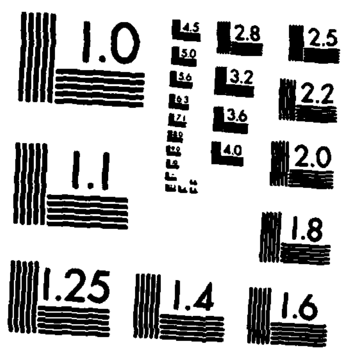
AD-A172 177 THE ESTIMATION OF THE EARTH'S GRAVITY FIELD(U) OHIO STATE UNIV COLUMBUS DEPT OF GEODETIC SCIENCE AND SURVEYING B SZABO JUN 86 OSU/DGSS-369 AFGL-TR-86-0125 1/2

UNCLASSIFIED F19628-82-K-0017 F/G 8/5 NL



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THE ESTIMATION OF THE EARTH'S GRAVITY FIELD

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JUNE 1986

SCIENTIFIC REPORT NO. 11

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFB, MASSACHUSETTS 01731

This technical report has been reviewed and is approved for publication.

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FOR THE COMMANDER

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-86-0125	2. GOVT ACCESSION NO. AD-A172 177	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE ESTIMATION OF THE EARTH'S GRAVITY FIELD		5. TYPE OF REPORT & PERIOD COVERED Scientific Report No. 11
		6. PERFORMING ORG. REPORT NUMBER Report No. 369
7. AUTHOR(s) Bela Szabo		8. CONTRACT OR GRANT NUMBER(s) F19628-82-K-0017
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Geodetic Science and Surveying The Ohio State University Columbus, Ohio 43210-1247		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62101F 760003AM
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory Hanscom AFB, Massachusetts 01731 Contract Monitor: Christopher Jekeli/LWG		12. REPORT DATE June, 1986
		13. NUMBER OF PAGES 110
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Geopotential Models; Satellite Altimetry; Gravity Gradiometry; Integral formulae; Gravity at Altitudes; Regional, Local Gravity Field; Satellite to Satellite Tracking; Spectral properties of Gravity Data; Collocation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report reviews the various methods for the description of the Earth's gravity field from direct and/or indirect observations. Geopotential models produced by various organizations and in use during the past 15 years are discussed in detail. Recent and future programs for the improvement of global gravity fields are reviewed and the expected improvements from new observation and data processing techniques are estimated.

The regional and local gravity field is also reviewed. The various data types and their spectral properties, the sensitivities of the different

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20. (cont.) gravimetric quantities to data types are discussed.

The techniques for the estimation of gravimetric quantities and the achievable accuracies are presented (e.g. integral formulae, collocation). The results of recent works in this area by prominent authors are reviewed. The prediction of gravity outside the earth from surface data is discussed in two forms: a) prediction of gravity disturbance at high altitudes and b) upward continuation of gravity anomalies.

The achievable improvements of the high frequency field by airborne gradiometry is summarized utilizing recent investigations.

In the section of "Summary and Recommendations" the expected short- and long-term improvements are summarized and some recommendations are made for data processing and observation method.

FOREWORD

This report was prepared by Mr. Bela Szabo, consultant, under Air Force Contract No. F19628-82-K-0017, The Ohio State University Research Foundation, Project No. 714255, Project Supervisor, Urho A. Uotila, Professor, Department of Geodetic Science and Surveying. The contract covering this research is administered by The Air Force Geophysics Laboratory (AFGL), Hanscom Air Force Base, Massachusetts, with Dr. Christopher Jekeli, Contract Manager.

CONTENTS

1. INTRODUCTION

The description of the earth's gravity field, or in other words, its approximation with mathematical models, is an old interest of physical geodesy and more recently it is a common goal with geophysics in general and with satellite orbit determinations in particular. Before the development of satellite techniques the observation techniques were restricted to terrestrial gravimetry; adequate gravity coverage existed only over industrially well-developed areas. The gravity profiles over the oceans were inadequate in coverage, inefficient, and expensive, in addition to doubtful accuracy of the results. Under these circumstances, the development of a meaningful gravity model was impossible.

The satellite tracking technology opened the potential of geopotential modeling by addition of new types of efficient data collection techniques with global coverage capabilities. With the improvements in the quality, quantity, and distribution of satellite tracking data (Smith 1982); as well as the increase in coverage, refinements in resolution, and improvements in accuracy of surface data (Rapp 1978, 1981, 1983; Rowland 1981); the geopotential models showed significant progress. Satellite tracking contributed to the long wavelength part of the spectrum expressed by improved low degree coefficients; satellite altimetry and improved terrestrial surface information contributed to the medium lengths of the spectrum. New developments in data analysis and combination techniques helped to produce improved geopotential models.

Despite the improvements, large discrepancies still exist between the various models, even between those derived from nearly the same observations. Some are "tailored" for specific satellite orbits, others represent better surface data in specific areas. A general model which is equally adequate for orbit determinations and geodetic use is lacking at the present time; the principal reason is the lack of observation material covering the full spectral range.

In the preceding years programs were organized by various institutions both in the United States and in Europe for a systematic approach with specific goals for the improvement of the geopotential. The Committee on Geodesy of the National Research Council, National Academy of Sciences, U.S.A., in "Applications of a Dedicated Gravitational Satellite Mission", Washington D.C., 1979 suggested research goals for the geodynamic program of NASA. These include a gravitational satellite mission (GRAVSAT) to provide data necessary for the achievement of the objectives of the program (Lerch, 1983). The accuracies required with resolutions at 100 to 3000km were 2.5 to 10mgal for gravity anomalies and 10cm accuracy for the oceanic geoid. Later NASA developed a "Geopotential Research Program Plan" (NASA 1982) with specific goals for gravity: ±1mgal accuracy with 100km resolution for the gravity field; 5cm accuracy with 100km resolution for the geoid (Murphy, 1983). This program is divided into three phases: a) interim field model improvements, b) geopotential research mission, and c) advanced mission. Currently all three phases are progressing simultaneously and they will be discussed later in this report.

The European Space Agency (ESA) also prepared a plan called Space Laser Low Orbit Mission (SLALOM) to obtain global data coverage for improved geopotential modeling.

Current geopotential models contain the low and medium frequency parts of the spectrum, because the data types (satellite tracking data and $1° \times 1°$ mean anomalies) do not contain high and very high frequency information. Theoretically each data type contains the total frequency range, however in reality the measuring process acts as a bandpass filter limiting the range of the spectrum (Schwarz, 1984). Therefore, the combination of different types of measurements is necessary to obtain a homogeneous spectral resolution. Data types containing high frequency are: $5' \times 5'$ anomalies, deflections of the vertical, inertial survey data, and airborne gradiometry. The very high frequency field can be obtained from point gravity measurements, inertial surveys, airborne gradiometry, height data, and surface density information.

The available high frequency gravity data covering a limited area are usually used for the determination of some functionals of the anomalous potential in a local area or at some selected points. These functionals can be expressed mathematically by integral formulae (space domain) or by spherical harmonic series (frequency domain). Theoretically the two types of formulae are equivalent, but in reality they are different because of the differences in the types and other characteristics of the gravity information used for the computations. The quality of the estimation of a local or regional gravity field depends on several factors such as: a) the sensitivity of the gravimetric quantity to be estimated to the given type of data set, b) the density of the data set, and c) area of the coverage and measurement accuracy. Various functionals are sensitive to different frequency ranges represented by different types of data: for example, in accurate geoid determination the low frequency data are dominant; in accurate determination of the radial component of the second order gravity gradient the high and very high frequency data are required.

Recently local gravity data sets and elevation data are available in gridded form and high degree geopotential models can be used as reference fields. As a result, the higher frequency features of the gravity field can be analyzed in local areas. The problem in this case, as in the case of combination of global data sets, is the weighting of data sets in a combination solution.

In the following sections of this report the approximation of the external gravity potential of the earth is discussed in detail. The description of the global gravity field by spherical harmonic series includes most of the gravity models published since the late 1970's. The observation programs and techniques utilized now and in the planning stage for future use are described. The present techniques and potential improvements for adjustment and data combination for gravity modeling are summarized.

Regarding the local and regional gravity field estimation the following topics are discussed:

(a) the spectral properties of various data types

(b) estimation of gravimetric quantities in local and regional areas.

The estimation techniques covered by the reviews of several recent studies include the use of modified integral formulae and of the collocation method.

Due to the specific interest in this topic, the prediction of gravity disturbance at high altitude is also covered.

Considering that the high and very high frequency local and regional field improvement is expected from airborne gradiometry, several accuracy and feasibility studies in this area are also reviewed.

2. GEOPOTENTIAL MODELS

The gravitational potential of the earth can be expressed in an earth fixed and earth centered coordinate system by the well known harmonic series:

$$V(r,\phi,\lambda) = \frac{GM}{r}\left[1 + \sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\left(\frac{a}{r}\right)^{\ell}\left(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda\right)\cdot \bar{P}_{\ell m}(\sin\phi)\right]$$

where:

- r,ϕ,λ — are the geocentric coordinates of a point,
- GM — is the product of the gravitational constant (G) and the mass of the earth (M),
- a — is a scale factor, usually the equatorial radius of the reference ellipsoid,
- $\bar{C}_{\ell m},\bar{S}_{\ell m}$ — are a set of fully normalized harmonic coefficients,
- $\bar{P}_{\ell m}$ — are fully normalized Legendre functions.

The coefficients $\bar{C}_{\ell m}$ and $\bar{S}_{\ell m}$ are then determined up to a certain degree and order from analysis of satellite orbit perturbations with or without combination of surface gravity data.

In the following sections some of the recent contributions to the global geopotential model development will be recapitulated in groups arranged according to the originating institutions.

2.1 Goddard Earth Models (GEM) of the Goddard Space Flight Center (NASA), Greenbelt, Maryland, 20771

2.1.1 GEM 9 and GEM 10 Models

GEM9 (Lerch and al. 1979) was derived by using laser tracking data from GEOS-3, LAGEOS and Starlette satellites; S-band measurements on LANDSAT I; and various tracking data on 26 other satellites used in previous GEM models.

These represent about 840,000 satellite tracking measurements, 200,000 of which are laser ranging data. Because of the sensitivity of laser ranging to satellite perturbations observation partials on nine satellites were computed complete through degree and order 22 for the harmonic coefficients. For other satellites, measurements are complete only to degree and order 16. The orbital data and the list of satellites observed are given in Lerch and al. (1979). GEM9 harmonic coefficients are complete to order and degree 20 with selected higher degree terms.

GEM10 (Lerch and al. 1979) contains surface gravity data in the form of 5° × 5° mean anomalies (Rapp 1977). Of the 1654 5° × 5° blocks, 1507 are computed from 1° × 1° values within each block, and 174 5° × 5° mean values were interpolated. These surface date were combined with the satellite information of GEM9. GEM10 is complete to degree and order 22 with selected higher degree terms to degree and order 30.

In the development of GEM9 and GEM10, changes were made in the modeling technique used for previous GEM solutions. One of these changes was the use of a "modified least squares solution" (Lerch and al. 1979). By using Kaula's rule ($\sigma_s = 10^{-5}/\ell^2$) for the coefficients as a constraint, the extension of the solution to order and degree 20 was achieved. The use of least squares collocation not only permitted a larger field but the application of least squares collocation was also required to achieve acceptable results due to the high correlation between several of the higher degree and order coefficients (Lerch and al. 1981). Another modification was the use of less weight for the surface gravity data in GEM10.

Geocentric coordinates for 146 tracking stations were also determined with estimated accuracies of 1-2m in each direction. The equatorial radius of the reference ellipsoid was derived by three different methods, all agreeing within 1m to 6378139m; the value for GM was also determined from laser ranging as GM = 398,600.64 km^3/sec^2. The value of the flattening 1/f = 298.257 (updated by the recovery of J_2).

The GEM9 and 10 gravity fields were evaluated for orbit determination accuracy and compared with other independent information for geoid heights and gravity anomalies. GEOS-3 orbital accuracies are about 1m in radial components for 5 day arc. The accuracies of the coefficients imply commission errors in geoid height ±1.9m RMS for GEM9 and ±1.5m RMS for GEM10.

The harmonic coefficients, station coordinates, results of comparisons and evaluations are tabulated or plotted in the paper: Lerch and al. (1979).

2.1.2 GEM 10B and 10C Models

GEM10B (Lerch and al. 1981) is a model derived by the combination of GEM10 data with about 700 altimetry passes of GEOS-3 globally distributed with a 2° spacing. GEM10B solution was a least squares adjustment without any constraints; it is complete to order and degree 36. The geoid was also computed from the harmonic coefficients using Brun's formula.

GEM10C is an extension of GEM10B beyond degree and order 36 and it is complete to degree and order 180; it has the same harmonic coefficients as GEM10B through degree 36. The extended field was derived from: a) 38,000 1° x 1° mean gravity anomalies from surface measurements (Rapp 1979a) (The geophysically predicted surface anomalies in the Rapp file, predicted by DMA Aerospace Center, were not used); b) GEOS-3 altimeter data in the form of 28,000 1° x 1° mean sea surface values (geoid heights). Over 3000 passes have been used to produce the set of mean surface values. In combining the data, a single value per 1° block was used and altimeter data was preferred over surface measured values. In total, about 50,500 1° x 1° blocks with gravity or altimeter observations were used in the adjustment of GEM10C. Residual geoid heights were computed by subtraction of GEM10B geoid from the 1° global set of geoid heights. The computer program developed by Rizos (1979) was used for the harmonic analysis.

The use of altimeter data resulted in significant improvements in the GEM models. Extensive tests have been made to determine the improvements in terms of radial orbit and geoid accuracies.

The radial accuracy of the models were evaluated by computing GEOS-3 orbits from the same laser ranging data using different gravity models. The difference of the altimeter residuals at the crossover points is a measure of the radial orbital error. A crossover test comparing GEM10 and GEM10B using 348 altimeter crossover points resulted in orbital RMS errors for GEM10 of 1.37m and for GEM10B of 1.00m.

To test the accuracy of the global geoids obtained from these models, the RMS fit between the GEOS-3 altimeter profiles and those computed from the models are listed for several GEM models in Lerch and al. (1981). Residual RMS of fit for 10 "trench arcs" and for 10 "non-trench arcs" are respectively:

	Trench arcs	Non-trench arcs
GEM10	2.87m	1.8m
GEM10B	2.47m	0.94m
GEM10C	1.22m	0.75m

Only a little improvement can be seen for the "trench" passes until high degree coefficients are computed (GEM10C). For the "non-trench" passes, the improvement is 100% in GEM10B versus GEM10. SEASAT profiles were also used for comparison with GEM10B values. The two geoids match within ±1m.

The geodetic parameters computed from GEM10B are very close to current values adopted by the IAG: GM = 398600.44 ± .02 km³/sec², a_e = 6378138 ± 1m, 1/f = 298.257 ± 0.001, and g_e = 978031.5 ± 0.3 mgal.

Considering that GEM10C is an extension of GEM10B by harmonic analysis of the residual 1° × 1° geoid heights, the authors (Lerch and al. 1981) compared these residual geoid heights with values computed from the harmonic coefficients of the "residual" gravity field. The RMS difference was 44cm, which is a very good agreement. The power spectrum of the 180 degree field "shares an excellent continuity" with the GEM10B spectrum for terms beyond degree 36.

2.1.3 GEM L-2 model

A new Goddard Earth Model, GEM L-2, was developed from "satellite only" observations utilizing 2 1/2 years of LAGEOS satellite laser ranging data in combination with tracking data on 30 other satellites in GEM9 (Lerch and al. 1982a). The shape, high density and high altitude (about one earth radius) of LAGEOS eliminates, for all practical purposes, errors due to uncertainties in modeling non-gravitational forces. This model, complete through degree and order 20, contains more that 600,000 laser measurements with more than half of them on LAGEOS. Due to the high altitude of LAGEOS, the long wavelength features are substantially improved; however, the coefficients beyond degree 7 are dominated by GEM9 data. The main purpose of this model was to obtain more precise station positions and in turn, better distances between the stations for NASA's crustal dynamics program and to improve the long wavelength gravity field. The same modified least squares method that was employed for the GEM9 model was utilized for this solution. The method is described in Lerch and al. (1977)

Simultaneously with the harmonic coefficients and station coordinates, the GM term was computed as 398600.607 km^3/sec^2, using c = 299792.5 m/sec as the speed of light. Polar motion and A1-UT1 variations were estimated from 5-day segments of LAGEOS data. Finally earth tides were modeled with Love numbers h = .60, ℓ_2 = .075, and k = .29. The solid earth phase angle was obtained as 2.018°.

The improved accuracies of GEM L-2 coefficients versus those of the GEM9 model through degree and order four are significant. They are tabulated in (Lerch and al. 1982a). An independent verification of these accuracies has been performed by Wagner (1982) using independent long term 24 hour satellite longitude accelerations. GEM L-2 satisfies these accelerations. The 'by degree' contribution of the harmonic coefficients to the total longitude accelerations show that: "while the accelerations are sensitive to harmonics as high as degree 6, the GEM L-2 uncertainties beyond degrees 3 or 4 are not directly tested by these longitude accelerations" (Lerch and al. 1982a).
The improved GEM L-2 resulted in more accurate orbits, and in turn, more accurate station positions. Consequently, the capabilities to improve monitoring tectonic motions by satellite laser ranging were also improved. The polar motion and A1-UT1 variation estimates contributed to the stability of harmonic coefficient solutions and improved positioning accuracy. The polar motion values have a precision of about 10 cm.

The effects of the GEM9 and GEM L-2 models and of the polar motions from B1H and GEM L-2 on station positioning were compared. For the comparison the entire LAGEOS data set (1979 through 1981) has been divided into two independent sets by splitting each month's data into two 15-day segments. This way the two solutions contain independent data, span the same time interval, and average out any resulting plate motion over these base lines (distances). Comparison of 28 distances between eight stations were made using GEM9 and GEM L-2 station coordinates and with GEM L-2 and B1H polar motion. The RMS error for the 28 lines from GEM9 was 7.2 cm; the same from GEM L-2 model and polar motion was only 1.8 cm.

In summary, the laser ranging on LAGEOS and the GEM L-2 model resulted in: a) an improved method for tracking station positioning, b) a great improvement of the harmonic coefficients of the gravity field through degree and order 4, and c) an improvement of LAGEOS orbit errors from 1m with GEM9 to about 30 cm with GEM L-2 (F.J. Lerch and al. 1982a, 1984).

2.2 The Ohio State University (Rapp) Models

2.2.1 The Rapp 1978 Model

In this combination solution the following satellite, terrestrial gravity, and altimeter data were used (Rapp 1978):

— The potential coefficients of GEM9 (Lerch and al. 1979) derived from satellite tracking data up to degree and order 20 were used in two adjustments: the first adjusted the coefficients up to degree 8 and the second to degree 12.

— The second type of data was a set of 1° × 1° mean anomalies consisting of 39,405 land anomalies and 29,478 1° × 1° mean values derived from GEOS-3 altimeter data (Rapp 1978a). After editing and replacing oceanic estimated values with altimeter anomalies, where available, a total of 50,650 1° × 1° block values were retained in the merged set. For the missing 14150 anomalies, zero values were used with a standard deviation of ±30 mgal.

The combination method essentially is a weighted adjustment of the harmonic coefficients computed from the mean anomalies and those obtained from the satellite model. The results are a set of consistent harmonic coefficients and mean anomalies. Details of the method are given in Kaula (1966), Rapp (1968), Snowden and Rapp (1968), and Rapp (1978).

Each solution resulted in the adjusted harmonic coefficients (to degree 8 and 12 respectively) and a set of 64800 1° × 1° adjusted mean anomalies. Each set of the mean anomalies were developed into harmonic coefficients to degree 30 and compared to the coefficients of the GEM9 solution. The differences are tabulated in terms of: root mean square coefficient differences, percentage differences, root mean square undulation, and root mean square anomaly differences. For the degree 8 solution the maximum undulation difference is 67cm at degree 8 and for the 12 degree solution it is 91cm at degree 11. The coefficients between 9 and 12 of the degree 8 solution disagree with the GEM9 coefficients more that those of the 12 degree solution. Beyond that the differences are about the same for both solutions.

The degree 8 and 12 solutions in terms of the implied coefficients show maximum differences at degree 9 through 14; beyond that the differences are small. The percentage differences are: at degree 8, 3%; at degree 10, 16%; at degree 12, 26%; at degree 14, 11%; at degree 16, 8%; and at degree 30, 5%. This shows that the higher degree terms are not sensitive to the highest degree used in the adjustment.

The adjusted anomalies of the degree 12 solution were developed into harmonic coefficients to degree 60. The degree variances and the root mean square coefficient variations implied by these coefficients were compared to GEM9 adjusted $1° \times 1°$ mean anomalies (Rapp 1977), and to the geoid spectrum from altimetry by Wagner (1978) Kaula's rule ($10^{-5}/\ell^2$) gave variations too large with respect to the Rapp solution. An excellent agreement was shown with Wagner's results from the analysis of altimeter data. The author notes that an adjustment to degree 20 would improve the solution, especially for satellite orbit computation,s but it would take too much space and time on the OSU computer. The effects of the correction terms due to the spherical approximation, neglect of the topography in computing the mean anomalies, and the atmosphere were not considered. Rapp considers as most critical the terrain correction, which reached 6% at degree 39 in his previous study of 5° anomalies solution (Rapp 1977).

The 12 degree adjustments' mean anomalies were converted later to potential harmonic coefficients to degree 180 using an efficient algorithm of Rizos (1979) as discussed by Rapp (1979a). The deficiency of this 180×180 set of coefficients was that the coefficients above 12 degrees (from GEM9) were omitted from the adjustment, therefore making it inadequate for orbit computations. Studies and experimental computations were carried out (Rapp 1980) to facilitate the incorporation of higher degree coefficients without significant additional computer requirements. An approximate procedure using certain assumptions was used to generate a set of 180×180 coefficients; this was compared to the rigorous solution (Rapp 1978). The average percentage difference was 8.6%, the root mean square undulation difference was $\pm$ 80cm and the root mean square anomaly difference was 2 mgal. The percentage difference was 6.0% for degrees 2 through 12, 15.4% for 25 through 36, and 8% for higher degrees. Relative to the data noise percentage error (about 50%) the difference between the rigorous and the approximate solutions was considered very small. The approximate combination method was used for Rapp's following model (Rapp 1981).

2.2.2 <u>The Rapp 1981 Model</u>

This model used the most recent available input material in an approximate type of adjustment. A selected set of harmonic coefficients up to order and degree 36 were combined with the new 64800 1° × 1° surface mean anomalies. The results were the adjusted 1° × 1° anomalies and the adjusted coefficients corresponding to the a priori coefficients. The adjusted anomalies were developed into spherical harmonic coefficients through order and degree 180.

<u>The satellite observations</u> are represented by a set of harmonic coefficients merged from selected solutions. For these "a priori" coefficients all available solutions were considered. These models are briefly described in Rapp's report (1981). Likewise the considered zonal coefficients and resonance terms are identified in the report. Before selection and merging of the coefficients, 1° × 1° mean anomalies were computed from each set and compared with a terrestrial and GEOS-3 1° × 1° data set. The differences between the terrestrial and harmonic coefficient values are due mainly to the difference in the frequency constraint of the two types of data. The small differences between the various harmonic solutions was considered not to be significant. The merge procedure was to form a weighted average for a coefficient from the PGS S-2, PGS 1331, PGSL-1, and the "miscellaneous" coefficient. The weighting of the PGSL-1 and miscellaneous coefficients was done by the use of the standard deviations for each coefficient. For the other two sets the standard deviations were taken to be 0.9 of the corresponding GEM9 coefficient. If it did not exist, the standard deviation of the miscellaneous coefficient was used.

The final set of coefficients, called "SET 1", was checked by comparing the computed 1° × 1° anomalies with the corresponding surface data. Rapp's (1981) report shows the mean square anomaly differences between GEM9, PGS S-2, PGS S-4, PGS 1331, PGSL 1, and "SET 1" harmonic coefficient fields and the Terrestrial/GEOS-3 Data Set. There is no significant change. There are also no significant differences between the anomaly degree variances for the GEM9 and "SET 1" coefficients (Table 3 of Rapp's 1981 report).

<u>The 1° × 1° mean gravity anomalies</u> used in the combination were obtained by merging an updated set of land 1° × 1° anomalies (42585 values) and 1° × 1° anomalies computed from adjusted SEASAT altimeter data (Rowlands 1981). The merged set contained 56751 anomalies. All anomalies were referred to the 1980 Geodetic Reference System (Moritz 1980). In

comparing the new set to the previous set merged with GEOS-3 mean anomalies, a root mean square difference of ±7.5 mgal was found on the basis of 52972 common values. The standard deviation of all values in the new set is ±10 mgal (Rapp 1981).

In the application of equations relating coefficients and anomalies or the quadrature formula used for the estimation of the potential coefficients, 64800 1° × 1° anomalies are required. For the missing 8049, values the mean anomalies implied by the coefficients to degree 36 of the model "SET 1" were used. Considering the standard deviation of the implied anomalies to be ±30 mgal, the root mean square standard deviation of the complete set is about ±15 mgal. Rapp used ±20 mgal, allowing for the errors in the anomalies computed by geophysical correlations in the unsurveyed areas (the adjustment technique used requires that the accuracy figure for all blocks be assumed the same).

The process of combination: If a set of mean anomalies is given, the coefficients and anomalies can be related by a quadrature formula derived by Colombo (1980). He also showed that the formula gives results almost as good as those of more complicated optimal estimation techniques for the harmonic coefficients (Rapp 1981). The combination procedure used by Rapp was, in essence, the computation of a weighted average of the starting coefficients and those implied by the anomaly data. The advantage of this combination technique is that normal equations are not required; however, simplifications were made relative to the observation weights for the anomalies in order to make a solution to degree 36. It was necessary to assume that all the 1° × 1° mean anomalies have the same accuracy. The principle of the adjustment was initiated by Kaula (1966), and the adjustment equations are given in Rapp's reports (1978 and 1981).

Results: The first results of the combination were the 1° × 1° mean anomalies and the adjusted coefficients up to order and degree 36. The results were compared with other recent models and evaluated to assess the quality and the improvements of the new model. A summary of these assessments are given as follows: (1) The accuracy of the geoid undulation by degree, implied by the accuracy of the adjusted coefficients from degree 2 through 36, was ±152cm for the starting coefficients and ±87cm for the adjusted set, (2) the differences between the a priori and the adjusted coefficients are tabulated in the form of average percentage, root mean square undulation and anomaly differences by degrees. The percentage differences are largest at the higher degrees (60% to 80% between degrees 29 to 36), (3) the anomaly degree variances are somewhat higher at the higher degrees of the new solutions.

The adjusted 1° × 1° anomalies were compared to the anomalies of Rapp's 1978 solution. The new - old mean anomaly difference was 0.5 mgal, the RMS difference was ±11 mgal, and the maximum difference was 215 mgal. The absolute value of the residuals of the adjusted anomalies ranges between 0 and 18 mgal; 30884 of the 64800 1° × 1° blocks are between 0 and 2 mgal with only 3827 above 7 mgal. The large residuals are generally correlated with the locations where the 1° × 1° mean values have been geophysically predicted.

The harmonic coefficients were compared to those of: Rapp (1978), GEM9, GEM10B, GEM10C and GRIM3. The percentage difference between the Rapp 1978 and 1981 models is 10% at degree 7 and 23% at degree 10, increasing gradually to 60% at degree 180. The difference between Rapp's 1981 and GEM10C (the other expansion to degree 180 published by Lerch and al., in 1981), is 7% at degree 7, 14% at degree 10, and 120% near degree 180. A large difference in the RMS anomaly of ±7.3 mgal is tabulated between these two 180° × 180° solutions. The anomaly differences between Rapp's (1981) model and the other compared expansions range from ±3.6 to ±9.1 mgal.

The standard deviation of the adjusted coefficients (order and degree 36) is obtained in the adjustment; for the higher degree coefficients (not part of the adjustment), accuracy estimates were assigned composed from data noise (±20 mgal) and sampling error due to the finite size of the anomaly blocks. At degree 50 the percent error due to data noise is 52% while the sampling error is only 4%; at degree 175 the data noise part of the percent error is 138% and the sampling error part is 52%.

Geoid undulation accuracy and gravity anomaly accuracy by degree and cumulativity are plotted on graphs in the report. These are compared to GEM9 and to the a priori coefficients respectively. The percentage error of the adjusted coefficients are shown on a graph with the same errors of GEM9 and the a priori coefficients. This graph shows that the Rapp 1981 solution reaches 100% error at degree 120; the a priori coefficients do so at about degree 30 and GEM9 at about degree 20.

Anomaly degree variances are plotted for GEM10C, GRIM3, and Rapp 81 adjusted coefficients and those implied by Kaula's rule. GRIM3 values are higher than the other two after degree 20. The summations of the degree variances from degree 2 to 36 for each model are the following: GEM10C, 239 mgal; GRIM3, 265 mgal; Rapp 81, 228 mgal. After about degree 50, the degree variances of the GEM10C expansion are lower than Rapp's solution by about 1 mgal.

For testing the model for orbital computations, a number of orbital comparisons were made at Goddard Space Flight Center by Frank Lerch and James Marsh. The results of these tests indicate that Rapp's model performed comparably with some solutions in existence at that time, such as GEM9, GEM10B, and PGSL 1. However the author points out that additional testing is needed to obtain a better picture of the performance of this model in orbital work. It is also stated in the report that the intent was to produce a general field and not one specifically tailored to a satellite.

In the conclusions are mentioned the required improvements generally given also by other authors such as: surface topography corrections for the altimetry data; better mean anomalies for the replacement of geophysically predicted values; and more rigorous combination procedures.

2.3 The Smithsonian Astrophysical Observatory (SAO Gaposchkin) Models

2.3.1 SAO-79 model

This is a description of the earth's gravity field in spherical harmonics through degree and order 30 (Gaposchkin 1980). Three types of data have been used in the combination: satellite tracking, terrestrial gravity observations, and satellite-altimeter data.

Satellite laser-ranging data on 10 satellites from 15 globally distributed stations were used. The accuracy of the observations ranges from 5m (1971) to 5cm for the 1975 data. The normal equations from a previous model (Gaposchkin and Mendes 1977) involving the same satellites and observations were used in the adjustment of this solution. The procedure for orbit analysis is given in Gaposchkin (1979a)

The $1^\circ \times 1^\circ$ mean gravity anomaly data was obtained from the Defense Mapping Agency, Aerospace Center. These data were merged with existing SAO data. In total, 1504 550km - 550km block anomalies were computed by collocation.

The base for altimeter data was 2116 tracks with 442,411 data points. The processed data was sorted into 1° × 1° averages by linear regression and the RMS of the residuals were computed. The bad data with a residual greater than 3σ were removed. This process removed 65 tracks and 15,762 data points. The average σ of the remaining 25,295 1° × 1° geoid heights was 2.1m. The 1° × 1° geoid heights were then formed into 550km × 550km block values by collocation, resulting in 1218 values.

About seven solutions with different weights were made, and the one that gave the best agreement with all data types was adopted. Increasing the influence of one particular type of data usually weakens the agreement with the others; therefore, the determination of the relative weights is a difficult task.

Detailed comparisons of the various solutions with surface gravity data, geoid heights, and satellite orbits are tabulated in Gaposchkin's report (1980). In addition to Gaposchkin's seven solutions, the GEM10 model by Lerch and al. (1977) is also included in these comparisons. The harmonic coefficients of the selected solution are listed in the report. It is also concluded that the model agrees satisfactorily with the surface gravity, altimetry, and satellite ephemerides available at the time of the development. The author also lists deficiencies of the basic information used, such as: a) the satellite normal equations date back to 1976, with "poorer quality in both accuracy and distribution" with respect to "significant new satellite laser-ranging data"; b) it is stated that better reference models, orbit computation programs, and station coordinates would improve the model; c) only a fraction of the altimeter data has been included, and some time variations could be reduced by averaging the data; d) many of the ocean data (observed or predicted) should be replaced by altimeter data and geophysically interpreted data on land areas must be handled with caution.

2.4 The European (GRIM) Models

2.4.1 The GRIM 3 Model (Ch. Reigber and al. 1983a)

For the development of this model the following data were used:

(1) Orbital perturbations for 22 satellites, consisting of: optical tracking data on 18 satellites, laser ranging on 9 satellites, and Doppler measurements

on one satellite. These measurements were combined in 106 arcs of 5 to 25 days, corresponding to 1028 days of continuous tracking.

(2) Condition equations for zonal and resonant harmonics of order 11 to 15, derived from other studies.

(3) Surface gravity data in the form of $1^\circ \times 1^\circ$ mean free air gravity anomalies consisting of (a) 25001 values computed from measured or predicted point values and taken from Rapp's Oct. 1979 file; (b) 27916 values over ocean areas derived from GEOS-3 altimetry, (Rapp's 1980 file).

The solution was obtained by combining the normal equation systems and introducing constraints for some station positions of "special importance" (larger weights). Absolute constraint was introduced to impose well known short inter-station vectors and the connections of the station network of Doppler data (MEDOC) to the laser and optical sites.

The results are spherical harmonic coefficients up to order and degree 36 and coordinates of 95 tracking stations. A detailed gravimetric geoid was computed from the $1^\circ \times 1^\circ$ surface gravity data and the GRIM3 model by superimposition of the geoid computed by the integration of Stocks formula to the long wavelength geoid from the GRIM3 coefficients.

The model has been extensively compared to and evaluated with respect to other recent solutions, such as GEM10B, Rapp 81, and SAO79. The comparisons show that the model is reasonably close to these models, especially to SAO79, in terms of zonal and resonant harmonics of order 15. RMS coefficient differences, anomaly differences, and undulation differences by degree are given in plots between GRIM3 and each of GEM10B, SAO79, and Rapp 81 solutions. The comparisons show larger differences in the long wavelength geoid (up to ℓ = 10); the anomaly differences by degree increase with ℓ. The authors state that: " there is no evidence that one model is better than the other".

The long wavelength differences were analyzed by comparing potential differences at high altitude. This analysis concludes that terms of degree 2 and 3 of GRIM3 are mostly involved. The authors suspect that the low weight given to the predicted gravity data is one of the reasons, in addition to a lack of enough high and well observed satellite data (such as LAGEOS). The geoid heights were compared with those of GEM10B. The results show the following differences: maximum, 14.7m; minimum, 15.8m; mean, 0.03m; RMS, 3.7m. The extreme differences occur where only predicted data was available. The differences over ocean areas are small (RMS ≈ 1.5m). Both solutions depend in these areas on GEOS 3 data, but they also indicate the good quality of the anomalies converted from GEOS-3 geoid heights.

The geoid height over ocean areas were compared with SEASAT independent geoid heights along selected profiles. For these comparisons, profiles from GEM10B and SAO79 were also used. The conclusion of this test is that GEM10B fits the SEASAT geoid better than GRIM3 and that the differences have long wavelength components due to the error in the low degree harmonics previously discussed.

For orbit computation GRIM3 solution with GEM10B model using GRIM3 and Tapley (1980) station coordinates respectively. GEOS 1, GEOS 3, LAGEOS, BEACON 3, and Starlette were used. The conclusions of these tests are that orbital errors are quite large with both models; radial and cross-track errors are very close, but errors in the along-track component are smaller with the GRIM3 model.

The authors conclude that the model is satisfactory globally, although they recognize deficiencies - due to poor or missing gravity data - of some low degree and order terms.

2.4.2 <u>GRIM 3B Model (Ch. Reigber and al. 1983b)</u>

The data used for the original solution of GRIM3 was supplemented with the following additional data:

(a) 16 months of LAGEOS laser tracking data

(b) $1^\circ \times 1^\circ$ SEASAT altimeter data (Rapp)

(c) $1^\circ \times 1^\circ$ land gravity anomalies from Rapp's 1983 file

(d) recent zonal and resonant harmonics.

The new solution resulted in a complete set of spherical harmonic coefficients to degree and order 36, and the coordinates of 109 tracking sites.

A total of 151 arcs of laser and optical measurements to 21 satellites were used in this solution. 63 arcs were LAGEOS arcs observed from 20 laser tracking stations between January 1980 and June 1981. For the LAGEOS orbit analysis in addition to the earth's gravity and lunar-solar attractions models for earth tides, ocean tides, the earth albedo and along track accelerations were used. For the coefficients all partials were computed according to the sensitivity of a particular orbit to perturbations related to specific coefficients. For LAGEOS all partials for harmonics of degree and order 16 were computed, and for the zonals up to degree 19.

Observation equations for even and odd zonal harmonics as derived by various authors were included in the solution. In total, 112 condition equations were used for zonal harmonics. For resonant harmonics of order 11 to 15, 236 observation equations were added as derived by various authors between 1974-1981.

As was mentioned previously, the surface gravity data was contributed by Rapp, and consisted of 18504 land mean anomalies for 1° × 1° blocks and 38,345 1° × 1° sea anomalies determined from SEASAT altimetry covering oceanic and coastal areas between 72°N and 72°S latitudes. The general computation method used for the GRIM models is described in detail in Reigber and al. (1983a). Essentially, it is the combination of separate normal equation systems derived from: a) satellite orbit perturbations computed from tracking data of close earth satellites; (b) observation equations for zonal and resonant terms obtained from analysis of secular and long period perturbations of various satellites; and c) surface gravity data in form of 1° × 1° mean anomalies of both land measurements and ocean surface altimetry observations. The normal equations were computed by directly using the 1° × 1° values and their respective uncertainties (5 - 15 mgal).

The results of the solution are: a) fully normalized harmonic coefficients of the potential complete to degree and order 36; b) the geoid computed form the obtained coefficients; and c) the coordinates of 109 tracking stations.

The harmonic coefficients were compared with the coefficients of the GEM10B, GEM L-2, Rapp 81, and SAO79 models; the RMS coefficient, undulation, and anomaly differences by degree are plotted on graphs. Comparisons were made also with observed land gravity anomalies, SEASAT altimetric geoid, gravity derived from satellite to satellite tracking, and for satellite orbit determinations.

From the detailed evaluations and comparisons given in the paper, the authors conclude that due to the large quantity of precise LAGEOS observations, the longest wavelength part of the potential (harmonic coefficients up to order and degree 7) has been considerably improved. A better fit to observed longitude accelerations of 24 hour satellites and to the SEASAT geoid has been obtained as compared to GRIM3. The utilization of GRIM3B improved LAGEOS orbits; however, for Starlette and GEOS 3 the orbits are better with GRIM3 than GRIM3B. A small improvement is seen in the fit to surface gravity data. The coordinates of the 109 tracking sites range in accuracy from 10 meters to several centimeters. The large errors are for the old optical stations and the most accurate are the coordinates of the 20 LAGEOS laser tracking sites. These are compared with the GEM L-2 set. No major difference was found between the two sets of coordinates.

The authors feel that the longest wavelength part of the model is "reliably determined". As far as the rest of the spectrum is concerned, improvements are necessary.

2.4.3 The GRIM3-L1 and the GRIM3-MIP models

Since the derivation of the GRIM3B model, two additional models were derived by the same group: GRIM3-L1 (Reigber and al. 1984), and GRIM3-MIP (Reigber and al. 1984a). These models are essentially recombinations of the observation material used for GRIM3B, with different weighting of particular data sets according to the dedication of the model. GRIM3-MIP (preliminary model) contains Doppler tracking data of TRASIT (three months data from the MEDOC-1 tracking campaign). This model was tailored for the processing of MEDOC-2 Doppler data. Although the description of GRIM3-L1 is still not available in the open literature, in Reigber and al.(1984) it is stated that this model has a well-balanced weighting and it is not tailored to a particular satellite. GRIM3B (Reigber and al. 1983b) is considered a LAGEOS model. The latest three GRIM models: GRIM3B, GRIM3-L1, and GRIM3-MIP, have been compared with GEM10B and GEM L-2 models, with observed gravity data and for orbit determinations. The conclusion is that GRIM3-L1 and GRIM3-MIP give a better fit to any data source than previous solutions (GRIM models). A significant improvement was achieved in the determination of low order harmonics. Further improvements are expected from more accurate and better data coverage (Reigber and al. 1984a)

2.5 Special Models

The Geopotential models reviewed in the preceding sections belong in the group of "general purpose" models intended for both terrestrial use and satellite orbit computations. Most of these models were derived by combination of satellite tracking and various surface gravity data. The weights of the contributing information were usually determined by experimental solutions with the goal that the resulting coefficients satisfy the best all types of input data and fit several satellite orbits. An exception is the GEM9 model, which contains only satellite observations; however, this model provided the satellite tracking data for many subsequent general and special combination solutions. Therefore, the GEM9 and its refined model, the GEM L-2, are discussed in the preceding group of the general models. Special models are developed for the computation of satellite orbits or more specifically "tailored" for the orbit of specific satellites (Gravity models used for the computation of the Navy's navigation satellites, or those developed for GEOS-3 and SEASAT).

2.5.1 Special Models of the Goddard Space Flight Center: PGS-1, PGS-S2, PGS-S3 and PGS-S4 (Lerch and al. 1982b)

The purpose of this development was to improve the geopotential model for the computation of SEASAT ephemerides to achieve a 10cm accuracy for the radial component of the satellite position. The altimeter data precision of SEASAT is somewhat better than 10cm; therefore, the full utilization of the altimetric precision for geodetic and oceanographic work required the 10cm accuracy for the radial position of the spacecraft.

The ephemerides computed with the GEM9 model and the tracking station coordinates by Marsh and al. (1977) indicated an error of 3-5m RMS in the radial position. First, the normal equations of GEM9 were combined with eight 3-day laser orbits of SEASAT. This solution, called PGS-S1, was complete to degree and order 30 with additional terms to degree 36. This solution provided some improvement, but the radial errors remained several meters in magnitude. In the second step of development, the Unifid S. Brand (USB) data for the same eight arcs and for one additional arc of SEASAT data was added to form PGS-S2 model. The radial accuracy of this model was improved; however, large differences showed in accuracy between the regions with adequate tracking station and those with sparse tracking sites or none at all. In the Northwest Atlantic region the accuracy was about 1m, and in the South Atlantic and Pacific areas about 3m. The RMS value was 1.8m.

Analyses indicated that tracking data accuracy is adequate to obtain a 50cm radial accuracy, provided a very accurate force model is given (Lerch and al. 1982). There was no more ground tracking data for the improvement of the geopotential model, thus it was decided to use GEOS-3 altimeter data.

The GEM10B model consisting of GEM9 + GEOS-3 + 5° × 5° surface mean anomalies was used. The normal equations of this model were combined with SEASAT tracking normal equations into a new model, PGS-S3, developed to degree and order 36. The authors state that "a quasi-stationary sea surface topography model was not used, therefore there may be some aliasing within a few low-degree and low-order coefficients of the geopotential due to these unmodeled effects". They consider this effect significant for SEASAT orbit computation at the 10cm level. It is recognized that the global validity of the coefficients may be sacrificed, but it is pointed out that the purpose of the development was not to create a general model, but a specific one for the SEASAT orbit requirements.

The errors of the SEASAT radial positions improved to about 1.2m RMS, a substantial improvement over PGS-S2. This model has been used by the Jet Propulsion Laboratory for the computation of the ephemerides for the final set of the released SEASAT altimeter data. The PGS-S3 coefficients and the coordinates for the laser and USB stations are listed in the report (Lerch and al. 1982b).

The GEOS-3 altimeter data has a number of limitations as compared to the SEASAT data (altimeter precision, lack of data recording system restricting the altimetry to shorter arc lengths). The SEASAT altimeter, with 10cm precision and with a data recording system, permitted continuous coverage of the oceans. A set of 9600 globally distributed observations covering 12 days were combined with the data of PGS-S3 model. This combination resulted in PGS-S4, a set of coefficients (36 × 36) and adjusted station coordinates. This model reduced the SEASAT radial error to about 70cm RMS

The basic technique for the combinations, like for the other GEM models, was a weighted least squares adjustment. The subsequent solutions of this set of models have been progressively produced by the addition of new data. For PGS-S1 and PGS-S2, where only laser and USB data were added, it was necessary to use a "modified least squares approach". This technique is described as "using the a priori statistics of the gravity field to provide stability for the recovery of higher degree coefficients and to reduce the effect of aliasing". The method minimizes both the signal and the noise, thus controlling the over-adjustment which occurs in the case of simple least squares adjustment, when high correlation exists between high degree and order coefficients in the presence of noise (observation residuals). The mathematics is described in Lerch and al. (1979), in the discussion of GEM9. [In the paper "Goddard Earth Models for Oceanographic Applications (GEM10B and 10C, Lerch and al., 1981) the "modified least squares" technique, called "least squares collocation", was also used in the adjustment of GEM10.]

The SEASAT gravity models have been tested extensively regarding their performance in the determination of the radial position of the satellite. One group of these tests was the use of altimeter data at track intersections for the determination of radial position errors; the other was the comparison with the independent orbital computations for SEASAT by the Naval Surface Weapons Center (NSWC) from TRANET/Georeceiver-Doppler data (Colquitt and al. 1980). The altimeter crossover test gave 1.4m RMS for the NSWC "smoothed" ephemeris versus the 1.1m RMS of PGS-S4.

2.5.2 The Rapp 1977 model

Another "special model" is the set of potential coefficients to degree 52 computed by Rapp (1977a) from terrestrial anomalies only. The purpose of this experiment was not to produce the best possible model but to analyze certain aspects of the terrestrial gravity field.

From 38,406 1° x 1° mean free air anomalies, 1507 5° equal area anomalies were computed, with an additional 147 predicted values - a total of 1654 globally distributed anomalies were available. The summation formulae were used to derive the harmonic coefficients. A smoothing operator was used in these computations and was found to significantly effect the higher degree coefficients. For the determination of the terrain effect several different terrain correction models were used. It was indicated that the terrain correction to low degree coefficients is on the order of 10% to 25%. It was found that the corrected potential coefficients did not agree as well as the uncorrected coefficients with the satellite derived GEM7 coefficients. The percentage accuracy by degree: 8.6% at degree 3, 53% at degree 12, 86% at degree 30, and 88% at degree 52.

Mean anomalies were computed from the potential coefficients and compared to the original anomalies. The difference was ±8.6 mgal at degree 16, ±4.7 mgal at degree 36, and ±2.6 mgal at degree 52. Obviously, better coverage and better quality gravity data would have given better results.

2.5.3 Special Models of the Naval Surface Weapons Center

These models consist of harmonic coefficients computed through nineteenth order and degree with some additional terms or deletions in different variations. The models are optimized for the polar orbits of the Navy Navigation Satellites and generally were not intended for use for other satellites and/or terrestrial purposes. The principal geodetic use of these models was for the computation of Doppler satellite ephemerides, and they were revised many times from the original inception of the system. Between 1967 and 1972, the gravity fields designated as NWL-8D, NWL-8H, and NWL-9B have been used for the Doppler system. Some information regarding these models can be found in (Anderle and al. 1976). The gravity fields used since 1972 are listed below with information available in the open literature. Some additional information was obtained from Richard J. Anderle of Naval Surface Weapons Center (NSWC).

2.5.3.1 World Geodetic System 1972 (WGS72) Gravity Model. This model was derived simultaneously with the development of the World Geodetic System 1972 (WGS72) by a Department of Defense (DOD) working group (Seppelin 1974).

The model consists of harmonic coefficients complete through degree and order 19, with additional zonal coefficients through degree 24 and resonance terms through order 27. The data used for the development was that of Doppler observations of the Navy Navigation Satellites (NAVSATs) taken at the semi-permanent Doppler network (TRANET stations). This is about 85% of the Doppler data; 15% was observed by equipment in mobile vans at over 120 worldwide distributed locations; optical observations, laser data and surface mean anomalies in the form of normal equations of the Smithsonian Earth Model II, derived by the Smithsonian Astrophysical Observatory (SAO); and surface gravity data in the form of 410 $10^\circ \times 10^\circ$ equal area mean free air gravity anomalies. Only 45% of these anomalies were computed from observed data. The remaining 55% were derived by "gravity-geophysical correlation techniques".

For each data set, a normal equation matrix was formed, then the normal equation matrices were combined and the resultant matrix solved.

There is no evaluation or comparison of this model with other models in the open literature.

2.5.3.2 NWL10-E Model. This model was introduced for computation of all precise NAVSAT ephemerides in January 1973, replacing the previous gravity field designated at NWL9-B. It is based on satellite observations used for the development of WGS72, and gravity coefficients are computed up to order and degree (n,m) = (19,19), but selected terms below 19th degree were suppressed and higher order coefficients to (n,m) = (29,27) were added (Anderle 1984). This model produces the same NAVSAT orbit as WGS-72 to an accuracy of 1m.

2.5.3.3 NWL10E-1. This model replaced the previous NWL10-E model in June 1977. It is the same as NWL10-E except for the modification of two 27th order resonance coefficients.

2.5.3.4 NWL-IG6. This gravity field was a revision of two pairs of coefficients of each order from the NWL10-E model to obtain orbits which fit GEOS-3 Doppler observations. It was expected that comparison of altimeter measurements on intersecting tracks will reduce the bias to 70cm below the precision of the altimeter (Anderle and al. 1976). This set of coefficients was used for a year or two for GEOS-3 and was subsequently replaced. All previous ephemerides were recomputed using a Goddard model, GEM9 (Anderle 1984).

NWL10E[1]: This model is the NWL10E model with improved 14th and 15th order resonance coefficients. This model was adopted for precise computations of SEASAT orbit by NSWC because of a slight improvement in radial residuals over those obtained with the GEM10 model (Anderle 1984).

2.5.3.5 World Geodetic System 1984 (WGS84). This gravity model is part of the new geodetic system derived by the Department of Defense to replace the WGS72. This model is a combination adjustment of Doppler tracking data of near-earth satellites, laser-ranging data on LAGEOS satellite altimetry data over the oceans, and mean gravity anomalies of $3^\circ \times 3^\circ$ derived from surface observations. The harmonic coefficients complete to degree and order are given in Macomber (1985). The model, however, is complete through degree and order 42 with some higher degree terms (Anderle 1984). A Technical Report describing the WGS84 is in preparation and will be published in 1985.

3. PROGRAMS AND TECHNIQUES FOR THE IMPROVEMENT OF THE GRAVITY FIELD MODELS

The analysis of satellite orbital perturbations and satellite altimetry substantially advanced the description of the earth's gravity field in terms of spherical harmonic series and oceanic geoid undulations and/or mean anomalies. These new types of data covering large areas combined with surface measurements over land areas facilitated the derivation of many different sets of harmonic coefficients expanded to various degrees according to the data used and the purpose of the model. Some of these models are discussed in the previous sections.

One of the principal error sources in satellite orbit computations, and in turn in satellite altimetry, is the gravity model. An illustration of the need for more accurate gravity models is the accuracy of the current altimetry. The SEASAT altimeter instrumental precision is about 10cm, on the other hand, the best radial accuracy of the orbit computed with the best fitting gravity model is 70cm. This is largely due to the uncertainties in the gravity model (Lerch and al. 1982a).

Years ago the large gaps over the oceans (gravimetrically empty areas) hindered the derivation of global gravity models with meaningful accuracy; today the gaps are - excluding polar regions - over land areas such as: (1) physically inaccessible areas (high mountains and deserts) in Africa, parts of Asia, Alaska, and South America; (2) countries where gravity data are classified, even mean values as large as $1^\circ \times 1^\circ$: USSR, Eastern European countries, and China are examples (Balmino 1983, Rapp 1985).

The latest and the best published $1^\circ \times 1^\circ$ mean free anomaly coverage is the "Rapp January 1983 $1^\circ \times 1^\circ$ Data Set" (Rapp 1983). Prof. Rapp and his staff collected, analyzed, and processed gravity data during the past ten years. The data include: direct observations over land and ocean areas, predicted values by topographic-isostatic techniques and other geophysical correlations.

The Jan 83 Rapp file contains 44,513 $1^\circ \times 1^\circ$ mean anomalies on land and sea derived from direct observations or predicted. These data merged with approximately 38000 $1^\circ \times 1^\circ$ anomalies estimated from SEASAT altimeter data resulted in a combined set of 56000-57000 $1^\circ \times 1^\circ$ values used in recent harmonic expansions.

There is no hope that the policy might change in the areas where gravity data is classified. The physically inaccessible areas may be surveyed by airborne gradiometry and/or by aircraft carrying accelerometers of appropriate design and GPS receivers for position and aircraft acceleration measurements. In other areas terrestrial gravity data is likely to be available as local or regional surveys are progressing; however, it cannot be expected that large amounts of data substantially assisting global models will be produced by direct ground/surface techniques.

Substantial improvements of the gravity models can be expected from additional gravity data delivered by satellite programs. Several programs and various sensor instruments are under development with very good potentials. These programs are briefly reviewed in the sequel.

3.1 Satellite Altimetry

The sea surface information provided by satellite altimetry contributed dramatically to the knowledge of the geoid, the gravity anomalies over the oceans and in turn to the improvement of the spherical harmonic geopotential models. In addition to the geodetic aspects, altimeter measurements substantially contributed to oceanography in the fields of: current detection, ocean tides, winds and waves, etc. Through the experience with the Skylab, GEOS-3, and SEASAT, the follow on hardware will permit about 2cm precision over a period of several years (Marsh 1983).

Satellite altimetry over the past 12 to 15 years has become very rich in literature. Several special issues have been published devoted to the various scientific fields related to satellite altimetry. A comprehensive review and a long reference list is given by J.G. Marsh in the Reviews of Geophysics and Space Physics (1983). From the point of view of geodesy, the shape of the mean sea surface approximating the geoid and the recovery of mean gravity anomalies were the primary objectives of the analyses of altimeter data. An estimate of the sea surface relative to the geoid for the North Atlantic Ocean was published by Winch (1981). Similar work was done by Stommel and al. (1978), and for the global ocean surface by Levitus (1982). Numerous analyses have been done to obtain the sea surface heights (geoid) and mean gravity anomalies from both GEOS-3 and SEASAT data. Some of these analyses are:

Rapp (1977, 1979a, 1979b, 1979c, 1982a, 1982b, 1983b), Cruz (1983), Hadgigeorge and al. (1979, 1981), Kearsley (1977), Kahn and al. (1979a, 1979b), Marsh, J.G. and al. (1980), Marsh, J.G. and Martin (1982), Liang (1983), Rowlands (1981), Marsh, J.G. and al. (1983).

Satellite altimeter data were used in combination solutions for global gravity models by: Rapp (1978, 1981), Gaposchkin (1980), Hadgigeorge and Blaha (1981), Lerch and al. (1981, 1982a), Reigber and al. (1983a, 1983b, 1984), and others. These models are reviewed in the preceding sections of this report.

The most extensive gravity anomaly work from altimetry data was performed by Rapp and his research associates at the Ohio State University. Some results of their work is reviewed below: The computation of mean anomalies and geoid heights from GEOS-3 data is described in Rapp (1977, 1979a, 1979b). The altimeter data were received from NASA Wallops Flight Center up to Sept. 1977. After editing, the data set contained 419,216 frame measurements in 1976 arcs. The orbital error and altimeter bias were removed by an adjustment using a first degree polynomial in time for the orbit and bias error, and introducing crossover observation equations (Rapp 1979b). From a set of adjusted *geoid undulations* geoid undulation maps, mean gravity anomalies and mean undulations were produced by least squares collocation in 12,144 $1^\circ \times 1^\circ$ blocks and for 377 blocks of 5°. The accuracy of a $1^\circ \times 1^\circ$ mean anomaly was predicted as ± 7 mgal and it was ± 3 mgal for a 5° block. These figures represented an improvement factor of 2 in relative accuracies obtainable from the GEM9 model.

Rapp and his research associates at the Ohio State University (OSU) analyzed SEASAT altimeter data covering all ocean areas between 70° North and South latitudes. The analysis consisted of: the editing of the basic data produced by the Jet Propulsion Laboratory and provided to OSU by the National Geodetic Survey; the adjustment of the data using the crossing arc criteria; and the recovery of mean gravity anomalies and sea surface heights above the reference ellipsoid. Based on a set of criteria established from previous experience, 1,085,006 values were deleted from the total of 4,429,491 values. A second edit was performed by fitting the altimeter sea surface heights to the geoid undulations derived from Rapp's geopotential model to degree 180 (Rapp 1978). This edit deleted 6420 data points. 1667 duplicate observations were found, so that after editing, 3,336,398 observations remained in 5036 arcs (Rapp 1982b) The adjustment was performed by Rowlands (1981) using the crossing arc technique. Due to the limitations of the computer, first a primary set of global arcs were adjusted, then holding these fixed, four regional areas followed. The average crossover difference before the adjustment was ±1.5m, after the adjustment it was reduced to ±28cm. The adjusted SEASAT surface heights, by ignoring the effects of sea surface topography, can be regarded as geoid undulations. From the undulations 34,973 1° × 1° and 1178 5° × 5° mean anomalies and geoid heights were computed by least squares collocation, previously utilized for GEOS-3 data and described in Rapp (1979a). The comparison with common GEOS-3 values resulted in ±7.8 mgal difference (standard deviation between 27221 common, 1° × 1° anomalies) for the anomalies, which corresponds to ±87cm for the undulations. Comparisons of the common 5° × 5° values gave ±2.2 mgal and ±76cm differences for the anomalies and undulation, respectively (Rapp 1983b). Point sea surface heights computed from the adjusted data were compared with similar GEOS-3 values. The mean difference was 1.3m with standard deviation of ±37cm. Later it was discovered that some crossover data was not used in the adjustment. The northeast Pacific area was primarily affected by this error. It was determined that the effect on 1° × 1° mean anomalies was of the order of ±6 mgal, and on the surface heights ±40cm. Outside this area the errors are significantly smaller (Rapp 1983b). A number of regional adjustments repeating the original Rowland (1981) adjustment were carried out in specific areas. Information on these adjustments are given in Rapp (1982a)

Another work accomplished at OSU with GEOS-3 and SEASAT altimeter data is the "Adjustment and Combination of GEOS-3 and SEASAT Altimeter Data" by Liang (1983). The previous GEOS-3 adjustment of Rapp (1979b) included only 3275 arcs. In 1981 OSU received from NOAA-NGS the 3.5 year GEOS-3 data set including 10,520 arcs from April 1975 to December 1978. This data set is described in detail in Agreen (1982). This revised data set was processed at OSU with the goal that the "new" GEOS-3 data be on the same system with the SEASAT adjustment so that the two sets of data can be combined. A new editing and crossover computing procedure was used which has several advantages versus the previously used procedure at OSU.

The combined set of data is a very dense coverage including all available sea surface height data from both GEOS-3 and SEASAT missions. The combined data set also has a very good distribution due to the different inclinations of the two satellites.

The differences between the adjusted GEOS-3 and adjusted SEASAT 1° × 1° surface heights in the sense of GEOS-3 - SEASAT is 5cm with standard deviation ±52cm (28,810 1° × 1° blocks compared).

The comparison of the combined data set with the adjusted GEOS-3 and the adjusted SEASAT data gave the following results: The average 1° × 1° mean surface height difference, GEOS-3 - combined, considering two different areas, is 3cm with an average standard deviation of ±32cm. The average 1° × 1° mean surface height difference, SEASAT - combined, is -1cm with an average standard deviation of ±27cm.

For the areas of the "old" GEOS-3 coverage Kearsley (1977) prepared 26 geoid undulation maps from the adjusted data by Rapp (1977). The final adjusted data of SEASAT (Rowland 1981, Rapp 1982a) were used by Rapp (1982b) to construct 53 maps of sea surface heights at a contour interval of 2 meters. The heights are referred to the 1980 Geodetic Reference System Ellipsoid: a = 6378137m, f = 1/298.257222. The map is centered of data on a 1° × 1° grid predicted from the adjusted data by a least squares collocation procedure.

The Air Force Geophysical Laboratory (AFGL), formerly the Air Force Cambridge Research Laboratories (AFCRL), developed a short arc adjustment method for the determination of the geoid and the gravity field from satellite altimeter data. This method does not require the precise orbit of the altimeter satellite (Brown, 1973, Blaha 1979, Hadgigeorge and al. 1981). This method was used by AFCRL for the processing of both GEOS-3 and SEASAT altimeter observations, and it was upgraded to combine altimeter and gravity anomaly data. GEOS-3 measurements were combined with a set of 1654 equal area 5° mean anomalies (Rapp 1977a), resulting in a 14 × 14 solution in spherical harmonics and geoid and gravity anomalies on a 5° × 5° grid (Hadgigeorge and Blaha 1979). These adjustments (14 × 14) are termed as "first phase" adjustments and they are used as reference fields for "second phase" solutions consisting of point mass and collocation techniques (Blaha 1984, Blaha and al. 1984). The results of the two "second phase" adjustments (using as observations the residuals of the "first phase" 14 × 14 adjustment) represent a 2° resolution, corresponding "very approximately" to a spherical harmonic expansion of 90 × 90. The results on a 2° × 2° equilateral grid were densified into data on a 1° × 1° geographical grid by an "errorless collocation". The comparison of the results for geoid undulations and gravity anomalies was accomplished in 13 large oceanic blocks, containing 12,934 grid points (Bessette and Hadgigeorge 1984). The r.m.s. difference in geoid undulation is 0.45m and the anomaly r.m.s. difference is 2.8 mgal. The average magnitudes of the differences are 33cm and 2.0 mgal respectively.

Efforts are in progress for the improvement of the precision of the altimeter system. These systems and the expected improvements are described in detail by McGoogan and al. (1982), and others in the field of radar and radio sciences.

One essential requirement, to match the altimeter precision (currently 10cm), is the improvement of the radial orbit accuracy of the altimeter satellite. This depends on the improvement of current gravity potential models, improvement in the global distribution and accuracy of tracking of the satellite, and on the elimination of non-gravitational force effects (air drag, radiation pressure, etc.).

A number of studies have performed and published in a special issue of the Journal of the Astronautical Sciences [28(4) Oct.-Dec 1980] on the orbital accuracy of SEASAT. The results can be summarized as follows: The orbital errors of GEOS-3 have been systematically reduced for SEASAT by the improvements of the geopotential models, due to satellite altimetry data. The several meters error in GEOS-3 radial distance was reduced to 75cm - 1.5m r.m.s. in the case of SEASAT. Analyses by Marsh, J.G. and Williamson (1980) and other studies also confirmed an apparent 4 meter difference in the Z components of tracking station coordinates of Goddard Space Flight Center and Naval Surface Weapons Center SEASAT ephemerides. The same study indicated that additional analyses, in correlation with laser and unified S-band tracking data, are expected to yield an orbital accuracy of 50cm r.m.s.

There are two new altimeter missions in development in the U.S.: The GEOSAT satellite of the Navy (Kilgus and al. 1981) and the TOPEX of NASA. The GEOSAT, launched in 1985, carried a SEASAT type of altimeter with a three years' lifetime and 18 months of nominal mission time. The TOPEX mission according to the plans will carry an altimeter with a ±2cm precision and will yield an accuracy of ±14 cm along a measurement grid (J.G. Marsh, 1973). A detailed analysis of the expected TOPEX mission, contemplated fro 1990, is given in Christensen (1982). For the tracking of TOPEX and other future altimeter missions the development of a new improved TRANET Doppler network is planned. A new tracking system is under development at the Jet Propulsion Laboratory (JPL) using the Global Positioning system (GPS). These, together with some improvements in the modeling of the force fields, may achieve a 10cm accuracy in the the radial component of the orbits of altimeter satellites.

3.2 Satellite-to-Satellite Tracking (SST)

The technique of the satellite-to-satellite tracking for direct measurement of the earth's gravity field originated in the Apollo program. Muller and Sjogren (1968) used the range rate tracking data of the lunar orbiters as "direct observations" of the acceleration of gravity along the line of site between the tracking station on Earth and the satellites orbiting the moon. These observations showed circular mass concentrations ("mascons") over the flatlands of the moon (Colombo 1981b). The adaptation of this concept to the Earth's gravity field, was the "high-low" two satellite configuration, realized by the "low" orbiting GEOS-3 (8400km) tracked by Doppler from ATS6 (40,000km) geosynchronous satellite. Wolf (1969) proposed a pair of satellites in a "low -low" configuration, where two satellites on a low, circular polar orbit following as close as possible, would measure the range rate between the two spacecraft. This arrangement will yield a high density global set of gravity data covering land and sea without any gaps. The measurements will provide the medium wavelength spectrum of the gravity field, (higher degree spherical harmonics) which cannot be determined accurately from ground based tracking. The enormous potential of this technique generated interest and research to resolve theoretical and practical problems. Studies were made to optimize satellite arrangements, develop data reduction methods and to estimate achievable accuracies. Some of these studies are: Schwarz (1970, 1972), Hajela (1974, 1978, 1979), Douglas and al. (1980), Rummel and al. (1976), Marsh and al. (1981, 1984). The above studies have been concerned with the recovery of medium and short wavelength gravity field in regional or local areas, utilizing GEOS-3 and ATS-6 "high - low" observations. As the advantages of the "low - low" configuration became obvious, more and more studies and error analyses were prepared on this topic.

Rummel (1980) estimated simultaneously orbital parameters and the gravity field in a study of "low - low" SST experiment, assuming an "optimal" situation: *range rate precision* ± 10^-6 ms^-1, satellite distance of 250km, and an orbit altitude of 200km. The a posteriori standard deviations were: ±0.9m for point geoid heights, ±0.7m *for* geoid height *differences* and 6 *to* 7 mgal *for* 1° × 1° mean gravity anomalies. These values compare well with the results of GEOS-3 altimetry. A study by Jekeli and Rapp (1980) estimated the accuracy of mean anomalies and geoid undulations for various block sizes based on an assumed mission. The accuracy was defined as a commission error due to measurement noise propagation and a truncation error. For a "low - low" mission of six months duration, at an altitude of 160km and range rate data noise of ±1 micrometer/sec. for four second integration time, it was estimated ±2.3 mgal accuracy for 1° × 1° mean anomalies and 4.3cm for 1° × 1° mean geoid undulations.

Colombo (1981b) made an error analysis of the global geopotential model from a "low - low" SST mission. It was assumed that two drag compensated (DISCOS system) satellites were used. The differentiated range - rate signal was considered as equal to the line of sight component of the gravitational acceleration. Both the least-squares adjustment and the least squares collocation adjustments were used. The main results are summed up by Colombo as follows:

IF

(1) the two satellites move on the same polar circular orbit at 160km altitude, at a distance of 300km from each other;

(2) the accuracy of the averaged range rate is $\sqrt{2} \times 10^{-6}$ ms^{-1}, the averaging interval is 4s;

(3) residual data are used with respect to a reference model of specified accuracy, complete to degree and order 20,

THEN:

(1) The relative error in the potential coefficients could be better than 1% up to degree n = 130, better than 10% up to n = 210, and better than 50% up to n = 270;

(2) The accuracy of point geoid undulation implied by the coefficients could be better than 5cm RMS in the band from 300km to 40030km (total error) and better than 10cm in the band from 140km to 3000km (total error).

It is noted, that above degree 200, the accuracies predicted by collocation are significantly better than those by least squares. The report considers that the principles of the study can be applied to actual SST data, to obtain a high resolution harmonic model. Colombo recommends as adequate technique the method in Colombo (1981a) and that more attention is to be paid to global data reduction.

To escape from the large inversion problems of the least squares collocation, Kaula (1983) developed an analytic scheme for inferring the variations of the gravity field from SST. Each revolution is Fourier analyzed separately, from north pole to north pole and east-west variations are inferred by requiring that the potentials at the poles agree. The method for data analysis is outlined in the paper and it is applied as a test to a pair of satellites at 160km and 100km spacing, with 5° data point interval (72 data points per revolution). The assumed gravity field (Gaposchkin 1980) of tesseral harmonics to the eight degree was completely recovered in three iterations over 64 revolutions. It is demonstrated that data points at regular intervals provide the opportunity for utilizing techniques without massive matrix inversions.

On the basis of the successful demonstration of the SST technique by the results of the GEOS-3/ATS-6 and of the "low - low" simulation studies the planning of gravitational satellite missions started in the mid 1970's. NASA in the US prepared the GRAVSAT program, mentioned earlier and the European Space Agency (ESA) came up with a plan called Space Laser Low Orbit Mission (SLALOM) which involves the simultaneous tracking by laser interferometry, from the space shuttle, of two reflecting spheres to measure their relative velocities with respect to the shuttle and to each other (Colombo 1981b)

The GRAVSAT mission was reformulated into the NASA Geopotential Research Mission (GRM) with more ambitious accuracy and resolution goals. The GRM mission will be discussed later in this report.

3.3 Satellite Gradiometry.

An approach with a very good potential for the determination of the Earth's global gravity field with higher resolution and accuracy is a gradiometer of high precision capable to operate from a satellite. At least two instruments with different approaches and a number of simulation studies for the assessment of their performances are available. It is expected that an operational system may be available sometime in the 1990's.

One of the instruments under development by Paik (1981) is the cryogenic gravity gradiometer. Superconductivity and the availability of SQUID (Superconducting Quantum Interference Device) technology for low noise amplifiers allowed the construction of a very sensitive and low drift gravity gradiometer, operating at liquid helium temperatures. By employing accelerometer pairs and differencing the outputs along each coordinate axis, a "tensor gravity gradiometer" is obtained, which measures all the independent components of the gradient tensor simultaneously (five independent plus one component). The goal for the sensitivity of the instrument is 10^{-4}E [1E (Eötvös unit) = 10^{-9} S^{-2}]

Another satellite-gradiometer is under study by a group of French scientists, under the name of PROJECT GRADIO. It is described by Balmino and al. (1984). This concept employs a gradiometer composed of micro-accelerometers to measure the components of the gravity gradient tensor from a specially constructed satellite with a disturbance compensation system. The satellite will be launched to a circular, polar orbit of 200 to 250km altitude for a six month's mission. The accuracy obtainable for the terrestrial gravity field is estimated to be 2 to 5 mgal for a resolution of 150 to 300km from measurement precision of 0.01E. The expected launch data is 1991 - 1992.

Simulation studies were carried out to estimate the accuracy and resolution of the gravity field from satellite gradiometry. Some of these studies are: Breakwell (1979), Jekeli and Rapp (1980), Colombo and Kleusberg (1983), Rummel and Colombo (1983), Rapp (1985).

Assuming a satellite gradiometer with an accuracy of 10^{-4}E (rms), Colombo and Kleusberg (1983) estimated the attainable accuracies for the gravity field and for the distance of the satellite to the center of the Earth. A six month mission in polar orbit at 200km altitude with data taken every three seconds, would provide data for computation of the harmonic coefficients up to degree and order 300 with less than 50% error and improve the coefficients up to degree 30 by up to four orders of magnitude compared to existing values.

A simulation study suggested that an adjustment based on gradiometer data could produce orbital accuracy in radial distance 10cm or better, if the orbits are 2000km high and first an improved gravity model to degree 30 is achieved. This may substantially improve satellite altimetry.

Rummel and Colombo (1983) in their study assume a satellite-gradiometer measuring all six second-order derivatives of the potential. With this information it is possible to separate: gravity field recovery, altitude of the satellite, and orbit determination. This allows the use of fast spherical harmonic analysis like in Colombo (1981a). The simulation study explains the separation of the orbit and the gravity parameter estimation process. The gravity field was generated only from Zonal potential coefficients up to n = 300 (The resulting gravity becomes invariant in longitude). The actual orbit displacement and the potential coefficients were almost exactly recovered after two iterations.

Recently Rapp (1985) computed the expected accuracies for anomalies and undulations for a six month mission with a radial component gradiometer ($\pm 10^{-4}$E) at an altitude of 130km. The results in terms of accuracy versus resolution are plotted on two graphs and compared to an SST mission (± 1 μm/sec.) and to two models, OSU (Rapp) 81 and GEM L-2. On a global average, the SST mission improves the current models by a factor of 10. The gradiometer mission improves the SST results by about 60% with the exception of the very short wavelengths (i.e., < 20km). Taken from the anomaly graph, the gradiometer could obtain an accuracy of ± 3 mgal with 50km resolution, the SST error is three times as large. From the geoid undulation graph for 50km resolution, the accuracy of the gradiometer geoid is ± 2cm versus the SST geoid accuracy of ± 18cm for the same resolution.

3.4 NASA's Geopotential Research Program (1982)

As it was mentioned in the introduction, the program is divided into three areas: a) interim field model improvements; b) geopotential research mission; and c) advanced mission. All three areas will be discussed briefly based mainly on J.P. Murphy's paper (1983), presented at the I.A.G. Symposium in Hamburg Germany (1983).

The objective of the "Interim Field Model Improvements" phase is the achievement of a "factor of two" improvement in the current GEM models. In essence, this is a rework and analysis of older observations by utilizing current and improved techniques, updated force models, recently obtained direct or indirect gravity observations, etc. In 1982 a Gravity Field Workshop was held at the Goddard Space Flight Center to specify the required actions for the achievement of the improvements. For the improvement of the long wavelength features of the models, the recommended steps were: 1) reprocessing of selected laser, S-band, Doppler, and SST data using improved models for measurement corrections and for satellite motions; 2) incorporation of orbital information obtained from the analysis of the motion of synchronous and of satellites in resonant orbits; 3) processing of additional Doppler data from satellites with different orbital inclinations; 4) acquiring and processing laser data from four satellites in a new tracking program; 5) reprocessing available optical data with improved star catalogue (F5); and 6) incorporation of orbit perturbation information on the Zonal coefficients.

For the improvement of the short wavelength features, the recommendations of the workshop were: 1) improvement of data collection in surface gravimetry and incorporation of data into the models; 2) full utilization of satellite altimetry data with state of the art ephemerides, ocean topography, tidal and other environmental corrections. In addition, the use of optimal estimation and adjustment techniques and of the latest in computer technology is recommended.

The purpose of the Geopotential Research Mission (GRM) is to map the gravity and magnetic fields with accuracies and resolutions necessary for requirements in geodetic solid earth and ocean science applications. Considering only gravity requirements the geomagnetic aspects of the program are omitted from this summary.

The GRM system will consist of two satellites to be launched by the space shuttle and maneuvered into a circular polar orbit of 160km altitude. The separation along the orbit between the two satellites could vary between 100 and 600km. The two satellites are equipped with a Doppler satellite-to-satellite tracking system, with a design capability of one micron per second relative range rate measurements. When all the components of the system error budget are taken into account, the overall performance is better than .1 micrometer per second for four second averages, one order of magnitude smaller than the design goal of one micrometer per second (Murphy 1983). Air drag and other non-gravitational forces are cancelled by the Disturbance Compensation System (DISCOS). this system keeps the proof mass and the satellite in a purely gravitational orbit. The mission is planned for six months, during which time there will be six tracks through each 1° x 1° square. The tentative launch date is 1992-1993 (Rapp 1985). As it was stated previously, the goal of the mission is to obtain the gravity field globally to ±1 mgal and the geoid accurate to 5cm with 100km resolutions for both. Most significantly, this data set will cover the currently inaccessible and gravimetrically "unknown" areas. The resolution and the accuracy of the gravity field will allow the computation of 0.5° x 0.5° mean anomalies globally, and in turn, the expansion of harmonic coefficients, to high degree. A better gravity model will produce more accurate orbits for low satellites used for satellite altimetry and for other gravity sensors as well. An improved geoid will help studies of sea surface topography and the conversion of ellipsoidal heights, produced by space techniques, into geoid heights. In the field of geophysics, the wavelength features of the gravity field, between 100 and 1000km will help the understanding of the mass variations in the upper mantle (lithospheric plate motions). Shorter wavelength gravity anomaly features may permit the study of mantle convection and density inhomogeneities (Taylor and al. 1983).

The third phase of NASA's Geopotential Research Program Plan is the "Advanced Mission". This phase consists of advanced instrument and system studies for the purpose of achieving the "ultimate" in gravity and magnetic field surveys from space. According to present concepts, the space vehicle for these surveys will be a Tethered Satellite System (TSS) and one of the candidates for the gravity sensor is the cryogenic gradiometer under development by Paik and al. (1981). With the potential of the space shuttle, the tethering concept is under investigation for a number of applications. Currently, NASA, in a cooperative program with Italy, is working on the advanced development of a tethered vehicle and tether mechanism for possible test flight in 1987. A number of potential applications are described by Bekey (1983). Gullahorn and al. (1984) gave a presentation on: Feasibility of Gravity Gradient Measurements from a tethered Subsatellite Platform (EOS Vol. 65, No. 28 Abstract). NASA's objective with the gradient measurements from a tethered satellite is the achievement of higher resolution in gravity measurements.

3.5 Data Processing and Adjustment Techniques

Most of the algorithms for estimation of harmonic coefficients from various gravity related observations are based on the standard least squares adjustment. From the observations of each type of measurements, normal equations are formed and these are combined by a weighted least squares adjustment. The increasing quantity of the observations and the requirements for higher degree of coefficients made the use of rigorous least squares procedures impractical. The same situation applies to the use of the simple least squares collocation, due to the large matrices which must be inverted.

Various authors designed and used different simplified approaches (see previous sections on global gravity models) to alleviate the computer burden; however, this could not be done without paying some price for the shortening of the computations. If the expansion is truncated, say at degree 36, which can be handled comfortably by the average computer, the finer detail of the gravity field is not adequately represented in the results. Therefore, the usefulness of the model is limited. If the developments are carried to higher degree and order, say 180, the percentage errors of the coefficients usually attain 100% around degree 100 of the expansion.

Recently several studies have been accomplished to find solutions which could improve the present situation and together with the improvement of the computer technology (supercomputers) would be capable to handle the ever-increasing amount of observations and to produce accurate high degree solutions.

Colombo (1979, 1981a), utilizing the symmetries of data on spherical grids and relations between spherical harmonics and Fourier series, developed efficient algorithms for the estimation of spherical harmonics to high degree. Algorithms are derived for the evaluation of harmonic coefficients by numerical quadratures, and it is shown that the number of operations is the order of N3 for equal angular grids (N is the number of lines of latitude, or the "Nyquist frequency" of the grid). For the error estimation of the coefficients, Colombo utilizes the error measure of least-squares collocation and derives efficient algorithms for implementation on the sphere. The principle is that for "regular" grids, the variance-covariance matrix of the data consist of "Toepliz-circulant blocks", so it can be both set up and inverted very efficiently (Colombo 1981). Efficient methods for computation of covariances between area means is described, and numerical examples are demonstrated in the report.

A real data set of 5° × 5° mean gravity anomalies obtained from Rapp's (1978) 1° × 1° mean values was used for the demonstration of the use of optimal estimators. The results are compared with Rapp's coefficients obtained by numerical quadratures up to order 36 from the original 1° × 1° data. The collocation solution (Colombo 1981) follows very closely the values of Rapp's solution, considered "true" values because of the much finer 1° × 1° grid.

The Colombo (1981) algorithms for harmonic analysis have been implemented for global 1° × 1° anomalies by Hajela (1984). The original Colombo algorithms were modified before their use for the optimal estimation of the coefficients complete to degree and order 250 and for their error estimates. This was necessary due to the large array size requirements for the 1° × 1° anomalies.

To retain the efficiency in the computation of the coefficients, it was required that the variances of all anomalies, in a latitude band, be equated to the average value for that band. This gives the correct values only for a pair of coefficients $(\bar{C}_{nm}, \bar{S}_{nm})$ for any particular degree and order. The variance for each coefficient in the pair is arbitrarily chosen equal (Colombo 1981, Hajela 1984).

The data set used in this study is the set of harmonic coefficients to degree 180 developed by Rapp (1981), termed data set A. From Rapp's coefficients, Hajela computed a global set of 1° × 1° mean anomalies. These anomalies then were used to derive potential coefficient sets using optimal estimation procedures. For the anomaly data, several different error estimates were assumed. Rapp (1981) also computed a global 1° × 1° anomaly set (64800 values), adjusting a combined set of satellite derived coefficients to degree 36 with a combined terrestrial set of 1° × 1° mean anomalies. This second set was also used in Hajela's study, termed anomaly set B. Several sets of harmonic coefficients were developed from the two sets of global mean anomalies using generally two sets of anomaly error estimates. One set termed "realistic" (error estimate A) was computed as the average variance in each latitude band from estimates of 1° × 1° anomaly errors; anomalies computed from potential coefficients being assigned standard error of 30 mgal. The average anomaly error in a latitude band ranged from 30 to 4 mgal. Error estimate B (idealized version): with no variation in latitude and an assigned values of ±5 mgal. Several other anomaly error estimates were also used in various tests.

Anomaly set B with error estimates A and B was used for the computation of potential coefficients complete to degree and order 250. These are compared with Rapp's (1981) set of coefficients (data set A). The improvement of the coefficients, per degree, due to the use of optimal estimation over Rapp's coefficients using de-smoothing factors is about: 8% at degree 60, 11% at degree 120, and 33% at degree 180. In optimal estimation, the total percentage error per degree does not exceed 100% at degree 250.

There is no discontinuity in the degree variances of optimally estimated coefficients at degree 60 and 180. Colombo (1981a) tested his algorithms for optimal estimation of potential coefficients with a global set of 5° x 5° anomalies. He estimated that two hours of CPU times will be required for the computation of coefficients to degree 180 with a global set of 1° x 1° anomalies on an Amdahl 470 V/6-11 computer. With a faster 470 V/8 computer, Hajela used about 60 minutes for the computation of coefficients to degree 250.

S.C. Bose and al. (1983) Applied Science Analytics, Inc., Canoga Park, California, examined the optimal estimation of harmonic coefficients to high degree from gravity anomalies available globally. The basic method is the least-squares collocation and similarly to Colombo's (1979,1981) approach, consist of the exploitation of the grid structure of the covariance matrix, thereby dramatically reducing the computational requirements. This study is an extension of Colombo's work. The authors fully explore the structure and effects of "rotational symmetry", i.e., the sample need not be uniform over all latitudes in the grid; and consider meridianal and equatorial symmetries for the reduction of the computations. The study shows that each parallel should not have the same number of data samples. At high latitude the distances between sample points become less and less. This crowding of sampling is ill-conditioning of the data covariance matrix. When data will be available over polar regions, a complete global solution of the gravity field will be necessary; therefore the "thinning" of the sample data and avoiding the ill-conditioning problem without increasing computational cost is significant. The study presents the concept and outlines the general approach to the solution. Computational cost comparisons for different types of symmetries are tabulated.

Rapp (1984) reviewed some of the methods that can be used for the combination of satellite and terrestrial gravity data for the development of high degree spherical harmonic coefficients. After a short outline of an adjustment process using satellite data only and a discussion of gravity anomalies and the boundary condition, (defining gravity anomalies) the report outlines a combination method of satellite and terrestrial data by combining the normal equations of the two types of data in a least squares sense. In this case no corrections are applied to the terrestrial anomalies due to the topography; they are interpreted as surface free-air anomalies and the series are being evaluated at the surface.

The method for the combination of satellite and terrestrial measurements is based on the orthogonality relationship between gravity anomalies and harmonic coefficients. The concept was initiated by Kaula (1966) and utilized by Rapp (1978, 1981). In the previous applications of this method, two assumptions were made, namely the spherical approximation and that the gravity anomalies are on the surface of the reference ellipsoid. Using Pellinen's (1983) relationship between the ℓ and m component of the disturbing potential and gravity anomaly on the ellipsoid in terms of the coefficients $C_{\ell m}$, $S_{\ell m}$, Rapp derived expressions for corrections to the coefficients computed by the orthogonality relationship from the anomalies on the ellipsoid. Applying the corrections, the results will be consistent with the coefficients obtained from satellite data.

The assumption that the gravity anomalies are on the surface of the ellipsoid is also inaccurate. They are free-air anomalies on the surface of the earth. Therefore, for the application of the procedure described in the previous paragraph, they should downward continued to the ellipsoid. To avoid the problems regarding the validity of the analytical continuation technique, the study suggests upward continuation of the surface anomalies to a bounding sphere (The Brillouin sphere) enclosing all the topography. This would be an alternate approach to both the downward continuation and the ellipsoidal problems. Correction terms were derived to the orthogonality formula for the effect of the upward continuation. For the 180 × 180 Rapp (1981) coefficients, using a global set of 1° × 1° mean elevations, correction terms were computed. The comparison of the results with the 1981values gave the percentage error in the coefficients caused by the neglect of elevation effects (use of uncorrected anomalies) and of the ellipticity (use of spherical approximation formulas). From the plot of the errors it can be seen that the elevation effect is small: 1% at low degrees and slowly rising to 3% at degree 180. The effect of spherical approximation is large: 10% at degree 75, 20% at degree 130, and 31% at degree 180. The anomaly and undulation errors caused by the elevation and ellipticity effects are tabulated. The undulation error by degree is 20cm at degree 2 and drops below 3cm for all degrees above 20. The cumulative undulation error increases slowly from 20 to 34.4cm at degree 180. Anomaly errors gradually increase to 3.12 mgal at 180 degrees. The report points out that undulation errors are the largest in the polar regions and in high mountain areas or regions of high vertical anomaly gradients (Aleutian region), when the errors can reach 80 cm.

Some details and alternate adjustment techniques are discussed for the combination of satellite and terrestrial data using the orthogonality relationship and gravity anomalies on the bounding sphere. The end products are the adjusted potential coefficients and adjusted anomalies on the bounding sphere. These anomalies can be developed into high degree coefficients by Colombo's procedure (Hajela 1984). The combination solution can be accomplished with the rigorous formation of the normal equations. It is suggested in the report that the large computer requirement can be managed by the use of a supercomputer; or Colombo's (1981a) technique, where the unknown coefficients are ordered in a way that the normal equations are block diagonal, reducing the computer time substantially. After the combination solution, a new set of harmonic coefficients can be computed. This expansion can be developed to degree of 250 (Hajela 1984).

Another recommendation of the Rapp (1984) report is to use $0.5^\circ \times 0.5^\circ$ block size for the mean anomalies globally where it is available. This set of anomalies should be upward continued to the bounding sphere. A global set of $0.5^\circ \times 0.5^\circ$ mean anomalies can produce harmonic coefficients to about 360 degrees. Obviously the solution will be deficient in areas lacking data.

4. THE UTILIZATION OF SPHERICAL HARMONIC MODELS IN GEODESY

The availability of spherical harmonic expansions to high degree (Rapp 1978, 1981; Lerch and al. 1981; Hajela 1984) made possible the use of these expansions for many geodetic and geophysical applications (Tscherning 1983). These expansions, together with the expansion of topography and of the potential of the isostatically compensated topography (Rapp 1982) give many computational advantages as compared to the techniques without the use of the series.

The first obvious use of the coefficients of the harmonic expansions of the gravity potential is the computation of quantities of the gravity field such as: gravity anomalies, height anomalies, components of the deflection of the vertical, etc. For the computation of these quantities, very efficient algorithms have been derived and are in general use today (Rizos 1979; Colombo 1981; Rapp 1982; Tscherning and Poder 1982; and Tscherning and al. 1983). In Tscherning and al. (1983), the four above referenced programs are briefly described and intercompared for results and to obtain timing comparisons.

The Rizos (1979) program computed the height and gravity anomaly from harmonic coefficients for points of a two dimensional evenly spaced geographic grid. The area of the computation can be local, regional, or global.

The Colombo (1981) computer program calculates the height and gravity anomalies from potential coefficients for a global grid only at specified latitude and longitude intervals. A subroutine was designed for the computation of area mean or point values. The Legendre functions are first computed for the grid interval. Due to the grid symmetry with respect to the equator, the values are computed for latitudes north of the equator. Then by a Fast Fourier Transform (FFT), sums of series are computed along the latitude rows. This procedure is implemented by subroutine FFTCC of the IMSL subroutine library.

The Rapp (1982) program is for the computation of the height anomaly, gravity anomaly, gravity disturbance, and the components of the deflection of the vertical from spherical harmonic coefficients. The program has been tested with coefficients up to degree 180 and can be extended higher. The program calculates the above listed quantities on a point to point basis.

The Tscherning/Goad Program (Tscherning and al. 1983): The original

Tscherning and Poder (1982) program extends the derivatives of the potential to the second derivatives and uses the Clenshaw summation, which has a numerical advantage compared to the usual methods, consisting of a decrease in the loss of significant digits during the summation. With this algorithm it was possible to evaluate the sum of spherical harmonic series with degree and order 180 on a computer using only 10 1/2 significant digits (Tscherning and al. 1983).

The original Algol programs given in Tscherning and Poder (1982) was translated into Fortran by Goad, adding subroutines needed for the actual computations.

The aforementioned four programs have been intercompared by test runs on the Amdahl 470 V/8 computer. Timing comparisons are tabulated in Tscherning and al. (1983). Considering point to point calculation times, Rapp's and the Tscherning-Goad programs are comparable. Rizos' is fastest, with 0.46 seconds, and Tscherning/Goad next with 1.91 seconds and the Rapp program a distant third with 15.59 seconds (no provision is made in the Rapp program for data given on a uniform longitudinal grid). The time comparison of the computation of height anomalies on a global $1^\circ \times 1^\circ$ grid gave 47 seconds for the Colombo program and 66 seconds for Rizos. In the other two point by point programs (Rapp and Tscherning/Goad) the results computed at one latitude were extrapolated, resulting in relatively poor time.

It can be seen that efficient algorithms exist for the computation of gravimetric quantities from spherical harmonic expansions of high degree. For limited or global coverage of one or all five quantities, the appropriate best fitting program can be selected.

Using high degree and order coefficients, the height anomalies can be computed globally with ± 1.2m standard error. If the degree and order of the coefficients is increased to N = 360, and with some improvements of the coefficients, the standard error of the height anomaly could be improved to 0.5m (Tscherning 1983). The height anomalies can be used for conversion of ellipsoidal heights into normal heights, reduction of distances, etc.

High degree spherical harmonic expansions are used as the reference-base for local gravity field determination, e.g. Forsberg and Tscherning (1981), Lachapelle and Rapp (1982), and Sunkel (1983). In the future, the harmonic series may replace the role of normal potential.

Due to the easy computation of geodetic quantities of the gravity field from harmonic series, the contribution of the high degree expansions can be subtracted from the point values of these quantities. The residuals will have empirical covariance functions for which the first zero value is located at a spherical distance of about 7° for height anomalies, 45' for gravity anomalies, and 20' for the longitude component of the deflection of the vertical (Tscherning, 1983). Therefore, the spherical caps used for the evaluation of integral formulae will be much smaller, and local collocations require a much smaller data collection area. In addition, many elements in the normal equation matrices may be set equal to zero, resulting in only about 1% error and substantial savings in computation.

Another beneficial effect of using high degree harmonic coefficients is that the long wavelength part of the topographic effects are included in the model. For the remaining effects, the topography must be considered only for a short distance from the computation point.

5. THE LOCAL AND REGIONAL GRAVITY FIELD

The description of the gravity field of a limited area (local or regional) usually consists of the determination of some functionals of the anomalous gravity potential in a particular area and/or at selected points. These functionals or gravimetric quantities are: the components of the deflection of the vertical (ξ,η); the height of the geoid (N); the gravity anomaly (Δg) etc. Mathematically these functionals can be represented by integral formulae or by spherical harmonic series. The two types of formulas are equivalent theoretically, but in the practical work they are different due to the differences in the types, spacing, distribution, and accuracy of input data used for the computations (mean or point anomalies for the integral formula and geopotential coefficients for the series of spherical harmonics).

In this section the following topics are reviewed: spectral properties of data types; representation and estimation techniques of the local gravity field by integral formulae and collocation; prediction of gravity outside the earth from surface data and improvement of local and regional gravity fields by airborne gradiometry. Representative recent works in each subject area are reviewed and discussed. It is relied on lectures of the Beijing International Summer School, China, 1984, Schwarz, K.P. (1984) and on the published works of other authors in this subject area, e.g. Forsberg, Tscherning, Sünkel, Cruz, etc.

5.1 "Data Types and Their Spectral Properties"

In his lectures at the Beijing Summer School K.P. Schwarz (1984a) under the above quoted title presents and analyzes a number of topics relevant to the representation of the local, regional and global gravity fields. Various parts of the lecture are summarized and freely quoted in the sequel.

The quality of local gravity field estimation depends on the following factors:

- the density of the available data
- area coverage (global or local)
- measurement accuracy
- the sensitivity of the gravimetric quantity (functional), to be estimated,

to the given data set.

This can be assessed by transforming the measurements into the frequency domain and comparing the approximation obtained to the rigorous representation of series of spherical

harmonics. In view that all functionals and data types can be represented in the form of the series of spherical harmonics, the approximation error is due only to the factors listed above.

The spectral sensitivity of some specific functionals are evaluated with respect to the following frequency ranges:

$$2 < \text{low} < 36$$
$$37 < \text{medium} < 360$$
$$361 < \text{high} < 3600$$
$$36001 < \text{very high} < 36{,}000$$

The spectral sensitivities of geoid height (N), gravity anomaly (Δg) and of the second order radial gradient of the anomalous potential (T_{rr}) are computed in percentage of the total value of each spectral range (Table 5.1). The computations were made by the use of the global covariance model of Tscherning and Rapp (1974).

	FREQUENCY RANGES			
FUNCTIONAL	**low**	**med.**	**high**	**v. high**
N	99.2%	0.8%	0.0%	0.0%
Δg	22.5%	41.9%	32.7%	2.8%
T_{rr}	0.0%	0.8%	39.0%	60.2%

Table 5.1. Spectral sensitivity of some functionals. After K.P. Schwarz (1984)

It can be seen from table 5.1 that for accurate determination of N the low and medium frequencies are required with high accuracy, and for the T_{rr} the high and very high frequency ranges are dominant. There is an exact correspondence between the frequency and the spacial domain; from the spectral sensitivity of a functional, the spacial representation of the frequency range in gridded form can be determined (data density).

The local gravity field estimation involves a specific data distribution, and the question is how well a specific functional can be determined from it. For example, consider a set of errorless gridded data. The spectral representation of this spacial data set is a truncated series of spherical harmonics. The estimated coefficients will contain systematic errors because the data contains the effects of all frequencies. This effect is the aliasing and it is a source of errors of the spectral representation. If there is only a local data set, the low frequencies cannot be properly determined. This is known as spectral leakage. The integration formulas will have this problem if the integration is extended only over a limited area. The measuring errors of the data add additional complications.

The steps to obtain a good estimation of a local gravity field according to Schwarz's lecture are: first, the spectral sensitivity of the functional to be computed should be analyzed; second, data types which contribute most to the sensitive part of the spectrum should be selected. The aliasing and spectral leakage problems can be minimized by existing standard procedures. The effect of measuring errors can be reduced if their statistical behavior is known. In addition to the methods listed above, the problems of combining several data types with different spectral and statistical characteristics must be solved. This leads to the characterization of current and future data types.

The current and future data types are characterized (Table 5.2) according to: spectral resolution, data density, data coverage, data distribution, and noise spectrum. These characteristics are useful for optimal data combination for the gravity field spectrum. Theoretically, each data type contains the total spectrum; however, in practice the measuring process acts as a bandpass filter limiting the range of the spectrum. Therefore, a single data type cannot resolve the complete spectrum and it is necessary to combine different types of measurements to obtain a homogenous spectral resolution.

It can be seen from Table 5.2 that the low frequency information comes from satellite observations and no other source is in sight for this frequency range. The medium range is currently determined from $1^\circ \times 1^\circ$ mean gravity anomalies on land combined with satellite altimeter data over the oceans. This frequency range will be improved by satellite to satellite gradiometry, if these programs will become operational. The above types are regular in distribution and global in coverage, therefore their use in integral formulas (space domain)

DATA TYPE / CHARACTERISTICS	CURRENT							FUTURE					
	Satellite Perturbation	*Altimeter Data*	*Mean Gravity Anomalies* 1°×1°	*Mean Gravity Anomalies* 5'×5'	*Point Gravity Anomalies*	*Deflection of Vertical*	*Height Data*	*Satellite to Satellite*	*Satellite Gradiometry*	*Inertial*	*Airborne Gradiometry*	*GPS Level*	*Surface Densities*
SPECTRAL RESOLUTION													
low	*												
medium		*	*					*	*			*	
high				*		*				*	*		
very high					*		*			*	*		*
DATA DENSITY													
low	*												
medium		*	*			*			*			*	
high				*	*		*	*			*		*
DATA COVERAGE													
global	*	*	*					*	*				
regional				*	*		*				*	*	
local						*				*			*
DATA DISTRIBUTION													
regular	*	*	*	*			*	*	*	*	*		
irregular					*	*						*	*
DATA ACCURACY (% of RMS value)													
< 5%		*			*		*					*	
5% 20%	*		*	*		*	*	*	*	*	*		*
> 20%													

Table 5.2. Characteristics of Geodetic Data (From K.P. Schwarz 1984)

or in series of spherical harmonics (frequency domain) should yield the same accuracy. All the other data types cover only part of the space domain, therefore integral formulas will give better results.

The high frequency spectrum is currently resolved by mean anomalies of 5' × 5', or deflections of the vertical. In the future, airborne gradiometry and inertial surveys may contribute to this frequency range, provided the projected accuracies of 1 mgal and 0."3 will be realized. The very high frequency range currently is very little known. Airborne gradiometry and height data may improve the situation in the future. Some of the very high frequency range is removed from gravity anomalies by terrain corrections computed from height data on a grid (1km × 1km). This removes the frequencies dependent on the topography. The remaining part represents the anomaly variations caused by density changes. At present, very little is known on the spectral power of density changes in the very high frequency range. For the purpose of optimal spacing of gravity and height measurements, a detailed numerical analysis is recommended by K.P. Schwarz. A spectral analysis of high density gravity and height data would indicate whether or not the upper part of the very high frequency spectrum can be computed with sufficient accuracy from height data only. If yes, the interval of gravity measurements can be larger than those of height data, and the existing height data can be used with benefit. Airborne gradiometer and surface density data would provide in the future material for spectral analysis of this frequency range.

The following parts of the lecture (Chapters 5 and 6) discuss the concepts and mathematical treatment of spectral analysis of discrete data which apply to local gravity field approximation. The one dimensional case is discussed first, then the extension to a two dimensional surface. Correlation and spectral density functions are described. The main formulas are given with derivations and some of the literature is listed for more details. For the one dimensional case: Popoulis (1965), Bendat and Miller (1982). For a good discussion of all essential formulas for the two-dimensional planar case, I recommend Sideris (1984), and for the spherical case, Colombo (1981a). These topics have also been treated in textbooks and in the related literature and will not be discussed here. It should be noted, however, the use of convolutions theorems in gravity field approximations. The convolutions of two functions, in general, can be interpreted as the filtering of one function by the other.

Examples are the Stokes and Vening-Meinesz integrals. These two integrals can be considered as two different filters applied to the same gravity anomaly function (Δg). The theorem states that "convolution in the space domain can be replaced by multiplication in the spectral domain and vice versa". All of the basic integrals in gravity field estimation can be expressed as convolution integrals; therefore this theorem permits the replacement of integration (space domain) by multiplication in the frequency domain. This technique is very advantageous because recently gravity and height data are available frequently in gridded form, and Fast Fourier Transform (FFT) allow performance of discrete Fourier transformations and convolutions much more rapidly than integration. Detail for the Fourier approach to Stokes' and Vening-Meinesz' integral are given in Jordan (1978), and details of the terrain correction integral in Sideris (1984)

The Spectral Analysis of Global Gravity Data is discussed next in connection with the use of a set of harmonic coefficients, as a reference field for local or regional gravity data.

The concepts of spectral analysis extended to the sphere, the series of spherical harmonics, and their convergence, is widely covered in the literature, e.g. Moritz (1980). The application of the concepts to the geodetic problems are given in detail in Meissl (1971). Detailed numerical formulas and extension of FFT to the sphere is discussed in Colombo (1981). In principle, the transform pair for the representation of the anomalous gravity field on the sphere is equivalent to the transform pair representing the gravity field on the plane; however, the topology differences of the two surfaces cause the direct application of FFT techniques to the sphere to be impossible. In Colombo (1981a), it is shown that spherical harmonics are finite sums of two-dimensional Fourier harmonics, and this can be used to design efficient algorithms for spectral analysis.

Global data available at the present time for spectral analysis are: Satellite orbital perturbations, satellite altimeter data, and mean gravity anomaly data. The combination of these three types of very different information on the gravity field is still not on the desired optimal level. In a typical procedure, the coefficients are determined in steps, e.g. Rapp (1981). The weighting is of crucial importance in the combination of global and local gravity data. To obtain proper weighting for a global model, the determination of its error spectrum is necessary; this is difficult because the global model is a combination of different types of data with different error characteristics. If a truncated series of harmonic coefficients is used as a reference for local approximation to obtain a proper weighting between the two data sets, the following questions arise (Schwarz 1984):

(a) "How much of the total spectral power of the anomalous gravity field is contained in a solution of degree and order N?"

(b) "How good is the approximation given by a specific model; i.e., which error spectrum is associated with the global solution?"

Practical procedures for weighted combination of harmonic coefficients and local data are discussed in Wenzel (1982) and Sjoberg (1981).

Returning to the data sets used in a global solution, the characteristics of these data is briefly discussed. Orbital perturbations can be considered to be upward continued geoid undulations. Due to the attenuation, the short wavelength part is smoothed out with the altitude. This effect and measurement distribution problems are the principal reasons why only low order harmonics can be determined from this type of data.

From altimeter data, the geoid over the oceans is obtained, provided the sea surface topography can be neglected. The data are regular and the measurement precision is sufficient for about 50km half wavelength resolution (Cruz 1983). This would be adequate for a series expansion of 360 degree and order, but a similar set of data is not available over land areas. Currently $1^\circ \times 1^\circ$ mean anomalies are developed from the altimeter data and merged with $1^\circ \times 1^\circ$ mean gravity anomalies over land. Two particular problems emerge with this data set: The effect of averaging and aliasing. Averaging changes the coefficients. To recover the spherical harmonic coefficients of the point gravity anomaly function the smoothing by averaging has to be reversed. This 'desmoothing' can be done by optimal factors derived by Colombo (1981a). The other is the aliasing, which is not eliminated by "desmoothing". These errors are about 50% of the actual coefficient at the Nyquist frequency and it is larger above. Therefore, there is a need to replace mean values with smoothed values free of aliasing. Simulation studies to used filters to estimated mean values performed by Jekeli (1981) eliminated the leakage problem for the mean values, but not the aliasing. Schwarz's suggestion for a possible way to obtain a set of smoothed values free of aliasing is: "to use a bandpass filter on the power spectrum, obtain the autocovariance function by

Fourier Transforming the results to the space domain, and estimate smoothed values in the center of the block using this function". In addition to aliasing, missing mean anomaly blocks and the errors in the data affect the coefficients.

Recently, local data sets and height data are frequently available in gridded form, and high degree geopotential models as reference fields are available to provide low frequency features, therefore, the analysis of higher frequency features in local areas became possible. Theoretically if a perfect 36 × 36 solution is available, a 10° × 10° field of local data centered on the computation point is required for the resolution of all frequencies above degree 36. To improve the wavelengths in the medium range and to help spectral leakage, a radius of 20 degrees would be required instead of the 5 degree radius for a 10° × 10° local field. Studies by Lachapelle and Rapp (1982) indicate, however, that the extension of the local area does not improve the long wavelength features.

Here, like in the case of combination of global data sets, the weighting or accounting for the actual accuracy of data sets in a combination solution is a problem. Wenzel (1982) proposed the use of the error covariance functions of various data types for the derivation of spectral weighting functions for an optimal data combination. This approach is, in the judgment of Schwarz, "most promising". An advantage of the method is that it can be extended to other combination methods, i.e. it is not restricted to the integration method only.

Schwarz analyzed the covariance function behavior obtained by different methods over the same test areas, using 5' × 5' mean anomalies referenced to GEM-10 5° area blocks, in North America. The analysis was made by both the space domain and spectral methods. The difference in the parameters of autocovariance functions are tabulated for 16 sample areas in Schwarz (1984). Two dimensional FFT methods have been used, which, in addition to the efficiency, provide information on the local spectrum. New results on the degree variances in the $200 < n < 2900$ are also presented.

5.2 Estimation of Gravimetric Quantities in Local and Regional Areas.

The estimation of the functionals of the anomalous gravity potential such as deflections of the vertical, geoid undulations, mean gravity anomalies, etc.

from the limited local or regional information usually is obtained by the use of integral formulae. Best known examples are the Stokes and the Vening-Meinesz integrals. Theoretically, the integration should be extended over the whole earth and the gravity anomaly should be known at every point. Practically, these conditions cannot be satisfied; therefore, these integral formulae are usually modified to accommodate in combination, other gravity related data, contributing in some form the information outside of the zone of integration. The types of these information are: harmonic coefficients of a series expansion of the global field; information on the topography in the form of digitized elevation data, or information on the isostatic compensation of the topographic masses. By the use of topographic information in the form of gridded digital terrain models (DTMs), the local gravity field can be smoothed by removing the effect of the topography calculated from the DTM. Terrain corrected, gravity data are of course applicable not only in the integral methods, but also in the collocation technique for the estimation of the gravity field.

Since spherical harmonic expansions to degree and order 180 are available (Rapp 1978, 1981; Lerch and al 1981) and expansions of the topography, of the rock equivalent topography, and of the isostatically compensated topography have also become available (Rapp 1982, Grasegger and Wotruba, 1983), these, together with a fairly good coverage of DTM data, allowed a substantial number of studies and numerical demonstrations for the estimation of regional and local gravity fields. Some of these are: Lachapelle (1984, 1979); Lachapelle, G. and K.P. Schwarz (1980); Lachapelle, G. and A. Mainville (1979); Tscherning (1984, 1983, 1983a); K.P. Schwarz (1984a, 1983); Schwarz et al (1983); Schwarz, K.P. and G. Lachapelle (1980); Sünkel (1984, 1984a, 1983a, 1983b); Sünkel, H. and G. Kraiger 1983); Forsberg (1984, 1984a); Forsberg and Tscherning (1981); Rapp and Wichiencharoen (1984); and Moritz (1983, 1980, 1977).

5.2.1. Estimation by the use of Modified Integral Formulae.

Lachapelle (1984), in his lecture at the Beijing Summer School, discussed the modifications of the basic integral formulae of Stokes and Vening Meinesz, and of the topography integration formula for the deflection components, to allow for the combination of all available data.

The free air gravity anomalies used in the Stokes and Vening-Meinesz integrals are reduced from the earth's surface to the geoid by the use of the gradient of the normal gravity (0.3086 mgal/m). In mountain areas, the vertical gradient of the actual gravity can differ as much as 0.1 mgal/m. This can seriously affect the accuracies of the results. In the Molodensky approach, the measured gravity remains on the surface of the earth and the normal gravity is computed on the telluroid by applying the normal gradient upward from the ellipsoid. The modified formulae (Moritz 1980, Sec. 48; and Lachapelle 1984) for the height anomaly and for the surface deflection components contain a first-order correction term (g_1) to the anomaly on the earth's surface ($\Delta g^* + g_1$). It is shown in Moritz (1980) that the formula $(g_1)_P$ for an arbitrary point P can be replaced by a given terrain correction formula, if a constant density of topography and a linear correlation between gravity anomaly and height are assumed.

The terrain correction formula (replacing the formula for g_1) reveals that the terrain correction is significant only over a terrain with rough topography; therefore, the uncorrected Stokes and Vening Meinesz formulae (neglecting the term g_1) are fairly accurate approximations to the Molodensky approach in flat areas. The correction terms are significant in mountain areas, particularly for the computation of the deflection of the vertical. For details see Moritz (1980, section 49). Numerical results for terrain corrections for geoid undulations are in Section 3 of the Lachapelle (1984) paper. Four different methods for the combination of Stokes' formula with spherical harmonic expansions are discussed and the results of the computations are tabulated. The spherical harmonic models GEM10B, and GEM10C by Lerch and al (1981) and two models of Rapp (1978, 1981) were used. All except GEM10B are complete to 180 × 180. GEM10B is complete to order and degree 36. The gravity data in form of 5' mean anomaly coverage of North America was used (Lachapelle 1978). Mean topographic heights of 5' were used in the Western Cordillera mountain region for the computation of the terrain effect. The undulations obtained by various methods are compared with satellite Doppler undulations (Tables 4, 5, and 6 of Lachapelle 1984). To show the improvement in accuracy due to the combination of surface gravity data with harmonic expansions, a comparison of Doppler undulations and those computed only from geopotential coefficients is given in Table 3 of the referenced paper. The rms values show improvements when models complete to 180 degree are used. Best results of rms residuals of the order of 1 meter were obtained using Rapp's

1978 model. The GEM10B solution (36 × 36) gave an rms agreement of 1.7 meters. Rapp's 1981 180 × 180 set agreed to the order of 1.2 meters.

An analysis of various modifications of Stokes' function for the improvement of geoid undulation computation accuracy was performed by Jekeli (1981). Another recent study on data requirements and accuracy of geoid undulation computation from gravity data is reported by Kearsley (1984).

The deflections of the vertical are much more sensitive to the gravity anomalies in the vicinity of the computation point than undulations. Therefore, deflections computed from harmonic coefficients only will be less accurate than geoid undulations. A comparison of deflection components of 820 points in Canada computed from harmonic coefficients were compared with astrogeodetic values. (Table 7 in Lachapelle 1984). It can be seen from the comparison of astrogeodetic rms values with harmonic coefficient derived deflection components that the deflections from harmonic coefficients only are very poor. About 20% of the deflection signal comes from the coefficients, while for the undulation signal the contribution of the harmonic coefficients is above 90%.

Several methods for the computation of the components of the deflection of the vertical from the combined effects of gravity potential coefficients, surface gravity and topographic data are reviewed in Lachapelle (1984).

The Vening Meinesz formula was combined with collocation using harmonic coefficients, gravity anomalies, and optionally, astrogeodetic data (Lachapelle 1977). In flat areas of Canada, accuracies of $1''.0$ to $1''.5$ were obtained as rms differences with astrogeodetic values. The radius of the inner zone was 1° containing about 200 5' × 5' mean anomalies. The Vening Meinesz function was unmodified. The modified function does not improve the accuracy in the case of deflections, as it is discussed in Jekeli (1982). The computation software is described in Lachapelle and Mainville (1982).

In mountain areas where local topographic data are available, the visible topography and its isostatic compensation can result in about 80% of the deflection signal, provided that the selected isostatic model is in fair agreement with the reality. Results in the Swiss Alps by Elmiger (1969) and in the North American Western Cordillera by Lachapelle and Mainville (1979) confirm the estimate.

In mountain areas, where astrogeodetic deflections exist in close intervals, a combination of these deflections with topographic-isostatic ones improves the accuracy. Elmiger (1969) obtained about $1''.5$ to $2''.0$ in the Alps. Topographic-isostatic deflections alone may be significantly biased. Lachapelle (1975) combined topographic-isostatic deflections with global data, such as harmonic coefficients of the geopotential for the purpose of reducing large scale systematic effects. The deflection components were the sums of the contributions of: the low degree potential coefficients, the corresponding coefficients of the isostatic reduction potential, and the topographic-isostatic deflection components computed from the detailed topography using the topography integration formula. Tables (9 and 10) in Lachapelle (1984), extracted from Lachapelle and Mainville (1979) give some results obtained by this method in the Canadian and New Mexico (White Sands) mountain areas at selected astrogeodetic deflection stations. The rms differences with astrogeodetic values are given for the topographic-isostatic components and for the total components. In the Canadian Cordillera, mean differences between the topographic-isostatic and astrogeodetic components are $-0''.28$ (ξ) and $0''.21$ (η) and between the total and astrogeodetic components are $0''.26$ (ξ) and $-0''.22$ (η). This indicates that the harmonic coefficients of the geopotential and isostatic reduction potential do not remove significantly regional trends. In the White Sands area, the predicted total deflection components agree better with the astrogeodetic values than with the topographic-isostatic components. Thus, the geopotential and the isostatic reduction potential coefficients contribute substantially to the removal of regional biases. The mean differences between the topographic-isostatic and astrogeodetic components are: $-1''.37$ (ξ) and $-1''.92$ (η). These are reduced to $0''.51$ (ξ) and $-0''.02$ (η) between the total combined and astrogeodetic deflection components.

5.2.2. Comparison of Gravimetric and Satellite Derived Undulations

Gravimetric geoid undulations were computed by three different methods and compared with geoid undulations obtained from satellite Doppler data at the same sites (Rapp and Wichiencharoen 1984). Twenty Doppler sites were selected from the 65 in the United States from the Lachapelle (1979) study, for the comparison of Doppler-derived and gravimetric geoid undulations in North America. Ten of these sites were in the mountainous western United States, where large residual differences were found between the Doppler derived and gravimetric undulations. The other 10 stations were in the relatively flat eastern part of the United States. The original Doppler undulations were corrected for a -0.4 ppm scale change and for a 4m Z axis bias. The gravimetric geoid undulations were computed by: a) combining Rapp (1981) potential coefficients to degree 36 with uncorrected $1^\circ \times 1^\circ$ mean free air gravity anomalies in a 10° cap surrounding each station, b) using Helmert's second condensation procedure for anomaly reduction where terrain corrections and topographic indirect effects are computed (10 stations in the western U.S.), c) the use of potential coefficients to degree 180 (Rapp 1981). The components of the gravimetric undulations and the differences between these and the Doppler-derived values are tabulated. The effect of the terrain correction is of the order of 1.8m and of the indirect effect is 15cm. The mean difference between Doppler and gravimetric undulations for the 10 western stations excluding the terrain correction and indirect effect was 1.65m with standard deviation of the difference of ±0.96m. With the corrections, the mean systematic difference decreased to 0.1m ± 0.93m.

In the eastern U.S. stations, where no terrain corrections are applied, the mean difference between Doppler and gravimetric undulations is 0.42 ± 0.55m.

The spherical harmonic coefficients to degree 180 gave results close to those when uncorrected $1^\circ \times 1^\circ$ mean anomalies were used. For the 10 stations in the western U.S., a difference of 1.27 ± 1.07m was obtained. For the eastern U.S. stations it was 0.45 ± 0.62m.

5.2.3. On the Accuracy of Height Anomalies and Deflections of the Vertical Obtained from Combination of Mean Anomalies and Harmonic Coefficients.

The accuracies of height anomalies and deflections of the vertical obtained by the combination of harmonic coefficients and mean gravity anomalies was investigated by Heck (1983).

It is known that the applications of the Stokes and Vening Meinesz integrals require continuous gravity data over the entire earth. Since the existing global coverage of gravity anomalies is neither continuous nor homogeneous, in practice the integration is limited to a spherical cap around the computation point. Because the gravity data usually is known in terms of mean anomalies of surface blocks, the integrals are replaced by summations.

The outer zones are replaced or represented by a geopotential model. Four types of errors in such combinations are considered in the Heck (1983) paper: 1) error due to the lack of higher degree coefficients, 2) errors in the used set of coefficients, 3) error due to the loss of information by using finite block size, and 4) errors of the mean anomalies. Error analyses reveal that at the zeros of the kernel functions within the spherical integrals, the error functions show local minima. This was the motivation for modifications of the kernel functions by: Molodensky and al (1962), Wong and Gore (1969), Meissl (1971), and Colombo (1977). The various approaches have also been studied by Jekeli (1981), Rapp (1980), Chen (1981), Fell and Karaska (1981) and Heck and Gruninger (1983).

The investigations of Heck show that the errors caused by the first two error sources listed above show distinct minima if the integration radius is confined to the zeros of the kernel functions. This characteristic suggests that the Wong and Gore modification procedure be used for combining gravity anomalies of a spherical cap and geopotential coeff· ients. It is emphasized that the integration radius be extended exactly to the first zero of the modified Stokes and Vening Meinesz kernel functions.

Using a recent geopotential model and mean anomalies of 2 to 10km side length, absolute undulations have a global rms of ±1 to 3cm; relative undulations between points 50 to 100km apart have an accuracy of a few centimeters. The deflections of the vertical have an accuracy to an order of magnitude of 0".2 to 0".3 with 2×2 km^2 mean anomaly elements. The above numerical estimates have been obtained by the use of hypothetical error models; therefore, the results must be considered to represent only the order of magnitude of the errors.

The study emphasized the importance of the consideration of terrain corrections for gravity anomalies in rugged mountainous terrain. Substantial systematic errors may occur using uncorrected anomalies for computation of gravimetric quantities.

5.2.4. Estimation of Gravity Field by Collocation.

The integral procedures, such as Stokes' and Vening Meinesz', use one type of data for the approximation of other functions of the gravity field. Different types of data are frequently available, containing useful information regarding the gravity field. As it is well known, the collocation method is capable of using gravity dependent heterogeneous data to predict any other gravity field quantity. It is also known that there is a strong correlation of the free air anomalies with elevation. This correlation results in a trend in the deflections of the vertical, in the gravity anomalies, etc. which must be removed before the use of the data for interpolation or before applying them in collocation.

5.2.4.1. <u>Forsberg and Tscherning (1981).</u> This paper describes various methods for the computation of terrain effects applicable to the estimation of gravity field quantities by collocation. The various "reduction" methods were tested in two mountainous $1° \times 1°$ areas in New Mexico. Both areas are characterized by a North-South mountain chain 800-1500m above the plateau which is about 1200-1400m high. In these areas, known gravity anomalies and deflections of the vertical were predicted. The predictions were made by the use of stepwise collocations, described by Tscherning (1974, 1978) and previously used in Tscherning and Forsberg (1978). In the first step, the GEM10B set of harmonic coefficients was used. In the second step a set of $1° \times 1°$ mean gravity anomalies was used. In the third step, a local approximation was computed using anomaly spacing of about 10km and a few deflections 40km apart. For this step, local empirical covariance functions were estimated for anomalies and deflections.

The predictions were carried out with the unchanged original data, and with the data reduced with the following methods:

a) fixed sector topographic/isostatic reduction for a $6° \times 7°$ area around the test areas, Airy isostatic model, crustal thickness 32km and density contrast (crust/mantle) 0.4 g/cm^3
b) fixed sector ($6° \times 7°$) reduction for the visible topography above sea level
c) residual terrain model (RTM) reduction with mean topography defined by a bilinear interpolation in a $30' \times 30'$ mean height grid, residual topography taken into account in a fixed $3° \times 4°$ sector.
d) RTM reduction with $15' \times 15'$ mean height grid, residual topography taken into account out to a distance of only 60km.

Both the point observations and mean gravity values were terrain corrected. The RTM effects on the mean anomalies were very small. comparing the original and the reduced observations, it is apparent that the major par' of the change in the observed quantities is due to the effect of the isostatically compensated topography. The choice of the isostatic parameters have little effect on the variation. It is pointed out that some isostatic effect should always be included to avoid the bias on gravity anomalies (and height anomalies) which occur when only the topography is removed. (Refer to Table 1 of Forsberg and Tscherning (1981). A total of 110 deflection pairs and 150 gravity free air anomalies were predicted and compared to the known (observed) values. The mean values of the predicted deflection components and gravity anomalies in the two 1° × 1° areas are tabulated with their standard deviations. The values are tabulated in three variations according to the point observations used: A) gravity plus deflections, B) gravity alone, and C) no point values, only the 1° × 1° mean anomalies and GEM10B coefficients were used. The tabulation demonstrates the improvement of the prediction when the effect of the terrain is considered. The rms error decreased with a factor of close to 3 when terrain corrected data is used. It is also shown that better deflection results are obtained using topographic/isostatic reduction without any local gravity data (variation "C" above) than by the use of all the gravity data without terrain effect correction. It is concluded that it is possible to predict deflections of the vertical and gravity anomalies in areas of rugged terrain with accuracies of 1" and 3-4 mgal respectively from anomaly data spaced 6' apart, and when the gravity field is smoothed by terrain corrections computed on the basis of 0.5 × 0.5 minute point heights.

If only rough height information is available (such as 5' × 5' mean elevations), substantial improvements occur in the predictions. An example is the geoid prediction in Greenland by Forsberg and Madson (1981). The study shows that the collocation method yields results in areas of rough topography comparable in accuracy to the results obtained in flat areas.

5.2.4.2. <u>"The Geoid of Austria" by Sünkel.</u> Another example of the use of global geopotential and digital terrain models in combination with local gravity information for the computation of the regional geoid is "The Geoid of Austria" (Sünkel 1983).

For this work, the following data was used: The Rapp 180 × 180 geopotential model, 1° × 1° mean values of the global Digital Terrain Model (DTM), a 20" × 20" digital terrain model of point values interpolated from a 1:500,000 topographic map of Austria [a more sophisticated program for digital models of topography and rock densities is described in Steinhauser and al (1983).], and 521 observed astrogeodetic deflections of the vertical. The deflections are the results of an extensive program started in 1975 with new instruments. The formal accuracies of ± 0.2 - 0.3 seconds were changed to

± 0.5 - 0.7 seconds as a priori estimates in the collocation. The deflections, originally on a local system, have been converted to the geocentric Doppler system of the Naval Weapons Lab (NWL 9D) and to the ellipsoid of the Geodetic Reference System 1980.

The harmonic coefficients of the gravitational potential of the topography and its isostatic compensation were determined from the DTM data. Then the coefficients of the Rapp geopotential model and the topographic-isostatic coefficients have been subtracted from each other and the effect on the surface deflections determined.

The topographic-isostatic data reduction was calculated in four steps. In the inner zone ($\Gamma < 2$km), the exact parallelpiped formula was used, and in the second zone ($2 < \Gamma < 38$km),a much simpler formula. In zones 3 and 4, the mass point formula replaced the exact method. The effect of the remote zones ($38 < \Gamma < 150$km) and ($\Gamma > 150$km) were determined by interpolation from a grid of 15' × 15' and 45' × 60' respectively.

The two parameter "attenuated white noise model" of Jordan and Heller (1978) was chose for the model of covariance functions. Height anomaly covariance functions have been determined for 5 regions within the whole area. A variance of $0.05 m^2$ and a correlation length of 35 km was obtained for the area of investigation.

For the collocation process, a stepwise collocation solution was chosen, with the following arrangements:

"At step zero, a subset of 30 approximately homogeneously distributed vertical deflections have been processed. For this subset, the covariance matrix, its inverse, and the solution vector have been calculated. With this data,the non-processed vertical deflections have been predicted and compared to the measurements. Those (10) vertical deflections, which showed the largest discrepancies, were used as additional data in the next step. In this way, the covariance matrix and its inverse and the solution vector successively improved using block partitioning methods."

It is interesting to note the improvement of the prediction by the stepwise approach with the increase of the amount of data. The rms vertical deflection error is almost constant (about 2."0) for up to 25% of the total data, with 50% of the data the error drops to 1."0, and with 90% the error is about 0."3. It is also noted that the external prediction error estimates agree with the collocation prediction error estimates for almost all points with 20%. This is an indication of the reliability of collocation error estimates.

The collocation contribution to the geoid height or the height anomaly have been predicted for a (ϕ,λ) grid of 3' in latitude and 5' in longitude (2240 points). Due to the long CPU time for the estimation of the prediction error, this was computed only for a few sample points. The estimated rms errors of the relative geoid heights range between ±4 and ±14cm. The representative estimate for the whole area was ±8cm.

By adding the results obtained from the topographic-isostatic reduction and from the geopotential model to the contribution of the collocation, and applying the indirect effect, the geoid height and the actual height anomalies are obtained. The contributions from the topographic-isostatic model and from the geopotential model vary from 41.5 to 47.5m, and the collocation contribution is between ±0.5m after removal of a trend on the order of 3m. The effect of the analytical continuation on the isostatically and gravity model reduced potential is of 13cm in the Central Alps. The maximum difference between height anomalies and geoidal heights is located at the "Grosslockner" massif (3800m), and has a value of 35cm, which is in excellent agreement with the formula given in Heiskanen and Moritz (1967, p.325).

A number of additional papers were presented on the geoid in Austria at the XVIII General Assembly of IUGG/IAG in Hamburg, August 1983: Bretterbauer (1983), Steinhauser and al (1983), and Erker (1983). A comprehensive description of the work on "Geoid in Austria" is given in German by Lichtenegger and al (1983).

Moritz (1983) in his report on "Local Geoid Determination in Mountain Regions" reviews the methods for regional determination of the geoid or of height anomalies from deflections of the vertical or by combination of deflections and anomalies by collocation. Basic definitions and geometry are reviewed, geoid determinations, reduction for curvature of the plumb line and topographic-isostatic reduction of vertical deflections are discussed in the classical and modern (Molodensky) approaches. The application of the collocation is outlined and a summary of the geoid computation for Austria (Sünkel 1983) is given.

At the IAG Symposium in Hamburg during the XVIII General Assembly of IUGG/IAG, Aug. 1983, a paper was presented by Gurtner and Elmiger (1983) summarizing the work done in Switzerland, in the past 15 years and currently, regarding geoid and vertical deflection computations.

5.2.4.3. Other Gravity Field Estimations by Collocation. An algorithm for the prediction of free air anomaly is presented in Sunkel and Kraiger (1983). The algorithm was tested with 4 data sets in Austria. The computation method used was least-squares collocation with parameters assuming a linear correlation between terrain corrected free-air anomaly and elevation. The computations showed that: a) point free air anomalies can be predicted with a standard deviation of ±3 mgal (or better) from terrain corrected free-air anomalies with data density of better than 5 data per 100 km², b) the collocation determined Bouger factor deviates from the standard value of 0.112 mgal/m less than 5% with an rms error of ±3%; c) the covariance function of terrain corrected and trend reduced free-air anomalies agrees very well with the gravity anomaly covariance function obtained from topographic-isostatically reduced vertical deflections.

The actual accuracies of the predictions, by comparing predicted values to non-process measurements were: in the north-east foothills of the Alps the prediction error for the free-air anomalies was ±3 - 4 mgal; in the eastern foothills ±2 mgal; and in the central Alps ±10 mgal. The ±10 mgal error is probably due to the incomplete terrain correction of free air anomalies.

Tscherning (1984) reviews the theory and the steps of the implementation of the use of least squares collocation for computation of the disturbing potential and related parameters. It is shown how to reduce the number of observations under certain conditions in local solutions. The effect of the smoothing of the gravity field, achieved by subtracting out the contribution of the topography and eventually of the geological structures, is one of the topics discussed in detail. The computer processing is described, dividing it into separate steps. Each step is illustrated in Tscherning (1982). It is shown that the collocation method is a very powerful tool to solve geodetic problems and has two major advantages, one being that it can process heterogeneous data, and the other being its ability to compute error estimates. Tscherning (1981) compared collocation with other methods and concluded that other methods "having a sufficiently solid theoretical basis" would give comparable results.

Forsberg (1984) treats and analyzes a number of topics, all related to the modeling of the local and regional gravity field. The first topic is the combined collocation and geophysical inversion procedure for modeling. The term "gravity field modeling" in contrast to the geodetic interpretation of representing the external potential of the earth, in geophysics "stands for the process of determining internal density distribution of the earth consistent with the observed outer field". The geophysical modeling of the gravity field, therefore turns out to be the determination of the external gravity potential due to density distributions. Our current knowledge of density anomalies contributes only to the shorter wavelength part of the gravity field. The longer wavelength parts can be obtained from spherical harmonic expansions, termed "external" modeling in the report. The "external" and "internal" modeling could be carried out simultaneously by combined versions of collocation and geophysical inversion. The results: the external gravity field and density information could be obtained simultaneously. The theoretical background is outlined regarding density anomalies and an outline is given in principle on the combination of inversion and collocation for gravity modeling to include density (non-gravity) information in gravity field approximation.

The basic terrain reduction methods are reviewed in the follow-on sections. It is pointed out that the conventional gravimetric "terrain correction" is not a terrain reduction; the term is used in the report for a small non-linear correction to the Bouger reduction. A FORTRAN program for the computation of terrain reductions for gravity anomalies, geoid undulations, and deflections of the vertical is listed in the appendix.

The accuracy of the "linear approximation" (an approximative terrain reduction), used for error studies and FFT techniques, is investigated and it is found that the approximation is usually acceptable for theoretical models and for actual data. By assuming the validity of linear approximation FFT methods for terrain effect computation are outlined.

Error studies are made on the Digital Terrain Model (DTM) resolution requirements. Formulae and error diagrams are derived for the computation of rms errors of terrain effects on gravity anomalies, deflections of the vertical, and height anomalies on the basis of spacing of elevation data and on the covariance functions of the topography. The effect of topography is studied by investigation of empirical covariance functions in five different areas of the United states. These studies resulted in actual empirical information on covariance functions for topography and gravity. In addition, information on magnitude of terrain corrections and of RTM geoid effects are obtained. Information is given on the degree of smoothing obtained from terrain reductions of actual gravity data.

Finally, from isostatic reductions of satellite altimetry data in two areas of the Pacific, a relationship is derived between ocean geoids by altimetry and those computed from bathymetric data.

5.2.5. Prediction of Gravity Outside the Earth From Surface Data.

The prediction of the gravity disturbance vector at high altitudes is investigated for several technical reasons, such as high altitude gravity experiments, inertial navigation, etc. The task is to estimate the gravity disturbance vector components, gravity anomalies, or other functions of the gravity potential at altitudes from surface gravity data, with a theoretically valid and computationally manageable and feasible procedure. In addition, it is desired to account for all error sources involved during the process, determine the propagation of these errors into the results, and obtain at the end a reliable error estimate for the final products. Basic steps of possible procedures such as gravity reduction methods, computations of mean anomalies, an techniques for upward continuation of gravity are well known, however, the actual work performed in this area to date, is experimental in nature.

Some of the recent studies on this subject, selected for more detailed description are Sünkel (1984a, 1981), and Cruz and Laskowsky (1984).

5.2.5.1. Prediction of the Gravity Disturbance at Altitudes. Sunkel (1984a) examines four approaches: The Green's approach, by representing the external disturbing potential with a linear combination of a single and double layer potential; the surface layer approach; a combination of analytical continuation down to sea level with the Pizzetti-Stokes method; and the collocation approach. Each method has its advantages and disadvantages. The integral formulae have the common assumptions that: a) gravity anomalies are given at every point of the earth, b) only gravity anomalies are used as data, and c) all data are exact. Obviously, these assumptions are never met. The advantage of the integral formulae is that the inversion process is performed analytically, resulting in a very fast and simple process for the computation of gravity field quantities. Conceptually, the Least Squares Collocation (LSC) is superior to all other methods. It combines heterogeneous input data, downward continuation of surface data, smooth interpolation, upward continuation, noise filtering, and estimation of parameters. "Theoretically the whole apparatus of LSC can be directly applied to the unreduced data as input yielding any estimable gravity field quantity as output" (Sunkel 1984a). Theoretically LSC seems to be the ideal tool for the tasks under discussion, however, the solution of a very large system of equations with a fully occupied matrix makes it infeasible in practice (Nice example for the "balance of difficulties", Sunkel 1984a).

The logical solution, therefore, is to combine the advantages and avoid the disadvantages of each method. See Lachapelle (1975, 1977) for the computation of deflections of the vertical, height anomalies, and gravity anomalies. Sunkel formulates the "ultimate" combination as the combination of LSC with integral formulae, high degree earth model, and the best possible topographic information.

Turning to the currently used procedures for the determination of the disturbing potential above the surface of the earth, the problem is usually treated in two steps: a) prediction of gravity anomalies at the surface or at the sea (zero) level, in most cases, in the form of mean anomalies of various block sizes; b) application of one of the boundary value solutions for the computation of the disturbing potential. Gravity anomalies are used, not only because they are the most frequently available form of gravity information, but also for the reason that the boundary value solutions are based on gravity anomalies. Concerning the prediction of gravity anomalies at the earth's surface, we are reminded of two facts from the literature and experience. One is that the quality of prediction (interpolation) depends on the variance and the correlation length of the free-air anomaly covariance function, in addition to the data density. The second fact is that free-air anomalies are strongly correlated with elevation. In areas of rugged topography, large variance and small correlation length can be expected. This situation will result in unreliable interpolated values. The data should be reduced to decrease the variance and increase the correlation length. In addition, the gravity data in areas of rough topography are usually scarce; this further aggravates the situation. The procedure in mountainous areas, therefore, will be: removal of the effect of topography, (reduction) interpolation along the smoothed surface of the reduced data using LSC and restoration of the removed topographical effect. Two of several prediction techniques are recommended for the mean anomalies at the earth's surface.

The first is least square collocation with the parameter model. The iterative procedure is based on terrain corrected (TC) free-air anomalies; details are in Sünkel and Kraiger (1983). It was tested in Austria both in flat and rugged areas, and it is considered the "most practicable" for free-air anomaly prediction along the earth's surface. A fast algorithm by Sideris (1984) computes terrain corrections on a grid using gridded topographical data and FFT technique. The second technique is LSC and topographic-isostatic reduction (TIR). This approach is well known and is frequently used. This method also assumes a compensation model and an average density value, which are wrong in most cases. The differences between the two methods are as follows: In the first approach the effect of the topography is removed using a best possible estimate for the average density in terms of the parameter b (Bouger factor), and in the second approach a standard density is used; in method one the trend is optimally estimated, and in method two the isostatic compensation (a user selected model) takes over the parameter model of the first method. It is well known that the actual density compensation is largely different from an ideal model, especially in small local areas. Lastly, the first technique can process only gravity anomalies and requires an iteration process, and the second technique can process any type of data with no iteration required. Both approaches have advantages and drawbacks; the first method is considered equal or superior for most circumstances, except when combination of heterogeneous data is required.

The Stokes-Pizetti procedure for the determination of gravity field quantities requires gravity anomalies at zero level. The analytical continuation of all types of anomalies to the zero or sea level can be performed by various methods. Several of these are discussed in the report. In the opinion of the author of the report (Sunkel 1984), the LSC is the ideal tool for analytical continuation of anomalies, or any other gravity field quantity, to zero level. Used in combination with the quasi-isostatic anomaly of previously described approach one, or applied to the isostatic anomaly of approach two, the results will be analytically continued quasi-isostatic or analytically continued isostatic anomalies at zero level, respectively. Both of these can be directly used in the Stokes-Pizetti integral to calculate the disturbing potential outside the earth's surface (after adding the indirect effect). Any other gravity field quantity can be obtained from the disturbing potential by applying the corresponding linear functional to the disturbing potential formula.

In the chapter "Error Considerations", the errors entering into the prediction of the disturbance vector are discussed in detail. The errors are divided into four major groups, each containing the relevant error sources.

1.) Data reduction errors
  a) errors due to terrain correction compensation
  b) errors due to the DTM sampling rate
  c) errors due to errors in the DTM data
  d) errors due to the computation of TIRs
  e) errors due to approximation of the analytical continuation
2.) Mean anomaly errors
  a) errors due to data density and distribution
  b) errors due to data errors
3.) Representation and errors due to mean anomaly errors
  a) errors due to use of mean anomalies
  b) errors due to mean anomaly errors
4.) Errors of data reduction effects on the estimated quantities, breakdown as in 1.)

All of the above error sources are discussed in detail and their contributions to the error budget of the disturbance vector analyzed. Methods of computations and numerical estimates are give for most contributing error components.

Another study by Sunkel (1981) estimates the accuracies of the gravity disturbance vector components at high altitudes from a given set of free-air mean anomalies and their rms errors at sea level.

The arrangement of the mean anomalies is symmetrical with respect to the computation point. The innermost rectangular zone, consisting of $5' \times 5'$ blocks of mean anomalies, covers an area of $2° \times 2°$, $3° \times 4°$, and $7° \times 9°$ according to varying cases of data distribution. The next zone of $15' \times 15'$ mean anomalies covers an area varying in size from $6° \times 8°$ to $16° \times 18°$. The following rectangular area consisting of $1° \times 1°$ mean values covers and area changing from $26° \times 30°$ to $38° \times 42°$. Another zone of $5° \times 5°$ mean anomalies has two variations in coverage area: $50° \times 70°$ and $60° \times 100°$. The remaining surface of the sphere ($180° \times 360°$) is covered by $5° \times 5°$ blocks, having larger rms errors. In total, there are nine variations of data distribution. The assumed rms errors for the $5' \times 5'$ blocks are ± 8 or ± 10

mgal; for the 15' × 15' blocks ±7 and ±8 mgal; for the 1° × 1° blocks ±4 and ±5 mgal; and for the 5° × 5° outer zones ±3 and ±5 mgal. In total, 18 cases (according to distribution and rms errors) were considered in the numerical computations.

Five altitudes for the computation point were used: 30,000, 40,000, 70,000, 100,000, and 200,000 feet.

Two methods were used for the computation of the disturbance vector components. These are the integral formula for the gradient of the disturbing potential, in terms of free-air anomalies and least squares collocation. For the integral solution, the representation error was calculated in the frequency domain. The results of the two methods differ by less than 10%. It was expected that the results will differ only slightly if the gravity coverage is reasonably good. This was confirmed by the numerical computations. An optimal algorithm was developed for the collocation solution to take advantage of the regularity and symmetry of the data distribution. From the numerical investigations the following conclusions were made.

For radial components:

a) The error decreases rapidly if the number of 5' × 5' mean anomalies around the computation point increases;
b) The larger the 5' × 5' data set, the smaller the representation error;
c) The effect of data errors decreases with increasing altitude;
d) The size of data area of strong contribution increases with the increase of the prediction height, i.e. if the number of data blocks are constant, the prediction error will increase with altitude.

For horizontal components:

a) For the same data, the error is more than twice as large than the error of the radial component and decreases only slowly, if the number of 5' × 5' anomalies increases;
b) The larger the data set, the smaller the representation error;
c) The influence of data errors decreases with increasing altitude;
d) The remote zone has a very significant effect on the results.

The results of the error estimates of gravity disturbance vector components at the considered altitudes for the 18 cases are tabulated. The main numerical results are: The radial component can be estimated with ±1 mgal accuracy at

50,000 ft altitude (on the basis of the stated data sets); to obtain the same accuracy at 30,000 ft, the data accuracy (especially the 5' × 5' set) has to be increased by 60%. Regarding the horizontal component, with the best data distribution ±2.3 mgal accuracy can be achieved at 30,000 ft altitude (0.5" in the direction of the gravity vector). For the achievement of ±1 mgal accuracy, the block sizes must be reduced by a factor of 2 out to a spherical distance of about 30°, and the overall data errors must be reduced by 30%.

5.2.5.2. Upward Continuation of Gravity Anomalies. The upward continuation of gravity anomalies was recently investigated by Cruz and Laskowsky (1984). Three procedures were compared in a numerical study with real data. The original gravity data consisted of about 18,386 free air anomaly values irregularly distributed over a 7° × 9° area in New Mexico. The simple Bouger anomaly and the terrain correction were also furnished. Elevation data in the form of 30" × 30" grid point values were also available. 5' × 5' mean anomalies were developed in the test area in steps including: data thinning, terrain reduction, tailoring of covariance function, and usage of only ten of the closest data points to the point of computation in the collocation prediction. "Collocation from the ten closest points" was used to reduce the size of the computation load. Mean elevations for 5' × 5' blocks were also computed in the 7° × 9° test area from the 30" × 30" grid data.

The given mean anomalies are referred to the earth's surface, the upward continuation of these anomalies without reductions would be possible by collocation, or by a Bjerhammer-type solution; however, these were not feasible due to large matrix inversion requirements. If the discrete estimation procedures are ruled out, the use of upward continuation of continuous anomaly functions require that the data be continued downward to the level surface (sphere). To avoid the "major difficulties" associated with this reduction, the investigators designed and tested a procedure named "the indirect method". In this method the gravity anomaly field is separated into three frequency ranges. The high frequency part, the effect of the shallow topography, is modeled by directly integrating the gravitational effects without any reduction to a level surface (by prism integration). From the field left after the removal of the effect of the topography, the low frequency component was also removed by using the Rapp 180 × 180 (1981) field. This eliminated truncation errors due to the neglect of remote zone data. In the removal process it was assumed that the data points are on a common level surface instead of their actual vertical position. The justification for this assumption is that the vertical gradient of a 180-field is "expected to be small" and that it was necessary to keep the computation time reasonable. After the removal of the high and low frequency fields, the residual medium field, much smoother than the original, however, it is still located on the earth's surface. To improve the situation, an approximative "reduction" is used. This is the terrain correction of an expansion of the topography to degree 180, implicitly applied to the residual anomalies. Detailed discussion of this procedure is in Moritz (1966). After all these operations, the final position of the level surface, to which the data are "reduced", is unknown and assumed to coincide with the mean elevation of the topography in the area of upward continuation. This affects the upward continuation distance and the results. The sensitivity of profile anomalies to 1.5 km. change in upward continuation distance (H_0) for various heights are tabulated in the report. The percentage ratios of rms change in profile anomalies, when H_0 is reduced by 1.5 km, to the rms values of profile anomalies at height H_0 are: 3.8%, 6.8%, and 9.1% at ho = 30km, 10km, and 5km, respectively.

The final value of the upward continued anomaly is the sum of the three components discussed above. The results of the "indirect method" was compared with the results obtained by the two other procedures. These were the "direct Poisson" integration of the original uncorrected surface anomalies assuming that they are on a level- surface; and ~~the second,~~ the Poisson integration of terrain corrected surface anomalies (Faye anomalies).

Test profiles were computed by the three procedures at 30, 10, and 5km altitudes. The results show that the Poisson integration of terrain uncorrected data are too low by 0.5, 0.6, and 0.7 mgal at 30, 10, and 5km, respectively, compared with the results of the Poisson integration of terrain corrected data (this bias is the upward continued terrain correction). No bias was found between the two terrain corrected methods. Finally, the standard deviation of the differences of all three methods are the order of (0.5, 0.6, 1.3) mgal at (30, 10, 5) km altitudes.

Test computations were made by the use of FFT technique and compared to the Poisson integration results. At 30 and 10km altitudes, the agreement between the two methods was 0.1 and 0.3 mgal respectively.

5.2.6. Airborne Gradiometry Improvement of the Local and Regional Gravity Field)

As it is shown in Table 2, section 5.1, the high and very high frequency parts of the gravity field can be obtained today almost exclusively by point gravity anomalies. 5' x 5' mean gravity anomalies and deflections of the vertical contributed to the high frequency field; however, the distribution of these data is mostly irregular. The coverage of the deflections is local and the density of data in most cases is medium. Height data can furnish the effect of the visible topography and can furnish high and very high spectral resolution in combination with airborne gradiometer data. An economically feasible and efficient technique for observation of the high frequency field with high data density, regional coverage, regular data distribution and high data accuracy will be the airborne gradiometry. The research and development of a feasible moving base gradiometer system is continuing during the past 15 - 20 years. The literature of these developments is quite extensive, well-known, and therefore will not be discussed here. Nevertheless, an operational system is expected to be completed soon and actual flight testing can begin. Survey accuracy, simulation and geodetic application studies have been made as early as 1975 and still continuing today. Some of these studies are: Moritz (1975), Schwarz (1976, 1977), White (1980), Jekeli (1983, 1984), TASC (1984).

A recent study on achievable accuracies for geodetic use by an airborne gravity gradiometer system is by Jekeli (1983). The accuracy of the point gravity vector on the earth's surface, estimated from the five independent components of the second order gradient tensor, observed at altitude, was evaluated by a numerical analysis. For the method of the analysis the least-squares collocation was chosen as the ideally suited optimal process. In addition to the advantages of the method, especially the immediate by-product, the prediction error, and the disadvantages are also discussed. Among other problem areas, Jekeli points out that the covariance model may not be adequate because the gravity field is not known in sufficient detail for the development of a model at very high frequency. It is stated, however, that the error analysis based on least-square collocation, in spite of some negative aspects, should provide "a fair indication" of the performance of the gradiometer system. The analysis was formulated in both the space and frequency domains with respect to either one or two horizontal dimensions.

Approximations for a manageable analysis were: flat earth surface, infinitely long tracks, continuous data, and infinitely many parallel tracks. The analysis was carried out with a cutoff wavelength of 500km implying that the error estimates refer to a high frequency reference field corresponding to degree $n > 80$ in harmonic expansion. Other parametric values were: aircraft speed, 300 km/hr; altitude, 600 m; gradiometer white noise $35E^2/H^2$, corresponding to 1.9E standard error for 10 seconds averaging time; and track spacing, 5km.

From the results of the analyses, it is concluded that the accuracy of gravity estimated from airborne gradiometer data depends on the density and coverage of the survey tracks. With track spacing of > 5 km and unidirectional-track survey, an accuracy of 1 mgal cannot be achieved. A bidirectional-track survey substantially improves the accuracy of the estimated gravity. Track spacing and survey altitude are significant contributors to total error. A diagram depicts accuracies of the gravity disturbance (δg) and components of the deflection of the vertical (ξ,η), estimated from gradient components observed at altitude along bidirectional tracks, in function of track spacing. A second diagram shows the same in function of survey height above the earth's surface. The accuracy of δg at 5 km track spacing is about 0.4 mgal, at 10 km spacing it is about 0.7 mgal, and at 15 km track spacing it is about 1 mgal. the deflection of the vertical components accuracies are $0\overset{''}{.}06$, $0\overset{''}{.}1$, and $0\overset{''}{.}22$ (arc seconds) respectively.

From the survey height diagram, at 600 m the accuracies are as above. At 2000 m, δg has an error of 0.6 mgal, and at 4000 m 1 mgal. The deflection components at 2000 m have an error of $0\overset{''}{.}09$ and at 4000 m $0\overset{''}{.}14$ (arc seconds).

It is noted that the above figures will be spoiled somewhat by the used approximations, and they are relative to the reference field. If the reference model would be error-free, the above accuracy figures would represent the absolute accuracy. The current models have quite large errors, especially at the shorter wavelengths; therefore, the attainment of the 1 mgal accuracy goal cannot be firmly substantiated.

Another analysis by Jekeli (1984) concerns the techniques for processing of gradiometry data. An airborne gravity gradiometer system will produce a very large amount of data, especially if the 1 mgal accuracy for an area survey is the desired goal and all the six gradients at each point are used. In a grid of 300 km with 5 km point density, there will be 21600 observations.

If the optimal method, the least-squares collocation, is used, the inversion of the autocovariance matrix (21600 $\times$ 21600) generally would require $O(10^{13})$ operations. Therefore, an unmodified application of the least-squares collocation is not feasible, under the present and foreseeable future computing capabilities. As in many other applications of this technique, the computational problem can be improved without hurting the optimality of the solution. An example is applying certain restrictions to the pattern of the observations.

If single observations are uniformly spaced on a straight line, the auto-covariance matrix will have a Toeplitz structure. Such a simple Toeplitz matrix can be inverted in $O(n^2)$ operations, instead of $O(n^3)$ operations required for the inversion of a general auto-covariance matrix (n = number of observations). Using Fast Fourier Transform, this can be reduced to $O[n(\log n)^2]$ operations according to the claim of Bitmead and Anderson (1980).

If the measurement pattern is a grid of parallel tracks in both directions, with uniform point spacing, and with one observation at each point, the auto-covariance matrix is a "Toeplitz - block Toeplitz matrix". If the number of observations at each point is greater than one, we have a "blocked Toeplitz - block Toeplitz matrix". An algorithm for the inversion of a block Toeplitz matrix in $O(k^3n^2)$ operations is given in Wiggins and Robinson (1965). The dimension of the block is $k \times k$. This can be reduced to $O[k^2n(\log n)^2]$ (Bitmead and Anderson 1980). An algorithm by Wax and Kailath (1983) requires $O(k^3n^2)$ operations to invert a Toeplitz block Toeplitz matrix. This may be transformed into one requiring $O(n^3k^2)$ operations, and it is beneficial if $n \ll k$. Applying the Bitmead and Anderson (1980) algorithm to the example of a 300 km grid with 5 km uniformly spaced points and six gradients at each point, the inversion will require $O(2 \times 10^7)$ operations, which is feasible.

If rectangular coordinates are replaced by cylindrical polar coordinates, then the covariance matrix for uniformly spaced single observations on a circular track, in addition to its Toeplitz structure, is also circulant. Its Fourier transform is a diagonal matrix; its inversion requires $O(n \log n)$ operations. If we have multiple uniformly spaced concentric tracks with multiple observations at each point, then we have the case of blocked circulant-block Toeplitz matrix. The transformation of rectangular pattern to circular pattern is an additional processing of data.

The second type of technique discussed by Jekeli (1984) under the title of "Non-optimal Estimation" is reduction of the number of observations by averaging. The Analytic Sciences Corporation (TASC) under contract to AFGL is working on a method where the survey area around the computation point is divided into zones. The size of the zones increases with the distance from the point. On the basis of the average observations in each zone, least-squares collocation is used for the estimation of the gravity vector. It is expected that the differences between this and the optimal solution will be small. Simulation tests and comparative analyses lie in the future.

Another approach in the "non-optimal" category is the use of integral formulae derived from the second derivatives of the disturbing potential. The formulae are given in Jekeli (1984). The formulae are exact except for the planar approximation of the earth (this is usually negligible considering the extent of the survey). The error sources are the discrete and noisy observations and the finite extent of the data coverage. The effect of this can be improved by the modification of integration kernel (Jekeli, 1982). Using the integration formulae the disturbance vector would be at altitude, and a simple least-squares collocation procedure would compute the corresponding surface values.

In estimating anomalous gravity or gradients, two or three digit accuracy in the final values are generally satisfactory. Therefore, under certain circumstances, a large covariance matrix can be inverted by an approximate method which is computationally feasible. The solution is non-optimal, but the numerical results are indistinguishable from the results of the optimal solution. Jekeli terms this type of estimation as "virtually optimal estimation". An example is TASC's GEOFAST algorithm (Tait 1979). A class of methods for approximation of the inverse of a large matrix is to improve sequentially on initial estimates of the inverse by iteration. These techniques, called the class of relaxation methods, converge satisfactorily; a good initial estimate will save computer time. The techniques are described in Schwarz et. al (1973). Jekeli investigated the conjugate gradient method for the inversion of the covariance matrix. From the several options he chose a separable matrix approximation suggested by Tait (1979) for the initial estimate.

A numerical analysis was performed for the purpose of comparison of several data processing techniques. The assumed measurement grid was a regular one with data points 5 km apart in each direction. The extent of the grid was variable from 50 km to 300 km, and the altitude above the ground was 600 m. The assumed accuracy of the gradient observation was 2E (uncorrelated random error). The average accuracies of estimating the residual gravity disturbances, using the (1) integral formulas, (2) integral formulas with modified kernel, (3) conjugate gradient method with iterations, and (4) the least squares collocation method, are compared in one tabulation and five diagrams. The overall conclusions are that the least-squares collocation is the optimal method in terms of accuracy, but requires "inordinate" amount of computation "in the most general case". The integral formulae may be sufficient in some applications, but not for the achievement of 1 mgal accuracy.

The conjugate gradient technique for the inversion of the covariance matrix of the least-squares collocation looks promising regarding reductions in computations. It was experienced that convergence to 1 or 2 digits is achieved in much fewer iterations than the theoretically required n iterations, where n is the size of the matrix.

An earlier study regarding achievable accuracies from airborne gradiometry was performed by Schwarz, K.P. (1976), and a simulation study also by Schwarz (1977).

The accuracy study investigates the achievable accuracies of mean gravity anomalies for various block sizes from 5' × 5' to 5° × 5° from observed first and second order gradients at flying altitudes. Gravity anomalies are derived by integrating the first order gradients observed by accelerometers simultaneously with the gradiometer system. In other words, this concept can be interpreted as a combined set of measurements where the integrated first order gradients provide the reference field (so to speak) for the fine structure observed by the gradiometer system. The theoretical background of separation of gravitation and inertia and the derivation of related mathematical formulae are given in several publications of Moritz (1967, 1971, 1975). In Moritz (1975) and Schwarz (1976), a system of linear second order differential equations are given interrelating the gravity disturbance vector with the observed second order gradients and accelerometer outputs (gravitational plus inertial force combined). For the combination of the measured second order gradients and integrated values, their downward continuation from the flight altitude to the surface of the earth and for the computation of mean values, the method of least squares collocation was used. All of the above steps were performed by appropriate selection and changes of the covariance function. In the particular case of Schwarz's study, the combination with satellite altimeter data was also included in the algorithm.

Due to the decisive role of the covariance function in the collocation method, after a careful study, two models: Hirvonen's (1962), derived from local data; and Kaula's (1966), derived from local and regional data, were selected. Using the characteristic parameters of both types of functions (Hirvonen's called "local" and Kaula's called "regional"), the covariance functions of the gravity anomalies, second order radial gradients, and geoid undulations have been computed for 0 km (sea level) and 10 km (flight) altitudes.

The measurements used are, as previously mentioned, gradiometer data in a (ϕ,λ,r) - system and Δg values from integration along the profile. Mean values at 10 second intervals were used as gradiometer measurements. For the consideration of the measurement errors, the covariance matrix of the measurements (C_{xx}) was augmented by the covariance matrix of the noise. To obtain information on the sensitivity of the estimation to the errors of the different measurements, the standard errors of each type of measurements have been varied separately. The first values estimated were the Δg values in any arbitrary position between two flight profiles. This can be done by combination of various data of the respective flight profiles. The following combinations were considered: (1) Δg values interpolated between the profiles, (2) use of the three first order gradients condensed from the five gradiometer measurements, (3) five second order gradients alone, and (4) a combination of Δg and different second order gradients. The numerical problem is the inversion of the covariance matrix of the measurements. After some test computations it was found that an interpolation point between the flight profiles can be satisfactorily determined by using a range of observation points along the respective flight profiles. The length of the range is about three times the distance from the estimated point to the profiles. It was determined that the standard error decreases only slightly if more points are used. The interpolated point should be an equal distance between the two profiles. Computations were performed using both the "local" and "regional" covariance functions. For the purpose of the accuracy study, oniy the error covariance matrix was computed.

The downward continuation from flight altitude to zero level was performed by the selection of a covariance function that can be continued analytically to the sea level, so that the downward continuation became a problem of spatial interpolation. Like in the planar interpolation at the flight altitude, the accuracy of the interpolated point depends on the distance to the observation points. In both cases (planar and spacial interpolation) the best data combination is Δg-values combined with gradiometer data. The optimal solution is the combination of Δg and five independent gradients. For east-west profiles, the combination of Δg, $T_{r\phi}$, and $T_{\phi\phi}$ is very close to the best combination.

To asses the influence of the measurement errors, two accuracy diagrams are given in the report. One shows the standard error of the estimation in the function of the standard error of the observed gravity anomalies and errorless gradient measurements; the second diagram shows the estimation error in the function of the standard error of the gradient measurements. Both error diagrams give the estimation errors for profile spacings of 0.2°, 0.5°, 0.7°, and 1.0°. For the computation of mean gravity anomalies for various block sizes, the basic least-squares collocation formula used for the point anomalies was modified. For the derivation of the required cross-covariance function between point and mean anomalies smoothing operators were computed for blocks of different sizes according to Meissl (1971). Using different assumptions for the measurement errors and various profile spacings mean gravity anomaly accuracies were computed for several block sizes.

The conclusions of Schwarz's accuracy study are summarized as follows:

1. The accuracy of an interpolated anomaly depends on its distance from the nearest profile. Therefore, the dense spacing of the observation profiles is more desirable than cross profiles. Cross profiles should be used for updating only.
2. A high density of observed points along a profile is necessary only for the integration of first order gradients. For interpolation of Δg values, a density between 1/4 and 1/2 of the profile spacing is sufficient.
3. Optimal data combination for profile spacing larger than 0.3° should include: Δg, $T_{r\phi}$, $T_{r\lambda}$, $T_{\phi\phi}$, $T_{\lambda\lambda}$; for spacings smaller thar 0.3°, Δg, $T_{r\phi}$, $T_{r\lambda}$, and T_{rr} are sufficient.
4. The effect of measurement errors is larger for small profile spacing.
5. The combination with altimeter data is significant for large (5° × 5°) mean

values. The contribution to $1^\circ \times 1^\circ$ blocks is marginal.

6. The attainable accuracies under the assumption made about the covariance functions were:

   a) with 1° profile spacing, an accuracy of ± 3 mgal standard error for $5^\circ \times 5^\circ$ blocks and ± 5 mgal for $1^\circ \times 1^\circ$ blocks can be achieved (for the $5^\circ \times 5^\circ$ blocks, only with the combination of altimeter data).

   b) with 0.3° profile spacing for $15' \times 15'$ and $5' \times 5'$ blocks, ± 3 mgal can be obtained.

In the "Simulation Study of Airborne Gradiometry" (Schwarz 1977) simulated gravity anomalies and second order gradients are used for the recovery of the anomalies at ground level from the simulated measurements at flight level. The purpose of the study was to obtain an algorithm for airborne gradiometry, and to design a method for efficient handling of very large amounts of data expected from the measuring system. For covering an area of $20^\circ \times 25^\circ$ with profiles spaced at 1° and assuming 250 observations per profile degree, there will be 130,000 measurements; if 20' profile spacing is used, the number of observations will be 390,000.

The study consists of four distinct steps: (1) gravity and gradiometry data are generated at ground and flight level, (2) the flight level data are corrupted by an error model, (3) the ground level gravity anomalies are estimated from the flight level data, and (4) estimated and generated ground level data are compared.

The gravity field data was generated from a grid of mass points of 10' spacing at 40 km depth for an area of $10^\circ \times 15^\circ$. A positive or negative mass of constant size, or a zero mass, was selected according to a normal distribution (the data have been generated by DMAAC, St. Louis).

For the estimation of gravity anomalies at ground level from the simulated observations at flight level (10 km), the formula of least-squares collocation was used. According to the spacing of the profiles and the size of the mean anomalies to be estimated an optimal point configuration was chosen. It was assumed that the observation profiles are parallel and the observation points are at constant intervals (At 500 knots speed and 10 seconds integration time, the observation rate was a uniform 2.6 km). With the above conditions a "moving operator" was computed (from the cross-covariances between signal and observation and from the auto-covariance matrix of the observations). This unchanged operator was moved along the flight profiles from one set of data points to the next, according to the pattern described previously in Schwarz (1976), using all available observations.

The covariance function used in this study was that used in Schwarz (1976). Its parameters are different somewhat from the covariance function directly derived from the simulated data. This was also used for the determination of the effects of assumption errors in the covariance on the results of the estimation. For the statistical error models "normal models" (numbers taken from a normal distribution), and Markov sequences of first and second order were used. The difference between the normal and Markov model is that normal deviates are uncorrelated, elements of Markov sequences are not. To illustrate the accuracy of the estimated point anomaly, a 3° long "interpolation profile" on the ground level obtained from "flight profiles" spaced at 20' is compared to the exact profile of the simulated field. The normal error model was used, and the standard errors of the observed gravity anomalies and of the second order gradients were assumed as ±1 mgal and ±1E respectively (too optimistic assumptions). The agreement between the two plotted profiles is very good. The standard error is 3.3 mgal for the point estimation. If the interpolated profile is determined form the same data as before, but is located directly below one of the flight profiles, the standard error drops to 1.96 mgal. It is concluded that a measuring error of ±1 mgal at flight altitude (10 km) is amplified to about ±2 mgal by downward continuation, and to about ±3 mgal when interpolated between two profiles 20' apart.

The results for the mean anomalies are tabulated and compared to the accuracies obtained by the accuracy study (Schwarz 1976). With profile spacing of 20', the accuracies of the 15' × 15', 30' × 30', and 1° × 1° blocks agree well with the values obtained by the accuracy study. The average of the differences between the two sets of accuracy figures is about 0.4 mgal. For the 1° × 1° blocks obtained from 1° profile spacing, the simulation study gave ±4.6 mgal, versus the ±5.6 mgal obtained by the accuracy study. The normal error model was used for the above computations.

In the conclusions it is pointed out that the two studies (accuracy and simulation) agree well regarding the obtained accuracies, however, it is also pointed out that correlated errors in the measurements will strongly effect the accuracy of the results. Depending on the size of the correlations and on the variances, the mean-square errors may double as compared to the uncorrelated case.

6. SUMMARY AND RECOMMENDATIONS

The principal requirement for the derivation of a model which satisfactorily approximates the earth's gravity field is a uniform, dense, and global coverage of several types of gravity data. The techniques to provide the instrumentations necessary for such coverage already exist or are in their final stages of development. The achievement of ambitious goals, like NASA's Geopotential Research Mission (GRM), would require additional substantial efforts in data collection programs and data processing techniques.

Current geopotential models are derived, generally, from the combination of three types of information: perturbation of satellite orbits, satellite altimeter data, and terrestrial mean anomalies.

Satellite orbital data represent the long wavelength part of the gravity spectrum, the short wavelength parts are smoothed out by the attenuation of the gravity field with the altitude. Models containing well distributed and precise orbital perturbation data determine the low order coefficients very well (i.e. degree and order 4) and they are very useful for satellite orbit computations. Examples are GEM9, GEM-L2, GRIM-3B, and GRIM-MPI.

From satellite altimetry, the geoid over the oceans is obtained. These data are regularly distributed and could yield about 50 km half wavelength resolution. If so, this would give an expansion of 360 degree and order, provided the coverage extends over the globe. currently, $1^\circ \times 1^\circ$ mean anomalies are developed from the altimeter data and merged with gravity anomalies of $1^\circ \times 1^\circ$ obtained over land. One of the two problems with this type of data, the effect of averaging, can be reversed by the use of "desmoothing" factors (Colombo 1981a). The other problem, that of aliasing, remains after desmoothing or filtering. In the case of $1^\circ \times 1^\circ$ mean anomalies, the aliasing errors are about 50% of the coefficient at the Nyquist frequency, and larger above. Therefore, there is a need to replace the current mean values with smoothed values free of aliasing. It is recommended to study this problem and to test K.P. Schwarz's (1984) suggestion "to use a bandpass filter on the power spectrum, obtain the auto-covariance function by Fourier transforming the results to the space domain and estimate smoothed values in the center of the block using this function".

In the combination of various data sets for global models, or using global models as reference fields for regional or local gravity field approximations, the proper weighting of the various data sets, or models, would require their actual accuracies. Wenzel (1982) proposed the use of the error covariance functions of various data types for the derivation of spectral weighting functions for use in data combinations. This approach can be used in connection with any combination technique and it is "most promising" (Schwarz 1984). It is recommended to investigate the possibility of whether the spectral weighting functions of a particular data set could establish a link between the frequency range of the data sets and the accuracy of the combined product, or harmonic coefficients of a certain degree and order derived from the data.

From the inspection of the percentage errors of the coefficients of various solutions (e.g. Rapp 1981), it can be seen that the coefficients above degree 12 have larger errors. It is also apparent that up to degree 15, the coefficients are influenced mainly by satellite perturbation data, and above degree 15 the mean anomalies, including altimeter data, dominate the accuracy of the combination.

The accuracy of current models are illustrated by the intercomparison of the result obtained by various groups. It is true that the various groups frequently used the same observed material, but their combination and adjustment methods are different. The comparisons indicate only the order of magnitude of the errors in the various solutions. Generally the data errors, lack of data continuity and aliasing are the principal sources of error. For details see section 2.

6.1. Satellite Programs for the Improvement of the Gravity Field

6.1.1. Satellite Altimetry

The most productive program contributing to the improvement of global gravity coverage was satellite altimetry. GEOS-3 and SEASAT results, their contributions to the geodetic aspects of the program, and future plans for satellite altimetry are discussed in detail in Section 3.1 of this report.

As it was mentioned earlier, the disadvantage of satellite altimetry from the point of view of geopotential modeling, is that the data coverage, due to its very nature, is restricted only to the ocean areas. This requires the combination with another type of data (terrestrial mean anomalies). Therefore, the global data homogeneity is lost. Nevertheless, a homogeneous, regular, and fairly accurate coverage was possible over the areas by this technique.

The results of satellite altimetry can be improved upon by improving three related factors: a) sea topography and tidal corrections, b) accuracy of the radial component of the satellite orbit, and c) precision of the altimeter measurements.

Currently, two new altimeter missions are in some stages of their development: The GEOSAT and the TOPEX. The GEOSAT of the Navy was scheduled for the mid 1980's and the TOPEX of NASA is contemplated for the 1990's. GEOSAT will have a SEASAT type of sensor and about 18 months nominal mission time. The plans for TOPEX call for an altimeter accuracy of ±2 cm and for a sea surface height accuracy along a grid of ±14 cm.

For the improvement of the radial component of the orbit, an improved TRANET Doppler network is planned, and the Jet Propulsion Laboratory (JPL) intends to develop a new tracking system (Séries X) using the GPS system for the determination of the tracking sites' coordinates.

6.1.2. Satellite to Satellite Tracking and Satellite Gradiometry.

These techniques, discussed in detail in sections 3.2 and 3.3, are similar in the respect that both have a capability to provide a high density, regular and global coverage of gravity data. The "global coverage" in both cases are real, i.e. covering land and sea, pole to pole, without any gaps. The frequency range for both will be in the medium field, corresponding to the frequencies of $1^\circ \times 1^\circ$ mean anomalies, provided by satellite altimetry, and by $1^\circ \times 1^\circ$ land values. The possibility, depending on the survey configurations, exists for extension into higher frequency ranges, particularly for the gradiometer survey. The data accuracies will be superior to the accuracy of the current altimeter derived data.

The results of several accuracy and simulation studies are summarized in section 3.2 and 3.3. The results of some of these are given as follows:
For a low-low satellite to satellite tracking mission of six months, at an altitude of 160 km and a range-rate noise of ±1 micrometer/sec with 4 sec integration time, the accuracy of the derived mean anomalies of $1^\circ \times 1^\circ$ is estimated as ±2.3 mgal, and of the $1^\circ \times 1^\circ$ geoid undulations as 4.3 cm (Jekeli and Rapp 1980)

Colombo (1981b) from the error analysis of the global geopotential, obtained from a low-low SST mission, under certain conditions, the following accuracies:

(1) The relative error in the potential coefficients could be better than: 1% up to degree n = 130; better than 10% up to n = 210; and better than 50% up to degree n = 270.

(2) The geoid point value accuracy could be better than 5 cm rms in the band from 3000 km to 40,000 km and better than 10 cm in the band from 140 km to 3000 km (both total errors).

For satellite gradiometry two instruments are under development: the cryogenic gravity gradiometer by Paik (1981) and "Project Gradio" by Balmino et al (1984). The instrument precision goals are 10^{-4}E (1E = $10^{-9}s^{-2}$). An operational system is expected for the 1990's.

Simulation studies for five or one (radial) component systems are available. Some are summarized in section 3.3. The most recent is by Rapp (1985). The SST and satellite gradiometry is intercompared with two recent geopotential models. (Rapp 81 and GEM-L2). The SST mission improves current models by a factor of 10. The gradiometer mission improves the SST results by about 60% with the exception of wavelengths shorter than 20 km.

6.2. Data Processing and Adjustment Techniques

The processing of a large amount of observations and the combination of several types of gravity data for estimation of gravity field models is today not on the optimal level. It is widely recognized that the least squares collocation is the optimal method for the solutions of many problems involving functionals of the gravity potential. It is also widely known that in many cases the advantages of this method is balanced out by the problem of inverting large matrices, the sizes of which, normally, equals the number of observations.. The problem of the number of observations will be more difficult if the data from satellite to satellite tracking and satellite gradiometry will be available.

The problem can be improved with utilization of the symmetries of data and with some restrictions to the pattern of the observations. Colombo's (1981) algorithms for harmonic analysis is an example where the symmetries of data and relations between spherical harmonics and Fourier series was used for the evaluation of harmonic coefficients (Section 3.5).

Hajela (1984) implemented Colombo's (1981) algorithms, originally designed for 5° × 5° anomalies, for global 1° × 1° anomalies with some necessary modifications. The optimal estimation of the coefficients was carried to degree and order 250. The use of optimal estimation resulted in improvement versus Rapp's coefficients obtained by a good approximative process (Section 3.5).

Jekeli (1984) discussed how the computational load can be reduced without hurting the optimal solution of the least squares collocation. According to the various patterns of observations, the auto-covariance function will have different Toeplitz structures. The number of the required operations for the different Toeplitz matrices are given in the paper (see Section 5.2.6.).

S.C. Bose et al. (1983), like Colombo's approach, recommend the more extensive exploitation of the grid structure of the data and of the structure of the covariance matrix for the reduction of the computations. They explore the equatorial and rotational symmetries to avoid the ill-conditioning by crowded data in polar regions. It is shown that at high latitudes thinned-out data samples can be used (Section 3.5)

Rapp (1984) reviewed some of the adjustment techniques for the combination of satellite and terrestrial data for the development of harmonic coefficients. For the method based on orthogonality relation between anomalies and harmonic coefficients, correction terms were derived for the effects of the assumptions of spherical approximation and of the zero elevation of the anomalies. The effects of the elevation is small (1% - 3%); the spherical approximation error is large, 10% at degree 75 and 31% at degree 180. Some alternate adjustment techniques are discussed using the orthogonality, with anomalies on a bounding sphere enclosing all the topography. It is recommended to use 0.5° × 0.5° mean anomalies globally when this data becomes available over significantly large areas (Section 3.5).

6.3. Estimation of Gravimetric Quantities in Local and Regional Areas.

The estimation of the local or regional gravity field usually consist of the determination of some functionals of the anomalous gravity potential in a particular area or at selected points.

The quality of the estimated gravimetric quantities will be determined by:

(1) the density of the available data

(2) the extent of the data coverage

(3) the sensitivity of the function to be estimated to the frequency of the given data set

(4) measurement accuracy.

The spectral sensitivities of geoid height (N), gravity anomaly (Δg) and of the second order radial gradient (T_{rr}), as computed by Schwarz (1984) are listed in Table 5.1. The current and future data types are characterized according to spectral resolution, density, coverage, distribution, and accuracy in Table 5.2 (after Schwarz 1984).

If a truncated series of harmonic coefficients is used as a reference field for some local estimation, for the proper weighting of the two data sets, it is desirable to know: a) the part of the total spectral power contained in a solution of degree and order N, b) which error spectrum is associated with the global solution? This subject area should be investigated in detail (Section 5.1).

The local or regional gravimetric quantities can be computed by integral formulae or by spherical harmonic series. The two are equivalent theoretically. Practically, however, they are different due to the differences in input data characteristics. Since the theoretical data requirements for the integration formulae cannot be fully satisfied, these formulae have been modified and used in combination with other data contributing the information outside of the zone of integration. Spherical harmonic expansions of the potential to degree and order 180 (Rapp 1978, 1981; Lerch et al 1981), expansions of the topography, of the rock equivalent topography and of the isostatically compensated topography (Rapp 1982) are available. Gridded digital terrain models, (DTMs) are also available for many areas for smoothing the local gravity field, necessary especially in topographically rough areas.

A number of studies for the estimation of local and regional gravity fields are listed and some are reviewed in detail in Section 5.2.1. The modifications of the Stokes and Vening-Meinesz formulae, their combination with harmonic coefficients and topographic data are discussed in Lachapelle (1984), Lachapelle (1978), and Jekeli (1981).

Gravimetric and satellite derived undulations are compared by Rapp and Wichiencharoen (1984) using 20 Doppler sites in the U.S., ten in the mountains and ten in flat areas. The results of the study are given in Section 5.2.2.

The accuracies of height anomalies and deflections of the vertical obtained from the combination of harmonic coefficients and mean gravity anomalies was studied by Heck (1983). The integration, replaced by summation, is limited to a cap around the computation point, the outer zones are replaced, or represented, by a geopotential model. The errors of various sources are analyzed and it is revealed that at the zeros of the kernel functions within the spherical integrals, the error functions show local minima. Therefore, the integration radius should be extended exactly to the first zero of the modified Stokes and Vening Meinesz kernel functions (Section 5.2.3).

Various methods of the consideration of terrain effects in the estimation of local gravity field quantities and the use of the stepwise collocation for the estimation are discussed in two studies. The first, by Forsberg and Tscherning (1981), is summarized in Section 5.2.4. A number of observed gravity anomalies and deflections of the vertical were predicted in the mountainous area of New Mexico. The original unchanged gravity data and their corrected versions by several reduction methods were used to illustrate the effect of the reductions. The conclusion of the study is that it is possible to predict deflections of the vertical and gravity anomalies in areas of rugged topography with accuracies of 1" and 3-4 mgal respectively from anomaly data spaced at 6' apart, if terrain corrections are computed from 0.5 × 0.5 minutes point elevation data. The other study, "The geoid of Austria" by Sunkel (1983), determined the geoid from 521 observed deflections of the vertical and a 20" × 20" digital terrain model as local data. In addition, the Rapp 1981 geopotential model and a 1° × 1° mean digital terrain model (DTM) were used as global information. The results from a stepwise collocation, topographic-isostatic reduction, and of the geopotential model were combined to obtain the final values. The differences between height anomalies and geoid heights are in very good agreement with the results obtained from Bouger anomalies and topographical elevation data (Section 5.2.4.2).

Some other collocation studies for prediction of free-air anomalies (Sunkel and Kraiger 1983), combination of collocation and geophysical inversion, and studies of terrain reduction accuracies in the report of Forsberg (1984) are reviewed in Section 5.2.4.3.

6.4. Gravity at Altitudes

Several of the more recent studies on this subject are described in sections 5.2.5.1 and 5.2.5.2.

In Sunkel (1984a), four different approaches are discussed for the prediction of gravity disturbance at altitudes, with the advantages and disadvantages of each. The recommended most feasible method is a combination of least squares collocation, integral formulae, high degree earth model, and the best topographic information (the last item is crucial in areas of rugged topography). A detailed error analysis is given for each step of the process, with the following groups: (1) data reduction errors, (2) mean anomaly errors, (3) representation errors, and (4) errors of data reduction effects on the estimated quantities.

In Sunkel (1981), the accuracies of the disturbance vector components at high altitude are estimated from a given set of free-air mean anomalies and their accuracies at sea level. The general conclusion from the study is that the radial component can be estimated with ±1 mgal accuracy at 50,000 ft altitude from the given, reasonable, data set. To obtain the same accuracy at 30,000 ft, the accuracy of the data (especially the 5' × 5' mean anomaly field) has to be increased by 60%. For the horizontal components, the best given data distribution yields only ±2.3 mgal accuracy at 30,000 ft (0."5 in the direction of the gravity vector). for the achievement of ±1 mgal, the given block sizes must be reduced by a factor of 2 out to a spherical distance of 30°, and the overall data errors reduced by 30%. Two methods, the Poisson upward continuation integral and least squares collocation, were used. The results agree within 10%.

The upward continuation of gravity anomalies was recently investigated by Cruz and Laskowski (1984). Three procedures were used and the results intercompared: (1) the Poisson integral using uncorrected anomalies, (2) the Poisson integration using terrain corrected surface anomalies, and (3) a procedure called "indirect method" where the given anomaly field is divided into three frequency ranges. The three ranges (low, medium, and high) are treated separately. Values at altitudes of 30, 10, and 5 km were computed along test profiles. The profiles obtained from terrain-uncorrected anomalies have a negative bias of 0.6, 0.5, and 0.7 mgal, respective to the above altitudes, versus the results from terrain corrected data. The standard

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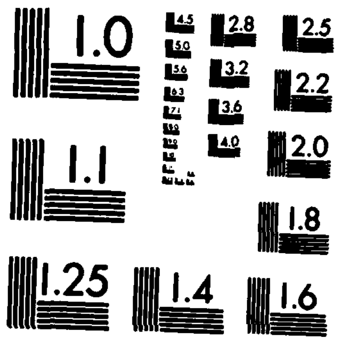

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deviation of the differences among the three methods are on the order of 0.5, 0.6, and 1.3 mgal at 30, 10, and 5 km elevations. A FFT was also tested and the results agree with Poisson integration on the level of 0.1 and 0.3 mgal at 30 and 10 km elevations.

With reference to the various reductions of gravity data and to the studies regarding the gravity disturbance vector at high altitudes, it is recommended to perform the following study:

(1) Considering that digital terrain models (DTMs) are already available in a gridded form for many regional and local areas, and harmonic coefficients of the potential up to order and degree 180, and 250 are also available, the feasibility of obtaining free-air mean anomalies for small blocks (5' x 5', 15' x 15', 30' x 30') from the above information should be investigated.

(2) The various steps of data reductions and the feasible methods for prediction of the gravity disturbance vector at high altitudes 30,000 to 200,000 ft) should be evaluated by numerical analyses. For the optimal and feasible procedure, an algorithm should be prepared consisting of a series of existing or new subroutines for the various steps of the procedures. The computations should provide the accuracies of each step involved and the errors of the final values, i.e. of the vector components of the gravity disturbance at the altitude.

6.5. Airborne Gradiometry.

In section 5.2.6., two recent studies by Jekeli (1983, 1984) and two earlier reports by K.P. Schwarz (1976,1977) on this subject are described. Each author prepared a study on the accuracies achievable with an assumed instrumentation, measuring accuracies, and observation pattern. The second analyses by the authors are concerned with techniques for processing of the large number of measurements.

Jekeli (1983) evaluates the accuracy of the point gravity vector from the five independent components of the gradient tenser observed at altitude. A uniform bidirectional track spacing at 600 m altitude was used. The gradiometer accuracy was assumed to be 1.9E standard error. The accuracy of the obtained gravity disturbance at 5 km track spacing was about 0.4 mgal, at 10 km spacing about 0.7 mgal, and at 15 km spacing about 1 mgal. The deflection components were accurate to $0\overset{''}{.}6$, $0\overset{''}{.}1$, and $0\overset{''}{.}22$ respectively. The change of the accuracies in the function of altitude is also give in section 5.2.6. The accuracies refer to a reference field corresponding to a harmonic expansion of about degree 80; therefore, the figures in an absolute sense, will be downgraded by the errors of such field. The method of analysis was the least squares collocation.

In Jekeli (1984) the possible arrangements for the auto-covariance functions (Toeplitz structures) to reduce the required operations for the inversion are discussed. These arrangements (possible under certain restrictions for the pattern of the observations), will not affect the optimal solution. Several "non-optimal" and "virtually optimal solutions" are also discussed. The various processing techniques are intercompared in a numerical analysis.

The accuracy study by Schwarz (1976) investigates the achievable accuracies of mean anomalies for various block sizes. The observed quantities are the first and second order gradients of the potential, measured by a combined accelerometer-gradiometer system (six measurements at each point). The inertial and gravitational forces can be separated from the output of the accelerometers by a system of linear second-order differential equations interrelating the gravity disturbance vector with the observed second order gradients and the accelerometer's output. By the integration of the first order gradients along the flight profile gravity anomalies (Δg) were obtained. The measurements used were Δg and the five components of the second order gradients in a ϕ,λ,r system. The least squares collocation technique was used for the analysis. At equal distance between two flight profiles, interpolation points were computed using a range of observations along both flight profiles. From the interpolated profiles, the mean anomalies were computed. With $0\overset{\circ}{.}3$ spacing of east-west profiles, ± 3 mgal accuracy was obtained for $5' \times 5'$ and $15' \times 15'$ blocks. With 1° spacing, ± 5 mgal was the accuracy of the $1^\circ \times 1^\circ$ blocks. The flight altitude was 10 km.

The simulation study of Schwarz (1977) consisted of four steps: (1) gravity and gradiometry data were generated at ground and flight level, (2) the flight level data were corrupted by an error model, (3) the ground level anomalies are estimated form the flight level data, and (4) estimated and generated ground level data were compared. Again the least squares collocation was used for the analysis. Parallel observation profiles with a constant intervals for the observations points (2.6 km) was the pattern of the measurements. This permitted computation of a "moving operator" (from the cross-covariance between signal and observation and from the auto-covariance matrix of the observations). This unchanged operator was moved along the flight profiles from one set of data points to the next. This arrangement allowed an efficient processing. The standard errors of the observed anomalies and gradients were assumed to be ±1 mgal and ±1E respectively. The standard error of a point along the interpolated profile on the ground was 3.3 mgal. Comparing the mean anomaly accuracies obtained from the accuracy study and simulation, the average of the differences is 0.4 mgal.

Recommendation: The conclusions of the experimental work and theoretical studies in the area of airborne gravimetry conducted by AFGL and OSU in the late 1960's and early 1970's was that the desired resolution and accuracy cannot be attained using accelerometer-type sensors on a moving platform. The reason for this was that the gravimeters or accelerometers measured not only gravity but also inertial disturbances, due to the various aircraft motions. According to the principle of equivalence, the two forces cannot be rigorously separated. Attempts to separate by the use of the frequency differences between gravitation and inertia was only partially successful. A rigorous separation is possible in case of second order gradients and linear inertial accelerations. This experience initiated the research to produce a moving base gravity gradiometer. The Global Positioning System (GPS) currently under full development changes the above situation. The GPS can provide instantaneously the position and velocity of a moving object with high accuracy. If an aircraft equipped with accelercmeters measuring the combined gravitational and inertial forces, the GPS system can furnish independently the inertial part, i.e. no second order gradient sensors are required. This would greatly simplify the process of airborne gravimetry. It is recommended that a study be performed for the determination of the feasibility, system parameters, and expected accuracies of this approach.

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