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T. Henningsen, M. Garbuny, and R. H. Hopkins

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T. Henningsen, M. Garbuyn, and R. H. Hopkins

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ABSTRACT

Work during this period evaluated analytically the concept of multistage optical transistors based on two spectroscopically complementary materials. It is shown that very high total gains can be obtained with either of two alternatives, viz., (1) a chain of discrete complementary transistor units, or (2) a complementary transistor continuum (a concept which has no electronic analog). Whereas the total gain grows exponentially with the number of stages, the required constant photon fluxes grow only linearly with that number. High signal-to-noise ratios can be obtained without the need of cooling. However, the spectroscopic matching conditions of the two complementary materials are stringent. Under the guidance of the theoretical work, a search for suitably matching materials is now underway.
1. INTRODUCTION

The first semi-annual report\textsuperscript{(1)} (R&D Document No. 85-9F42-NUTRN-R1) derived the rules governing the material selection for optical switches and transistors (OST). It was shown that particularly high values for speed of response and gain, and low demands on control and signal power, could be obtained with small amounts of free-atom media. An outstanding example of such media are the vapors of the alkali metals atoms, for which spectroscopic and other physical data are well known. Calculations for the case of lithium as well as of sodium, yielded, as expected, relatively high efficiencies in the use of control and signal power and of the required material amounts. They also predicted relatively high speeds of response and transistor gains up to a factor of 12.

At this point it would have already been possible to demonstrate experimentally the first optical transistor operation. However, for reasons of cost-effectiveness, it appeared preferable to postpone such relatively time-consuming experimentation and, instead, to extend the theoretical study to quasi-free atom media and other optical transistor systems. Among the latter, the most important concept is that of optical transistor cascading. This means, in its broadest definition, the interaction of beams in two different optical transistor materials to achieve exponential signal gain or to perform other operations which may, or may not, occur in electronic transistor practice. The two types of transistor differ, after all, in the nature of transported interacting particles: photons of two different frequencies in one case, holes and electrons in the other. As a consequence, the feasibility of optical transistor cascades depends on the success of searching for, and finding, materials with properly matching energy term schemes. This problem existed for electronic transistor combinations to a far lesser
extent. On the other hand, optical transistor systems, if successful, offer opportunities of operation not available in the purely electronic domain.

This report discusses the operation of multiple optical transistors, proceeding from the concept of "complementary" (optical) materials. The discussion limits itself to the objective of achieving maximum signal gain. Even with this limitation, it will be seen that there exists a multitude of transistor systems, differing qualitatively as well as quantitatively, with which gain, growing exponentially with chain length, can be achieved. In general, these different system concepts also impose different requirements on the selection of the complementary materials. It will be shown that such selection is, at best, no trivial task. Therefore, the procedure adopted is first to identify the transistor system which is most advantageous in terms of power demand or signal-to-noise ratio, or both. Only then will it pay to search among the considerable range of alternatives for the complementary transistor materials which fulfill the requirements of the optimum system.
2. COMPLEMENTARY OPTICAL TRANSISTOR CHAINS

2.1 Basic Unit of Complementary Optical Transistors

The preceding semi-annual progress report\(^{(1)}\) dealt with single transistors and switches. Both devices are based on selected materials in which the essential energy structure consists of three levels excited by two beams: (1) the "control" beam which excites the intermediate level 2 by resonance absorption from the ground state 1; and (2) the (then called) "signal" beam which is partially, or (in switches) completely, absorbed by the excited population of level 2 in transistors to the level 3. In the case of transistor operation, materials and power levels are chosen in such a way that the dynamic (ac) gain is maximized. It was shown\(^{(1)}\) that such gain may reach values of up to nearly 12 in the alkali metal vapors.

Figure 1 shows the energy term schemes of two transistor materials (I and II) which are complementary to each other. By this is meant that the energy difference of levels 1 and 2 in the first transistor is equal to that of levels 2 and 3 of the second transistor; and similarly, the energy difference between levels 2 and 3 of the first equals that between levels 1 and 2 of the second transistor. Although drawn separately, transistors I and II are in good optical contact, i.e., radiation passes from one to the other with negligible loss. Thus, as depicted in Figure 1, a control beam A excites transitions from the ground state to level 2 in the first in transistor I, but the portion of beam A not absorbed in this process excites transitions of a population in level 2 of transistor II to level 3. This population is generated by a beam B which, in analogy to electronic transistor nomenclature, we now call the "collector" beam (replacing the previous term "signal"). The collector beam undergoes a process...
complementary to that of the control beam, i.e., it excites transitions 2-3 in I and 1-2 in II. The assembly of two transistors as just described may be considered as a basic complementary optical transistor unit within a transistor chain. However, as will be seen in the following discussion, in general it is necessary to add (to what is shown in Figure 1) a network of constant radiation power to maintain the gain of the alternating radiation signal.

2.1.1 Gain Considerations of the Complementary Unit As First Stage

Assume a small signal \( \Delta A \) (in number of photons) is introduced into the first transistor (I) as control power (see Figure 2). The fraction of \( \Delta A \) absorbed by transitions from level 1 to level 2 can be adjusted, of course, by a choice of the transistor pathlength \( L' \) or by the density of the absorption centers. Figure 2 indicates the important, and mathematically convenient, case that \( \Delta A \) is completely absorbed. A collector beam \( B_1 \) of sufficient power is provided to achieve a gain \( G' \) in the number of transitions from level 1 to level 2. Thus, beam \( B_1 \) loses \( G'\Delta A \) photons by multiple absorptions and becomes the control beam of strength \( (B_1 - G'\Delta A) \) for the complementary transistor (II). On the other
Figure 2. Gain operation of complementary transistor unit (example of first stage in transistor chain).

Hand, beam $\Delta A$, completely absorbed in transistor I, has nothing to contribute to transistor II. Thus, a collector current $A_1$ has to be supplied in sufficient strength for the transitions between level 2 and 3 in transistor II. Here again the pathlength $L''$ is adjusted for complete absorption of the control beam, resulting in a gain $G''$ for transistor II. $G''$ thus serves as multiplier for the power $(B_1-G'\Delta A)$ so that the collector beam $A_1$ emerges from the complementary optical transistor unit with the power

$$A = A_1 - G''B_1 + G'G''\Delta A.$$  

The first two terms on the right of Equation 1 represent constant (dc) power supply contributions. The third term, however, represents the alternating (ac) signal $\Delta A$ multiplied by a gain $G$ where

$$G = G'G''.$$
These considerations show that the two complementary transitors operate synergistically; the original signal photons are reproduced with the gain given by Equation 2. Furthermore, multiple complementary units can be arranged in cascade resulting in an exponential increase of total gain, as will be discussed in a later section in greater detail. The capability of exponential gain is of particular importance for the choice of materials. Such gain requires, at least in principle, only that \( G > 1 \), even if \( G' \) or \( G'' \) are smaller than unity.

2.1.2 Search for Suitable Complementary Transistor Materials

Given a material of very narrow absorption lines for one of the transistors, the probability of finding a complementary transistor material is sharply decreasing, if one expects it to approach a similar linewidth. On the other hand, the prospects for matching energy levels are much improved provided one is content with broader widths for the spectrum of the second material. Even materials exhibiting partly overlapping spectral quasi-continua can have transistor gains larger than unity as has been projected,\(^2\) e.g., for uranyl.

Concurrently, we are in the process of searching among the spectra of rare earths (Types 4f and 5f) in various host materials for suitable candidates of a mutual match. This search presents a prolonged task since the pertinent data must be gathered from individual publications rather than from table collections as they are available for spectra of free atoms.\(^3\)

2.2 Discrete Complementary Units in Optical Transistor Chain

The construction of a transistor chain of discrete units, such as shown in Figure 2, requires the periodic injection of dc optical power to maintain transitions between levels 2 and 3 for each transistor for sufficient absorption gain. Such optical systems are analogous to the Darlington circuits of electronic transistor amplifiers in which the emitter current of stage \( n \) becomes the base current for stage \( (n+1) \),
while for each stage a dc input ("collector") flux must be provided. In
the optical case under discussion, of course, the dc supply radiation
for material I must have the frequency of \( B_1 \) and, alternatively, for
material II that of \( A_1 \).

Thus, it is necessary to inject successively \( B_1, A_1, B_2, A_2, B_3, A_3, B_n, A_n \) for \( n \) complementary units. To determine the size of these dc
components, it is permissible (in small-signal theory) to ignore the
effect of the ac components, but it must be considered that the
relatively large dc components also undergo amplification. This is
shown in Figure 2 for the first stage. Beam \( B_1 \), which maintains the
gain in the first transistor and is then injected as control into the
second detector, reduces the collector beam \( A_1 \) by the amplified amount
of absorption \( (-)G'B_1 \). The reduction may severely depress gain \( G'' \),
unless \( A_1 \) is made large enough to compensate for this loss.

The relationship between gain and the collector beam power is
shown in Figure 3. The latter is a reproduction of Figure 3 (see also
Figure 5) of the previous report,(1) although the \( G \)-values in the
present report represent photon multiplication factors \( (G_{ph}) \). It will
be seen that the gain as a function of the collector flux input \( S_o \) equals

\[
G = G_{sat} \left(1 + S_h/S_o \right)^{-1},
\]

where \( S_h \) is the flux input at which \( G \) reaches \( G_{sat}/2 \), i.e., half the
gain saturation. Thus, the gain rises at first relatively fast before
it approaches saturation asymptotically. In the following calculations
we will, for convenience, assume dc fluxes high enough so that
variations of the gain due to the absorption process can be ignored,
i.e., \( G' \) and \( G'' \) are constants for the materials I and II, respectively.
This requires a minimum collector flux \( B_M \) in material I and \( A_M \) in
material II. Thus, in the first stage, \( A_1 = A_M + G''B \). The output \( A_M \)
then enters the second stage as the control beam of transistor I
(neglecting again the ac component). There it will produce amplified
absorption \( (-)G'A_M \) for the collector input beam \( B_2 \) so that the latter,
by similar reason as before, should be \( B_2 = B_M + G'A_M \).
Maximum Theoretical Amplification \( G_{\text{max}} = G_f \times G_{\text{ph}} = \left( \frac{V_{23}}{V_{12}} \right) \times \left( \frac{\gamma_{23}}{\gamma_{12}} \gamma'_{2} \right) \)

![Graph showing the relationship between \( G(S_0)/G_{\text{max}} \) and \( S_0/S_n \)]

Figure 3. Normalized gain as a function of normalized control power for a transistor.

Figure 4 shows the extension of this method to a transistor chain of \( n \) stages. Figure 4 displays the fact that, under the assumed special conditions, the input dc supply fluxes A and B remain the same for each stage except the first, where the input control beam is just the initial signal \( \Delta A \). However, although the dc input fluxes return periodically to the same values, \( \Delta A \) continues to be amplified as it passes through the various stages. This is shown in Table 1 for the example of the input and output fluxes in transistor II of the second stage.

**TABLE 1**

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<th>Transition</th>
<th>Input</th>
<th>Output</th>
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<tr>
<td>1-2</td>
<td>( B-G'A+G'B_H+\left( G' \right)^2G''A )</td>
<td>0</td>
</tr>
<tr>
<td>2-3</td>
<td>( A_H+G''B-G'C+G''A+G'A )</td>
<td>( A_H+\left( G'G'' \right)^2\Delta A )</td>
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Figure 4. Example of dc flux flow in optical transistor chain.

In Table 1, the special relationships \( A_1 = A_2 = \ldots = A \), \( B_1 = B_M \), and \( B_2 = B_M \) have been used, where

\[
A = A_M + G''B_M \tag{4}
\]

\[
B = B_M + G'A_M. \tag{5}
\]

It will then be seen in Table 1 that when the control input is multiplied by \(-G''\) and added to the input of the collector, the output of the collector is just \( A_M + (G'G'')^2 A \). By similar reasoning it is obvious that the \( n \)-stage signal photon gain equals

\[
G_n = (G'G'')^n. \tag{6}
\]

2.2.1 DC Flux Demands

As seen from Figure 4, the required supply of dc flux amounts to

\[
P_{DC} = n(G'' + 1)B_M + n(G' + 1)A_M - G'A_M. \tag{7}
\]
The last term of Equation 7 results from the fact that the control input of the first transistor does not contain the dc component $A_M$, so that the term $G' A_M$ has to be deducted from the second term on the right.

A comparison of Equations 6 and 7 shows that the dc radiation demand increases linearly with the number $n$ of stages, whereas the gain increases exponentially with $n$.

It will be understood that the condition of constant $G'$ and $G''$, chosen for mathematical convenience, is unnecessary in practice. It is entirely possible to use flux quantities well below the quasi-asymptotic region (see Figure 3) so that the gains $G'$ and $G''$ vary from a maximum, when the collector flux enters the transistor, to a minimum, when absorption has reduced the flux to $A_M$ and $B_M$, respectively. The only difference is that in Equations 4 to 7, $G'$ and $G''$ represent average values which are somewhat complex to evaluate. Clearly, there will be values for $A_M$ and $B_M$ for which the efficiency of the dc fluxes is optimized.

Similarly, the complete absorption of the control beams represents only a special case. It is also possible to operate with the flux flow shown in Figure 1, although dc flux supply has to be added.

2.2.2 Signal-to-Noise Ratio (S/N)

For many applications of optical transistors, S/N values will be of critical importance. The noise in such transistors is photon noise arising mainly from two contributing sources: (1) fluctuations of the photon fluxes at the input of the first transistor, and (2) noise of the spontaneous emission which follows the absorption processes at the same frequency (although it is absent in radiationless transitions).

(1) We will at present assume (subject to a later further extension) that we are dealing with simple photon random noise as in (electronic) photomultipliers. Assume one observes an average number $<N>$ of photons during an observation time $\Delta t$. Such observations will reveal fluctuations $\Delta N$ for which there exists the stochastic relationship
\[\langle (\Delta N)^2 \rangle = \langle N \rangle. \quad (8)\]

Photon noise, which is the root-mean-square of deviations \( \Delta N \), therefore equals the square root of the average number of photons counted. The energy of a photon at 1 \( \mu \text{m} \) equals \( 2 \times 10^{-19} \) Joule. At the first transistor an input collector flux \( B_M \) of 100 \( \mu \text{W} \) at 1 \( \mu \text{m} \) consists of \( 5 \times 10^{14} \) photons per second, and these have a fluctuation of \( 2 \times 10^7 \) photons.

Thus, during an observation time of one second, the noise of the dc input equals \( 4 \times 10^{-12} \) Joule. A signal input of this energy yields then \( \text{S/N} = 1 \). More generally, the noise equivalent power (NEP) equals \( 4 \times 10^{-12} \) watt-sec\(^{1/2} \) under the assumed conditions which include a relatively small signal, hence a correspondingly small photon noise.

Just as for the dynode multiplication in electronic photomultipliers, succeeding stages do not add materially to the noise since the signal increases much faster than the dc supply.

(2) Even smaller is the noise contributed by fluctuations of spontaneous emission. Only a small fraction is within the acceptance angle of the beams. The accepted fluorescence noise is therefore small compared to that of the supply flux.

2.3 Optical Continuum Transistors

The transistor chain treated in the preceding sections consisted of discrete complementary transistor units. Among the possible alternatives of such chains, we selected the important extreme case in which the beam exciting the transition from level 1 to level 2 is completely absorbed in each pass through a transistor.

However, there exists another limiting case in which absorption is continuously compensated by dc supply (limit of thin complementary units). This extreme leads to a new type of transistor, with no electronic analog, in which materials I and II are homogeneously mixed. There results the "continuum transistor" shown schematically in Figure 5.
Beams $A_0$ and $B_0$ are inputs to the device and pass through it in the $x$-direction. In addition, dc supply beams $A_I$ and $B_I$ enter the material mixture, distributed over the length of the device. This "feed leakage" can be provided in various ways, such as evanescent waves or by other types of insertion. The excitation paths of beams $A$ and $B$ are those shown in Figure 1 (where, however, the dc supplies have been omitted). It is clear that beam $A$ can only be absorbed by transitions between levels 1 and 2 in medium component I and, similarly, beam $B$ only between levels 2 and 3 of component II of the mixture. Furthermore, the photon multiplication gains $G'$ and $G''$ remain the same as defined before.

We now assume that a concentration mixture of materials I and II is selected for a convenient absorption coefficient $\alpha$. One obtains then for the incremental changes of $A(x)$ and $B(x)$ in an incremental interval $dx$ as the balance of absorption and the constant feed supply $A_I$ and $B_I$ the following differential equations:

\[
\frac{dA}{dx} = \frac{d(A - A_I)}{dx} = -\alpha(A - A_I) - \alpha G''(B - B_I) \tag{9}
\]

\[
\frac{dB}{dx} = \frac{d(B - B_I)}{dx} = -\alpha (B - B_I) - \alpha G'(A - A_I). \tag{10}
\]
Multiplication of these equations by $\sqrt{G'}$ and $\sqrt{G''}$, respectively, yields the expressions

\begin{align}
&d[\sqrt{G'} (A-A_I)]/dx = -\alpha\sqrt{G'}(A-A_I) - \alpha\sqrt{G''}/(G'G'')(B-B_I) \\
&d[\sqrt{G''} (B-B_I)]/dx = -\alpha\sqrt{G''}(B-B_I) - \alpha\sqrt{G'}/(G'G'')(A-A_I).
\end{align}

The solution of Equations 11 and 12 is given by

\begin{align}
&A(x) = A_I + (1/2) \{ (A_0-A_I) - (B_0-B_I)/(G''/G') \} e^{-\alpha x} [1 - \sqrt{(G'G'')}] \\
&\quad + (1/2) \{ (A_0-A_I) - (B_0-B_I)/(G''/G') \} e^{-\alpha x} [1 + \sqrt{(G'G'')}] \\
&B(x) = B_I - (1/2) \{ (A_0-A_I)/(G''/G') - (B_0-B_I) \} e^{-\alpha x} [1 - \sqrt{(G'G'')}] \\
&\quad + (1/2) \{ (A_0-A_I)/(G''/G') + (B_0-B_I) \} e^{-\alpha x} [1 + \sqrt{(G'G'')}] .
\end{align}

The gain of $A(x)$ and $B(x)$ with respect to the input $A_0$ is now, respectively,

\begin{align}
&dA(x)/dA_0 = (1/2)e^{-\alpha x}[1 - \sqrt{(G'G'')}] + (1/2)e^{-\alpha x}[1 + \sqrt{(G'G'')}] \\
&dB(x)/dA_0 = -(1/2)/\sqrt{(G''/G')}\{e^{-\alpha x}[1 - \sqrt{(G'G'')}] - e^{-\alpha x}[1 + \sqrt{(G'G'')}] \}.
\end{align}

Equations 15 and 16 state that, for $\sqrt{(G'G'') > 1}$, the gain for $A(x)$ is always larger than unity for a positive increment $\Delta A_0$, while that for $B(x)$ is negative (as is also the case for the basic complementary transistor unit). The inverse results for a positive $\Delta B_0$. For practical purposes, the second term on the right of the two equations becomes negligible so that it is seen that the gain of $A(x)$ grows exponentially as the beam passes along the x-direction.
2.3.1 DC Flux Demands in Continuum Transistor

The requirement for the dc photon supply to maintain the gains \( G' \) and \( G'' \) is, as readily determined, per unit length of the medium:

\[
P_A = aA_I + aG''B_I \quad (17)
\]

\[
P_B = aB_I + aG'A_I \quad (18)
\]

or

\[
P_A + P_B = a(G'+1)A_I + a(G''+1)B_I. \quad (19)
\]

Note that \( a \) is the reciprocal depletion length \( L' \) in the absence of gains \( G' \) and \( G'' \). Because of the added absorption, the "effective" depletion length is by the factor \((G'+1)\) or \((G''+1)\) smaller. This is brought out by Equation 19. The total dc flux required for a continuum device of length \( L \) equals

\[
P = L(P_A + P_B). \quad (20)
\]

2.3.2 Comparison of Continuum and Discrete Unit Chains

It is of interest to compare the merits of the two limiting cases, although they differ apparently in specification, except for the molecular gains \( G' \) and \( G'' \). The continuum device is characterized by a length \( L \) and absorption coefficient \( a = 1/L' \), whereas the discrete chain device is specified by the number \( n \) of complementary units. Nevertheless, a comparison is possible by interpreting a number \( n \) also for the continuum device.

2.3.2.1 DC Flux Demand

For the continuum device, one obtains with Equations 19 and 20 for the total dc demand

\[
P = (L/L')(G' + 1)A_M + (L/L')(G''+1)B_M \quad (21)
\]
where, for purposes of comparison, we have replaced the index I by the index M.

The power required for the discrete unit chain is given by Equation 7, which contains the term \(-G'M\) based on zero dc signal input of the first transistor, whereas an input \(A_0\) (dc input \(A_M\)) was postulated for the continuum. One can do either for both devices, but for comparison we omit the term \(-G'A_M\) in Equation 7 and obtain for the discrete unit chain of \(n\) stages

\[
P = n(G'+1)A_M + n(G''+1)B_M. \tag{22}
\]

It is seen that Equation 21 for the continuum device is identical with Equation 22 if we define

\[
n = L/L', \tag{23}
\]

i.e., as the number of depletion lengths \(L' (=l/a)\) contained in the length \(L\) of the continuum transistor. This plausible result implies that the two transistor multiplier types require the same dc power.

2.3.2.2 Total Gain

Using Equation 23, we can now rewrite Equation 15 for \(x = L\) of the continuum device:

\[
G_n = \frac{dA(L)}{dA_0} = \frac{1}{2}e^{n(G_g-1)} + \frac{1}{2}e^{-n(G_g+1)} \tag{24}
\]

where we have substituted the geometric mean \(G_g\):

\[
G_g = \sqrt{(G'G'')}. \tag{25}
\]
For the discrete units device, we have from Equation 6 with Equation 25:

\[ G_n = G_g^{2n} \]  

(26)

We now compare the total gains obtained with Equation 24 for the continuum and with Equation 26 for the discrete units chain for \( n=10 \). Table 2 presents this comparison for various values of the molecular mean gain \( G_g \).

**TABLE 2**

Total Gain \( G_n \) for Continuum and Discrete Units Chains (\( n=10 \))

<table>
<thead>
<tr>
<th>( G_g )</th>
<th>( G_n ) - Discrete</th>
<th>( G_n ) - Continuum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>6.7</td>
<td>1.36</td>
</tr>
<tr>
<td>1.5</td>
<td>3.3 ( \times ) 10^3</td>
<td>74</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0 ( \times ) 10^6</td>
<td>1.1 ( \times ) 10^4</td>
</tr>
<tr>
<td>3.0</td>
<td>3.5 ( \times ) 10^9</td>
<td>2.4 ( \times ) 10^8</td>
</tr>
<tr>
<td>3.7</td>
<td>2.3 ( \times ) 10^{11}</td>
<td>2.7 ( \times ) 10^{11}</td>
</tr>
<tr>
<td>4.0</td>
<td>1.1 ( \times ) 10^{12}</td>
<td>5.0 ( \times ) 10^{12}</td>
</tr>
</tbody>
</table>

Table 2 demonstrates that for 10 stages, the discrete units chain yields much higher photon multiplication than the continuum device for \( G_g < 3.0 \), but begins to fall behind the latter for \( G_g > 3.7 \). From the practical viewpoint, the continuum transistor has the often decisive advantage of monolithic structure. Thus, \( n=30 \) stages with a \( G_g = 1.5 \) would pose considerable fabrication difficulties for the discrete complementary transistor chain, although it would yield a total gain of \( 3.7 \times 10^{10} \). However, continuum transistor construction with the same \( n \) and \( G_g \) would be a relatively easy task, yet have a still respectable gain of \( 1.6 \times 10^6 \).
Finally, it is of interest to compare the results for setting, instead of Equation 23, \(2n = L/L'\), for which the number depletion lengths in the continuum is twice the number of stages in the discrete chain. This also requires twice the dc supply power. However, for large \(n\), the gain approaches the magnitude of that of the discrete chain since in the large-\(n\) limit, \(e^{2n(G_g - 1)} = G_g^{2n}\).
3. CONCLUSIONS

Optical complementary transistor systems, whether as continua or as chains of discrete units, were shown to offer very high photon multiplication factors even with rather low basic (or molecular) mean gain factors $G_g$, such as 1.5. It was also shown that, while the gain increases exponentially with the number $n$ of stages, the dc supply power grows only linearly with $n$. Finally, the signal-to-noise ratio is limited only by photon (shot) noise so that it can reach excellent values without the need for cooling.

However, it is important to stress that the rigid demands of complementary spectroscopic energy terms still pose a difficult task for identifying suitable combinations of materials. As described in Section 2.1.2, a search for such materials is underway, guided by the theoretical results described in this report. Perhaps the most important result is that mean basic gains of 1.5 or even lower suffice for very high total gains.
4. REFERENCES


