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# Fatigue Life Analysis of a Turboprop Reduction Gearbox

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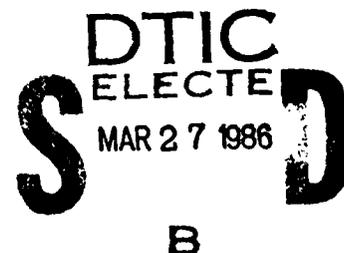
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SUMMARY

A fatigue life analysis of the reduction gearbox of a turboprop aircraft was developed. The Allison T56/501 gearbox, consisting of eleven rolling-element bearings and nine spur gears in the main power train, was used for the analysis. Using methods of probability and statistics, the life and reliability of the gearbox was evaluated based on the lives and reliabilities of its main power train bearings and gears. Cylindrical roller bearing lives were determined by NASA computer program CYBEAN. Ball bearing lives were determined by program SHABERTH and spherical roller planet bearing lives were determined by program SPHERBEAN. All programs use the Lundberg-Palmgren theory in calculating life. A combined material and material processing life adjustment factor of twelve was used for bearings made from vacuum-induction melted, vacuum-arc remelted (VIM-VAR) AISI M-50 steel. A factor of six was used for bearings made from consumable-electrode vacuum melted (CEVM) AISI 52100 or AISI 9310 steel. Lubrication life adjustment factors were also used. The Lundberg-Palmgren model was adapted to determine gear life. NASA computer program 1ELSGE was used to determine gear lubrication life adjustment factors. Miner's rule was used to determine mission life based on a theoretical mission profile. The analytical results with and without life adjustment factors were compared to field data.

The five planet bearing set had the shortest calculated life among the various gearbox components, which agreed with field experience where the planet bearing had the greatest incidences of failure. The analytical predictions of relative lives among the various bearings were in reasonable agreement with field data for both when the life adjustment factors were used and not used. The predicted gearbox life was in excellent agreement with field experience when the material life adjustment factors alone were used. The gearbox had

a lower predicted life in comparison with field experience when no life adjustment factors were used. This was caused by lower bearing lives. The gearbox had a lower predicted life in comparison with field experience when lubrication life adjustment factors were used alone or in combination with material factors. This was caused by a lower planet bearing life.

## INTRODUCTION

Increasing fuel costs have encouraged more fuel efficient propulsion systems for the aircraft industry. Ludemann (ref. 1) discusses the advanced propeller or prop-fan propulsion system as a possible candidate for future technology. References 2 and 3 discuss requirements and arrangements for reduction gearboxes of such prop-fan propulsion systems. Life and reliability play an important role in the development of these gear-boxes. Design requirements such as low weight and high power capacity should be balanced with high life and low maintenance costs. Analytical tools predicting the life of a gearbox can be a valuable asset in the design of a gearbox or in the comparison of different gearbox designs.

Bearing fatigue life is a major factor in the evaluation of gearbox life. The fatigue life model proposed by Lundberg and Palmgren (refs. 4 to 6) is the commonly accepted theory for predicting rolling-element bearing life and is used in handbook life equations. Recent work in the development of computer programs for thermal and mechanical performance predictions of ball, cylindrical, spherical, and tapered roller bearings also use the Lundberg-Palmgren theory for life calculations (refs. 7 to 9).

Another major factor in the evaluation of gearbox life is gear fatigue life. Mathematical models have been developed for surface fatigue life of spur and helical gears (refs. 10 to 14). These gear life models are based on a modified Lundberg-Palmgren theory due to the similarity in fatigue failures between bearings and gears.

Reliability models for planetary gear trains have been developed (refs. 15 and 16). These models are based on the individual reliabilities of the transmission's bearings and gears. Each bearing and gear life was calculated and the results were statistically combined to produce a system life for the total transmission.

The objective of the work reported herein was to provide a methodology for calculating life and reliability for typical reduction gearboxes for both present-day and future turboprop aircraft. The Allison T56/501 reduction gearbox was selected for an example in this study due to its high usage and large reliability database. The life model for the complete gearbox was based on the individual lives of each bearing and gear. A mission profile was used in determining loads on the components. Actual mission profiles vary from mission to mission. The profile used represents what may be considered a typical mission which most gear-boxes experience. The component lives were calculated using the Lundberg-Palmgren theory and assumed to follow the Weibull failure distribution. These lives were combined using methods of probability and statistics to produce a life and reliability model of the total gearbox.



t percent time  
V stressed volume,  $m^3$  ( $in^3$ )  
z depth to critical shear stress, m( $in$ )  
n millions of stress cycles  
 $\Sigma\rho$  curvature sum,  $m^{-1}$  ( $in^{-1}$ ) ( $= 1/R_1 + 1/R_2$ )  
 $\tau$  critical shear stress,  $N/m^2$  (psi)

Subscripts

a first load  
B bearing  
B1 front pinion bearing  
B2 rear pinion bearing  
B3 main drive bearing  
B4 carrier support bearing  
B5 prop thrust bearing  
B6 prop radial bearing  
B7 planet bearing  
b second load  
c third load  
d four load  
G gear  
G1 pinion gear  
G2 main drive gear  
G3 sun gear  
G4 planet gear  
G5 ring gear  
M mission  
T total gearbox  
t gear tooth

- t1 tooth of sun gear-planet gear mesh
- t2 tooth of planet gear-ring gear mesh
- 10 90-percent probability of survival
- 1 planet gear meshing with sun gear
- 2 planet gear meshing with ring gear

## ANALYSIS

### Force and Motion Analysis

The goal is to determine the life of each bearing and gear, and finally, the system life of the main power train gearbox assembly (fig. 1). Since load affects life, a force analysis of the gearbox is needed to determine the load on each component. The forces on each gear can be separated into tangential and radial components. The tangential force is related to the power transmitted through the gear from the engine to the propeller. The radial force on each gear is related to the tangential gear force by the pressure angle of the gear. The radial loads on the front and rear pinion bearings are the reactions from the pinion gear forces. The radial load on the main drive bearing is the reaction from the main drive gear forces. The radial load on each planet bearing equals twice the tangential load on a planet gear combined vectorially with the centrifugal force. It is assumed that all the planets share the load equally. The radial loads on the carrier support bearing and the prop radial bearing are the reactions from the propeller and the main drive gear. The prop thrust bearing reacts all the thrust load from the propeller loading. This bearing carries no radial load due to high diametral clearance of the bearing with the housing. It is assumed that all the bearings with the exception of the prop thrust bearing will carry radial loads only.

A motion analysis of the gearbox is needed to relate the number of load cycles on each component to input shaft revolutions. Table IV presents the motion analysis using principles of kinematics.

### Fatigue Life Model

The fatigue life model proposed by Lundberg and Palmgren (refs. 4 to 6) is the commonly accepted theory in predicting pitting fatigue life of rolling-element bearings. The relationship between the probability of survival of a bearing and the stress cycles is

$$\log \frac{1}{S} \propto \frac{\tau_c n^e V}{z^h} \quad (1)$$

where  $S$  is the probability of survival,  $n$  is the life in millions of stress cycles,  $V$  is the stressed volume,  $\tau_c$  is the critical shear stress, and  $z$  is the depth to the critical shear stress. The exponents  $c$ ,  $h$ , and  $e$  are material dependent exponents determined from experimental life testing.

Equation (1) is based upon the two-parameter Weibull cumulative distribution function where the probability of survival,  $S$ , is a function of millions of stress cycles,  $n$ . The exponent  $e$  is commonly called the Weibull exponent and is a measure of scatter of the distribution of lives. Defining  $n = n_{10}$  and  $S = 0.9$  for a 90 percent probability of survival and using equation (1) gives the following equation

$$\log \frac{1}{S} = \log \frac{1}{0.9} \left( \frac{n}{n_{10}} \right)^e \quad (2)$$

which applies for a given bearing with constant load. Equation (2) defines the probability of survival,  $S$ , as a function of life,  $n$ , in terms of its two parameters,  $n_{10}$  and  $e$ .

### Bearing Life and Reliability

A generalized life-reliability equation can be written for each of the bearings in the gearbox. For each bearing,

$$\log \frac{1}{S_B} = \log \frac{1}{0.9} \left( \frac{L}{L_{10B}} \right)^{e_B} \quad (3)$$

where

$$L_{10B} = \frac{n_{10B}}{k} \quad (4)$$

and

$$n_{10B} = a_B \left( \frac{C_B}{F_B} \right)^{p_B} \quad (5)$$

$n_{10B}$  is the number of millions of stress cycles of the bearing in which 90 percent will survive.  $n_{10B}$  can be determined from the Lundberg-Palmgren

theory using equation (5) where  $C_B$  is the basic dynamic capacity of the bearing;  $F_B$  is the equivalent load on the bearing;  $p_B$  is the load-life exponent (three for ball and four for roller bearings); and  $a_B$  is the life adjustment factor to account for improved materials, improved material processing, and lubrication effects (ref. 17).  $n_{10B}$  is converted from

millions of stress cycles of the bearing to millions of input shaft revolutions by using equation (4), where  $L_{10B}$  is the life of the bearing in millions of

input shaft revolutions, and  $k$  is the number of load cycles of the bearing per input shaft revolution as defined in table IV. In equation (3),  $S_B$  is the probability of survival of the bearing and  $L$  is the life of the bearing

in millions of input shaft revolutions.  $e_g$ , the Weibull exponent, has shown to be about 1.1 from experimental rolling-element fatigue testing (refs. 4 to 6).

### Gear Life and Reliability

Due to the similarity in the fatigue failure mechanism between gears and rolling-element bearings made from high-strength steel, the Lundberg-Palmgren life model for bearings has been adapted to predict gear life (refs. 10 to 14). Experimental research of AISI 9310 steel spur gears has shown gear fatigue life to follow the Weibull failure distribution with an average Weibull exponent of about 2.5 (ref. 13). A generalized life-reliability equation may be written for each of the gears in the gearbox. For each gear,

$$\log \frac{1}{S_G} = \log \frac{1}{0.9} \left( \frac{L}{L_{10G}} \right)^{e_G} \quad (6)$$

where

$$L_{10G} = \frac{N^{-1/e_G} \eta_{10t}}{k} \quad (7)$$

for all gears except the planet gear and

$$L_{10G} = \frac{N^{-1/e_G} \left( \eta_{10t1}^{-e_G} + \eta_{10t2}^{-e_G} \right)^{-1/e_G}}{k} \quad (8)$$

for the planet gear and

$$\eta_{10t} = a_t \left( \frac{C_t}{F_t} \right)^{p_G} \quad (9)$$

where

$$C_t = Bf^{0.907} \Sigma P^{-1.165} \eta^{-0.093} \quad (10)$$

$\eta_{10t}$  is the number of millions of stress cycles for which one particular tooth of a gear has a 90 percent probability of survival.  $\eta_{10t}$  can be determined using equation (9) where  $C_t$  is the basic dynamic capacity of the gear tooth;  $F_t$  is the normal tooth load;  $p_G$  is the load-life exponent based on experimental data (equal to 4.3); and  $a_t$  is the life adjustment

factor.  $C_t$  can be determined using equation (10) where  $B$  is the material constant based on experimental data and found to be  $\sim 1.39 \times 10^8$  when SI units are used (newtons and meters) and 21 800 when English units are used (pounds and inches) for AISI 9310 steel spur gears;  $f$  is the tooth face width;  $\Sigma\rho$  is the curvature sum at the start of single tooth contact; and  $l$  is the length of the involute surface during single tooth contact.  $L_{10G}$  is the life of the gear (all teeth) in millions of input shaft revolutions in which 90 percent will survive.  $L_{10G}$  can be determined by equations (7) or (8) where  $N$  is the total number of teeth on the gear;  $e_G$  is the Weibull exponent (2.5); and  $k$  is the number of load cycles of a gear tooth per input shaft revolution as defined in table IV.

For all the gears except the planet gear, each tooth will see contact on only one side of its face for a given direction of input shaft rotation. However, each tooth on a planet gear will see contact on both sides of its face for a given direction of input shaft rotation. One side of its face will contact a tooth on the sun gear and the other side of the face will contact a tooth on the ring gear. Equation (8) takes this into account.  $n_{10,t1}$  is the millions of stress cycles for a 90 percent probability of survival of a planet tooth meshing with the sun gear and  $n_{10,t2}$  is the millions of stress cycles for a 90 percent probability of survival of a planet tooth meshing with the ring gear.

#### System Life and Reliability

The life and reliability of the gearbox is based on the lives and reliabilities of all of its bearings and gears. Using the subscripts for the bearings as defined in table I and for the gears as defined in table II, the probability of survival of the gearbox,  $S_T$ , is

$$S_T = S_{B1} \cdot S_{B2} \cdot S_{B3} \cdot S_{B4} \cdot S_{B5} \cdot S_{B6} \cdot S_{B7}^n \cdot S_{G1} \cdot S_{G2} \cdot S_{G3} \cdot S_{G4}^n \cdot S_{G5} \quad (11)$$

where  $n$  is the number of planets, and the subscript  $T$  designates transmission assembly. Taking the logarithm of the inverse of equation (11) and with equations (3) and (6), the generalized system life-reliability equation is

$$\log \frac{1}{S_T} = \log \frac{1}{0.9} \left[ \left( \frac{L}{L_{10B1}} \right)^{e_B} + \left( \frac{L}{L_{10B2}} \right)^{e_B} + \left( \frac{L}{L_{10B3}} \right)^{e_B} + \left( \frac{L}{L_{10B4}} \right)^{e_B} + \left( \frac{L}{L_{10B5}} \right)^{e_B} \right. \\ \left. + \left( \frac{L}{L_{10B6}} \right)^{e_B} + n \left( \frac{L}{L_{10B7}} \right)^{e_B} + \left( \frac{L}{L_{10G1}} \right)^{e_G} + \left( \frac{L}{L_{10G2}} \right)^{e_G} + \left( \frac{L}{L_{10G3}} \right)^{e_G} \right. \\ \left. + n \left( \frac{L}{L_{10G4}} \right)^{e_G} + \left( \frac{L}{L_{10G5}} \right)^{e_G} \right] \quad (12)$$

where the probability of survival for the complete gearbox,  $S_T$ , is a function of millions of input shaft revolutions,  $L$ , and the lives at a 90 percent probability of survival of each bearing and gear in terms of millions of input shaft revolutions.

For a given load on the gearbox, the lives of each bearing and gear will be constants and can be determined by the appropriate equations (4), (7), or (8). Using equation (12), the system life for a given probability of survival can be calculated using an iterative process. A curve can be plotted on Weibull coordinates using a variety of  $S$ 's and corresponding  $L$ 's. These curves may not be straight lines due to the different slopes for bearings and gears. For any  $S$ , the system life is always less than the life of the shortest lived component at the same  $S$ .

#### Mission Life

A gearbox does not usually operate at one constant load in actual service. Miner's Rule is used to sum fatigue damage of a mission profile consisting of loads and time-at-loads. For a given probability of survival, the mission life for a component (or the mission life for the system),  $L_M$ , is

$$L_M = \left( \frac{t_a}{L_a} + \frac{t_b}{L_b} + \frac{t_c}{L_c} + \frac{t_d}{L_d} \right)^{-1} \quad (13)$$

where  $t_a$ ,  $t_b$ ,  $t_c$ , and  $t_d$  are the fraction of the total time at loads  $a$ ,  $b$ ,  $c$ , and  $d$ , respectively, and  $L_a$ ,  $L_b$ ,  $L_c$ , and  $L_d$  are the component (or system) lives at that probability of survival at loads  $a$ ,  $b$ ,  $c$ , and  $d$ , respectively. A mission life-survivability Weibull plot can be constructed using this method through a variety of probability of survivals.

#### Statistical Treatment of Field Data

Scatter in fatigue life of identical items run under identical conditions is an inherent characteristic of any rolling-element. Statistical analysis can estimate the behavior of life of a large group of items based on a finite number of failures, even if the number of failures is small. Johnson presents a method to construct a graphic picture of failure data (ref. 18). In Johnson's

method, the median rank of each failure is plotted versus the time at the failure. When plotted on special coordinates called Weibull coordinates, the percentage of failures can be approximated in a least squares sense as a linear function of life. The Weibull exponent  $e$  is the slope of the line that best fits the data. From the Weibull plot, the life at any probability of survival may be obtained. Early failures are of great interest. The life at a 90 percent probability of survival, designated the  $L_{10}$  life, is often used for comparison purposes. The life at a 50 percent probability of survival, called the median life and designated the  $L_{50}$  life, is also used for comparison. Weibull distributions are generally skewed and the mean (average) life does not coincide with the median life ( $L_{50}$ ). The location of the mean life in percent of items failed is a function of the Weibull slope. For slopes in the range from 1.0 to 2.5, the location of the mean is at a probability of survival of 36.8 to 47.6 percent (which is a probability of failure of 63.2 to 52.4 percent, respectively).

A common practice in determining the mean life of a component is to divide the total number of hours on all of those components (failed and unfailed) by the total number of failures. Arithmetic mean lives thus determined are only representative of the mean lives on the Weibull distributions if all items of the sample group have failed.

In this study, analytically predicted life is compared with field data. The predicted life using the Lundberg and Palmgren theory assumes the pitting fatigue failure mode. However, field data may have nonpitting failures, and these data points should be treated as so called "suspended items." The arithmetic mean as defined above is not valid with the addition of suspended items. Suspended items can be accounted for and a mean life can be determined using the methods of reference 18.

## RESULTS AND DISCUSSION

### Analytical

The theoretical mission profile of loads and time-at-loads for the gearbox is shown in table V. This data represents a typical mission profile for the Allison T56/501-022A gearbox, even though the actual profile varies from mission to mission. The loads on each component were determined from this data.

The lives for all the cylindrical roller bearings were determined by NASA computer program CYBEAN (ref. 7). The lives for the prop thrust ball bearing were determined by program SHABERTH (ref. 8). The lives for the planet spherical roller bearing were determined by program SPHERBEAN (ref. 9). The rollers and raceways of all bearings except the front and rear pinion bearings are made from consumable-electrode vacuum melted (CEVM) AISI 52100 or AISI 9310 steel. They were all given a combined material and material processing life adjustment factor of six (two for material and three for material processing (ref. 17)). The rollers and raceways for the front and rear pinion bearings are made from vacuum-induction melted, vacuum-arc remelted (VIM-VAR) AISI M-50 steel. The life adjustment factor for VIM/VAR processing is not stated in reference 17. Some experimental studies have shown bearings using VIM/VAR processing to have a higher life than those using CEVM processing (ref. 19). Other studies have shown bearing lives using VIM/VAR to be the same as CEVM (ref. 20). A combined material and material processing life adjustment factor of 12 was chosen for

the front and rear pinion bearings. For simplicity, the material life adjustment factor will refer to a combined material and material processing life adjustment factor in the remaining discussion. The lubrication life adjustment factor due to the ratio of elastohydrodynamic (EHD) film thickness to composite surface roughness (ref. 17) was also used.

Figure 2 shows the predicted mission lives as a result of these programs when plotted on Weibull coordinates with and without life adjustment factors. The bearing life lines were given a slope of 1.1. When no life adjustment factors were used, the predicted lives for the five planet system set, the front pinion bearing, and the rear pinion bearing were all about the same, and were predicted to be the shortest lived bearings of the main power train (fig. 2(a)). When life adjustment factors were used, the five planet bearing set was predicted to be the shortest lived component (figs. 2(b) to 2(d)).

The lives for the gears were determined by equations (7) and (8). These results seemed too low compared to the bearing lives. The material constant in equation (10) was determined from experimental tests in reference 13. Further investigation showed this constant contained the lubrication life adjustment factor for those tests. Treating this factor separately, the material constant became  $1.64 \times 10^8$  for SI units (25 700 for English units). The resulting lives using this new constant still seemed too low. The material constant was then determined from the experimental tests in reference 21 which turned out to be  $2.23 \times 10^8$  for SI units (35 000 for English units). This produced better results and this constant was used for the remaining gear life calculations. The EHD film thickness throughout the gear mesh contact cycle was determined by NASA computer program TELSGE (ref. 22). Using the minimum film thickness during single tooth contact, the composite surface roughness, and the resulting lubrication life adjustment factor from reference 17, the gear lives were modified. Figure 2 shows the mission lives with and without the lubrication life adjustment factors. The gear life lines were given a slope of 2.5. The sun gear was predicted to be the shortest lived gear of the main power train in all cases.

The predicted system lives can also be seen in figure 2. The system life curves were plotted using equation (12). When no life adjustment factors were used, the predicted system life curve is approximately a straight line with a slope of 1.1 (fig. 2(a)). This is due to the influence of the front and rear pinion bearings, the main drive bearing, and the planet bearing (and the fact that there are five planet bearings). The predicted system mean life is about 1600 hr. The bearing lives increased with the use of material life adjustment factors (fig. 2(b)). The predicted system life curve deviates slightly from a straight line due to the influence of the previously mentioned bearings and the sun gear. Using a least squares fit, the system life curve has a slope of 1.2 and a mean life of about 12 000 hr. Figure 2(c) shows the results when only lubrication life adjustment factors were used. Comparing with figure 2(a), the planet bearing life decreases while the lives of the front pinion bearing, rear pinion bearing, and main drive bearing increase. The predicted system life curve is approximately a straight line with a slope of 1.1 due to the influence of the planet bearing. The predicted system mean life is about 900 hr. Figure 2(d) shows the results with both material and lubrication life adjustment factors. The planet bearing is the most influential component and the predicted system life curve has a slope of 1.1 and a mean life of 5400 hr.

## Field Data and Analytical Comparison

Field failure data was collected for the main power train bearings and gears of the Allison T56/501-D22A gearboxes from January, 1975, through April, 1983. This data consisted of premature gearbox removal data and overhaul data. Included in the data were identification of components that failed, reasons of failure, gearbox serial numbers, total hours on gearboxes, and hours on gearboxes since last overhauls. This data was screened and only pitting fatigue failures were considered. Table VI shows the distribution of failures among the gearbox components. The planet bearing had the greatest incidences of failure, which is consistent with the analytical predictions.

A Weibull plot for planet bearing failures along with the analytical results are shown in figure 3. The method of Johnson (ref. 18) was used to plot the failure data and a least squares method fitted the line connecting the failure points. Ninety percent confidence bands were also drawn. The meaning of the 90 percent confidence bands is that if more field data is acquired and this exercise is repeated again and again, 90 percent of the tests will have failures falling within this band. The results of the field data for the planet bearing show a slope of 1.2 and a mean life of 110 000 hr. The analytically predicted life using the material life adjustment factors alone shows excellent correlation with the field data. When no life adjustment factors were used or when lubrication life adjustment factors were used, the analytical results are low compared to field data.

Figure 4 shows a Weibull plot for the front pinion bearing. The field data results show a slope of 1.6 and a mean life of 26 000 hr. The analytical predictions show a lower life than the field data when no life adjustment factors or when lubrication life adjustment factors alone are used. The analytical predictions show a higher life than the field data when material life adjustment factors are added. The best correlation of analytical predictions with field data results when material life adjustment factors alone are used.

The rear pinion bearing follows the same trend as the front pinion bearing. The field data results for the rear pinion bearing, depicted in figure 5, show a slope of 1.4 and a mean life of 37 000 hr.

For the main drive bearing (fig. 6), the slope is 1.1 and the mean life is 40 000 hr in field experience. The analytical predictions of the main drive bearing show good correlation with field data when either the lubrication life adjustment factors alone or the material life adjustment factors alone are used.

The rest of the bearings either did not have a pitting fatigue failure or not enough failures to construct a meaningful Weibull plot. From the foregoing discussion, it is concluded that the analytical prediction of relative lives of bearing components were in a reasonable agreement with field data for all cases.

From preliminary premature gearbox removal data, there was only one gear failure and that was a planet gear. Due to only one failure, overhaul data was not collected for gears. Thus, the complete history of a gear is not known. Assuming none of the gears were replaced at previous overhauls, the field data shows some gears have accumulated 30 000 to 40 000 hr (at 13 820 pinion rpm's).

At this number of hours, the analytical results predict the sun gear to have the highest probability of failure among the gears (fig. 2). This shows possible conservatism in analytical predictions since no sun gears have failed in premature gearbox removal data. Also, due to only one gear failure, lubrication life adjustment factors using reference 17, which were compiled for rolling-element bearings, could not be verified for gear use.

The system life curve for field data is shown in figure 7 along with the analytical results. The system life curve for the field data was plotted using an equation similar to equation (12). The field data lives at a 90 percent probability of survival for the planet bearing, front and rear pinion bearings, and main drive bearing were used along with their corresponding slopes (from figs. 3 to 6). Using a least squares fit, the system life curve for the field data has a slope of 1.3 and a mean life of 11 000 hr. The analytical system life shows excellent agreement with field experience when the material life adjustment factors alone are used. The analytical system results predict lower life in comparison to field experience when no life adjustment factors are used. This is caused by a reduction in predicted life for all the bearings. The analytical system results predict lower life in comparison to field experience when lubrication life adjustment factors are used alone, or in combination with the material factors. The reason is a reduction in planet bearing life caused by the lubrication life adjustment factor.

#### Additional General Remarks

Experience has shown that computer program SPHERBEAN predicts a low lubrication life adjustment factor for a planet bearing (see sample output in reference 9). This raises a question about the validity of the film thickness calculation in this program. It seems that more correlation with experimental work is needed. However, besides the lubrication life adjustment factors, other variables and assumptions can affect the calculated component life. For example, a study of aircraft bearing rejections (ref. 23) states that some of the failures due to pitting may have originated from stress concentrations caused by corrosion pits, dents, or nicks, and not classical fatigue. If this is the case, the field data would show higher lives and comparisons with analytical predictions would have to be re-evaluated.

The mission profile of loads and time-at-loads was an estimate and does vary among aircraft and airlines. Additional comparison between analytical predictions and field experience is needed to determine the sensitivity on life for varying mission profiles. Accurate recordkeeping of the mission profile is also required to validate inputs required for the analytical life predictions. Pitting fatigue failures are usually found by chip detectors in the lubrication system even though some are not noticed until overhaul. Many hours may be on a component since the start of a pit. If this is the case and since the Lundberg-Palmgren theory predicts the time to a pit, the analytical results would show conservatism.

The foregoing discussion and comparison of field data predicted life leads to the conclusion that predictions and field experience have the best correlation if only the material life adjustment factor is used. This may be true only for the current study and/or type of gear arrangement. When analytical life prediction methods are used in preliminary design of new aircraft gearboxes, the results should be used with caution, as the proper choice of life

adjustment factors is not clear. The authors believe that the predictive life equations are best used for relative ranking of competing designs, rather than predicting actual life. It is also believed, with the exception of the lubrication life adjustment factor for the planet bearing, the material and lubrication life adjustment factors used were reasonable.

#### SUMMARY OF RESULTS

A generic fatigue life analysis methodology was developed for a turboprop aircraft reduction gearbox. The methodology was applied using a theoretical mission profile for the Allison T56/501 gearbox. The life and reliability of the gearbox was based on the lives and reliabilities of its main power train bearings and gears. The bearing lives were determined by NASA computer programs CYBEAN, SHABERTH, and SPHERBEAN, which use the Lundberg-Palmgren theory in calculating life. A modified Lundberg-Palmgren model was used to determine gear life. Miner's rule was used to determine mission life based on a mission profile. The analytical results with and without life adjustment factors were compared to field data. The following results were obtained.

1. The five planet bearing set was the shortest lived component from the analytical predictions. In field experience, the planet bearing had the greatest incidences of failure.

2. The analytical predictions of relative lives of bearing components were in reasonable agreement with field data both when life adjustment factors were and were not used.

3. The analytical system life predictions showed excellent agreement with field experience when the material life adjustment factors alone were used. The analytical system results predicted lower lives in comparison with field experience when no life adjustment factors were used. This was caused by lower bearing lives. The analytical system results predicted lower lives in comparison with field experience when lubrication life adjustment factors were used alone, or in combination with material factors. This was caused by lower planet bearing life.

4. The gear life calculations indicated some gear failures could be expected within the time that the gearboxes were running. However, no significant number of gear failures were experienced. It was concluded that the gear life calculations were conservative.

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TABLE I. - ROLLING-ELEMENT BEARINGS USED IN MAIN POWER TRAIN OF ALLISON T56/501 GEARBOX

Bearing	Number per gearbox	Type	Bore, mm (in)	Subscript
	1	Cylindrical	75 (2.95)	B1
Input pinion	↓	↓	55 (2.17)	B2
Main drive			160 (6.30)	B3
Carrier support			160 (6.30)	B4
Prop thrust		Ball	125 (4.92)	B5
Prop radial		Cylindrical	125 (4.92)	B6
Planet	5	Spherical (double row)	55 (2.17)	B7

TABLE II. - GEARS USED IN MAIN POWER TRAIN OF ALLISON T56/501 GEARBOX

Gear	Number per gearbox	Type	Pitch radius, mm (in)	Number of teeth	Pressure angle, deg	Subscript
Pinion	1	Spur	67.7 (2.67)	32	25	G1
Main drive	1	↓	211.7 (8.33)	100	↓	G2
Sun	1		63.5 (2.50)	30		G3
Planet	5		74.1 (2.92)	35		G4
Ring	1	↓	211.7 (8.33)	100	↓	G5

TABLE III. - TYPICAL PROPERTIES OF A LUBRICANT CONFORMING TO MIL-L-23699 SPECIFICATIONS

Kinematic viscosity, cm <sup>2</sup> /sec (cS) at	
37.8 °C (100 °F)	28x10 <sup>-2</sup> (28)
100 °C (210 °F)	5.1x10 <sup>-2</sup> (5.1)
Density at 15.56 °C (60 °F), gm/cm <sup>3</sup>	1.010
Thermal conductivity, W/m °C	0.152
Thermal coefficient of expansion, °C <sup>-1</sup>	7.45x10 <sup>-4</sup>

TABLE IV. - MOTION ANALYSIS OF ALLISON T56/501 GEARBOX

[ $\omega$  = rotations of input shaft;  $R_A$  = pitch radius of pinion gear;  $R_B$  = pitch radius of main drive gear;  $R_S$  = pitch radius of sun gear;  $R_P$  = pitch radius of planet gear;  $R_R$  = pitch radius of ring gear; and  $n$  = number of planets.]

Component, $i$	Component rotation with respect to the ring gear, $\theta_i$	Component rotation with respect to the planet carrier, $\theta_{i/c}$	Component load cycles per gearbox input shaft revolution, $k$
Front pinion bearing Rear pinion bearing Pinion gear	$\omega$	$1 + \left(\frac{R_A}{R_B}\right)\left(\frac{R_S}{R_S + R_R}\right)\omega$	1
Main drive gear	$\left(-\frac{R_A}{R_B}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{-R_R}{R_S + R_R}\right)\omega$	$\frac{R_A}{R_B}$
Main drive bearing	$\left(-\frac{R_A}{R_B}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{-R_R}{R_S + R_R}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_R}{R_S + R_R}\right)$
Sun gear	$\left(-\frac{R_A}{R_B}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{-R_R}{R_S + R_R}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_R}{R_S + R_R}\right)n$
Planet bearing Planet gear	$\left(-\frac{R_A}{R_B}\right)\left(\frac{R_S - R_P}{R_S + R_R}\right)\left(\frac{R_S}{R_P}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_S R_R}{R_P(R_R + R_S)}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_S R_R}{R_P(R_R + R_S)}\right)$
Ring gear	0	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_S}{R_S + R_R}\right)\omega$	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_S}{R_S + R_R}\right)n$
Carrier support bearing Prop thrust bearing Prop radial bearing	$\left(-\frac{R_A}{R_B}\right)\left(\frac{R_S}{R_S + R_R}\right)\omega$	0	$\left(\frac{R_A}{R_B}\right)\left(\frac{R_S}{R_S + R_R}\right)$

TABLE V. - MISSION PROFILE OF ALLISON T56/501 GEARBOX USED FOR ANALYTIC PREDICTIONS

Mission segment	Percent time of segment	Prop shaft power, kW (HP)	Prop shaft moment, N·m (in-lb)
Takeoff	2.84	3132 (4200)	5875 (52 000)
Climb	17.02	2461 (3300)	5875 (52 000)
Cruise	66.08	1516 (2033)	5197 (46 000)
Descent	12.06	945 (1267)	3728 (33 000)

TABLE VI. - DISTRIBUTION OF PITTING FATIGUE FAILURES AMONG GEARBOX COMPONENTS

Component	Percent of total pitting fatigue failures
Planet brg.	43.6
Main drive brg.	25.6
Rear pinion brg.	15.4
Front pinion brg.	12.8
Carrier support brg.	1.3
Planet gear	1.3

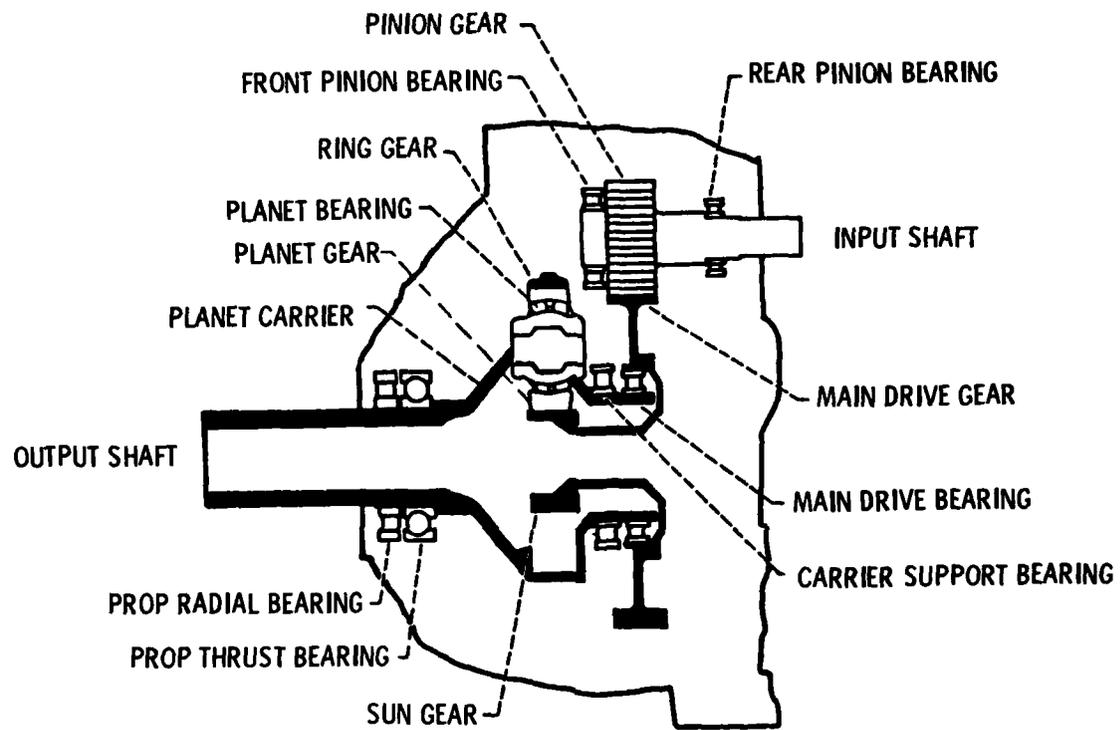


Figure 1. - Allison T56/501 reduction gearbox.

- |    |                         |    |                 |
|----|-------------------------|----|-----------------|
| B1 | FRONT PINION BEARING    | G1 | PINION GEAR     |
| B2 | REAR PINION BEARING     | G2 | MAIN DRIVE GEAR |
| B3 | MAIN DRIVE BEARING      | G3 | SUN GEAR        |
| B4 | CARRIER SUPPORT BEARING | G4 | PLANET GEAR     |
| B5 | PROP THRUST BEARING     |    |                 |
| B6 | PROP RADIAL BEARING     |    |                 |
| B7 | PLANET BEARING          |    |                 |

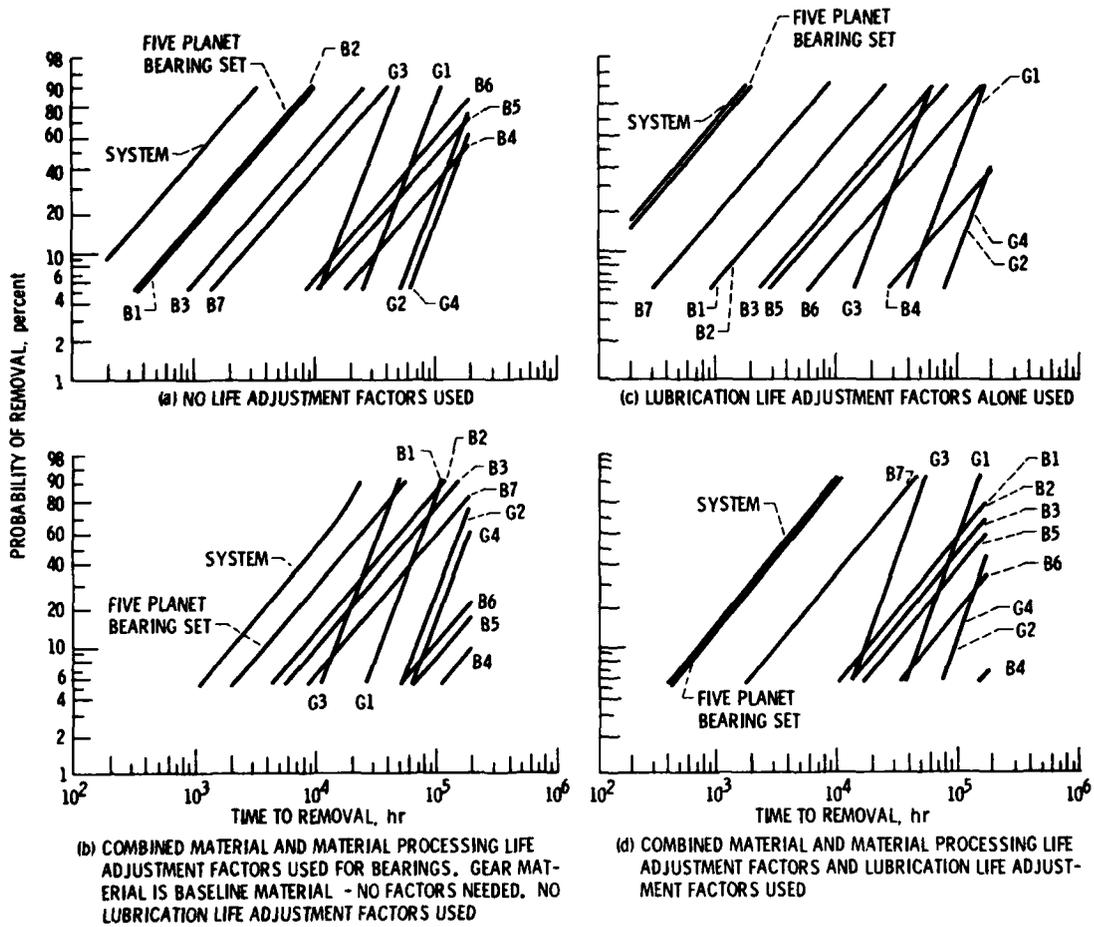


Figure 2. - Analytically predicted system and component pitting fatigue mission lives.

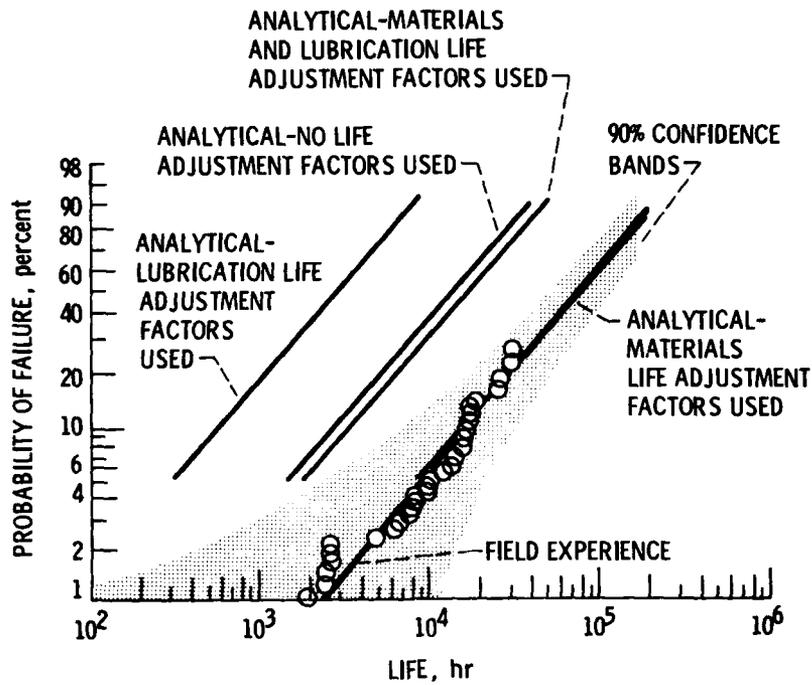


Figure 3. - Planet bearing pitting fatigue failures compared with analytically predicted mission lives. (Material factor implies combined material and material processing factor.)

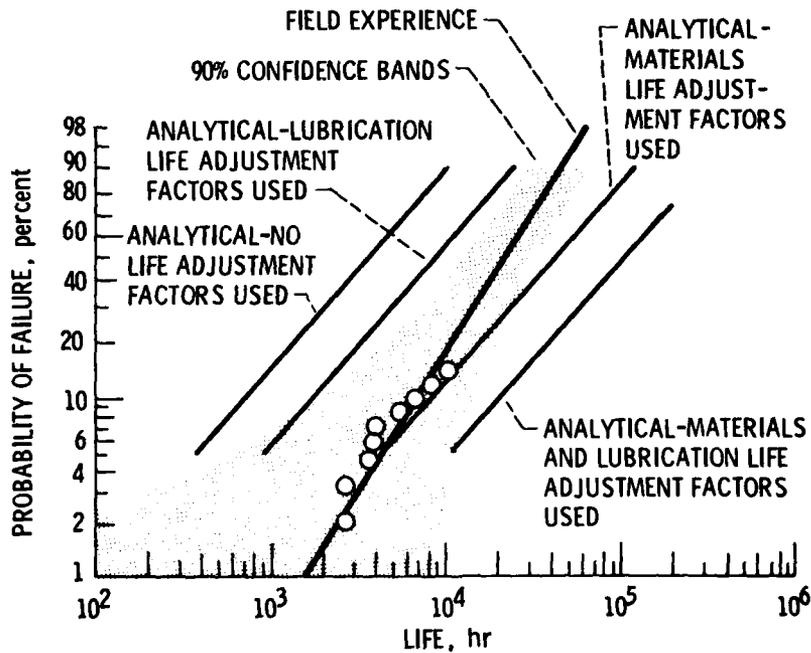


Figure 4. - Front pinion bearing pitting fatigue failures compared with analytically predicted mission lives. (Material factor implies combined material and material processing factor.)

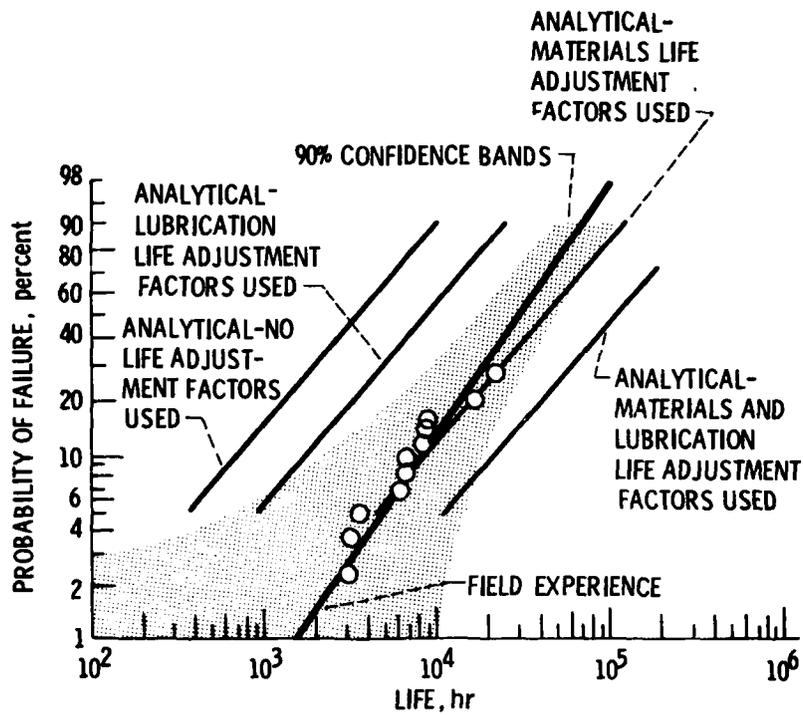


Figure 5. - Rear pinion bearing pitting fatigue failures compared with analytically predicted mission lives. (Material factor implies combined material and material processing factor.)

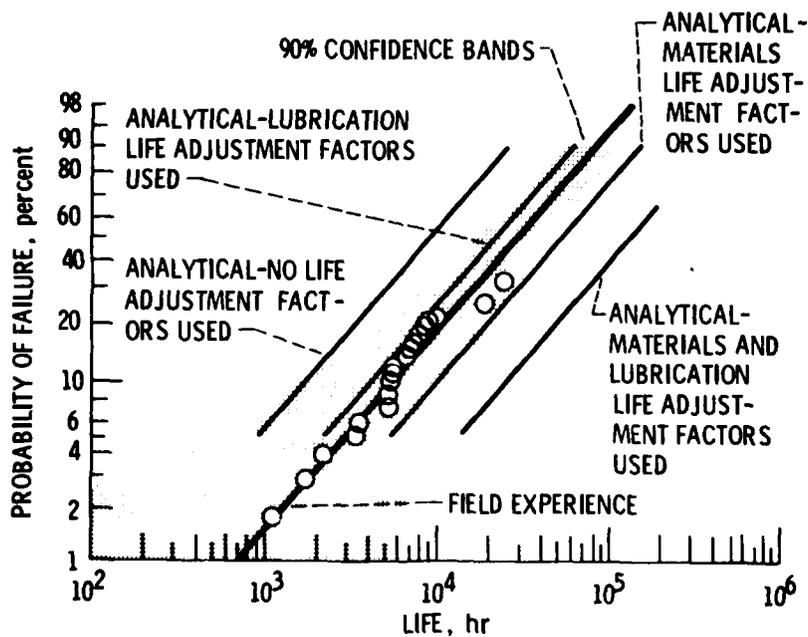


Figure 6. - Main drive bearing fatigue failures compared with analytically predicted mission lives. (Material factor implies combined material and material processing factor.)

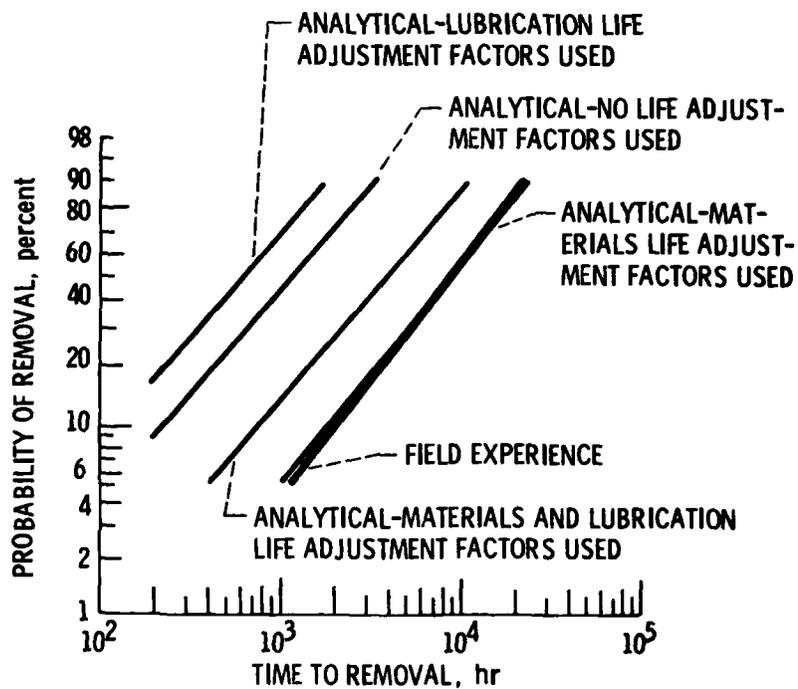


Figure 7. - Main power train system: pitting fatigue failures compared with analytically predicted system mission lives. (Material factor implies combined material and material processing factor.)

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