IMPROVEMENT OF MECHANICAL MODEL IN THE ENTRAINMENT METHOD FOR COMPRESSIBLE
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IMPROVEMENT OF MECHANICAL MODEL IN THE ENTRAINMENT METHOD FOR COMPRESSIBLE TURBULENT BOUNDARY LAYER

by

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HUMAN TRANSLATION

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This paper presents an improved entrainment method for the two-dimensional compressible turbulent boundary layer. This method is simple with short computer time and can be used for the development of calculations for the turbulent boundary layer behind a shock wave. Calculated results from six examples indicate that this method has excellent numerical accuracy. When compared with P. D. Smith's method, the results show a marked improvement.
1. Preface

The calculations of turbulent boundary layers at aircraft wings for the modification of flow field of transonic nonviscous flow is indispensable. Although many differential methods have been developed abroad, certain practical transonic computer programs (e.g., the three NASA full potential programs: FL-023, FL-027 and FL-030) still utilize the integral method as the counterpart. This is because the integral method is cost effective, convenient to use, and, at least at present, not inferior to the differential method in accuracy. Currently, the entrainment method appears to be the only integral method that can be extended into the application of three-dimensional aircraft wings. This method was developed from the basis of M. R. Head's mechanical model by P. D. Smith[1]. Based on our calculating experience, the accuracy for certain examples using the mechanical model of P. D. Smith is not, even for two-dimensional condition, satisfactory. This paper attempts to improve this method. Since examples used for evaluating accuracy of the three-dimensional turbulent boundary layer are extremely limited, this paper only validates the effects of the improved method for two-dimensional conditions.

M. R. Head and V. C. Patel were the first to present[2] the method of taking the effects of the non-equilibrium boundary layer into consideration to improve the mechanical model of the entrainment method. Their method only applied to a two-dimensional incompressible flow. In 1973, J. E. Green, et al developed the lay entrainment method[3]; i.e., to use one differential equation to determine the entrainment coefficient $C_E$, and thereby taking the upstream effects during entrainment into account. This method has been extensively applied abroad, yet so far no reports have been made public about extending it into the calculations for three-dimensional boundary layers. This paper adopts another approach and attempts to extend the improved method of Head and Patel to compressible boundary layer conditions. A method which takes the lag effects of the pressure...
gradient into consideration to improve the calculations for the entrainment coefficient under a strong adverse pressure gradient is also presented in order to calculate the development of the turbulent boundary layer behind a shock wave. The results of six examples indicate that the improvement effects of this method are not inferior to Green's method and that the calculation accuracy is also not inferior to a complicated differential method.

2. General Description of Method

The two-dimensional boundary layer entrainment method must solve the momentum integral equation and entrainment equation simultaneously, and they are

\[
\frac{d\theta_{11}}{dx} + \theta_{11}L(2 + H - M^2) = C_f/2
\] (1)

\[
\frac{d\theta_{11}}{dx} + \theta_{11}H(1 - M^2) = C_E
\] (2)

where \( \theta_{11} \) is the momentum thickness; \( C_E \) is the entrainment coefficient; \( C_f \) is the wall surface friction coefficient; \( H \) and \( H_1 \) are the shape factor and entrainment shape factor; \( L \) is the pressure gradient parameter: \( L = 1 - \frac{d\theta}{dx} \). and \( Me \) are the velocity outside boundary layer and Mach Number, respectively. Equations (1) and (2) can be closed by substituting several empirical and semi-empirical formulae related to the above parameters. The calculated results of the boundary layer are greatly affected by how \( C_E \) is determined. In P. D. Smith's method, \( C_E \) is completely dependent upon the local boundary layer's total parameters, and thus the history of upstream effects of turbulent flow cannot be considered. Based on the thoughts of Head and Patel, this paper expresses \( C_E \) as

\[ C_E = C_{E\text{eq}} F(r) \] (3)

where \( C_{E\text{eq}} \) represents the \( C_E \) value of the boundary layer under local equilibrium conditions; \( F(r) \) represents the non-equilibrium factor which shows the deviation situation of \( C_E \) under equilibrium conditions; \( r \) is the characteristic parameter. Head and Patel pointed out
where

\[
F(r) = \begin{cases} 
(5 - 4r)/(3 - 2r) & r < 1 \\
1/(2r - 1) & r \geq 1 
\end{cases}
\]  

(5)

Morkovin [5] assumed that turbulent flow structure is not affected by the Mach Number as long as the Mach Number based on typical pulse velocity is rather small. Therefore, we can regard \( F(r) \) as unchanged for compressible flow and only rewrite \( r \) as

\[
r = \frac{1}{\rho_e} \frac{d(p \cdot p, \theta u)}{dx} / \left[ \frac{1}{\rho,} \frac{d(p \cdot p, \theta u)}{dx} \right] 
\]

(6)

(here \( \rho_e \) is the flow density outside the boundary layer). The equilibrium formula can be rewritten as

\[
G = (4.8285(x + 1.0717)^{1/2} + 1.0438(1 + 0.04Me)^{1/2}) 
\]

(7)

where

\[
G = (\bar{H} - 1) / \sqrt{\frac{1}{C'}} \\
\bar{x} = -(2H/C')(L_{th})
\]

(8)

(9)

Here \( \bar{H} \) is the conversion shape factor. Utilizing equations (1), (2) and (6) to (9) as well as the formulae [1] for \( H, H_1 \) and \( \bar{H} \), it is not difficult to solve for \( r \) and \( C_{eq} \), and therefore \( C_E \) from equations (3) and (5).

When a shock wave is present on the wing surface, the effects of a pressure jump across the shock wave will sustain far into regions behind the wave. Therefore, in order to take these effects into account, it is appropriate to substitute \( L \) in equation (6) with the lag-pressure gradient parameter \( L_1 \). Inspired by Reference [6] and applying the functional form of lag model's weighting function therein, the expression for \( L_1 \) we substituted was

\[
L_1 = \frac{1}{\sigma} \int_{z_o}^{z} L(x_i) \exp(-{(x - x_i)^3/\lambda^3(H_1^3)})dx_i 
\]

(10)

where

\[
\sigma = \int_{z_o}^{z} \exp(-{(x - x_i)^3/\lambda^3(H_1^3)})dx_i 
\]

(11)
\( x_D \) represents the starting point of the shock wave; \( \lambda \) is a constant associated with the pressure jump value \( \Delta C_p \), and based upon experience it can be expressed as

\[
\lambda = 183\Delta C_p \tag{12}
\]

3. Calculated Results

Six examples of a profile with a compressible turbulent boundary layer Mach Number range of subsonic and transonic flows were calculated using the present method. The pressure distribution used in the calculations were measured in a wind tunnel.

Examples 1 and 2 are the boundary layers for upper and lower surfaces of RAE 2814 profile. In Figs. 1 and 2 the calculated results are compared with experimental results \((M=0.725, Re=1.5 \times 10^6)\). It can be observed from the figures that the results of P. D. Smith's method and those of the experiments for the upper surface of the profile differ little. They, however, differ very much for the lower surface of the profile. Results of this method show good agreement with experimental data for both upper and lower surfaces. For comparison purposes, calculated results of Green's method are also given in Figs. 1 and 2. It can be observed that, for these two examples, the improvement effects of Green's method is inferior to the present method and its initial values for the upper surface of the profile show greater deviation from the experimental points. Moreover, Green's method is more complicated than the present method and consequently, computation time is longer. The experimental results for this example are from Reference [3].
Fig. 1. Calculated results of $C_f$ for upper and lower surfaces of RAE 2814 profile $M_\infty = 0.725$, $Re=1.5\times10^7$
Key: (1) Experiment; (2) Smith's method; (3) Present method; (4) Green's method; (5) lower surface; (6) upper surface.

Fig. 2. Calculated results of $R$ for upper and lower surfaces of RAE 2814 profile $M_\infty = 0.725$, $Re=1.5\times10^7$
Key: (1) Experiment; (2) Smith's method; (3) Present method; (4) Green's method; (5) upper surface; (6) lower surface.
Figure 3 shows the two sets of calculated results for the upper and lower surfaces of RAE 2822 profile using the present method and P. D. Smith's method: (a) $M_\infty=0.676$, $Re=5.7\times10^6$, $\alpha=-2.18^\circ$; (b) $M_\infty=0.73$, $Re=6.5\times10^6$, $\alpha=3.19^\circ$. For condition (a), no shock wave is present on the wing surface and data from the present method, P. D. Smith's method and the experiments are in good agreement. For condition (b), a shock wave is present on the wing surface. The calculated results of $C_f$ and $H$ using Smith's method show marked deviation in the region immediately following the shock wave, whereas those using the present method are basically consistent with the experimental results, as each $H$ displays a peak value following the shock wave. The experimental results for this example are from Reference [4].

![Figure 3](image_url)

Fig. 3. Calculated results for upper surface of RAE 2822 profile
(a) $M_\infty=0.676$, $\alpha=-2.18^\circ$, $Re=5.7\times10^6$; (b) $M_\infty=0.73$, $\alpha=3.19^\circ$, $Re=6.5\times10^6$. Key: (1) Smith's method; (2) Present method; (3) Experiment.
Figure 4 is the calculated results for the low speed boundary layer of the NACA-65(216)-222 profile. The attack angle $\alpha = 10.1^\circ$. The calculated results using the present method and the Reynold's stress equation as well as the experimental data are compared. Even though the Reynold's stress equation is more stringent in theory than the present method, its computation time is much longer and the calculated results are not necessarily better than those from the present method. Except for points near the separation point, the results of the present method are very consistent with the experimental data. Both theoretical methods predicted the separation trend after $\frac{x}{c} > 0.55$.

![Graph](image)

**Fig. 4. Calculated results for upper surface of NACA-65(216)-222 profile $\alpha = 10.1^\circ$, Re=2.6x10^6**

Key: (1) Reynold's stress equation method; (2) Experiment; (3) Present method.

Figure 5 gives the calculated results for the upper surface of the super critical profile DSMA-523. The calculation conditions are: $M_{\infty}=0.83$, $Re=3x10^6$, $C_y=0.54$. A shock wave is also present on the wing surface. The calculated results of this example are not as good
as those of the previous five examples, and this is related to the characteristics of the super critical profile. The results of T. Cebeci's differential method are also given in the figure. Though this method consumes for more computation time than the present method, its calculated results are not necessarily more accurate than those of the present method. Especially for the shape factor H this method fails to show the peaks which appeared following the shock wave and the rising trend near the tail end.

![Figure 5](image)

**Fig. 5.** Calculated results for the upper surface of DSMA-523 profile. $M_\infty=0.83$, $Re=3\times10^6$, $C_\infty=0.54$

Key: (1) Present method; (2) Cebeci's method; (3) Experiment.

4. Conclusions

This paper presents an improved entrainment method for calculating a two-dimensional compressible turbulent boundary layer. When there is a strong adverse pressure gradient on the wing surface, especially when a shock wave is present, the lag effects of the pressure gradient near the shock wave are taking into account and rather satisfactory calculation results can still be obtained. Calculated results from
examples of six profile boundary layers show that the accuracy of the present method compares favorably with those of some of the outstanding calculation methods for a turbulent boundary layer. Also, the present method is simple and fast and easier to extend to three-dimensional cases. The mechanical model of the present method has currently been introduced into the calculation of a three-dimensional turbulent boundary layer on aircraft wings.

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LITERATURE


(A new improved entrainment method for calculation of compressible turbulent boundary layer is suggested, in which the effect of non-equilibrium turbulent boundary layer on determining entrainment coefficient is taken into account, and a kind of lag-pressure gradient parameter is introduced into calculation of flow field with strong adverse pressure gradient, such as that near shock waves. For evaluation and validation of the method six numerical examples are presented, and the results show good agreement with experimental data. Moreover, in comparison with other theoretical methods the present method is as good as J. E. Green's method, T. Cebeci's method and Reynolds's stress equation method, but more efficient in computation, and superior to P. D. Smith's method in approaching to the experimental data. The method is also extended to three-dimensional cases.)
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