CONVECTIVE STABILIZATION OF IONOSPHERIC PLASMA CLOUDS

J F Drake et al.

19 MAR 86 NRL-MR-5722
Convective Stabilization of Ionospheric Plasma Clouds

J. F. Drake
Science Applications International Corporation
McLean, VA 22102
and
University of Maryland
College Park, MD 20742

J. D. Huba
Geophysical and Plasma Dynamics Branch
Plasma Physics Division

March 19, 1986

This research was sponsored by the Defense Nuclear Agency under Subtask QIEQMXBB,
work unit 00005 and work unit title "Plasma Structure Evolution."

Approved for public release, distribution unlimited.
Convective Stabilization of Ionospheric Plasma Clouds

**Abstract**

We derive a stability criteria for the large-scale structuring of ionospheric plasma clouds due to the $E \times B$ gradient drift instability. For the equilibrium we consider a cylindrical 2D waterbag cloud aligned along a uniform magnetic field that is polarized by a neutral wind. We perform a stability analysis that allows three dimensional perturbations (in $r$, $\theta$, and $z$), and consider both local and global modes. We find that when the parallel drift velocity is increased further, the unstable modes localize at a finite angle away from the backside; at a point where the diamagnetic propagation number, $k_d^2$ exceeds a threshold value, exponentially growing global eigenmodes form which are localized on the “backside” of the cloud. This is in contrast to the $k_d^2 = 0$ limit in which there are no exponential solutions. As $k_d^2$ is increased further, the unstable modes localize at a finite angle away from the backside; at a point where the diamagnetic propagation number, $k_d^2$ exceeds a threshold value, exponentially growing global eigenmodes form which are localized on the “backside” of the cloud.

**Supplementary Notation**

- Science Applications International Corporation, McLean, VA 22102 and University of Maryland, College Park, MD 20742

**Subject Terms**

- Plasma clouds
- $E \times B$ gradient drift instability
- Ionospheric disturbances
- Striation freezing

**Distribution/Availability of Report**

Approved for public release; distribution unlimited.
<table>
<thead>
<tr>
<th>PROGRAM ELEMENT NO.</th>
<th>PROJECT NO.</th>
<th>TASK NO.</th>
<th>WORK UNIT ACCESSION NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOE 61153N</td>
<td>23</td>
<td>RR011-09-41</td>
<td>DN680-382</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DN320-105</td>
</tr>
</tbody>
</table>

**SECURITY CLASSIFICATION OF THIS PAGE**
CONTENTS

I. INTRODUCTION .......................................................... 1
II. GENERAL EQUATIONS AND EQUILIBRIUM .............................. 4
III. LINEARIZED EQUATIONS AND DISPERSION EQUATION ............... 7
IV. EVALUATION OF THE LOCAL DISPERSION EQUATION ............... 11
V. GLOBAL EIGENFUNCTIONS ............................................. 22
VI. SUMMARY AND CONCLUSION ......................................... 27

ACKNOWLEDGMENTS ...................................................... 32
REFERENCES ............................................................... 33
CONVECTIVE STABILIZATION OF IONOSPHERIC PLASMA CLOUDS

I. INTRODUCTION

The evolution of artificial plasma clouds (e.g., barium) in the earth's ionosphere continues to be of interest to space plasma physicists after more than two decades of research. The plasma dynamics associated with ionospheric clouds are of interest since they provide a diagnostic of the ambient ionospheric environment, and also provide experimental data for the study of plasma instabilities. Of particular interest is the onset and evolution of the $E \times B$ gradient drift instability (Simon, 1963; Hoh, 1963) which is believed to cause the gross structuring of plasma clouds, i.e., field-aligned striations (Linson and Workman, 1970). This instability is also believed to be responsible for, at times, the structuring of the ambient, high latitude F region ionosphere (Keskinen and Ossakow, 1983). The $E \times B$ gradient drift instability is an interchange mode and is driven by a neutral wind or dc electric field in an inhomogeneous, weakly collisional plasma. A substantial amount of theoretical and computational research has been carried out to understand the $E \times B$ gradient drift instability and its relevance to ionospheric structure (Volk and Haerendel, 1971; Perkins et al., 1973; Zabusky et al., 1973; Shiau and Simon, 1972; Perkins and Doles, 1975; Scannapieco et al., 1976; Chaturvedi and Ossakow, 1979; Keskinen et al., 1980; McDonald et al., 1980, 1981; Huba et al., 1983; Sperling, 1983, 1984; Overman et al., 1983; Sperling and Glassman, 1985; Sperling et al., 1984; Drake et al., 1985).

The bulk of theoretical analyses to date have been based on the local approximation in slab geometry. The local approximation is appropriate for unstable modes which have wavelengths much shorter than the scale length of

Manuscript approved November 19, 1985.
the density gradient associated with the cloud boundary (i.e., $kL >> 1$
where $k$ is the wavenumber and $L$ is the density gradient scale length) (Huba
et al., 1983, Huba and Zalesak, 1983). However, the observed gross
structuring of plasma clouds seems to suggest that the dominant modes have
$kL < 1$ so that the local approximation may not be valid. The slab or one-
dimensional [$n = n_0(x)$] models of plasma clouds do not include a number of
important physical effects which appear in more realistic two-dimensional
models. The polarization of large plasma clouds in 2D greatly reduces the
relative slip velocity of the cloud and the ambient neutral wind thereby
weakening the $E \times B$ gradient drift instability (Overman et al., 1983;
Zalesak and Huba, 1984). Furthermore, convection and/or propagation of the
perturbations from the unstable backside to the stable frontside of the
cloud may influence the overall stability of 2D models.

The most detailed linear stability analysis of 2D plasma clouds in the
long wavelength regime has been based on the waterbag model (Overman et
al., 1983). The purpose of this paper is to extend the analysis of Overman
et al. (1983) by including parallel dynamics, i.e., density and potential
fluctuations along the ambient magnetic field. Recently it has been shown
that the parallel effects can strongly influence the linear stability of
ionospheric plasma clouds by stabilizing the short wavelength modes
(Sperling, 1983; Sperling et al., 1984; Sperling and Glassman, 1985; Drake
et al., 1985). However, these investigations have been based on a 1D cloud
model and the need to consider a 2D cloud model is apparent. Thus, in this
paper we study the influence of parallel dynamics on the stability of 2D
ionospheric plasma clouds. In particular, as in Overman et al. (1983) we
consider a simple equilibrium consisting of a 2D cylindrical waterbag, but
in contrast to them we consider three dimensional perturbations. We find
that the finite parallel dynamics can dramatically alter the $E \times B$ gradient drift instability. Unlike the limit $k_z = 0$, exponentially growing global eigenmodes can exist when $k_z \neq 0$. As $k_z$ is increased above a threshold value, the unstable modes localize at a finite angle away from the backside; at a point where the diamagnetic propagation velocity ($V_d$) balances the convective flow velocity of the background plasma around the cloud ($V_b$). We find that the $E \times B$ gradient drift instability is stable when $V_d > V_b$ so that the cloud is no longer susceptible to large-scale structuring. We apply these results to ionospheric barium clouds and estimate that they will cease structuring when $L < (cT/eB)(M+2)/2V_n$ where $L$ is the transverse size of the cloud, $T = T_e + T_i$ is the total temperature, $M = n_c/n_b$, and $n_c$ is the cloud density, $n_b$ is the background density, and $V_n$ is the neutral wind velocity. For mid-latitude barium releases at $-180$ km we estimate $L \sim 160 \sim 480$ m which is consistent with observations.

The organization of the paper is as follows. In the next section we present the assumptions and general equations used in the analysis. In Section III we derive the dispersion equation and in Section IV we present analytical and numerical results. In Section V we investigate the possibility of exponentially growing global eigenmodes. Finally, in Section VI we summarize our findings and discuss the application of our theory to the evolution of ionospheric barium clouds.
II. GENERAL EQUATIONS AND EQUILIBRIUM

The general three dimensional equations for a warm plasma cloud in a uniform magnetic field \( \mathbf{B} = B \mathbf{e}_z \) and a background neutral wind \( \mathbf{V}_n = V_n \mathbf{e}_x \) (see Fig. 1) are given by (Drake et al., 1985)

\[
\frac{\partial \mathbf{n}}{\partial t} - \frac{B}{\mathbf{e}_z} \nabla \varphi \times \mathbf{V}_n + \frac{1}{\mathbf{e}_z} \frac{1}{\mathbf{n}_e} \left( \frac{3}{2} \frac{\mathbf{e}_\perp}{\mathbf{n}_e} \right) \frac{T_e}{\mathbf{n}_e} \frac{\partial \mathbf{n}}{\partial z} = 0 \tag{1}
\]

\[
\frac{\mathbf{V}_n}{\mathbf{e}_z} \nabla \cdot \mathbf{V}_n + \frac{\mathbf{V}_n}{\mathbf{e}_z} \cdot \nabla \mathbf{V}_n + \frac{\mathbf{V}_n}{\mathbf{e}_z} \cdot \mathbf{V}_n \cdot \nabla \mathbf{V}_n + \frac{1}{\mathbf{e}_z} \frac{1}{\mathbf{n}_e} \left( \frac{3}{2} \frac{\mathbf{e}_\perp}{\mathbf{n}_e} \right) \frac{T_e}{\mathbf{n}_e} \frac{\partial \mathbf{V}_n}{\partial z} = 0 \tag{2}
\]

where \( \eta_\parallel = m_e v_e / n e^2 \) is the parallel resistivity, \( v_e = v_{e1} + v_{en} \), \( D_\parallel = (v_{in} / \Omega_i) c T_i / e B \) is the perpendicular ion diffusion coefficient, \( v_{e1} \) is the electron-ion collision frequency and \( \Omega_\alpha \) and \( v_{an} \) are the cyclotron and neutral collision frequencies of the species \( \alpha \). Equation (1) is the electron continuity equation and (2) arises from charge neutrality \((\mathbf{V}.\mathbf{J} = 0)\). We have considered the electrostatic limit, have assumed \( v_e / \Omega_e, v_{in} / \Omega_i \ll 1 \), and have neglected the ion parallel diffusion \((D_\parallel)\) and perpendicular electron diffusion \((D_\perp)\). We therefore assume that

\[
\frac{\partial \mathbf{n}}{\partial t} \gg D_\parallel \frac{\partial^2 \mathbf{n}}{\partial z^2}, D_\perp \frac{\partial^2 \mathbf{n}}{\partial z^2}.
\]

It is convenient to change variables by defining a new potential

\[
\phi = \phi + \frac{T_i}{e \mathbf{e}_z} \ln(n). \tag{3}
\]
Fig. 1) Cylindrical geometry and plasma configuration used in the analysis.
Equations (1) and (2) then become

\[
\frac{3n}{\delta t} = \frac{c}{B} V_\phi \times z \cdot \dot{V}_n + \frac{3}{\delta z} \frac{1}{\eta_e} \left( \frac{\partial \phi}{\partial z} + \frac{T}{\eta_e} \frac{\partial n}{\partial z} \right) = 0
\]

\[
\frac{c}{B} \frac{\dot{V}_n}{\eta_1} \cdot V \cdot V \phi + \frac{V_i}{\eta_1} \frac{\partial}{\partial z} \times \frac{\dot{V}_n}{\eta_1} + \frac{3}{\delta z} \frac{1}{\eta_e} \left( \frac{\partial \phi}{\partial z} + \frac{T}{\eta_e} \frac{\partial n}{\partial z} \right) = 0
\]

where \( T = T_e + T_i \).

We consider a simple equilibrium consisting of a cylindrical waterbag of radius \( r_c \) and density \( n_c \) in a uniform background \( n_b \):

\[
n_0(r) = n_c H(r_c - r) + n_b
\]

where \( H \) is the Heaviside function. The solution of the equilibrium equations for this configuration are well known. The drag between the cloud and neutral wind polarizes the cloud. The resulting potential \( \phi_0 \) is given by (Smythe, 1950),

\[
\frac{c\phi_0}{B} = \begin{cases} 
- V \frac{M}{n} \frac{r \sin \theta}{M+2} & r < r_c \\
- V \frac{M}{n} \frac{r_c^2}{M+2} \sin \theta & r > r_c
\end{cases}
\]

where \( M = n_c/n_b \). The potential causes the cloud to drift with a uniform velocity

\[
\dot{V}_c = V \frac{M}{n} \frac{\dot{c}}{x^*}
\]
The linear stability analysis which follows is most easily carried out in the frame of reference of the moving cloud. In this frame the potential \( \phi_{0c} \) is given by

\[
\begin{align*}
\frac{c \phi_{0c}}{B} &= 0 & r < r_c \\
\frac{c \phi_{0c}}{B} &= V_n \frac{M}{M+2} (r - \frac{r_c^2}{r}) \sin \theta & r > r_c
\end{align*}
\]

III. LINEARIZED EQUATIONS AND DISPERSION EQUATION.

We investigate the stability of this equilibrium by considering small perturbations \( \bar{n}(r, \theta, z, t) \) and \( \bar{\phi}(r, \theta, z, t) \) around \( n_0 \) and \( \phi_{0c} \). Since the equilibrium is independent of \( z \), we can expand the perturbations in plane waves in the \( z \) direction \( (\bar{n}, \bar{\phi} = \exp ik_z z) \) without loss of generality. In the perpendicular plane such a simple expansion is not generally possible since the equilibrium depends on both \( r \) and \( \theta \). Equations (4) and (5) yield the linearized equations,

\[
\begin{align*}
\frac{\partial \bar{n}}{\partial t} - \frac{c}{B} \nabla \phi_{0c} \times \bar{z} \cdot \nabla \bar{n} + \frac{c}{B} \frac{n_c}{r_c} \frac{\partial}{\partial \theta} \delta (r - r_c) - \frac{k_z}{e n_e} \bar{\phi} - \bar{n} = 0 \quad \tag{9a} \\
\frac{c}{B} \frac{\partial}{\partial \eta_1} \left( \bar{v} \cdot n_0 \bar{\phi} + \bar{v} \cdot n \bar{\phi}_{0c} \right) + \frac{\partial}{\partial \eta_1} \left( \bar{v}_n - \bar{v} \right) \frac{\delta n}{\partial y} - \frac{k_z}{e n_e} \bar{\phi} - \bar{n} = 0 \quad \tag{9b}
\end{align*}
\]

Equations (9) are solved separately in the region \( r > r_c \) and \( r < r_c \), and the solutions are then matched using boundary conditions which are obtained from the equations for \( r = r_c \). In the limit \( k_z = 0 \), this procedure is straightforward since \( \bar{n} = 0 \) and \( \nabla^2 \bar{\phi} = 0 \) for \( r = r_c \). In the case \( k_z = 0 \), the equations for \( \bar{n} \) and \( \bar{\phi} \) in the region \( r = r_c \) are much more complicated so
further simplifications must be made. We limit the calculation to perturbations for which \( \theta/\theta' >> 1 \). In this limit the \( \theta \) dependence of \( \tilde{n} \) and \( \tilde{\phi} \) can be represented by the eikonal \( \tilde{\phi}, \tilde{n} = \exp[iS(\theta)] \) where

\[
\nabla S = ik_\theta(\theta) \tilde{\phi},
\]

(10)

In this limit the perturbations are strongly localized around the boundary \( r_c \) and decay exponentially away from this boundary. Thus, for \( r = r_c \) we write

\[
\tilde{\phi}_+ = \tilde{\phi}_+ \exp[iS(\theta)] \exp[ik_\theta(r - r_c)],
\]

(11)

where + and - refer to the region \( r > r_c \) and \( r < r_c \), respectively. The specific range of parameters for which the form of \( \tilde{\phi} \) given in (11) is valid will be presented later. Similar expressions can be written for \( \tilde{n}_+ (r, \theta) \).

Finally, we assume \( \tilde{n} \) and \( \tilde{\phi} \) grow exponentially in time with a growth rate \( Y \). For the present, \( Y \) must be considered a local growth rate \( Y(\theta) \).

Eventually, we will investigate under what circumstances exponentially growing normal modes of the cloud can exist.

With the form of \( \tilde{\phi} \) (and \( \tilde{n} \)) given in (11), (9a) and (9b) reduce to algebraic expressions for \( k_{r=\pm} \) in the regions \( r = r_c \),

\[
k_{r=}^2 - k_{\theta}^2 - \frac{\alpha k_{z}^2}{\gamma} \left[ Y + ik_\theta \frac{2M}{M+2} V_n \sin\theta \right. \\
- \frac{2}{M+2} V_n \frac{\sinh}{n} \left[ ik_\theta \cos\theta - k_{r=1}(1+\theta) \sin\theta \right] = 0
\]

(12a)

\[
k_{r=}^2 - k_{\theta}^2 - \frac{\alpha k_{z}^2}{\gamma} \left[ Y - \frac{2}{M+2} V_n \frac{\sinh}{n} \left[ ik_\theta \cos\theta + k_{r=1} \sin\theta \right] \right] = 0
\]

(12b)
\[ Y_+ = Y + k_z^2 D_{\parallel e} + i k_{\theta} \frac{2M}{M+2} v_n \sin \theta, \quad \text{(12c)} \]

\[ Y_n = Y + k_z^2 D_{\parallel e}', \quad \text{(12d)} \]

\[ \alpha = \frac{\rho_{\parallel 1}/v_{\parallel 0}}{\rho_{\parallel 0} v_{\parallel 0} \ln \gg 1}, D_{\parallel e} = T/m e v_{\parallel e} \] is the parallel electron diffusion coefficient and

\[ Y_z \hat{n}_z = k_z^2 \hat{\psi}_z/e n_{\parallel e}, \quad \text{(13)} \]

The matching conditions for \( \hat{n}_z, \hat{\psi}_z \) and \( k_{\parallel e} \) at \( r = r_c \) can be derived from (9). The radial \( E \times B \) convection of the cloud causes the density \( \bar{n} \) to be singular at the cloud boundary (third term in the continuity equation). It is convenient therefore to separate out the singular behavior of \( \bar{n} \) so that in the region \( r = r_c \)

\[ \bar{n}(r, \theta) = [\bar{N} \delta(r - r_c) + \hat{n}_z] \exp[iS(\theta)]. \quad \text{(14)} \]

The potential \( \hat{\psi} \) remains finite at the boundary so (9a) can be written as

\[ [Y + k_z^2 D_{\parallel e} + (c/B) \frac{i k_{\theta} \phi'_{0c}}{n_{\parallel e}}] \bar{N} + (c/B) \frac{i k_{\theta} n_{\parallel e} \bar{\phi}}{n_{\parallel e}} = 0. \quad \text{(15)} \]

for \( r = r_c \). Equation (15) now contains no singularities at \( r = r_c \). However, \( \phi'_{0c} \) is discontinuous across \( r = r_c \) since it is zero for \( r < r_c \) and finite for \( r > r_c \). The parallel diffusion coefficient \( D_{\parallel e} \) is also generally not continuous across the boundary. These discontinuities must be balanced by a corresponding jump in the potential \( \bar{\phi} \) at the boundary.
Thus, from (8) and (15), we find

$$\hat{\phi}_+/Y_+ = \hat{\phi}_- / Y_-.$$  \hspace{1cm} (16)

This condition can also be derived from the requirement that the tangential electric field across the boundary of the plasma cloud be continuous. The continuity equation then reduces to

$$Y_+ \tilde{N} + (c/B) i k_\theta n_c \hat{\phi}_- = 0.$$  \hspace{1cm} (17)

The matching condition for $k_{r+}$ can be derived by integrating (9b) across the boundary,

$$- n_b k_{r+} \hat{\phi}_+ - (n_b + n_c)k_{r+} \hat{\phi}_- + \frac{2V}{c k z (M+2)} [i k_\theta \cos \theta \tilde{N} + (1+M) \sin \theta \hat{n}_+ \]

$$- \sin \theta \hat{n}_-] + \frac{(a_+ + a_-)}{2} k^2 \frac{T}{e \tilde{N}} = 0.$$  \hspace{1cm} (18)

Equations (13) and (16)-(18) can then be combined into a single relation between $k_{r+}$ and $k_{r-}$,

$$k_{r+} Y_+ + (1+M) k_{r-} Y_- - \frac{2V}{M+2} [M k^2 \cos \theta + (1+M)(a_+ - a_-) \frac{V_{in}}{n_i} k_z^2 \sin \theta]$$

$$+ \epsilon (a_+ + a_-) \frac{M}{2} k_z^2 \epsilon \theta e B = 0.$$  \hspace{1cm} (19)

Equations (12) and (19) constitute a local dispersion equation for the $E \times B$ instability with $k_z = 0$ based on a 2D waterbag equilibrium.
IV. EVALUATION OF THE LOCAL DISPERSION EQUATION

We first evaluate the local growth rate \( \gamma(\theta) \) by solving the dispersion equation given by (12) and (19). We consider two limits: the cold plasma limit \((T = 0)\) and the warm plasma limit \((T \neq 0)\).

A. Cold Plasma Limit: \( T = 0 \)

In the cold plasma limit (19) simplifies to

\[
kr_+ \gamma_+ + (1+M)kr_- \gamma_- = \frac{2v_n}{M+2} \left[Mk_\theta^2 \cos\theta + (1+M)(\alpha_+ - \alpha_-) \frac{V_{in}}{n_1} k_z^2 \sin\theta \right] \tag{20a}
\]

where

\[
\gamma_+ = \gamma + 2ik_\theta MV_{in} \sin\theta/(M+2) \tag{20b}
\]

\[
\gamma_- = \gamma \tag{20c}
\]

since \( D_e T = 0 \). Equations (12c) and (12b) for \( kr_+ \) remain unchanged. In the limit of small \( k_z \), (12a) and (12b) yield

\[
k_{r+} = k_\theta \tag{21a}
\]

and the growth rate \( \gamma \) can then be calculated from (20),

\[
\gamma = m\gamma_0 (\cos\theta - i \sin\theta) \tag{21b}
\]

\[
\gamma_0 = \frac{2M}{(M+2)^2} \frac{V_n}{r_c} \tag{21c}
\]
where \( m = k_g r_c \) is the poloidal mode number. The growth rate peaks at \( \theta = 0 \) (the backside of the plasma cloud) and is linearly proportional to \( m \). The growth rate in (21) is consistent with previous rigorous and heuristic investigations of the stability of circular waterbag models of plasma clouds (Overman et al., 1983; Zalesak and Huba, 1984). For \( \theta = 0 \), the second term on the right side of (21b) causes the mode to propagate at a finite frequency. This propagation results as the fluid outside of the cloud convects around the circular boundary and carries the perturbation. The point \( \theta = 0 \) corresponds to a stagnation point of the flow so there is no finite frequency there. Finally, from (21a) the assumption that the modes are strongly localized around the cloud boundary requires \( m \gg 1 \).

To obtain analytic expressions for the growth rate of modes with \( k_z = 0 \), we consider only the case \( \theta = 0 \) which corresponds to the most unstable mode. Later we will present numerical solutions of the more general dispersion relation in which this restriction is relaxed. For simplicity, we also assume that \( \nu_e = \nu_en \gg \nu_ei \) so that \( \alpha_n = \alpha_+ = \alpha_0 \).

For \( \theta = 0 \), we have \( k_{r+} = k_{r-} = k_r \) and \( \gamma_+ = \gamma_- = \gamma \) and the dispersion equations are given by,

\[
\begin{align*}
\gamma k_{r+} r_c &= m^2 \gamma_0, \\
k_{r-}^2 r_c^2 &= m^2 + \lambda^2 \left( 1 - \frac{m+2}{M} \frac{m \gamma_0 \nu_{in}}{\nu_{in}} \right)
\end{align*}
\]  

with the solution

\[
\gamma = m \gamma_0 \left[ m^2 (m^2 + \lambda^2) - \lambda^4 \left( \frac{m+2}{4M^2} \frac{\nu_{in}^2}{\nu_{in}} \right)^{1/2} + \lambda^2 \frac{m+2}{2M} \frac{\nu_{in}}{\nu_{in}} (m^2 + \lambda^2)^{-1} \right]^{1/2}
\]  

(23a)
where

\[ \lambda = a_0^{1/2} k_{z} r_{c}. \]  

(23b)

As \( k_{z} \) is increased from zero, over the interval \( \lambda < m \) the growth rate is roughly given by its \( k_{z} = 0 \) limit. For

\[ 1 \ll \lambda^2/m^2 \ll \Omega_i^2/v_{in}^2 \]  

(24a)

the growth rate decreases with \( k_{z} \) as

\[ \gamma = m^2 \Omega_0/\lambda. \]  

(24b)

Over this range \( k_{z} = a_0^{1/2} k_{z} \gg k_{a} \) so that \( k_{z} \) causes the mode to become more localized near the cloud boundary. For

\[ \lambda^2 = a_0 k_{z}^2 r_{c}^2 > \frac{4 M^2}{(M+2)^2} \frac{a_i^2}{m^2} \]  

(25a)

the mode is stable. Note that (25a) implies that the lowest order poloidal modes are stabilized first as \( k_{z} \) is increased. The stability condition given in (25b) is only valid for \( v_{e1} \ll v_{en} \). In the opposite limit \( v_{e1} \gg v_{en} \), \( a_- = a_0/(1+M) \) and the stability condition can be similarly derived,

\[ \lambda^2 = a_0 k_{z}^2 r_{c}^2 > \frac{4 M^2}{[1 + (M+1)^{1/2}]^2} \frac{a_i^2}{m^2}. \]  

(25b)
To confirm the analytic results for \( T = 0 \) and to generalize the results for \( \theta = 0 \), we present Figs. 2 and 3 which are obtained by solving (12) and (19) numerically. In each figure we plot (a) the growth rate \( Y/Y_0 \) vs. \( \lambda = 1/2 k_z R \) and (b) the real frequency \( \omega_r/Y_0 \) vs. \( \lambda \) for the parameters \( m = 1, v_i/\Omega_i = 0.025, M = 2, \) and \( D_e = 0 \) (i.e., \( T = 0 \)), and several values of \( \theta \): (A) \( \theta = 0^\circ \), (B) \( \theta = 22.5^\circ \), (C) \( \theta = 45^\circ \), (D) \( \theta = 67.5^\circ \), and (E) \( \theta = 90^\circ \). In Fig. 2 we have taken the limit \( v_e = v_{en} >> v_{ei} \) while in Fig. 3 we consider the opposite limit, \( v_e = v_{ei} >> v_{en} \).

We note the following from Fig. 2 (the limit \( v_{en} >> v_{ei} \)). First, the growth rate \( Y \) is a maximum when \( k_z = 0 \), and decreases as \( \theta \) increases. For \( \theta = 90^\circ \) there is no growth. On the other hand, the real frequency \( \omega_r \) increases in magnitude as a function of \( \theta \) for \( \lambda = 0 \), i.e., \( k_z = 0 \). These points are consistent with (21). Second, for all values of \( \theta \), \( Y \) decreases as \( \lambda \) increases which is consistent with (24b), while \( \omega_r \) remains roughly constant. Finally, the modes become purely propagating for sufficiently large \( \lambda \), i.e., \( Y = 0 \) but \( \omega_r \neq 0 \). The value of \( \lambda \) denoted by the arrow in Fig. 2a corresponds to the stabilization point predicted by (25a) which is in excellent agreement with the numerical results. We also note that the stabilization point is independent of \( \theta \).

We now discuss Fig. 3 (the limit \( v_{ei} >> v_{en} \)). First, for \( \lambda = 0 \) we note that \( Y \) decreases as \( \theta \) increases, while \( \omega_r \) increases in magnitude; this is similar to the results in Fig. 2. Second, in contrast to Fig. 2, we find that a secondary maxima occurs in \( Y \) at \( \lambda = 36 \), and that for finite \( k_z \), modes at \( \theta = 90^\circ \) are unstable. This enhanced instability at finite values of \( k_z \) is due to the term proportional to \( (a_+ - a_-) \sin \theta \) in (20a). For \( v_{en} >> v_{ei} \) we have \( a_+ = a_- \) so that this term does not contribute to the growth of the mode. However, for \( v_{ei} >> v_{en} \) we note that \( a_+ = a_- \) so that
Fig. 2) Plot of $\omega/\omega_0$ vs. $\lambda = a_{1/2} k_{zc} r$ in the limit $v_{en} \gg v_{ei}$. The parameters used are $m=1$, $v_1/\Omega_1 = 0.025$, $M=2$, $D_{ze} = 0$ (i.e., $T = 0$), and several values of $\theta$: (A) $\theta = 0^\circ$, (B) $\theta = 22.5^\circ$, (C) $\theta = 45^\circ$, (D) $\theta = 67.5^\circ$, and (E) $\theta = 90^\circ$. The arrow denotes the value of $\lambda$ given by (25a) for marginal stability. (a) Plot of the growth rate $\gamma/\gamma_0$ vs. $\lambda$. (b) Plot of the real frequency $\omega_r/\gamma_0$ vs. $\lambda$. 
Fig. 3) Plot of $\omega/\omega_0$ vs. $\lambda = \alpha_{b}^{1/2} k_{z} r_{c}$ in the limit $v_{e1} \gg v_{en}$. The parameters are the same as in Fig. 2. The arrow denotes the value of $\lambda$ at marginal stability given by (25b). (a) Plot of the growth rate $\gamma/\gamma_0$ vs. $\lambda$. (b) Plot of the real frequency $\omega_r/\gamma_0$ vs. $\lambda$. 
this term is finite and positive for $0 < \theta < \pi$. Finally, the modes are stable for sufficiently large $\lambda$. The arrow in Fig. 3a denotes the stability condition given in (25b) for $\theta = 0^\circ$; again this value is in excellent agreement with the numerical results.

B. Warm Plasma Limit: $T = 0$

We only analytically investigate modes at $\theta = 0$ and again consider the limit $\nu_{en} >> \nu_{ei}$. Equations (12) and (19) reduce to

\[ k_r c_0 \gamma_r = m(m\gamma_0 - i \frac{M}{M+2} \frac{\nu_{in}}{\nu_{ei}} k_r^2 D) \]  \hspace{1cm} (26a)

\[ k_r^2 c_0 = m^2 + \lambda^2 (\gamma + k_r^2 D) \]  \hspace{1cm} (26b)

where $k_r c_0 = k_r$, $\gamma_+ = \gamma_- = \gamma + k_r^2 D$ and we have neglected the terms proportional to $\nu_{in}/\nu_{ei}$ in (26b) for $k_r$. This approximation is justified for sufficiently large $T$ (to be proven later).

For $k_r + 0$, the finite temperature corrections drop out of (26) and the growth rate reduces to $\gamma = m\gamma_0$ as in the cold plasma limit. As $k_r$ increases the second term on the right side of (26a) becomes comparable to the driving term $\gamma_0$ when $k_r^2 D \nu_{in} / \nu_{ei} \ll \gamma_0$. At this point $k_r^2 D$ can still be neglected compared with $\gamma = \gamma_0$ in the definition of $\gamma_+$. Thus, the eigenvalue $\gamma$ is given by

\[ \gamma = \frac{(m\gamma_0 - i\omega_0)m}{(m^2 + \lambda^2)^{1/2}} \]  \hspace{1cm} (27a)

where

\[ k_r^2 c_0 = m^2 + \lambda^2 \]  \hspace{1cm} (27b)
\[
\omega_0 = \frac{M c T}{M+2} \frac{\lambda^2}{r_c^2}
\]

(27c)

Thus, \( k_z \) decreases the growth rate as in the \( T = 0 \) limit and causes the mode to propagate at a finite frequency which increases with \( k_z \).

When \( k_{z \parallel}^2 e \gg Y_0 v_{\text{in}} / a_1 \), the modes to lowest order simply propagate at the frequency given in (27a). To calculate the growth rate in this limit, \( k_{z \parallel}^2 e \) can no longer be neglected compared with \( Y \) in \( Y_\perp \). In (26b) we assume \( k_{z \parallel}^2 e \ll Y \) so that

\[
k_{z \parallel} r_c = (m^2 + \lambda^2)^{1/2} - \frac{1}{2} \frac{k_{z \parallel}^2 e}{Y}
\]

and from (26a)

\[
\gamma = (-i \omega_0 + m Y_0) \frac{m}{(m^2 + \lambda^2)^{1/2}} - k_{z \parallel}^2 e \frac{m^2 + \lambda^2 / 2}{(m^2 + \lambda^2)}.
\]

(28a)

The \( E \times B \) gradient drift instability is stable for

\[
 k_{z \parallel}^2 e \frac{m^2 + \lambda^2 / 2}{(m^2 + \lambda^2)^{1/2}} > m^2 Y_0.
\]

(28b)

Note that this inequality again implies that the lowest poloidal mode numbers are stabilized first. The assumption that \( Y \gg k_{z \parallel}^2 e \) is valid at the stability point for

\[
k_y R_1 \gg (v_n / v_\text{in}) v_{\text{in}}^2 / a_1^2.
\]

(28c)

Also, the neglect of the terms proportional to \( v_{\text{in}} / a_1 \) in (12a) and (12b) near the stability point are valid in this same limit. Thus, when (28c) is
satisfied, finite temperature effects stabilize the $E \times B$ instability, and
the stability point is given in (28b). In the opposite limit

$$k_\theta p_1 \ll \left(\frac{v_n}{v_1}\right) \frac{\nu}{\nu_1},$$

(29)

the cold plasma effects stabilize the mode and the stability point is given
in (25). An expression similar to that given in (28b) can again be derived
in the limit $v_{ei} \gg v_{en}$.

To verify our analytic results for $T = 0$, and to extend these results
to $\theta = 0$, as well as to both $v_{en} \gg v_{ei}$ and $v_{ei} \gg v_{en}$, we present numerical
solutions to the complete dispersion equation [(12) and (19)]. In Figs. 4
and 5 we plot (a) the growth rate $\gamma$ vs. $\lambda$, and (b) the real frequency $\omega_r$
vs. $\lambda$ for the parameters $m=1$, $v_1/\Omega_1 = 0.025$, $M = 2$, $b \nu_{he}/a b_0^2 \gamma = 5.0 \times
10^{-4}$, and several values of $\theta$: (A) $\theta = 0^\circ$, (B) $\theta = 22.5^\circ$, (C) $\theta = 45^\circ$,
(D) $\theta = 67.5^\circ$, and (E) $\theta = 90^\circ$. In Fig. 4 we consider the limit $v_e =
v_{en} \gg v_{ei}$, while in Fig. 5 we take $v_{e} = v_{ei} \gg v_{en}$. Thus, Fig. 4 corre-
ponds to the warm temperature limit of Fig. 2 and Fig. 5 corresponds to
that of Fig. 3.

In Fig. 4 we note several similarities to Fig. 2. The growth rate $\gamma$
decreases as $\theta$ or $k_z$ increases and the mode is stable ($\gamma < 0$) for
sufficiently large $k_z$. On the other hand, for $T = 0$ (or $D_{ie} = 0$) the mode
stabilizes at a much smaller value of $k_z$ (other parameters being equal) and
the stability point is sensitive to $\theta$. The arrow denotes the value of $\lambda$
for marginal stability ($\gamma = 0$) based on (28b) for $\theta = 0^\circ$. This value
($\lambda = 15.8$) is in very good agreement with the numerical results ($\lambda = 15.5$).

In Fig. 5 there are also similarities to Fig. 3. For $k_z = 0$, the
growth rate $\gamma$ decreases and the real frequency $\omega_r$ increases in magnitude
Fig. 4) Plot of $\omega/\gamma_0$ vs. $\lambda = \alpha_{bc}^{1/2} k r_c$ in the limit $\gamma_0 \gg \gamma_1$. The parameters are the same as in Fig. 2 except we assume $T = 0$ and take $D_{bc}^b a_{bc} r_c^2 \gamma_0 = 5.0 \times 10^{-4}$. (a) Plot of the growth rate $\gamma/\gamma_0$ vs. $\lambda$. (b) Plot of the real frequency $\omega_r/\gamma_0$ vs. $\lambda$. 
Fig. 5) Plot of $\omega/\omega_0$ vs. $\lambda = a_0^{1/2} k_2 r_c$ in the limit $v_{ei} >> v_{en}$. The parameters are the same as in Fig. 4.
as $\theta$ increases. A secondary maxima (or plateau) in $\gamma$ occurs for a finite value of $k_z$. However, in contrast to Fig. 2, the value of $k_z$ at marginal stability increases with $\theta$ so that the most difficult mode to stabilize is at $\theta = 90^\circ$. Also, the value of $k_z$ at marginal stability is smaller than when $T = 0$.

It is also illuminating to calculate the damping rate of the instability in the limit of very large $k_z$. There are two modes of the system. The first damps at the ion diffusion rate,

$$\gamma = -i k_r \frac{k_z^2 D}{\perp}$$

(30a)

where

$$k_r = -i k_\theta \frac{\Omega_i}{\nu_{in} M^2}$$

(30b)

and $D = (cT/eB)\nu_{le}/\Omega_i$ is the ion diffusion coefficient based on the total temperature $T = T_e + T_i$. The second damps at the electron parallel diffusion rate

$$\gamma = -k_z^2 D_{le}.$$  

(31)

V. GLOBAL EIGENFUNCTIONS

In Section III we derived a local dispersion equation [(12) and (19)] which was solved to obtain the growth rate $\gamma(\theta)$ in Section IV. This local growth rate is peaked on the backside of the cloud ($\theta = 0$). We now investigate under what conditions the gradient drift instability forms a global eigenmode as it grows on the cylindrical waterbag.

22
Such global eigenmodes can exist if solutions $\tilde{\phi}(\theta)$ can be constructed which are localized around some angle $\theta_0$. To obtain these solutions we take the growth rate in (12) and (19) to be independent of $\theta$ so that the dispersion equation yields $k'_0(\theta, \gamma)$. We then make the identification $k_0 = -i r_c^{-1} \partial / \partial \theta$ to obtain a differential equation for $\tilde{\phi}(\theta)$.

In the limit $k^2 - \alpha k^2 r_c^2 << 1$, (12) yields $k_{r_a} = k_0$ and the local dispersion equation in (19) becomes

$$
\gamma_0 \partial^2 \tilde{\phi}/\partial \theta^2 - i \gamma \exp(i\theta) \tilde{\phi} + \omega_0 \tilde{\phi} = 0. \quad (32)
$$

Since this equation is first order in $\partial / \partial \theta$, there are no bounded solutions so that there are no exponentially growing global eigenmodes when $k_z$ is small. This result is consistent with previous calculations in the limit $k_z = 0$ where it was shown that the energy always cascades to lower poloidal mode numbers (Overman et al., 1983). The local growth rate [see (11)] is the approximate rate of increase of the amplitude of a broad spectrum of modes centered around $k_0$.

We now consider the case where $\lambda^2 >> 1$ while $k_z$ is sufficiently small so that the terms proportional to $\nu_{m}/Q_i$ in (12a) and (12b) can be neglected. We also assume that $\gamma >> k_{zD}^2 e_i$ and $v_{en} >> v_{ei}$. With these constraints (12a) and (12b) simplify to

$$
k_{r_a} = \alpha^{1/2} k_z
$$

and (19) yields the equation

$$
\gamma_0 \cos \theta \frac{\partial^2}{\partial \theta^2} \tilde{\phi} + \left[ \omega_0 + \gamma_0 \lambda \sin \theta \right] \frac{\partial}{\partial \theta} \tilde{\phi} + \gamma \lambda \tilde{\phi} = 0. \quad (33)
$$
We simplify this equation by defining a new dependent variable

\[ \tilde{\phi} = \phi \exp[-f(\theta)] \]

with \( \partial f/\partial \theta = \left[ \omega_0 + \gamma_0 \lambda \sin \theta \right]/2\gamma_0 \cos \theta \) and find that

\[ \partial^2 \tilde{\phi}/\partial \theta^2 + V(\theta) \tilde{\phi} = 0 \]

(34a)

where the potential \( V(\theta) \) is given by

\[
V(\theta) = \left[ \left( \frac{Y}{Y_0} \cos \theta - \frac{1}{2} \right) \lambda - \frac{\omega_0}{2\gamma_0} \sin \theta \right.
\]

\[ - \frac{1}{4\gamma_0^2} \left[ \omega_0 + \gamma_0 \lambda \sin \theta \right]^2 \left\{ \frac{1}{\cos^2 \theta} \right\}. \]

(34b)

We first consider the limit \( T = 0 \) and expand \( V(\theta) \) around \( \theta = 0 \),

\[ V = \lambda \left( \frac{Y}{Y_0} - \frac{1}{2} \right) - \frac{1}{4} \lambda^2 \theta^2. \]

(35)

The bound state solutions for this potential have eigenvalues

\[ \gamma = \gamma_0 (n+1), \]

(36)

where \( n \) is a non-negative integer. The mode is localized on the backside of the cloud and has an angular width \( \Delta \theta \sim \lambda^{-1/2} \ll 1 \), which decreases with increasing \( k_z \). Thus, the expansion of \( V \) around \( \theta = 0 \) is valid for \( \lambda \gg 1 \). Note also that higher order modes \( n \), which have more structure in the
poloidal direction, have larger growth rates. This result is consistent with local theory where \( Y \) increases with the poloidal mode number \( m \).

The potential \( V \) in (34) is modified by finite thermal effects when \( \omega_0 - Y_0 \lambda \). Comparing the magnitude of the various terms in (34), we find that the term proportional to \((4Y_0^2)^{-1}\) is of order \( \lambda > 1 \) larger than the remaining terms unless the terms within the bracket cancel. We therefore look for a mode localized around the angle \( \theta_0 \) defined by

\[
\sin \theta_0 = -\omega_0 / Y_0 \lambda. \tag{37}
\]

Near \( \theta_0 \) the potential assumes the form

\[
V(\theta) = \lambda \frac{Y}{Y_0} \left( \frac{1}{\cos \theta_0} - \frac{1}{2} \lambda - \frac{1}{4} \lambda^2 (\theta - \theta_0)^2 \right). \tag{38}
\]

The bounded solutions have eigenvalues

\[
\gamma = Y_0 (n+1) \left[ 1 - \frac{\omega_0^2}{Y_0^2 \lambda^2} \right]^{1/2}, \tag{39}
\]

where \( n \) is again a non-negative integer. When \( \omega_0 \ll Y_0 \lambda \), the growth reduces to the previous zero temperature result in (36). As \( \omega_0 \) increases the growth rate decreases until the mode becomes stable at \( \omega_0 = Y_0 \lambda \) or

\[
a^{1/2} k_z \frac{c T}{e B} > \frac{2}{M+2} \nu_n. \tag{40}
\]

The reduction of the growth rate in (39) is a consequence of the localization of the mode at an angle \( \theta_0 = 0 \). The growth rate in (39) can be rewritten as
\[ Y = Y_0(n+1) \cos \theta_0. \]

At the marginal stability point, \( \theta_0 = \pi/2 \), i.e., the mode localizes in a region where there is no driving force. Above the threshold in (40) there are no bounded solutions to (34) since \( \theta_0 \) moves into the complex plane.

The physics behind the localization of the mode can be readily understood. As we discussed previously in Sec. IV, the convection of the background plasma past the cloud causes the mode to propagate with a frequency [see (21b)]

\[ \omega_r = mY_0 \sin \theta. \]  \quad (41)

On the other hand, the thermal effects cause the mode to propagate with a frequency [see (27a)]

\[ \omega_r = m \omega_0 / \lambda. \]  \quad (42)

These two frequencies balance each other at the angle \( \theta_0 \) given in (37). The mode localized at this angle has no real frequency and grows at the local growth rate corresponding to this angle. When the propagation rate due to the thermal effects is everywhere larger than the convection flow no localized solutions exist. In this regime the \( E \times B \) gradient drift instability is effectively stable since, unlike the limit where \( k_z = 0 \), the rate of propagation of the mode from the unstable backside of the cloud to the stable frontside exceeds the rate of growth of the mode. Convective amplification of perturbations in this finite \( k_z \) limit is therefore not significant.
VI. SUMMARY AND CONCLUSION

We have investigated the influence of finite parallel wavelength on the stability of a cylindrical plasma cloud. We first derived a dispersion relation for the local growth rate $\gamma(\theta)$ of the $E \times B$ gradient drift instability, where $\theta$ is the poloidal angle ($\theta = 0$ on the "backside" of the cloud as shown in Fig. 1). For sufficiently large values of the parallel wavenumber $k_z$ the local growth rate is negative for all values of $\theta$. In the cold plasma limit,

$$ k_0^{n_1} < \left( \frac{\nu_n}{\nu_i} \right) \nu_{in}/n_i^2 $$

(43)

the stability criterion is given in (25a) while in the warm plasma limit [the inequality in (43) is reversed], the stability condition is given in (28b). In both cases the lowest poloidal mode numbers are stabilized at the smallest values of $k_z$.

We have also investigated under what conditions exponentially growing global eigenmodes can exist. In the limit

$$ \lambda^2 = \alpha k_z^2 \ll 1 $$

(44)

with $\alpha = \Omega_i e_i / \nu v_{in}$ there are no exponentially growing solutions. In this limit the energy cascades to lower poloidal mode numbers as the instability grows as found by Overman et al. (1983). When $\lambda^2 >> 1$ a localized mode of angular width $\theta - \lambda^{-1/2} \ll 1$ forms on the "backside" of the plasma cloud and grows exponentially in time. Thus, the cascade of energy to lower mode numbers no longer takes place when $k_z$ is sufficiently large. The structure of the mode for this case is illustrated in Fig. 6a. This figure is drawn
Fig. 6) Schematic of the plasma cloud, flow field, and position of a growing eigenfunction: (a) \( T = 0 \); (b) \( T \neq 0 \). See text for a detailed description.
in the rest frame of the circular cloud. The neutral wind is moving from
the left to right with a uniform velocity $2V_n/(M+2)$ while the background
plasma flows to the left with a velocity $V_b$. The dashed line illustrates
the amplitude of the lowest order mode. At still larger values of $k_z$ the
unstable mode becomes localized at a finite angle $\theta_0$ given by

$$\sin\theta_0 = -\frac{V_d}{V_b}$$  \hspace{1cm} (45)

where

$$V_d = M \frac{cT}{eB} \alpha^{1/2} k_z$$

is the effective diamagnetic velocity and

$$V_b = \frac{2M}{M+2} V_n$$

with $M = n_c/n_b$. For a waterbag distribution the density scale length which
usually appears in definition of the diamagnetic drift velocity is replaced
by the radial scale length, $\alpha^{-1/2} k_z$. For $V_d > V_b$ or

$$\alpha^{1/2} k_z \frac{cT}{eB} > \frac{2}{M+2} V_n$$  \hspace{1cm} (46)

the mode is completely stable. The physical mechanism which causes the
localization at $\theta_0$ as well as the stabilization is illustrated in Fig.
6b. In the local dispersion relation the diamagnetic effects cause the
mode to propagate in the poloidal direction with a frequency

$$\omega = k_\theta V_d/(M+2),$$
where $k_\theta$ is the poloidal wavenumber. The velocity of the background plasma just outside the cloud (in the reference frame of the cloud) is given by

$$v = v_b \sin \theta.$$

The convection of the background plasma past the cloud causes the mode to propagate with a frequency

$$\omega = k_\theta v_b \sin \theta.$$

These two frequencies balance to produce a non-propagating mode at the angle $\theta_0$ defined in (45). The dashed line in Fig. 6b illustrates the localization of the mode in this case. When the diamagnetic velocity everywhere exceeds the flow of the background plasma around the cloud the gradient drift instability is convectively stabilized.

Finally, we apply these results to the structuring of barium clouds and discuss their application to the so-called "striation freezing" phenomenon (Linson and Meltz, 1972). Basically, it has been observed that barium clouds released in the ionosphere structure because of the $E \times B$ gradient drift instability and develop field-aligned striations. The first generation of striations can also undergo further structuring, at times, and break up into even smaller striations (i.e., smaller in size transverse to $B_0$). This process, known as bifurcation, appears to continue until a minimum transverse scale size is reached (which we refer to as the "freezing scale length"). For barium clouds released at altitudes ~180 km the freezing scale length is roughly 400 m (Prettie, 1985). A number of
studies have been carried out which address this problem (Francis and Perkins, 1975; McDonald et al., 1981; Zalesak et al., 1984; Drake et al., 1985; Sperling and Glassman, 1985). Rather than describe the detailed processes proposed in these papers, it is sufficient to note that there is no generally accepted model of striation freezing at this time; each model has its merits and shortcomings. Although the theory presented in this paper does not explicitly predict a "freezing scale length", we can make an estimate of this size based on (46) and a simple physical argument. The free parameter in (46) is $k_z$; all other parameters are determined by ionospheric conditions. Thus, we need to make a reasonable estimate of $k_z$. We do this by noting that transverse perturbations can map parallel to the magnetic field. The relationship between parallel and perpendicular scale lengths is approximately given by $L_{||} - (a/L_\perp)^{1/2} = a^{1/2}L_\perp$ where $L_\perp$ and $a$ refer to the scale size and conductivity, respectively (Farley, 1959; Goldman et al., 1976). Assuming that $k_z = L_{||}^{-1} - (a^{1/2}L_\perp)^{-1}$ we find that (46) can be written as

$$L_{\perp} < \frac{M+2}{2} \frac{cT}{eB} V_n.$$  \hspace{1cm} (47)

Thus, (47) suggests that barium cloud striations with transverse dimensions smaller than $L_\perp$ would be stable to further structuring by the $E \times B$ gradient drift instability. For typical barium cloud ionospheric parameters at 180 km, i.e., $T = T_e + T_i = 0.2$ eV, $B = 0.5$ G, $V_n = 50$ m/sec, and $M = 2-10$, we find that $L_\perp = 160-480$ m which is consistent with observations. Of course this result is predicated on the assumption that $k_z = (a^{1/2}L_\perp)^{-1}$ which, although plausible, is somewhat ad hoc. In order to remove this assumption it is necessary to consider the finite length of a barium cloud.
which introduces a physical parameter which will remove the arbitrariness associated with $k_z$. We are presently developing such a model.

ACKNOWLEDGMENTS

We thank Steve Zalesak and Norm Zabusky for helpful discussions. This research has been supported by the Defense Nuclear Agency.
REFERENCES


ADA 141284
DISTRIBUTION LIST

DEPARTMENT OF DEFENSE

ASSISTANT SECRETARY OF DEFENSE
COMM, CMD, CONT 7 INTELL
WASHINGTON, DC 20301

DIRECTOR
COMMAND CONTROL TECHNICAL CENTER
PENTAGON RM BE 685
WASHINGTON, DC 20301
01CY ATTN C-650
01CY ATTN C-312 R. MASON

DIRECTOR
DEFENSE ADVANCED RSCH PROJ AGENCY
ARCHITECT BUILDING
1400 WILSON BLVD.
ARLINGTON, VA 22209
01CY ATTN NUCLEAR MONITORING RESEARCH
01CY ATTN STRATEGIC TECH OFFICE

DEFENSE COMMUNICATION ENGINEER CENTER
1860 WIEHLE AVENUE
RESTON, VA 22090
01CY ATTN CODE R410
01CY ATTN CODE R812

DIRECTOR
DEFENSE NUCLEAR AGENCY
WASHINGTON, DC 20305
01CY ATTN STVL
04CY ATTN TITL
01CY ATTN DDST
03CY ATTN RAAE

COMMANDER
FIELD COMMAND
DEFENSE NUCLEAR AGENCY
KIRTLAND AFB, NM 87115
01CY ATTN FCPR

DEFENSE NUCLEAR AGENCY
SAO/DNA
BUILDING 20676
KIRTLAND AFB, NM 87115
01CY D.C. THORNBURG

DIRECTOR
INTERSERVICE NUCLEAR WEAPONS SCHOOL
KIRTLAND AFB, NM 87115
01CY ATTN DOCUMENT CONTROL

JOINT PROGRAM MANAGEMENT OFFICE
WASHINGTON, DC 20330
01CY ATTN J-3 WMCCS EVALUATION OFFICE

DIRECTOR
JOINT STRAT TGT PLANNING STAFF
OFFUTT AFB
OMAHA, NE 68113
01CY ATTN JSTPS/JLKS
01CY ATTN JPST G. GOETZ

CHIEF
LIVERMORE DIVISION FLD COMMAND DNA
DEPARTMENT OF DEFENSE
LAWRENCE LIVERMORE LABORATORY
P.O. BOX 808
LIVERMORE, CA 94550
01CY ATTN FCPRL

COMMANDANT
NATO SCHOOL (SHAPE)
APO NEW YORK 09172
01CY ATTN U.S. DOCUMENTS OFFICER

UNDER SECY OF DEF FOR RSCH & ENGRG
DEPARTMENT OF DEFENSE
WASHINGTON, DC 20301
01CY ATTN STRATEGIC & SPACE SYSTEMS (OS)

COMMANDER/DIRECTOR
ATMOSPHERIC SCIENCES LABORATORY
U.S. ARMY ELECTRONICS COMMAND
WHITE SANDS MISSILE RANGE, NM 88002
01CY ATTN DELAS-EO, F. NILES
OTHER GOVERNMENT

INSTITUTE FOR TELECOM SCIENCES
NATIONAL TELECOMMUNICATIONS & INFO ADMIN
BOULDER, CO 80303
01CY ATTN D. CROMBIE
01CY ATTN L. BERRY

NATIONAL OCEANIC & ATMOSPHERIC ADMIN
ENVIRONMENTAL RESEARCH LABORATORIES
DEPARTMENT OF COMMERCE
BOULDER, CO 80302
01CY ATTN R. GRUBB
01CY ATTN AERONOMY LAB G. REID

DEPARTMENT OF DEFENSE CONTRACTORS

AEROSPACE CORPORATION
P.O. BOX 92957
LOS ANGELES, CA 90009
01CY ATTN I. GARFUNKEL
01CY ATTN T. SALMI
01CY ATTN V. JOSEPHSON
01CY ATTN S. BOWER
01CY ATTN D. OLESEN

ANALYTICAL SYSTEMS ENGINEERING CORP
5 OLD CONCORD ROAD
BURLINGTON, MA 01803
01CY ATTN RADIO SCIENCES

AUSTIN RESEARCH ASSOC., INC.
1901 RUTLAND DRIVE
AUSTIN, TX 78758
01CY ATTN L. SLOAN
01CY ATTN R. THOMPSON

BERKELEY RESEARCH ASSOCIATES, INC.
P.O. BOX 983
BERKELEY, CA 94701
01CY ATTN J. WORKMAN
01CY ATTN C. PRETIE
01CY ATTN S. BRECHT

BOEING COMPANY, THE
P.O. BOX 3707
SEATTLE, WA 98124
01CY ATTN G. KEISTER
01CY ATTN D. MURRAY
01CY ATTN G. HALL
01CY ATTN J. KENNEY

CHARLES STARK DRAPER LABORATORY, INC.
555 TECHNOLOGY SQUARE
CAMBRIDGE, MA 02139
01CY ATTN D.B. COX
01CY ATTN J.P. GILMORE

COMSAT LABORATORIES
LINTHICUM ROAD
CLARKSBURG, MD 20734
01CY ATTN G. HYDE

CORNELL UNIVERSITY
DEPARTMENT OF ELECTRICAL ENGINEERING
ITHACA, NY 14850
01CY ATTN D.T. FARLEY, JR.

ELECTROSPACE SYSTEMS, INC.
BOX 1359
RICHARDSON, TX 75080
01CY ATTN H. LOGSTON
01CY ATTN SECURITY (PAUL PHILLIPS)

EOS TECHNOLOGIES, INC.
606 Wilshire Blvd.
Santa Monica, CA 90401
01CY ATTN C.B. GABBARD
01CY ATTN R. LELEVIER

ESL, INC.
495 JAVA DRIVE
SUNNYVALE, CA 94086
01CY ATTN J. ROBERTS
01CY ATTN JAMES MARSHALL

GENERAL ELECTRIC COMPANY
SPACE DIVISION
VALLEY FORGE SPACE CENTER
GODDARD BLVD KING OF PRUSSIA
P.O. BOX 8555
PHILADELPHIA, PA 19101
01CY ATTN M.H. BORTNER
SPACE SCI LAB

GENERAL ELECTRIC TECH SERVICES
CO., INC.
HMES
COURT STREET
SYRACUSE, NY 13201
01CY ATTN G. MILLMAN
END FILMED

5-86

DTIC