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STATISTICAL MODEL FOR LASER SAFETY ANALYSIS

J.A. Hermann

Approved for Public Release

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Statistical model for laser safety analysis

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STATISTICAL MODEL FOR LASER SAFETY ANALYSIS

1. INTRODUCTION

The objective of this study is to investigate a statistically-based model of low-energy pulsed laser propagation within the atmosphere, and to apply it to the determination of ocular hazard distances.

Safety calculations based upon the Australian Laser Safety Standard [1] AS 2211 (1981) are primarily geometrical and deterministic. They attempt to allow for the effects of scintillation within the beam by applying a constant correction factor to the calculated mean irradiance. However, ocular hazard distances determined by this method may be conservatively high, and it is therefore desirable to compare the result with that obtained using a satisfactory model of the turbulence effects.

Procedures for protecting observers exposed to laser radiation within the 100 nm to 1 mm wavelength range are described in AS 2211. The laser hazard is defined in terms of the maximum permissible exposure (MPE); this is the quantity of radiant energy per unit area specified as being safe to the eye. The MPE for nanosecond-pulsed visible radiation given in AS 2211 is $5 \times 10^{-3}$ J/m², while for near infra-red radiation the corresponding value is an order of magnitude higher. These thresholds appear to have been derived from the 1978 paper by Sliney [2].

A nominal ocular hazard distance (NOHD) is described in the AS 2211 as the distance at which the beam irradiance corresponds to the MPE for a given pulse duration. Using the 'top-hat' model of the laser beam irradiance, the expression for NOHD is normally written as

$$\text{NOHD} = \frac{1}{\phi} \sqrt{\frac{E \cdot k_1 \cdot k_2}{1.2 \cdot \pi \cdot \text{MPE}}}$$

(1)

where $E$ is the laser pulse energy, $\phi$ is the half-angle beam divergence, $k_1$ is the beam homogeneity factor ($>1$), and $k_2$ is the lenticulation safety factor ($>1$). The value $k_2 = 10$ is recommended in AS 2211 as a suitable safety margin for calculating NOHD when atmospheric turbulence is present.
The model developed in this report is statistical, in line with previous US and UK work involving laser scintillations applied to safety clearance determinations [2-6]. An interactive computer program has been developed so that a range of input parameter options can be made available for investigating the physical processes attending beam propagation.

2. THE PROPAGATION MODEL

It is normally assumed in laser propagation studies that atmospheric attenuation, attributable to Rayleigh and Mie scattering and to water-vapor absorption, follows the exponential Bouguer law [7]. The extinction coefficient for a particular wavelength can be determined using Koschmieder's formula [7] for the 0.55 µm wavelength:

$$a_{0.55} = \frac{3.912}{V}$$

(2)

where $V$ is the prescribed visibility (in km). Extinction coefficients at other wavelengths $\lambda$ may be obtained by using eqn (2) in conjunction with an interpolation procedure applied to empirical data. Turbulent refractive effects associated with atmospheric propagation are often expressed by means of the refractive index structure constant $C_n^2$ [8,9] which exhibits a height-dependence often expressed in the form

$$C_n^2(z) = C_n^2(z_0)(z/z_0)^{-\beta}$$

(3)

where $z$ is the height above ground level, $z_0$ is a prescribed height (usually 1 m), and $\beta$ is an exponent determined by the environmental conditions (under the conditions considered in this study, it is valid to set $\beta = 1.3$ [8,9]).

The hazard range $D$ (in metres) for a laser aimed at a target can be estimated from a deterministic model which assumes a gaussian irradiance profile at all propagation distances [10]. This profile is characteristic of lasers operating in the lowest transverse mode ($TEM_{00}$). The maximum (central) irradiance of a laser beam is usually considered in safety calculations, since exposed viewers are taken to be located in the most hazardous position within the beam. An effective radius must be used even for irregular beams, hence the adopted "equivalent" gaussian shape provides a beam radius where the irradiance is $1/e$ of the maximum value. A threshold energy $T$ (joules), directly related to the MPE through the relation $T = MPE \times$ (eye-pupil area), can be shown (see the Appendix) to be given by

$$T = k_1k_2 E \exp(-a_0 D)(1 - \exp(-m^2w^2/r_D^2))$$

(4)

where
(a) \[ r_D = \left( (w_0 + D\phi)^2 + n^2\theta^2 \right)^{1/2} \] 

is the laser spot radius at the hazard distance D (metres),

(b) \( w_0 \) is the exit beam radius (metres),

(c) \( \phi \) is the geometric half-angle spread (radians),

(d) \( \theta = \lambda/(2\pi w_0) \) is the half-angle diffraction spread (radians),

(e) \( m \) is the magnification (\( > 1 \)) associated with any extra-ocular optics,

(f) \( w \) is an aperture parameter (metres) for the viewing optics as defined in the Appendix.

In the visible and near-infrared region, diffraction spread is sometimes an order of magnitude smaller than the spread attributable to other effects. On the other hand, with solid-state lasers inhomogeneities in the lasing medium may cause fluctuations in the output irradiance profile, leading to larger diffractive effects. Where it can be established that the laser is operating in the lowest spatial mode, it is normally appropriate to set \( k_1 = 1 \) when using the present model to predict the hazard distance. Equation (4) now can be solved easily for \( D \), using iterative root-finding techniques.

Statistical fluctuations in the irradiance (often called scintillations or lenticulation) occur in response to thermal fluctuations in the atmosphere. These fluctuations are on the order of magnitude smaller than the spread attributable to other effects. The atmospheric scintillation of laser beams poses an additional safety problem to that revealed in a geometrical/deterministic safety analysis. Atmospheric turbulence near the ground can generate "turbulons" (small turbulent eddies) and associated scintillation spots, or "hot spots", which can move in an uncertain way within the laser beam [5,11,12,13,14]. They can travel across the beam quite readily when a cross-wind is present, and are present to the greatest extent when the vertical temperature gradient is large. Clear sunshine, a cloudless sky, and hot ground are all conducive to turbulent effects of this sort. The scintillation fluctuations possess a temporal frequency spectrum ranging as high as several hundred hertz, and tend to increase in size with propagation distance until a range is reached at which "saturation" (leveling out of the intensity variance) occurs.

One of the difficulties in a laser-safety analysis which allows for atmospheric effects lies in the existence of a non-zero risk of subjecting an observer to a localized irradiance far larger than would occur in a non-turbulent environment owing to the possible presence of hot spots. The calculation of such a risk is complicated by the necessity to allow for turbulent beam spreading, scintillation-saturation, atmospheric attenuation, aperture averaging, wind speed, and other effects. It is also necessary to model the fluctuation statistics over a wide range of turbulence conditions. The present model assumes that the irradiance fluctuations obey modified log-normal statistics [15], although the method of calculation can accommodate a range of probability distributions of the irradiance if desired. A log-normal distribution arises when the receiver is located in a turbulent medium and the scattering cone is narrow, so that the received field
is the result of multiplicative effects instead of additive effects. Detailed analysis of this situation is given by Strohbehn et al [15], and references therein. The irradiance statistics used in the model studied in this report differs from a simple log-normal distribution by the inclusion of saturation effects in the log-intensity variance [16-18,19], as well as aperture averaging [12].

In place of the deterministic eqn. (4), we now have to use the statistical counterpart, which differs by virtue of two modifications:

(a) The lenticulation safety factor \( K_2 \) must be replaced with a variable scintillation factor \( F(\sigma^2) \), where the variable \( \sigma^2 \) is a modified log-intensity variance. This variance is ultimately derived from the Tatarski variance for fluctuations of a spherical wave propagating in the atmosphere, \( \sigma^2 \) [8,9] and is given by the integral

\[
\sigma_T^2 = 2.24 \frac{k^7}{6} \int_0^1 C^2_n(u) u^{5/6} (1-u)^{5/6} du \tag{6}
\]

where \( k = 2\pi/\lambda \) and \( u = x/D \), \( x \) being the distance variable. The scintillation factor is defined to be the ratio of the threshold intensity \( I_{th} \) to the mean intensity \( <I> \). Application of the theory of the log-normal distribution [8,15,20] in conjunction with this definition then gives:

\[
F(\sigma_s) = \exp\left(\sigma_s (b - \frac{1}{2} \sigma_s^2)\right) \tag{7}
\]

where

\[
\sigma_s = \left( \ln \left[ 1 - \frac{\ln (1 - \exp(-\sigma_T^2))}{1 + k\frac{D}{\lambda}} \right] \right)^{1/2} \tag{8}
\]

represents the aperture-averaged deviation. Saturation of the variance (which occurs when the turbulence significantly diminishes the beam quality) has been observed experimentally since the late 1960's [16,17,19-21]. An early analysis by de Wolf [18] gives the saturated variance as

\[
\sigma_1^2 = \ln \left( 2 - \exp(-\sigma_T^2) \right),
\]

which shows reasonable agreement with experimental data available at that time. Another more recent approach has been usefully represented by the equation [19]

\[
\sigma_1 = \sigma_T / (1 + \rho^\mu), \tag{9}
\]
the constants $p$ and $\mu$ being determined (for a prescribed wavelength) by empirically fitting relevant experimental data [19] to the assumed expression.

The saturation effect can be interpreted in terms of the relative effect of atmospheric turbulence upon the phase and amplitude of the beam's wavefront. At low $C_n^2$ or short range the effect is almost exclusively a phase aberration, while at higher $C_n^2$ or longer ranges the amplitude fluctuations play the dominant role and the beam tends to spread out and ultimately to break up into a multitude of small regions. The nominal range at which the division between weak and strong turbulence occurs has been given [8, 9] as

$$D_A = 0.969 \lambda^{7/11} (C_n^2)^{-6/11}$$

In eqn. (7) the parameter $b$ appears; this is the upper limit of the integral representing the probability of exceeding the prescribed threshold, $Q$:

$$Q = 1 - (2\pi)^{-1/2} \int_{-\infty}^{b} \exp \left( -\frac{1}{2} t^2 \right) dt.$$ \hspace{1cm} (10)

Equations (7) and (10) both emerge in the theory of the log-normal distribution. Series expansions for the inverse probability function $b(Q)$, with good convergence properties, are also available [22]. Note that large (small) values of $Q(b)$ correspond with values for the scintillation factor which are less than (greater than) unity. This tautology follows directly from eqns. (7) and (10), and may be expressed in the following way:

$$F(\sigma_g) = I_{th}/\langle I \rangle \leq 1 \longleftrightarrow Q(b) \leq Q(\frac{1}{2} \sigma_g).$$

(b) In place of the expression for $r_D$, eqn. (5), we substitute the turbulence-broadened spot size

$$r_D' = (r_D^2 + D^2 e_{st}^2)^{1/2},$$ \hspace{1cm} (11)

where $\theta_{st}$ is the short-term beamspread to the $e^{-1}$ point (radians), and represents the radial enhancement due to turbulence of the observed beam in the short-time exposure limit. Note that $r_D' = \sqrt{\langle e_{st}^2 \rangle}$, where $e_{st}$ is the short-exposure radius defined in section 5(c). The short-term beamspread has been evaluated as [23]

$$\theta_{st} = \begin{cases} \frac{0.427}{e} \theta_{st} & \varepsilon = 2w_0/r_0 < 3 \\ \theta_{st} \left[1 - 1.18/e^{1/3}\varepsilon^{1/2}\right] & \varepsilon > 3 \end{cases}$$ \hspace{1cm} (12a)
where $r_0$ is sometimes referred to as the Fried coherence length, or lateral coherence length, of a spherical wave. It is given by [23]

$$r_0 = 2.10 \left( 1.455 k^2 n^2 \int_0^1 c_n^2(u) (1-u)^{5/3} \, du \right)^{-3/5}. \quad (13)$$

The long-term turbulent beam spread to the e\(^{-1}\) point (radians) is the radial enhancement due to turbulence in the long-time exposure limit, and is given by [23]

$$\theta_{lt} = 0.604 \, \omega_0 / r_0. \quad (14)$$

Equations (11) and (12) for the infinite gaussian beam have been modified by Breaux [23,24] through the incorporation of new parameters which serve to match empirically the spot radius calculated via Fresnel integrals. Although this procedure should improve the accuracy of the model, it has not been employed in the present calculations.

The equations given in this section have been utilized in a computer program which calculates the hazard distance for a given set of input parameters. The program also can selectively treat any of the input parameters as a variable. Important input parameters are listed in Table 1, and numerical calculations of hazard distances using this program are discussed in Section 3.

A simplified flow chart for the program is depicted in Figure 1. Being an iterative calculation, it proceeds via the large circular loop, as shown, until the hazard distance has been calculated to within a prescribed accuracy. In order to initiate the iterative cycle, it is necessary to provide a rough estimate for the range, and this is given by the solution of the deterministic equation (4). Other quantities which are calculated by the program include the scintillation factor, the respective variances $\sigma^2_T, \sigma^2_s, \sigma^2_l$, the short-term and long-term beam spreads, the corresponding radii at the hazard distance, and the inverse probability variable $b$. 
TABLE 1

Summary of the input parameters

<table>
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<th>Viewer parameters</th>
<th>Propagation parameters</th>
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</thead>
<tbody>
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<td>1. Energy</td>
<td>1. Attenuation ((a_1))</td>
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<tr>
<td></td>
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<td>(Absorption, Scattering)</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>5. Height of laser ((z))</td>
<td>5. Magnification optics</td>
<td></td>
</tr>
</tbody>
</table>

3. NUMERICAL CALCULATIONS

Calculations of hazard distance under various conditions were carried out by means of the computer program described in section 2, using the following quantities (Table 2) as fixed parameters:

TABLE 2

Program Input Parameters

\[
\begin{align*}
V &= 23.5 \text{ km} \\
E &= 150 \text{ mJ} \\
T &= 2 \mu\text{J} \\
Q &= 0.01 \\
\phi &= \sqrt{2} \times 10^{-4} \text{ rad. (to } e^{-1}, \text{ half-angle)} \\
\lambda &= 1.064 \mu\text{m} \\
w_0 &= 9 \text{ mm} \\
w_C &= w_E = w_p = 3.5 \text{ mm (refer to Appendix)} \\
\alpha &= 0.1 \text{ km}^{-1} \\
z &= 2 \text{ m}
\end{align*}
\]
The value $T = 2 \mu J$ corresponds to the MPE at wavelength 1.064 $\mu m$ with a pupil diameter of 7 mm. The program has been given the facility for varying one of these input quantities, the remaining quantities then assuming the above values by default. The program determines the hazard distance at which a viewer, looking directly at the laser, has a probability $Q$ of exceeding the threshold energy $T$. It also calculates the scintillation factor, the variance $\sigma_s^2$, and the short-term and long-term beam radii.

The refractive index structure parameter $C_n^2$ was allowed to vary over the range $10^{-17}$ to $10^{-10}$ $(m^{-2/3})$. As a rough guide to the scaling, $10^{-11}$ corresponds to very strongly turbulent conditions and $10^{-15}$ to very weak turbulence. Figures 2 and 3 illustrate the results of the calculations and show hazard distances for different visibilities and pulse energies respectively. Hazard distances and scintillation factors in the present statistical model approach the fixed values from the corresponding deterministic model as $C_n^2$ becomes smaller. With very large values of $C_n^2$, the hazard distance is dominated by the effects of beam spreading and beam break-up. These processes ensure that hazard distances in a very strongly turbulent atmosphere fall far short of those predicted by a deterministic model. The hazard distances and scintillation factors usually both peak in the moderate turbulence regime.

4. ATMOSPHERIC TURBULENCE

The turbulence level of the atmosphere can be estimated if required from a knowledge of the following measurable meteorological parameters.

(a) Insolation: The intensity of sunlight striking the ground, or insolation strength, depends upon the sun's elevation angle and the attenuation of the atmosphere. An empirical formula for $C_n^2$ in terms of insolation strength can be determined [6].

(b) Aerosol Level: There is some experimental evidence to show [26] that $C_n^2$ can be expected to increase as the meteorological range (visibility) increases from 8 to 22 km.

(c) Wind Speed: For all values of the insolation, $C_n^2$ falls gradually with increasing surface wind speed. Experimental data is given in ref. [6] for wind speed ranges from 1.5-6 m/s.

(d) Terrain Properties: There is uncertain evidence concerning the effect of terrain upon the turbulence level. Bare earth, sand, rock etc can be expected to be hotter than grass, giving rise to more turbulence [6]. Cloud cover, time of day and season are clearly important considerations, since properties of the vegetation such as the reflectance and transpiration rate are involved.
Temperature: Refractive turbulence does not appear to be strongly related to the mean air temperature except through the correlation between insolation level, temperature lapse rate, and the mean air temperature [6]. In other words, the mean air temperature alone should not provide any indication of the turbulence level.

Humidity: Scintillation at far-infrared wavelengths may be affected by absorption of water vapour, but the effect is usually neglected for most other wavelengths.

Height above Ground: The refractive index structure constant decreases with height, e.g., as in equation 3. Relevant factors influencing this variation are macro-meteorological phenomena such as fronts, variations in terrain, etc., and micro-meteorological influences such as the diffusion time of heat from the surface to higher altitudes. In the computer program $C_n^2$ is prescribed for the standard height 1 m, and corresponding values for other heights are found using eqn (3).

5. DISCUSSION

The type of physical model that we have been considering is clearly more satisfactory than deterministic models, but also possesses the limitations listed below. Some of these have been discussed previously by Sliney [2,11] and by Sliney and Wolbarsht [13].

(a) Exposure Limit: The first difficulty concerns the relation between the aperture over which the visible or near-infrared radiation is averaged and the exposure limit. It is relevant to mention that irradiance fluctuations sampled by a small aperture (e.g., 1 mm diameter) can behave differently from those measured by larger apertures, since the larger aperture tends to average the fluctuations over its entire area, thereby reducing the variance of the distribution. The MPE, as specified in AS 2211 refers to an ocular pupil size of 7 mm. This is usually considered to be the largest (or worst-case) aperture size, which corresponds to evening or dusk conditions. It would appear desirable to permit a higher exposure limit for daylight conditions where the pupil is constricted, but there is some debate on the issue, and such a scheme has not been implemented. Sliney, in particular, has provided arguments and evidence [2,11] which suggest that the optical gain of the eye (the peak irradiance divided by the corneal irradiance) is only slightly affected by pupil size in practice. Other factors relevant to exposure limits include the age of the viewer, the duration and repetitiveness of pulses, and the laser wavelength's).

(b) Beam Profile: A further problem arises by reason of the assumed 'gaussian' irradiance cross-sectional profile over all propagation distances. This is a convenient shape to employ since many lasers operate in the lowest transverse mode, however it must be recognised that in highly turbulent situations, or at long distances (several km), the laser beam is likely to consist of a large number of unconnected spots whose distribution is asymmetric in the short term. At shorter distances the gaussian profile is
clearly preferable to the simpler 'top-hat' shape. For output from the laser cavity which does not already possess a gaussian shape, we may use diffraction theory in order to determine the shape of an 'equivalent' gaussian. Given a circular top-hat irradiance at the output aperture, the simplest method is to set the top-hat radius equal to the radius at the 1/e point on a gaussian of equal height. This procedure attempts to allow for diffraction in the far field, and ensures that radiant power is conserved. A more satisfactory method would be to assume that the irradiance profile by Fraunhofer diffraction possesses an 'Airy pattern' or 'sombrero' shape \[ J_1(2\theta/\xi)/\xi \] where \( \theta = r/w \) is the radial coordinate and \( \xi = D/(\pi k w^2) \); a gaussian curve is then easily fitted to the central peak, the ring structure being ignored.

(c) Exposure Time: When considering laser beam propagation in a turbulent medium it is necessary to distinguish between short-term and long-term beamspread. With pulsed beams, 'long-term' implies averaging over a large number of pulses. Large turbulent eddies tend to deflect the laser beam as a whole (i.e. produce beam wander), whereas small eddies mainly tend to broaden the beam. A photograph of the beam taken with an exposure time much longer than the characteristic wander time would reveal a broadened spot with mean-square radius \( r^2 = \langle r^2 \rangle + \langle r_{cen}^2 \rangle \); here \( r_{st} \) is the short-exposure radius and \( r_{cen} \) is the mean centroid of \( r_{st} \). This simple picture is of course incorrect when the turbulence is strong, since then the beam profile is discontinuous, and moreover the beam wanders only slightly by comparison with beams in lower turbulence. However, in strongly turbulent media the long-exposure photograph appears like the short-term exposure one, with roughly the same radius.

(d) Retinal Irradiance: The irradiance at the retina, \( I_r \), of a beam intercepted by an unaided eye, has been derived [14] as a function of the radiance (brightness) of the source \( L \), the pupil diameter \( d_p \), and the spectral transmittance of the ocular media, \( \tau(\lambda) \):

\[
I_r = 0.27 L \tau(\lambda) d_p^2
\]

and this works adequately for source angles greater than 5.7 mrad, corresponding to a retinal image diameter of 100 \( \mu m \). The laser radiance is therefore of importance in laser safety considerations. As a laser is effectively a point source, the rays are normally imaged on the retina as a minimal image. However, atmospheric turbulons can sometimes defocus the rays to such an extent that the retinal image is enlarged [2], thereby (hopefully) offsetting the increased power entering the pupil when the eye encounters a hot spot. At most, however, it has been found [2,11] that this effect has only a limited impact.

(e) Additional Factors: Other physical processes which tend to suppress the danger to the retina from hot spots in a highly turbulent atmosphere include beam-spreading and also the reduction in coherency and beam quality associated with saturation of the scintillations. Beam spreading is manifested in the strong turbulence region as a large negative gradient in the graphs of the hazard range vs \( C_n^2 \) (see figures 2 and 3). No such effect appears in corresponding graphs of the scintillation factor vs \( C_n^2 \) however. Based on experiments designed to study the above effects, Sliney et. al. [2,11; 13,14] have come to the conclusion that turbulence does not add to the
out-of-doors laser hazard as significantly as was believed by previous researchers. Another complicating factor lies in the assumption that laser-induced retinal injury only depends upon retinal irradiance. This is not a correct assumption for some CW and short-pulse laser exposures, where the injury threshold irradiance diminishes as the image size increases. Scintillation effects are disregarded for CW lasers since the exposure becomes averaged over a number of scintillations.

(f) Probability of Injury: The maximum probability of ocular injury attending laser irradiation of an individual outdoors will be realised when:

(i) the individual is looking in the direction of a pulsed laser,
(ii) the individual’s eye is relaxed (focused at infinity),
(iii) the pupil is located at the average irradiance maximum of the beam,
(iv) the fovea is directed at the propagation axis,
(v) the retina is more sensitive (absorbing) than average,
(vi) the eye intercepts a significant hot spot,
(vii) laser jitter and beam divergence are minimal,
(viii) there is a large visual range (weather dependent).

The probability that all of these conditions will occur simultaneously at distances beyond the NORD is extremely small. The probability of injury is enhanced, however, when the individual views the laser with the aid of a telescope or pair of binoculars.

Taking into account the multiplicity of effects working to reduce the significance of hot spots in laser safety estimates, and also taking cognisance of calculations involving scintillation statistics by authors in the UK [6], a preliminary value of $Q = 0.01$ (1%) can be assumed as a convenient statistical boundary for the probability of hazardous exposure in a colinear arrangement of the optical axes of laser and viewer. Apart from this choice, it is questionable whether the statistical model employed in the present study is valid in the extreme wings of the range-dependent probability distribution. In the sample calculations presented herein for standard clear visibility ($V = 23.5$ km), this value of $Q$ produces a maximum value for the scintillation factor slightly larger than 4, even the excessively cautious value (in the judgement of the author) $Q = 0.001$ has been found to produce a maximum which is only between 8 and 9. It can be concluded on this basis that the incorporation of the features of scintillation saturation and turbulent beam spreading (amongst others) in a statistically-based laser safety analysis will generate a safety factor (the maximum scintillation factor) somewhat lower than the value $k_2 = 10$ recommended in the AS 2211.

The quantity $Q$, which is the probability of exceeding a specified threshold in a colinear arrangement of transmitter and viewer, should not be
confused with the net probability of receiving ocular injury. In a realistic safety estimate, the value chosen for \( Q \) in practice will lead to a net injury probability which is lower than \( Q \) by several orders of magnitude. This reduction in magnitude is the consequence of allowing for geometrical exposure factors, as well as a number of biological and dynamic factors which (unfortunately) may be difficult to quantify.

It will be observed in figs. 4a,b that the hazard ranges and scintillation factors are reduced significantly, as expected, when \( Q \) increases in magnitude. Also apparent in figs. 4a,b is the expected result that the hazard range is almost independent of \( Q \) for very low and for very high turbulence. The latter case merely confirms that strong beam spreading and super-saturation override simple statistical considerations and tend to destroy the effectiveness of hot spots. Lastly, it should be emphasised that probabilistic models are not new to laser-safety calculations (see, for example, the 1969 paper by Deitz [25]), although these earlier models neglected important features such as beam spreading. It is also clear that buried in even a deterministic approach to laser safety are some aspects of the probabilistic approach, such as the relationship between MPE and the statistical distribution of responses from an ensemble of eyes.

6. SUMMARY

This analysis of laser safety for a pulsed laser beam allows for atmospheric turbulence effects by modelling the irradiance fluctuations in terms of a known (modified log-normal) statistical distribution. The formulation of the model is based in the main upon well-known theoretical expressions and empirical algorithms derived from a considerable quantity of experimental data.

An iterative computer program has been developed which can determine, for a prescribed maximum permissible exposure, either the spatial distribution of the probability of exceeding the exposure threshold or the ocular hazard range corresponding to a specific probability. Features of both UK and USA statistical models for determining hazard range have been incorporated into the present program, which is both simple to implement and fast.
7. REFERENCES


Equation (4) introduces an aperture parameter $w$, which will now be defined for an optically-aided eye. Let $w_p$ be the eye-pupil radius, $w_e$ be the exit-pupil radius, and let $w_R = w_c/m$ where $w_c$ is the collector-aperture radius (all in metres), and $m$ is the optical magnification. The appropriate expression for $w$ can be derived and interpreted by reference to the following simplified diagram of the optics:

![Diagram of optics](image)

It is assumed that the optical axes of the laser, the magnifying optics and the eye are all colinear. The irradiance distribution of an idealised gaussian beam at range $D$ is

$$I(r, D) = \frac{P}{\pi r_D^2} e^{-\frac{\alpha_D}{r_D}} e^{-\frac{r^2}{r_D^2}}$$  \hspace{1cm} A1$$

where $r$ is the radial coordinate, and $r_D$ is the beam width (to the $1/e$ position) at that range. The power transmitted at the objective (or collector aperture) is therefore

$$P_C = \int_0^{w_c} I(r, D) 2\pi r \, dr$$

$$= \int_0^{w_c} \frac{P}{\pi r_D^2} e^{-\frac{\alpha_D}{r_D}} e^{-\frac{r^2}{r_D^2}} 2\pi r \, dr$$

$$= \frac{P e^{-\alpha_D}}{\lambda} \left( \frac{2r_D}{w_c} \right)^2$$  \hspace{1cm} A2$$

$$= P e^{-\alpha_D} \left( \frac{2r_D}{w_c} \right)^2$$
At the receiver plane R the intensity profile $I_R(r)$ must satisfy

$$P_c = \int_0^{w_R} I_R(r) 2\pi r \, dr,$$

hence one easily obtains

$$I_R(r) = \frac{m^2 P}{\pi r^2 D} e^{-\alpha_D A} - \frac{m^2 r^2}{r_D^2}.$$  \hspace{1cm} \text{(A3)}$$

If $w_E > w_R$, then the power transmitted by the exit aperture is given by eqn. (A2); otherwise the transmitted power is

$$P_R = \int_0^{w_E} I_R(r) 2\pi r \, dr$$

$$= \frac{P e^{\frac{m^2}{r_D} - \frac{m^2 w_E^2}{r_D^2}}}{e^{\frac{m^2}{r_D} - \frac{m^2 w_E^2}{r_D^2}}}.$$  \hspace{1cm} \text{(A4)}$$

Equations (A2) and (A4) represent the power entering the eye pupil provided that $w_P$ is at least as large as $w_R$ in the former case or $w_E$ in the latter case. Equation (A2) clearly must be altered when $w_P < w_R$, and in both cases the power entering the pupil is

$$P e^{\frac{m^2}{r_D} - \frac{m^2 w_P^2}{r_D^2}}.$$  \hspace{1cm} \text{(A5)}$$

Hence it has been established that the optical power entering the pupil of the eye is generally given by

$$P e^{\frac{m^2}{r_D} - \frac{m^2 w^2}{r_D^2}}.$$  \hspace{1cm} \text{(A6)}$$

where

$$w = \min (w_P, w_E, w_R).$$  \hspace{1cm} \text{(A7)}$$
FIGURE 1  Flow chart for the iterative calculation of optical hazard distance.
FIGURE 2a  Ocular hazard distance of the naked eye as a function of the turbulence parameter $C_n^2$, for two different visibility levels: (a) $V = 23.5$ km (standard clear), (b) $V = 8$ km (light haze). The default values (Table 2) apply to all other parameters.

FIGURE 2b  The scintillation factor $F(\sigma_s)$ vs $C_n^2$, corresponding to the two visibility values in Fig. 2a.
FIGURE 3  The effect of varying the laser pulse energy $E$ upon the hazard distance for a $6 \times 42$-aided eye: (a) $E = 200 \text{ mJ}$, (b) $150 \text{ mJ}$, (c) $100 \text{ mJ}$. Default values in Table 2 again apply to other parameters.
FIGURE 4a  Ocular hazard distance of an unaided eye, for various values of the probability of exceeding the damage threshold $T = 2 \mu J$: (a) $\Omega = 0.01$, (b) $\Omega = 0.1$, (c) $\Omega = 0.5$, (d) $\Omega = 0.9$.

FIGURE 4b  Scintillation factors corresponding to the ranges depicted in Fig. 4a.