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WITH APPLICATION TO ASVAB DATA

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## Three and Four Mode Factor Analysis With Application to ASVAB Data

**Shin-ichi Mayekawa**

### Performing Organization Name and Address
The University of Iowa, Melvin R. Novick
224 Lindquist Center
Iowa City, Iowa 52242

### CONTROLLING OFFICE NAME AND ADDRESS
Office of Naval Research
536 South Clark Street
Chicago, IL 60605

### ABSTRACT
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of the common factor structure. A model which allows us to compare
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Shin-ichi Mayekawa
The University of Iowa

Abstract

The traditional factor analytic view of the PARAFAC model and its extension to a four mode situation with the derivation of the maximum likelihood estimation procedure by the generalized EM algorithm was presented. The four mode model was applied to six data matrices defined by three specialty, (clerical, mechanical, and electrical), times two services, (Air Force and Marine Corps) and successfully recovered the usual four dimensional structure without any rotation. The specialty and service differences was expressed in terms of different weighting of the common factor structure. A model which allows us to compare the factor score means was also investigated.

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Introduction

Factor analysis is a data reduction technique to represent a set of variables in a lower dimensional space. That is, the purpose of factor analysis is to represent a set of variables as a linear combination of a smaller set of latent variables, namely, factors and an overall mean. Given measurements on a set of variables, the analysis results in the estimate of factors, i.e., factor scores, the regression coefficients of each variables on the factors, i.e., factor loadings, and the error variances. When the factors are assumed to be oblique the estimate of the factor correlations is also provided.

When measurements of a set of variables are available from several data sources we could analyze each data matrix independently and compare the results. This approach results in as many sets of factor scores, factor loadings and error variances estimates as the number of data sources. A more parsimonious way is to analyze all the data matrices jointly, with some additional assumptions, reducing the number of parameters to be estimated.

Several methods have been proposed to accomplish this parsimony. For example, if we assume that all the data sources share the same factor loadings we have the factorial invariance model proposed by Lawley and Maxwell (1963) and Meredith (1964a, 1964b). A general estimation procedure under this model is found in Joreskog (1971) where he expresses the degrees of factor invariance in terms of the strength of the equality assumption among parameters. In this approach the main interest is to test whether each factor loadings matrix is the same or not. That is, the equality, or the factor invariance, is treated as an external assumptions to be tested rather than as part of structural model.
Another approach, proposed by Tucker (1966) and later by Harshman (1970), (also Harshman and Lundy (1984)), incorporates some equality of the parameters as an essence of the structural model. For example, Harshman's model, called the PARAFAC model, expresses each data matrix as a product of (common) factor scores, (common) factor loadings, and the weight matrix which represents the differences among the data sources. Tucker's model, called the Three Mode Factor Analysis (TMFA) model, introduces another set of parameters in a core matrix which describes the relationship or the interactions among the variables and the data sources. It is known, Harshman and Lundy (1984), that the PARAFAC model is a special case of Tucker's TMFA model. Several estimation procedures are available for both models. For the PARAFAC model, Harshman (1970, 1972) provides a least squares estimation method and Sands and Young (1980) provide a nonmetric extension of the least squares solution. For the TMFA model, least squares estimation is provided by Krooneberg and de Leeuw (1980) and, maximum likelihood estimation, by Bentler and Lee (1978, 1979).

One advantage of the PARAFAC model is that it provides an unique set of factors in the sense that there are no degrees of freedom for rotation. In contrast, the TMFA model, though it provides a better reproduction of the data because of its generality, does not have this uniqueness property. Naturally, the PARAFAC model is more parsimonious and easier to interpret. Its disadvantage is that it will generally yield larger error variances.

However, the PARAFAC model is not usually expressed in terms of traditional factor analysis terminology and not yet widely used.
The purpose of this paper is to provide a traditional factor analytic view of the PARAFAC model and its maximum likelihood estimation procedure. Also, an extension to the multimode case is discussed.

Model

Suppose that there are $p$ variables, $s$ data sources, $N_k$, $k=1,2,\ldots,s$, observations from each data source, and $r$ factors. The PARAFAC model can be expressed in terms of factor scores, factor loadings, data source weights and error terms as follows.

\[(1) \quad y_{ik} = m_k + A W_k f_{ik} + e_{ik}, \quad i=1,2,\ldots,N_k, \quad k=1,2,\ldots,s,\]

where

- $y_{ik}$ is the $p \times 1$ vector of observations on the individual $i$ in the data source $k$,
- $m_k$ is the $p \times 1$ grand mean vector for the data source $k$,
- $A$ is the $p \times r$ factor loadings matrix common to all the data sources,
- $W_k$ is the $r \times r$ diagonal weight matrix for the data source $k$,
- $f_{ik}$ is the $r \times 1$ factor score vector of individual $i$ in the data source $k$,
- $e_{ik}$ is the $p \times 1$ error vectors of individual $i$ in the data source $k$.

Also, the parameters associated with each individual are assumed to have the
following properties.

(2) \( \mathbb{E}[f_{ik}^k] = 0, \ i=1,2,\ldots,N_k, \ k=1,2,\ldots,s, \)

\[ D[f_{ik}^k] = C_P^k, \ i=1,2,\ldots,N_k, \ k=1,2,\ldots,s, \]

\[ \mathbb{E}[e_{ik}^k] = 0, \ i=1,2,\ldots,N_k, \ k=1,2,\ldots,s, \]

and

\[ D[e_{ik}^k] = D_k, \ i=1,2,\ldots,N_k, \ k=1,2,\ldots,s, \]

where \( \mathbb{E} \) and \( D \) are, respectively, expectation and dispersion operators and \( D_k, \ k=1,2,\ldots,s \) is assumed to be diagonal.

Also, it is assumed that each \( f \) and \( e \) are uncorrelated. To avoid the multiplicative indeterminacy it is assumed that \( \text{diag}[ C_P^k ] = I_r \) and the average of squared \( W_k \) matrices is equal to the identity matrix of size \( r \).

The additive indeterminacy

(3) \( m_k + A W_k f_k - A W_k ( f_{ik} - f_k ) \)

or

\[ m_k + A m + A W_k ( f_{ik} - W_k^{-1} m ) \]

is taken care of by setting the mean of the factor scores equal to zero.

Usually the PARAFAC model assumes that each individual is measured \( s \) times on the same set of variables, (i.e., under \( s \) occasions), but here we assume that each data source has different set of individuals. By assuming that each factor score has the same factor dispersion matrix, \( C_P \), the uniqueness property of the original model is retained.

In terms of the dispersion matrix, the model can be written as
(4) \( C_k = A W_k C F W_k A' + D_k, \quad k=1,2,\ldots,s, \)

where \( C_k \) is the \( p \times p \) sample dispersion matrix of the data source \( k \).

The uniqueness property is the result of the fact that after any nonsingular transformation of the factors the model cannot be expressed as the product of the \( p \times r \) matrix, the \( r \times r \) diagonal matrix and the \( r \times 1 \) vector without changing the goodness of fit. That is, the set of \( s \) identities

\[
A W_k f_{ik} = A W_k T^{-1} T f_{ik}, \quad k=1,2,\ldots,s,
\]

where \( T \) is the \( r \times r \) nonsingular matrix cannot always be expressed as

\[
B V_k (T f_{ik}), \quad k=1,2,\ldots,s,
\]

where \( V_k \), \( k=1,2,\ldots,s \) is a diagonal matrix.

The model stated above can be interpreted in various ways. First, by defining a new variable

(5) \( z_{ik} = W_k f_{ik}, \quad i=1,2,\ldots,N_k, \quad k=1,2,\ldots,s, \)

we have

(6) \( y_{ik} = m_k + A z_{ik} + \epsilon_{ik} \)

or

(7) \( C_k = A C_{zk} A' + D_k \)

where \( C_{zk} = D[z_{ik}] = W_k C F W_k \),

which is a special case of the factorial invariance model where the differences among the data sources are expressed in terms of the differences among the factor dispersions and the error variances. Interestingly, this form of factorial invariance model has not been proposed to this author's knowledge.

The reason seems to be that the usual selection theorem does not imply this
form of relationship among the factor correlation matrices.

Another way to interpret the model is as follows. By defining a new factor loadings matrix

\[ B_k = A W_k, \quad k=1,2,\ldots,s, \]

we have

\[ y_{ik} = m_k + B_k f_{ik} + e_{ik}, \]

or

\[ C_k B_k C_F + D_k, \]

where the differences are explained in terms of the differences among the factor loadings or the regression coefficients of each variable on the factors.

In either case, the elements of \( W_k \) matrix are considered to be the relative weights/importance of the factors. That is, when the (11) element of the \( W_k \) matrix has a relatively high value, the implication is that the \( l \)th factor is relatively more important than the rest of the factors in the \( k \)th data source. However, since the mean of each factor across the individuals are set equal to zero, this does not mean that those variables whose factor loadings are high on the \( l \)th factor have higher values. Instead, the larger weight generally implies larger variances of those variables whose factor loadings are high on the \( l \)th factor. For example, if the \( k \)th data source is highly selective on the basis of those variables whose factor loadings are high on the \( l \)th factor, we should expect that the mean of those variables is high and that the weight of the \( l \)th factor in the \( k \)th data source is low, resulting in the high-mean and the small-variance of those variables in the
As mentioned before, the model we are dealing with does not allow us to compare the factor score means of each data source. When it is desired, a slightly different formulation of the model must be used. Several methods which enables the comparison of the factor score means will be discussed in the later section.

**Extension to the Four Mode Situation**

Suppose that each data source can be expressed as a combination of two categories. For example, if a test battery is administered to a set of individuals we could divide the entire sample into six subsamples defined by sex (male, female) and race (black, white, other). In this case, it may be more parsimonious to express the variations among the data sources by the product of two weight matrices, namely, one associated with sex and another with race.

Generally, if the s data sources can be regarded as the result of s1 x s2 classification we could write that

$$W_k = W_{k1} W_{k2}, \quad k1=1,2,...,s1, \quad k2=1,2,...,s2,$$

where $k = (k1-1) s2 + k2$.

The idea is similar to the usual decomposition of the cell means into the combination of the column and row effect in $s1 \times s2$ factorial experimental design where the decomposition is additive rather than multiplicative. General form of this decomposition is known as the Canonical Decomposition of N-Way Tables. (See Carroll and Pruzansky 1984). Those weight matrices can be interpreted similarly as before.
If it is desired to decompose the means we could have the usual ANOVA decomposition such as

\[(12) \quad m_k = m + m_{k1} + m_{k2}, \quad k1=1,2,\ldots,s1, \quad k2=1,2,\ldots,s2,\]

where \(k = (k1-1)*s2 + k2.\)

The Maximum Likelihood Estimation by the EM Algorithm

With the additional assumption of normality

\[(13) \quad f_{ik} : N_i(0, C_p), \quad i=1,2,\ldots,N_k, k=1,2,\ldots,s, \quad iid,\]

and

\[(14) \quad S_k : W_p(Q_k, N_k), \quad k=1,2,\ldots,s,\]

where \(S_k\) is the sample mean corrected SSCP matrix,

\[Q_k = A W_k C_p W_k A' + D_k,\]

and

\[W_p(A, df)\] denotes the \(p\)-variate Wishart distribution with the degrees of freedom \(df\) and the mean \(df x A.\)

Here the parameter \(m_k, k=1,2,\ldots,s,\) is estimated by the sample mean and treated as a constant when deriving the Wishart distributions.

The MLE based on this model can be found by differentiating the product of \(s\) Wishart likelihood functions with respect to \(A, C_p, D_k,\) and \(W_k, \)
\(k=1,2,\ldots,s.\) However, noticing that the Wishart likelihood presented above is
the marginal likelihood of \( f_{ik}, A, C_p, W_k, D_k, i=1,2,\ldots,N_k, \)
\( k=1,2,\ldots,s, \) with all the \( f_{ik} \)'s integrated out, we could instead use the

following EM algorithm where the factor scores are treated as missing data.

This approach, originally advocated by Rubin and Thayer (1982) in the standard
factor analysis context, has the definite advantage of simplicity of the
calculation involved due to the linear (tri-linear) nature of the complete data
likelihood.

The application of the General EM algorithm scheme in this context is
outlined below. For further discussion of the EM algorithm see Mayekawa (1985).

In the E-step, the expectation of the log complete data likelihood with respect
to the conditional distribution of the factor scores given data, factor
loadings, data source weights, and the error variances is calculated. The
complete data likelihood is given by

\[
L = f(Y | F, A, W, D) \times f(F | C_p),
\]

where

\[
(15) \quad f(Y | F, A, W, D) = \\
\frac{1}{\prod_{k=1}^{s} \left[ f(Y_k | F_k, A, W_k, D_k) \right]},
\]

where

\[
(16) \quad f(Y_k | F_k, A, W_k, D_k) = \\
\frac{1}{\prod_{i=1}^{N_k} \left[ f(Y_{ik} | f_{ik}, A, W_k, D_k) \right]},
\]

where

\[
(17) \quad -2 \ln f(Y_{ik} | f_{ik}, A, W_k, D_k) = \\
\left( Y_{ik} - AW_{ik}f_{ik} \right)'D_k^{-1}\left( Y_{ik} - AW_{ik}f_{ik} \right)
\]
+ \ln |D_k| + \text{constant which does not involve the parameters.}

and

(19) \( f(F_k | C_F) = \prod_{k=1}^{s} \left[ f(F_k | C_F) \right] \),

where

(20) \( -2 \ln f(F_k | C_F) = \)

\[ \sum_{i=1}^{N_k} \left[ -\frac{1}{2} \cdot C_F^{-1} \cdot f_{ik} \right] \]

\[ + N_k \times \ln |C_F| \]

\[ + \text{constant.} \]

It should be noted that, in order to avoid notational complexity, the mean deviation score, \( \hat{Y}_{ik} - m_k \), is denoted by \( y_{ik} \) in the above expressions and throughout this section. Also, the \( N_k \times p \) column centered data matrix of the data source \( k \) is denoted by \( Y_k \), and the \( N_k \times r \) factor score matrix, by \( F_k \).

The conditional distribution of the factor score is

(21) \( f_{ik} | Y, A, W, D, C_F : N_r \left( f_{ik}^*, \Sigma_k \right) \),

\[ i = 1, 2, \ldots, N_k, \text{i.i.d., } k=1, 2, \ldots, s, \]

where

(22) \( \Sigma_k = (W_k A_k^T D_k^{-1} A_k W_k + C_k^{-1})^{-1}, \text{ k=1, 2, \ldots, s,} \)

and

(23) \( f_{ik}^* = Y_k D_k^{-1} A_k W_k V_k, \text{ k=1, 2, \ldots, s,} \)

and the expectation of \( \ln L \) is given by substituting \( f_{ik}^* \) for \( f_{ik} \) in (18).
and adding a term

\[(24) \text{tr } W_k A' D_k^{-1} A W_k V_k^* \]

and

\[N_k \text{ tr } C_F^{-1} V_k^*\]

to (18) and (20), respectively. The result can be expressed as

\[(25) \mathbb{E}[\ln L] = \sum_{k=1}^{s} \left[ \right.
\]

\[\text{tr} \left[ \left( Y_k - F_k W_k A' \right) D_k^{-1} \left( Y_k - F_k W_k A' \right)' \right] \]

\[+ N_k \ln |D_k| \]

\[+ N_k \text{ tr } W_k A' D_k^{-1} A W_k V_k^* \]

\[+ \text{tr } C_F^{-1} F_k' F_k + N_k \text{ tr } C_F^{-1} V_k^* \]

\[+ N_k \ln |C_F| \]

\[+ \text{constant which does not involve the parameters.} \]

Since the conditional expectation can be expressed as a function of

\( F_k' F_k \) and \( Y_k' F_k \), the \( N_k \times r \) matrices, \( F_k' \)'s, need not be stored in the course of calculation. Thus, the E-step can be summarized as

The E-step.

\[(26) V_k^* = \left( W_k A' D_k^{-1} A W_k + C_F^{-1} \right)^{-1}, k=1,2,\ldots,s, \]

\[(27) Y_k' F_k = S_k D_k^{-1} A W_k V_k^*, k=1,2,\ldots,s, \]

and

\[(28) F_k' F_k = V_k W_k A' D_k^{-1} Y_k' F_k', k=1,2,\ldots,s. \]
In the N-step the conditional expectation of the log complete data likelihood is maximized with respect to $A, W_k, D_k', k=1,2,...,s$, and $C_F$ treating $F_k^*$ and $V_k^*$ as constant.

The N-step.

\[ a_j = \sum_{k=1}^{s} \left( (W_k (F_k^* W_k + N_1 V_k^*) W_k / d_{jk})^{-1} \right) \]
\[ x \sum_{k=1}^{s} \left[ W_k F_k^* W_k / d_{jk} \right], j=1,2,...,p, \]

\[ d_{jk} = (\text{RSS}_{jk} + N_k a_j W_k V_k^*) / N_k, \]
\[ j=1,2,...,p, k=1,2,...,s, \]

where $\text{RSS}_{jk} = (Y_{jk} - F_k^* a_j)'(Y_{jk} - F_k^* a_j)$,

\[ W_k = \text{diag}[c_k], k=1,2,...,s, \]

where $c_k = T^{-1} h_k$.

\[ h_k = \sum_{j=1}^{p} [a_{jk} / d_{jk} (Y_{jk} F_k^*)], j=1,2,...,r, \]

\[ t_{lm} = \sum_{j=1}^{p} [a_{jk} a_{jm} / d_{jk}] \]
\[ x (F_k^* F_k^* + N_k V_k^*)_{lm}, l,m=1,2,...,r, \]

and

\[ C_F = (1/N_+) \sum_{k=1}^{s} [F_k^* F_k^* + N_k V_k^*], \]

where $N_+ = \sum_{k=1}^{s} (N_k)$.

The above formulæ are derived by taking the partial derivative of the
conditional expectation of the complete log likelihood with respect to each parameter and solving the resulting normal equations independently of the normal equations for the rest of the parameters. Therefore, strictly speaking, they do not provide the maximum of the conditional expectation but merely increase the value of the conditional expectation.

In order to avoid multiplicative redundancy, the diagonal elements of the $C_F$ matrix should be normalized to the identity matrix and the $W_k$ matrix should be also normalized so that the average of the squared $W_k$ matrices is the identity matrix.

As in standard factor analysis, the mean vectors and the SSCP matrices of each data source are sufficient to estimate the parameters $A, W_k, k=1,2,...,s,$ and $D_k, k=1,2,...,s.$

Optionally, we could enforce some equality restrictions such as

\begin{equation}
D_1 = D_2, \ldots, D_s = D,
\end{equation}

\begin{equation}
D_k = d_k I_p, k=1,2,...,s,
\end{equation}

or

\begin{equation}
D_1 = D_2, \ldots, D_s = d.
\end{equation}

With the last restriction the MLE is equivalent to the least squares estimates. Also, when the factors are assumed to be orthogonal we simply skip the estimation of $C_F$ holding it to the identity matrix.

The $W$-step for the $W_1$ and $W_2$ matrices can be derived using a similar linearization technique. That is, noticing that the conditional distribution of $f_{ik}$ and the conditional expectation of $\ln L$ is the same as (21), (22),
(23) and (25) with \( W_k \) matrix and the subscript \( k \) defined by (11), all the E-step and the N-step except for (31) remain the same. For the data source weight matrix, (31) should be modified as follows:

\[
(36) \quad W_{1k1} = \text{diag}[c], \quad k1=1,2,\ldots,s1,
\]

where \( c = \left( \sum_{k2=1}^{s2} [T_{k2}] \right)^{-1} \sum_{k2=1}^{s2} [h_{k2}], \)

\[
h_{k2 l} = \sum_{j=1}^{p} \left[ a_{j1} a_{j2} w_{k2} \right] \left[ 1/d_{jk} \right] (Y_{k1 k2} F_{k1 k2})_{jl}
\]

\[
1=1,2,\ldots,r,
\]

\[
t_{k2 lm} = \sum_{j=1}^{p} \left[ a_{j1} a_{j2} w_{k2} \right] \left[ 1/w_{k2} \right] \left[ 1/m_{jk} \right]
\]

\[
x (F_{k1 k2} V_{k1 k2} + N_{k1 k2})_{lm}
\]

\[
1,m = 1,2,\ldots,r,
\]

and

\[
(37) \quad W_{2k2} = \text{diag}[c], \quad k2=1,2,\ldots,s2,
\]

where \( c = \left( \sum_{k1=1}^{s1} [T_{k1}] \right)^{-1} \sum_{k1=1}^{s1} [h_{k1}], \)

\[
h_{k1 l} = \sum_{j=1}^{p} \left[ a_{j1} a_{j2} w_{k1} \right] \left[ 1/d_{jk} \right] (Y_{k1 k2} F_{k1 k2})_{jl}
\]

\[
1=1,2,\ldots,r,
\]

\[
t_{k1 lm} = \sum_{j=1}^{p} \left[ a_{j1} a_{j2} w_{k1} \right] \left[ 1/w_{k1} \right] \left[ 1/m_{jk} \right]
\]

\[
x (F_{k1 k2} V_{k1 k2} + N_{k1 k2})_{lm}
\]

\[
1,m = 1,2,\ldots,r.
\]

Also, normalizations such as setting the average squared \( W_k \) matrices and the average squared \( W_{1k1} \) matrices to the identity matrix shall be enforced.
Initial Configuration

The most efficient way to calculate the initial configuration seems to be the application of the SUMSCAL algorithm advocated by de Leeuw and Pruzansky (1978). The method, which assumes orthogonality of the factors and is restricted to the three mode situation, has been used in Novick, et. al. (1983). When a four or higher mode model is used, the log additive decomposition of the initial $W_k$ matrices should provide a reasonable estimate of each weight matrix.

Standardization of the Raw Data

Since the MLE has a nice property of scale invariance we may be able to rescale each variable to a desired form. Usual practice in standard factor analysis is to scale each variable so that each has zero mean and unit variance. However, as pointed out by Joereskog (1971) and Harshman and Lundy (1984), standardization within each data source changes the form of the likelihood. That is, the rescaling must be performed, after subtracting each within data source mean, by multiplying a common constant across all the data sources to each variable. The most convenient approach is to rescale the variables so that the average of the rescaled dispersion matrix has unit diagonal elements. The number of individuals may or may not be used to weigh the averaging process.
Analysis of ASVAB Data Set

The method proposed in the previous sections is applied to a subset of ASVAB Form 8.

The variables analyzed are:

1. General Science (GS)
2. Arithmetic Reasoning (AR)
3. Word Knowledge (WK)
4. Paragraph Comprehension (PC)
5. Numerical Operations (speeded) (NO)
6. Coding Speed (speeded) (CS)
7. Auto-Shop Information (AS)
8. Mathematics Knowledge (MK)
9. Mechanical Comprehension (MC)
10. Electronics Information (EI)

The means and the standard deviations are shown in Table 1. This set of variables are known to have the following four factors, see, for example, Ree, Mullins, Mathews and Massey (1981):

1. Verbal: variables 1, 3, 4
2. Technical: variables 7, 9, 10
3. Mathematical: variables 2, 8
4. Speeded: variables 5, 6

It is also known that these factors are positively correlated, with most correlation in the .4 range.

The scores of these ten variables are available for the following six data sources which are the combinations of two different armed services, (Marine
Corps, Air Force) and three different specialties, (Clerical, Mechanical, Electrical). The number of individuals in each data source are:

1. MC - CLE 3285
2. MC - NEC 3118
3. MC - ELE 1415
4. AF - CLE 8963
5. AF - NEC 16884
6. AF - ELE 7897

In the analysis we assume only that there are four factors and attempt to demonstrate that the usually accepted pattern of factor loadings can be found using the PARAFAC model.

The six sample dispersion matrices are first rescaled so that the diagonal elements of the weighted average are equal to unity. The resulting rescaled dispersion matrices are shown in Table 2.

The four mode analysis of this data set by the maximum likelihood method proposed in the previous sections, with \( r = 4 \), resulted in the parameter estimates shown in Table 3. The error variances are assumed to be equal across the data sources. The diagonal elements of each \( W \) matrix are arranged to form data source x factor matrix in Table 3.

First, it should be noted that, without any rotation, the four mode analysis recovered those four dimensions found by the standard two mode analysis. According to Ree, et.al.(1981), we could name the first factor Technical, the second, Speeded, the third, Verbal, and the fourth, Mathematical. The major difference is in the factor dispersion matrix: their solutions is more oblique whereas the highest factor correlation in our
solution is about 0.2. As a result, the Technical factor, first factor, is more influential than their corresponding factor. Once again, we emphasize that NO rotations are performed on the final result.

Second, the inspection of the W1 matrices confirms the fact that the Air Force is more selective in general. This is shown by the smaller value of the W1 weight matrix which represents the difference between the Air Force and the Marine Corps. In particular, the Air Force is highly selective on the Speeded Factor, second factor. The means of variables 5 and 6, which are highly loaded on the factor, in Table 1 shows that in all three specialty areas their means are higher than those of the Marine Corps.

Also, the W2 matrix shows that the mechanical specialty and the clerical specialty area has a smaller weight on, respectively, Technical and Speeded factors. This, combined with the inspections of the means and the standard deviations in Table 1, shows that mechanical specialists are homogeneous in those variables which are highly loaded on the Technical factor and also have higher scores on those variables. The same argument should be applied to the clerical specialist with respect to the Speeded factors. As the result, the Air Force - Clerical specialist has the smallest variances of the variables 5 and 6, which can be confirmed by Table 1 and Table 2.

Discussion

In this section we discuss a slightly different formulation of the model which enables us to compare the factor score means. Consider the additive indeterminacy in (3). The formula says that the subtraction of the mean factor score, $f_k$, can be compensated by the addition of corresponding quantity to
In the previous section we removed this redundancy by setting the factor score mean equal to zero. This approach is equivalent to defining $m_k$ as the mean of the observation,

$$E[Y_{ik}] = m_k + A W_k f_k$$

$$= m_k.$$

The reason why we chose to use this method is that this is the usual constraint in standard factor analysis where the factor score mean is not of interest. There are, however, other ways to remove this redundancy, especially, in the PARAFAC situation. For example, we could set all the $m_k$'s equal to zero,

$$E[Y_{ik}] = A W_k f_k,$$

and treat $f_k$'s as additional parameters to be estimated. The implication of this formulation is that, within each data source, if a subset of variables has the similar factor loadings they must have similar means. That is, if variable $j$ and $j'$ have identical factor loadings, i.e., if the $j$th and the $j'$th row of the $A$ matrix are identical, their mean must be identical within each data source. This may not be a realistic assumption in practice. For example, when a test and its half-test is analyzed together we expect that the means of the half-test is about half of the mean of the full-test while expecting that the both tests have similar factor loadings. Another way to reduce the redundancy is to assume

$$E[Y_{jk}] = m + A W_k f_k.$$

Since there are some redundancies left in (40) we further define $m$ as the grand mean across all the data sources. (The restriction that the average of $f_k$ is
equal to 0 can also remove the remaining redundancy.) Note that this is more restrictive than (39) since (39) does not enforce any structural restrictions on each mean while (40) assumes that each mean is a sum of the grand mean and a vector which lies in the column space of $A W_k$. Therefore, we may say that this approach is a compromise between our original approach, (38), and the more restrictive case (39).

The analysis under this assumption may be performed by modifying the conditional distribution of $f_{ik}$ in (21) and resulting expectation of the log complete data likelihood. The factor score means should be estimated in the M-step.

**Summary**

The traditional factor analytic view of the PARAFAC model and its extension to a four mode situation with the derivation of the maximum likelihood estimation procedure by the generalized EM algorithm was presented. The four mode model was applied to six data matrices defined by three specialty, (clerical, mechanical, and electrical), times two services, (Air Force and Marine Corps) and successfully recovered the usual four dimensional structure without any rotation. The specialty and service differences was expressed in terms of different weighting of the common factor structure. A model which allows us to compare the factor score means was also investigated.
References


Mayekawa, S. Bayesian factor analysis. (ONR Technical Report 85-3). Iowa City, IA: The University of Iowa, College of Education.


Table 1. Means and Standard Deviations of the Original Variables

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Table 3: Parameter Estimates

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W1-MATRIX: Weight matrix associated with each armed services

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W2-MATRIX: Weight matrix associated with each specialties

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W-MATRIX: Product of W1 and W2 matrices

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CF-MATRIX: Factor dispersion matrix

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Personnel Analysis Division, AF/MPA
5C360, The Pentagon
Washington, DC 20330

Air Force Human Resources Lab
AFHRL/MPD
Brooks AFB, TX 78235

AFOSR,
Life Sciences Directorate
Bolling Air Force Base
Washington, DC 20332

Dr. William E. Alley
AFHRL/MOT
Brooks AFB, TX 78235

Dr. Earl A. Alluisi
HQ, AFHRL (AFSC)
Brooks AFB, TX 78235

Technical Director, ARI
5001 Eisenhower Avenue
Alexandria, VA 22333

Special Assistant for Projects, OASN(M&RA)
5D800, The Pentagon
Washington, DC 20350

Dr. Meryl S. Baker
Navy Personnel R&D Center
San Diego, CA 92152

Dr. R. Darrell Bock
University of Chicago
Department of Education
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijinstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code N-095R
NAVTRAEEQUIPCEN
Orlando, FL 32813

M.C.S. Jacques Bremond
Centre de Recherches du Service
de Sante des Armees
1 Bis, Rue du
Lieutenant Raoul Batany
92141 Clamart, FRANCE

Dr. Robert Brennan
American College Testing Programs
P. O. Box 168
Iowa City, IA 52243

Mr. James W. Carey
Commandant (G-PTE)
U.S. Coast Guard
2100 Second Street, S.W.
Washington, DC 20593

Dr. James Carlson
American College Testing Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd.
Chapel Hill, NC 27514

Dr. Robert Carroll
NAVOP 01B7
Washington, DC 20370

Mr. Raymond E. Christal
AFHRL/MOE
Brooks AFB, TX 78235

Director,
Manpower Support and Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Chief of Naval Education and Training
Liaison Office
Air Force Human Resource Laboratory
Operations Training Division
Williams AFB, AZ 85224
Assistant Chief of Staff
for Research, Development,
Test, and Evaluation
Naval Education and
Training Command (N-5)
NAS Pensacola, FL 32508

Dr. Stanley Collyer
Office of Naval Technology
800 N. Quincy Street
Arlington, VA 22217

Dr. Lee Cronbach
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Atherton, CA 94205

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CDR Mike Curran
Office of Naval Research
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Arlington, VA 22217-5000

Dr. Dattprasad Divgi
Syracuse University
Department of Psychology
Syracuse, NY 13210

Dr. Hei-Ki Dong
Ball Foundation
800 Roosevelt Road
Building C, Suite 206
Glen Ellyn, IL 60137

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Richard Elster
Deputy Assistant Secretary
of the Navy (Manpower)
OASN (M&RA)
Department of the Navy
Washington, DC 20350-1000

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. Marshall J. Farr
2520 North Vernon Street
Arlington, VA 22207

Dr. Pat Federico
Code 511
NPRDC
San Diego, CA 92152

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P.O. Box 168
Iowa City, IA 52240
Dr. Myron Fischl
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Alfred R. Fregly
AFOSR/NL
Bolling AFB, DC 20332

Dr. Bob Frey
Commandant (G-P-1/2)
USCG HQ
Washington, DC 20593

Dr. Robert D. Gibbons
University of Illinois-Chicago
P.O. Box 6998
Chicago, IL 60680

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Gene L. Gloye
Office of Naval Research
Detachment
1030 E. Green Street
Pasadena, CA 91106-2485

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

H. William Greenup
Education Advisor (E031)
Education Center, MCDEC
Quantico, VA 22134

Dr. Ronald K. Hambleton
Laboratory of Psychometric and
Evaluative Research
University of Massachusetts
Amherst, MA 01003

Dr. Ray Hannapel
Scientific and Engineering
Personnel and Education
National Science Foundation
Washington, DC 20550

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 62
San Diego, CA 92152

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 90010

Mr. Dick Hoshaw
NAVOP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Earl Hunt
Department of Psychology
University of Washington
Seattle, WA 98105

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Douglas H. Jones
Advanced Statistical
Technologies Corporation
10 Trafalgar Court
Lawrenceville, NJ 08148
Dr. G. Gage Kingsbury  
Portland Public Schools  
Research and Evaluation Department  
501 North Dixon Street  
P. O. Box 3107  
Portland, OR 97209-3107  

Dr. William Koch  
University of Texas-Austin  
Measurement and Evaluation Center  
Austin, TX 78703  

Dr. Leonard Kroeker  
Navy Personnel R&D Center  
San Diego, CA 92152  

Dr. Patrick Kyllonen  
AFHRL/ME  
Brooks AFB, TX 78235  

Dr. Anita Lancaster  
Accession Policy  
OASD/MI&L/MP&FM/AP  
Pentagon  
Washington, DC 20301  

Dr. Daryll Lang  
Navy Personnel R&D Center  
San Diego, CA 92152  

Dr. Michael Levine  
Educational Psychology  
210 Education Bldg.  
University of Illinois  
Champaign, IL 61801  

Dr. Charles Lewis  
Faculteit Sociale Wetenschappen  
Rijksuniversiteit Groningen  
Oude Boteringestraat 23  
9712GC Groningen  
The NETHERLANDS  

Science and Technology Division  
Library of Congress  
Washington, DC 20540  

Dr. Robert Linn  
College of Education  
University of Illinois  
Urbana, IL 61801  

Dr. Robert Lockman  
Center for Naval Analysis  
200 North Beauregard St.  
Alexandria, VA 22311  

Dr. Frederic M. Lord  
Educational Testing Service  
Princeton, NJ 08541  

Dr. William L. Maloy  
Chief of Naval Education and Training  
Naval Air Station  
Pensacola, FL 32508  

Dr. Gary Marco  
Stop 31-E  
Educational Testing Service  
Princeton, NJ 08451  

Dr. Kneale Marshall  
Operations Research Department  
Naval Post Graduate School  
Monterey, CA 93940  

Dr. Clessen Martin  
Army Research Institute  
5001 Eisenhower Blvd.  
Alexandria, VA 22333  

Dr. James McBride  
Psychological Corporation  
c/o Harcourt, Brace,  
Jovanovich Inc.  
1250 West 6th Street  
San Diego, CA 92101  

Dr. Clarence McCormick  
HQ, MEPCOM  
MEPCT-P  
2500 Green Bay Road  
North Chicago, IL 60064  

Mr. Robert McKinley  
University of Toledo  
Department of Educational Psychology  
Toledo, OH 43606  

Dr. Barbara Means  
Human Resources  
Research Organization  
1100 South Washington  
Alexandria, VA 22314  

University of Iowa/Novick NR 150-521
Assistant for Personnel
Logistics Planning,
NAVOP 987H
50772, The Pentagon
Washington, DC 20350

Leadership Management Education
and Training Project Officer,
Naval Medical Command
Code 05C
Washington, DC 20372

Technical Director,
Navy Health Research Ctr.
P.O. Box 85122
San Diego, CA 92138

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Oklahoma City, OK 73069

Dr. William E. Nordbrock
FMC-ADCO Box 25
APO, NY 09710

Dr. Melvin R. Novick
356 Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152

Fleet Support Office,
NPRDC (Code 301)
San Diego, CA 92152

Library, NPRDC
Code P201L
San Diego, CA 92152

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. James Olson
WICAT, Inc.
1875 South State Street
Orem, UT 84057

Director, Technology Programs,
Office of Naval Research
Code 200
800 North Quincy Street
Arlington, VA 22217-5000

Director, Research Programs,
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217-5000

Mathematics Group,
Office of Naval Research
Code 41MA
800 North Quincy Street
Arlington, VA 22217-5000

Office of Naval Research,
Code 442
800 N. Quincy St.
Arlington, VA 22217-5000

Office of Naval Research,
Code 442EP
800 N. Quincy Street
Arlington, VA 22217-5000

Group Psychology Program,
ONR Code 442GP
800 N. Quincy St.
Arlington, VA 22217-5000

Office of Naval Research,
Code 442PT
800 N. Quincy Street
Arlington, VA 22217-5000

(6 Copies)

Psychologist
Office of Naval Research
Branch Office, London
Box 39
FPO New York, NY 09510
Special Assistant for Marine Corps Matters,
ONR Code 100M
800 N. Quincy St.
Arlington, VA 22217-5000

Psychologist
Office of Naval Research
Liaison Office, Far East
APO San Francisco, CA 96503

Dr. Judith Orasanu
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dr. James W. Pelligrino
University of California,
Santa Barbara
Department of Psychology
Santa Barbara, CA 93106

Military Assistant for Training and Personnel Technology,
OUSD (R & E)
Room 3D129, The Pentagon
Washington, DC 20301

Administrative Sciences Department,
Naval Postgraduate School
Monterey, CA 93940

Department of Computer Science,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MP
Brooks AFB, TX 78235

Dr. Bernard Rimland
Navy Personnel R&D Center
San Diego, CA 92152

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Funiko Samejima
Department of Psychology
University of Tennessee
Knoxville, TN 37916

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152

Dr. Robert Sasmor
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Lowell Schoer
Psychological & Quantitative Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MR&M)
2B269 The Pentagon
Washington, DC 20301
Dr. Joyce Shields  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Dr. Kazuo Shigemasu  
7-9-24 Kugenuma-Kaigan  
Fujusawa 251  
JAPAN

Dr. William Sims  
Center for Naval Analysis  
200 North Beauregard Street  
Alexandria, VA 22311

Dr. H. Wallace Sinaiko  
Manpower Research  
and Advisory Services  
Smithsonian Institution  
801 North Pitt Street  
Alexandria, VA 22314

Dr. A. L. Slafkosky  
Scientific Advisor  
Code RD-1  
HQ U. S. Marine Corps  
Washington, DC 20380

Dr. Alfred F. Smode  
Senior Scientist  
Code 7B  
Naval Training Equipment Center  
Orlando, FL 32813

Dr. Richard Sorensen  
Navy Personnel R&D Center  
San Diego, CA 92152

Dr. Peter Stoloff  
Center for Naval Analysis  
200 North Beauregard Street  
Alexandria, VA 22311

Maj. Bill Strickland  
AF/MPXOA  
4E168 Pentagon  
Washington, DC 20330

Dr. Hariharan Swaminathan  
Laboratory of Psychometric and Evaluation Research  
School of Education  
University of Massachusetts  
Amherst, MA 01003

Mr. Brad Sympson  
Navy Personnel R&D Center  
San Diego, CA 92152

Dr. John Tangney  
AFOSR/NL  
Bolling AFB, DC 20332

Dr. Kikumi Tatsuoka  
CERL  
252 Engineering Research Laboratory  
Urbana, IL 61801

Dr. Maurice Tatsuoka  
220 Education Bldg  
1310 S. Sixth St.  
Champaign, IL 61820

Dr. David Thissen  
Department of Psychology  
University of Kansas  
Lawrence, KS 66044

Dr. Ledyard Tucker  
University of Illinois  
Department of Psychology  
603 E. Daniel Street  
Champaign, IL 61820

Dr. James Tweeddale  
Technical Director  
Navy Personnel R&D Center  
San Diego, CA 92152

Dr. Vern W. Urry  
Personnel R&D Center  
Office of Personnel Management  
1900 E. Street, NW  
Washington, DC 20415

Headquarters, U. S. Marine Corps  
Code MPI-20  
Washington, DC 20380
Dr. Frank Vicino  
Navy Personnel R&D Center  
San Diego, CA 92152

Dr. Howard Wainer  
Division of Psychological Studies  
Educational Testing Service  
Princeton, NJ 08541

Dr. Ming-Mei Wang  
Lindquist Center  
for Measurement  
University of Iowa  
Iowa City, IA 52242

Mr. Thomas A. Warm  
Coast Guard Institute  
P. O. Substation 18  
Oklahoma City, OK 73169

Dr. Brian Waters  
Program Manager  
Manpower Analysis Program  
HumRRO  
1100 S. Washington St.  
Alexandria, VA 22314

Dr. David J. Weiss  
N660 Elliott Hall  
University of Minnesota  
75 E. River Road  
Minneapolis, MN 55455

Dr. Ronald A. Weitzman  
NPS, Code 54Wz  
Monterey, CA 92152

Major John Welsh  
AFHRL/MAH  
Brooks AFB, TX 78223

Dr. Rand R. Wilcox  
University of Southern California  
Department of Psychology  
Los Angeles, CA 90007

German Military Representative  
ATTN: Wolfgang Wildegrube  
Streitkraefstamt  
D-5300 Bonn 2  
4000 Brandywine Street, NW  
Washington, DC 20016

Dr. Bruce Williams  
Department of Educational Psychology  
University of Illinois  
Urbana, IL 61801

Dr. Hilda Wing  
Army Research Institute  
5001 Eisenhower Ave.  
Alexandria, VA 22333

Dr. Martin F. Wiskoff  
Navy Personnel R & D Center  
San Diego, CA 92152

Mr. John H. Wolfe  
Navy Personnel R&D Center  
San Diego, CA 92152

Dr. Wendy Yen  
CTB/McGraw Hill  
Del Monte Research Park  
Monterey, CA 93940