The following researches were conducted under this contract:

1. Measurements of the total and wake-survey viscous resistance of the Wigley ship form, with and without sinkage and trim restraints.
2. Measurements of the boundary layer and wake of the Wigley hull at various Froude numbers; comparison with computed boundary layer; application to computation of ship wave resistance including viscous effects.
3. Added moment-of-inertia of a rolling ship, low-frequency limit.
4. Procedures for investigating existence and finding solutions for slender-body centerplane distributions; derivation of an exact slender-body solution for an ellipsoid.
FINAL REPORT
ON
RESEARCH ON SHIP HYDRODYNAMICS,

Sponsored by
Office of Naval Research
Department of Naval Research
Contract N00014-82-K-0069 (NR-062-183)

by
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INTRODUCTION

In the interval from January 1982 through December 1984, research in two areas of ship hydrodynamics was performed at the Institute of Hydraulic Research of the University of Iowa. One was an extended experimental study, conducted in the IIHR towing tank, of the boundary layer and wake of a ship model at various Froude numbers. The other consisted of the development of procedures for calculating more accurately the potential flow and the wave resistance of sharp-edged bodies such as the Wigley ship form.

The staff working on these problems included a Postdoctoral Visitor, L.K. Forbes, three graduate students, A. Shahshahan, P.-P. Hsu and S. Ju, and Professor Emeritus L. Landweber, who served as the Principal Investigator.

Since the detailed content of our accomplishments is presented in the eleven publications listed at the end of this report, rather than repeat these, it will be more useful and interesting to comment on the reasons for selecting the course of the research, the significance of the results, and to indicate directions of future research on these problems that we believe would be fruitful.

Resistance of the Wigley Ship Form

In the first Workshop on Ship Wave Resistance Computations at DWTNSRDC in 1979, it was found that computed values of the wave resistance of the Wigley form could not be directly compared with experimental results, because the latter were obtained with the model free to sink and trim; but, for the former, zero trim and sinkage were assumed. Consequently, a study of the resistance of the Wigley form, with and without trim and sinkage restraints was undertaken. It was found that the effects of the restraints on the wave resistance were large and that the corrections for these restraints, introduced by the Workshop Committee for the Wigley form, and estimated separately by C.Y. Chen and F. Noblesse (JSR V. 27, No 4, 1983), were inadequate. This work served as the basis of the M.S. thesis of S. Ju and was published as an IIHR report [1].

* Numbers in [ ] refer to the list of publications at the end of this report.
Boundary-Layer Measurements on the Wigley Form

Pressure taps on the hull, a five-hole pitot tube, five pressure transducers, and a digital-analog computer were used to survey the boundary layer and wake at four Froude numbers. Additional data at "zero" Froude number were taken in the IIHR wind tunnel on a double model of the Wigley form. This painstaking towing-tank investigation was performed by A. Shahshahan as part of his Ph.D. thesis [2], an abbreviated version of which has appeared as IIHR Report No. 302 [3].

The original motivation for this work was to obtain measured values of the displacement thickness, which are required in a method of correcting wave-resistance calculations for viscous effects [4]. Shahshahan did apply his data for this purpose and found that the correction greatly improved the agreement between the calculated and "measured" wave resistance [2,3], showing that it was important to take into account the effect of the boundary layer and wake on the wavemaking. Of equal interest, however, are the effects of the ship waves on the boundary layer. Shahshahan discusses this in his thesis [2], and a comparison of his data with the results of boundary-layer calculations is given in two papers [5,6]. The huge mass of data recorded by Shahshahan, which is stored on tape, can be furnished to researchers working on methods for computing turbulent, three-dimensional boundary layers, for comparison with their predictions.

Irrotational Flows

1. Ship rolling

The problem of determining the added moment-of-inertia of a rolling ship section was the subject of P.-P. Hsu's MS thesis [7]. The low-frequency limit was expressed in terms of the coefficients of the Laurent series which maps the ship section into a circle. Results were given for Lewis forms and a square section. This work supplemented a previous one by L. Landweber (JSR, Sept. 1979) in which an expression for the high-frequency limit of the added moment-of-inertia was derived.
2. Viscid-Inviscid interaction

In this work, the displacement effect of a boundary layer and wake is represented by a source distribution, either on the hull surface or on its centerplane. It was presented by Landweber in the 1978 David Taylor Lectures and in the 1981 Weinblum Memorial Lectures. The latter were published under the present contract in 1982 [4].

The continuation of the outer flow as an irrotational flow field within the boundary-layer and wake regions has been applied by Kang (University of Iowa, Ph.D. thesis, July 1978) to a very thin form (Weinblum et al, TMB Report 840, Nov. 1952). He found that the calculated wave resistance was in much better agreement with the residuary resistance when the centerplane distribution was corrected for displacement effect. An interesting result was that a consistent first-order calculation was in better agreement than inconsistent second-order results, and that only a consistent second-order calculation yielded a superior comparison.

The Wigley hull was selected for trial of the aforementioned procedure on a more shiplike form. This was the original motivation for undertaking to measure the boundary-layer and wake characteristics of the Wigley model. An important difference from the Weinblum form is that the Wigley form must be considered as slender, rather than thin. Thus it was not evident that the previously used procedures, employing centerplane distributions to correct for the displacement effect, were applicable. This led to the investigation of centerplane distributions for slender bodies described in the next section. As has been mentioned, Shahshahan has already applied the displacement thickness derived from his measurements to calculate the associated centerplane source distribution and the corrected wave resistance for the Wigley model, with promising results. This is a tentative result, however, since only first-order calculations were performed, and a thin-ship formula was used to obtain the displacement-effect correction as a centerplane distribution.

3. Slender-body theory

Our approach to solving slender body problems differs from that of Newman or Tuck and Von Kerczek. By interpreting the two-dimensional singularities, which satisfy the 2-D Laplace equation and the slender-body Neumann boundary condition of a transverse section, as elements of 3-D singularities, one
obtains immediately a velocity potential which vanishes at infinity. The usual procedure of matching inner and outer solutions then becomes unnecessary.

Our motivation for developing this alternative approach was that we wished to apply our method for correcting ship wave-resistance computations for viscous effects to a form more shiplike than that of Weinblum, Kendrick and Todd, but with its singularities lying on the centerplane. With the latter requirement satisfied, one could immediately use the formulas of [4] to correct the source distribution for the displacement effect of the boundary layer, and the wave-resistance calculations would be greatly simplified.

In order that a centerplane distribution be a slender-body solution, each transverse section must have a 2-D axial distribution. A procedure in which the ship section is mapped conformally into the unit circle, and the solution of the exterior slender-body Neumann problem is written as a complex Poisson integral in the circle plane, was developed for determining whether an axial distribution existed. This existed if the complex potential was singularity-free within the region of the circle which is the mapping of half of the ship section. This procedure was illustrated by application to an ogival section in [7].

Application of the aforementioned procedure to the Wigley ship form showed that a centerplane distribution did not exist, but that, with a slight modification of its shape, it would exist. For this modified Wigley form, the axial distribution at each section could then be obtained by analytical continuation into the section to the axis. The analytical details of this work and the resulting centerplane distribution are given in [8].

As is well known, the internal singularities of an ellipsoid in a uniform stream parallel to its largest principal axis lie on its centerplane. We have found an exact solution of the slender-body boundary condition for the ellipsoid, a result that may be new. For an ellipsoid having the same principal dimensions as the Wigley ship form, the agreement between the exact and the exact-slender-body solutions were very good. Substitution of the latter into an iteration formula for solving the integral equation for the ellipsoid yielded even better agreement. This suggests that, when a slender-body centerplane distribution has been found for a ship form, it can be improved by iteration in the integral equation for that form. This work will be reported
in a nearly completed Ph.D. thesis by P.-P. Hsu. The derivation of the slender-body solution for the ellipsoid is given in the Appendix.

4. Calculation of ship wavemaking

Dr. L.K. Forbes was a Post-Doctoral Visitor at the Institute of Hydraulic Research from January 1982 to September 1983. During the first half of this interval he was supported by the present contract on the development of numerical procedures for calculating ship wavemaking, using the nonlinear boundary conditions. His thesis on the 2-D problem of the wavemaking of a semicircular obstruction on the bottom of a channel [9] had already treated a free-surface problem with its exact, nonlinear boundary condition. This thesis problem was extended, during this period, to include the effects of surface tension [10].

On the ship wavemaking problem, he considered the numerical treatment of the integral equation for a distribution of Havelock sources over a ship hull. The kernel of this integral equation, the normal derivative of the Havelock Green function, requires a very large amount of computer time, and he developed a number of closed-form approximations which are reported in [11]. Forbes eventually decided to abandon the Havelock Green function because of its excessive computer-time requirements and concentrated instead on methods using Rankine sources and the exact free-surface boundary conditions. His paper last year at the Numerical Ship Hydrodynamics Conference at DWTNSRDC, on the wavemaking of a pressure distribution on the free surface, indicates the direction of his current research interests. Several other papers which he published during his stay at IIHR are not listed since they were supported by other contracts.

Possibly more important than Forbes' specific output is that we have attracted a talented and productive researcher to the field of ship hydrodynamics.

Acknowledgement

This final report on the last three-year renewal of ship-hydrodynamic research at the University of Iowa concludes more than three decades of continuous ONR support. This is keenly appreciated by the present author, who takes this opportunity to express heartfelt thanks to Choung Lee, Bob Whitehead, and Ralph Cooper, and dedicates this paper to the memory of Phil Eisenberg.
Lists of Publications


APPENDIX

Exact Slender-Body Centerplane Distribution for Longitudinal Flow about an Ellipsoid in a Uniform Stream

The exact centerplane distribution for longitudinal flow about an ellipsoid in a uniform stream in the direction of its largest principal axis may be expressed in the form (T.H. Havelock, 1931; and T. Miloh, 1972 and 1974).

\[ m(x,y) = \frac{abc(1+K_1)U_x}{2\pi k^3 (b^2 - c^2)^{1/2}} \left[ 1 - \frac{x^2}{k^2} - \frac{y^2}{b^2 - c^2} \right]^{-1/2} \]  \hspace{5cm} (1A)

Here \( m(x,y) \) denotes source strength per unit area distributed over the area of the fundamental ellipse

\[ \frac{x^2}{k^2} + \frac{y^2}{b^2 - c^2} = 1 \]  \hspace{5cm} (2A)

of the ellipsoid

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a > b > c \]  \hspace{5cm} (3A)

with

\[ h^2 = a^2 - b^2, \quad k^2 = a^2 - c^2 \]

\( U \) is the free-stream velocity in the positive direction of the \( x \)-axis, and \( K_1 \) is the longitudinal added-mass coefficient, given by

\[ \frac{1}{1+K_1} = 1 - \frac{abc}{kh^2} \left[ F(a,\lambda) - E(a,\lambda) \right] \]  \hspace{5cm} (4A)
where $F$ and $E$ denote the Legendre incomplete elliptic integrals of the first and second kinds, and

$$\lambda = \frac{h}{k}, \quad \alpha = \arcsin \frac{k}{a} \quad (5A)$$

At a transverse section of the ellipsoid, $x$ - constant, we obtain the ellipse

$$\frac{y^2}{B^2} + \frac{z^2}{C^2} = 1, \quad \frac{B}{b} = \frac{C}{c} = \left(1 - \frac{x^2}{a^2}\right)^{1/2} \quad (6A)$$

with the boundary condition [8]

$$\frac{\phi}{\bar{n}} = U_\bar{z} \left(1 + \frac{z^2}{y^2}\right)^{-1/2} = U_x \left| \frac{dy}{ds} \right| \quad (7A)$$

where $\phi$ is the velocity potential, $n$ is distance in the direction of the outward normal to the ellipse in the plane of the transverse section and $s$ denotes arc length along the ellipse. This section is mapped into the unit circle in the $\zeta$-plane by

$$Z = y + iz = \frac{1}{2} \left[(B+C) \zeta + \frac{B-C}{\zeta}\right] \quad (8A)$$

where $i = \sqrt{-1}$ and $\zeta = \rho e^{i\theta}$.

In the $\zeta$-plane, the boundary condition (7A) becomes

$$\frac{1}{U} \left(\frac{\phi}{\bar{p}}\right)_{p=1} = \frac{1}{U} \left(\frac{\phi}{\bar{n}}\right) \left| \frac{d\bar{Z}}{d\bar{\zeta}} \right| = z_x \left| \frac{dy}{ds} \right| \left| \frac{ds}{d\bar{Z}} \right| = z_x \left| y_\theta \right| \quad (9A)$$

$$= Bz_x \left| \sin \theta \right|$$

since, by (8A), $y = B \cos \theta$ along the ellipse. Also we have

$$z_x = -\frac{cx}{a^2} \left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right]^{-1/2} = -\frac{c}{a^2} \left| z \right| = -\frac{a^2}{c} \left(1 - \frac{x^2}{a^2}\right)^{-1/2} \left| \csc \theta \right| \quad (10A)$$

Hence (9A) becomes

$$\left(\frac{\phi}{\bar{p}}\right)_{p=1} = -\frac{bc}{a^2} U_x \quad (11A)$$
Since $x$ is constant for that section, the solution is that for a source at the origin, given by

$$\phi = -\frac{bc}{a^2} U x \ln \rho \quad (12A)$$

The axis of the ellipse, $z = 0$, is mapped by the transformation (8A) into an interior concentric circle of radius

$$\rho_0 = \left(\frac{b-c}{b+c}\right)^{1/2} \quad (13A)$$

with

$$y = \frac{1}{2} \left[ \rho_0 (B+C) + \frac{1}{\rho_0} (B-C) \right] \cos \theta = (B^2 - C^2)^{1/2} \cos \theta \quad (14A)$$

On this circle, we have by (11A)

$$\frac{\partial \phi}{\partial \rho} = \frac{bc}{a^2} \rho_0 \quad (15A)$$

which yields

$$\left. \left( \frac{\partial \phi}{\partial z} \right) \right|_{z=0} = \left. \left( \frac{\partial \phi}{\partial \rho} \right) \rho_0 \right|_{\rho_0} = -\frac{U b c x \cos \theta}{a^2 \rho_0 (B+C)}$$

or, applying (6A), (13A) and (14A),

$$\left. \left( \frac{\partial \phi}{\partial z} \right) \right|_{z=0} = -\frac{U b c x}{a^2 \left( b^2 - c^2 \right)^{1/2}} \left[ 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2 - c^2} \right]^{-1/2} \quad (16A)$$

Hence there is a slender-body centerplane distribution

$$m_s(x, y) = -\frac{U b c x}{2\pi a^2 \left( b^2 - c^2 \right)^{1/2}} \left[ 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2 - c^2} \right]^{-1/2} \quad (17A)$$

extending over the area bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 - c^2} = 1 \quad (18A)$$
which is slightly larger than that for the exact solution (2A). Clearly, the agreement with the exact solution would be improved by introducing the added-mass factor $(1+k_1)$ in (18A).