TO: L. Lustik, Dr. Weiss, Naval Biodynamics Laboratory

FROM: Paul Ibanez, ANCO Engineers, Inc. Culver City, Calif. 90232

DATE: 3 January 1986

RE: "Radiographic Determination of Mass of Inertial Tensors of Anatomical Segments", Progress Reports 1-2, 0001AA, 0001AB

1.0 INTRODUCTION

ANCO is investigating the use of radiographic techniques to determine properties of anatomical segments - mass, center of gravity, and the inertial tensor. As shown in our proposal, three orthogonal projections (x-ray photographs) through the segment will determine these properties if sufficiently high energy x-rays are used so that attenuation is independent of atomic number and just dependent on mass density. This occurs above a few hundred keV energy.

The intent of this study is to investigate the practicality and potential accuracy of the method. Progress to date is described herein. All work will be performed by the end of February and the work plan is described here as well.

2.0 X-RAY SOURCE

The decision must be made as to the source of the x-rays, either an x-ray tube can be used or an x-ray (gamma ray) source. Most commercial x-ray tubes operate below several hundred keV. Special industrial x-ray tubes can reach several MeV. All tubes provide a distributed energy source containing energies above and below their specified energy (the bulk being below).

X-ray (vs. gamma ray) sources can provide single energy x-rays at high energies. Examples include Cesium 137 at 0.66 MeV (33 year half life, 0.39 r/hr/Curie at 1 meter dosage) and cobalt 60 (5.3 year half life, 1.35 r/hr/Curie).
Our studies indicate that a radioactive source is preferable to an x-ray tube for the following reasons:

- The source can have higher energy than the more commonly available tubes.
- The source energy is monochromatic (single energy) while the tube has much radiation at lower energies where absorption is atomic number dependent. This is true even for high energy tubes.
- Sources are less expensive and smaller than tubes, making the exposure set up less expensive and simpler. Also the smaller sources can be placed closer to the subject without blurring due to source size and are easier to shield.
- The source is more easily shielded than is a machine because of the small size of the source.

3.0 EXPOSURE TIME AND DOSE

Preliminary calculations indicate, for example, for Agfa D-7 film (with 5 mg Pb screens and X10 intensifier) and the source placed 1 meter from the segment and screen, the following exposure time and doses would be required (the source strength below one of those currently available to us and could, of course, be varied in future work).

<table>
<thead>
<tr>
<th>Source</th>
<th>Strength</th>
<th>Exposure Time (min)</th>
<th>Dose (roentgens)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cs137</td>
<td>.146 Ci</td>
<td>343</td>
<td>2.2</td>
</tr>
<tr>
<td>Co60</td>
<td>.80 Ci</td>
<td>26</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The exposure time can be reduced by placing the source closer to the object or increasing source strength (dose will not be significantly changed). Thus, for example, use of a one curie source at 1/2 meter would result in an exposure time of 12.5 minutes for Cs137 and 5.2 minutes for Co60. Both this distance and source strength are practical. As several exposures (typically 3) must be made.
we are looking at total exposure times of 15 - 40 minutes. It appears feasible to hold an anesthetized animal still for this length of time. A dose of 2-7 roentgens should not cause the animal harm on a one or two time basis. It is possible that alternate films may further reduce these times and dosages. Larger sources could be used to reduce exposure time.

4.0 EXPOSURE CONFIGURATION

As anticipated, multiple exposures are desired. The issues to be considered are:

- Can non-orthogonal views be used?
- Can more than 3 views be useful?
- Can exposure be made simultaneously using multiple sources?
- How can the anatomical segment be "spotted" to establish a coordinate reference?

The first three points above have a common solution. Non-orthogonal views are useful so as to better isolate a given segment. For example, the head is connected to the body by the neck. If the mass properties are desired above a given reference (say a specified vertebrae) and if one could sever the head at that point and "float" it in space, one could easily take 3 orthogonal views, each encompassing the entire head. Since we wish to do non-destructive testing, this can not be done. Orthogonal views of an unsevered head will result in some non-coverage and error, as illustrated in Figure 1.

Non-orthogonal views, as shown in Figure 1, reduce this error. The error could be reduced to zero by having the views in the same horizontal plane. However they would no longer be independent and this would lead to an inability to estimate certain mass properties. (Remember that, as shown
ORTHOGONAL VIEWS OF A SEVERED HEAD HAVE COMPLETE COVERAGE

ORTHOGONAL VIEWS OF AN ATTACHED HEAD CAN INTRODUCE ERRORS

Possible effect of body

NON ORTHOGONAL VIEWS REDUCE THIS ERROR

(for simplicity these examples assume one grit two dimensional)
in our proposal, two orthogonal views in a horizontal plane could determine mass, center of gravity, and all but the off-diagonal terms of the inertia tensor. Remember also that if many views, each differing by a small angle, were taken in a horizontal plane, we would have a "CAT" scan and could determine the density distribution and hence all properties. We are however trying to determine all properties with a few scans.) Hence some angle, however small, must exist between all scans. Multiple scans produce redundancy and help to make up for almost dependent scans and are consequently potentially useful.

In addition to these considerations, we must concern ourselves with beam spreading (parallax, fan beam) as we plan to use a source close to the segment. A source closer than about 2 meters will have a significant non-parallel beam. Non-orthogonal views, multiple views (more than 3) and beam speed all introduce transformations of the data that must be accounted for. Fortunately there is a simple technique for doing so.

Assume that any number of views are taken in any number of orientations. The views are discretely converted to give a vector \( R \) of measurements. For example, if 5 views were taken and a 20x20 cm grid used for each, then there would be 5x20x20 or 2000 measurements, and \( R \) would be a vector of length 2000. Assume also that in a convenient orthogonal coordinate system, the space containing the segment is discretely converted into cells of assumed uniform density \( P \). Thus if the space is 20x20x20 cm, we can define a vector \( P \) of size 8000. Depending on the geometry of the views and the spreading ray path, each element of \( R \) is linearly dependent on some (in fact a very few) of the elements of \( P \).

\[
R = TP
\]
where \( T \) is a large \((2000 \times 8000)\) matrix with most terms equal to zero. The \( P \) can not be solved for as there are too few equations \((2000)\) for the unknowns \((8000)\) and, further, all the equations may not represent linearly independent data. We do not, however, wish to know \( P \) but rather certain linear combinations of \( P \) \( (\text{i.e. mass, center of gravity, inertia tensor - see equations 1-4 of our proposal}) \). For example the elements of the inertia tensor can be placed in a six component vector \( J \) and equation 3 and 4 of our proposal written as

\[
J = UP
\]

where, for example \( U \) is a \( 6 \times 8000 \) matrix. For the moment, assume that the matrix \( T \) can be inverted and call this inversion \( TI \). Then

\[
P = TIR
\]

and

\[
J = UP
\]

or

\[
J = UTIR
\]

where \( UTI \) is a \( 6 \times 2000 \) matrix. We would in fact have an over constrained system \( (\text{more equations than unknowns}) \). Note that \( UTI \), if it existed, is only a property of the geometry of the sources and views and is independent of the properties of the segment under study.

This form of problem occurs frequently in various processing and estimation tasks. In fact a matrix \( TI \) can be defined which has many of the properties of the \((\text{non-existent})\) inverse of \( T \). It is called the Pseudo Inverse \( (\text{also the Penrose inverse, or singular value decomposition inverse}) \). Without further elaboration here,
note that \( J = UTIR \) should provide a good estimate of the inertial properties. We know that in an orthogonal parallel scan, the answer is exact (as provided in our proposal) and feel that non-orthogonal scans with spreading beams should also yield a good estimate (as long as the angles are not too far from 90 degrees) and that multiple views (beyond 3) can help. Thus the pseudo inverse allows solution in one formalism, accounting for

- Non-orthogonal views
- Spreading beams
- Redundant views (in the limit - a CAT Scan)

The evaluation of UTI, while a lengthy computation, needs only be done once and will not change once a view geometry is chosen.

These are two other issues mentioned above - simultaneous exposure and spotting for coordinate reference. In principal all views could be exposed at once, thus reducing the time the animal must be held still (anesthetized). There is no mathematical reason why the data could not be analyzed even if one view received direct exposure from more than one source. However, the effect of scattering would be greater in such a situation. This is because with one source, most of the scattered radiation is directed away from the film. With multiple simultaneous sources and views, most of the scattered radiation would fall upon one of the views. This effect will be quantified but will probably suggest against simultaneous exposure. Note that if used, simultaneous exposures reduce exposure time but not dosage.

The spotting of a coordinate reference is required. Our idea is to place a small dense object (ball bearings) at known points on the segment (e.g. vertebrae, top of head,
temples, etc.) and then locate these in each of the view exposures. The data reduction computer program would then relate these to the coordinate system in which the mass properties are defined. The weight of the objects would have little effect on the calculated mass properties.

5.0 ACCURACY

A primary goal is to determine the potential accuracy of this technique. Note that researchers using a CAT scanner have, at great cost, produced results with a 5% accuracy. It is unlikely that the accuracy of our few scan techniques would be better.

There are many factors affecting accuracy.

- Discretization - A coarser grid will give greater error than a finer grid. We are tentatively working with a 1 cm grid. Computer simulations will be used to suggest the relative errors between a 1 cm and 2 cm grid.

- Attenuation sensitivity - Even at several hundred keV and higher, there is some variation in attenuation that depends on atomic number (not just density). Computer simulations will also be used to suggest the effect of this error for typical variations.

- Scattering - Not all the radiation is either passed or absorbed. Some is scattered. If the scattered radiation did not reach the film plate, it could be combined with the absorbed radiation for an effective absorption. However, some of the scattered radiation may reach the film, perhaps after multiple scattering and perhaps downgraded in energy. Our initial computations suggest that about 10% of the radiation reaching the film may be scattered radiation. Moving the film away from the source and segment can reduce this fraction. It is not yet clear how much error this 10% will introduce but it is felt to be less than 10%. The experiments planned should suggest the magnitude of the error.

- Calibration and Dynamic Range - All films have certain sensitivity and dynamic range. The sensitivities are non-linear functions of exposure.
and have a threshold and saturation. Thus too small an exposure will produce a zero signal regardless of variation of the density. Too large an exposure will produce a "100%" signal regardless of the variations of density. A film and exposure time must be chosen to produce a result in the quantitative region of sensitivity and the resulting radiograph must be capable of being quantitatively read.

Initial simulations suggest that the attenuation of the beam will vary between 0% and 90% (leaving 100% to 10% of the beam). Thus a dynamic range of about 10 is required. The maximum attenuation is expected to equal about 1" of steel (the approximate equivalent path density of a ray through the head). To provide calibration of the film, a 1" steel and 1" aluminum step wedge have been constructed with 1/6" steps. As the aluminum is about 1/3 as dense as the steel, the steps provide a 24 fold dynamic range calibration in the region of interest. These wedges will be used in every view of exposure. The data reduction photodensitometer will read the wedge radiographs for each view to provide a direct density calibration. It is felt that the calibration and dynamic range error can be kept below 10%. Variation of film sensitivity across a single piece of film is felt to be negligible.

- Reading error - a photodensitometer has been constructed and appears to give repeatable results to within about 5%. Its accuracy will be further evaluated based on our planned experiments.

- Photon statistics - a radioactive source does produce a time and space varying radiation field. As we are calibrating every view, the time canceled out. There is the possibility of spatial variations but as we are dealing with a very large number of photons, this effect is negligible.

- Distributed source - the distributed source problem is small as the sources in mind are less than 1 cm in size (less than the grid size). This effort is felt to be negligible, but will be quantified.

- Slicing error - As discussed earlier, the radiography of any segment that is still connected to the rest of the body will cause some error due to non-overlap of the various views at the interface between the segment and the rest of the body. It is felt that by use of shallow angle views (but not zero angle), this effect can be kept below 5%. This will be quantified.
6.0 PHOTODENSITOMETER

A photodensitometer has been constructed. It allows sweeping a photocell over a radiograph. The position of the photocell is measured using two orthogonal CeleSCO lanyard transducers (.005" accuracy). The transducers have +5 volt output for +10" travel. The photocell reads light intensity from a reflected light source next to the photocell. The area of illumination and sensing is approximately a 1 cm diameter circle. The photocell output is 0 to 5 volts with adjustable sensitivity.

The three signals generated (two coordinates and one intensity) are continuously read by an IBM-PC based digital data acquisition program, ANFILM. Each 1/10 second, the data is sampled and averaged. The intensity is then assigned to the geometric grid point given by the two measured coordinates. The value is also displayed on a CRT screen grid. Thus the photocell can be swept by hand over the film at random without regard to grid lines to "fill in" the picture. Moving slowly in significant areas results in multiple measurements and averaging of the same grid point, hence improving accuracy. Once the grid is filled in, it is written in a standard data set for future processing (called an "R" data set).

ANFILM also allows positioning of the photocell on the radiographic wedge steps for calibration. Thus the photocell readings are directly and immediately converted to actual mass densities, cancelling out many potential errors.

ANFILM also allows positioning on the "spot" points on the segment so as to establish their exact relation to the measurement coordinate system.

7.0 SIMULATION PROGRAMS

Two other programs, ANRAY and ANSEG, have been designed and are about to be used. ANRAY has the purpose of solving
the equations of section 4.1.2 of our proposal. Thus, given three "R" data sets, ANRAY calculates the corresponding mass and inertial properties. (Initially for Phase I, we are assuming 3 orthogonal views and parallel beams.)

The second program ANSEG, allows the user to arbitrarily define the mass density of a segment in a 3 dimensional space grid. It then calculates the mass and inertial properties and the 3 resulting "R" data sets. This program will be used in the simulations to evaluate grid size error, absorption variation (with atomic number) error, and dynamic range requirements. Thus the various "R" sets from ANSEG will be analyzed by ANRAY to see how well the mass and inertial properties can be reconstructed after various error sources are introduced.

8.0 PLANNED EXPERIMENTS

A series of film exposures are planned to establish exposures, dynamic range, and accuracies. The films will be exposed to both low energy (80 keV) and higher energy (660 keV C137) radiation (for purposes of comparison). In all exposures, the steel and aluminum stepped wedges will be used. Two phantoms will be employed. One, for qualitative evaluation, is an approximately 8"x6"x6" plastic container filled with water and selected objects (spotting ball bearings and animal bones). Its radiograph will be reviewed to determine correct exposure and dynamic range. It is meant to roughly simulate a primate head. The second phantom is a 6" homogeneous plastic sphere. Its radiograph will be used to quantitatively evaluate the ability of the data reduction procedure to estimate mass and inertial properties.
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