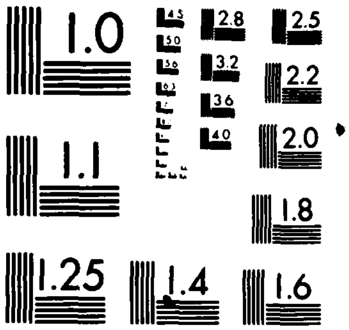
AD-A171 787 TWO DIMENSIONAL SAMPLING BY THE RETINAL LATTICE(U) YALE UNIV NEW HAVEN CT DEPT OF OPHTHALMOLOGY AND VISUAL SCIENCE J HIRSCH ET AL. SEP 83 TR-8303 1/1

UNCLASSIFIED AFOSR-TR-86-0601 F49620-83-C-0026 F/G 6/16 NL



AD-A171 787

DTIC FILE COPY

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 86-0 587
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): TWO DIMENSIONAL SAMPLING BY THE RETINAL LATTICE
5. TYPE OF REPORT & PERIOD COVERED: Technical Report
6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): Joy Hirsch, Ph.D. and Ron Hylton, Ph.D.
8. CONTRACT OR GRANT NUMBER(s): F49620-83-C-0026

9. PERFORMING ORGANIZATION NAME AND ADDRESS:
Yale University School of Medicine, Dept. of Ophth.
310 Cedar Street, BML 225
New Haven, CT 06510

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2313 A5

11. CONTROLLING OFFICE NAME AND ADDRESS:
Air Force Office of Scientific Research/NL
Bolling AFB, DC 20332

12. REPORT DATE: September 1983
13. NUMBER OF PAGES: 16

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report)
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

SEP 1 5 1986

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

retinal sampling, aliasing

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A discussion of sampling in two dimensions and its implications for vision is presented. We show that aliasing in two dimensions will change both the apparent frequency and orientation of a grating. Further, we demonstrate that the Nyquist frequency depends on orientation and is higher than the usual estimate of 60 c/deg.

DIVISION OF IMAGING SCIENCE

Two Dimensional Sampling by the Retinal Lattice

JOY HIRSCH[1] AND RON HYLTON[2]

Technical Report Number 8303

September 1983

DIAGNOSTIC IMAGING
YALE UNIVERSITY
NEW HAVEN, CONNECTICUT

86 9

Two Dimensional Sampling by the Retinal Lattice

JOY HIRSCH[1] AND RON HYLTON[2]

Technical Report Number 8303

September 1983

1 Yale University School of Medicine
Department of Ophthalmology and Visual Science
310 Cedar Street, P.O. Box 3333
New Haven, CT 06510

2 Columbia University
Department of Physics
New York, New York 10027

ABSTRACT

A discussion of sampling in two dimensions and its implications for vision is presented. We show that aliasing in two dimensions will change both the apparent frequency and orientation of a grating. Further, we demonstrate that the Nyquist frequency depends on orientation and is higher than the usual estimate of 60 c/deg.

Key words: retinal sampling, aliasing

There is a growing recognition that the characteristics of the retina considered as a two dimensional sampling system have significant consequences for vision (Williams and Collier, 1983; Yellott, 1982, 1983; French, Snyder and Stravenga, 1977; Hirsch and Hylton, 1983; Miller and Bernard, 1983). Although sampling in one dimension is generally understood, the fundamental differences between sampling in one and two dimensions have not been widely appreciated in the vision literature. In this report we will briefly discuss the effects of retinal sampling in two dimensions in order to clarify the fundamental notions related to visual science. Many of the following ideas are well known in other areas of imaging science (Goodman, 1968; Ziman, 1979; Ripley, 1981).

A common but mistaken notion is that if the retinal photoreceptor spacing is 1/120 (i.e. that the retinal sampling frequency is 120/deg), then aliasing due to sampling will cause a 100 c/deg sinusoidal grating to appear as a 20 c/deg grating (see for example Yellott, 1982, 1983). This results from applying a one dimensional answer to a two dimensional problem without respecting the fundamental differences between one and two dimensions. The most fundamental difference is that in one dimension spatial frequency is a <u>scalar</u> quantity whereas in two dimensions spatial frequency is a <u>vector</u> quantity. Thus, aliasing in two dimensions generally involves a vector subtraction in frequency, which almost always changes the orientation of the grating and certainly cannot be considered as a subtraction in the magnitudes of the frequencies.

The second major difference between one and two dimensions is that whereas a one dimensional system is fully characterized by a single (scalar) aliasing frequency, a two dimensional system, which possesses two independent directions, is fully characterized by two linearly independent (vector) aliasing frequencies. While the two aliasing frequencies may have the same magnitude, they always have different directions. In the case of a square lattice their directions differ by 90° while for a hexagonal lattice their orientations can be chosen to differ by either 60° or 120°. The two aliasing frequency vectors form the basis of a lattice in the spatial frequency domain known as the reciprocal lattice which contains all possible aliasing frequencies. That is, the sampled values of a sinusoid of frequency f are indistinguishable from the sampled values of a sinuoid of frequency $f + f_{RL}$ where f_{RL} is one of the points in the reciprocal lattice (see appendix). The only case in which aliasing corresponds to subtracting the magnitudes of the frequencies occurs when the orientation of the grating lines up with the orientation of one of the aliasing basis vectors, a very special case. These arguments are made more concrete below:

The essence of the sampling problem is how well the original continuous function can be recovered from a discrete set of sample points. In one dimension the answer is contained in the well known sampling theorem: if all the spatial frequency components of the continuous function vanish outside the region in frequency $-f_s/2$ to $f_s/2$, where f_s is the sampling frequency, then the continuous function can be recovered exactly from the sampled function (Bracewell, 1956). This result is easily visualized in terms of aliasing (Bracewell, 1965). Aliasing allows us to add to or subtract from the true frequency f any multiple of f_s. If the total range of frequencies in the true Fourier spectrum exceeds f_s, then aliasing can connect frequencies within the range, resulting in ambiguities that make recovery of the true Fourier spectrum impossible. However, if the total range of frequencies is less than or equal to f_s (as for $-f_s/2$ to $f_s/2$), then aliasing cannot connect two frequencies within the region and ambiguities do not occur. Note that the Nyquist criterion, which is usually thought of as the frequency $f_s/2$, is actually the boundary of the region $-f_s/2$ to $f_s/2$. The boundary of a one dimensional region is, of course, just the end points. The Nyquist interval is conventionally chosen to include the origin, other choices are possible.

The result for sampling in two dimensions is similar to the result in one dimension with the differences previously noted. To avoid ambiguities due to aliasing all two-dimensional spatial frequency components must vanish outside a region in the frequency plane. This region is determined by the requirement that no two frequencies within it can be connected by any combination of multiples of the two aliasing frequency basis vectors. This region turns out to be a polygon in the frequency plane (a square for a square spatial lattice and a hexagon for a hexagonal spatial lattice, if the region is chosen to include the origin). The perpendicular distance from the origin to the sides of the polygon is 1/(2d) for the square lattice and 1/(d SQRT(3)) for the hexagonal lattice, where d is the center-to-center spacing of the spatial lattice points. These frequency polygons have the same orientation as the space lattice for the two cases given. Figure 1 shows the basis vectors for the sampling (space) lattice and the reciprocal lattice for the cases of a square lattice and a hexagonal lattice. Figure 2 illustrates the construction of the polygon (known as the first Brillouin zone or primitive cell in the reciprocal lattice) which gives the conditions to avoid aliasing. The polygon is constructed by drawing lines from the origin to the points in the reciprocal lattice and then drawing the perpendicular bisectors of these lines. The region inside the perpendicular bisectors is characterized by the property that no two points can be connected by an aliasing vector. Note that the Nyquist criterion is the boundary of the polygon, and the magnitude of the Nyquist frequency depends on the orientation of the presented frequency relative to the sampling lattice. The orientation dependence of the Nyquist frequency can be regarded as due to an effective one dimensional sampling frequency which depends on orientation. For the square lattice, aliasing sets in at a frequency between 1 and SQRT(2) times 1/(2d), depending on orientation, while for the hexagonal lattice aliasing sets in at frequencies between 2/SQRT(3) and 4/3 times 1/(2d). Snyder and Miller (1977) have also noted that the effective sampling frequency of a hexagonal lattice is higher than the one dimensional estimate of the Nyquist limit. Thus since the retinal photoreceptors are hexagonally packed with center-to-center spacing of approximately 1/120°, aliasing sets in at frequencies ranging from 69 to 80 c/deg, depending on orientation, not 60 c/deg as is usually assumed. Thus an optical cutoff frequency of 60 c/deg is well within the sampling limit for the fovea.

Anatomical results have shown that the photoreceptor lattice is hexagonally packed (e.g. Miller, 1979). Sampling with a hexagonal lattice is preferable to sampling with a square lattice for several reasons. First, hexagonal packing has a greater number of samples per unit area than square packing, leading to a higher average Nyquist frequency (greater area in the Brillouin zone). Second, the hexagonal Brillouin zone is more nearly circular than the square zone resulting in less severe dependence on orientation. A third reason is somewhat speculative. We have recently suggested that hyperacuity involves short range interpolation between photoreceptors, possibly quadratic interpolation (Hirsch and Hylton, 1982). Since a two-dimensional quadratic has six parameters, such a mechanism requires at least six inputs. Then for a hexagonally packed lattice, a photoreceptor plus its six neighbors forms a natural grouping for short range interpolation, but a square lattice would require both the nearest and the more distant second nearest neighbors as input for quadratic interpolation.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the contributions of Drs. Gene Gindi and Stelios Orphanoudakis, Department of Diagnostic Radiology, Division of Medical Imaging, Yale University School of Medicine. This work was partially supported by grants from the National Eye Institute (EY 00785 and EY 00167), Research to Prevent Blindness, the Connecticut Lions Association, and the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under grant no. 49620-83-C-0026. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon.

APPENDIX

Mathematical Details

We present here a brief mathematical derivation for aliasing in two dimensions. The derivation is closely related to a basic result in solid state physics, where Fourier expansion of periodic systems (crystals) are quite important. (See, for example, Kittel, 1971, for a discussion of periodic systems, the reciprocal lattice, and Brillouin zones.)

The analysis starts from the definition of a lattice. An N dimensional lattice is fully described by the position of one lattice point (which we take to be the origin) and a set of N vectors, called the basis vectors of the lattice, which essentially give the magnitude and direction of the steps necessary to move from one lattice point to the next. An N dimensional lattice has N linearly independent such steps, and every lattice point has a position given by

$$\vec{P} = \sum_{i=1}^{N} M_i * \vec{d}_i$$

where the M_i are integers and the $\vec{d}_i$ are the basis vectors of the lattice. If we present a frequency $\vec{f}$, we measure the values of the sinusoid only at the set of points given by $\vec{P}$ above. Aliasing occurs when the values measured at the lattice points for a frequency $\vec{f}$ are the same as for a frequency $\vec{f} + \vec{f_a}$ for all lattice points. Thus aliasing requires:

$$e^{i2\pi \ \vec{f}\cdot\vec{P}} = e^{i2\pi(\vec{f}+\vec{f}_a)\cdot\vec{P}} \qquad \text{or} \qquad e^{i2\pi\vec{f}_a\cdot\vec{P}} = 1$$

for all lattice points $\vec{P}$. (The i appearing in front of 2π is the SQRT(-1).) This leads to sets of simultaneous linear equations of the form

$$\vec{f}_a \cdot \vec{P} = K_p \ .$$

where $\vec{P}$ is the set of points given above and K_p is an integer which varies from point to point. Substituting the form for $\vec{P}$ given above leads to the equations

$$\vec{f}_a \cdot \vec{d}_i = K_i$$

where i takes on the values 1 to N and K_i is an integer which can be different for each $\vec{d}_i$. Although we do not prove it here, there are an infinite number of frequencies $\vec{f}_a$ which satisfy these conditions. In fact, the solutions form a lattice in the spatial frequency domain known as the reciprocal lattice, and all the $\vec{f}_a$ are given by

$$\vec{f}_a = \sum_{i=1}^{N} M_i * \vec{f}_{ai}$$

where the M_i are integers and the $\vec{f}_{ai}$ are the N basis vectors for the reciprocal lattice. We will solve for the $\vec{f}_{ai}$ for the two dimensional case.

The aliasing conditions for two dimensions can be written as

$$\vec{f}_{a1} \cdot \vec{d}_1 = 1 \qquad \vec{f}_{a1} \cdot \vec{d}_2 = 0$$

$$\vec{f}_{a2} \cdot \vec{d}_1 = 0 \qquad \vec{f}_{a2} \cdot \vec{d}_2 = 1$$

Thus we require that $\vec{f}_{a1}$ be perpendicular to $\vec{d}_2$ and have a projection of $1/d_1$ onto the $\vec{d}_1$ direction, and similarly for $\vec{f}_{a2}$. These vectors can easily be found by using the cross product $\vec{d}_1 \times \vec{d}_2$. This is a vector perpendicular to both $\vec{d}_1$ and $\vec{d}_2$ (ie, it is perpendicular to the plane of the lattice). Then $\vec{d}_2 \times (\vec{d}_1 \times \vec{d}_2)$ is a vector in the lattice plane perpendicular to $\vec{d}_2$ but not $\vec{d}_1$, and its length can be chosen to have the correct projection on $\vec{d}_1$. Similarly, $\vec{d}_1 \times (\vec{d}_1 \times \vec{d}_2)$ is perpendicular to $\vec{d}_1$ but can be chosen to have the correct projection on $\vec{d}_2$. Thus we have found the basis for the reciprocal lattice in two dimensions, and all aliasing frequencies have the form

$$\vec{f}_a = M_1 * \vec{f}_{a1} + M_2 * \vec{f}_{a2}$$

where the M_i are integers. The value due to one of the aliasing frequencies at one of the sample points is then

$$e^{i2\pi(M_1 * \vec{f}_{a1} + M_2 * \vec{f}_{a2}) \cdot (N_1 * \vec{d}_1 + N_2 * \vec{d}_2)}$$

$$= e^{i2\pi(M_1N_1 + M_2N_2)} = 1$$

for all sample points. (N_i and M_i are integers). Thus the aliasing condition is met.

FIGURE CAPTIONS

Figure 1: Sampling and reciprocal lattices. The left hand figures show the basis for the sampling (space domain) lattice for square and hexagonal symmetries. The dots represent the points in space at which measurements are taken. The right hand figures show the corresponding reciprocal (spatial frequency domain) lattices. The points in the reciprocal lattice are aliasing frequencies which have the property that two spatial frequencies differing by any multiple of an aliasing frequency give rise to identical values at all the sample points and hence are indistinguishable.

Figure 2: Construction of the first Brillouin Zone. In order to avoid aliasing spatial frequencies must be restricted to a region in the reciprocal lattice, the first Brillouin zone, such that no two points in this region differ by one of the aliasing vectors. The boundary of this region is the equivalent of the Nyquist criterion in one dimension. The zone can be constructed by taking the region around the origin bounded by the perpendicular bisectors of lines drawn from the origin to the aliasing points. (See Kittel, 1971).

REFERENCES

Bracewell, R.M. (1956) Strip integration in radio astronomy. Australian Journal of Physics, 9, 198-217.

Bracewell, R.M. (1965) The Fourier transform and its applications. McGraw Hill, N.Y., 197-198.

French, A.S., Synder, A.W. and Stavenga, D.G. (1977) Image degradation by a random retinal mosaic. Biol. Cybernetics, 27, 229-233.

Goodman, J.W. Introduction to Fourier Optics. McGraw-Hill, New York, 1968.

Hirsch, J. and Hylton, R. (1982) Limits of spatial-frequency discrimination as evidence of neural interpolation. J. Opt. Soc. Am. 72, 1367-1374.

Hirsch, J. and Hylton, R. (1983) Quality of the primate photoreceptor lattice and limits of spatial vision. Vision Res., in press.

Kittel, C. (1971) Introduction to Solid State Physics (4th Edition) John Wiley and Sons, Inc.

Miller, W.H. Handbook of Sensory Physiology, Vol VII/6A, Fig. 30, p.118, Springer, Berlin, 1979.

Miller, W.H. and Bernard, G.D. (1983) Averaging over the foveal receptor aperature curtails aliasing. Vision Res., in press.

Ripley, Brian D. Spatial Statistics. John Wiley & Sons, New York, 1981.

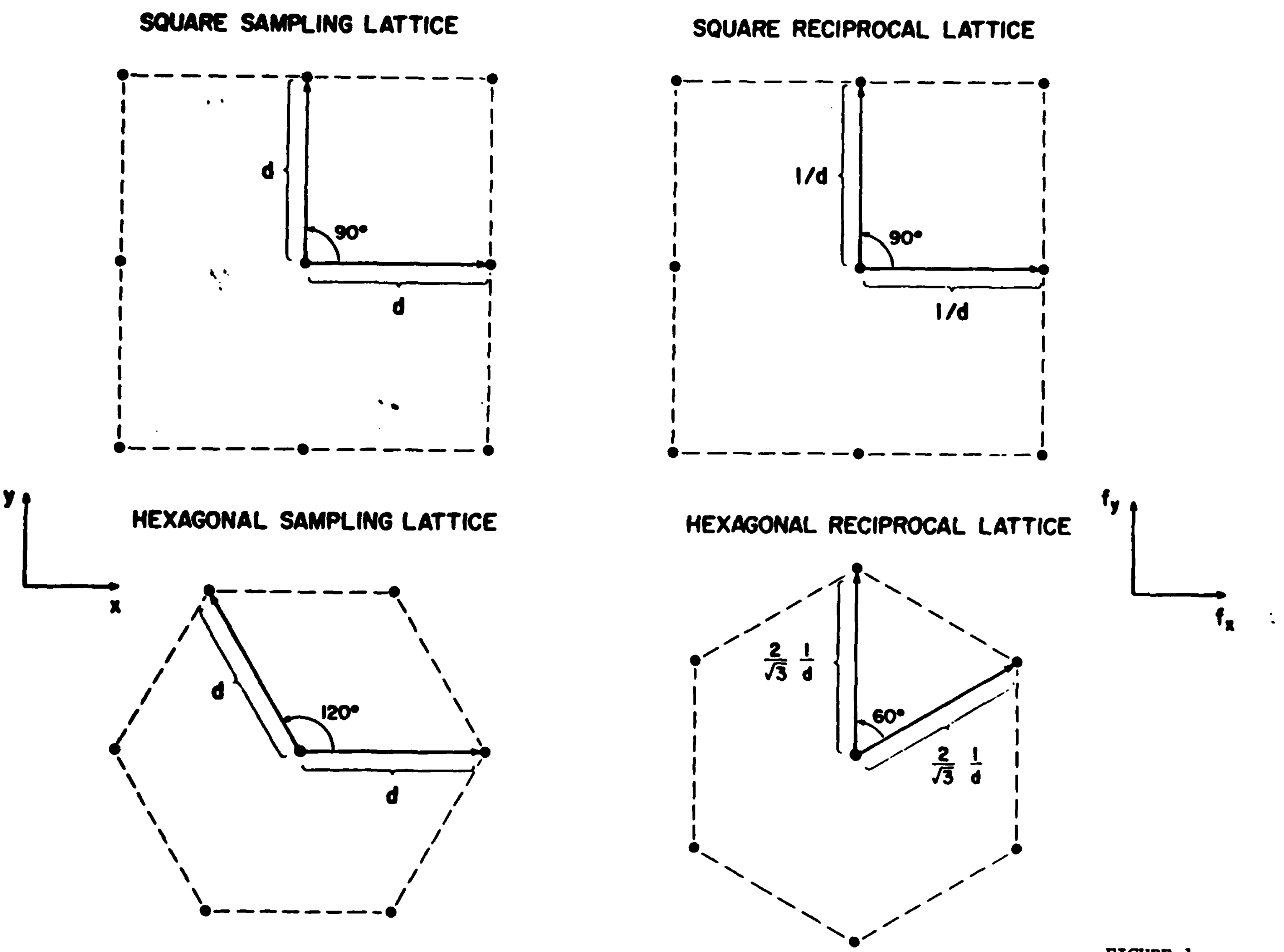
Snyder, A.W. and Miller, W.H. (1977) Photoreceptor diameter and spacing for highest resolving power. J. Opt. Soc. Am., 67(5).

Williams, D.R. and Collier, R. (1983) Consequences of spatial sampling by a human photoreceptor mosaic. Science, 221, 385-387.

Yellott, Jr., J.I. (1982) Spatial analysis of spatial sampling by photoreceptors: topological disorder prevents aliasing. Vis. Res. 22, 1205-1210.

Yellott, Jr., J.I. (1983) Spectral Consequences of Photoreceptor Sampling in the Rhesus Retina. Science, 221, 382-385.

Ziman, J.M. Models of disorder. Cambridge University Press, 1979.



SQUARE SAMPLING LATTICE
d
90°
d
SQUARE RECIPROCAL LATTICE
1/d
90°
1/d
y
x
HEXAGONAL SAMPLING LATTICE
d
120°
d
HEXAGONAL RECIPROCAL LATTICE
f_y
f_x
$\frac{2}{\sqrt{3}}\ \frac{1}{d}$
60°
$\frac{2}{\sqrt{3}}\ \frac{1}{d}$

FIGURE 1

CONSTRUCTION OF BRILLOUIN ZONE IN THE RECIPROCAL LATTICE

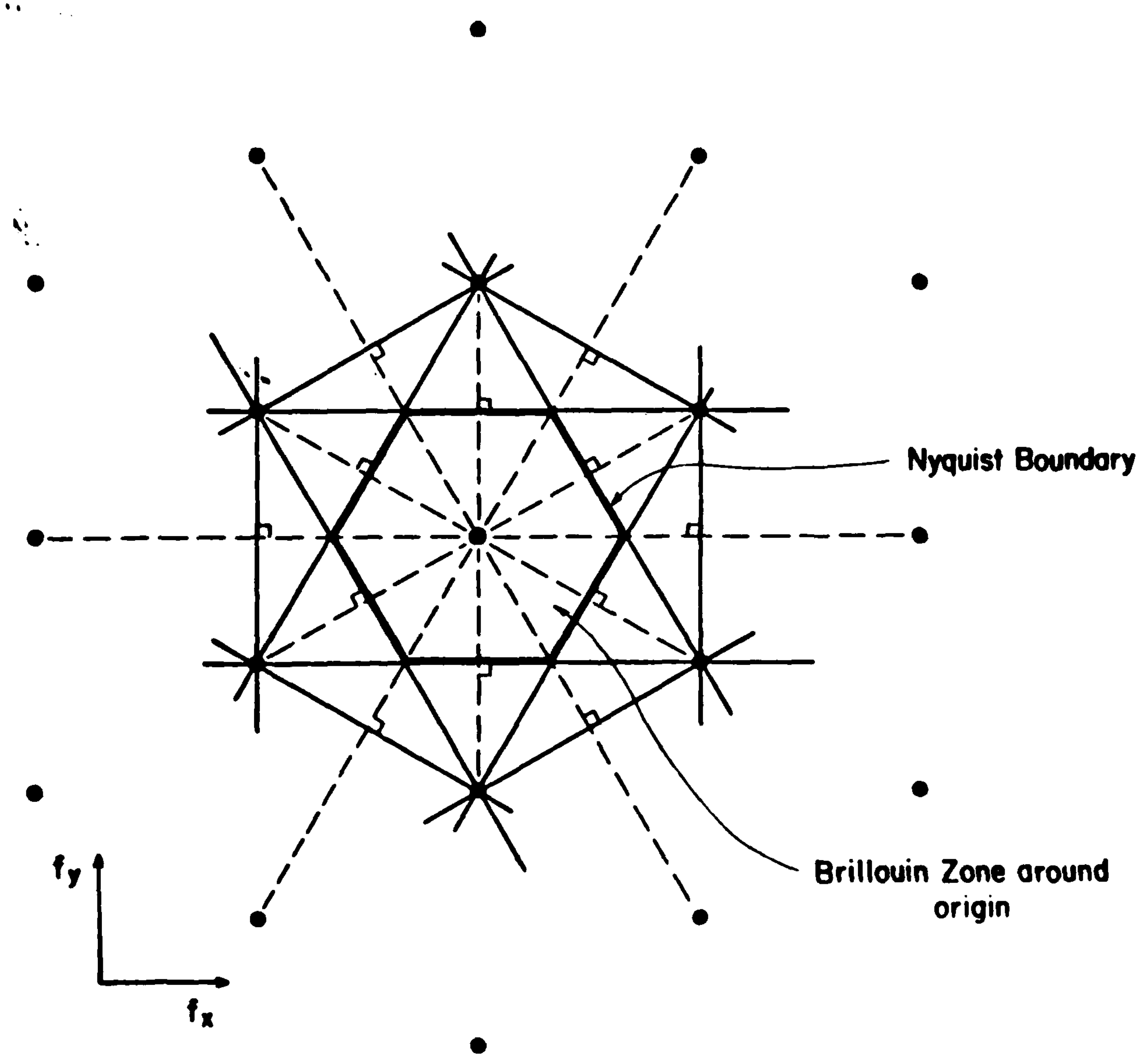


FIGURE 2

END
10-86
DTIC