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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1965-A
MODELING FOR THE REVISED OPMS

BY

COLONEL JAMES L. KAYS, FA

DISTRIBUTION STATEMENT A:
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2 APRIL 1986

US ARMY WAR COLLEGE, CARLISLE BARRACKS, PA 17013
In September 1984 the Army Chief of Staff approved initiatives recommended by a select ORMS Study Group. These initiatives, to be implemented over a five-year period, require revision to Army personnel management policies, procedures, and associated computer models and data bases. One of the computer programs facing revision is the Asset Utilization Model (AUM). AUM determines the distribution of the officer inventory, by grade and skill(s), to meet force structure authorizations. This paper provides an overview of the (continued)
alternatives for modifying or replacing AUM. Based on a study of AUM, numerous discussions with managers at MILPERSEN, and an analysis of plausible alternatives, it is concluded that AUM should be replaced in the short term by a modified storage array which retains essential AUM coding logic. It is also concluded that in the long term a vector storage methodology should be used for maximum flexibility and efficiency. These alternatives can be implemented using the current computer capability at MILPERSEN and address both the short- and long-term needs of Army personnel managers.
MODELING FOR THE REVISED OPMS
AN INDIVIDUAL STUDY PROJECT
by
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Carlisle Barracks, Pennsylvania 17013
2 April 1986

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ABSTRACT

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In September 1984 the Army Chief of Staff approved initiatives recommended by a select OPMS Study Group. These initiatives, to be implemented over a five-year period, require revision to Army personnel management policies, procedures, and associated computer models and data bases. One of the computer programs facing revision is the Asset Utilization Model (AUM). AUM determines the distribution of the officer inventory, by grade and skill(s), to meet force structure authorizations. This paper provides an overview of the alternatives for modifying or replacing AUM. Based on a study of AUM, numerous discussions with managers at MILPERCEN, and an analysis of plausible alternatives, it is concluded that AUM should be replaced in the short term by a modified storage array which retains essential AUM coding logic. It is also concluded that in the long term a vector storage methodology should be used for maximum flexibility and efficiency. These alternatives can be implemented using the current computer capability at MILPERCEN and address both the short- and long-term needs of Army personnel managers.
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CHAPTER I

INTRODUCTION

One of the major responsibilities of the U.S. Army Military Personnel Center (MILPERCEN) is to assure that the officer inventory meets the requirements of the force structure. Ideally, at any point in time the number of officers on active duty generally should meet the grade and skill requirements of officer positions authorized in the current force.

Meeting this critical, yet fundamental, responsibility for matching officer assets with officer requirements is not a simple task. The problem of properly distributing officers is characterized by a multitude of time-variant phenomena which complicate the search for solutions and, in fact, may even blur the distinction between good and bad solutions.

Of particular concern is that the force structure continues to be in a state of constant flux. Rapidly changing technology, changes in doctrine, and modernization of the force impact heavily
on Army structure, and ultimately, on those charged with assuring that officers are available with the proper skills and grade at the proper time to meet Army needs. In short, the officer requirements of today differ from those of yesterday as well as those of tomorrow, a phenomenon representing a formidable challenge to MILPERCEN.

The officer inventory itself is constantly changing, as personnel change rank and skills, as people enter and leave the force, and as the inventory goes through normal aging. Finally, decisions such as those affecting promotion rates and distribution of skills by grade (e.g., scheduled adjustments from CVI (Conditional Voluntary Indefinite) programs), as well as those made at high levels for a variety of nonpersonnel reasons (e.g., the Gramm-Rudman bill, changing Army needs and missions, etc.), often inject additional changes into the mix of skills and grades in the inventory, and hence, into the problem of matching officer needs and assets over time.

**BACKGROUND**

As might be expected MILPERCEN uses a wide variety of computer data bases, models, and methodologies to manage Army officer
personnel. An example is the Officer Force Implementation Plan (OFIP). The purpose of OFIP is to project an officer inventory by specialty code, grade, and years of service, but also constrained by DOPMA (Defense Officer Personnel Management Act) and end strength limitations, that best satisfies officer requirements.

An important part of OFIP is the Asset Utilization Model (AUM). AUM is a computer program designed to provide personnel managers with gross roll-up data indicating how many officers, by grade and specialty codes (SC’s), should be assigned to positions authorized in the force structure, also identified by grade and SC. The problem addressed by AUM may be based on current officer inventory and force structure or future inventory and structure. As an example, AUM might indicate at a given point in time how many LTC’s with SC’s 11/49 should be assigned to SC 11 and how many to SC 49. At the same time, AUM might indicate to personnel managers how many positions of each SC (11 and/or 49) remain to be filled or how many LTC 11/49’s remain unassigned after all SC 11 and SC 49 positions had been filled.

Under current OPMS (Officer Personnel Management System) guidelines, the Army (and AUM) identifies and manages the officer inventory using grade and two two-digit SC’s per officer, as in...
the preceding example of the LTC 11/49’s. Positions in the force structure are currently identified by grade and a single two-digit SC. However, these procedures will change in the not too distant future.

As a result of the findings of a select OPMS Study Group approved by the Chief of Staff in September 1984, and also as a result of subsequent decisions by the Vice Chief of Staff, beginning in 1987 the Army will manage officers using up to ten SC’s (in lieu of the current two SC’s) of three digits each (in lieu of the current two-digit SC’s). Moreover, some positions in the force structure may be identified by more than one three-digit SC.

The changes to OPMS also include different terminology to describe officer skills. Examples of these include functional areas, branches, and areas of concentration. For simplicity and to avoid confusion between the current and future OPMS guidelines, this paper will use SC generically to refer to officer skills.

Because of these decisions, particularly the increase of SC’s from two to ten for managing officer inventory and the change from two to three-digit SC’s to identify officer skills, computer models such as AUM will lose their utility, and input data bases
will require considerable revision.

PURPOSE AND ORGANIZATION

The primary purpose of this paper is to examine alternatives to current procedures involving AUM which will enable MILPERCEN to continue to ascertain the proper distribution of officers to meet force requirements under the new OPMS guidelines. The paper is intended to provide a general overview of the approaches which seem most plausible, given the time and resources available to MILPERCEN. The paper is not intended to be an all encompassing, technically detailed study, but a think piece from which MILPERCEN might proceed toward adapting AUM to the new OPMS or, possibly, adopting a new approach.

The paper begins by discussing the distribution problem faced by MILPERCEN. A general, conceptual distribution model is used to outline the basics of the problem. This simplistic introduction to distribution problems is then followed by a more explicit discussion of the challenge faced by managers at MILPERCEN. Alternative solutions are then described in detail. These solutions include both short- and long-range approaches, as well as a strictly mathematical model for solving the distribution
problem. After the alternatives are discussed, they are then analyzed and compared so as to draw conclusions and recommendations.

PROCEDURES

In gathering the information and data for the support of this paper, a thorough study of the current AUM was completed, to include an interview and discussions with its author, LTC N.T. O’Meara. Several visits to the Distribution Division, OPMD, MILPERCEN, were also conducted to determine as clearly as possible the purposes which were to be served by changes to AUM or any new model to replace AUM. In addition, the visits to MILPERCEN served to obtain an appreciation for any limitations imposed by in-house computer software and hardware, as well as a sense of urgency for a solution.

Research of appropriate mathematical models was also completed. As will become apparent in the ensuing discussion, there are a number of ways to approach the distribution problem, at least one of which is strictly mathematical.
Before considering general alternatives for modeling the distribution of officer inventory to meet force requirements, it is important to understand the nature of the problem. Specifically, it is essential to define as completely as possible, but in clear, simplistic terms, the essence of what must be accomplished. It is also important to understand how underlying assumptions and managerial constraints shape the alternatives and affect the models and their solutions.

CONCEPTUAL PROBLEM

A conceptualization of the distribution problem is depicted in Figure 1. The column of boxes on the left represents an inventory. Each box on the left contains a set of inventory items grouped according to identical characteristics. These items are depicted by smaller boxes, unshaded and/or shaded here to simulate the common characteristics of the items in each large box.
addition to the common characteristics of the items, there is a number associated with each box on the left indicating the number of items in the box.

The column of boxes on the right represents the potential destinations for the inventory items. Each box on the right will accept only inventory items of predetermined characteristics. For example, the box at the top right in Figure 1 will accept only unshaded small boxes. A shaded small box from the large box second from the top on the left could not be transported to the top box on the right. On the other hand the small boxes in the large box third from the top on the left, which have both the shaded and unshaded characteristics, could be transported to either of the top two boxes on the right. A number is associated with each box on the right to indicate the maximum number of items which can be placed in each box.

The fundamental problem is to displace each small box from its large box on the left to one of the large boxes on the right. As each small box is displaced, its characteristics must include (or match) the characteristics of the large box in which it is placed. If the large box on the right has more than one characteristic (shading), the characteristics of each small box entering from the
left must include all of those associated with the large box on
the right. All boxes on the left must be emptied, and the number
of small boxes entering any box on the right may not exceed the
capacity of the large box.

A simple observation is that there may be many solutions to the
problem. Perhaps more fundamental is that the "best" solution may
not be identifiable, depending on how the problem has been defined
and modeled, even if all solutions could be listed. Indeed,
"best" is more often in the eyes of whoever is using the results
and is a function of the desired pattern of distribution.

The needs of management also constrain the problem, normally to
the point that it would be difficult, if not impossible, to
display graphically the complete problem and the intricacies of
the required distribution. Hence, there is considerable utility
in using a simple conceptualization before moving to more
realistic situations.

Normally, just any distribution of inventory into positions
will not satisfy management. Some boxes may require specified
levels of fill; some may require complete fill. Moreover, some
small boxes in the inventory may be preassigned to specified large
boxes on the right or be prohibited from being placed in other
boxes, regardless of characteristics.

THE CURRENT PROBLEM

At Figure 2 is a depiction of a portion of the current problem faced by personnel managers at MILPERCEN. Most of the constraints to the distribution problem, generated by the current needs of personnel management, have been excluded for simplicity. On the left and similar to Figure 1, the column of boxes now represents officer inventory. Each box represents an SCC (a Specialty Code Combination of one or more SC's) which might be indexed by i. Associated with each box i is an inventory I_i, the number of officers in the box at any point in time in a particular grade who possess SCC i.

Officers may be distributed to requirements represented by boxes on the right. Each box on the right represents a SCC (indexed by j) authorized in the force structure. Therefore, associated with each box on the right is an authorization A_j, the number of positions in the force structure requiring SCC j at a particular grade. Although most authorizations are currently identified with only one SC, as explained earlier some authorized positions eventually will be identified by more than one SC, or an
The arrows in the middle of Figure 2 represent the distribution flow of officers from inventory $I_i$ to authorization $A_j$. Associated with each arrow is a variable $x_{ij}$, the number of officers with SCC $i$ assigned to authorization $A_j$. The ultimate purpose of any model for the distribution problem is to assign values to the variables $x_{ij}$, the output of the model and the solution to the distribution problem.

Not depicted in Figure 2 are the detailed constraints which apply to the current problem faced by MILPERCEN. These constraints are critically important to both management and the utility of any model to provide valid solutions to the problem. Specifically, the model for solving the flow problem in Figure 2 must accommodate realities of the current military personnel environment such as:

- predetermined fill levels
- transient, holdee, and student projections (THS)
- excepted authorizations
- substitutability by grade and skill.

In addition to the above the model must accommodate new constraints generated by the new OPMS guidelines. These include
the following:

- 3-digit Area of Concentration (AOC) (a new term analogous to SC)
- officer inventory will be characterized by 1 to 10 AOC skill designations
- authorizations may have more than one skill designation
- branching will be applied to the officer inventory (i.e., some officers will remain in a single fundamental SC for management purposes)
- requirements and inventory for special branches must be accommodated.

An additional requirement on any model has to do with computer support systems and data bases at the disposal of MILPERCEN. A model which exceeds the data storage capacity of the computers at MILPERCEN is of little or no use. A model which relies on prepackaged software not amenable to the unique management needs of MILPERCEN is also of very limited value. Models which do not permit relatively simple modifications or adjustments to accommodate changing needs of the Army are both costly and of limited utility over time.
CHAPTER III

ALTERNATIVES

In the context of this paper, the term "model" is used in the broadest sense. It may refer to a particular method or format for data storage, but also imply the inclusion of the computer coding necessary to manipulate the data so as to solve the distribution problem. It may also be represented by a particular analytical formulation of the distribution problem.

CURRENT AUM

Any discussion of alternatives logically begins with the current AUM. The primary reason for this conclusion is the simple fact that AUM works. Indeed, AUM works extremely well, providing an invaluable service to the managers at MILPERCEN. Thus, it follows that any consideration of modifications to AUM or the creation of a completely new model should begin with an understanding of AUM, what it does and does not do, and the basis of its success in satisfying its MILPERCEN users.
The current AUM uses a two-dimensional array to store and monitor both the officer inventory and assignments from the inventory to positions in the force structure. Figures 3 and 4 depict such arrays. Using AUM, the computer reads and stores information critical to the distribution problem. For example, inventory by grade and SC (see Figure 3) and authorizations by grade and SC (see Figure 4) are fed into the computer at the beginning of each computer run.

After initial data are read into the computer, AUM makes assignments one at a time by determining the most critical SC for the next fill and then determining the least critical specialty pair in the inventory from which the fill is made. After arrays are updated the procedure is repeated.

The two-dimensional arrays, with SC’s identifying both rows and columns, are particularly suitable, since under current OPMS guidelines officers are managed using two SC’s. For example, in Figure 3, the number 2200 at position (11,11) (i.e., at the intersection of the row identified by 11 to its left and the column identified by 11 above it) represents the number of officers in a particular grade with SC 11, but no other SC. The number 50 at position (11,49) represents the number of officers
with SC 11 and SC 49 in the same grade as the 2200 SC 11's.

Referring to Figure 4, as well as the numbers from Figure 3 described above, the number 2200 at position (11,11) represents the number of SC 11 officers (all of them) assigned to SC 11. Similarly, the number 40 at position (11,49) represents the number of SC 11/49 officers assigned to SC 11, and the number 10 at position (49,11) represents the number of SC 11/49 officers assigned to SC 49. That is to say, in the assignment array the row identifier determines the SC to which the officers are assigned. Also, the sum of a row represents the number of officers assigned to the SC associated with that row.

The constraints generated by the needs of management are entered into AUM using programming techniques. For example, minimum fill levels can be entered by simply programming the computer to fill to that level. Priority SC's can be accommodated by filling these SC's at the beginning of a run. Grade and specialty substitution, excepted authorizations, and THS personnel are addressed in a similar manner. Another appealing feature of AUM is that after all assignments have been through the iterative process described above, cross leveling between either SC's or authorized positions can be accomplished as desired.
The key decision feature of AUM, and the secret to its effectiveness, is the manner in which the SC for next fill and the SC pair to accomplish the next fill are determined. As mentioned above, the most critical specialty is identified for next fill and the least critical specialty pair is selected to make the next fill. The most critical SC for next fill is determined by considering two quantities, the inventory remaining to be assigned as a percentage of the number of positions remaining to be filled (work remaining to be done) and the number of positions filled so far as a percentage of the number of positions yet to fill (work accomplished to date). The two percentages are determined for each SC and then multiplied, yielding a specific value of what is called the critical specialty test function. The smallest product from among all SC's determines the most critical SC for next fill. Although admittedly heuristic in nature, the logic of the method seems inherently sound.

A similar, but related procedure, is used to determine the SC pair to be used for the fill. Having determined the SC for next fill, the computer considers all SC pairs which contain the SC designated for next fill. Then, using the same product of percentages described above, the criticality of the second SC in
each of the pairs is determined. That SC pair with the least
critical second SC (i.e., the second SC has the highest critical
specialty test function value) is used for the next fill.

Thus, in an iterative, heuristic fashion, the computer
systematically assigns officers from the inventory to authorized
positions. At each step, the most critical SC is chosen for the
next fill, and then the SC pair with the least critical second SC
is used to make that fill.

As stated previously, the strength of AUM is that it meets the
needs of those who use it, the most important criterion of all.
It is also readily adaptable to management decisions such as
changes in criteria for fill and substitutability rules. From a
purist point of view, however, it should be mentioned that its
output does not necessarily represent the "best" solution.
Indeed, as is the case for most heuristic models, it is impossible
to determine the "best" solution or to compare solutions.

Another shortcoming of AUM, although not critical under current
OPMS procedures, is the inefficient use of storage space.
Specifically, the inventory and assignment arrays, while useful
for the task at hand, are filled predominantly with zeroes. Thus,
considerable computer storage space is taken by entries which have
no bearing on the problem and its solution.

The current AUM is considered as the first alternative, because it is an established, working model which meets the needs of MILPERCEN. It would be possible to continue to use AUM under the new OPMS guidelines, if the officer inventory input could somehow continue to be identified with only two specialty codes, and if all positions in the force structure could somehow continue to be identified with only one specialty code. Such an approach obviously would require a transformation of an inventory of officers identified with up to ten SC's to an inventory using only two SC's, as well as transforming force structure authorizations identified with more than one SC to a single SC.

**ARRAY EXTENSION OF AUM**

Another alternative is to extend the concept of a two-dimensional array, as currently used in AUM, to a ten-dimensional, or even higher-dimensional, array. The logic is twofold. First, if a two-dimensional array is inherently convenient for an officer inventory using two SC's, a ten-dimensional array should serve the same purpose for an inventory which uses ten SC's per officer. The number of officers
in each SCC could be carried in exactly the same way as in AUM, and assignments to specific SC's could also be monitored using current AUM procedures.

An example of a three-dimensional array is depicted at Figure 5. At position (11,11,11) is the number 2200, which denotes the number of officers in the inventory with SC 11, but no others. At position (11,49,51) is the number 50, which might denote the number of officers in the inventory with SC's 11/49/51. Other SC's and strengths would be denoted in a similar fashion.

Theoretically, the number of dimensions of an array is unlimited, although dimensions above three cannot be displayed graphically.

Unfortunately, the use of such a large array places even greater demands on the storage capacity of the current computer system at MILPERCEN by exacerbating the storage inefficiencies already inherent in AUM. For example, a ten-dimensional array with 120 possible entries (roughly the number of SC's after the transition from two- to three-digit SC's) for each position would translate into 120 raised to the tenth power for the number of entries in the array! Users of the current AUM at MILPERCEN confirm that such an approach would exceed the storage capacity of their computer system.
VECTOR EXTENSION OF AUM

Another approach for addressing the needs to store the data necessary for an iterative program such as AUM is the use of vectors. A typical vector would carry the information pertaining to one combination of grade and SCC. Additional information, such as the current inventory level or the number assigned from the vector could also be carried in a designated location in the vector.

At Figure 6 are examples of vector extensions of AUM. In the inventory case, if 250 LTC's in the inventory were identified with SC's 11B/49A/51B and if at some point during a computer run, 115 had been assigned to authorized positions, the information might be carried in vector form as shown in Figure 6. The places in the vector notation would be reserved for specific information. In this example, the first entry always represents initial inventory level, the second position represents rank, the last position represents the number assigned, and the positions between rank and the number assigned represent skills. Assignment vectors could be used also, as explained at the bottom of Figure 6.

With the information carried in vectors in lieu of arrays, the
essential features of AUM could be retained using programming techniques. As changes in the inventory and force structure occur, the vectors could be readily modified to carry the information without any inefficiencies in storage.

It should be noted that the use of vectors is a distinctly different approach which would undoubtedly take a considerable period of time to develop, test and implement. The data bases which feed the model more than likely would have to be indexed or otherwise modified to accommodate the vector approach. Also, the programming effort to implement the vector model would be quite extensive and time consuming, a major consideration when faced with a near term suspense to produce a working model. As a long term approach to the personnel management problem, however, the use of vectors offers considerable flexibility and efficiency.

ANOTHER TWO-DIMENSIONAL ARRAY

Keeping in mind the strong appeal of the approach used by the current AUM, but faced with the inherent problems of a straightforward extension to higher-dimensional arrays or vectors, an alternative form of a two-dimensional array seems to hold promise for addressing the current needs of MILPERCEN. An example of such
an array is depicted at Figure 7.

In this particular array, the different SCC’s are listed across the top and identify the columns of the array. The SCC’s of officers in the current inventory are listed down the left side and identify the rows of the array. An additional row (top left corner) has been added to record the authorized remaining level for each SCC identified in the force structure. An additional column (top left corner) has been added to record the inventory available in each SCC to meet fill requirements.

The array permits all logic operations and manipulations in the current AUM. For example, the array will permit iterative tabulation of levels of fill in each of the authorized SCC’s and the number of officers remaining in each of the SCC’s in the inventory, values critical to retaining the iterative features of the current AUM.

Adaptation of the new two-dimensional array and continued use of the logic of the current AUM would require an adjustment for determining which SCC for the next fill and then which SCC in the inventory to use for the fill. As discussed in an earlier paragraph, the current AUM uses a heuristic technique which provides excellent results. A similar approach could be used for
the new array, although testing would surely be required to verify its utility.

At Figure 8 is a brief description of the current and modified procedures which underlie the iterative fill process. In both cases, the procedure considers both the "fill completed to date" and the "fill remaining to be completed." The major change in the modified array occurs when more than two SC's exist in the SCC's under consideration to complete the next fill. In this case, the critical test functions are combined in much the same manner as the current AUM combines the two percentages, corresponding to "fill completed" and "fill to be completed."

It should be noted that the modified two-dimensional array retains the storage inefficiencies of the current AUM. However, personnel at MILPERCEN indicate that their storage requirements should be sufficient to accommodate the new two-dimensional array. The question of storage remains relevant, however, since the implementation of three-digit SC's and the increase in the number of SC's with which the officer inventory is to be managed dramatically increases the demands on computer storage. The problem will worsen, as more and more combinations of SC's appear in the inventory and force structure authorizations.
The distribution problem depicted in a simplified manner in Figure 2 is well known to mathematicians as a linear programming problem. (See, for example, Reference 1, pp. 4-10.) More specifically, since the solutions must always be integers (officers cannot be distributed in fractions), it is commonly known as an integer programming problem. (Reference 1, pp. 136-148.)

The purpose of any linear programming problem is to maximize (or minimize) some quantity which is subject to a set of constraints describing limited resources. The quantity to be maximized is called the objective function, and in business it may be an analytical expression for profit or loss. Here, however, the distribution function might be the number of officers assigned. Thus, the objective function could be defined as simply the sum of the variables $x_{ij}$. Maximizing such a sum would assure that as many officers as possible were assigned to valid positions. Clearly, other objective functions could be written for the officer distribution problem, and the problem might even be written in terms of more than one objective function (Reference 2, pp. 144-148), depending on the goals of management. The main
point is that linear program models can be written to address the
distribution problem, and in fact, numerous software packages
exist for solving such problems.

At Figure 9 is a geometric representation of a simple linear
program to illustrate basic ideas. In this case, the objective is
to maximize the function \( z = 0.5x + 2y \), subject to the constraints \( x + y \leq 6 \), \( x - y \leq 1 \), \( 2x + y \geq 6 \), \( 0.5x - y \geq -4 \), \( x \geq 1 \), and \( y \geq 0 \). The cross-hatched
area inside the dashed lines represents the area in which feasible
solutions exist. The successive lines representing values for \( z \)
show that the maximum value for \( z \) is 10, which occurs when \( x = 4/3 \)
and \( y = 14/3 \).

Linear programs are not restricted to just two variables and
can be written for distribution problems, as mentioned earlier.
The inequalities would represent constraints such as the flow from
any SCC not exceeding the original inventory level and the flow
into an SCC in the force structure not exceeding the authorized
level. Thus, referring to Figure 2, a simple linear program for
the officer distribution problem might be constructed as follows:

\[
\text{maximize } \sum_{j} \sum_{i} x_{ij}, \\
\text{subject to } \sum_{j} x_{ij} \leq I_i
\]
Specific needs and constraints injected by managers would appear as additional inequalities.

Using a properly formulated mathematical model, such as a linear program, has several advantages. The first of these is that it is possible to discern an optimal solution, since by definition, the optimal solution is the solution to the linear program model. Another advantage of an analytical approach is that it becomes possible to do sensitivity analyses. The "what if" questions become answerable in both quantitative and qualitative terms, rather than resort to computer runs which simply display outputs resulting from modified inputs (the "what if's") without coming to grips with specific causes.

Linear program models also have their shortcomings, particularly when used improperly or when overconstrained. For example, an improper use would occur if the model was incompletely defined due to constraints being omitted or overlooked. Another improper use could occur when using prepackaged software. As stated previously, most distribution problems have many solutions, some more suitable to management than others. A software package
might distribute the 50 11/49's, used in an earlier example, to only 30 11 positions. This would meet the criteria for the linear program, but would undoubtedly be of little use to MILPERCEN which would seek a more balanced distribution. Of course, added constraints could be formulated to address such gross imbalances, but this could easily lead to overconstraining the problem. That is, a solution would not exist to the linear program.

Figure 10 depicts geometrically the same linear program as Figure 9, except that the constraint \(x+y \leq 1\) has been added. Whereas the original problem in Figure 9 had an optimal solution of 10, the addition of the constraint \(x+y \leq 1\) in Figure 10 has created a problem which has no solution. That is, there is no value for \(z\) which can meet all of the constraints on \(x\) and \(y\). Geometrically, this means simply that the shaded area in which solutions could be found in Figure 9 has been reduced to zero area in Figure 10.

Since unrealistic solutions are not uncommon when using linear program models, because problems can become overconstrained to the point they have no solutions, and since it may be difficult to list all of the constraints to a distribution problem, linear programming models must be used with care when applied to
situations such as the officer distribution problem faced by MILPERCEN.
In comparing the alternatives, several criteria are pertinent. The most important are meeting the needs of MILPERCEN, feasibility for implementation, and time required for implementation.

Attempting to use the current AUM by modifying the officer inventory to one which contains only two SC's would certainly enable MILPERCEN to continue with a most successful computer model. Unfortunately, assuming that one could modify a ten-SC inventory to a two-SC inventory in a way that made sense, one probably would never really know if the output of the model was any good. More to the point, any preliminary adjustment to the input would inevitably render the output questionable. For example, if a position in the structure, identified by two SC's, went unfilled at the end of a run, it would be impossible to ascertain if the shortage was legitimate or one caused by the consolidation of the inventory to two SC's. More fundamental,
MILPERCEN would have circumvented the guidance to manage the
officer inventory using ten SC's. In summary, then, the first
alternative could be implemented relatively quickly and is
feasible. However, use of the alternative would undoubtedly raise
questions with regard to meeting the guidance for managing the
officer inventory with ten SC's.

A direct extension of AUM to a higher dimensional array is
feasible, but only if the computer storage capacity at MILPERCEN
is enlarged. The larger dimensional array would allow a virtually
direct extension from the current AUM and the retention of its
highly effective logic. An array approach would also be easy to
monitor and manage for the same reasons that the current AUM is a
simple and easy to use model. The time to implement this
alternative would be minimal. Even then, the use of an approach
which was dominated by such gross storage inefficiencies would be
somewhat questionable and difficult to justify.

The move to another form of two-dimensional array is feasible
and could probably be accomplished in the near time frame (six
months or less). However, the storage inefficiencies of AUM are
increased with this approach, although not to the degree of the
larger-dimensional array. Thus, should any increase in SC's, or
combinations thereof, occur in the future, the utility of the modified array might be short-lived. The modified two-dimensional array would also retain the easy-to-use features inherent in the current AUM.

The strong appeal of the vector approach is that it uses only the computer storage space it needs to solve the problem, and the logic of the current AUM could be carried over in the coding. Thus, the storage inefficiencies inherent in the current AUM and the array alternatives described above would be avoided with the use of vectors. Moreover, the use of vectors would create a system completely flexible to adapt to future changes to OPMS and expansions in the SC-combinations used to describe officer inventory and positions in the force structure. However, it is likely that the use of vectors would require considerably more time to code, test, and implement than would, say, the modified two-dimensional array. In short, shifting to vectors might take more time to implement than is available if the January 1987 deadline established by the Chief of Staff is to be met.

The linear program approach would also take a considerable amount of time to reach implementation, since original coding would probably be required to assure realistic and useful
solutions. Moreover, the problem of defining all constraints without overconstraining the problem remains a dilemma. However, the linear program approach offers distinct and unique advantages unavailable with the other approaches. As explained earlier, if the distribution problem and all management constraints could be properly formulated, the personnel managers would be able to conduct true sensitivity analyses in ways not possible with the heuristic computer models. The linear problem model would also have little difficulty with the computer storage at MILPERCEN.

To summarize, since the current AUM cannot be used without violating new OPMS guidelines and higher level decisions and since higher-dimensional arrays exceed the computer storage capacity at MILPERCEN, there remain only three plausible alternatives. These are the modified two-dimensional array, the use of vectors, and a linear program model.

The vector approach offers the greatest efficiency and flexibility for the future, but undoubtedly will take a lengthy period to implement. The linear program model runs the risk of ultimately not satisfying MILPERCEN and also would require considerable time for implementation. The modified two-dimensional array can be adapted in a reasonably short period
and should meet MILPERCEN's needs, at least for the near time frame. Each of these approaches can be implemented using current MILPERCEN computer facilities.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

Based on the foregoing discussion the following conclusions are summarized:

a. Use of the current AUM will not meet the new OPMS guidelines.

b. Direct extension of AUM to higher-dimensional arrays is not feasible without an increase in the computer storage capacity at MILPERCEN.

c. Modification of the two-dimensional array in AUM to a different format is feasible, meets new OPMS guidelines, can be accomplished in the near time frame, but may be questionable as a long-term solution due to potential computer storage inefficiencies in the future.

d. A vector approach is feasible and reduces to a minimum the demands on computer storage; however, such an approach probably
will require a considerable period of time for implementation.

e. A linear program approach is feasible, but would require a considerable period for implementation and runs the risk of not providing useful results.

RECOMMENDATIONS

The following recommendations are made:

a. That a modified two-dimensional array, such as the one described herein, be used in the near time frame as a replacement for AUM.

b. That work begin toward formulating, testing, and implementing an approach using vectors similar to the one described herein.
BIBLIOGRAPHY


CONCEPTUAL PROBLEM

COMMON CHARACTERISTICS

INVENTORY GROUPINGS

DESTINATIONS

REQUIRED CHARACTERISTICS

permissible assignment

nonpermissible assignment

Figure 1
### CURRENT PROBLEM

<table>
<thead>
<tr>
<th>Index</th>
<th>Inventory</th>
<th>SCC Distribution</th>
<th>SCC Authoriz</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Ii</td>
<td>xij</td>
<td>Aj</td>
<td>j</td>
</tr>
</tbody>
</table>

1. 897 11A11B x12 x11 11A 1433 1
2. 68 11A11B49A x21 x22 11B 1521 2
3. 11A49A x23 26 3
4. 13A 1400 4
5. 49A x2j 50 j
6. 49A54A

Note: Detailed constraints not shown.

Figure 2
CURRENT MODEL (AUM)

INVENTORY

<table>
<thead>
<tr>
<th></th>
<th>11</th>
<th></th>
<th>13</th>
<th></th>
<th>49</th>
<th></th>
<th>54</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>2200</td>
<td>...</td>
<td>0</td>
<td></td>
<td>...</td>
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<td>...</td>
<td>0</td>
<td></td>
<td>...</td>
<td>0</td>
<td>...</td>
<td>5</td>
</tr>
</tbody>
</table>
**CURRENT MODEL (AUM)**

**ASSIGNMENTS**

<table>
<thead>
<tr>
<th>Grade</th>
<th>11</th>
<th>13</th>
<th>49</th>
<th>54</th>
</tr>
</thead>
<tbody>
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<td>2200</td>
<td>0</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>13</td>
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</tr>
<tr>
<td>49</td>
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<td>0</td>
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<tr>
<td>54</td>
<td>5</td>
<td>16</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Notes:
1. An array for each grade.
2. Sum of entries in a row = total assigned in the SC for that row. (e.g., 2200 SC 11 officers are assigned to SC 11; of 50 SC 11/49 officers in Fig. 3, 40 are assigned to SC 11 and 10 are assigned to SC 49)

Figure 4

Page 40
3-D ARRAY
EXTENSION OF AUM

Figure 5
VECTOR EXTENSION
OF AUM

INVENTORY
If there are 250 LTC’s in the initial inventory with skills 11B, 49A, and 51B, of which 115 have been assigned to authorized positions, information could be depicted using the vector

(250, LTC, 11B, 49A, 51B, 115),

where it is understood that the positions of the inventory vectors carry the following information in the places as indicated:

(initial inventory, rank, skills, inventory assigned).

ASSIGNMENTS
If there are 400 authorized positions in the force structure which require SCC 11A11B49A in the grade of LTC, of which 138 have been filled, the information could be depicted using the vector

(400, LTC, 11A11B49A, 138),

where it is understood that the positions of the assignment vectors carry information in the places as indicated:

(authorization, rank, SCC, level of fill).

Figure 6
## MODIFIED ARRAY

<table>
<thead>
<tr>
<th>Authorized</th>
<th>Inventory Available</th>
<th>Authorized SCC’s</th>
<th>Total Assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11A</td>
<td>11B</td>
<td>... 11A49A</td>
</tr>
<tr>
<td>11A11B</td>
<td>870</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>11A11B49A</td>
<td>60</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>13A49A54A</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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<td>...</td>
</tr>
<tr>
<td>Fill Level</td>
<td>12</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

### Notes:

1. An array for each grade.
2. Labeling of rows and columns will change according to SCC’s in inventory and authorizations.
3. An entry represents the number of officers assigned to a SCC authorization (i.e., a column) from a SCC inventory (i.e., a row).
4. The sum of entries in a column = total fill in a SCC at some point in a run.
5. The sum of entries in a row = total assigned from a SCC at some point in a run.

---

**Figure 7**

---

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ITERATIVE FILL PROCESS

1. CURRENT (AUM)
   a. For each SC in the authorizations, compute the product
      \[(R1)(R2) = \frac{\text{inventory remaining/authorized remaining}}{\text{total positions filled/authorized remaining}}\]
   b. SC with lowest \((R1)(R2)\) is next fill.
   c. SC pair to be used for next fill is that pair for which the second SC has the highest \((R1)(R2)\).
   d. Assign, then update the inventory and fill levels.

2. PROPOSED
   a. For each SCC in authorizations, compute the product
      \[\{(R1)(R2)\} \cdots \]
      as in 1.a. above, where each \((R1)(R2)\) corresponds to a distinct SC within the SCC.
   b. The SCC with lowest product determined in 2.a. is selected for next fill.
   c. Identify all SCC's in the inventory which contain the SCC selected for next fill. Eliminate from the inventory SCC's the SCC selected for next fill and repeat step 2.a. The inventory SCC with the highest product, after the SCC for next fill has been eliminated, is the SCC to be used for next fill.
   d. EXAMPLE:
      1) The computations in 2.a. result in SCC 11A49A having the lowest product.
      2) SCC's 11A11B49A and 11A11B49A51A are available to make the next fill. The computations in 2.a. are repeated using 11B and 11B51A, respectively. If 11B has the highest product, SCC 11A11B49A is used for next fill. Otherwise, SCC 11A11B49A51A is used for next fill.
   e. Assign, then update the inventory and fill levels.

Figure 8
LINEAR PROGRAM
GEOMETRIC REPRESENTATION

Figure 9
LINEAR PROGRAM

GEOMETRIC REPRESENTATION

Figure 10

\[
\begin{align*}
.5x - y & \geq -4 \\
2x + y & \geq 6 \\
x + y & \leq 6 \\
x & \geq 1 \\
x - y & \leq 1 \\
x + y & \leq 1
\end{align*}
\]
END
DTIC
9-86