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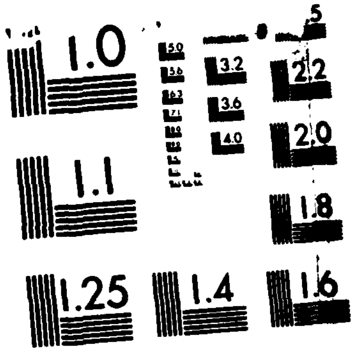
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TECHNICAL REPORT ARCCB-TR-86004

YOUNG'S MODULUS AND POISSON'S RATIO OF STEEL AS STRESS DEPENDENT QUANTITIES

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JANUARY 1986

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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Second and third-order elastic constants of 4340 steel obtained from ultrasonic measurements of specimens under zero and moderate uniaxial stresses are used under a continuum mechanics analysis of finite deformation to predict Young's modulus and Poisson's ratio, not only in the stress range of the measurement, but at stresses near the yield point (150,000 psi), where Young's modulus changes by about 2.5 percent, Poisson's ratio by 5 percent (both decrease in tension and increase in comparison).

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INTRODUCTION

Hooke's Law, the linear relationship between stress σ, and strain ϵ, (Young's modulus E defines the proportionality) is not a law of nature but a useful relationship which has innumerable experimental confirmations at small (infinitesimal) strains. At larger (finite) strains the relationship is no longer between stress and strain but between stress and strain to higher powers. An analysis of the acoustoelastic effect in gun steel (ref 1), carried out as a generalization of our work on the determination of residual stresses in gun steel (ref 2), caused us to ask the question of if and how the elastic properties of steel would change at high uniaxial stresses in tension or compression.

We note that we are not taking into account here the dislocation motion which takes place under applied stress or dislocation effects; we consider the continuum to be nondispersive. The results obtained here are purely from continuum mechanics using finite elasticity. If dislocation motion, which would be elastic or reversible (e.g. loops bowing out), comes into play at these high stresses, our results would represent a lower limit to changes in Young's modulus or Poisson's ratio.

We have only extended this work to third order elastic constants, and we are only considering the effect of uniaxial stresses. It remains for future

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[1]W. Scholz and J. Frankel, "Acoustoelastic Effects in Autofrettaged Steel Cylinders," paper presented at the Ultrasonics International 85, Kings College, London, July 1985.

[2]J. Frankel, W. Scholz, G. Capsimalis, and W. J. Korman, "Residual Stress Measurements in Circular Steel Cylinders," ARDC Technical Report ARLCB-TR-84018, Benet Weapons Laboratory, Watervliet, NY, May 1984.

investigation to consider the effect of higher order elastic constants or more complex (but more realistic) loading in gun steel.

STRESS-STRAIN RELATIONS

For an isotropic solid, the relations between stresses σ_{ij} and infinitesimal strains ε_{ij} can be written as (ref 3)

$$\sigma_{ii} = \lambda\Delta + 2\mu\varepsilon_{ii}$$

$$\sigma_{ij} = \mu\varepsilon_{ij} \tag{1}$$

where $\Delta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ and λ and μ are the second-order or so-called Lamé constants. The indices i,j refer to the three coordinate axes. Applying a uniaxial stress along the 1-direction allows one to solve for the strains in Eq. (1) in terms of σ_{11}. Thus, one obtains for Young's modulus

$$E = \frac{\sigma_{11}}{\varepsilon_{11}} = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \tag{2}$$

and for Poisson's ratio

$$\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}} = \frac{\lambda}{2(\lambda+\mu)} \tag{3}$$

The Lamé constants can be determined from ultrasonic-velocity measurements. The velocity of longitudinal waves is given by

$$\rho {v_L}^2 = \lambda + 2\mu \tag{4}$$

while for the transverse or shear wave one obtains

$$\rho {v_T}^2 = \mu \tag{5}$$

where ρ is the density of the material.

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[3]H. Kolsky, Stress Waves in Solids, First Edition, Dover Publications, NY, 1963.

For the case of finite strains the linear relations of Eq. (1) are no longer strictly valid. Murnaghan (ref 4) has developed the corresponding second-order approximation to Eq. (1) for the case of finite deformations. For isotropic materials, one requires, in addition to the Lamé constants λ and μ, three more constants ℓ, m, and n to describe the material. These three constants are usually referred to as third-order elastic constants or Murnaghan constants.

Hughes and Kelly (ref 5) have derived explicit expressions for the stress-strain relations in second-order theory for the case of an infinitesimal strain superimposed on a finite strain. For the situation of uniaxial stress in the 1-direction, their general equations reduce to

$$\begin{aligned} \sigma_{11} &= \sigma_{11}^{o} + c_{11,11}\epsilon_{11} + c_{11,22}\epsilon_{22} + c_{11,33}\epsilon_{33} \\ o &= c_{22,11}\epsilon_{11} + c_{22,22}\epsilon_{22} + c_{22,33}\epsilon_{33} \\ o &= c_{33,11} + c_{33,22}\epsilon_{22} + c_{33,33}\epsilon_{33} \end{aligned} \qquad (6)$$

Where σ_{11}^{o} is the stress producing the finite strains, ϵ_{ii} are the infinitesimal strains caused by $\sigma_{11} - \sigma_{11}^{o}$, and the coefficients $c_{ii,ii}$ are given by rather involved expressions containing the finite strains and the Lamé and Murnaghan constants.

Equation (6) can be solved analogous to Eq. (1) to determine Young's modulus and Poisson's ratio. The values obtained in this fashion correspond to the situation where the finite and the infinitesimal stresses are parallel.

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[4] F. D. Murnaghan, Finite Deformation of an Elastic Solid, Dover Publications, NY, 1967.

[5] D. S. Hughes and J. L. Kelly, "Second Order Elastic Deformation of Solids," Phys. Rev., Vol. 92, 1953, p. 1145.

Finally, by replacing the finite strains in the coefficients $c_{ii,jj}$ with the finite stress $\sigma_{11}{}^o$ through the use of linearized relations formally identical to Eq. (1), one obtains for Young's modulus

$$E = \frac{\sigma_{11}-\sigma_{11}{}^o}{\epsilon_{11}} = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu} \cdot \{1 + \frac{\sigma_{11}{}^o}{3K_o}[\frac{2\ell\mu}{3K_o(\lambda+\mu)} + \frac{3K_o(2\lambda+2\mu+m)}{\mu(\lambda+\mu)} + \frac{3n}{6K_o}(\frac{\lambda}{\mu})^2]\} \quad (7)$$

and for Poisson's ratio

$$\nu = -\frac{\epsilon_{22}}{\epsilon_{11}} = \frac{\lambda}{2(\lambda+\mu)}\{1 + \frac{\sigma_{11}{}^o}{3K_o} \cdot$$

$$[-\frac{n}{\mu} - \frac{2\lambda+3\mu+2m}{\lambda+\mu} + \frac{\mu(2\ell-m-\mu)}{\lambda(\lambda+\mu)} + \frac{3(\lambda+\mu)(\lambda+\mu+m)}{\lambda\mu}]\} \quad (8)$$

where $K_o = (1/3)(3\lambda+2\mu)$ is the bulk modulus. Equations (7) and (8) contain the dependence of E and ν on the applied finite stress as given by second-order elastic theory. As expected, they reduce to Eqs. (2) and (3) for zero applied finite stress.

DISCUSSION

The second and third-order elastic constants of steel are given in Table I. They are obtained from exact measurements of longitudinal and transverse sound velocities under zero and finite stress. Absolute velocities were measured by the pulse-echo overlap (ref 6) technique at two frequencies, with accuracies approaching one part in 10^4. The Lamé constants λ and μ were determined from the absolute velocities v_L = 5871 m/sec and v_T = 3192 m/sec using a density ρ = 7.84 x 10^3 kg/m^3. Relative velocity changes were measured to better than one in 10^4 by observing the change in return time of an echo,

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[6] E. P. Papadakis, J. Acoust. Soc. Am., Vol. 42, 1967, p. 1045.

typically the fifth, with applied stress. The third-order elastic constants derived from these measurements are also given in Table I. Inserting the elastic constants given in Table I into Eqs. (7) and (8), we obtain for Young's modulus the expression

$$E = 20.62[1 - 0.256\ \sigma_{11}^{o}(10^5\ \text{bar})](10^5\ \text{bar})$$

$$= 29.92 \times 10^6[1 - 1.77\times10^{-7}\ \sigma_{11}^{o}(\text{psi})](\text{psi}) \qquad (9)$$

and for Poisson's ratio

$$\nu = 0.290[1 - 0.489\ \sigma_{11}^{o}(10^5\ \text{bar})$$

$$= 0.290[1 - 3.37 \times 10^{-7}\ \sigma_{11}^{o}(\text{psi}) \qquad (10)$$

Equations (9) and (10) predict that tensile stresses near the yield point (150,000 psi) will reduce Young's modulus by less than three percent and Poisson's ratio by about five percent.

TABLE I. SECOND AND THIRD-ORDER ELASTIC CONSTANTS OF 4340 STEEL (in 10^5 bar)

λ	μ	ℓ	m	n
11.04	7.99	-38.8 ± 3.6	-62.4 ± 2.4	-74.7 ± 1.6

CONCLUSION

Ultrasonic velocity measurements with stress can be interpreted to obtain second and third-order elastic constants of steel. By using finite elasticity in continuum mechanics, we have shown how elastic properties can be predicted at high stresses when accurate velocity data can be obtained at low stresses. The Poisson's ratio and Young's modulus were calculated and shown to be affected by small percentages near the yield point. This, however, does not exhaust the problem of elastic properties in gun steels at very high elastic stresses. Questions remain in two fields. In continuum mechanics we ask what happens near the yield point if the stress fields are more complex than the ones calculated here. In materials science, we ask what is the effect of anelasticity (elastic properties which are time dependent) on the Young's modulus and Poisson's ratio. These considerations are tied in with dislocations bowing out and are not included in the consideration of continuum mechanics. The velocities are considered to be nondispersive.

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1. W. Scholz and J. Frankel, "Acoustoelastic Effects in Autofrettaged Steel Cylinders," paper presented at the Ultrasonics International 85, Kings College, London, July 1985.
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5. D. S. Hughes and J. L. Kelly, "Second Order Elastic Deformation of Solids," Phys. Rev., Vol. 92, 1953, p. 1145.
6. E. P. Papadakis, J. Acoust. Soc. Am., Vol. 42, 1967, p. 1045.

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