHOO
WRITE(4,203) VEL,RHOO
QBAR=0.5*RHOO*VEL**2
IF (VEL.EQ.0.0) THEN
  QBAR=0.0
  VEL=0.001
ENDIF
IF (NRB.EQ.0) GOTO 29
MODE(1)=HO
MODE(2)=ALPHAO
29 CONTINUE
IF (NS.EQ.0) GOTO 1
DO 1 I=1,NS
  J=I+NRB
  MODE(J)=ZETAO(I)
1 CONTINUE
IF (NC.EQ.0) GOTO 2
DO 2 I=1,NC
  J=I+NRB+NS
  MODE(J)=DELTAO(I)
2 CONTINUE
C
C Computation of all the dimensional derivatives is accomplished
C below. They are based on the paper by Rodden, AIAA
C and the formula for the generalized forces from FASTOP
C
IF (NRB.EQ.0) GOTO 20
ZALPHA=REAL(GF1(1,2))*QBAR*SREF/(MODE(1)*MODE(2))
MALPHA=REAL(GF1(2,2))*QBAR*SREF*CBAR/(MODE(2)**2)
ZALPHDT=-REAL(GF2(1,1))*(SREF*QBAR*BR**2/(K2**2*VEL))/
  O(MODE(1)**2)
MALPHDT=-REAL(GF2(2,1))*(SREF*CBAR*QBAR*BR**2/(K2**2*VEL))/
  O(MODE(1)*MODE(2))
ZQ=AIMAG(GF2(1,2))*(SREF*QBAR*BR/(K2*VEL))/(MODE(1)*MODE(2))-
  0ZALPHDT
MQ=AIMAG(GF2(2,2))*(SREF*CBAR*QBAR*BR/(K2*VEL))/(MODE(2)**2)-
  OMALPHDT
20 CONTINUE
IF (NS.EQ.0) GOTO 22
DO 3 I=1,NS
  J=I+NRB
  IF (NRB.EQ.0) GOTO 21
  FZALPHA(I)=REAL(GF1(J,2))*QBAR*SREF/(MODE(J)*MODE(2))
  FZALPDT(I)=-REAL(GF2(J,1))*(SREF*QBAR*BR**2/(K2**2*VEL))/
    O(MODE(1)*MODE(J))
  FZETQ(I)=AIMAG(GF2(J,2))*(SREF*QBAR*BR/(K2*VEL))/(MODE(J)*
    OMODE(2))-
  FZETDT(I)-FZALPDT(I)
  ZZETA(I)=REAL(GF1(1,J))*QBAR*SREF/(MODE(1)*MODE(J))
  MZETA(I)=REAL(GF1(2,J))*QBAR*SREF*CBAR/(MODE(2)*MODE(J))
  ZZETADT(I)=AIMAG(GF2(1,J))*(SREF*QBAR*BR/(K2*VEL))/
    O(MODE(1)*MODE(J))
  MZETADT(I)=AIMAG(GF2(2,J))*(SREF*CBAR*QBAR*BR/(K2*VEL))/

O(MODE(2)*MODE(J))
ZZETDDT(I)=-(REAL(GF2(1,J))*(SREF*QBAR/(MODE(1)*MODE(J)))
O-ZZET(A(I))/((K2*VEL/BR)**2)
MZETDDT(I)=-(REAL(GF2(2,J))*(SREF*CBAR*QBAR/
O(MODE(2)*MODE(J)))-MZETA(I))/((K2*VEL/BR)**2)
21 CONTINUE
    DO 4 K=1,NS
    L=K+NRB
    FZETZET(I,K)=REAL(GF1(J,L))*QBAR*SREF/(MODE(J)*MODE(L))
    FZETZDT(I,K)=AIMAG(GF2(J,L))*(SREF*QBAR*BR/(K2*VEL))/
O(MODE(J)*MODE(L))
    FZEDDDT(I,K)=-(REAL(GF2(J,L))*(SREF*QBAR/(MODE(J)*)
20 O(MODE(L)))-ZDELTA(I))/((K2*VEL/BR)**2)
4 CONTINUE
IF (NC.EQ.O) GOTO 5
DO 5 K=1,NC
L=K+NRB+NS
FDDELZET(K,I)=REAL(GF1(L,J))*QBAR*SREF*CBAR/(MODE(L)*MODE(J))
FDZET(Z(I,K)=REAL(GF1(J,L))*QBAR*SREF*CBAR/(MODE(J)*MODE(L))
FDDELZDT(K,I)=AIMAG(GF2(L,J))*(SREF*QBAR*QBAR*BR/(K2*VEL))/
O(MODE(L)*MODE(J))
FDZETDDT(I,K)=AIMAG(GF2(J,L))*(SREF*QBAR/(K2*VEL))/
O(MODE(J)*MODE(L))
FDDELDDT(K,I)=-(REAL(GF2(J,L))*(SREF*QBAR/(MODE(J)*)
O(MODE(L)))-DELTA(I))/((K2*VEL/BR)**2)
5 CONTINUE
3 CONTINUE
22 CONTINUE
IF (NC.EQ.O) GOTO 23
DO 6 I=1,NC
J=I+NRB+NS
IF (NRB.EQ.O) GOTO 25
ZDELTA(I)=REAL(GF1(1,J))*QBAR*SREF/(MODE(1)*MODE(J))
MDELET(I)=REAL(GF1(2,J))*QBAR*SREF*CBAR/(MODE(2)*MODE(J))
FDALPHA(I)=REAL(GF1(J,2))*QBAR*SREF*CBAR/(MODE(J)*MODE(2))
FDALPDT(I)=-REAL(GF2(1,J))*(SREF*CBAR*QBAR*BR**2/
O(K2**2*VEL))/((MODE(1)*MODE(J))
FDETQ(I)=AIMAG(GF2(J,2))*(SREF*CBAR*QBAR/(K2*VEL))/
O(MODE(J)*MODE(2)-FZALPDT(I)
ZDELDT(I)=AIMAG(GF2(J,2))*(SREF*QBAR*BR/(K2*VEL))/
O(MODE(J)*MODE(2))
MDELET(I)=AIMAG(GF2(2,J))*(SREF*CBAR*QBAR*BR/(K2*VEL))/
O(MODE(2)*MODE(J))
ZDELD(I)=-(REAL(GF2(1,J))*(SREF*QBAR/(MODE(1)*MODE(J)))
OZDELTA(I))/((K2*VEL/BR)**2)
MDELD(DT(I)=-(REAL(GF2(2,J))*(SREF*CBAR*QBAR/
O(MODE(2)*MODE(J)))-MDELET(I))/((K2*VEL/BR)**2)
25 CONTINUE
IF (NC.EQ.O) GOTO 24
DO 6 K=1,NC
   L=K+2*NS
   FDDELDEL(I,K)=REAL(GF1(J,L))*QBAR*SREF*CBAR/(MODE(J)*MODE(L))
   FDDELDDT(I,K)=AIMAG(GF2(J,L))*(SREF*CBAR*QBAR*BR/(K2*VEL))/
                      (MODE(J)*MODE(L))
   FDDDEDT(I,K)=-(REAL(GF2(J,L)))*(SREF*CBAR*QBAR*BR/(K2*VEL))
                      *(2*VEL/CBAR)
   O
C Computation of the non-dimensional stability derivatives of interest
C from the dimensional derivatives. These can be used for comparison
C against another aerodynamic code to check for accuracy.
C
   IF (NRB.EQ.0) GOTO 26
   CMALPHA=MALPHA/(0.5*RHO0*VEL**2*SREF*CBAR)
   CZALPHA=ZALPHA/(0.5*RHO0*VEL**2*SREF)
   CMALPDT=MALPHDT/(0.5*RHO0*VEL**2*SREF*CBAR)*(2*VEL/CBAR)
   CZALPDT=ZALPHDT/(0.5*RHO0*VEL**2*SREF)*(2*VEL/CBAR)
   CMQ=MQ/(0.5*RHO0*VEL**2*SREF*CBAR)*(2*VEL/CBAR)
   CZQ=Q/(0.5*RHO0*VEL**2*SREF)*(2*VEL/CBAR)
   WRITE(*,200) CMALPHA,CMALPDT,CMQ
   WRITE(*,201) CZALPHA,CZALPDT,CZQ
   WRITE(*,'(A)') ' The control surface derivatives are:'
   WRITE(4,200) CMALPHA,CMALPDT,CMQ
   WRITE(4,201) CZALPHA,CZALPDT,CZQ
   WRITE(4,'(A)') ' The control surface derivatives are:'

6 CONTINUE
23 CONTINUE
24 CONTINUE
6 CONTINUE

26 CONTINUE
DO 7 I=1,NC
   CMDELTA(I)=MDELTA(I)/(0.5*RHO0*VEL**2*SREF*CBAR)
   CZDELTA(I)=ZDELTA(I)/(0.5*RHO0*VEL**2*SREF)
   WRITE(*,202) I,CMDELTA(I),I,CZDELTA(I)
   WRITE(4,202) I,CMDELTA(I),I,CZDELTA(I)

7 CONTINUE
IF(NSTAB.EQ.0) GOTO 8
   WRITE(*,'(A)') ' Do you want to change any of the derivatives?'
   READ(*,204) QCHANGE
IF(QCHANGE.NE.'Y') GOTO 8
   IF(NRB.EQ.0) GOTO 27
   WRITE(*,'(A)') ' CMalpha:'
   READ(*,*) CMALPHA
   MALPHA=CMALPHA*QBAR*SREF*CBAR
   WRITE(*,'(A)') ' CZalpha:'
   READ(*,*) CZALPHA
   ZALPHA=CZALPHA*QBAR*SREF
   WRITE(*,'(A)') ' CMq:'
   READ(*,*) CMQ
   MQ=CMQ*QBAR*SREF*CBAR**2/(2*VEL)
   WRITE(*,'(A)') ' CZq:'
READ(*,*) CZ
ZQ=CZ*SREF*CBAR/(2*VEL)
WRITE(*,'(A)') ' CMalphadt:'
READ(*,*) CMALPDT
MALPHDT=CMALPDT*QBAR*SREF*CBAR**2/(2*VEL)
WRITE(*,'(A)') ' Czalphadt:'
READ(*,*) CZALPDT
ZALPHDT=CZALPDT*QBAR*SREF*CBAR/(2*VEL)
CONTINUE
IF (NC.EQ.0) GOTO 9
DO 9 I=1,NC
WRITE(*,205) I
READ(*,*) CMDELTA(I)
MDELTA(I)=CMDELTA(I)*QBAR*SREF*CBAR
WRITE(*,206) I
READ(*,*) CZDELTA(I)
ZDELTA(I)=CZDELTA(I)*QBAR*SREF*CBAR
9 CONTINUE
IF (NRB.EQ.0) GOTO 28
WRITE(*,'(/A)') 'The new derivatives are:'
WRITE(.,200) CMALPHA,CMALPDT,CMQ
WRITE(.,201) CZALPHA,CZALPDT,CZQ
WRITE(*,'(A)') 'The new control surface derivatives are:'
WRITE(4,200) CMALPHA,CMALPDT,CMQ
WRITE(4,201) CZALPHA,CZALPDT,CZQ
WRITE(4,'(A)') 'The new control surface derivatives are:'
CONTINUE
IF (NC.EQ.0) GOTO 10
DO 10 I=1,NC
CMDELTA(I)=MDELTA(I)/(0.5*RHOO*VEL**2*SREF*CBAR)
CZDELTA(I)=ZDELTA(I)/(0.5*RHOO*VEL**2*SREF)
WRITE(.,202) I,CMDELTA(I),I,CZDELTA(I)
WRITE(4,202) I,CMDELTA(I),I,CZDELTA(I)
10 CONTINUE
CONTINUE
FORMAT(' CMalpha=',E11.4,' CMalphadot=',E11.4,' CMq=',E11.4)
FORMAT(' CZalpha=',E11.4,' CZalphadot=',E11.4,' CZq=',E11.4)
FORMAT(' CMdelta(',I1,')=',E11.4,2)
FORMAT(//,' For a velocity of',F10.4,' ft/sec, and a density of',0F8.5,' slugs/ft**3')
FORMAT(A)
FORMAT(' CMdelta(',I1,'):')
FORMAT(' CZdelta(',I1,'):')
RETURN
END
Bibliography


John Joseph Cerra II was born on 3 October 1960 in Rome, New York. He attended the United States Air Force Academy from June 1978 to June 1982, receiving his Bachelor of Science Degree in Aeronautical Engineering. From July 1982 until May 1985 he worked as a Stability and Control Engineer in the Flight Control Division of The Flight Dynamics Laboratory of the Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base. In May 1985 he entered the Graduate Aeronautical Engineering program at the Air Force Institute of Technology.

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