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A STUDY OF THE FEASIBILITY OF
CONSTRUCTING A HIGH PRESSURE (4kBAR)
MULTIAXIAL MATERIAL TEST DEVICE

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Prepared for:

OFFICE OF NAVAL RESEARCH
800 N. Quincy Street
Arlington, Virginia 22217-5000
A Study of the Feasibility of Constructing a High Pressure (4 kBar), Multiaxial Material Test Device

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Final

From 85-9-15 to 86-9-14

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See Next Sheet

High Pressure Testing  Concrete Testing, Multiaxial

Constitutive Relations

22a NAME OF RESPONSIBLE INDIVIDUAL

22b TELEPHONE (Include Area Code)
Abstract

A study was conducted into the feasibility of constructing a fluid cushion, multiaxial loading device for testing concrete or other materials to high pressures. The requirements for the device are: 1) three independently controlled stress axes, 2) maximum pressures in each axis to 4 kBar, 3) load application through fluid cushions to minimize undesirable boundary effects, and 4) cubical specimen sizes of 4 inches (10 cm) or greater.

A number of possible design concepts were developed and critiqued. Of these two appear to hold promise for eventual design and construction. The most promising concept requires use of a preexisting conventional testing machine of at least 2 million lbs (8.9 MN) capacity. The second concept would be feasible for facilities not having a large loading machine.

Both proposed concepts depend on the successful development of a bladder/seal fluid cushion patterned after that of Michelis' but having a larger size and increased pressure capacity. Means to measure specimen displacement following the approach of Michelis would also need to be developed.

A development program together with time and cost estimates is presented for an effort to, 1) develop the fluid cushion design, 2) design and construct the multiaxial device and, 3) perform an initial series of calibration and material tests.
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SECTION 1
INTRODUCTION

1.1 IDENTIFICATION OF PROBLEM

Present and future U.S. defense needs will require the survival of critical structures from very large blast generated pressure loadings. These loadings are far in excess of the loadings encountered in conventional civil construction.

Structure response and failure level prediction for these structures are estimated analytically using complex structural analysis computer codes. These codes require an accurate representation of material behavior under stress states of interest if an accurate estimate of overall structural response is to be provided. The need for material testing at high confining pressure levels and the critical role that data from such tests play in forming a suitable database for model formulation and verification is discussed in proceedings and papers from conferences held at Northwestern University in 1983 and 1984 (Ref. 10 and 11).

These material models should be capable of describing the stress-strain-failure behavior of the materials (soil, rock, concrete) for the stress states and stress levels likely to be encountered in the structure. The stress states are likely to be multiaxial \( \sigma_1 \neq \sigma_2 \neq \sigma_3 \) and to pressure levels in the 2 to 4 kBar range. Presently available true multiaxial test equipment is limited to at best a 1.0 kBar level with most cells operating below this level. A need exists, therefore, to develop equipment and test methods to define material behavior at these significantly higher pressure levels so that accurate predictions of structural response of critical structures can be
provided by both present and future computer codes.

1.2 OBJECTIVE

The objective of this study was to assess the technical feasibility of designing and manufacturing a fluid cushion multiaxial material test device. The original pressure level for the study was to be in the range of 2-3 kBar, but to better cover current stress levels of interest for constitutive property determination of concrete the pressure range has been increased to 2-4 kBar. The study will be limited to the use of fluid cushions as the specimen loading mechanism.

Specimen size (4" cubical or greater) will be sufficiently large to include reasonable aggregate in concrete specimens. This study will determine the technical feasibility of building this device by evaluation of several conceptual designs, preliminary stress analysis, and identification of areas needing development. The report will present a proposed design concept for the testing device together with preliminary time and cost estimates.

Specific technical objectives of this study were:

- to establish a conceptual design for the device which appears capable of meeting the design objectives.

- to establish the availability of commercial instrumentation, seals, and hydraulic equipment for the pressure ranges of interest.

- to define what unproven design concepts and instrumentation may require a proof-of-concept test program to
establish design validity and to outline such a test program.

to provide an overall assessment of the technical feasibility and manufacturing cost of multiaxial test cell in the 2-4 kBar pressure range.

1.3 APPROACH

The approach taken in this study is to first briefly review in Section 2 past efforts in constructing fluid cushion devices. Next in Section 3 a number of different concepts for possibly achieving the desired loading levels and deformations are outlined in sketch form. These are critiqued to identify problem areas that require additional engineering development or problem areas that pose difficult or insurmountable problems in terms of pressure containment, instrumentation or safe operation.

In Section 4 various approaches to the problem of obtaining a uniform applied surface traction on the specimen face are presented and discussed. Several approaches to fluid cushion design are presented. In Section 5 the problem of measuring the deformations of the material specimen under the applied loading is presented. A number of different methods to measure deformation are discussed in terms of their applicability to multiaxial testing.

Finally in Section 6 the feasibility of constructing a 2-4 kBar multiaxial cell is discussed. Two proposed design approaches are presented. For each design approach an outline of a development and design program is given. This includes a listing of technical problems which require solution before
design and construction of the test device should be undertaken. A very preliminary estimate of time and cost involved with each approach is also given.
SECTION 2
PREVIOUS WORK

Foppl (1900) early in the development of materials research recognized the influence of interfacial load platen friction on the measured strength of rock and concrete materials. He utilized a multiaxial loading frame for his experiments on 7.1 cm cubical specimens (Fig. 2.1). His initial experiments used wax or grease between the platen and specimen as a friction reducer. Strength reductions from ungreased to greased of 44% were observed for cement mortar cubes. Even greater strength reductions were observed when testing granite specimens. Foppl observed that the lubricant was injected into the specimen pores at high pressure levels leading to premature failure. To overcome this, he jacketed his specimens with a 0.08 mm (.003") brass sheet.

In a second series of tests, Foppl (1902) developed a water pillow (wasserkissen) fluid cushion loading device which he evaluated in a series of uniaxial compression tests, (Fig. 2.2.) Stress was applied using a rubber balloon pressure membrane and a rubber sheet between the balloon and specimen. Axial stresses to approximately 2,650 psi (18.3 MPa) were achieved with this device.

Ko (1967), designed a low pressure test device for sand which applied a $\sigma_1 \neq \sigma_2 \neq \sigma_3$ stress state to a soil specimen through a fluid cushion (platen) so that all six surfaces of the cubical specimen were subjected to only normal pressures. The fluid cushion loading method eliminated the interfacial shear stresses which would otherwise create a significant confining stress state in the specimen and in turn would influence deformational and failure behavior.
FIGURE 2.1
Multiaxial Loading Device (Foppl, 1900)
FIGURE 2.2
Waterpillow Device, (Foppl, 1902)
Atkinson (1972) designed a multiaxial, cubical, fluid cushion test cell for use with rock and concrete. This device loads a 4.0 inch (10 cm) cubical specimen through flexible, fluid filled platens to working pressures from approximately .54 kBar hydrostatic to approximately .75 uniaxial. This cubical cell used a fluid cushion design consisting of a leather seal with a vinyl back-up seal and a thin latex membrane. Deformations were measured using proximity type transducers.

Sture (1973) by improving certain design details raised the working pressures to 0.8 kBar hydrostatic and 1.2 kBar uniaxial. Sture replaced the fluid cushion design of Atkinson with a single piece injection molded polyurethane membrane. He also was the first to use immersed LVDT's for measuring specimen deformations. Meier (1983) developed equipment and experimental procedures for converting one axis of Sture's cubical cell to apply a tensile stress to the specimen while the other two axes apply compressive stresses.

All of the high pressure multiaxial cells described above employ a frame machined from a solid billet of steel to both form the individual pressure chambers and to serve as a reaction frame for the loads applied to the cubical specimens (Fig. 2.3). The pressure is applied to the individual faces of the cubical specimen through flexible membranes. Specimen deformation under applied pressures are measured using either LVDT on proximity type transducers which are mounted on the exterior wall. All of these devices can apply a loading in compressive stress space, and with the tensile device of Meier, can provide one axis of tensile stress. An excellent discussion of the advantages and disadvantages associated with stress-controlled, strain-controlled or mixed boundary conditions for geologic materials is provided by Sture and Desai (1979).
FIGURE 2.3

Exploded View of Cubical Cell
Ideally, for determination of material constituent relations, one should be able to apply to the specimen uniform known boundary stresses or uniform known surface strains or stresses, respectively. The application of known surface displacements (strain-controlled boundary conditions) to all six sides of a prismatic specimen of a stiff material such as concrete is difficult. The relative rigidities between the specimen and loading platens and the presence of surface shear stresses between the specimen and platen makes determination of the stress state in the specimen very difficult. If moderate to large strains are experienced a complicated apparatus is necessary to adjust the platens to accommodate the changing dimensions of the specimen (Kirsten, 1985).

The mixed boundary condition apparatus is best represented by the conventional cylindrical triaxial test in which the axial load is supplied through a rigid strain controlled boundary while the lateral confining pressures are applied through a flexible membrane providing a stress controlled boundary. This mixed type boundary situation can result in non uniform stress and strain fields within the specimen. The conventional cylindrical triaxial test does not provide any information on the influence of the intermediate principal stress with the result that material models developed from triaxial data may be inaccurate.

A stress controlled test apparatus typically uses air or fluid filled membranes to apply a pressure to the surfaces of the specimen. When the stiffness of the membrane material is low compared to that of the specimen, the fluid filled membranes apply a known normal surface traction to the specimen. At the edges of a prismatic specimen, the membranes, however, must be
sufficiently stiff so that the membrane from the higher pressure side (σ₁) does not intrude and rupture into the lower pressure (σ₃) cavity. This problem can become critical at high levels of pressure difference, σ₁ - σ₃, and at the large strain levels often associated with the large pressure difference levels.

The mixed boundary condition cell of Michelis (1985) has achieved the highest fluid cushion multiaxial pressures reported to date for tests on concrete. His device has achieved pressures to 29,000 psi (200MPa) using fluid cushions to load one of the three axes, (Fig. 2.4). His device is designed for a specimen size of approximately 2.0 x 2.0 x 4.0 inch (5.0 x 5.0 x 10.0 cm). Each of the three axes of Michelis' cell employ a different loading mechanism. The vertical axis uses a conventional steel load platen. One lateral axis uses a fluid cushion bearing directly on the specimen while in the other lateral axis the fluid cushion loads a series of steel prisms which in turn load the specimen. Michelis' system has two innovative design features, namely, the design of the fluid cushion and the incorporation of a deformation measurement scheme into the fluid cushion. These will be discussed in later sections.
FIGURE 2.4
Cross Section of Michelis' Mixed Boundary Condition Cell (Michelis, 1985)
SECTION 3
REACTION SYSTEM DESIGN CONCEPTS

3.1 INTRODUCTION

One of the critical problems to be overcome in the design of a high pressure cubical cell is to react by some method the high forces which are applied to the cubical specimen. The magnitudes of these forces are listed in Table 3.1 for 4 inch (10.16 cm), 6 inch (15.24 cm) and 10 inch (25.40 cm) cubical specimens. Even for the smallest four inch specimen a reactive force of 960 kips (4.27 MN) is generated for a 60 ksi (414 MPa) pressure.

Adding to the difficulty of design is the requirement that the high reactive forces must be provided in three orthogonal directions if the material specimen is to be loaded to high stress levels in hydrostatic loading. If the orthogonal loads are to be carried by an independent external structure, the resulting structure will be quite massive. This leads to the requirement for special foundations, overhead cranes and a large enclosed area. Very quickly one is designing a facility rather than an individual piece of test equipment. On the other hand, designing the high pressure cell such that both the containment of the six fluid cushions and the reaction to the applied specimen loads are provided by the same structure poses considerable difficulties when stress concentration factors are included even if the strongest commercially available steels are employed.

In this section a number of individual design concepts are presented. These have been developed to the point where preliminary design and resulting stress analysis are possible.
### TABLE 3.1

**REACTION FORCES VS CUBE SIZE AND PRESSURE**

(Reactive Forces in lbs x 10 and MN)

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>Specimen Face Size</th>
<th>Specimen Face Size</th>
<th>Specimen Face Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4&quot; x 4&quot;</td>
<td>6&quot; x 6&quot;</td>
<td>10&quot; x 10&quot;</td>
</tr>
<tr>
<td></td>
<td>(10.16 x 10.16 cm)</td>
<td>(15.24 x 15.24 cm)</td>
<td>(25.40 x 25.40 cm)</td>
</tr>
<tr>
<td>30,000</td>
<td>0.480</td>
<td>1.080</td>
<td>3.000</td>
</tr>
<tr>
<td>(207)</td>
<td>(2.14)</td>
<td>(4.80)</td>
<td>(13.35)</td>
</tr>
<tr>
<td>40,000</td>
<td>0.640</td>
<td>1.440</td>
<td>4.000</td>
</tr>
<tr>
<td>(276)</td>
<td>(2.85)</td>
<td>(6.41)</td>
<td>(17.79)</td>
</tr>
<tr>
<td>50,000</td>
<td>0.800</td>
<td>1.800</td>
<td>5.000</td>
</tr>
<tr>
<td>(345)</td>
<td>(3.56)</td>
<td>(8.01)</td>
<td>(22.24)</td>
</tr>
<tr>
<td>60,000</td>
<td>0.960</td>
<td>2.160</td>
<td>6.000</td>
</tr>
<tr>
<td>(414)</td>
<td>(4.27)</td>
<td>(9.61)</td>
<td>(26.69)</td>
</tr>
<tr>
<td>70,000</td>
<td>1.120</td>
<td>2.520</td>
<td>7.000</td>
</tr>
<tr>
<td>(483)</td>
<td>(4.98)</td>
<td>(11.21)</td>
<td>(31.14)</td>
</tr>
</tbody>
</table>
Each concept is critiqued in terms of applied pressure capacity, instrumentation problems, interaction with fluid cushion requirements, anticipated manufacturing and construction difficulty and operational characteristics. Of the various concepts presented two appear to provide the best opportunity to achieve the required objectives of a multiaxial, fluid cushion test device for pressures to 4 kBar (60,000 psi). These are discussed in Section 6.

3.2 DESIGN CONCEPTS

3.2.1 Existing Cubical Cell Design

The present design of the cubical, multiaxial, fluid cushion test cell uses a 10.0 inch (25.4 cm) cubical steel frame to contain the fluid cushions and to react the specimen loads (Fig. 2.3). The present design is very inefficient in that for multiaxial loadings the total reactive force in any one direction includes both the specimen load as well as the lateral loads exerted by the four fluid cushion membranes located in orthogonal directions. For a hydrostatic test only 31% of the total force to be reacted along a given axis is produced by the specimen load.

The fluid cushion membrane applies an internal pressure to the sides of the square hole which is machined into the steel cube with a 0.050 inch (1.3 mm) corner radius. Finite element studies (Appendix A) show that stress concentration factors of approximately 5.4 are present for this case.

The present cell is manufactured from a VASCOMAX T-250 maraging steel billet and has operational pressure limits of
11,600 psi (80 MPa) in hydrostatic loading. One attempt was made to increase the capacity by increasing the strength of the steel from the 250 ksi provided by the VASCOMAX T-250 to 350 ksi provided by a VASCOMAX T-350 steel. This attempt ended in failure of the frame at load levels below those which had been previously obtained using the T-250 steel. This early failure was attributed to the very brittle nature of the T-350 steel and the very small size of the critical flaw length for the steel. The frame manufactured from the T-250 steel very likely is subject to localized yielding at high pressure levels.

Even if the fluid cushions of the present cell were modified to the Michelis type discussed in Section 4, the present design would have significant pressure capacity limitations for the two to four kBar pressures under consideration. Increasing the steel strength has been shown only to provide increased risk of failure due to brittle crack propagation. Increasing the overall size of the frame would provide little benefit as the stress concentration factor produced by the 0.050 inch corner radius would be only slightly reduced.

3.2.2 Pressure Vessel Concept

One means to provide both the containment of the individual fluid cushions and the reactive force required is to construct a cylindrical, multi-walled pressure vessel shown in Figures 3.1 and 3.2. In this concept the two load axes, X & Y, would be provided by the pressure vessel while the third, Z axis, would be carried by a conventional, large capacity testing machine on load frames. This axis would also contain fluid cushions as shown in (Fig. 3.3).
FIGURE 3.1
Horizontal Section View of Pressure Vessel Concept
FIGURE 3.2
Vertical Section of Pressure Vessel Concept
FIGURE 3.3
Section View of Z-Axis Platen, Pressure Vessel Concept
The principal advantage of the containment vessel concept is the relative efficiency with which the specimen forces are reted. This will result in a test device with a much smaller overall size with resulting savings in manufacturing and operational cost. Additional capacity can be obtained by manufacturing the containment vessel as a double or triple compound vessel in which concentric cylinders are assembled with a prescribed interference fit. The effect of assembly with an interference fit is to create at the interior radius of the vessel a zone of compressive tangential stress. The compressive stress zone falls in the same zone as the high tangential tensile stresses produced by the internal pressure loading. Under ideal conditions a 40% or greater increase in internal pressure capacity can result from the prestressing effects of compound vessel design.

Another version of compound vessel design occurs when the internal liner is assembled from radial segments and is press fit into an external cylinder (Fig. 3.4). This design permits a high internal pressure to be transferred to the load-carrying cylinder at a greater radius thus reducing the effective magnitude of the applied pressure. Use of segments for the internal liner provides distinct advantages in machining and instrumentation. This is illustrated in Figure 3.5 which presents results of a finite element analysis of a pressure vessel in which the triangular shapes required to provide fluid cushion cavity separation are machined on the inner surface of the pressure vessel. An increase in $J_2$ stress of approximately 10% is produced by this geometry compared to a cylindrical inner surface.

The fluid cushion for the X and Y direction would be fabricated from a flexible bladder and a metal seal as described in Section 4. The bladder can be manufactured using rotational
FIGURE 3.4
Segmented Liner Design Concept
FIGURE 3.5
Stresses \( (J^1_2) \) Associated with Hydrostatic Loading of Pressure Vessel
molding technology to have a flat surface next to the specimen and a curved surface that conforms to the pressure vessel walls.

The third or vertical axis load would be reacted by a large capacity testing machine or by a load frame/hydraulic jack combination. The test machine or load frame should have a capacity in the range of 1 million to 2.16 million pounds (4.4 GN to 10.0 GN). In order to avoid difficulties that would arise from machine flexibility the operational concept using the large testing machine would be to apply, prior to the test, a load in excess of the total vertical reactive load from a test. This vertical preloading would be carried by the pressure vessel as a compressive stress in the vertical direction. Application of fluid pressures in the vertical axis would cause a transfer in the vertical load from preloading the vessel to reacting the applied specimen stresses. By requiring the preload force to be in excess of the expected reactive load one is always working with the axial stiffness of the pressure vessel rather than with the stiffness of the testing machine which would likely be much lower.

The requirement for an existing large capacity testing machine effectively limits the development of this concept to a few locations with suitable machines. These include the U.S. Bureau of Reclamation, Denver, the Waterways Experiment Station, Vicksburg, the University of Texas - Austin and the U.S. Bureau of Standards, Washington which have machines in excess of 2 million lbs (8.8 GN) capacity. Use of smaller machines down to the 1 million lb (4.4 GN) capacity would limit specimen cube size to 4 inches. This in turn limits the maximum size aggregate that can be used to prepare the specimen with the result that the concrete tested may not be truly representative of field placed mixes.
The most serious disadvantage to this concept is the difficulty foreseen in incorporating deformation measuring transducers in the individual pressure chambers contained in the pressure vessel. Any transducer used will require a penetration of the pressure vessel wall to bring the instrumentation lead to the exterior. A circular hole will produce a stress magnification factor close to 3.0 at the inside surface of the pressure vessel. This will reduce the pressure rating of the containment vessel as the inside radius is the location of the high tensile stresses in the tangential direction that govern maximum pressure levels. Any design which incorporates an instrumentation hole will likely be subject to localized yielding around the hole if high stress levels are to be carried. This implies use of precision machining techniques to avoid presence of stress concentrations and use of a steel having sufficient ductility to accommodate such localized yielding.

The manufacture of the pressure vessel will require care in the machining and press fit of the components of the multi-wall cylinder. Assembly and material requirements suggest that a press fit of a tapered interference surface be used rather than a temperature shrink fit procedure.

3.2.3 External Reactive Forces Supplied by Hydraulic Actuators

One means to supply the reactive forces required would be to provide a large three axis external load frame which together with a system of hydraulic servo-controlled actuators would provide the needed forces to the test cell. This is illustrated in schematic form in (Fig. 3.6).
FIGURE 3.6

Schematic of Actuator System
The actual dimensions of such a loading system are illustrated in (Fig. 3.7) using approximate dimensions provided by the MTS Corporation for a one million pound (4.45 MN) actuator design. An actuator with a swivel head, swivel base and load cell has a length of eight feet (3.43 m), a diameter of three feet (0.91 m) and a weight of 14,000 lbs (6,400 kg). If the swivel base and load cell are deleted from the system the length is shortened to just under six feet (1.8 m) and the weight drops to approximately 10,000 lbs (4540 kg). This design concept will require an external reaction frame capable of carrying the one million pound loads acting in the three orthogonal directions. A sizeable steel framework structure will be required to support the million pound force and to provide sufficient stiffness to minimize deflections under the load.

Specimens whose size is larger that 4 inch (10.1 cm) on a side loaded to the 60 ksi (414 MPa) pressure range of interest will require actuators and reaction frames in the 1 million lbs (4.45 GN) to 2 million lb (8.9 GN) range. Actuators in this range are not normal production items and hence would be very expensive. Similarly the reaction frames required for this load would be large, massive and expensive.

While only one actuator is required in each axis to provide the reactive force, the need to avoid misalignment of actuators due to the external frame deflections is seen as a potential problem. If only one actuator is provided per axis then application of a one million pound force in one direction would almost certainly produce deflections which would cause misalignment in the other two directions. Considering that these other two directions may also be loaded to one million pounds (4.45 MN) non-aligned load axes should be avoided for structural and safety reasons. This situation is illustrated in...
FIGURE 3.7

Single Axis of Hydraulic Actuator

Reaction System
(Fig. 3.8) in a two dimensional schematic drawing. A rotational instability mode is also possible.

To maintain alignment using a deformable external load frame would require use of two actuators for each axis as shown in (Fig. 3.6) together with a sophisticated servo control system. One proposed control system would operate one actuator in force control and the other in displacement control to obtain the desired load and alignment control.

3.2.4 Modification of the Michelis Cell

The cell of Michelis discussed in Section 2 has achieved the highest fluid cushion multiaxial pressures reported to date for tests on concrete. His device has achieved pressures to 29,000 psi (200 MPa) using fluid cushions to load two of the three axes. The device is designed for a specimen size of approximately 2.0 x 2.0 x 4.0 inch (5.0 x 5.0 x 10.0 cm). Michelis fabricated his cell from a high strength steel having a yield/ultimate strength of 261,000 psi (1800 MPa) and drilled access holes for instrumentation and hydraulic lines,(Fig. 2.4.)

The feasibility of enlarging the pressure cell design of Michelis can be investigated in an approximate manner by assuming his cell functions as a cylindrical pressure vessel with an outer radius of 11 cm and an inner radius equal to the largest radial distance of the cell interior, 6.3 cm. Using basic pressure vessel equations from Spain and Paauwe (1977) the peak tangential stress at the inner radius is given by:

\[
\sigma_t = \frac{p}{\left(\frac{w+1}{w-1}\right)}
\]
FIGURE 3.3
Actuator Position Control Instability
Problem
where \( w \) is the ratio of outer radius to inner radius,

\[
\begin{align*}
11.0 & \\
1.75 & \\
6.3 & \\
\end{align*}
\]

If the design of Michelis' cell is enlarged to accommodate a four inch by four inch (10 cm x 10 cm) cubical specimen, then by using the same dimensions for the fluid cushions and steel loading prisms an equivalent inner radius of 9.7 cm (3.8 inches) is obtained. If the ratio of steel tangential stress to applied pressure is kept equal then the value of \( w = 1.75 \) must be maintained. For an inner radius of 9.7 cm (3.8 inches) this will require an outer radius of 17 cm (6.7 inches). Thus, based on the very approximate analysis just presented, it appears that this design could be enlarged to accept a four inch (10 cm) cubical specimen and could operate at the 29,000 psi (200 MPa) pressure level reported by Michelis. A detailed finite element analysis would be required to verify this statement as stress concentrations from small corner radii will govern the actual cell behavior.

The possibility of maintaining constant the tangential stress of the original design while increasing the pressure capacity of the cell by increasing the outer radius was investigated. With a starting wall ratio, \( w \) of 1.75, only a limited increase in pressure capacity can be gained by increasing the outer radius while keeping the inner radius constant. This is illustrated in (Fig. 3.9) which shows that for \( w = 1.75 \), an increase in pressure by a factor of 2.07 (29,000 psi to 60,000 psi) is not possible using monobloc cylinder design.

By increasing the wall ratio to 3.0 and by using a two piece shrink fit design in which both the inner and outer cylin-
FIGURE 3.9
Summary Plot of the Limiting Pressure of Cylinders
ders have the same wall ratio, a 2.07 increase in pressure can be obtained keeping the design tangential stress in the steel at a constant value.

The above discussion should only be considered as a very approximate guide. In operation the pressure applied in the two directions will be unequal in true multiaxial loadings which will create bending stresses in the cell which are not accounted for in this simple pressure vessel theory. As stated before, reentrant corners with small radii will produce stress concentrations. From results presented in Appendix A, it is shown that stress concentration factors can have values of 5.0 or greater. For applied pressures of 60,000 psi this would result in steel stresses of approximately 300,000 psi which implies that the steel selected must be able to accommodate localized yielding. Unfortunately, steels with yield points above 200 ksi often have very limited ductilities.

3.2.5 Externally Pressured Cubical Cell

All cubical cells constructed thus far have been designed to contain the internal pressures which are applied to the specimen in each of the orthogonal directions. If a cubical cell were also designed to seal against an external pressure, then application of external pressure would provide compressive preloading of the cell. This concept is illustrated in (Fig. 3.10).

Assuming a four inch (10.16 cm) cubical specimen and a 60 ksi (414 MPa) applied specimen stress a net outward force of 960 kips (427 MN) results. If the cubical cell wall was designed to
FIGURE 3.10
Section View of Externally Pressured Cubical Cell Design
FIGURE 3.11
Wall and Seal Design for Externally Pressured Cubical Cell Concept
have a 12 inch by 12 inch (30.5 x 30.5 cm) size as shown in (Fig. 3.11) with an "O-ring" seal groove inside the attachment bolts, a net effective sealed area of 87.5 in\(^2\) (565.5 cm\(^2\)) results. A 10,000 psi external hydraulic pressure will result in an 875 kip (3.89 MN) inward force on the wall.

The net unbalanced outward force of 85 kips (0.38 MN) can easily be accommodated by the 20 3/4" bolts used to attach the wall to the internal frame. A nominal bolting torque of 70% of rated proof load will result in a net inward force of 794 kips (3.53 MN) assuming 20 Grade 8, 3/4-10 bolts. It is evident that the combination of the external pressure plus bolt preload can provide sufficient reaction to the applied specimen loads.

The pressure chamber to house the cubical cell will need to have an internal diameter of approximately 24 to 30 inches (61 to 76 cm) and a length of approximately 36 inches (91 cm) to contain the cubical cell. The pressure chamber would have to be equipped with a minimum of 20 instrumentation and 3 hydraulic feed-thrus for connections to the cubical cell. The end of the pressure chamber would need to be designed to facilitate easy removal so that the cubical cell can be quickly removed from the cell to permit changing specimens. The design and manufacture of a pressure cell of this size and pressure rating is within the capability of commercial manufactures of pressure vessels, although it would require specialized forgings and would be expensive to produce.

This concept will require development of a fluid cushion design that is limited in area to the specimen face area. The thickness of the fluid cushion should be minimized to reduce as much as possible lateral pressures on the cubical cell frames. These items are discussed in Section 4.
This concept provides sufficient room to accommodate up to three normal sized LVDT's per wall using a spacing pattern similar to the present cubical cell design. With the 10,000 psi external pressure acting as the preload the internal frame will have a large amount of unused tensile capacity. The present cubical design with its inefficient fluid cushion design has a demonstrated hydrostatic capacity of 11,600 psi. Hydrostatic test pressures to 75,000 psi would seem to be feasible from the standpoint of gross overall reactive force capacity. Stress concentrations at these pressure levels will likely be limits in design factors. As mentioned previously, design of the fluid cushion and the stress concentration in the frame under the fluid cushion must be studied for these very high pressures.
SECTION 4
FLUID CUSHION DESIGN

4.1 INTRODUCTION

The unique aspect of the proposed high pressure cubical cell is the use of flexible fluid cushions to transmit the applied loads to the specimen. The purpose of the fluid cushion is the application of a known uniform normal stress to the surface of the specimen. Surface shearing stresses should either be eliminated or reduced to such a low level that they have no significant effect on specimen deformation or failure.

The fluid cushions are designed to eliminate the large surface shear stresses and non-uniform normal stresses that result when the specimen is loaded directly using rigid steel platens. For cubical specimens loaded multiaxially by steel platens the influence of surface shear stresses and non-uniform normal stress will extend throughout the volume of the specimen. Since the magnitudes of shear stress and non-uniform normal stresses are not known to any acceptable precision the derivation of basic material constitutive equations from the resulting strain data will contain considerable error.

Friction reduction schemes and the use of brush platens have also been employed in an attempt to achieve better control of applied specimen stresses. The friction reduction schemes often consist of a layer of lubricant (grease) contained between two flexible sheets (often teflon), Zimmerman (1965). While very low coefficients of interface friction and hence low lateral stresses can be achieved by this method evidence suggests that the normal stresses are not uniform. This is suggested by both analytical studies and by visual inspection of
the teflon-grease-teflon pad after a test which shows evidence of uneven grease flow and irregular surface bearing pressure.

The brush platen loading method was originally developed by Kupfer, et.al. (1969) in Germany and has been used to apply both compressive and tensile tractions. Each brush actually comprises a very small conventional steel platen. Individual brushes are designed to flex laterally by cantilever action to accommodate surface shear strains. The end of the brush will rotate with the result that a flat contact surface before compressive loading will become an edge on line loading resulting in a localized stress concentration.

While the advantages of fluid cushion loading have been recognized for a long time, until recently material and design limitations have prevented application for high pressures. Recent developments in plastic materials and manufacturing technology have made high pressure fluid cushion possible.

4.2 REVIEW OF PREVIOUS FLUID CUSHION DESIGNS FOR HIGH PRESSURE

The first use of fluid cushions by Foppl is described in Section 2. The first fluid cushions for high pressures were developed by Atkinson (1972) for use in his multiaxial test device. These consisted of a leather pad placed next to the specimen surface (Fig. 4.1). On top of the leather pad a soft vinyl back-up ring is placed and the actual fluid seal is formed by a 0.025 inch thick latex membrane. The leather pad provides good resistance to extrusion and flow under pressure while the vinyl back-up ring serves as a restraint and seal to prevent rupture of the latex at low pressure levels. The leather/vinyl combination was eventually replaced by an injection molded polyurethane seal for lower pressure tests. The leather/vinyl seal
FIGURE 4.1

Fluid Cushion Design of Atkinson (1972)
was effective to pressures of 12,000 psi (83 MPa) while the molded seal was effective to 3000 psi (21 MPa).

The principal drawback of the original cell designed by Atkinson was the design of the O-ring seal between the wall and the frame. Under increasing internal pressures the wall would move away from the O-ring creating a gap which would eventually leak. Sture (1973) designed the cell to provide a cylindrical sealing surface between the wall and the frame. The leather/vinyl/latex assembly previously used was replaced by a single injection molded membrane, (Fig. 4.2). When a high durometer polyurethane was used to mold the membrane, pressure differentials \((\sigma_1 - \sigma_3)\) in excess of 8,000 psi (55 MPa) have been obtained. The circular portion of the seal has successfully sealed to 22,000 psi (152 MPa) with no indication of leaks or membrane extrusion.

The seal designs illustrated in Figures 4.1 and 4.2 produce net reactive loading requirements much greater than those represented by the 16 square inch (103 cm \(^2\)) specimen face area. The circular seal between the wall and the frame has a diameter of 6.0 inches (15.2 cm) resulting in a net pressured area on the wall of 28.3 square inches (182 cm \(^2\)). Thus only 57% of the total wall loading is produced by the pressure on the specimen. These seal designs also apply pressures to the sides of the square opening in which they are placed. These pressures create tensile frame stresses in directions orthogonal to the axis being pressured. Under a hydrostatic loading condition only 31% of the tensile load carries by the frame in a given direction represents reactions to loads carried by the specimen.

Michelis (1985) has developed an unique fluid cushion design in which roto-molded PVC bags are used to apply the pressures.
FIGURE 4.2
Polyurethane Membrane Seal of Sture (1973)
(Fig 4.3). The bags are flexible, 2-3 mm thick and are equipped with small fittings to accommodate hydraulic fluid connections and permit passage of displacement measuring rods which transmit specimen deformation to LVDT's placed external to the pressurized fluid cushion. The bags are reported to have sustained operating pressures to 36,250 psi (250 MPa). Since the bags are self contained pressure bladders no external O-ring or similar type seal is required. Thus the bags can be designed to load only the face area of the specimen and avoid the large areas of wall loaded by previous designs which used circular O-rings. The depth of the PVC bags is approximately 2.5 cm (1 inch) which is less than half that of previous designs. This will reduce considerably the net forces at right angles to the loaded axis.

Michelis was able to employ the PVC bag to attain the high pressure seal because of his mixed boundary condition design which provided rigid containment surfaces for all sides of the bag. One horizontal axis provided a true fluid cushion loading on the specimen. The sides of the PVC bags in this axis were supported by small steel loading prisms which provide the loading mechanism on the other horizontal axis. In this second horizontal axis PVC bags were used to apply pressure to the steel prisms which loaded the surface of the specimen. This method of loading is somewhat similar to the brush platens of Kupfer et.al. Michelis' design, however, avoids the problems associated with loading through a rigid steel base and with cantilever bending action.
FIGURE 4.3
PVC Fluid Cushion Design of Michelis (1985)
4.3 POSSIBLE FLUID CUSHION DESIGNS FOR HIGH PRESSURE

Any fluid cushion design for a high pressure cubical multiaxial cell should satisfy several sometimes conflicting criteria:

1. The cushion design must be sufficiently soft and flexible to apply essential normal stresses to the specimen while eliminating or reducing to a satisfactory level surface shearing stresses.

2. The fluid cushion must provide a competent seal between adjacent faces of the specimen so that extrusion and failure of a fluid cushion from a high pressure chamber to a lower pressure chamber is prevented. This requirement will require that the fluid cushion be thicker and stiffer at the edges which works at cross purposes to the first requirement.

3. The fluid cushion must accommodate large specimen strains. Strain values to 3-4% are seen as a minimal requirement. Strain capacity to 10% would be desirable to study the behavior of materials undergoing large plastic strains. Strain capacity to this level may be extremely difficult to obtain, however, in a high pressure apparatus.

4. The fluid cushion must be compatible with the deformation measurement transducer employed. Characteristics of various transducers are discussed in Section 5.

The fluid cushion seal between adjacent cavities must retain sufficient strength and stiffness to prevent plastic flow.
under high pressures. The seal should at a minimum be able to provide an effective seal for pressure differential levels equal to the unconfined failure strength of the materials to be tested. For concretes of current interest this would be from 15,000 to 20,000 psi (103 to 138 MPa). Pressure differentials of this magnitude would rule out the class of injection molded plastics employed in previous fluid cushion designs as they show considerable extrusion and failure at 8,000 psi (55 MPa) pressure differential.

The method proposed to provide the fluid cushion is similar to that utilized by Michelis (1985) but modified and improved to allow it to be used in a true multiaxial, fluid cushion device. The proposed seal is shown in section view in (Fig. 4.4). It uses a PVC bag equipped with rods to transmit specimen deformation to LVDT's. To prevent extrusion of the PVC bag into other pressure cells a triangular shaped metal or very stiff plastic corner seal element is provided. The design and material selected for this seal must provide an effective seal at low pressures yet must function to total pressures of 60,000 psi (415 MPa) and to pressure differentials of approximately 20,000 psi (140 MPa). Under the pressure differential loading the seal should be able to bridge over a gap of at least 0.10 inch (2.5 mm) or greater to be able to accommodate specimen strains.

The advantage that this fluid cushion concept appears to have is the use of a completely enclosed fluid pressure bladder which solves the difficult problem of providing a fluid seal at the rear of the fluid cushion. A second advantage is that the relatively small thickness dimension of the bladder will reduce considerably the magnitude of pressure induced loads orthogonal to the loaded specimen face. This will serve to reduce the total magnitude of forces to be resisted by an external structural frame.
FIGURE 4.4
Proposed Seal Design Concept
The use of a bladder type fluid cushion creates serious difficulties in providing methods to measure specimen deformations under load. The proximity or capacitive type transducers (discussed in detail in Section 5) require accurate placement of large diameter flat discs close to the target surface. This requirement plus the need to bring lead wires or coaxial cables from the inside of the pressured bladder pose considerable practical difficulties. The manufacture of the bladder may also pose difficulties. Michelis (1986) reported that his group spent considerable time and effort selecting the proper combination of bladder design, bladder material and manufacturing techniques before achieving success.

An alternative to the completely enclosed bladder technique is to provide a membrane/seal design similar in concept to that described in Section 4.1 in which the rear of the seal is open. The present design practice of making the seal using a circular O-ring while providing for easy sealing results in a large increase in the net pressured area. The alternative then is to provide a seal around the inside of the square opening which would have the same approximate size as the loaded face area of the specimen. No existing high pressure seal design exists for this type of seal nor have any feasible designs been advanced.
SECTION 5
METHODS OF DEFORMATION MEASUREMENT

5.1 INTRODUCTION

A critical and necessary feature of the design of any cubical test system is the measurement of specimen deformations under the applied multiaxial stresses. The deformation measurement system must possess sufficient sensitivity and accuracy to determine elastic deformations of stiff materials such as concrete and rock under initial loadings. The measurement system must also have sufficient measurement range for specimen strains of at least 3 to 4 percent. A measurement range of up to 10 percent would be desirable.

The incorporation of deformation transducers in the design of a cubical cell poses a number of significant problems. First the actual transducing element must be compatible with the high fluid pressures being considered. Except for simple capacitive type devices most transducers employ coils, sliding elements, strain gages or other electrical or mechanical elements which may be sensitive to pressure. Methods to de-air coil potting compounds are, for example, necessary to prevent high stress concentrations and resulting damage.

With the transducers in the pressurized fluid a means to pass signal wires from the pressured cavity the ambient exterior must be provided. Some transducers require coaxial cables for their signal leads while others may require multiple leads per transducer. A LVDT, for example, may require up to 6 leads per individual transducer. Most commercially available electrical feedthrus are designed to maximum operating pressures of 10,000 psi (69 MPa) or less. Those specifically designed for pressures
in excess of 10,000 psi typically accommodate only a single wire and may require a large mounting area.

The cubical cell designed by Atkinson (1972) employed proximity type transducers which were immersed in the pressurized fluid. These proximitors were supplied by the Bentley-Nevada Corporation and were manufactured as a special item to meet the high pressure operational needs. The transducers have a non-linear response curve which requires a computer fitting procedure to reduce output voltage to deformation. These transducers have given many years successful performance in this application.

Recently manufactured cubical cells for soils, rock and concrete at moderate pressures have employed LVDT's manufactured by the Schaevitz Engineering Co. These are of the AC type and are provided with special vent holes to equalize pressures inside the transducer to the high surrounding fluid pressure. These LVDT's provide a linear output and can be supplied with various effective measurement ranges. LVDT's have been employed to pressures above 20,000 psi (138 MPa) in existing cubical cell applications. The Schaevitz company has reported successful operation above 50,000 psi (345 MPa) for this type of transducer.

In the remainder of this section each transducer type is described in terms of its operating principal, commercial sources and apparent advantages and disadvantages in a high pressure cubical cell system.
5.2 EDDY CURRENT PRINCIPAL (PROXIMITOR)

5.2.1 Principal

A coil in the sensor head is driven at RF frequency. The oscillating field will produce eddy currents in a metallic target which will couple with the coil producing an impedance variation (Fig 5.1). Associated electronic circuits produce a DC signal which is linear in magnitude with the gap width between the coil and target over a range which is governed by the physical size of the coil among other factors.

5.2.2 Typical Commercial Sensors

<table>
<thead>
<tr>
<th>Model</th>
<th>Sensor Diameter mm</th>
<th>Sensor Thickness mm</th>
<th>Min. Gapwidth mm</th>
<th>Linear Range mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaman-3U</td>
<td>8.4</td>
<td>20.8</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>Kaman-6U</td>
<td>14.2</td>
<td>25.7</td>
<td>0</td>
<td>6.0</td>
</tr>
<tr>
<td>Kaman-15</td>
<td>38.1</td>
<td>12.7</td>
<td>0</td>
<td>15.0</td>
</tr>
<tr>
<td>B-N 7200-8</td>
<td>8.0</td>
<td>20.3</td>
<td>0.25</td>
<td>2.0</td>
</tr>
<tr>
<td>B-N 7200-11</td>
<td>11.0</td>
<td>25.4</td>
<td>1.0</td>
<td>4.1</td>
</tr>
<tr>
<td>B-N 7200-25</td>
<td>25.4</td>
<td>12.0</td>
<td>1.3</td>
<td>12.7</td>
</tr>
</tbody>
</table>
Figure 5.1 Eddy Current Principal Transducer (Proximitron)
(Source: Bently - Nevada Corp.)

Figure 5.2 Capacitance Principal Transducer
(Source: MTI Instruments)
5.2.3 Discussion

The proximiter is a non-contacting type sensor with no physical contact with the target material required. This is a very attractive feature for cubical cell application involving use of fluid cushion loading pads. The target is typically a five mil (.12 mm) thick metal foil placed on the specimen surface. The sensor head is placed in the pressurized fluid with the magnetic field passing through the fluid cushion. Thus no penetration of the fluid cushion is required.

The sensor must be placed in the pressurizing fluid to be sufficiently close to the target material. This requires that the coil be potted in a nobubble material to avoid failure at high fluid pressures. Bentley Nevada has previously supplied sensors that have provided successful operation to 145 MPa (21,000 psi) in cubical cell operation.

The sensors operate at RF frequency which necessitates use of coaxial cable. Electrical signal feedthrus must provide sufficient shielding for the RF signal.

5.2.4 Commercial Sources:

a. Bentley Nevada Corporation, P.O.Box 157, Minden NV 89423

b. Kaman Instrumentation Corporation, P.O. Box 7463, Colorado Springs, CO 80933-7463
5.3 CAPACITANCE TRANSDUCER

5.3.1 Principle

Electrical capacitance between the probe and a conducting (metallic) target is converted into a linear measure of distance. The target must be connected to a ground reference. A coaxial cable connection to the probe is required to isolate any capacitance effects on the cable from surrounding metal (Fig. 5.2). The linear range is a function of probe diameter. The target is placed on the specimen surface similar to the proximitor type transducer except that an electrical connection to an external ground must be provided. With most fluid cushion membranes having a minimum thickness of 2-3 mm, to obtain a reasonable linear measurement range of approximately 10 mm (.4 inch) will require that a sensor with a diameter of 70 mm (2.75 inches) be used. This could pose a problem with many cubical cell designs.

5.3.2 Available Commercial Sensors

<table>
<thead>
<tr>
<th>Model</th>
<th>Sensor Diameter</th>
<th>Min. Gap</th>
<th>Linear Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>MTI - ASP-100</td>
<td>22.3</td>
<td>.12</td>
<td>2.5</td>
</tr>
<tr>
<td>MTI - ASP-200</td>
<td>34.8</td>
<td>.25</td>
<td>5.0</td>
</tr>
<tr>
<td>MTI - ASP-500</td>
<td>69.8</td>
<td>.62</td>
<td>12.5</td>
</tr>
</tbody>
</table>
5.3.3 Discussion

Range and gapwidth requirements require that the sensing element be placed in the pressurizing fluid. The sensing element is essentially a stainless steel disc which would have inherent resistance to pressurizing fluid.

The specimen must be covered by a conductive metal target which must be electrically connected to the reference ground of the sensor. Thus a physical connection between the specimen and the cubical cell is required.

5.3.4 Commercial Source:

MTI Instruments, 968 Albany-Shaker Rd., Latham, NY 12110

5.4 LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

5.4.1 Principle

The LVDT is a variable-inductance transducer which provides an A.C. voltage output proportional to the displacement of a core in a set of three coils. The center coil is driven by an A.C. source and the two end coils serve as pickup coils (Fig. 5.3). Linear displacement of the core is converted by signal processing equipment into a D.C. voltage which is linear with core displacement.
FIGURE 5.3

Linear Variable Differential Transducer
(LVDT)
5.4.2 Typical Commercial Sensor

Information provided below is from the Schaevitz Engineering Company who manufacture a wide range of precision transducers. Other manufacturers of LVDT's are available.

<table>
<thead>
<tr>
<th>Model</th>
<th>Size - mm</th>
<th>Linear Range (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA-121-125</td>
<td>104.6  19.0</td>
<td>6.25</td>
</tr>
<tr>
<td>GPA-121-250</td>
<td>126.7  19.0</td>
<td>12.70</td>
</tr>
<tr>
<td>100 MHR</td>
<td>25.4    9.5</td>
<td>5.00</td>
</tr>
<tr>
<td>250 MHR</td>
<td>47.0    9.5</td>
<td>12.50</td>
</tr>
<tr>
<td>099XS-B</td>
<td>22.0    5.0</td>
<td>5.00</td>
</tr>
<tr>
<td>249XS-B</td>
<td>48.0    5.0</td>
<td>12.00</td>
</tr>
</tbody>
</table>

5.4.3 Discussion

Models GPA & MHR listed above can be vented to permit operation in pressurized liquids and gases. Schaevitz Engineering (1985) reports that modified LVDT's have been used in pressurized liquids and gases to 60,000 psi (418 MPa). LDVT's presently employed in cubical cell operations have given reliable service for several years.

Each LVDT requires at least four connecting leads which must be passed through the pressure barrier. Modified LVDT's for operation in pressurized fluids cost approximately $800./unit. If the design approach used by Michelis is employed, then inexpensive unmodified LVDT's can be used.
5.4.4 Commercial Sources

a. Schaevitz Engineering, U.S. Route 130 and Union Ave., Pennsauken, NJ.

b. Sangamo Transducer, 1875 Grand Island Boulevard, Grand Island, NY 14072.

5.5 OPTICAL

5.5.1 Principle

Light is transmitted through a fiber optic bundle to the target surface (Fig. 5.4a). Intensity of light reflected from the surface is converted to an electrical signal which is proportional to the gap width between the end of fiber optic and the target. The response curve has a sensitive "front slope" curve to an optical peak and a less sensitive "back slope" curve which has a considerably greater range than the front slope curve.

5.5.2 Typical Characteristics

<table>
<thead>
<tr>
<th>Model</th>
<th>Tip Diameter</th>
<th>Back Slope Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Standoff Distance</td>
</tr>
<tr>
<td>MTI-3816</td>
<td>3.2 mm (.125)</td>
<td>3.5 mm (.14 in)</td>
</tr>
<tr>
<td>MTI-3814</td>
<td>3.2 mm (.125)</td>
<td>5.1 mm (.20 in)</td>
</tr>
</tbody>
</table>
FIGURE 5.4

Optical Principal Transducer
a) Transducer Characteristics (Source: MTI)

b) Scheme to Provide Increased Measurement Range for Optical Transducer
5.5.3 Discussion

Because the device is optical in operation an opaque fluid or even slightly opaque fluid is not possible between the target and sensor. The MTI - 381 model operating in the less sensitive back slope mode has a sensitivity of $30 \times 10^{-6}$ inch per milli-volt output.

This sensor could function to determine the movement of a Michelis type mechanical sensor which is incorporated into a pressure bladder. An increased range of displacement could be obtained by observing a flat surface on the rod which has been cut at an angle to its axis as shown (Fig. 5.4b).

5.5.4 Commercial Source:

Instruments Division, Mechanical Technology, Inc.
968 Albany - Shuker Road, Latham, NY 12110
SECTION 6
RECOMMENDATIONS

6.1 SYSTEM DEFINITION

Previous multiaxial test systems for concrete and other materials have been generally limited to testing specimens having a maximum dimension of four inches (approx. 10 cm) or less. This size was selected for some systems because the specimens were to be obtained by core drilling using six inch (15 cm) diameter core barrels. For other systems the specimen size was undoubtedly selected with a view of keeping the system to a reasonable size thus reducing total reactive loads to be carried and in turn reducing size, weight, material and machining costs. One notable exception was the work of Zimmerman (1965) who tested standard 6 inch (15 cm) diameter by 12 inch (30cm) long concrete cylinders in a triaxial loading apparatus to confining pressure of up to 75 ksi (517 MPa) at the U.S. Bureau of Reclamations, Denver Laboratory.

Ideally, laboratory tests should be conducted on a concrete mix identical in all respects to that to be placed in the field. The size limitations of laboratory equipment, however, usually requires that specimen be prepared using a mix with significantly smaller maximum size aggregate. For a given water/cement ratio, the maximum size of aggregate used will also affect the strength of the concrete and its shrinkage.

Another advantage of testing larger specimens is the possibility of embedding in the specimen strain and stress gages. A fluid cushion cubical cell device provides an ideal means to calibrate the response of embedded transducers to known
stress or strain states. Transducer response can be determined to both direct stress or strain fields (i.e. in line with the transducer axis) and to stress fields at an angle to the transducer axis. It would also be possible to evaluate transducer response to changing stress fields.

The size specimen around which a high pressure cubical multiaxial system is designed should therefore be as large as is practical. The effective limitations on specimen size is the amount of reactive force to be supplied and the size, cost and complexity of the cell and the required reaction systems.

The multiaxial cubical cells presently operational have hydrostatic capacities of approximately 15,000 psi (103 MPa) with a peak capacity in a single axis of approximately 22,000 psi (152 MPa). While these cells have proven satisfactory for testing conventional strength concretes, they would be inadequate for tests on the new high strength concretes. Concretes employed in building construction have reported strengths in the 8,000 to 10,000 psi (55 to 69 MPa) range. Concretes used or being considered for specialized military structures have reported field strengths in the 10,000 to 15,000 psi (69 to 103 MPa) range. These new concretes which employ silica fume additives, and superplasterizers have produced strengths in the laboratory to 20,000 psi (138 MPa).

The evaluation of the failure envelope of these concretes over a reasonable range of the compressive stress quadrant will require the ability to provide confining (i.e. \( \sigma_2 \) and \( \sigma_3 \)) stresses whose magnitude is at least 3 to 4 times the unconfined compressive strength of these materials. This would suggest that any planned multiaxial test device should be able to apply stresses up to 60,000 psi (413 MPa) if a ratio of peak stress to unconfined strength of 3 is desired for a 20,000 psi (138 MPa)
material.

While a detailed study of a proposed pressurizing fluid was not conducted as part of this effort, the selection of this fluid must be an integral part of any design. The fluid must be compatible with any membrane or bladder material it encounters. Some organic solvents, for example, can dissolve the plasticizer from certain classes of vinyl compounds. The fluid employed must also be compatible with the metal parts of the cell which it encounters. This is especially true when certain types of high alloy steels vulnerable to hydrogen induced stress embrittlement cracking are used. The fluid selected must also maintain a sufficiently low viscosity at high pressures. Many commercial hydraulic oils for example become gels at pressures greater than about 50 ksi (344 MPa). Finally the fluid should pose no toxicity explosive hazards.

Hydraulic pumping equipment and plumbing is commercially available from a number of sources for pressures to 60 ksi (344 MPa). These pressures can either be obtained directly using air driven pumps or can be obtained using a lower pressure source which operates thru a pressure intensifier system. Servo controlled equipment for these pressure ranges is limited to custom designed and manufactured equipment.

6.2 PROPOSED SYSTEMS

Of the various concepts examined in the earlier sections of this report two are seen as candidates for eventual design and construction of a high pressure, multiaxial cubical system. These are the pressure vessel concept discussed in Section 3.2.2 and the externally pressured cubical cell concept discussed in Section 3.2.5.
The pressure vessel concept will require development of a fluid cushion system and associated deformation measurement system based on the approach of Michelis. The externally pressured cubical cell concept would function best and would have the greatest pressure capacity in a Michelis type fluid cushion were employed. The second design concept could be developed, but at a lower rated pressure capacity, using the current polyurethane membrane and deformation measurement system of Sture (1973). The actual pressure capacity would be determined by the ratio of specimen face area to total pressured area. The sealing capability from the major to minor principal stress axes will need to be improved if the Sture sealing system is to be employed for high pressure. A development program for the fluid cushion and deformation measurement transducer is presented in Section 6.3.

6.2.1 PRESSURE VESSEL CONCEPT

The pressure vessel concept discussed in Section 3.2.2 is felt to offer the best and least expensive opportunity for designing and constructing a high pressure multiaxial system provided a testing machine having 2 million pounds (8.9 MN) axial load capacity is available.

This concept uses the efficient properties of a double shelled pressure vessel to contain 2 axes. An interference fit assembly of the two shells will provide a compressive tangential stress at the inner radius of the assembled vessel for the case of zero internal pressure. This compressive preload stress will offset the high tensile tangential stress at the inner radius which will produced by the internal pressure and multiplied by the presence of a radial hole through the vessel for instrumen-
tation purposes. The division of the inner cavity into individual pressure chambers would be accomplished most likely by installing a segmented inner liner. This concept develops its efficiency by placing the reaction structure as close to the loaded specimen as possible. Even with this efficient design it may be necessary to allow localized yielding around the instrumentation hole. This implies careful attention to material selection.

The use of an existing large capacity loading machine is an efficient and inexpensive method to provide load resistance in the third axis. The deformation response of the loading machine at high loads can be eliminated by preloading the pressure vessel to an axial load in excess of the maximum reactive load so that increasing applied pressures on the specimen serve to reduce the preload rather than applying new loads to the testing machine.

Listed below are technical problems and concerns that need to be answered before this concept is reduced to practice:

1. Detailed stress studies of the multiple walled vessel. The influence of differential pressures in the two pressure axes needs to be investigated.

2. Material selection for the inner segmental liner and the two components of the pressure vessel will be critical given the need to provide both high levels of resistance and large material ductility factors to accommodate possible localized yielding.

3. The design and installation of a segmented inner liner needs to be determined along with an understanding of how loads are transmitted through this inner liner to
the pressure vessel.

4. The design of the fluid cushion and associated instrumentation requires a development program. This is described in Section 6.3.

5. The press fit of the two components of the pressure vessel will be a critical item in the manufacture of the multiaxial cell.

6.2.2 Externally Pressured Cubical Cell Concept

This concept discussed in Section 3.2.5 would be the recommended solution in the situation where a large capacity loading machine is not available. This concept may also provide a better definition and isolation of individual pressure chambers than provided by the segmented inner liner approach.

In this design the multiaxial cubical cell is essentially dropped in a large pressurized tank. The cell is then subject to internal pressures from the loads on the specimen and to external pressures from the tank fluid. Each of the six walls can be imagined as a piston in a pressure intensifier with the small area (specimen face area) of the inside of the wall loaded to high pressures while the larger outside surface of the wall is subject to a lower pressure. An area ratio of approximately 5 to 6 would suggest that a tank pressure of 10,000 psi (69 MPa) would balance a peak specimen stress of 50,000 psi (344 MPa) to 60,000 psi (413 MPa).

This concept requires that each wall provide two seals, the first operating at the applied specimen pressure of up to 60,000 psi (413 MPa) and the second sealing at the applied tank pres-
sure. The inner high pressure seal will be discussed in the next section. The outer seal will require adapting some existing commercial high pressure seals to the specific design requirements of the cell.

This design can use conventional LVDT's contained in the wall. The instrumentation leads will need to be taken from the wall cavity to the pressurized tank environment and then from the tank to the ambient exterior. While commercial feedthrus exist for this pressure range each additional seal provided adds to the complexity of the system.

The main reacting structures for this concept requires a high pressure (10,000 psi, 69 MPa) tank of sufficient size to accommodate the cubical cell and the hydraulic and instrumentation leads connecting to it. For convenient operation the lid on the tank should provide for convenient opening and closing and should accommodate the various electrical and hydraulic feedthrus required. A pressure tank of this size should be manufactured by a commercial source with experience in construction of large chamber pressure vessels for the chemical industry. A tank with the required pressure capacity, size, lid design and electrical and hydraulic feedthrus will be very expensive to manufacture.

6.3 REQUIRED SEAL AND TRANSDUCER DEVELOPMENT PROGRAM

Both of the proposed concepts will require development of the Michelis type fluid cushion. This type of cushion has the ability to load the face of the specimen while only loading a small area in the lateral or normal direction. The thickness of the bladder should be kept to 1/2 inch (1.27 cm) or less to minimize off axis loads.
For use in the proposed cell this fluid cushion must be increased in size from the present 2 inch by 4 inch (5 cm x 10 cm) to a size of 4 inch by 4 inch (10 cm x 10 cm) or 6 inch by 6 inch (15 cm x 15 cm). The pressure capacity will have to be increased from 29,000 psi (200 MPa) to 60,000 psi (413 MPa).

To achieve the size increase in the bladder will require development of manufacturing techniques in the United States. When contacted on this matter Dr. Michelis offered to manufacture the bladders in Greece. He did not provide any offer of assistance for manufacture in the U.S. (Michelis, 1986).

A development program could be conducted by using a single axis pressure chamber in which proposed bladder and seal combination could be evaluated. The pressure chamber should have the same size and shape as that proposed for the full scale device. The chamber should be designed so that the gapwidth over which the bladder/seal bridges in the corner can be varied from essentially no gap to gaps of up to 0.100 inch (2.54 mm) or greater. This would simulate the gap created by specimen deformation in the specimen plane described by the major and minor principal stress axes. The chamber should also be designed so that the response of the deformation measuring system can be evaluated under pressure.

6.4 ESTIMATED DEVELOPMENT COSTS

Presented below are preliminary cost estimates for developing, manufacturing and testing a high pressure multiaxial test system. The basis for these costs is primarily derived from experience in designing and building other types of high pressure test equipment. Where possible actual costs from manu-
factures of instrumentation, hydraulic and other equipment have been used.

These costs are necessarily approximate as it is impossible at the start of such a development program to foresee all the specific problems to be overcome. The development of the fluid cushion, i.e. the design and manufacture of the bladders, seals, etc. required, is one area in which a great deal of trial and error effort will likely be required.

Incorporation of suitable instrumentation into any fluid cushion design will also require considerable effort. The actual design of the cell for either of the two options presented will depend to some extent on details of the fluid cushion design and hence costs estimated for the cell must be considered to be approximate.

The program outlined below is divided into three logical phases. The first phase is the development of a suitable fluid cushion design which meets the design requirements in terms of pressure levels and deformation capacity. After the details of the fluid cushion design are firmly established the design and manufacture of the multiaxial cells can proceed. The specific option selected must depend on the availability of a large testing machine if the pressure vessel option is selected. The costs listed for this phase include estimates for associated items including hydraulic pumps and plumbing, instrumentation, facility improvements and safety equipment items. The last phase of the project is to install the multiaxial testing system and to conduct a sufficient number of material tests to adequately debug the system. Another task in this phase would be to accurately calibrate the system and to evaluate the assumptions of stress and strain uniformity.
6.4.1 Fluid Cushion and Instrumentation Development Program

Objective: To develop a fluid cushion design meeting at a minimum a 60 ksi (413 MPa) and 2% specimen strain deformation criterion.

Estimated Time Period: 9 – 12 Months

Estimated Costs:

A. Equipment & Materials
   - Test Vessel
   - Hydraulic Equipment
   - Instrumentation
   - Molding Charges
   - Seal Development

   37,000.00

B. Labor
   - Senior Engineer, 3 man-months
   - Staff Engineer, 3 man-months
   - Technician, 2 man-months

   50,600.00

C. Misc.
   - Travel, Phone charges, etc.

   3,000.00

D. Subtotal

   90,800.00

E. Fee

   9,080.00

F. Total

   $99,880.00
6.4.2 Pressure Vessel Concept

Objective: Design and manufacture multiaxial test cell utilizing the pressure vessel concept of Section 3.2.2

Estimated Time Period: 12 months

Estimated Costs:

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6.4.3 Externally Pressured Cubical Cell Concept

Objective: Design and manufacture multiaxial test cell utilizing the externally pressured cubical cell concept of Section 3.2.5.

Estimated Time Period: 12 months

Estimated Costs:

A. Equipment & Materials
- Inner Frame
- Walls - six
- Instrumentation
- Hydraulics - high pressure
- Bladders & Seals
  \[97,500.00\]

B. Large 10,000 psi Tank
- Tank
- Facility Modification
- Tank Hydraulics
  \[137,500.00\]

C. Labor
- Senior Engineer 6 man-months
- Staff Engineer 6 man-months
- Technician 2 man-months
  \[96,500.00\]

D. Misc.
- Travel & Project Related Expenses
  \[5,000.00\]

E. Subtotal
  \[336,500.00\]

F. Fee
  \[33,650.00\]

G. Total
  \[370,150.00\]
Objective: To install, calibrate and conduct a sufficient number of tests to verify satisfactory operation of the system.

Estimated Time Period: 12 months

Estimated Costs:

A. Equipment & Materials
   - Bladder & Seals
   - Equipment Modification Expenses
   - Instrumentation Expenses
   - Expendable Supplies 40,000.00

B. Labor
   - Senior Engineer 6 man-months
   - Staff Engineer 8 man-months
   - Technician 10 man-months 125,900.00

C. Misc.
   - Travel & Project Related Expenses 5,000.00

D. Subtotal 170,900.00

E. Fee 17,090.00

F. Total $187,990.00
SECTION 7
REFERENCES


APPENDIX

STRESS CONCENTRATION AT THE CORNER OF A STEEL TRIAXIAL JACKET

By Brant Lahnert
Atkinson-Noland & Associates, Inc.

Introduction

This study was performed to determine the magnitude of the concentration of stress for corners of varying radii of a steel triaxial jacket using the finite element method of analysis.

Analysis

The jacket was modeled using the elastic version of VISCOT. Geometry, fixity conditions and loading conditions are shown in Fig. A1a with a close view of the area of interest in Fig A1b. Eight-noded quadrilateral elements were used. The jacket was pressured internally with 10 ksi. Four cases of radii varying from 0.025 to 0.100 inches were modeled.

Results

The stress plotted in Figs. A2 through A6 is \( J_2 \) which is analogous to the deviatoric component of the total stress vector. For the biaxial case:

\[
J_2 - k^2 = 0 \quad \text{where} \quad k = (1/\sqrt{3})\sigma_y
\]
A typical deviatoric stress distribution for the entire model is shown in Fig. A2. This distribution is quite similar for all cases. Close views of the stress distribution at the corner for each case is depicted in Figs. A3 through A6. Fig. A7 plots the extrapolated stress at the surface of the jacket for each radius.

Discussion

Von Mises' yield criteria, frequently used for steel material models, assumes failure occurs when the octahedral shear stress, \( \tau_o \), exceeds a given plane which resembles a cylinder of radius equal to the ultimate deviatoric shear strength and of axis equivalent to the hydrostatic axis (Fig. A8). The stress quantity \( J^2 \) is related to the octahedral shear stress \( \tau_o \) by:

\[
\tau_o = \sqrt{(2/3)} \cdot \sqrt{J^2}
\]

Von Mises' criteria is written:

\[
J_2 = \frac{1}{6} ((\sigma_x - \sigma_y)^2 + \sigma_y^2 + \sigma_x^2) + \tau_{xy}^2
\]

Therefore the quantity plotted in the stress contours of Figs. 2 through A6 is related to the uniaxial yield strength by:

\[
\sigma_y = \sqrt{3} \cdot \sqrt{J_2}
\]
By example: For the case in which the radius is 0.025" and the internal jacket pressure is 10 ksi, the required yield strength is:

\[ \sigma_y(\text{reqd}) = \sqrt[3]{(58)} = 100.5 \text{ ksi} \]

For internal pressures other than 10 ksi, the required yield strength must be scaled.
SiMS
CONCMTUON
(up : 9.1w INOIU)
Eight-odd Quadrilaterals

FIGURE A1a

FINITE ELEMENT MESH
FIGURE A1b

FINITE ELEMENT MESH, CORNER DETAIL

A5
FIGURE A2
DEVIATORIC STRESS DISTRIBUTION,
Radius = 0.100 Inch
FIGURE A3

DEVIATORIC STRESS DISTRIBUTION,

Radius = 0.100 Inch
FIGURE A4

DEVIATORIC STRESS DISTRIBUTION,

Radius = 0.075 Inch
FIGURE A5

DEVIATORIC STRESS DISTRIBUTION,

Radius = 0.050 Inch
Stress Concentration (Radius = 0.025 inches)
Tangential Stress Intensity - Total (ksi)

Extrapolated
Max = 60 ksi

FIGURE A6
DEVIATORIC STRESS DISTRIBUTION,
Radius = 0.025 Inch

A10
Internal Pressure = 10 ksi

FIGURE A7
EXTRAPOLATED STRESS AT SURFACE VERSUS RADIUS

FIGURE A8
Von MISES YIELD SURFACE
### STRESSES AT GAUSS POINTS (RADIUS = 0.100 IN.)

#### Sampling Point Coordinates

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#### XX-Stress YY-Stress XY-Stress Max P.S. Min P.S. Angle E.Str.

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#### TABLE A1

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**TABLE A2**
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**XX-Stress YY-Stress XY-Stress** Max P.S. Min P.S. Angle E.Str.

**Element No. = 24** (at gauss pts. 1-4)

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<tr>
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<th>Min P.S.</th>
<th>Angle E.Str.</th>
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**Element No. = 27** (at gauss pts. 1-4)

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<th>Min P.S.</th>
<th>Angle E.Str.</th>
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**Element No. = 30** (at gauss pts. 1-4)

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**Element No. = 33** (at gauss pts. 1-4)

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<th>Min P.S.</th>
<th>Angle E.Str.</th>
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**TABLE A3**

A14
STRESSES AT GAUSS POINTS (RADIUS = 0.025 IN.)

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<td>3</td>
<td>402E+01</td>
<td>391E+01</td>
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</tr>
</tbody>
</table>

XX-Stress YY-Stress XY-Stress Max P.S. Min P.S. Angle E.Str.

<table>
<thead>
<tr>
<th>Element No.</th>
<th>24</th>
<th>(1-4)</th>
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</thead>
<tbody>
<tr>
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<td>.163E+02</td>
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<td>.703E+02</td>
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<tr>
<td>.299E+02</td>
<td>.169E+02</td>
<td>-.141E+02</td>
<td>.390E+02</td>
<td>.787E+01</td>
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<tr>
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<td>.577E+02</td>
<td>-.284E+02</td>
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<td>.151E+02</td>
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<td>-.313E+02</td>
<td>.823E+02</td>
<td>.185E+02</td>
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<table>
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<th>Element No.</th>
<th>33</th>
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<tr>
<td>.169E+02</td>
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TABLE A4