Spacecraft Sheath Modification During Beam Ejection

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Spacecraft Sheath Modification During Beam Ejection


Numerical solutions to the equations relating the charge ejection to spacecraft potential have been used to study the response of spacecraft potential to ion and electron beam emissions. Ionization occurs within the sheath due to the return current. The charge density and potential profile in the sheath are altered. Non-monotonic behavior of spacecraft potential as a function of beam current is obtained. Parametric studies on the effects of neutral density, electron temperature, mean free path, and spacecraft size have been performed. Experimental evidence of spacecraft sheath modification during beam ejections is discussed.
16. (Continued)  

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1. INTRODUCTION

In recent space experiments,\(^1,2\) the potential difference created between spacecraft ground and the ambient plasma during the ejection of a beam of electrons from a sounding rocket payload in the ionosphere has been found to be much less than had originally been theoretically predicted.\(^3\) To determine the reasons for this limited potential difference, large vacuum chamber tests were conducted in which electron and ion currents were ejected from a payload into a simulated ionosphere.

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Motivated by the experimental results, sheath ionization models\(^4,5\) for small spacecraft have been studied for a plausible explanation of the observed current-voltage behavior. When an electron beam is emitted from a spacecraft, ambient electrons are attracted by the charged spacecraft.\(^6\) They collide with the neutral atmospheric atoms or molecules in their paths, and may be energetic enough to ionize the neutrals to form new electrons and ions.\(^7\) These newly created charges alter the space charge current arriving at the spacecraft, shifting the potential to a lower value. The beam electrons are assumed to be energetic enough to leave the spacecraft completely and to play a negligible role in the ionization. This mechanism is capable of explaining the low potential difference observed, and a possible non-monotonic current-voltage behavior.

2. MATHEMATICAL FORMULATION

The method of approach used is to study an analytical 'plasma probe' model\(^4,5,8,9\) with space charge flow of electrons accelerating through the sheath surrounding a spherical 'probe' that represents a spacecraft in an ionizable plasma environment. Magnetic field effect is ignored in this model.

The beam is assumed to be energetic enough to leave the spacecraft completely, and is not stopped by its own space charge at all. As the beam electrons leave, the spacecraft becomes charged oppositely. A polarization region (sheath) is formed in the vicinity of the spacecraft. In our model, ions are assumed depleted due to charge repulsion inside the sheath (Figure 1).

The depletion radius \(r_0\) will be defined by the balance of the outgoing beam current with the incoming ambient current. For a beam current \(I_b\), the depletion radius \(r_0\) is determined by

\[ r_0 = \left( \frac{2I_b}{\sigma N} \right)^{1/2} \]

Figure 1. Sheath Formation During Beam Emission

\[ I_b = 4 \pi r_0^2 n_e v_{th} \]  \hspace{1cm} (1)

where \( v_{th} \) is the thermal velocity, and \( n_e \) is the number density of ambient electrons. Some typical values of sheath radius as calculated by means of Eq. (1) are shown in Figure 2.

![Figure 2. Parametric Dependence of Sheath Size](image-url)
The potential $\phi$ at any point inside the sheath is governed by the Poisson equation

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}, \quad (2)$$

where $\rho$ is the space charge density, and $\varepsilon_0$ is the permittivity of empty space.

3. SPHERICAL SYMMETRIC SYSTEM

To simplify the geometry, we assume spherical symmetry in the spacecraft and sheath system. Equation (2) becomes simply a radial equation:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi(r)}{\partial r} \right) = -\frac{\rho(r)}{\varepsilon_0}, \quad (3)$$

where the gradient of the potential $\phi$ gives the electric field $E$

$$\frac{\partial \phi(r)}{\partial r} = -E(r). \quad (4)$$

Taking into account the electron and ion pairs created as a result of ionization, the charge density $\rho$ at any point $r$ in the sheath is given by the sum of charge densities (Figure 3):

$$\rho(r) = e \left[ n^+(r) - n^-(r) - n_e(r) \right], \quad (5)$$

where $n_e$ is the return current (primary) electron density, while $n^+$ and $n^-$ are the ionization ion and electron densities, respectively, due to return current electron collisions with neutrals.

The ionization electron density $n^-(r)$ is due to all ionizations that occur outwards of $r$, and the density $n^+(r)$ of ions at $r$ is due to all ionizations that occur inwards of $r$. Thus, for a spacecraft of radius $R$,

$$n^-(r) = \frac{1}{r^2} \int_{r}^{R} \frac{dn}{dr} r^2 dr', \quad (6)$$

and

$$n^+(r) = \frac{1}{r^2} \int_{R}^{r} \frac{dn}{dr} r^2 dr', \quad (7)$$
Figure 3. Ionization Pair-Creation in Sheath

where the rate of ionization is a function of the mean free path $\lambda$, the electron current, and the velocity-dependent probability $P$ of ionization:

$$\left[ \frac{dn}{dt} \right]_{r'} = \lambda^{-1} P \left( v_e(r') \right) n_e(r') v_e(r').$$

(8)

Figure 4 shows the ionization cross-section of atomic oxygen as a function of energy. It has a peak at $\sim 90$ eV and a cutoff at $\sim 14$ eV for typical atmospheric neutrals.

4. NUMERICAL METHOD

To solve the system of equations [Eqs. (3) to (8)], one divides the space of the sheath into $N$ concentric shells, and sets up $N$ equations for the $N$ unknowns $\phi_i$ (see Figure 5). In view of the complexity of the ionization terms in Eqs. (6) and (7), it is impossible to solve these equations exactly. Instead, one seeks the approximate solutions that minimize a function $F$, the mean square of $f_i$, constructed from the radial Poisson equation [Eq. (3)] for the $i$-th cell, where $i = 1, \ldots, N$. 
\[ f_i(E_1, \ldots, E_N) = \left( r^2 E \right)_{i+1} - \left( r^2 E \right)_i - \frac{1}{\epsilon_\infty} \left[ r^2 \rho \left( E_1, \ldots, E_N \right) \right]_i \Delta r , \]  

(9)

where the electric field \( E \) [Eq. (4)] is constructed in a finite difference scheme:

\[ \phi_i - \phi_{i+1} = \Delta r \left( E_i + 2E_{i+1} + E_{i+2} \right) / 4 . \]  

(10)

The numerical method used to solve Eqs. (8) to (10) is the standard Newton-Ralphson method of iteration:

\[ E_i^{(j+1)} = E_i^{(j)} - \frac{f_i(E_1^{(j)}, \ldots, E_N^{(j)})}{\partial f_i(E_1^{(j)}, \ldots, E_N^{(j)}) / \partial E_i} . \]  

(11)

Figure 5. Decomposition of Sheath Into Shells
To start, a set of trial solutions is used in the Newton-Ralphson iteration process, and a convergent set of solutions is sought for each set of input parameters such as beam current, ambient electron density, ambient electron temperature, mean free path, and spacecraft radius.

5. RESULTS AND DISCUSSION

Figure 6 shows the computed results of spacecraft potential as a function of electron beam current for various electron density, electron temperature, and mean free path. The non-monotonic behavior of potential current curves shows up. At low currents, the potential increases with beam current. When the current increases further, ionization occurs inside the sheath. The potential then turns around as the current of the electron beam increases.

![Figure 6](image-url)  
**Figure 6.** Typical Non-Monotonic Current Voltage Behaviors for (a) Spherical and (b) Cylindrical Spacecraft

The ion and electron charges created by ionization alter the behavior of the space charge flow, originally governed by the single charged Poisson equation. The potential turns to a lower value, and stays approximately constant as current further increases.

In the rest of the paper, spherical symmetry will be assumed.
To examine the mechanism of space charge modification in the sheath in greater detail, the sheath potential profiles for various beam currents are displayed in Figure 7. As the potential in the I-V curve turns to a lower value, the potential profile, as a function of radial distance, shows a locally flat gradient. This is due to ions created inside the sheath being unable to move out quickly due to their heavy masses. If a local ion charge build-up forms a potential hump, ion motion would be two-way, and the theory would then break down. To overcome this difficulty, a sweep velocity $v_s$ is added to the ions, and Eq. (7) becomes

$$n^+(r) = -\frac{1}{r^2} \int_r^R \left[ \frac{dn}{dt} \right] \frac{r^2}{\left[ 2e \left( \phi(r) - \phi(r') \right)/m_i + v_s^2 \right]^{1/2}} \, dr.$$  

(12)

Figure 7. Relation Between Potential Profile and I-V Behavior

It is argued that the motion of a spacecraft relative to its plasma environment can provide such a sweep velocity $v_s$ [Eq. (12)]. The value of $v_s$ is of the order of spacecraft velocity, and is an arbitrary input to the computation. However, at a higher current, a potential hump again shows up, the computation fails to converge, and the technique breaks down. It is conjectured that two-way space charge flows should be accommodated when a potential hump appears.
The effect of mean ionization distance $\lambda$ on the potential of a spacecraft emitting an electron beam is shown in Figure 8. In the region where ionization is negligible ($\lambda = 10^3$), the potential curves are flat, and the potential increases with beam current. As the mean ionization distance decreases (i.e., the probability of ionization increases), the potential curve turns down, and stays approximately constant as the ionization distance decreases further.

![Figure 8. Spacecraft Potential as a Function of Mean Ionization Length](image)

Ambient electron density, $n_e$, is an important parameter affecting spacecraft potential. In Figure 9, starting with large values of electron density, the potential increases as $n_e$ decreases. This is well expected, since the spacecraft potential depends on how much the ambient electrons can provide in the incoming current. However, as $n_e$ decreases, the depletion radius $r_o$ increases ($r_o \propto n_e^{-1/2}$ in Eq. (1)). Hence, the total distance travelled by an incoming electron increases, thereby producing more ionization. As a result, the potential is eventually lowered when ionization becomes important.

The ambient electron temperature, $T_e$, plays a role analogous to that of electron density, $n_e$, but in an opposite manner. At high temperatures, the ambient electron current is high, and keeps the spacecraft potential low. As temperature decreases, the ambient current decreases, but the depletion radius, $r_o$, increases. Then, ionization becomes more probable. Eventually, at sufficiently low temperature $T_e$, the potential curve turns down because of ionization (Figure 10).
For increasing spacecraft radii, the non-monotonic current-voltage behavior still persists (Figure 11). However, increased spacecraft radius lowers the maximum spacecraft potential induced by beam emission. Also, the amplitude of the difference between the maximum potential and the minimum (beyond the turnaround) diminishes. Figure 12 shows a plot of the envelope of maximum and minimum potentials for various spacecraft radii.
Figure 11. Persistence of Non-Monotonic I-V Behavior. The parametric conditions are as in Figure 12.

Figure 12. Envelope of the Potential Extreme in Figure 11.
For a given beam current $I_b$ [Eq. (1)], the sheath surface area remains constant and is unaffected by the increase in spacecraft radius. The sheath thickness (defined as the sheath radius minus the spacecraft radius), however, diminishes. As a result, a lower spacecraft potential is sufficient to attract ambient electrons through the sheath for the compensation of electron beam current leaving the spacecraft.

Beyond the turn-around point in a current-potential curve, the minimum potential is limited by the minimum energy required to ionize a neutral molecule in the atmosphere. Since such a minimum energy is generally of the order of 14 eV, the minimum potential in a current-potential curve is expected to approach 14 eV asymptotically, depending on the model of ionization used. For the same reason, if the maximum potential induced by beam emissions is below $\sim 14$ eV, no non-monotonic behavior is expected.

Figure 12 shows the calculated envelopes of the maximum and minimum (beyond turn-around) potentials for various spacecraft radii in a given ambient environment. The amount of ionization becomes very small as the sheath potential approaches the minimum ionization potential. The amplitude of the potential drop beyond the turn-around also approaches the value of minimum ionization energy.

There is another critical beam current that manifests itself for large spacecraft but not for small ones. This current is determined by equating the sheath radius to the spacecraft radius. If the sheath radius is too small, the spacecraft would receive enough ambient electrons to compensate beam emissions without being charged up. The potential of the spacecraft in this case is that of natural charging. Beyond this critical current, the beam emission is able to swing the spacecraft to an opposite potential and, hence, control the charging of the spacecraft. This phenomenon shows up in the calculations (Figure 11).

In the model studied, as the radius of a spacecraft increases, three regimes of physical behavior can be identified. Figure 13 shows these regimes clearly. The potential vs spacecraft radius curve is relatively flat in the small radius regime. This is the regime in which saturated ionization occurs, i.e., the regime beyond the minimum potential in a current-voltage curve. The second regime is characterized by the presence of the potential maximum, which is the main feature of non-monotonic behavior. The third regime occurs when the spacecraft is so large that its radius exceeds the sheath radius (measured from the spacecraft center) for a given current. The beam loses its control of the spacecraft potential, and natural charging dominates.

For ion beam emissions from a spacecraft, the return current of ions is not capable of turning around the spacecraft potential. This is because the cross-sections of ion impact ionization of neutral atoms or molecules are much too low,
Figure 13. Non-Monotonic Behavior of Spacecraft Potential as a Function of Spacecraft Radius, for a Given Electron Beam Current

in contrast to the electron case. The current-voltage characteristics for ion beam emissions are expected to be monotonic, as demonstrated in space. There seems to be no bound for how high the spacecraft potential can be induced if the current of the ion beam, which is assumed to be energetic enough to leave completely, is increased.
References


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