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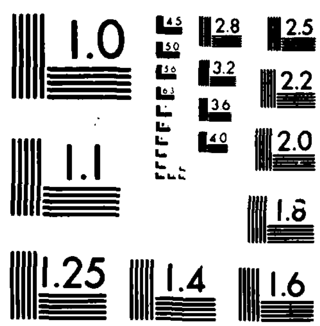


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CONTROL OF A LARGE SPACE STRUCTURE
USING DIRECT OUTPUT FEEDBACK
AND MODAL SUPPRESSION

THESIS

Eric E. Keller
Second Lieutenant, USAF

AFIT/GAE/AA/85D-10





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# CONTROL OF A LARGE SPACE STRUCTURE USING DIRECT OUTPUT FEEDBACK AND MODAL SUPPRESSION

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

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Master of Science in Aeronautical Engineering

Eric E. Keller

2Lt, USAF

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Eric E. Keller

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## Notation

$A$ – Plant matrix
$B$ – Control matrix
$C$ – Output matrix
$D$ – Modal actuator
$E$ – Damping matrix
$G$ – Optimal constant full–state feedback matrix
$\bar{g}$ – Generalized coordinate
$I$ – Identity matrix
$J$ – Quadratic performance index
$K$ – *Suboptimal gain matrix, stiffness matrix*
$M$ – Generalized mass matrix
$n_a$ – Number of actuators
$n_s$ – Number of sensors
$n_m$ – Number of modes
$Q$ – Modal weighting matrix
$R$ – Control Weighting matrix
$T$ – Transformation matrix
$\bar{u}$ – Control vector
$V$ – Matrix of right singular vectors
$U$ – Matrix of left singular vectors
$x$ – State vector
$y$ – Output vector
$\nu$ – Transformed control vector
$\Gamma$ – Transformation matrix
$\kappa$ – Sum of K's
$\Phi$ – Modal matrix
$\Psi$ – Output matrix

$\sigma$ – Singular value

$\Sigma$ – Summation/singular value matrix

$\zeta$ – Damping ratio

$\omega$ – Natural frequencies

Subscripts

c – critical/controlled modes

s – suppressed modes

i – index for variable

j – index for variable

m – index for variable

p – position elements

v – velocity elements

Superscripts

$*$ – Transformed matrix

$+$ – Pseudo Inverse

## Abstract

Direct output feedback control using one or two controllers is applied to the NASA Ground Test Facility offset antenna model. This is a test structure designed to have the vibration characteristics associated with large space structures.

The control problem is transformed from physical variables into modal variables and reformulated into a first order system. This system is truncated to a reduced order model with residual modes used only in performance evaluation. Optimal linear quadratic regulator techniques are used to design the gain matrices, and full state feedback is approximated by use of generalized inverses of the observation matrices. Spillover is eliminated through the use of transformation matrices.

The structure is shown to be controllable with this method. Alternative sensor placement is explored, and found to cause improvement in performance. The torsion modes are found to be particularly important to the performance of the structure, but need more sensing and actuation to be adequately controlled. Two controller systems require more sensors and actuators than available to achieve acceptable performance with this structure.

# CONTROL OF A LARGE SPACE STRUCTURE USING DIRECT OUTPUT FEEDBACK AND MODAL SUPPRESSION

## I. Introduction

Mankind now has the capability to place large useful objects into space. Seeking energy, military advantage, financial gain, or to satisfy scientific curiosity, activities in space will continue to increase in the future. Such missions tend to require large and flimsy devices, with ambitious performance requirements that tend to conflict with a need for low vibration levels. When Large Space Structures (LSS) such as these are subjected to control energy for pointing or maneuvering, it is difficult to avoid exciting their resonant frequencies. The classical method of dealing with this problem is by constructing stiffer structures or by limiting the bandwidth of the commands coming into the system. Since these vehicles will have to be lifted into space, it is desirable to actively control vibration rather than avoiding the problem through passive methods of material selection and construction. From another point of view, given a LSS, active vibration control can be used to increase the performance level of that vehicle.

The LSS control problem is characterized by: Infinite dimensional and nonlinear in theory, large dimensional in practice, many low resonant frequencies, often grouped in "clumps", passive damping is very light, and performance requirements can be very stringent, particularly in pointing applications. The LSS is represented by a discretized model obtained through a finite element analysis. This technique yields errors in the model, particularly in the higher frequency modes. Thus it is undesirable to try to control all of the modelled modes even if it were possible. One of the major concerns in LSS control is how to best reduce the system size. From a synthesis veiwpoint, this means the determination of which modes are critical to system performance and need control, which modes need only be protected from the effects of the controller, and which modes need not be considered in the design.

Many control strategies are appropriate for the LSS problem. Most center on modal space, and use constant gain optimal full state feedback [7,8,9,10]. Since the full state is rarely available to the controller, observers or estimators are used to provide the missing information. The resulting dimensionality problem has been mitigated by use of decentralized control [1,9,10,11]. This technique uses multiple controllers, each responsible for controlling a group of modes using lower order algorithms than would be posible using one controller. The controller structure is shown in figure 1. Each controller receives the same information from the sensors. The effectiveness of decentralized cost optimal control has been demonstrated by several researchers [1,9,10]. They found that transformation matrices applied to gain matrices eliminates spillover and enhances controller performance.

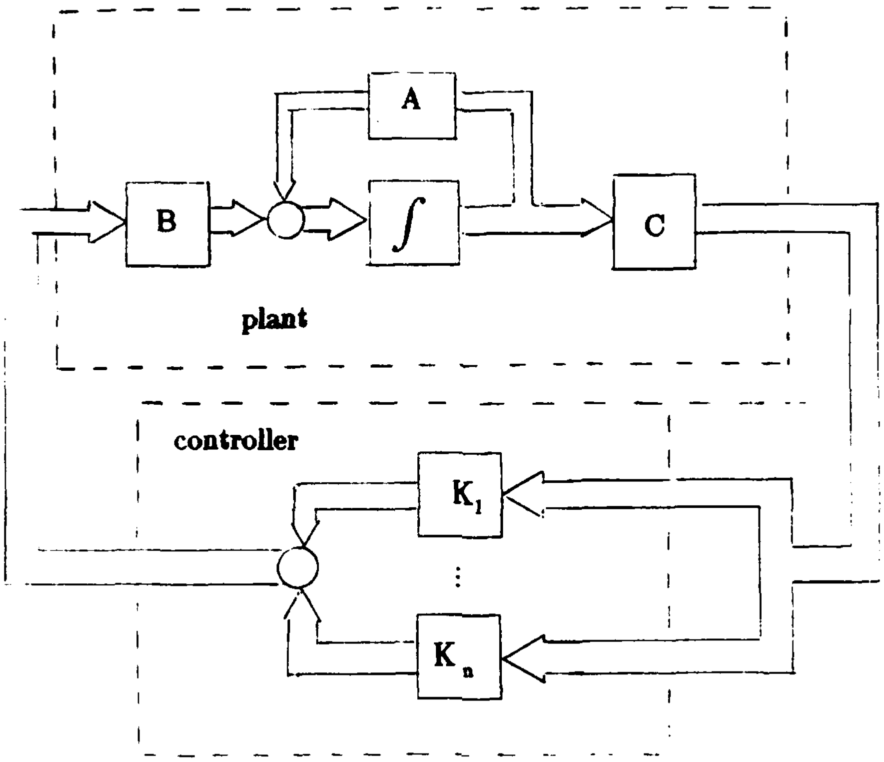



Figure 1. Decentralized Output Feedback Structure

However, maintaining four states for each mode raises concern for the capability of computers to handle the control of such a large structure on line. This concern led Calico to propose decentralized direct output feedback (DOFB). Thyfault [10] developed this technique for three or four controllers. The advantage of DOFB is that it does not require that an estimate of the state be kept on line, so the computational burden on each controller is reduced or the number of controlled modes can be increased. Since the number of modes that can be controlled is always many fewer than exsist in the structure, this may be an overwhelming advantage.

This thesis applies the method of direct output feedback to the NASA Ground Test Facility test article. This is a laboratory test article that has been designed to have properties characteristic of large space structures. One controller and two controller sytems were designed. This thesis seeks to show that this control method can be successfully applied to this structure. The object of the control effort was limited to increasing the damping ratios of the controlled modes as much as possible while maintaining the stablility of the uncontrolled (residual) modes. The time response of the system was also explored.

## II. LSS SYSTEM CONFIGURATION

The system model used in this study is a finite element model of NASA's Ground Facility for Large Space Structures Control Verification at Marshal Space Flight Center. This is a facility designed to test LSS control schemes, and was not designed to develop structures that will actually be put in space. In order to accomplish these objectives, the test structure was designed with the pathologies associated with large space structure behavior.

The test structure consists of an ASTROMAST beam suspended from a set of three gimbals, the angular pointing system (APS). The APS is in turn suspended from an x–y shaker table, the Base Excitation Table (BET), that can be used to introduce disturbances into the system. In order to simulate typical LSS behavior, two tip attachments have been built: an antenna and a cruciform. These attachments provide the closely packed natural frequencies associated with typical LSS behavior. Figure 2 shows the structure with the antenna attached, as was used in this study, and figure 3 shows the structure with the cruciform attached. Future plans call for the antenna configuration to include evaluation of line of sight control capabilities.

In the original configuration, the only actuators available for control are the three torque motors associated with the APS. Under the VCOSS II program (Vibration Control of Space Structures), four additional actuators

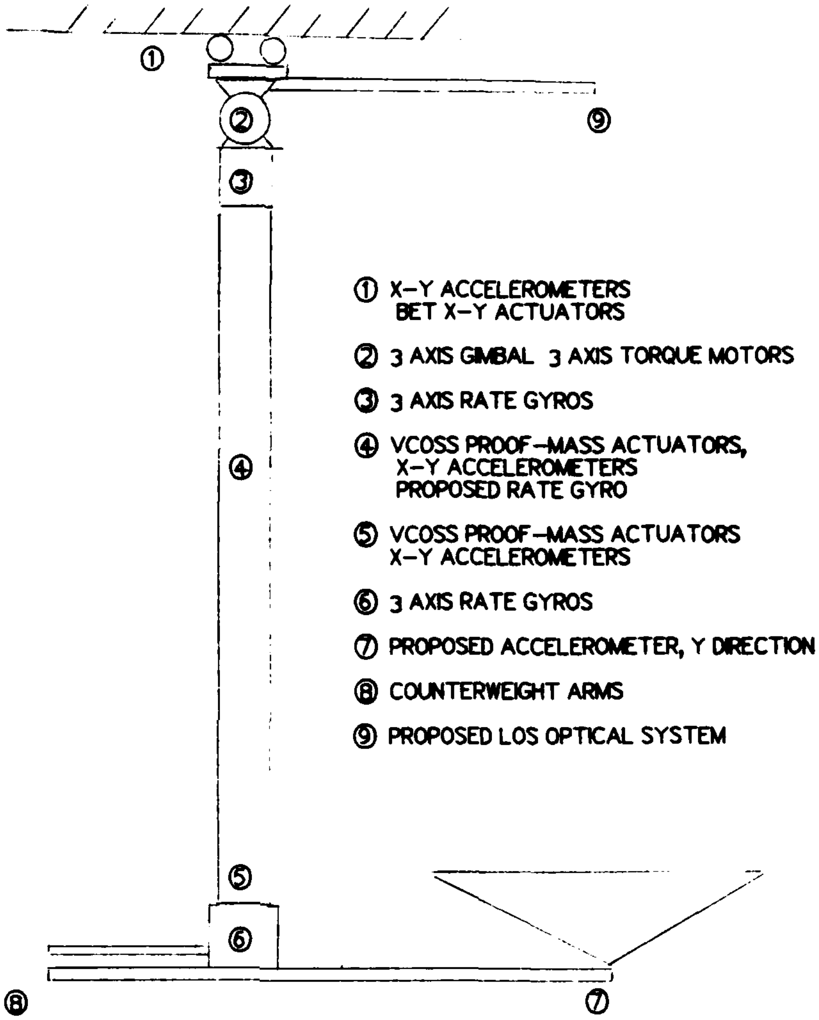



**Figure 2. NASA Test Model**

will be added. These will be proof–mass x–y actuators. Four additional accelerometers will be added co–located with these new actuators. As it stands, the apparatus has x–y accelerometers at the BET and x–y–z accelerometers at the mast tip. There are three rate gyros attached to the APS base plate and three rate gyros at the mast tip. Thus after the VCOSS modifications the structure will have 15 sensors and 7 actuators.

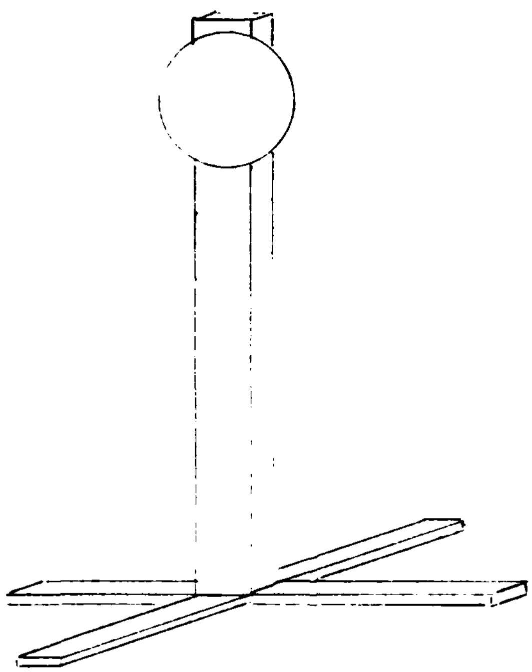


Figure 3. Test Structure With Cruciform Attachment

The sensors outputs are processed by a COSMEC−I microcomputer, which then sends the data to a Hewlett Packard 9000 series microcomputer for control law calculations. The COSMEC−I system can be used to process the control law while the HP collects the data for post processing. The outputs of the accelerometers are integrated in the COSMEC−I system. The control output is sent by the COMSEC−1 in analog form, from $\pm 10$ volts, which is the saturation level of the torquer servo amplifier.

The x and y pendulum gimbals can exert 51 N.m of torque, and the azimuth gimbal can exert 10 N.m of torque. The azimuth gimbal was included to facilitate various test scenarios, and because there is significant coupling between the torsional and bending modes.

The beam is a spare Voyager ASTROMAST. The astromast weighs approximately 5 pounds and is 45 feet long. It is constructed of S−GLASS. The beam was launched in a folded state and untwisted in space. This design produces a longitudinal twist of about 280 degrees, which couples the bending and torsional modes.

The tip package containing rate gyros and accelerometers weighs approximately 45 pounds. This gives the structure the characteristics of being pinned at both ends rather than pinned at the top and free at the tip.

The analytical model was generated by the Interactive Structures Matrix Interpretive System (ISMIS) finite element program. The model included 28 nodes and, where appropriate, each node had 6 generalized coordinates. The position of the modes on the structure is shown in figure 4. A 133 mode model was generated which was truncated to a 30 mode model for control purposes. The modal frequencies and a characterizaton of the modes can be found in table 1. Unfortunately, generalized mass and stiffness matrices were not available. However, since the eigenvectors were normalized, the eigenvalues and eigenvectors provide sufficient information to construct a control model.

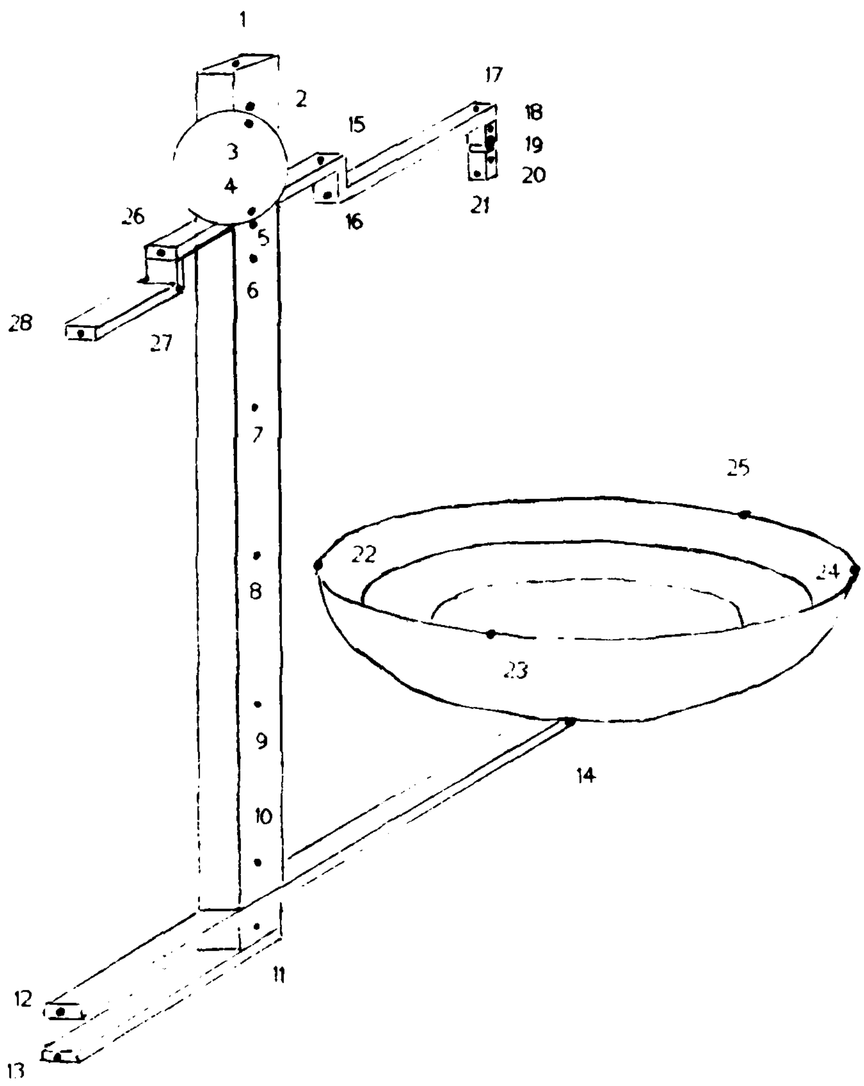



Figure 4. Test Structure Node Placement

Table 1

NASA Test Model Modes

| Mode | Frequency (Hz) | Characterization |
|---|---|---|
| 1 | 0.0 | X Direction Free Body Mode |
| 2 | 0.0 | Z Axis Free Body Rotation |
| 3 | 0.0 | Y Direction Free Body Mode |
| 4 | 0.0573 | Torsion |
| 5 | 0.1307 | Y–Plane Pendulum |
| 6 | 0.1338 | X–Plane Pendulum |
| 7 | 0.3366 | Y–Plane bending |
| 8 | 0.4243 | Bending X–Plane + Torsion |
| 9 | 0.4364 | Y–Plane bending |
| 10 | 0.4513 | Bending X–Plane + Torsion |
| 11 | 0.4696 | Dish Rim Motion |
| 12 | 0.4702 | Dish Rim Motion |
| 13 | 0.5736 | Bending X–Plane + Torsion |
| 14 | 0.6213 | Y–Plane bending |
| 15 | 1.0289 | Bending X–Plane + Torsion |
| 16 | 1.0387 | Dish Rim Motion |
| 17 | 1.0387 | Dish Rim Motion |
| 18 | 1.2579 | Y–Plane bending |
| 19 | 1.2978 | Local CW Arms |
| 20 | 1.2995 | Local CW Arms |
| 21 | 1.3944 | Bending X–Plane + Torsion |
| 22 | 1.5022 | Y–Plane bending |
| 23 | 1.7572 | Bending X–Plane + Torsion |
| 24 | 1.7893 | Local Antenna |
| 25 | 2.1078 | Bending X–Plane + Torsion |
| 26 | 2.1863 | Y–Plane Bending |
| 27 | 2.6632 | Y–Plane Bending |
| 28 | 2.7438 | Bending X–Plane + Torsion |
| 29 | 2.8745 | Dish Rim Motion |
| 30 | 2.8745 | Dish Rim Motion |

The main controller performance criteria was the increase of damping as well as the maintenance of stability. Time response was also studied, but not used in the controller design process. The time response of the antenna attachment point, node 14, was used as representative of structure time response. The control method used is presented in the next chapter.

# III. SYSTEM MODEL

## EQUATIONS of MOTION

Generally, the equations of motion for a large space structure are given as

$$M\ddot{\bar{g}} + E\dot{\bar{g}} + K\bar{g} = D\bar{u} \qquad (1)$$

where M is a nxn symmetric mass matrix, E is a nxn symmetric damping matrix, K is a nxn symmetric stiffness matrix, D is a nxm matrix of actuator locations, $\bar{g}$ is a nx1 generalized coordinate vector, and u is a mx1 control input vector. Equation 1 represents a second order eigenvalue–eigenvector problem. The nxn matrix of eigenvectors, $\Phi$, is normalized with respect to the mass matrix, M, such that

$$\Phi^T M \Phi = I \qquad (2)$$

which can be shown to yield [6]

$$\Phi^T E \Phi = \begin{bmatrix} \diagdown & & \\ & 2\zeta\omega & \\ & & \diagdown \end{bmatrix}$$

$$\Phi^T K \Phi = \begin{bmatrix} \diagdown & & \\ & \omega^2 & \\ & & \diagdown \end{bmatrix} \qquad (3)$$

where

$I$ = nxn identity matrix

$2\zeta\omega$ = nxn diagonal damping matrix

$\omega^2$ = nxn diagonal matrix of eigenvalues of Eq (1)

Defining the modal coordinates, $\bar{\eta}$, by

$$\bar{g} = \Phi\bar{\eta} \tag{4}$$

Eq (1) can be transformed to modal coordinates by pre-multiplying by $\Phi^T$, yielding

$$\ddot{\bar{\eta}} + 2\zeta\omega\dot{\bar{\eta}} + \omega^2\bar{\eta} = \Phi^T D\bar{u} \tag{5}$$

Define $\Phi_a$ as

$$\Phi_a = \Phi^T D \tag{6}$$

This can be put into state space representation of the form

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \tag{7}$$

where A is the 2nx2n plant matrix, B is the 2nxm input matrix, $\bar{x}$ is the 2nx1 state vector, and $\bar{u}$ is the mx1 control vector. These are related to Eq (5) by

$$A = \begin{bmatrix} 0 & I \\ -\omega^2 & -2\zeta\omega \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \Phi_a \end{bmatrix} \tag{8}$$

$$\bar{x} = \begin{bmatrix} \bar{\eta} \\ \dot{\bar{\eta}} \end{bmatrix}$$

The output equation is given by

$$y = \Psi_v \dot{\bar{g}} + \Psi_p \bar{g} = \Psi_v \Phi \dot{\bar{\eta}} + \Psi_p \Phi \bar{\eta} \tag{9}$$

In this study, only rate sensors were considered, so the output equation is given by

$$y = \Psi_v \Phi \dot{\bar{\eta}} = \Phi_s \dot{\bar{\eta}} = [\ 0 \mid \Phi_s ] \bar{x} \tag{10}$$

In modal coordinates, the A matrix contains no coupling between modes. The only coupling is in the actuator and sensor matrices. Thus the set of system model equations can be split into multiple sets of equations, one for the controller, one for the suppressed modes or, in a two controller system, the second controller, and one for the residual modes. A controller has the dual responsibility of controlling the modes assigned to it and "protecting" its modes from the effects of observation or control spillover.

Modal Control

The advantage of modal control is that before control there is no interaction between modes. The only linkage is through the output equation and through the applied control. This means that an n mode system could be arbitrarily partitioned into from 1 to n uncoupled second order differential equations. For the purposes of this study, the system equation is partitioned into three sets of modes. These are the controller 1 modes, the suppressed modes or the controller 2 modes, and the residual modes.

Thus the system model can be written as

$$\begin{aligned} \dot{\bar{x}}_c &= A_c \bar{x}_c + B_c \bar{u} \\ \dot{\bar{x}}_s &= A_s \bar{x}_s + B_s \bar{u} \\ \dot{\bar{x}}_r &= A_r \bar{x}_r + B_r \bar{u} \end{aligned} \tag{11}$$

for which the output equation is

$$y = C_c\bar{x}_c + C_s\bar{x}_s + C_r\bar{x}_r \qquad (12)$$

Thus it can be seen that the output equation cannot be easily decoupled and that the three differential equations that make up the suystem are coupled by a common control vector, $\underline{u}$.

The control method employed in this study is direct output feedback, which feeds back the sensor outputs multiplied by a constant gain matrix as the control input. Thus the controller gain must deal with the coupling of the modes by the control (control spillover) or by the sensors (observation spillover). It will be shown later that one controller need not eliminate both forms of spillover. If multiple controllers are used, one controller will eliminate control spillover, one controller will eliminate observation spillover, and any other controllers will eliminate both types of spillover. The next part of this chapter will deal with the gain selection method. The approach taken in this study is suboptimal control using direct output feedback. The method for selecting the gains will be discussed in the next section.

## Suboptimal Control

Optimal control theory is used to select the controller feedback gains. Linear Quadratic cost optimal control (LQ control) has many advantages in the synthesis of multiple–input, multiple output control laws which suggest LQ techniques for the LSS control problem. However, LQ control requires full state feedback. Since direct output feedback doesn't provide access to all of the states, we cannot use the optimal gain directly. Thus it is necessary to operate on the optimal gain matrix to get an appropriate suboptimal gain.

To obtain the feedback gain matrix G, a quadratic cost function is defined as

$$J = 1/2\int_0^{\infty} \bar{x}_c^T Q \bar{x}_c + \bar{u}^T R \bar{u} dt \tag{13}$$

where

Q is an n x n positive semidefinete weighting matrix

R is an m x m positive definite weighting matrix

and $\bar{x}_c$ is constrained by Eq 11a. For this cost functional the optimum feedback gain is

$$G = -R^{-1}B_c^T S \tag{14}$$

where S is the solution to the algebraic Riccati equation

$$SA_c + A_c^T S - SB_c R^{-1} B_c^T S + Q = 0 \tag{15}$$

Thusly the applied optimal control would be

$$\bar{u} = G\bar{x}_c \tag{16}$$

yielding the closed loop system

$$\dot{\bar{x}}_c = (A_c + B_c G)\bar{x}_c \tag{17}$$

This solution requires full state feedback for proper application. Since the output y cannot provide the full state, an approximation to this optimal gain must be made for use with direct output feedback. Equating the closed loop system equations for direct output feedback and full state feedback

$$(A_c + B_c K C_c)\bar{x}_c = (A_c + B_c G)\bar{x}_c \tag{18}$$

$$A_c + B_c K C_c = A_c + B_c G \tag{19}$$

$$B_c K C_c = B_c G \tag{20}$$

$$K C_c = G \tag{21}$$

If the $C_c$ output matrix is full rank (making $C_c C_c^T$ invertable), the suboptimal gains can be solved for directly

$$KC_c C_c^T = GC_c^T \tag{22}$$

$$K(C_c C_c^T)(C_c C_c^T)^{-1} = GC_c^T(C_c C_c^T)^{-1} \tag{23}$$

$$K = GC_c^T(C_c C_c^T)^{-1} \tag{24}$$

When $C_c$ is not full rank, K is found using the inverse of the singular value decomposition of $C_c$, $C_c^+$, which is called the generalized or Penrose inverse. From (21)

$$KC_c C_c^+ = GC_c^+ \tag{25}$$

$$K = GC_c^+ \tag{26}$$

Using one of these methods, the suboptimal gains can be determined for an output matrix of any rank.

## Direct Output Feedback Control

The system as now equipped can most reasonably be controlled using a one or two controller model. Then there are four classes of modes: controlled/critical modes, suppressed or second controller modes, residual modes, unmodelled residual modes. The residual modes are used in the truth model if at all. Suppressed modes are not actively controlled, but the feedback gains are selected to reduce observation and control spillover between these modes and the controlled modes. In a two controller system, the feedback gains are selected to cause minimal interaction between modes.

The system equation of motion is rearranged as follows

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_r \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_r \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_r \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_r \end{bmatrix} |\bar{u}| \tag{27}$$

The output equation is

$$\dot{\bar{x}} = \begin{bmatrix} C_1 & C_2 & C_r \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_r \end{bmatrix} \tag{28}$$

The proposed control for one controller is of the form

$$\bar{u} = Ky \tag{29a}$$

For two controllers, this becomes

$$\bar{u} = \kappa y \tag{29b}$$

Where $\kappa = K_1 + K_2$. $K_1$ and $K_2$ are the direct output feedback gain matrices for system 1 and 2, respectively. Thus the closed loop state equation is

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_r \end{bmatrix} = \begin{bmatrix} A_1+B_1KC_1 & B_1KC_2 & B_1KC_r \\ B_2KC_1 & A_2+B_2KC_2 & B_2KC_r \\ B_rKC_1 & B_rKC_2 & A_r+B_rKC_r \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_r \end{bmatrix} \tag{30}$$

It is obvious that the above control method would not guarantee stability, even without the residuals. Since direct output feedback is suboptimal, there are no guarantees of stability, but stability can normally be attained through the gain selection process. Although it is not possible to avoid changing the eigenvalues of the residual modes, stability can be maintained by proper selection of modal weightings and proper selection of which modes to treat as residual. To guarantee that the eigenvalues of system 1 and 2 are as designed (ignoring the residuals), the gain matrix K or $\kappa$ must meet a number of conditions. The method by which Eq. 30 is put in the proper form is detailed in the next section.

## Transformed DOFB Control

As was stated in the last section, the feedback gains must be chosen such that Eq 30 is upper triangular. This can be acheived by eliminating control and observation spillover between the controlled and suppressed modes (for multiple controllers, the suppressed modes of one controller are the controlled modes in the other controllers). For one controller this means that the 1−2 and 2−1 partitions of Eq (direct 4) be zero, or

$$KC_2 = 0$$

and (31)

$$KC_c \neq 0$$

For two controllers

$$K_1C_2 = 0$$

$$B_1K_2 = 0$$

while (32)

$$B_2K_2 \neq 0$$

$$K_1C_1 \neq 0$$

While it would be desirable to force $K_iC_r = 0$ and $B_rK_i = 0$, this is an impossibility since these modes are assumed to be unmodelled. They are present here to provide a system truth model.

From Eq (30), it is obvious that any control that ignores the residual modes cannot guarantee overall system stability, since the spillover may cause instability. This is true of all methods, since by nature of the problem (infinite dimensionality) there will always be unmodelled modes. The only approach to this is to try to avoid expending any high frequency control energy and to include the modelled residual modes in any performance evaluation.

The method of acheiving the conditions given in Eq (31) will be to find transformation matrices T and $\Gamma$ and define a transformed gain matrix

$$K_1 = K_1^* \Gamma$$

and (33)

$$K_2 = T K_2^*$$

Then the feedback matrix for a single controller is given by $K_1$ and the feedback matrix for a two controller matrix is given by $\kappa = K_1 + K_2$

The $K_i^*$ matrices are given by

$$K_1^* = G_1 (\Gamma C_1)^+$$

and (34)

$$K_2^* = G_2^* C_2^+$$

$G_2^*$ is the Riccatti solution when $B_2^* = TB_2$ is used instead of $B_2$. The modifications to the gain matrices are necessary to make the gain and transformation matrices conform. This is discussed further in the next chapter.

For one controller the closed loop system equation is given by

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} A_c + B_c K C_c & 0 & B_c K C_r \\ B_s K C_c & A_s & B_s K C_r \\ B_r K C_c & 0 & A_r + B_r K C_r \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_s \\ \bar{x}_r \end{bmatrix} \tag{35}$$

For a two controller system the state equation is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_r \end{bmatrix} = \begin{bmatrix} A_1 + B_1 \kappa^* C_1 & 0 & B_1 \kappa^* C_r \\ B_2 \kappa^* C_1 & A_2 + B_2 \kappa^* C_2 & B_2 \kappa^* C_r \\ B_r \kappa^* C_1 & B_r \kappa^* C_2 & A_r + B_r K C_r \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_2 \\ \bar{x}_r \end{bmatrix} \tag{36}$$

The sensor and actuator requirements to acheive the above closed loop systems are given in the next section of this chapter. The transformation technique is outlined in the next chapter.

## Sensor/Actuator Requirements

To satisfy the conditions of Eq (31), the column space of $K^T$ must be in the null space of $C_c^T$. The effect of suppression is that the eigenvalues of the suppressed modes are unchanged. In a one controller system this can be acheived by suppressing either control spillover, where $B_s K = 0$, or by suppressing observation spillover, where $KC_s = 0$. It is possible to have two sets of suppressed modes. Then K would have the property that it eliminated control spillover into one set and it eliminated observation spillover into the other set. It is also possible to eliminate control and observation spillover into one set of modes. This means that these modes would not be excited by the controller. However, for this study it made the most sense to suppress observation spillover into the suppressed modes, because there are many more sensors available than actuators. In a three or more controller system, it is necessary to eliminate both control and observation spillover for some of the controllers.

The number of modes that the controller suppresses is directly related to the number of sensors available. There must be more sensors than suppressed modes using the suppression method outlined below. Another concern is that the observability of the controlled modes goes down as more modes are suppressed. This is not demonstrated rigorously here, but trial and error computer runs demonstrate the point. Direct output feedback has more problems than observer methods in this regard. Similarly, using actuators to suppress modes reduces the controlability of the controlled modes.

If there are $n_s$ suppressed modes and $n_{sen}$ sensors, then $C_s$ will be $n_{sen} x n_s$. However, for velocity feedback, $C_s = [\ 0 \mid C_{vs}\ ]$, so the only condition needed for suppression is that $KC_{vs} = 0$. The rank of K cannot be greater than the dimension of the null space of $C_{vs}$. The null space of $C_{vs}$ has dimension $n_{sen} - r$ where $r \leq n_s$. Thus the designer of such a controller would choose to suppress fewer modes than he has sensors.

As has been mentioned, the number of modes that are suppressed has an effect on the observability of the system. Thus it is usually good practice to suppress only those modes that are affected significantly by the controller.

The development of the suppression method given in the next chapter will further demonstrate these requirements.

## IV. Suppression Technique

The method used to transform the gain matrix such that it is orthogonal to $C_s$ is the same one as used by Aldrige, Miller, Thyfault and others [1,9,10,11]. The optimal gains are modified such that the closed loop system, Eq (30), yields Eq (31), which is in block triangular form. The modification is carried out by a linear transformation that maps the gain matrix K into the null space of $C_s$. This linear transformation has been called $\Gamma$ in the previous development.

$\Gamma$ rids the system of observation spillover between the suppressed and controlled modes. It is also possible to force the columns space of K to be in the null space of $B_s$, which eliminates control spillover between the suppressed and controlled modes. The transformation matrix that suppresses control spillover has been called T in the previous discussion.

In a one controller system, either type of suppression has the effect that, for no residuals, the eigenvalues of the suppressed modes are unchanged from their uncontrolled values. Suppression with one controller requires only one transformation matrix. In contrast, multiple controller models require suppression of both control and observation spillover.

For the suppression of a set of modes while controlling another set of modes, the transformation matrix, $\Gamma$ or T, must meet the conditions

$$\Gamma C_s = 0 \quad \text{and} \quad \Gamma C_c \neq 0 \tag{37}$$

For two controllers this becomes

$$\Gamma C_2 = 0$$

$$\text{and} \tag{38}$$

$$\Gamma C_1 \neq 0$$

while

$$B_1 T = 0$$

$$\text{and} \tag{39}$$

$$B_2 T \neq 0$$

As was previously discussed, there will always be residual modes that cannot be suppressed, since it is impossible to have an infinite number of sensors or actuators. It is also likely that a large structure will have noncritical modes that need not be controlled to meet the control objectives. In any case, there are limits on the number of actuators and sensors in any problem of interest. The effect of the controller on the residual modes can be minimized by limiting the bandwidth of the controller and by reducing the control energy that is fed into the higher frequency controlled modes. Other studies [1,2,9,10] have considered multiple controller models where all modes were either controlled or residual. Thus control energy was applied to modes very close in frequency to the residual modes. One controller models have the property that the suppressed modes form a barrier between the controlled modes and the residual modes where very little control energy is expended.

The gist of this discussion is that a means of finding a basis for the null space of $B_s$ or $C_s$ is necessary to find the T and $\Gamma$ matrices. One convenient source of these vectors is provided by the technique of singular value decomposition. Briefly stated [3], for all mxn matrices C, there exist unitary matrices U and V such that

$$\Sigma = U^T C V$$

where for m>n

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \ldots 0 \\ 0 & \sigma_2 \ldots 0 \\ & \ldots \\ 0 & 0 \ldots \sigma_n \\ 0 & 0 \ldots 0 \\ & \ldots \\ 0 & 0 \ldots 0 \end{bmatrix} \tag{40}$$

Where $\sigma_i^2$ are the eigenvalues of $C^TC$. The positive square roots of $\sigma_i^2$ are called the singular values of C. U is the mxm matrix of eigenvectors of $CC^T$ and V is the nxn matrix of eigenvectors of $C^TC$. If C is of rank k, then there will be k non−zero singular values of C. The bottom m−k rows of U map the columns of C to zero. These m−k rows of U can be used to suppress the C matrix.

Normally, C will be of full rank so $k=n_{sup}$. C will be less than full rank if its columns are not linearly independent. In this case $k < n_{sup}$. This corresponds to redundant sensors.

The above development shows that $\Gamma = [u_{k+1}^T \ldots u_m^T]^T$ can be used to suppress the C matrix given that the system has more sensors than suppressed modes. However, since $\Gamma$ is $(n_{sen}-n_{sup})xn_{sen}$, this means that the K matrix must be $n_{act}x(n_{sen}-n_{sup})$ in order to conform with $\Gamma$. The gain matrix can be modified to meet this requirement as will be shown later in this section.

As has been stated before, optimal control using a linear quadratic regulator requires full state feedback. However, direct output feedback gets around this requirement by using a pseudo inverse as was discussed in the last chapter. For a one controller model, the calculation of the gain matrix $K^*$ is only slightly different than that of the untransformed gain matrix K. If $C_c^*$ is defined as

$$C_c^* = \Gamma C_c \tag{41}$$

it can be seen that

$$K^* = GC^{*+} \tag{42}$$

where the optimal gain matrix G is the same as given in Eq (14), which gives

$$G = -R^{-1}B_c{}^TS \tag{43}$$

S is the solution to the algebraic Riccati equation

$$SA + A^TS - SBR^{-1}B^TS + Q = 0 \tag{44}$$

Since Eq (44) does not involve the C matrix, which is all the transformation technique changes for one controller, it is reasonable to use the same G matrix as before transformation. With multiple controllers it is necessary to recalculate the G matrix [10].

Then, as in the previous chapter

$$(A + BK^*C^*)\bar{x} = (A + BG)\bar{x} \tag{45}$$

$$A + BK^*C^* = A + BG \tag{46}$$

$$BK^*C^* = BG \tag{47}$$

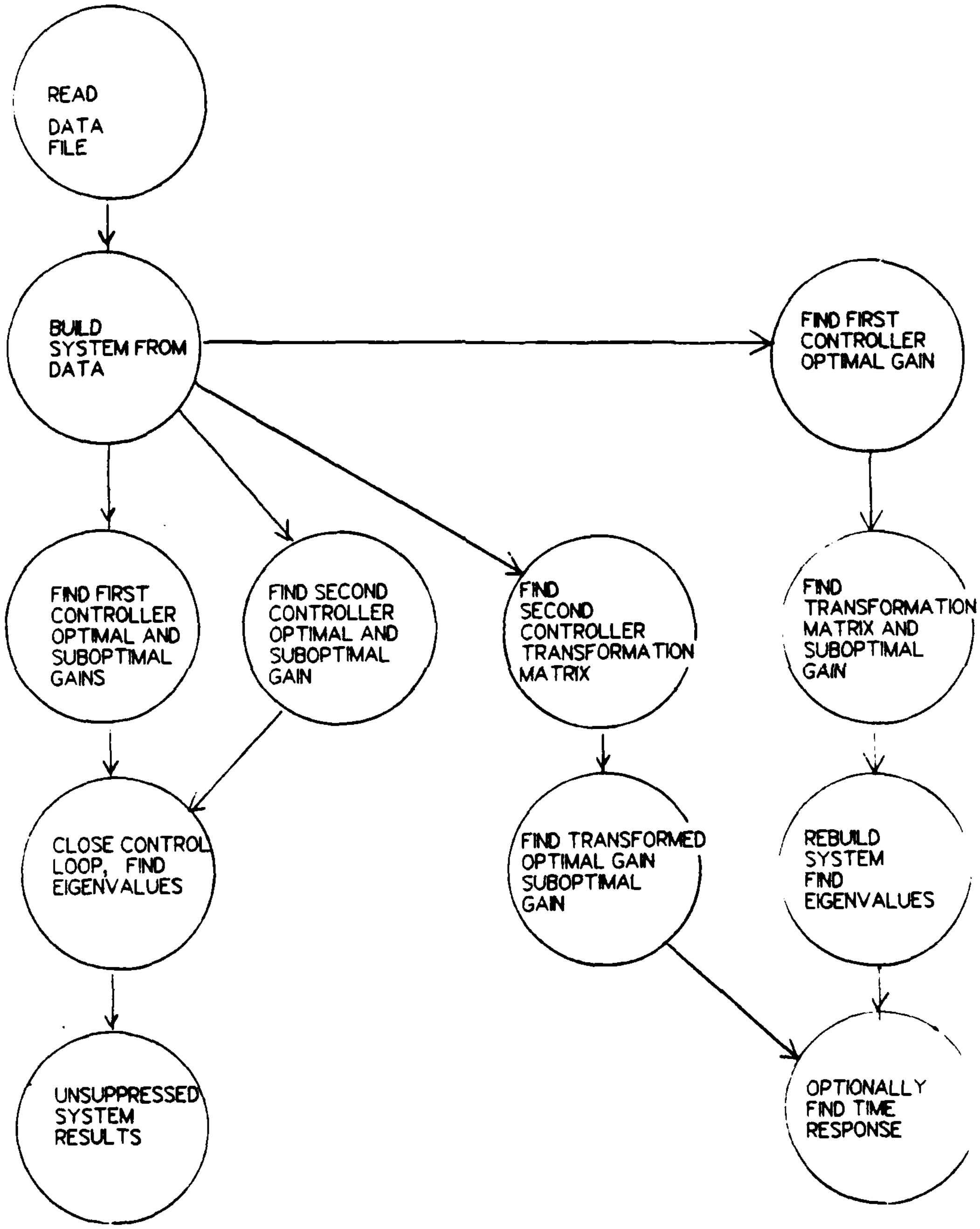
$$K^*C^* = G \tag{48}$$

Which yields Eq (42).

## V. Computer Analysis.

A modified form of the ACOSS program [1,9,10,11] was used to design the controller. A data flow chart of this program can be found on the next page, and a listing can be found in Appendix A. After the system model is set up and partitioned, the optimal full state feedback gain matrix is calculated. As an aid to controller design, the loop is closed with this gain (with no suppression).

The suppressed modes C matrix is then input to an International Mathmatical and Statistical Library (IMSL) routine that finds its singular value decomposition. This routine returns the gamma matrix. The gamma matrix is then multiplied by the controlled modes C matrix, and the resulting matrix, $C^*$, is sent to the gain finding routine. The new gain is multiplied by $\Gamma$ to form the transformed gain $K^*$. The control loop is closed with this gain and the results are sent to an eigenvalue analysis routine. Finally, the time response is optionally calculated.




Program Data Flow Diagram

## VI. Investigation

The investigation mainly proceded in pursuit of a stable system. The first cut at controlling the structure consisted of putting the first 11 bending modes in the controller, suppressing the next 8 modes, and treating the remaining 8 bending modes and the free body modes as residuals. These choices were made because there is a large frequency difference between modes 14 and 15 and another large difference between modes 22 and 23. However, it soon became obvious that modes 11, 12, 16, 17, 19, 20, 24, 29 and 30 were not affected by control. These modes are local to the antenna and the counterweight arms and thus are not affected by actuators on the beam. This makes physical sense, since the antenna is isolated from the beam by the relativly massive tip package. These modes were put into the residuals. It should be noted that modes 11 and 12 are fairly important in the motion of the dish rim motion, but are nearly unobservable and uncontrollable from anywhere else on the structure, including the dish attachment point. Similarly, modes 19 and 20 only affect the motion of the couterweight arms. Modes 16 and 17 are less local, but nontheless are only slightly affected by control. Modes 29 and 30 are local to the antenna dish, and are not touched by the controller.

Also immediately obvious after a few runs was that mode 28 was very important to system stability. Thus it was made into a suppressed mode. After finer adjustments, mode 27 was suppressed. It was very interesting that although both modes 27 and 28 became much more stable when treated as residual, other modes lost stability unless these two modes were suppressed. Similarly, the eigenvalues of modes 22, 23, and 25 moved significantly when treated as residual, so these modes were suppressed. As has been discussed previously, suppression lowers the observability of the controlled modes. This was evident in three cases. Mode 13 is a torsion/x bending mode, as is mode 15. Many runs were made with mode 15 suppressed and for these runs, mode 13 lost damping and was nearly unstable. Once mode 15 was moved into the controller, mode 13 became controllable. Similarly, modes 14 and 18 are both y bending modes. If mode 18 is suppressed, mode 14 gains very little damping by being controlled. Thus mode 18 was moved into the controller. The final example of loss of observability, and the most difficult case is the interaction between the first bending mode and the z−axis rotational free body mode. These modes are difficult to stabilize simultaneously. The real problem is that the mode 4 is not very observable. If the free body mode is made a residual, it goes unstable and the torsion mode is stable with fairly good damping. If the free body mode is suppressed, the torsion mode is not controllable. If both modes are controlled, the gains can be set so that one or the other is controllable, but not both. This problem can be solved by dropping the z−axis torque motor from the control loop and making the z−axis free body mode a residual. This is not a very palatable solution, although a stable configuration. Mode 4 can be adequately controlled by the other actuators in the system since there is strong coupling between the torsion modes and the x plane bending modes

. Also a problem with the controller is that if the sensors at the attachment to the ceiling are included in the model, the x and y free body modes are unstable. This is because the only actuators that can control these modes are those in the base exitation table (BET). Since these two sensors don't contribute information on any bending modes, they can be left out of the control loop. If they are included in the control loop, the BET must also be in the loop.

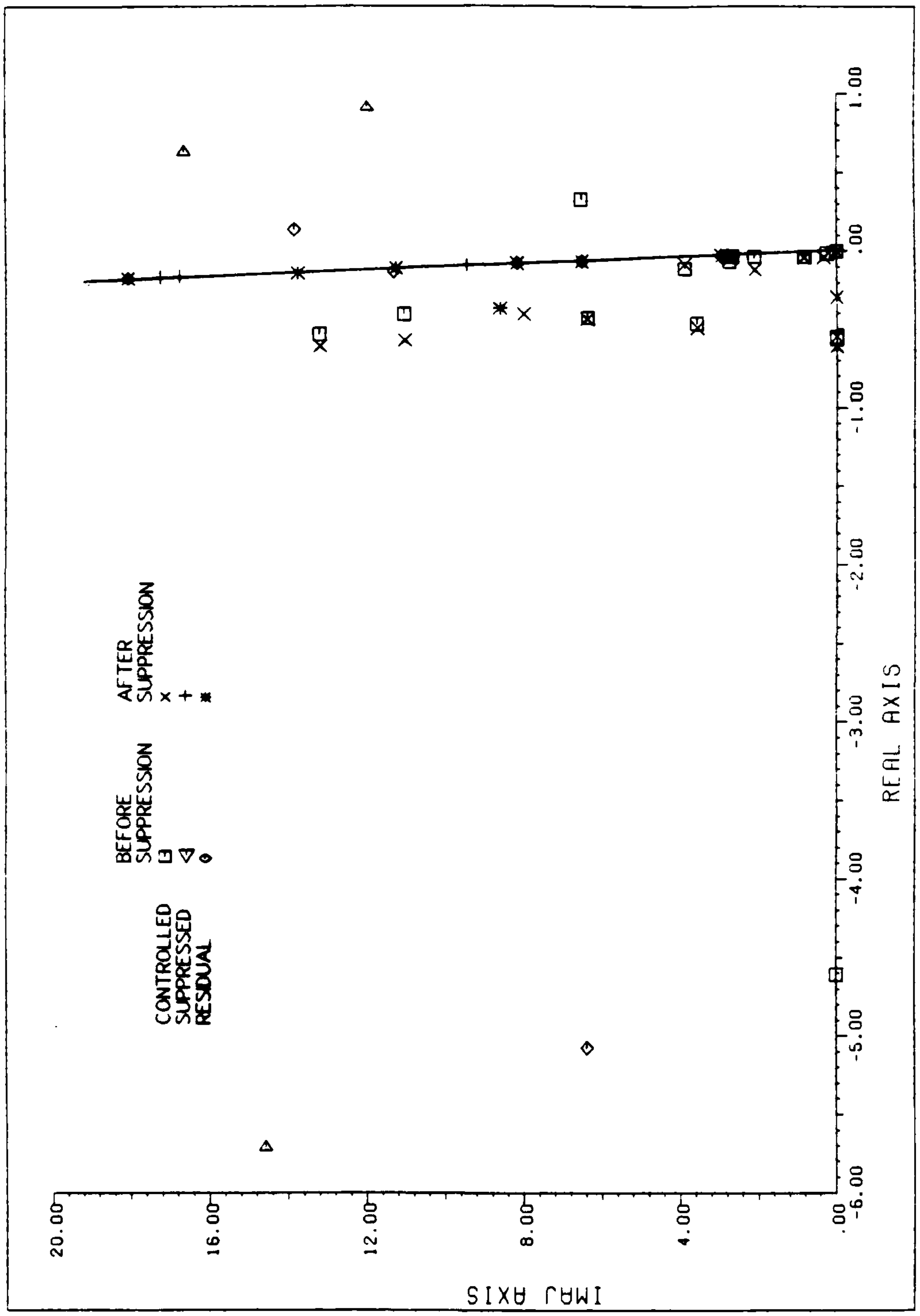
Unfortunately, using the BET and leaving the sensors in the loop doesn't contribute to the solution of the problem with the first torsion mode. This problem is intensified by the fact that modes 23 and 25 are torsion/bending modes. Thus if these are suppressed, the observability of all the torsion modes is severely reduced. An inspection of the $\Phi_s$ observation matrix shows that mode 4 only shows up significantly in two sensors. This demonstrates that it is important to have enough torsion sensors with this structure.

The final iteration with the available configuration has modes 23 and 25, both torsion/bending modes, in the controller. This allows mode 4 to be controlled. The insight that lead to this assignment was that the 4th mode really only has two sensors that measured it well, thus when these modes were suppressed they took all of the obsevability away from mode 4. It still requires the highest cost of any mode in order to gain significant damping. The other problem with this is that it is not prudent to control a mode of that high a number in the model. Higher frequency modes are normally modelled less accurately than lower frequency modes when using finite element techniques. Figure 5 is a plot of the eigenvalues of this control configuration showing the eigenvalues both before and after suppression. Before control, all eigenvalues are on the straight line associated with $\zeta$ = 0.01, the assumed passive damping ratio. As can be seen from figure 5, the eigenvalues of the suppressed modes lie almost directly on this line. The importance of suppression to system stability can be seen in all eigenvalue plots generated during this study. In every case, the pre–suppression system is unstable.

It was then proposed that additional rate gyros be placed at the momentum exchanger positions. In application, this could be implemented by realigning the two accelerometers that are already at these positions, taking sum and difference calculations to produce both acceleration and angular information. Since the bottom proof–mass actuator is very close to a rate gyro, an accelerometer was placed at the antenna attachment point pointed in the circumferencial direction. With these sensors in place modes 23 and 25 could be suppressed while still adequately controlling the first torsion mode. Figure 6 is a plot of the eigenvalues of this control configuration.

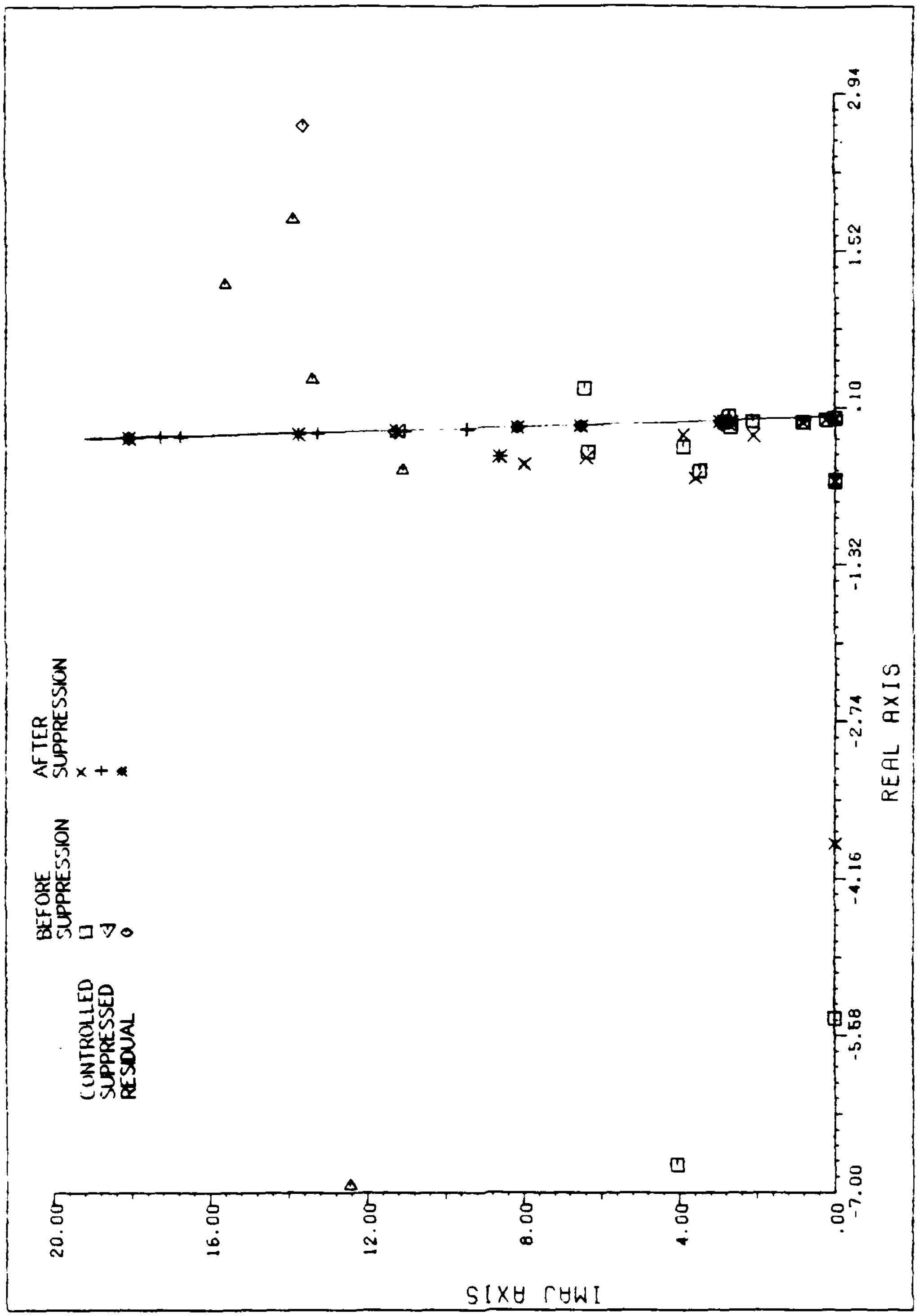
Another aspect of this study was to explore the performance of a two controller system. Since both controlability and observability are given up in order to produce decoupled controllers, it should not be expected that a two controller system would perform as well as a one controller system given sufficient computer speed. In fact the application of two controllers to this system did result in a stable system, but the eigenvalues of the system were moved very little. In order to stabilize the system, mode 21 had to be controlled. In the other designs, mode 21 had been treated as a residual. However, since in the two controller model, modes of higher frequency than mode 21 were being controlled rather than suppressed, mode 21 was being subjected to control energy and thus went unstable. Figure 7 is a plot of the eigenvalues of this control design.

The final part of the investigation explored the application of an actuator to the tip package. This was done in order to demonstrate its desirability, not to show that the results could actually be accomplished. The actuator was assumed to be of zero weight thus not changing the mode shapes. The results show that the torsion modes were easily controlled, with the first torsion mode needing a control weight a factor of 300 less in order to achieve a higher damping ratio. The eigenvalues of this system are plotted in figure 8.



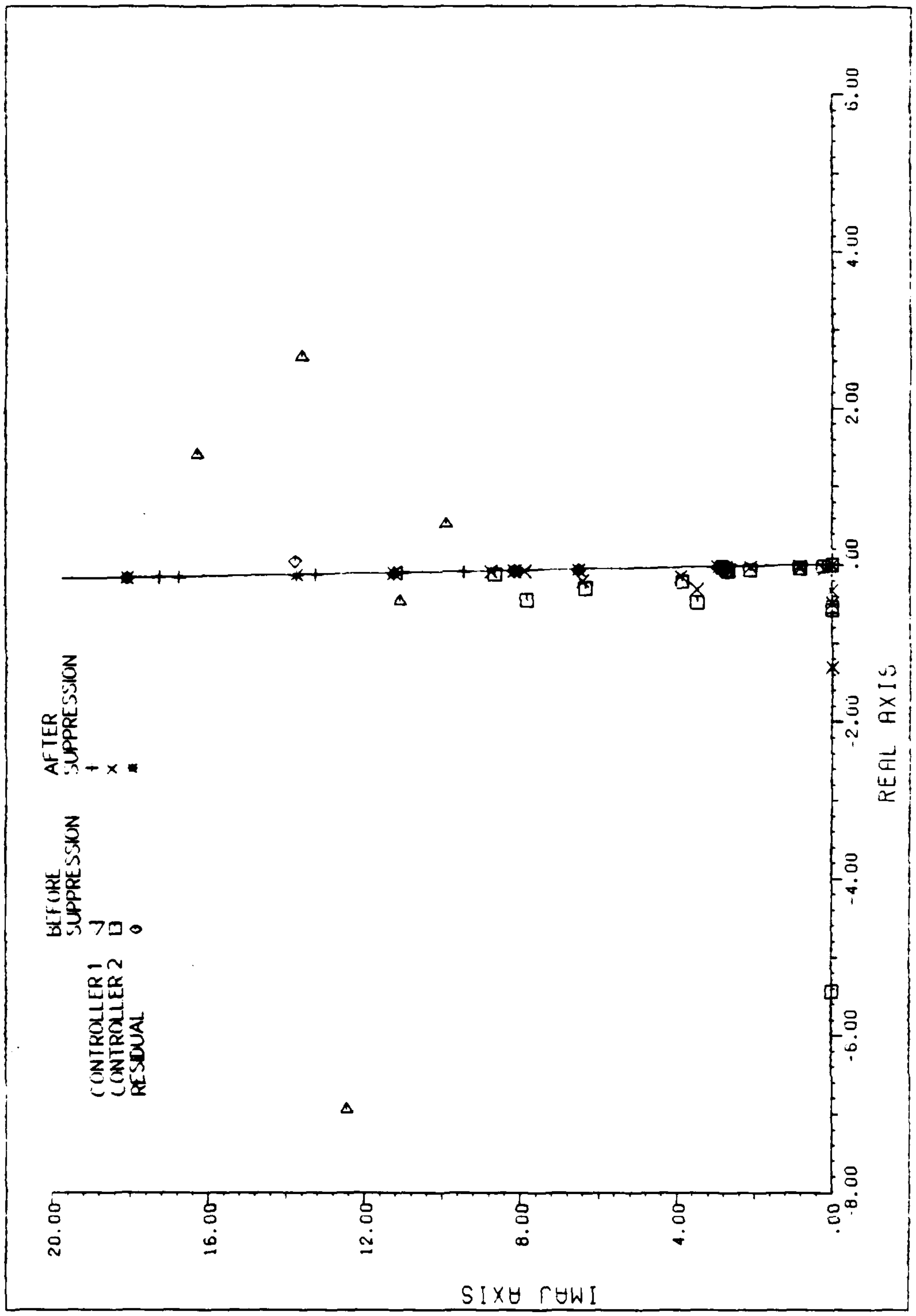

EIGENVALUES, 1 CONTROLLER, 15 SENSORS, 9 ACTUATORS

**Figure 5.**



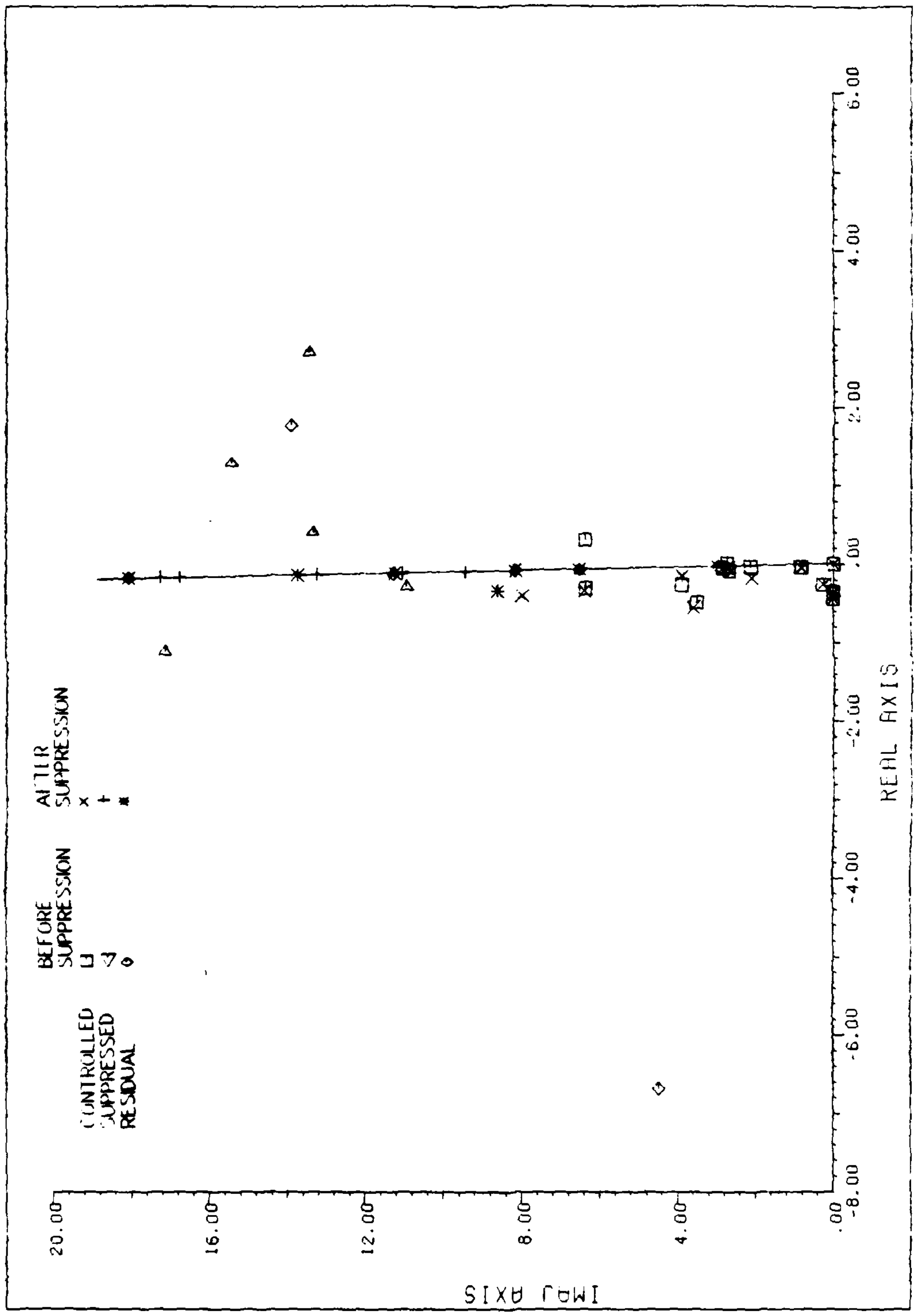

EIGENVALUES, 17 SENSORS 9 ACTUATORS, 1 CONTROLLER

**Figure 6.**




EIGENVALUES. 2 CONTROLLERS. 17 SENSORS 3 ACTUATORS

**Figure 7.**




EIGENVALUES, 1 CONTROLLER, 10 ACTUATORS

**Figure 8.**

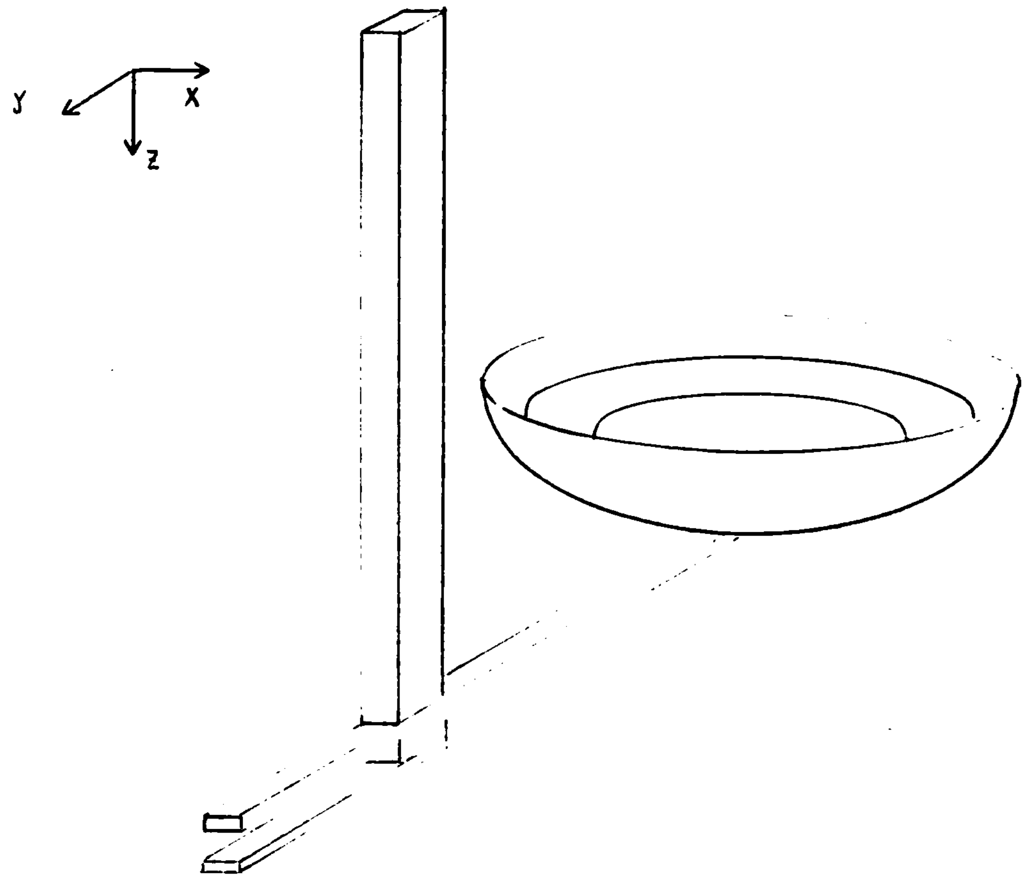



**Figure 9.** Coordinate System Used in Modelling

The time response of the system to various initial conditions was considered next. The time response for the antenna attachment point was considered representative of the overall system response and so was plotted for all cases. Figure 9 shows the coordinate system used for this discussion. Figures 10 and 11 are the response of the 17 sensor and 15 sensor systems to a .68 m/s velocity in the y direction. The curve with the largest magnitude is the y position and the other curve is the z position in both plots. The significant vibration in both plots occurs in the y pendulum mode. The z position plots show evidence of both the pendulum mode and the first bending mode.

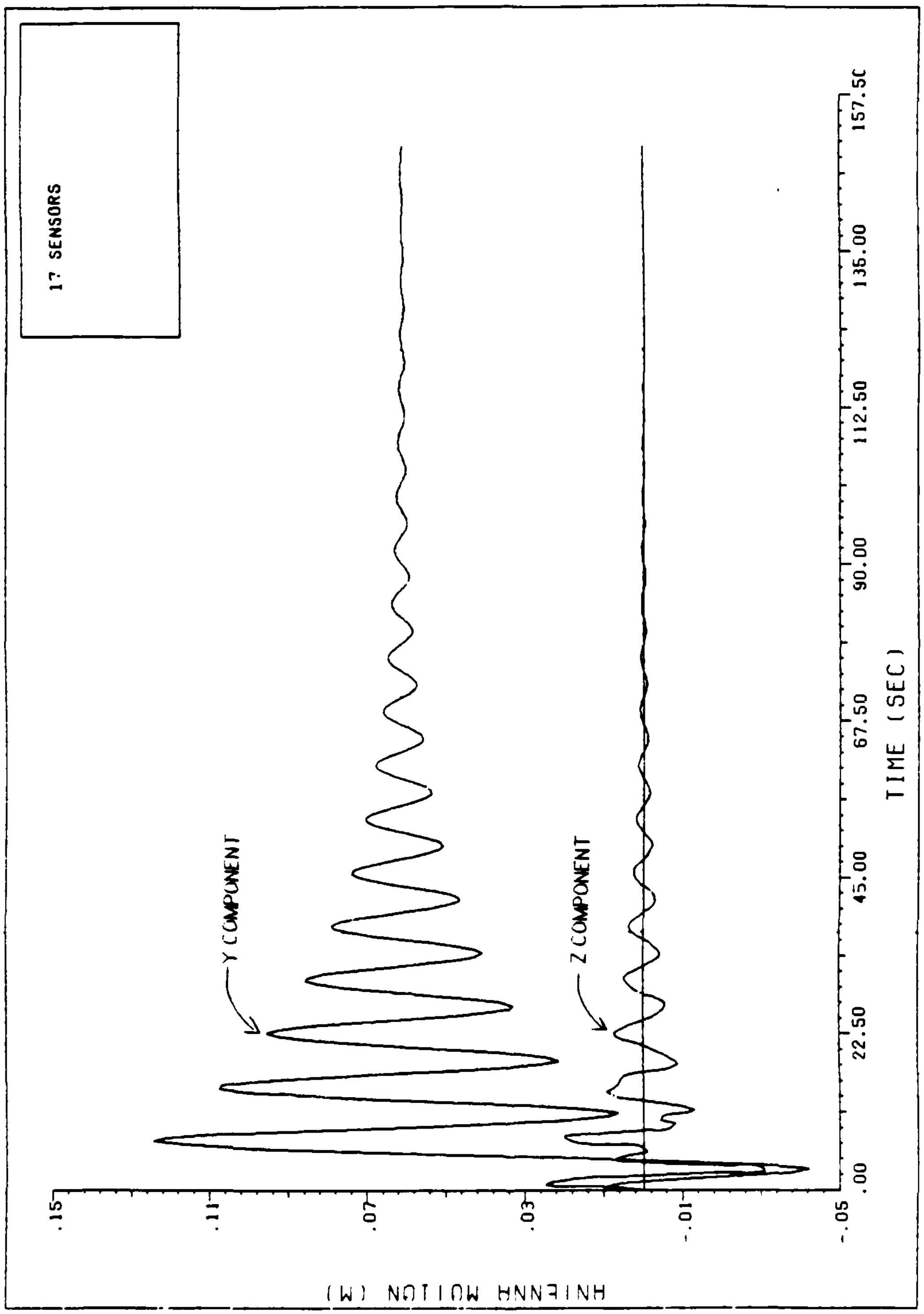
Figures 12 and 13 are the response to an initial .36 m/s velocity in the x direction. Figure 12 shows the response of the 17 sensor model, and the significant vibration is in the x pendulum mode with some evidence of the first torsion mode. The 15 sensor system shows much more vibration in the torsion mode with some pendulum mode.

Figures 14 and 15 show the response of the system to a velocity in the x plane. This again shows mainly vibration in the y pendulum mode. The 15 sensor model again shows a bit of vibration in the torsion mode.

Figures 16 and 17 show the response of the 17 and 15 sensor models to an initial rotational velocity about the z axis. Although the 15 sensor model shows somewhat higher overshoot, it quells the system down much faster. This is to be expected because there are more controlled modes in this model than in the 17 sensor system. Since the observability of the system is degraded to suppress more modes in the 17 sensor system, the 15 sensor model should be expected to perform better in this situation. However, there is more uncertainty that the simulated 15 sensor response could actually be achieved.

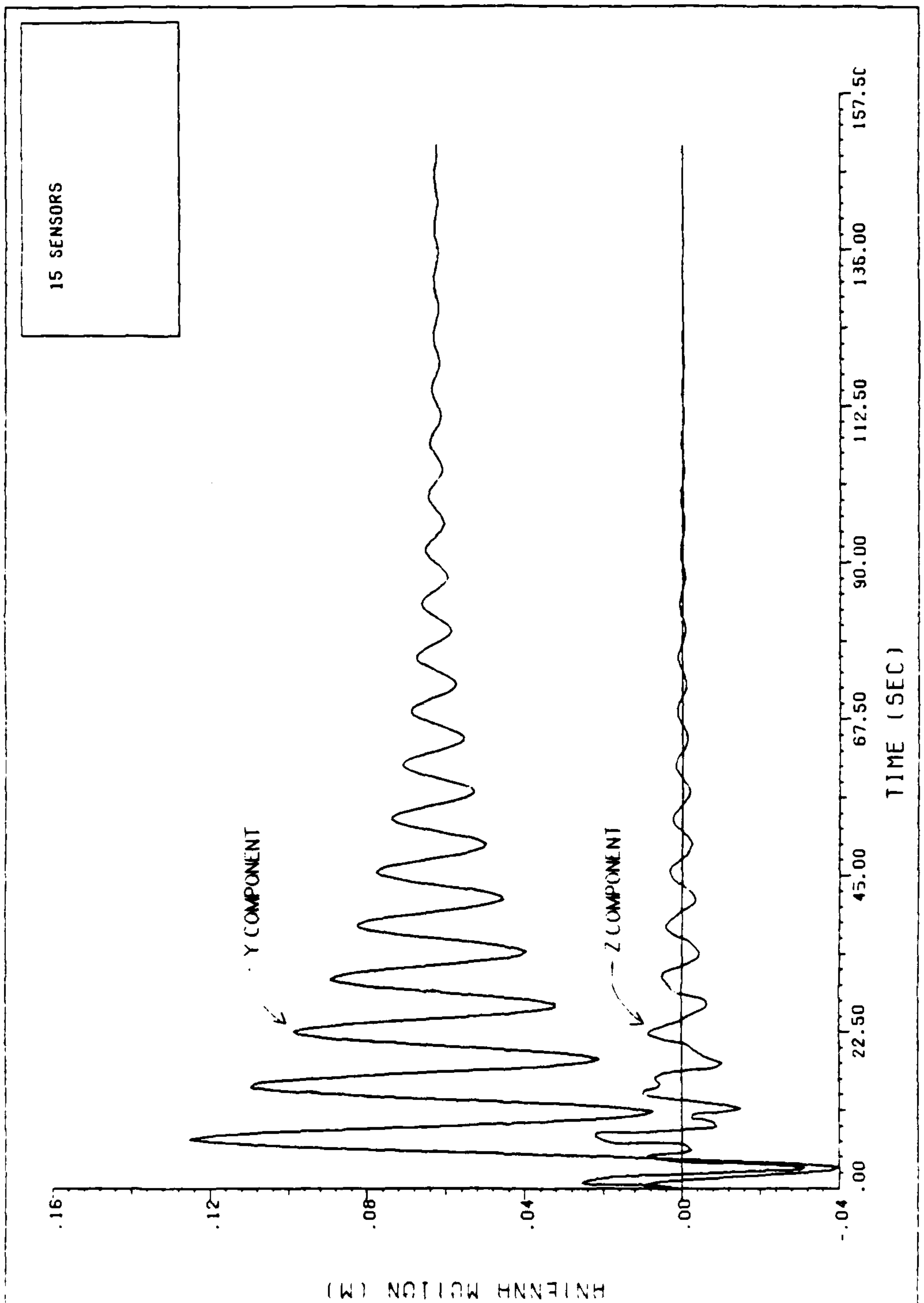
Figure 18 is the response to the same initial conditions of the system model with the extra assumed actuator at the tip package position. As can be seen, this effectivly stops the system from rotating in much less time than either of the other one controller model.

Finally, figure 19 is the response of the two controller system. The system response is poor as would be expected for this sensor actuator configuration. The two controller model would have also benefitted from an additional actuator, but this case was not studied.



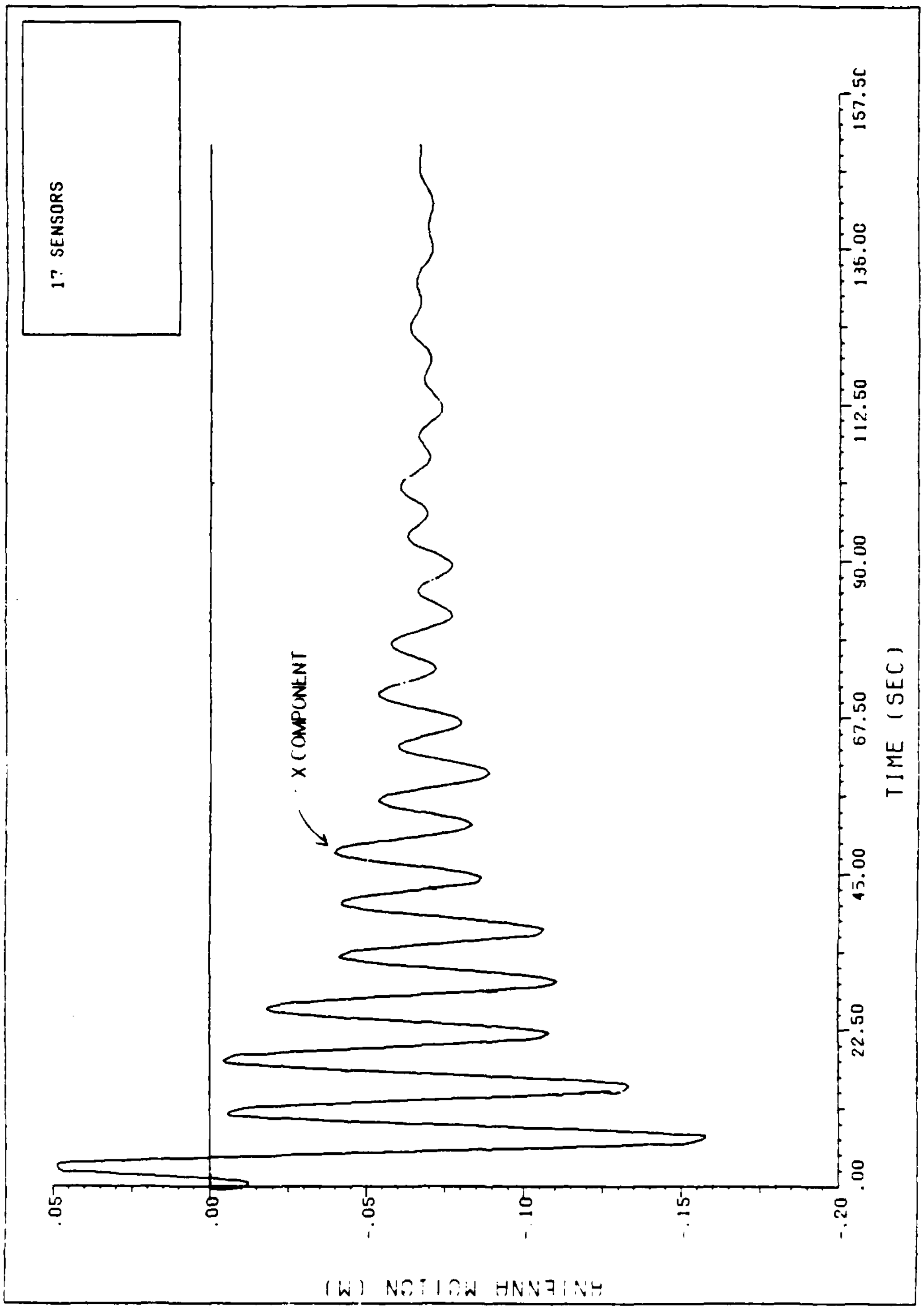

ANTENNA MOTION Y VELOCITY INITIAL CONDITIONS

**Figure 10.**



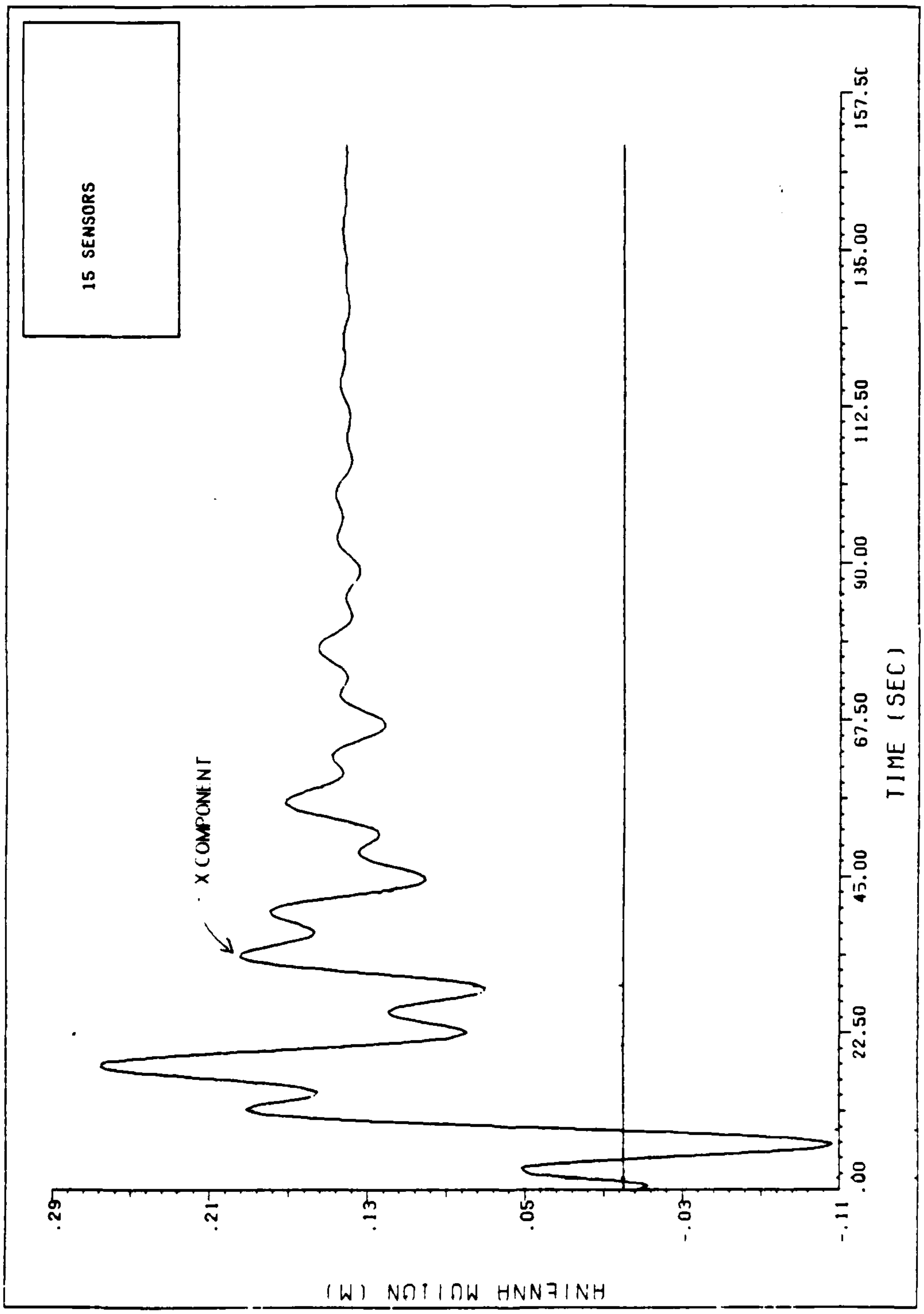

ANTENNA MOTION Y VELOCITY INITIAL CONDITIONS

**Figure 11.**



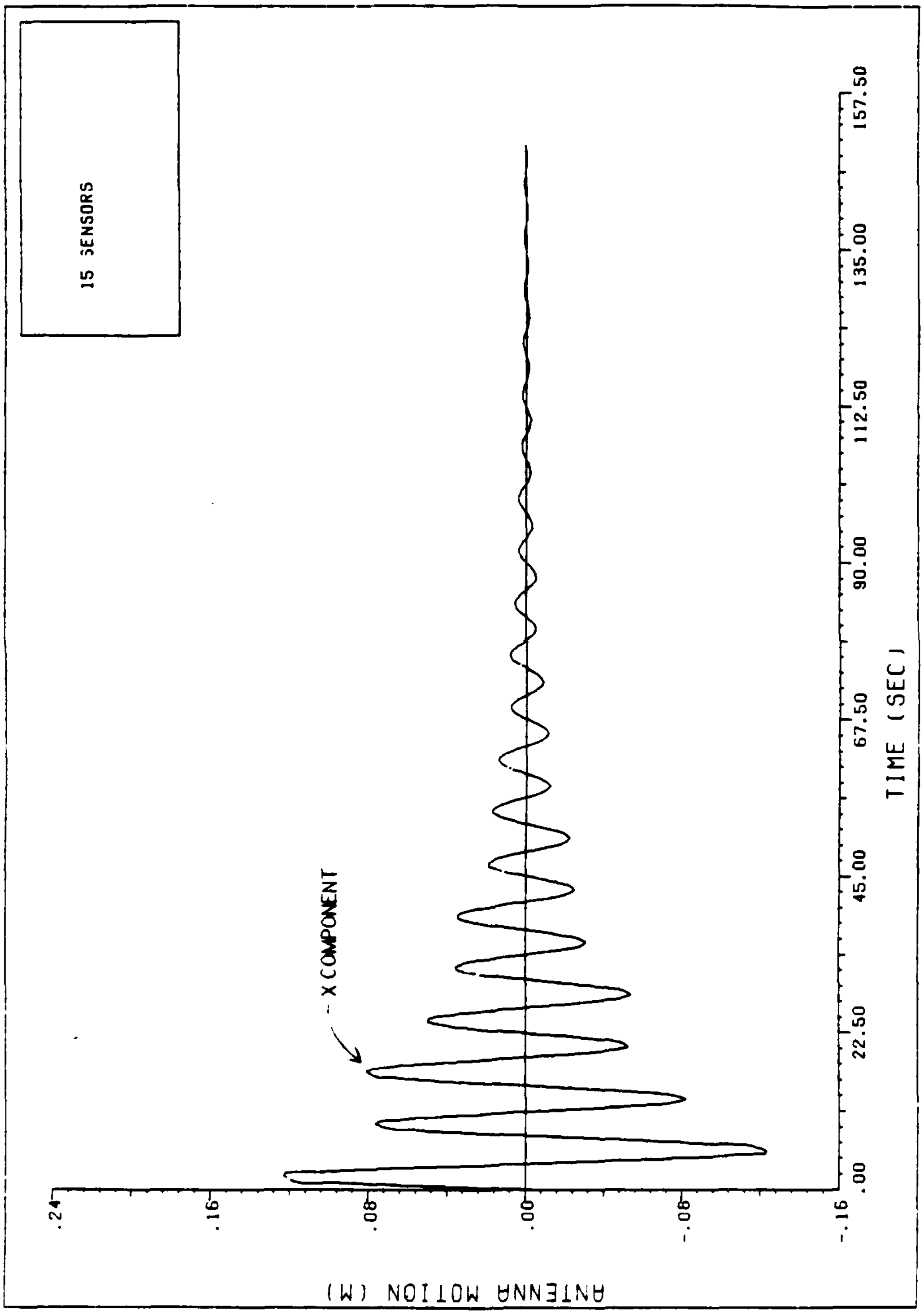

ANTENNA MOTION X VELOCITY INITIAL CONDITION

Figure 12.

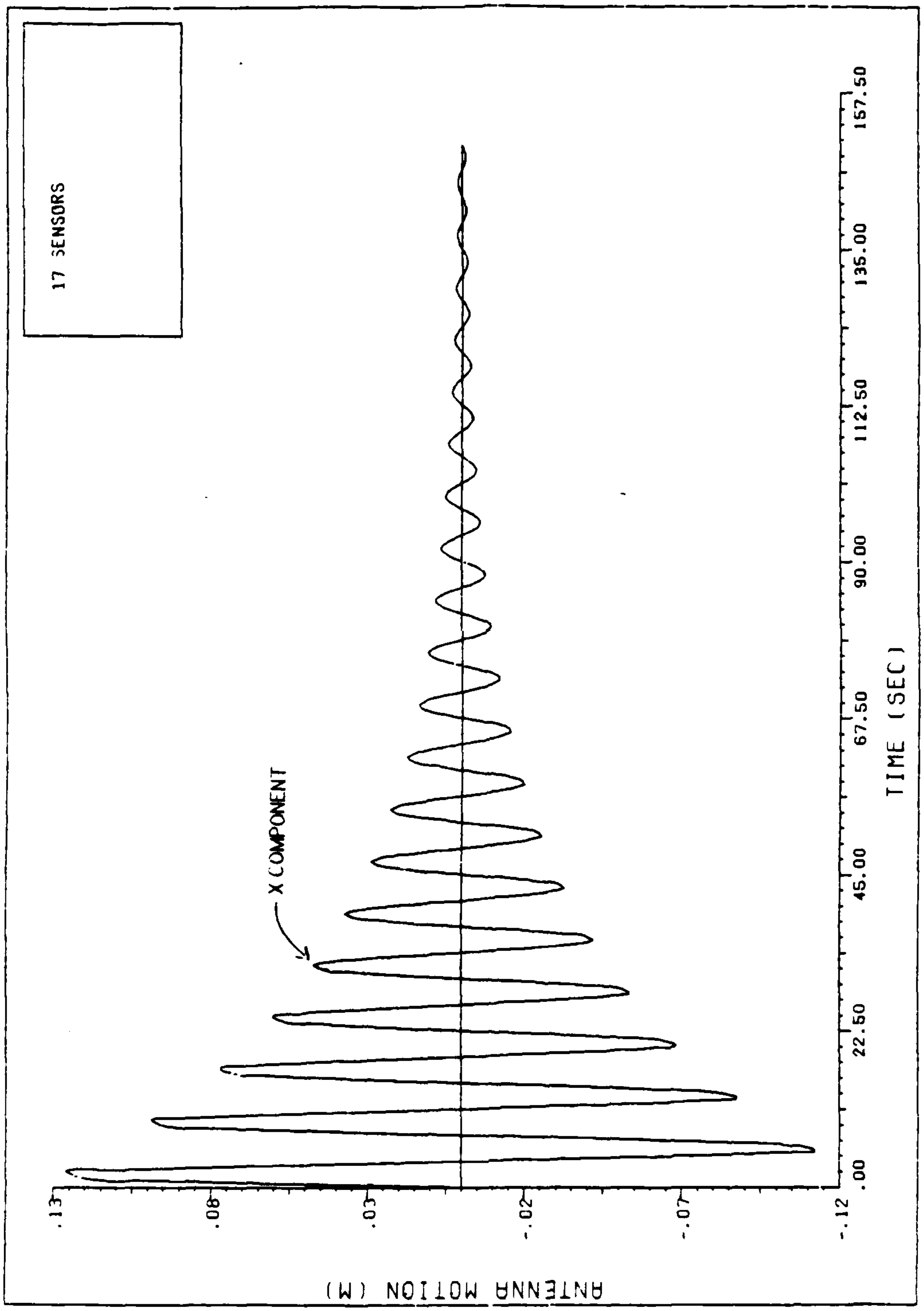



ANTENNA MOTION X VELOCITY INITIAL CONDITIONS

**Figure 13.**

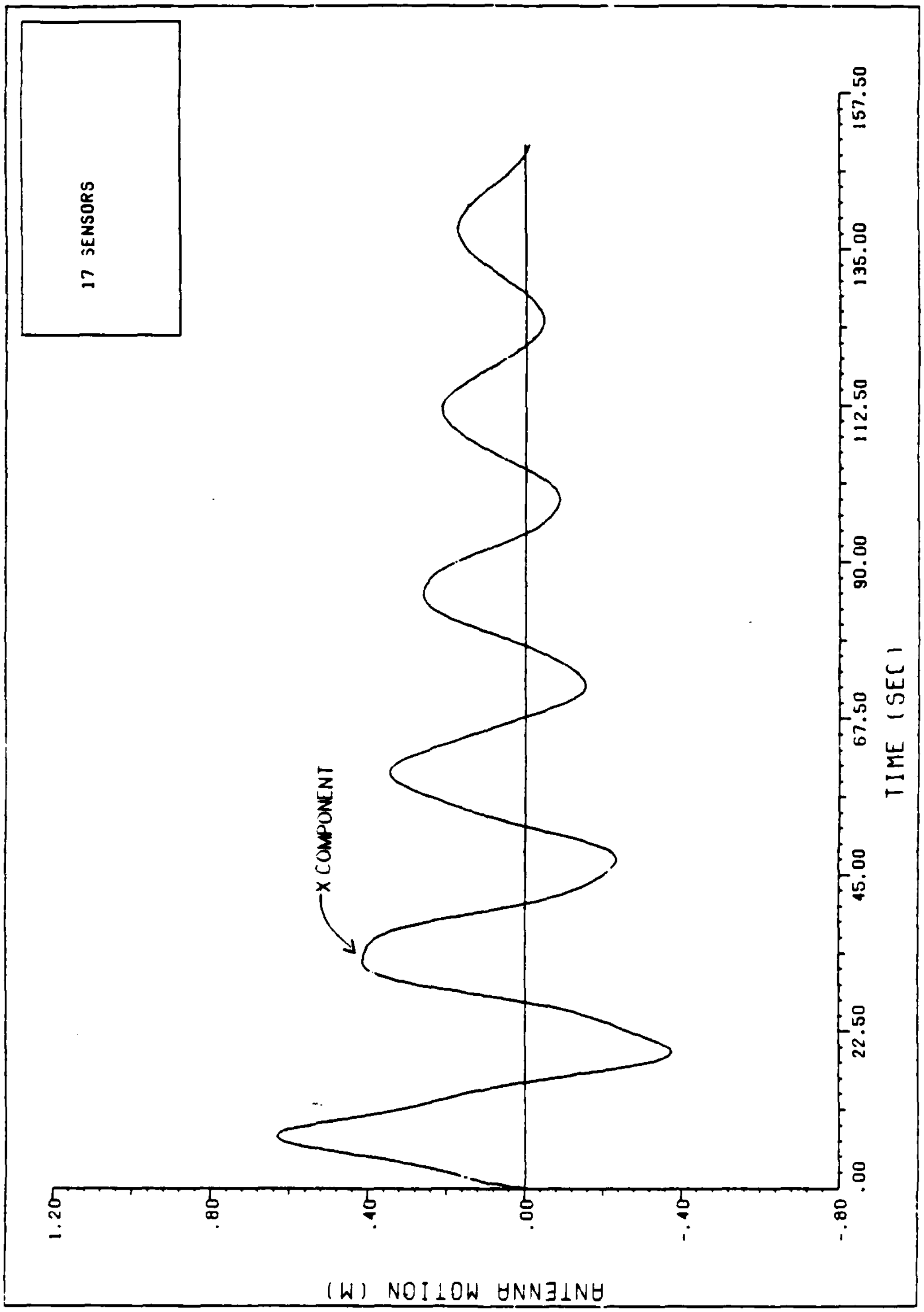



ANTENNA MOTION X PENDULUM INITIAL CONDITIONS
**Figure 14.**



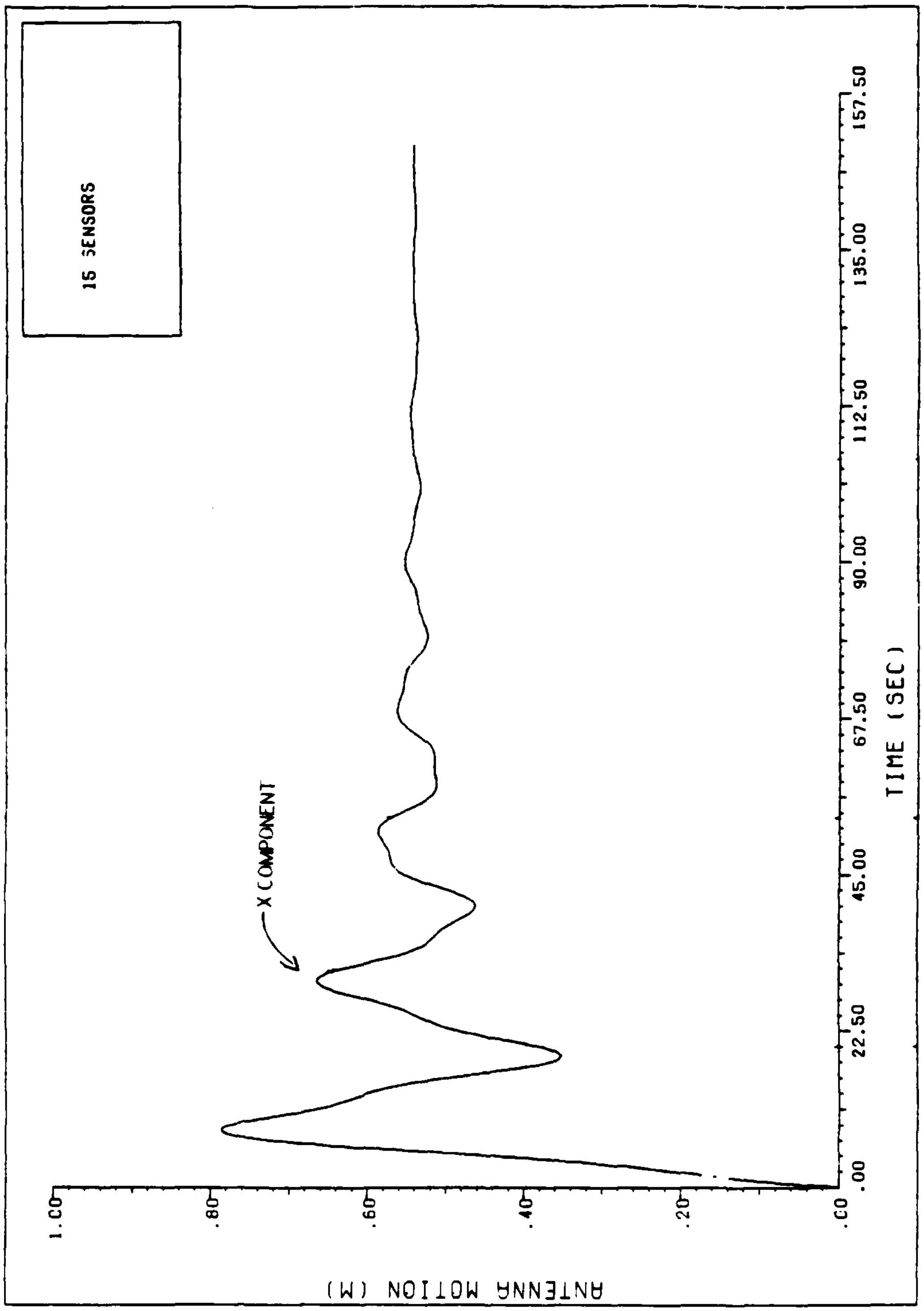

ANTENNA MOTION X  PENDULUM INITIAL CONDITION

**Figure 15.**




ANTENNA MOTION ROTATION INITIAL CONDITION

**Figure 16.**




ANTENNA MOTION ROTATION INITIAL CONDITION
**Figure 17.**

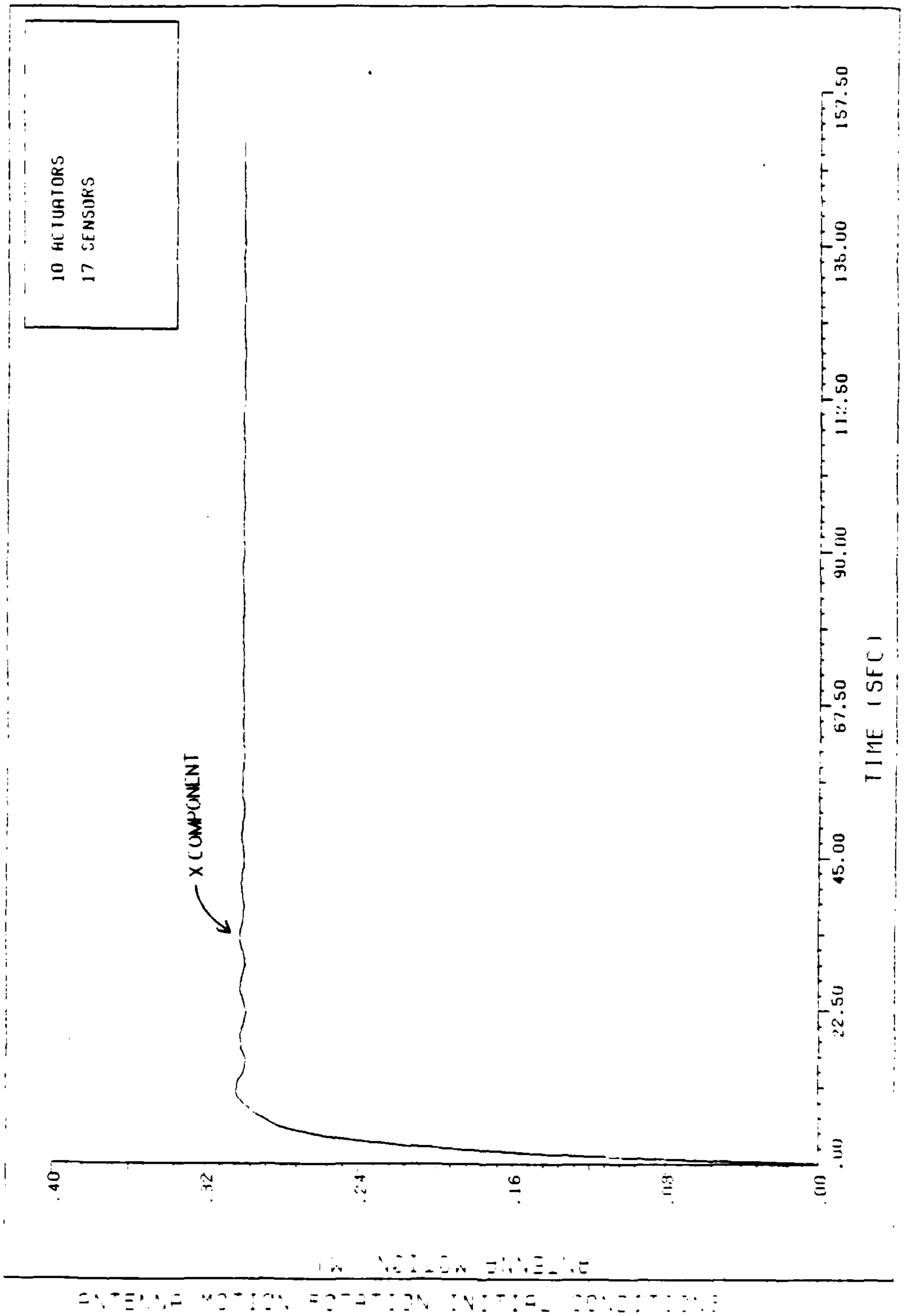
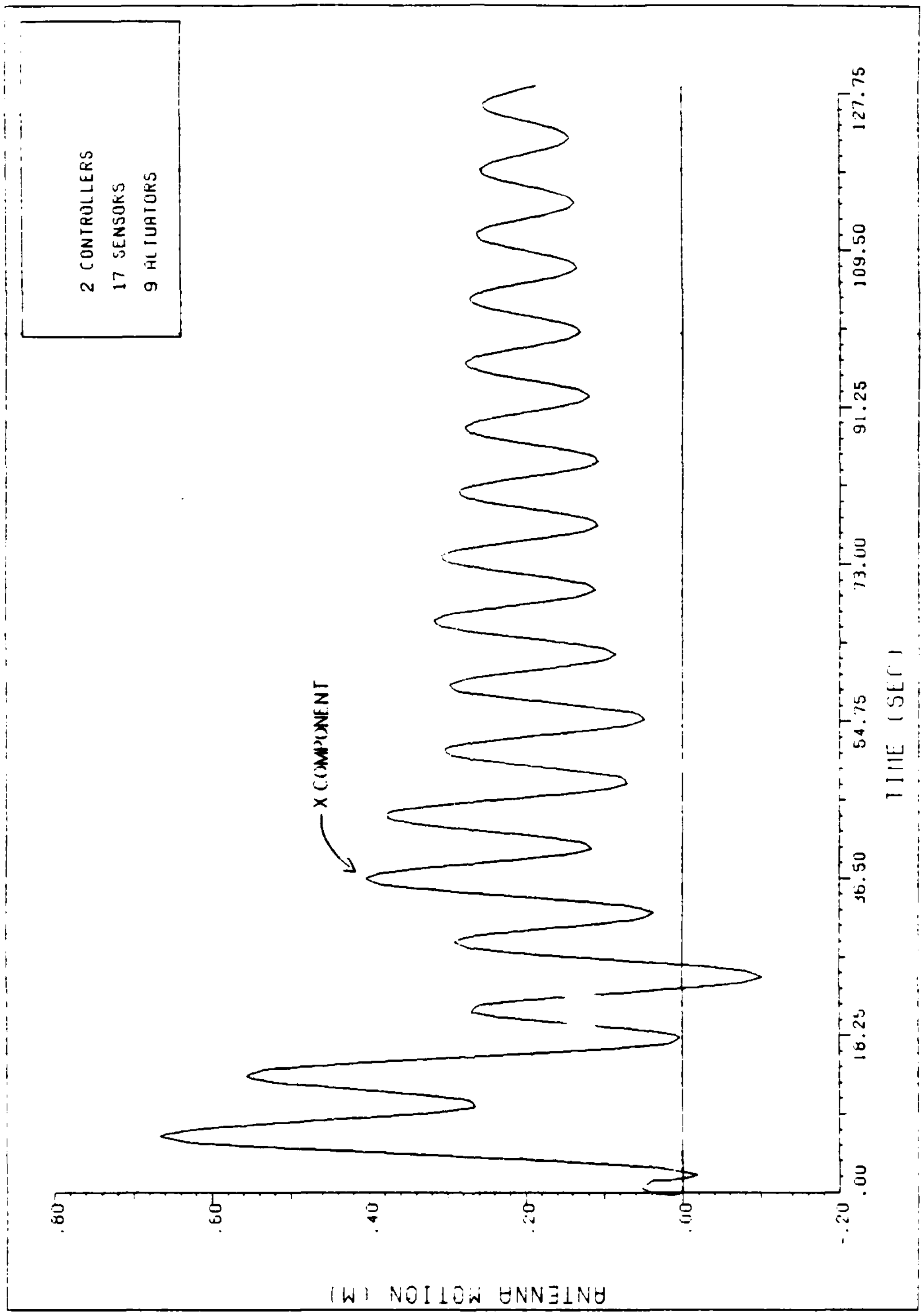

10 ACTUATORS
17 SENSORS
X COMPONENT
TIME (SEC)
.00
22.50
45.00
67.50
90.00
135.00
157.50
.40
.32
.24
.16
.00

Figure 18.




TWO CONTROLLER ANTENNA MOTION ROTATION INIT. COND.

**Figure 19.**

## VII. Recommendations and Conclusions

Since the object is to actually run a control law (in basic) the more benign configurations should be studied. I feel confident that the cruciform is controllable using direct output feedback, if only because they have been controlling it using loop closing methods.

Other practical issues are the fact that we have a barely stable system under computer control. Since we have designed a continuous time controller assuming high enough bandwidth, this should be checked. I have worked out the math to digitize the system, and then the eigenvalues can be evaluated using the same IMSL routines as presently used, with stability ensured if the eigenvalues are inside the unit circle. The only element of the digitized system not presently implemented is the convolution integral. The MEXP subroutine can be used to digitize the A matrix. Since the convolution has to be done to provide a forced system response, this is a logical and necessary extension.

The continuous time transformation techniques have exact parallel techniques in discrete time. The suppression of observation spillover is exactly the same. The technique to suppress control spillover is exactly the same, except that the technique must be performed on $B_d$, the digitized control input matrix. The only real problem is that the discrete time and continuous time algebraic Riccati equations are not the same for t greater than zero. The LQGLIB library available on the ASD Cyber has routines that will solve the discrete time Riccati equation.

One final issue that must be resolved is that of real actuator dynamics. This and past thesis efforts have treated actuators as perfect devices with no dynamics. This is probably fairly good for the torque motors, but proof mass actators have first order dynamics at best, and are nonlinear in reality. TRW has a non–linear actuator truth model. Some actuator dynamics would be included in a reasonable performance evaluation. The incorperation of actuator dynamics into the system model in order to suppress and control means resolving the eigenvalue–eigenvector problem.

## Appendix A

## Program Source Code

```
      PROGRAM ACONE
C   ATEST1 WITH ONE TRANSFORMATION
C   THIS PROGRAM IS A CLONE OF DOFB33
C   IT ONLY USES ONE CONTROLLER
C THIS PROGRAM IS MODIFIED TO WORK WITH THE NASA GROUND TEST VEHICLE
C IT USES ONE CONTROLLER, ALTHOUGH IN THE FUTURE MORE MAY BE
C AVAILABLE
C
C TO CHANGE THE PROBLEM TO OTHER CONFIGURATIONS
C THE DIMENSION OF ALL THESE ARRAYS HAS TO BE GREATER THAN
C     MAX( # OF SENSORS, # OF ACTUATORS, # OF MODES PER CONTROLLER)
C UNFORTUNATELY, YOU MUST ALSO DIMENSION MAJM EXACTLY THE
C SAME SIZE AS YOUR PROBLEM.
C OTHER UNFORTUNATES:
C ABCG DESTROYS THE K MATRIX
C FINDK IS HARDWIRED FOR NCOL = 41
C IF YOU HAVE MORE THAN 10 MODES IN THE CONTROLLER, SUPPRESSED OF
C IN THE RESIDUALS YOU WILL HAVE TO MODIFY FINDK
C
C  THAT IS ALL BESIDES THE INITIALIZATION SECTION AFTER
C THE VARIABLE DECLARATIONS. (SEE THE FILE ASTART FOR A DISCUSSION
C OF WHY THESE CAN BE DIMENSIONED THIS WAY.)
C OTHERWISE, THE PROGRAM GETS ITS INFO FROM THE INPUT FILE
C
C       NEXT THREE LINES/ SYSTEM PARTITIONS
C       DIMENSIONED NCOL BY NCOL
      REAL A1(41,41),A2(41,41),A3(41,41)
      REAL B1(41,41),B2(41,41),B3(41,41)
      REAL C1(41,41),C2(41,41),C3(41,41)
C  QA1 = WEIGHTING MATRIX FOR THE CONTROLLER
      REAL QA1(41,41)
C   THESE THREE MATRICES ARE AFTER SUPPRESSION
      REAL CSTAR1(41,41)
      REAL GT1(41,41)
      REAL KHAT1(41,41),KSTAR1(41,41),KWORK(41,41)
      REAL SAT(41,41),AKC(41,41),ACT(41,41),BCG(41,41)
      REAL P(41,41),S(41,41),KCC(41,41)
      REAL ABG1(41,41),ABG2(41,41),ABG3(41,41)
      REAL ABG5(41,41),ABG6(41,41),ABG7(41,41)
      REAL GAIN1(41,41),KT1(41,41)
      REAL GAMMA1(41,41),T1(41,41),TKG1(41,41)
      REAL TRT(41,41),TEN(41,41),CT(41,41),V(41,41)
      REAL MAJM(60,60),D(33),X0(60),W(33),TOL,DT,XC(60)
      REAL Y(41)
      REAL ZETA,AA(33),BB(33),SING(41),XTR(41,41),X1(60)
      REAL EAT2(60,60),WORK(60,60),STOR(41,41)
      REAL PHIA(41,41),PHIS(41,41),PHILOS(3,60),INIT(2,33)
```

```
      INTEGER N,N2,NC1,NSUP,NC12,NSUP2,NRES,PDT
      INTEGER I,J,K,L,M,KK,LL,MM
      INTEGER Q,NACT,NSEN,IR(41),IER,SKIP,NCOL,NCOL1
      INTEGER NDA,NDIM,NDA1,NDIM1,ZZ,E1,P1
      COMPLEX Z(66),W1(33)
      COMMON/MAINA/NDA,NDA1
      COMMON/MAINB/NCOL,NCOL1
      COMMON/MAIN1/NDIM,NDIM1,TEN,X(3700)
      COMMON/MAIN2/STOR
      COMMON/MAIN3/XTR
      COMMON/SAVE/T(100),TS(100)
      COMMON/INOUT/KOUT,TAPE
      COMMON/IO/NSEN,NACT
      COMMON/PRO/Q
      COMMON/NUM/IC1(41),IC2(41),IC3(41),IR,NC1,NSUP,NRES
C
C
C   INITIALIZATIONS
C     THE NEXT 6 VARIABLES SHOULD NOT BE CHANGED UNLESS
C     THE DIMENSION OF YOUR PROBLEM IS SUCH THAT THE
C     NUMBER OF MODES IS GREATER THAN 33 OR YOU HAVE MORE THAN
C      41 ACTUATORS OR SENSORS. OR MORE THAN 15 MODES IN A CONTROLLER.
C
C      NDIM IS GREATER THAN THE NUMBER OF MODES
C
      NDIM = 41
      NDIM1 = 42
C
C NCOL IS THE LENGTH OF A COLUMN IN THE WORK MATRICES SUCH
C AS PHIA, PHIS, A1, A2, A3, &ETC.
C THESE ARRAY SIZES ARE NOT IMPORTANT AS LONG AS THEY ARE BIGGER THAN
C YOUR
C ACTUAL PROBLEM SIZE
      NCOL = 41
      NCOL1 = 42
C  NDA IS TWICE NDIM
C AND THUS GREATER THAN THE DIMENSION OF THE FIRST ORDER SYSTEM
C
      NDA = 60
      NDA1 = 61
C
C DON'T REALLY KNOW WHAT THESE DO...
      KOUT = 6
      TAPE = 9
      Q = 0
      IER = 0
      ZZ = 0
C
```

```
      OPEN(8,FILE='TAPE8')
      REWIND 8
      OPEN(6,FILE='TAPE6')
      REWIND 6
      CALL HEADER ()
C
C  HEADER PRINTS THE HEADER AT THE TOP OF THE OUTPUT
C
C %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C
C    READ IN INFORMATION FROM INPUT FILE
C
C %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C
C    IF Q EQ 2, THEN PRNT, PRNTXL, PRINTK DON'T WORK
      PRINT*,' TO PRINT ALL OF THE MATRICES ENTER 1, ELSE ENTER 0 >'
      READ(8,*) Q
      PRINT*,Q
C  PHI MATRICES AND CONTROLLER ENTRIES
C
      PRINT'(///)'
C
C NC1 IS THE NUMBER OF CONTROLLED MODES
C NSUP IS THE NUMBER OF SUPPRESSED MODES
C NRES IS THE NUMBER OF RESIDUAL MODES
C NACT IS THE NUMBER OF ACTUATORS
C NSEN IS THE NUMBER OF SENSORS
C ZETA IS A COMMON ASSUMED DAMPING RATIO FOR ALL MODES
C
      PRINT*,' ENTER NC1,NSUP,NRES,NACT,NSEN,ZETA  >'
      READ(8,*) NC1,NSUP,NRES,NACT,NSEN,ZETA
      PRINT*,NC1,NSUP,NRES,NACT,NSEN,ZETA
      PRINT*,' '
      PRINT*,' ENTER THE ',NACT,' ELEMENTS FOR EACH PHIA '
      PRINT*,' '
      N = NC1 + NSUP + NRES
C
C
      PRINT*,' ENTER PHIA, A ',N,' BY ',NACT,' MATRIX
     * BY ROWS'
      DO 1 I=1,N
      READ(8,*) (PHIA(I,J),J=1,NACT)
 1    CONTINUE
      CALL PRNT(PHIA,N,NACT)
      PRINT'(//)'
      PRINT*,' ENTER THE ',NSEN,' ELEMENTS FOR EACH PHIS '
      PRINT*,' '
      PRINT*,' ENTER PHIS, A ',NSEN,' BY ',N,' MATRIX
```

```
     * BY COLUMNS'
C    THE PHIS MATRIX IS FROM THE EQUATION
C    Y = C * X WHERE C IS GIVEN BY
C     C = ( O : PHIS TRANSPOSED )
C    IN THIS PROGRAM, BECAUSE ONLY VELOCITY
C    SENSORS ARE USED. THIS IS A FUNCTION
C   OF SUBROUTINE FORMC
      DO 2 I=1,N
      READ(8,*) (PHIS(I,J),J=1,NSEN)
 2    CONTINUE
      CALL PRNT(PHIS,N,NSEN)
      PRINT'(//)'
C
C   OMEGAS
C
      PRINT*, ' ENTER THE VALUE FOR EACH OMEGA '
      PRINT*,' '
      READ(8,*)(W(I),I=1,N)
      DO 3 I=1,N
      PRINT*,'                    ',W(I)
      D(I) = -2. * ZETA * W(I)
 3    CONTINUE
C
C
 20   CONTINUE
C
      PRINT'(//)'
C
C   MODE ASSIGNMENT TO CONTROLLERS
C
C
      PRINT'(///)'
      PRINT*,' THE FOLLOWING MODES ARE ENTERED ACCORDING TO THE '
      PRINT*,' ORDER IN WHICH THEY ARE ENTERED IN THE DATA FILE '
      PRINT*,' AND NOT ACCORDING TO THEIR ACTUAL MODE NUMBER. '
      PRINT'(//)'
      PRINT*,' ENTER THE ',NC1,' CONTROLLED MODES >'
      READ(8,*) (IC1(I),I=1,NC1)
      PRINT*,'      ',(IC1(I),I=1,NC1)
      PRINT*,' '
      PRINT*,' ENTER THE ',NSUP,' SUPPRESSED MODES >'
      READ(8,*) (IC2(I),I=1,NSUP)
      PRINT*,'      ',(IC2(I),I=1,NSUP)
      PRINT*,' '
      PRINT*,' ENTER THE ',NRES,' RESIDUAL MODES >'
      READ(8,*) (IC3(I),I=1,NRES)
      PRINT*,'      ',(IC3(I),I=1,NRES)
      PRINT*,' '
```

```
      PRINT*,' '
C
C
      NC12 = 2 * NC1
      NSUP2 = 2 * NSUP
      NRES2 = 2 * NRES
      N2 = 2 * N
      M =  NC12 +  NSUP2 +  NRES2
      NDA = M
      NDA1 = M + 1
      NDIM = N
      NDIM1 = N + 1
 100  CONTINUE
      PRINT'(///)'
C
C  READ IN THE WEIGHTING MATRIX DIAGONAL
C  VALUE FOR EACH MODE
C
      PRINT*,' ENTER THE DIAGONAL VALUES, IN MODE INPUT '
      PRINT*,' ORDER, FOR THE CONTROL WEIGHTING MATRIX >'
      READ(8,*) (AA(IQ.1) THEN
        PRINT*,' THE CONTROLLER A MATRIX IS '
        CALL PRNT(A1,NC12,NC12)
        PRINT*,' THE CONTROLLER B MATRIX IS '
        CALL PRNT(B1,NC12,NACT)
        PRINT*,' THE CONTROLLER C MATRIX IS '
        CALL PRNT(C1,NSEN,NC12)
        PRINT*,' THE C1 CONTROL WEIGHTING MATRIX IS '
        CALL PRNT(QA1,NC12,NC12)
        PRINT*,' THE SUPPRSD MODES A MATRIX IS '
        CALL PRNT(A2,NSUP2,NSUP2)
        PRINT*,' THE SUPPRSD MODES B MATRIX IS '
        CALL PRNT(B2,NSUP2,NACT)
        PRINT*,' THE SUPPRSD MODES C MATRIX IS '
        CALL PRNT(C2,NSEN,NSUP2)
C
        IF (NRES.GT.0) THEN
          PRINT*,' THE RESIDUAL A MATRIX IS '
          CALL PRNT(A3,NRES2,NRES2)
          PRINT*,' THE RESIDUAL B MATRIX IS '
          CALL PRNT(B3,NRES2,NACT)
          PRINT*,' THE RESIDUAL C MATRIX IS '
          CALL PRNT(C3,NSEN,NRES2)
        ELSE
          PRINT*,'  NO RESIDUAL TERMS '
        ENDIF
      ENDIF
C
```

```
C THIS SECTION GENERATES THE RICCATI SOLUTION
C AND THE GAIN MATRIX FOR THE CONTROLLER
C
      CALL VMULFP(B1,B1,NC12,NACT,NC12,NCOL,NCOL,SAT,NCOL,IER)
      IER = 0
      TOL = 0.001
      PRINT*,' THE FOLLOWING ARE THE MRIC A+BG 1 INPUTS '
      PRINT'(//)'
      PRINT*,' THE MATRIX A1 IS '
      CALL PRNT(A1,NC12,NC12)
      PRINT*,' THE MATRIX SAT (B1*B1T) IS '
      CALL PRNT(SAT,NC12,NC12)
      PRINT*,' THE MATRIX QA1 IS '
      CALL PRNT(QA1,NC12,NC12)
      PRINT*,' NC12 = ',NC12
      PRINT'(//)'
      CALL MRIC(NC12,A1,SAT,QA1,S,ABG1,TOL,IER)
C
C  ABG1 = A1 + B1G1
C
      PRINT*,'      THE EIGENVALUES OF A1 + B1G1 '
      PRINT'(/)'
      CALL EIGRF(ABG1,NC12,NCOL,0,W1,TEN,NCOL,STOR,IER)
      DO 629 I=1,NC12
 629  PRINT*,'    ',W1(I)
      PRINT'(/)'
C
C  IN A MULTIPLE CONTROLLER, SOME OF THE RICCATI GAINS
C  HAVE TO BE RECOMPUTED USING THE TRANSFORMATION MATRICES
C THIS IS NOT SO FOR ONE CONTROLLER BECAUSE WE SUPPRESS
C  THE  C MATRIX, AND THE C MATRIX IS NOT INVOLVED IN
C  THE  RICCATI SOLUTION.
C
       PRINT*,' THE RICCATI SOLUTION OF AC + BCG #1 IS '
       CALL PRNT(S,NC12,NC12)
C IN ORDER TO HAVE A CONTROL WIEGHTING MATRIX R, THIS CODE WOULD BE
C CHANGED. G = -R^-1*B^T*S, BUT OUR R = I UNTIL CODED DIFFERENTLY
      CALL VMULFM(B1,S,NC12,NACT,NC12,NCOL,NCOL,GAIN1,NCOL,IER)
      PRINT*,' THE G1 GAIN MATRIX IS '
      CALL PRNT(GAIN1,NACT,NC12)
C
C THIS IF STATEMENT GETS RID OF THE UNSUPPRESSED EIGENVALUE
C CALCULATIONS.  AFTER THE PROGRAM WORKS, THESE REALLY LOSE
C MOST OF THEIR SIGNIFICANCE, SO WHY WASTE THE TIME?
C
      IF (Q.LT.3) THEN
C
C FINDK  FINDS THE K GAIN MATRIX FROM THE OPIMAL FULL STATE
```

```
C FEEDBACK GAIN, G.
C
      CALL FINDK(C1,NSEN,NC12,WORK,GAIN1,NACT,KHAT1)
      PRINT*,'THE FEEDBACK MATRIX KHAT1 IS'
      CALL PRNT(KHAT1,NACT,NSEN)
C
      DO 13 I=1,NACT
      DO 13 J=1,NSEN
  13  KWORK(I,J) = KHAT1(I,J)
C
C
C  THIS SECTION GENERATES THE BLOCK SEGMENTS
C  OF MAJM AND PUTS THEM INTO THE MAJM MATRIX
C
C
C  THE SYSTEM MATRIX LOOKS LIKE:
C
C  ****                                    ****
C  *                                          *
C  *  A1+B1K1C1      B1K1C2      B1K1C3       *
C  *                                          *
C  *   B2KIC1      A2+B2KIC2     B2KIC3       *
C  *                                          *
C  *   B3KIC1        B3KIC2    A3+B3KIC3      *
C  *                                          *
C  ****                                    ****
C
C
      K=NC12
      L=NSUP2 + K
      MM = NC12 + NSUP2 + NRES2
C
      PRINT'(///)'
      DO 16 I=1,NACT
      DO 16 J=1,NSEN
  16  KHAT1(I,J) = KWORK(I,J)
C
      DO 200 I=1,MM
      DO 200 J=1,MM
 200  MAJM(I,J) = 0.0
C
C NOTE THAT ABGC DESTROYS KHAT1 EACH TIME!!!
C
      CALL ABGC(A1,B1,C1,NC12,KHAT1,NACT,NSEN,ABG1)
C
      DO 17 I=1,NACT
      DO 17 J=1,NSEN
  17  KHAT1(I,J) = KWORK(I,J)
```

```
C
C      PARTITION 1,1
C
       DO 201 I=1,NC12
       DO 201 J=1,NC12
 201   MAJM(I,J) = ABG1(I,J)
C
C      PARTITION 2,2
C
       CALL ABGC(A2,B2,C2,NSUP2,KHAT1,NACT,NSEN,ABG2)
C
       DO 18 I=1,NACT
       DO 18 J=1,NSEN
  18   KHAT1(I,J) = KWORK(I,J)
       DO 202 I=1,NSUP2
       DO 202 J=1,NSUP2
 202   MAJM(I+K,J+K) = ABG2(I,J)
C
C     PARTITION 3,3
C
       CALL ABGC(A3,B3,C3,NRES2,KHAT1,NACT,NSEN,ABG3)
       DO 24 I=1,NACT
       DO 24 J=1,NSEN
  24   KHAT1(I,J) = KWORK(I,J)
       DO 203 I=1,NRES2
       DO 203 J=1,NRES2
 203   MAJM(I+L,J+L) = ABG3(I,J)
C
C      PARTITION 1,2
C
       CALL MMUL(B1,KHAT1,NC12,NACT,NSEN,TEN)
       CALL MMUL(TEN,C2,NC12,NSEN,NSUP2,BCG)
       DO 208 I=1,NC12
       DO 208 J=1,NSUP2
 208   MAJM(I,J+K) = BCG(I,J)
C
C     ,NRES2
  405   PRINT*,'           ',W1(I)
        PRINT'(//)'
       ELSE
        PRINT*,' NO RESIDUAL TERM EIGENVALUES '
        PRINT'(/)'
       ENDIF
       ENDIF
C
C
C  THIS SECTION FORMS THE TRANSFORMATION MATRIX.
C SINCE THE OBJECT OF THIS PROGRAM IS TO DESIGN ONE
```

```
C CONTROLLER
C          U = K * GAMMA * C * X
C
C  AFTER THE TRANSFORMATION IS COMPLETE,
C  THE MAJM WILL LOOK LIKE:
C
C  ****                                                      ****
C  *                                                            *
C  *      (A1+B1K1C1)          0             B1K1CR             *
C  *                                                            *
C  *       B2K1C1             A2             B2K1CR             *
C  *                                                            *
C  *      (BRK1C1)         (BRK1C2)       (AR+BRK1CR)           *
C  *                                                            *
C  ****                                                      ****
C
C
C  WHERE THE NON-ZERO TERMS INCLUDE THE
C  TRANSFORMATION MATRIX.
C
C    GENERATE THE TRANSFORMATION MATRIX GAMMA1
C
      CALL TFR(CT,C2,NSEN,NSUP2,1,2)
      DO 500 I=1,NSUP
      DO 500 J=1,NSEN
 500  V(I,J) = CT(I+NSUP,J)
      NRV = NSUP
      PRINT*,' V (C2) IS '
      CALL PRNT(V,NRV,NSEN)
      CALL LSVDF(V,NCOL,NRV,NSEN,TEN,NCOL,-1,SING,STOR,IER)
      PRINT*,' '
      PRINT*,' V OUT OF LSVDF IS '
      CALL PRNT(V,NSEN,NSEN)
      P1 = NSEN - NRV
      IF (P1.LT.1) THEN
        DO 503 I=1,NSEN
 503    GAMMA1(I,1) = V(I,NSEN)
        P1 = 1
       ELSE
        DO 504 I=1,NSEN
        DO 504 J=1,P1
 504    GAMMA1(I,J) = V(I,J+NRV)
      ENDIF
C
      PRINT*,' TRANSFORMATION MATRIX GAMMA1 '
      CALL PRNT(GAMMA1,NSEN,P1)
C
C  CHECK TO SEE THAT GAMMA1 IS ORTHOGONAL TO C2
```

```
C
C NOTE:    AKC IN THIS SECTION IS JUST A WORK AREA TO TEST
C          THE ORTHOGONALITY OF CT * TR.  IN ALL CASES IT
C          SHOULD BE A BLOCK ZERO MATRIX.
C
      CALL TFR(CT,C2,NSEN,NSUP2,1,2)
      CALL MMUL(CT,GAMMA1,NSUP2,NSEN,P1,AKC)
      PRINT*,' C2T * GAMMA1 '
      CALL PRNT(AKC,NSUP2,P1)
C
      PRINT*,' C2 SINGULAR VALUES '
      CALL PRNT(SING,NRV,1)
C
      DO 411 I=1,NSEN
      DO 411 J=1,P1
 411  GT1(J,I)=GAMMA1(I,J)
      PRINT*,' GAMMA1 TRANSPOSE  '
      CALL PRNT(GT1,P1,NSEN)
C
C     CSTAR = GAMMA * C
C
      CALL MMUL(GT1,C1,P1,NSEN,NC12,CSTAR1)
      PRINT*,'  (GT1)(C1)   '
      CALL PRNT(CSTAR1,P1,NC12)
C
C     GSTAR1 = GAIN1
C
C    CALCULATE UT, V, QPLUS  WHERE CSTAR = U*Q*VT  AND
C                                  CSTAR+ = V*QPLUS*UT
C                                  KSTAR = GSTAR*CSTAR+
C
      CALL FINDK(CSTAR1,P1,NC12,WORK,GAIN1,NACT,KSTAR1)
      PRINT*,'  KSTAR1   '
      CALL PRNT(KSTAR1,NACT,P1)
C
C
C       FORM  TKG =  KSTAR * GAMMA
C
      CALL MMUL(KSTAR1,GT1,NACT,P1,NSEN,TKG1)
      PRINT*,'  (KSTAR1)(GT1)   '
      CALL PRNT(TKG1,NACT,NSEN)
C
      DO 348 I=1,NACT
      DO 348 J=1,NSEN
 348  SAT(I,J)=0.0
C
C    KT1 = -TKG
C    REMEMBER THAT ABGC TAKES CARE OF THIS MINUS SIGN FOR
```

```
C    THE DIAGONAL BLOCKS
C
        CALL UBOAT(SAT,TKG1,NACT,NSEN,KT1)
C
C
C
C REMEMBER THAT ABGC DESTROYS TKG1
        DO 21 I=1,NACT
        DO 21 J=1,NSEN
   21   KWORK(I,J) = TKG1(I,J)
        DO 350 I=1,M
        DO 350 J=1,M
  350   MAJM(I,J)=0.0
C
C       FILL IN PARTITION 1,1 OF MAJM
C
        CALL ABGC(A1,B1,C1,NC12,TKG1,NACT,NSEN,ABG5)
        PRINT*,'  A1 + B1(TKG1)C1   '
        CALL PRNT(ABG5,NC12,NC12)
        DO 351 I=1,NC12
        DO 351 J=1,NC12
  351   MAJM(I,J)=ABG5(I,J)
C
C    PARTITI[illegible],2
C
        DO 22 I=1,NACT
        DO 22 J=1,NSEN
   22   TKG1(I,J) = KWORK(I,J)
        CALL ABGC(A2,B2,C2,NSUP2,TKG1,NACT,NSEN,AB[illegible])
        PRINT*,'  A2 + B2(TKG1)C2   '
        CALL PRNT(ABG6,NSUP2,NSUP2)
        J=1,NRES2
  358    MAJM(I,J+L)=KCC(I,J)
        ENDIF
C
C ((((((((((((((((((((((((((((((((((((((((((
C  NEXT, FORM E TO THE AT
C  BECAUSE EIGRF DESTROYS MAJM
C
        IF (SKIP.EQ.1) THEN
         TOL = 0.001
         CALL MEXP(MM,MAJM,DT,EAT2)
         PRINT*,' MEXPOUT '
         PRINT'(//)'
         PRINT*,'  THE SOLUTION EAT2 IS '
         CALL PRNTXL(EAT2,MM,MM)
        ENDIF
C
```

```
C
C  EAT2 IS NOW THE SOLUTION TO E TO THE AT
C
C
C )))))))))))))))))))))))))))))))))))))))))))))
C
      PRINT*,'     LOWER TRIANGLE ONE CONTROLLER MAJOR MATRIX  '
      PRINT*,'    AFTER SUPPRESSION, WITH RESIDUALS'
      CALL PRNT(MAJM,M,M)
       PRINT*,'(///)'
      PRINT*,'   TRANSFORMED SYSTEM EIGENVALUES  '
      PRINT*,'    AFTER SUPPRESSION'
      CALL EIGRF(MAJM,M,NDA,0,Z,TEN,NCOL,WORK,IER)
      PRINT'(/)'
      DO 366 I=1,M
 366  PRINT*,'     ',Z(I)
      PRINT'(//)'
      PRINT*,'  EIGENVALUES OF  A1 + (B1)(TKG1)(C1)  '
      CALL EIGRF(ABG5,NC12,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT'(/)'
      DO 723 I=1,NC12
 723  PRINT*,'   ',W1(I)
      PRINT'(/)'
      PRINT*,'  EIGENVALUES OF  A2 + (B2)(TKG1)(C2)  '
      CALL EIGRF(ABG6,NSUP2,NCOL,0,W1,TEN,NCOL,STOR,IER)
      PRINT'(/)'
      DO 724 I=1,NSUP2
 724  PRINT*,'     ',W1(I)
      PRINT'(/)'
C
      IF (NRES.GT.0) THEN
       PRINT*,'  EIGENVALUES OF  A3 + (B3)(TKG1)(C3)  '
       CALL EIGRF(ABG7,NRES2,NCOL,0,W1,TEN,NCOL,STOR,IER)
       PRINT'(/)'
       DO 725 I=1,NRES2
 725   PRINT*,'    ',W1(I)
      ENDIF
C
      IF (SKIP.NE.1) THEN
       PRINT*,'      NO TIME RESPONSE PRINTOUT  '
      ELSE
       PRINT'(///)'
       PRINT*,'  STATE TRANSITION MATRIX, [illegible]2'
       CALL PRNT(EAT2,MM,MM)
       CALL TIMEX(EAT2,MM,DT,X0,PDT,TMAX,X1,XC,PHILOS,
     *IC1,IC2,IC3,NC1,NSUP,NRES)
      ENDIF
C
```

```
C
C  STM IS NOW THE SOLUTION TO ZDOT = MAJM * Z
C  WE NOW PROPAGATE THE STATE IN DT STEPS
C END OF TIME RESPONSE SECTION
C
      PRINT*,'  ***********************************  '
      PRINT*,'  **                               **  '
      PRINT*,'  **    END   DOFB335   PROGRAM    **  '
      PRINT*,'  **                               **  '
      PRINT*,'  ***********************************  '
      PRINT'(///)'
      END
C
C ))))))))))))))))))))))))))))))))))))))))))))))))))
C
C        START OF SUBROUTINES
C
C )))))))))))))))))))))))))))))))))))))))))))))
C
C                  ABGC
C     ABGC IS A BAD CITIZEN, IT DESTROYS KCC, TAKE HEED
C
C )))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE ABGC(A,B,C,N2,KCC,NACT,NSEN,ABG)
      COMMON/MAINB/NCOL,NCOL1
      REAL B(NCOL,1),A(NCOL,1),C(NCOL,1),KCC(NCOL,1),ABG(NCOL,1)
      INTEGER N2,NACT,NSEN
      CALL MMUL(B,KCC,N2,NACT,NSEN,ABG)
      CALL MMUL(ABG,C,N2,NSEN,N2,KCC)
      CALL UBOAT(A,KCC,N2,N2,ABG)
      RETURN
      END
C
C  )))))))))))))))))))))))))))))))))))))))))))))
C
C     FUNCTION DOT
C
C  )))))))))))))))))))))))))))))))))))))))))))))
C
      FUNCTION DOT(NR,A,B)
      DIMENSION A(1),B(1)
      DOT=0.
      DO 1 I=1,NR
 1    DOT=DOT+A(I)*B(I)
      RETURN
      END
C
```

```
C  FINDS K FROM THE RICCATI SOLUTION, G
C
C )))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE FINDK(C,M,N,W,G,L,K)
C  M=NSEN N=NC12 W=WORK SPACE L=NACT   (G IS NACT BY NC12)
      COMMON/MAINA/NDA,NDA1
      COMMON/MAINB/NCOL,NCOL1
      REAL UT(41,41),VP(41,41),QPLUS(41,41)
      REAL SI(42),CPLUS(41,41),QUT(41,41)
      REAL V(41,41),C(NCOL,NCOL),G(NCOL,NCOL)
      REAL K(NCOL,NCOL),W(NDA,NDA)
      DO 1 I=1,M
      DO 1 J=1,M
 1    UT(I,J)=0.0
      DO 2 I=1,M
 2    UT(I,I)=1.0
      DO 3 I=1,M
      DO 3 J=1,N
 3    VP(I,J)=C(I,J)
      CALL LSVDF(VP,NCOL,M,N,UT,NCOL,M,SI,W,IER)
      DO 4 I=1,N
      DO 4 J=1,M
 4    QPLUS(I,J)=0.0
      DO 5 I=1,N
 5    IF(SI(I).GT.0.00001)  QPLUS(I,I)=1/SI(I)
      DO 6 I=1,N
      DO 6 J=1,N
 6    V(I,J)=VP(I,J)
      CALL MMUL(QPLUS,UT,N,M,M,QUT)
      CALL MMUL(V,QUT,N,N,M,CPLUS)
      CALL MMUL(G,CPLUS,L,N,M,K)
      RETURN
      END
C
C ))))))))))))))))))))))))))))))))))))))))))))))))))))
C
C       FORMX2
C
C ((((((((((((((((((((((((((((((((((((((((((((((((((
      SUBROUTINE FORMX2(XO,INIT)
      COMMON/NUM/IC1(41),IC2(41),IC3(41),IR(41),NC1,NSUP,NRES,NR
      REAL XO(41),INIT(2,41)
      INTEGER M,I,J,K,L
      DO 1 I=1,NC1
      M = IC1(I)
      XO(I) = INIT(1,M)
  1   XO(I+NC1) = INIT(2,M)
      J = NC1*2
      DO 2 I=1,NSUP
      M = IC2(I)
      XO(I+J) = INIT(1,M)
```

```
 2    XO(I+J+NSUP) = INIT(2,M)
      K = J + NSUP*2
      DO 3 I=1,NRES
      M = IC3(I)
      XO(I+K) = INIT(1,M)
 3    XO(I+K+NRES) = INIT(2,M)
      RETURN
      END
C
C )))))))))))))))))))))))))))))))))))))))))))
C
C
C     SUBROUTINE FACTOR
C
C ))))))))))))))))))))))))))))))))))))))))))
      SUBROUTINE FACTOR(N,A,S,MR)
C       A=S'S
      DIMENSION A(1),S(1)
      COMMON/MAINB/ NCOL,NCOL1
      COMMON/INOUT/KOUT
      TOL=1.E-6
      MR=0
      NN=N*NCOL
      TOL1=0.
      DO 1 I=1,NN,NCOL1
      R=ABS(A(I))
 1    IF (R.GT.TOL1) TOL1=R
      TOL1=TOL1*1.E-12
      II=1
      DO 50 I=1,N
      IM1=I-1
       DO 5 JJ=I,NN,NCOL
 5    S(JJ)=0.
      ID=II+IM1
      R=A(ID)-DOT(IM1,S(II),S(II))
      IF (ABS(R).LT.(TOL*A(ID)+TOL1)) GO TO 50
      IF (R) 15,50,20
 15   MR=-1
      WRITE(KOUT,1000)
1000  FORMAT(37HOTRIED TO FACTOR AN INDEFINITE MATRIX    )
      RETURN
 20   S(ID)=SQRT(R)
      MR=MR+1
      IF (I.EQ.N) RETURN
      L=II+NCOL
      DO 25 JJ=L,NN,NCOL
      IJ=JJ+IM1
 25   S(IJ)=(A(IJ)-DOT(IM1,S(II),S(JJ)))/S(ID)
```

```
 50    II=II+NCOL
       RETURN
       END
C
C )))))))))))))))))))))))))))))))))))))))))
C
C          SUBROUTINE   FORMA
C
C )))))))))))))))))))))))))))))))))))))))))
C
       SUBROUTINE FORMA(A,D,W,N,N2,IC)
       COMMON/MAINB/NCOL
       REAL A(NCOL,NCOL),W(NCOL),D(NCOL)
       INTEGER IC(NCOL),I,J,N,M
       DO 1 I=1,N2
       DO 1 J=1,N2
       A(I,J)=0.0
 1     CONTINUE
       DO 2 I=1,N
       M= IC(I)
       A((I+N),(I+N))=D(M)
       A(I,(I+N)) = 1.0
       A((I+N),I) = -(W(M)**2)
 2     CONTINUE
       RETURN
       END
C
C )))))))))))))))))))))))))))))))))))))))))))))))
C
C                             SUBROUTINE FORMB
C
C )))))))))))))))))))))))))))))))))))))))))))))))
C
       SUBROUTINE FORMB(B,PHI,N,N2,NACT,IC)
       COMMON/MAINB/NCOL
       REAL B(NCOL,NCOL),PHI(NCOL,NCOL)
        INTEGER IC(N),NACT,N,M,I,J
        DO 1 I=1,N2
        DO 1 J=1,NACT
        B(I,J) = 0.0
 1      CONTINUE
        DO 2 I=1,N
        M = IC(I)
        DO 2 J=1,NACT
        B((N+I),J) = PHI(M,J)
 2      CONTINUE
        RETURN
        END
```

```
C
C ))))))))))))))))))))))))))))))))))))))))
C
C                 SUBROUTINE FORMC
C
C ))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE FORMC(C,PHIS,N,N2,NSEN,IC)
      COMMON/MAINB/NCOL
      REAL C(NCOL,NCOL),PHIS(NCOL,NCOL)
      INTEGER IC(N),M,NSEN,N,N2,I,J
      DO 1 I=1,NSEN
      DO 1 J=1,N2
      C(I,J) = 0.0
 1    CONTINUE
      DO 2 I=1,NSEN
      DO 2 J=1,N
      M = IC(J)
      C(I,N+J)=PHIS(M,I)
 2    CONTINUE
      RETURN
      END
C
C ))))))))))))))))))))))))))))))))))))))))))))
C
C      FORMQ
C
C ))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE FORMQ(Q,A,N,IC)
      COMMON/MAINB/NCOL
      REAL A(NCOL),Q(NCOL,NCOL)
      INTEGER I,J,K,M,N,N2,IC(NCOL)
      N2 = N * 2
      DO 1 I=1,N2
      DO 1 J=1,N2
      Q(I,J) = 0.0
 1    CONTINUE
      DO 2 I=1,N
      M = IC(I)
      Q(I,I) = A(M)
      Q(I+N,I+N) = Q(I,I)
 2    CONTINUE
      RETURN
      END
C
C   ))))))))))))))))))))))))))))))))))))))))
C
```

```
C        SUBROUTINE GMINV
C
C    ))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE GMINV(NR,NC,A,U,MR,MT)
      DIMENSION A(1),U(1)
      COMMON/MAIN1/ NDIM,NDIM1,S(1)
      COMMON/MAINB/NCOL,NCOL1
      COMMON/INOUT/KOUT
      TOL=1.E-12
      MR=NC
      NRM1=NR-1
      TOL1=1.E-20
      JJ=1
      DO 100 J=1,NC
      FAC=DOT(NR,A(JJ),A(JJ))
      JM1=J-1
      JRM=JJ+NRM1
      JCM=JJ+JM1
      DO 20 I=JJ,JCM
 20   U(I)=0.
      U(JCM)=1.0
      IF (J.EQ.1) GO TO 54
      KK=1
      DO 30 K=1,JM1
      IF (S(K).EQ.1.0) GO TO 30
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
 30   KK=KK+NCOL
      DO 50 L=1,2
      KK=1
            DO 50 K=1,JM1
      IF (S(K).EQ.0.) GO TO 50
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(NR,TEMP,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
 50   KK=KK+NDA
      TOL1=TOL*FAC
      FAC=DOT(NR,A(JJ),A(JJ))
 54   IF (FAC.GT.TOL1) GO TO 70
      DO 55 I=JJ,JRM
 55   A(I)=0.
      S(J)=0.
      KK=1
      DO 65 K=1,JM1
      IF (S(K).EQ.0.) GO TO 65
      TEMP=-DOT(K,U(KK),U(JJ))
      CALL VADD(NR,TEMP,A(JJ),A(KK))
```

```
 65   KK=KK+NDA
      FAC=DOT(J,U(JJ),U(JJ))
      MR=MR-1
      GO TO 75
 70   S(J)=1.0
      KK=1
      DO 72 K=1,JM1
      IF (S(K).EQ.1.) GO TO 72
      TEMP=-DOT(NR,A(JJ),A(KK))
      CALL VADD(K,TEMP,U(JJ),U(KK))
 72   KK=KK+NDA
 75   FAC=1./SQRT(FAC)
      DO 80 I=JJ,JRM
 80   A(I)=A(I)*FAC
      DO 85 I=JJ,JCM
 85   U(I)=U(I)*FAC
 100  JJ=JJ+NDA
      IF (MR.EQ.NR.OR.MR.EQ.NC) GO TO 120
      IF (MT.NE.0) WRITE(KOUT,110)NR,NC,MR
 110  FORMAT(I3,1HX,I2,8H M  RANK,I2)
 120  NEND=NC*NDA
      JJ=1
      DO 135 J=1,NC
      DO 125 I=1,NR
      II=I-J
      S(I)=0.
      DO 125 KK=JJ,NEND,NDA
 125  S(I)=S(I)+A(II+KK)*U(KK)
      II=J
      DO 130 I=1,NR
      U(II)=S(I)
 130  II=II+NDA
 135  JJ=JJ+NDA1
      RETURN
      END
C
C ****************************************************************
C
C      HEADER  (IS A SUBROUTINE SINCE I GOT TIRED OF LOOKING AT IT)
C
C ****************************************************************
C
      SUBROUTINE HEADER
C
      PRINT'(////)'
      PRINT*,'     ******************************************** '
      PRINT*,'     *****                                  ***** '
      PRINT*,'     *****      DIRECT OUTPUT FEEDBACK      ***** '
```

```
      PRINT*,'     *****                                        ***** '
      PRINT*,'     *****        ONE CONTROLLER VERSION          ***** '
      PRINT*,'     *****                                        ***** '
      PRINT*,'     *****       NASA GROUND TEST VEHICLE         ***** '
      PRINT*,'     *****  PROGRAM ACONE                         ***** '
      PRINT*,'     ********************************************** '
      PRINT'(//)'
      PRINT*,'  THIS PROGRAM GENERATES A SOLUTION',
     *      ' SUPPRESSING CONTROL AND OBSERVATION SPILLOVER'
      PRINT*,' FOR ONE SET OF SUPPRESSED MODES.'
      PRINT*,' RESIDUALS ARE ALLOWED '
      PRINT*,' '
      RETURN
      END
C
C  )))))))))))))))))))))))))))))))))))))))))))
C
C          SUBROUTINE INTEG
C
C )))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE INTEG(N,A,C,S,T)
C     S=INTEGRAL EA*C*EA FROM 0 TO T
C     C IS DESTROYED
      DIMENSION A(1),C(1),S(1)
      COMMON/MAIN1/ NDIM,NDIM1, X(1)
      COMMON/MAINB/NCOL,NCOL1
      COMMON/MAIN2/COEF(100)
      NN=N*NCOL
      NM1=N-1
      IND=0
      ANORM=XNORM(N,A)
      DT=T
 5    IF (ANORM*ABS(DT).LE.0.5) GO TO 10
      DT=DT/2.
      IND=IND+1
      GO TO 5
 10   DO 15 I=1,NN,NCOL
      J=I+NM1
      DO 15 JJ=I,J
 15   S(JJ)=DT*C(JJ)
      T1=DT**2/2.
      DO 25 IT=3,15
      CALL MMUL(A,C,N,N,N,X)
      DO 20 I=1,N
      II=(I-1)*NCOL
      DO 20 JJ=I,NN,NCOL
      II=II+1
```

```
      C(JJ)=(X(JJ)+X(II))*T1
   20 S(JJ)=S(JJ)+C(JJ)
   25 T1=DT/FLOAT(IT)
      IF (IND.EQ.0) GO TO 100
      COEF(11)=1.0
      DO 30 I=1,10
      II=11-I
   30 COEF(II)=DT*COEF(II+1)/FLOAT(I)
      II=1
      DO 40 I=1,NN,NCOL
      J=I+NM1
      DO 35 JJ=I,J
   35 X(JJ)=A(JJ)*COEF(1)
      X(II)=X(II)+COEF(2)
   40 II=II+NCOL1
      DO 55 L=3,11
      CALL MMUL(A,X,N,N,N,C)
      II=1
      T1=COEF(L)
      DO 55 I=1,NN,NCOL
      J=I+NM1
      DO 50 JJ=I,J
   50 X(JJ)=C(JJ)
      X(II)=X(II)+T1
   55 II=II+NCOL1
C     X=EXP(A*DT)
      L=0
   60 L=L+1
      CALL MMUL(X,S,N,N,N,C)
      II=1
      DO 90 I=1,N
      J=II
      IF (I.EQ.1) GO TO 75
      DO 70 JJ=I,II,NCOL
      S(JJ)=S(J)
   70 J=J+1
   75 DO 85 JJ=I,N
      KK=JJ
      DO 80 K=I,NN,NCOL
      S(J)=S(J)+C(K)*X(KK)
   80 KK=KK+NCOL
   85 J=J+NCOL
      DO 87 JJ=I,NN,NCOL
   87 C(JJ)=X(JJ)
   90 II=II+NCOL
      IF (L.EQ.IND) GO TO 100
      CALL MMUL(C,C,N,N,N,X)
      GO TO 60
```

```
 100  CONTINUE
      RETURN
      END
C
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C         LOS
C
C  $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
      SUBROUTINE LOS(E,
      C2=C2/FLOAT(L)
      C(L)=C2
      D(L+1)=0.
      II=N+1-L
      E(II)=W
      II=1
      DO 35 I=1,NN,N
      IL=I+NM1
      DO 30 J=I,IL
   30 X(J)=EA(J)
      X(II)=X(II)+C2
   35 II=II+NP1
      IF (L.EQ.N) GO TO 40
      CALL MMUL1(X,A,N,N,N,EA)
      W=W*T1/FLOAT(L)
      L=L+1
      GO TO 20
   40 CONTINUE
C****** CAN CHECK X 0 FOR ACCURACY
      J=N+63
      DO 50 L=N,J
      DO 45 K=1,N
      D(K)=(D(K+1)-W*C(K))*T1/FLOAT(L)
   45 E(K)=E(K)+D(K)
   50 W=D(1)
      II=1
      DO 60 I=1,NN,N
      IL=I+NM1
      DO 55 J=I,IL
   55 EA(J)=E(1)*A(J)
      EA(II)=EA(II)+E(2)
   60 II=II+NP1
      IF (N.EQ.2) GO TO 85
      DO 80 L=3,N
      CALL MMUL1(EA,A,N,N,N,X)
      II=1
      C2=E(L)
```

```
      DO 75 I=1,NN,N
      IL=I+NM1
      DO 70 J=I,IL
   70 EA(J)=X(J)
      EA(II)=EA(II)+C2
   75 II=II+NP1
   80 CONTINUE
C
C    THIS NEXT SECTION MULTIPLIES
C    THE MATRIX EXPONENT BY ITSELF UNTIL IT IS AT
C    THE RIGHT TIME STEP
C
   85 IF (IND.EQ.0) RETURN
      DO 100 L=1,IND
      DO 90 I=1,NN,N
      IL=I+NM1
      DO 90 J=I,IL
   90 X(J)=EA(J)
  100 CALL MMUL(X,X,N,N,N,EA)
      RETURN
      END
C
C )))))))))))))))))))))))))))))))))))))))))))))))))))))
C
C
C                    MLINEQ
C
C )))))))))))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE MLINEQ(N,A,C,X,TOL,IER)
C     SOLVES A'X+XA+C=0
C     A AND X CAN BE IN SAME LOCATION
C     ANSWER RETURNED IN C AND X
      DIMENSION A(1),C(1),X(1)
      COMMON/MAINB/ NCOL, NCOL1
      COMMON/MAIN3/F(1)
      ADV=TOL*1.E-6
      DT=.5
      DT1=0.
      NN=N*NCOL
      DO 5 II=1,NN,NCOL1
 5    DT1=DT1-A(II)
      DT1=DT1/N
      IF (DT1.GT.4.0) DT=DT*4.0/DT1
      II=1
      DO 20 I=1,N
      DO 15 JJ=I,NN,NCOL
 15   X(JJ)=DT*A(JJ)
```

```
      X(II)=X(II)-.5
   20 II=II+NCOL1
      CALL GMINV(N,N,X,F,MR,0)
      IER=4
      IF (MR.NE.N) RETURN
      CALL MMUL(C,F,N,N,N,X)
C     INITIALIZATION OF X,F
      I=1
      DO 40 II=1,NN,NCOL
      J=II
      IF (I.EQ.1) GO TO 30
      DO 25 JJ=I,II,NCOL
      C(J)=C(JJ)
   25 J=J+1
   30 ID=J
      DO 35 JJ=II,NN,NCOL
      C(J)=DT*DOT(N,F(II),X(JJ))
   35 J=J+1
      F(ID)=F(ID)+1.0
   40 I=I+1
      DO 90 IT=1,20
      NEZ=0
      CALL MMUL(C,F,N,N,N,X)
      I=1
      II=1
      J=1
      GO TO 70
   60 J=II
      DO 65 JJ=I,II,NCOL
      C(J)=C(JJ)
   65 J=J+1
   70 ID=J
      DT1=C(J)
      DO 75 JJ=II,NN,NCOL
      C(J)=C(J)+DOT(N,F(II),X(JJ))
   75 J=J+1
      J=J-1
      DO 80 JJ=II,J
   80 X(JJ)=F(JJ)
      IF (ABS(C(ID)).GT.1.E150) GO TO 95
      IF (ABS(C(ID)-DT1).LT.(ADV+TOL*ABS(C(ID)))) NEZ=NEZ+1
      I=I+1
      II=II+NCOL
      IF (I.LE.N) GO TO 60
      IF (NEZ.EQ.N) GO TO 150
      CALL MMUL(X,X,N,N,N,F)
   90 CONTINUE
   95 IER=1
```

```
      RETURN
 150  CONTINUE
      NM1=N-1
      DO 155 I=1,NN,NCOL
      II=I+NM1
      DO 155 JJ=I,II
 155  X(JJ)=C(JJ)
      IER=0
      RETURN
      END
C
C ))))))))))))))))))))))))))))))))))))))))))))))))))))))))
C
C                           MMUL
C
C ))))))))))))))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE MMUL(X,Y,N1,N2,N3,Z)
      COMMON /MAINB/ NCOL
      DIMENSION X(NCOL,1),Y(NCOL,1),Z(NCOL,1)
      DO 3 J=1,N3
      DO 2 I=1,N1
      S=0.
      DO 1 K=1,N2
   1  S=S+X(I,K)*Y(K,J)
   2  Z(I,J)=S
   3  CONTINUE
      END
C
C
C )()()()()()()()()()()()()()()()()()()()()()()()()()()()()
C
C       -MMUL1-
C
C  09090909090909090909)()()90)(0)9)909)9)9)9)(09090909
C
      SUBROO TO 40
      CALL GMINV(N,N,Z,F,MR,0)
      CALL MMUL(S,X,N,N,N,Z)
      DO 30 I=1,NN,NCOL
      II=I+NM1
      DO 30 J=I,II
 30   Z(J)=A(J)-Z(J)
      IF (MR.NE.N) WRITE(KOUT,35)MR
  35  FORMAT(27HORICCATI SOLN IS PSD--RANK ,I3)
      GO TO 65
 40   CONTINUE
      WRITE(KOUT,45) MAXIT
```

```
 45   FORMAT(27HORICCATI NON-CONVERGENT IN ,I2,11H
      GO TO 60
 50   WRITE(KOUT,55)IT,T1
C
C
 55   FORMAT(30HORICCATI BLOW-UP AT ITERATION ,I2,12H
 60   IER=1
 65   RETURN
 200  IF (IND.EQ.2) GO TO 250
      IF (COUNT.GE.10.) RETURN
      T1=T1/(2.**COUNT)
      IND=2
      GO TO 300
 250  T1=T1*(2.**COUNT)
      IND=1
      GO TO 300
      END
C
C )))))))))))))))))))))))))))))))))))))))))))))))
C
C                                    PRNT
C
C ))))))))))))))))))))))))))))))))))))))) ))))
C
      SUBROUTINE PRNT(MAT,N,M)
      COMMON/MAINB/NCOL
      COMMON/PRO/Q
      REAL MAT(NCOL,NCOL)
      INTEGER N,I,J,K,M,Q
      IF (Q.GT.1) THEN
       RETURN
      ENDIF
      PRINT*,' '
      IF (M.GT.12) GOTO 2
      DO 1 I=1,N
      PRINT'(1X,12F10.4)',(MAT(I,J),J=1,M)
 1    CONTINUE
      GOTO 10
 2    CONTINUE
      IF (M.GT.NCOL) THEN
      CALL PRNTXL(MAT,N,M)
      RETURN
      ENDIF
      DO 3 I=1,N
      PRINT'(1X,12F10.4)',(MAT(I,J),J=1,12)
 3    CONTINUE
      PRINT'(//)'
      IF (M.LE.24) THEN
```

```
          DO 4 I=1,N
          PRINT'(1X,12F10.4)',(MAT(I,J),J=13,M)
 4        CONTINUE
          PRINT'(///)'
          RETURN
         ENDIF
         IF (M.LE.36) THEN
          DO 5 I=1,N
          PRINT'(1X,12F10.4)',(MAT(I,J),J=13,24)
 5        CONTINUE
          PRINT'(//)'
         DO 6 I=1,N
         PRINT'(1X,12F10.4)',(MAT(I,J),J=24,M)
 6       CONTINUE
         ENDIF
         IF (M.GT.36) THEN
          PRINT*,' MATRIX PRINTING HAS BEEN TRUNCATED '
         ENDIF
 10       PRINT'(///)'
         RETURN
         END
C
C ))))))))))))))))))))))))))))))))))))))))))))))
C
C            PRNTK
C     PRINTS SMALL MATRICES WIDER
C
C ))))))))))))))))))))))))))))))))))))))))))))))
C
         SUBROUTINE PRNTK(MAT,N,M)
         COMMON/MAINB/NCOL
          COMMON/PRO/Q
         REAL MAT(NCOL,NCOL)
         INTEGER N,I,J,K,M,Q
         IF (Q.GT.1) THEN
          RETURN
         ENDIF
         PRINT*,' '
         IF (M.GT.10) THEN
          CALL PRNT(MAT,N,M)
          RETURN
         ENDIF
         DO 1 I=1,N
         PRINT'(1X,10F15.4)',(MAT(I,J),J=1,M)
 1       CONTINUE
         PRINT'(///)'
         RETURN
         END
```

```
C
C )))))))))))))))))))))))))))))))))))))))))))))))
C
C                 PRNTXL
C
C )))))))))))))))))))))))))))))))))))))))))))))))
C
      SUBROUTINE PRNTXL(MAT,N,M)
      COMMON/MAINB/NCOL
       COMMON/PRO/Q
      COMMON/MAINA/NDA
      REAL MAT(NDA,NDA)
      INTEGER I,J,K,L,M,N,Q
      IF (Q.GT.1) THEN
       RETURN
      ENDIF
      PRINT*,' '
      DO 1 L=1,M,12
      K = L + 11
      IF (M-L.LT.11) K = M
      DO 2 I=1,N
      PRINT'(1X,12F10.5)',(MAT(I,J),J=L,K)
 2    CONTINUE
      PRINT'(//)'
 1    CONTINUE
      PRINT'(///)'
      RETURN
      END
C
C  ))()()()()()()()()()()()()()()()()()()()()()()()(
C
C          RFMPHIL THIS SUBROUTINE REFORMS
C           THE LINE OF SIGHT PHI MATRIX
C   ()()()()()()()()()()()()()()()()()()()()()()()((
C
      SUBROUTINE RFMPHIL(PHIL,RPHIL,NC1,NC2,NC3,NR,IC1,IC2,IC3,IR,
     *PETA,CPHIL,MM)
      COMMON/MAINA/NDA
      COMMON/MAINB/NCOL
      REAL PHIL(3,NCOL
C
      DIMENSION X(1),A(1)
      COMMON/MAINB/NCOL
      JS=(K-1)*NCOL*M
      JEND=M*NCOL
      GO TO (10,30,50,70),I
 10   DO 20 II=1,N
      DO 20 JJ=II,JEND,NCOL
```

```
 20     X(JJ)=A(JJ+JS)
        RETURN
 30     DO 40 II=1,N
        KK=(II-1)*NCOL
        DO 40 JJ=1,M
        LL=(JJ-1)*NCOL+II
 40     X(KK+JJ)=A(LL+JS)
        RETURN
 50     KK=0
        DO 60 II=1,JEND,NCOL
        LL=II+N-1
        DO 60 JJ=II,LL
        KK=KK+1
 60     X(KK)=A(JJ+JS)
        RETURN
 70     KK=M*N+1
        DO 80 II=1,M
        LL=(M-II)*NCOL+1
        DO 80 IJ=1,N
        KK=KK-1
        JJ=LL+N-IJ
 80     A(JJ+JS)=X(KK)
        RETURN
        END
C
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
C       TIMEX
C
C $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
C
        SUBROUTINE TIMEX(STM,MM,DT,X0,PDT,TMAX,X1,XC,PHILOS,
       *IC1,IC2,IC3,NC1,NSUP,NRES)
        COMMON/MAINA/NDA,NDA1
        COMMON/MAINB/NCOL,NCOL1
        REAL DT,TMAX,ABT,STM(NDA,NDA),X0(NDA),X1(NDA),XC(NDA)
C       OK,SO THIS NEXT LINE IS NOT THE ELEGANT WAY OF DOING
C      THINGS IS YOUR THESIS DUE TOMORROW?
C   E IS MY OUTPUT VECTOR WHEN YOU CALL LOS(     ) THE NUMBER IS
C     THE SIZE OF YOUR OUTPUT VECTOR. MINE IS THREE
C
        REAL E(3)
        INTEGER I,J,MM,PDT,EK
        EK=3
        ABT=0.
C MAKE A COPY OF THE I.C. VECTOR AND USE THE COPY.
        DO 10 I=1,MM
        XC(I) = X0(I)
```

```
   10 CONTINUE
      PRINT'(//)'
      PRINT*,' INIT STATE (TIME = 0.0)'
      PRINT'(1X,10E12.4)',(XO(I),I=1,MM)
      CALL LOS(E,PHILOS,EK,XO,IC1,IC2,IC3,NC1,NSUP,NRES)
      PRINT*,ABT,E(1),E(2),E(3)
C CHECK THAT MAX TIME, 'TMAX', IS NOT REACHED AND CONTINUE TO
C PROPAGATE THE STATE VECTOR AND PRT/PLT IT EVERY 'PDT' TIME
C INCREMENTS
   20 DO 40 I=1,PDT
      CALL VMULFF(STM,XC,MM,MM,1,NDA,NDA,X1,NDA,IER)
      DO 50 J=1,MM
      XC(J) = X1(J)
   50 CONTINUE
   40 CONTINUE
C INCREMENT THE ABSOLUTE TIME AND OUTPUT THE STATE VECTOR
      ABT=ABT+DT*PDT
      CALL LOS(E,PHILOS,EK,XC,IC1,IC2,IC3,NC1,NSUP,NRES)
      PRINT*,ABT,E(1),E(2),E(3)
      IF(ABT.LE.TMAX) THEN
        GOTO 20
      ENDIF
      RETURN
      END
C
C    *(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(*(
C
C           -TOUT-
C    THIS SUBROUTINE PRINTS THE TIME RESPONSE AS REPRESENTED
C    BY SENSOR OUTPUTS
C
C   )*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*)*
C
      SUBROUTINE TOUT(C1,NC1,C2,NSUP,C3,NRES,XC,Y)
C NEED COMMON BLOCKS FOR NSEN
      COMMON/MAINA/NDA,NDA1
      COMMON/MAINB/NCOL,NCOL1
      COMMON/IO/NSEN,NACT
      REAL C1(NCOL,NCOL),C2(NCOL,NCOL),C3(NCOL,NCOL)
      REAL XC(NDA),Y(NSEN)
      DO 1 I=1,NSEN
       Y(I)=0.0
    1 CONTINUE
      DO 2 I=1,NSEN
C
       DO 3 J=1,NC1
        Y(I) = Y(I) + C1(I,NC1+J) * XC(NC1+J)
    3  CONTINUE
```

```
C
        K = (NC1*2) + NSUP
        DO 4 J = 1,NSUP
         Y(I) = Y(I) + C2(I,NSUP+J) * XC(K+J)
    4   CONTINUE
C
        K = (NC1*2) + (NSUP*2) + NRES
        DO 5 J = 1,NRES
         Y(I) = Y(I) + C3(I,NRES+J) * XC(K+J)
    5   CONTINUE
    2 CONTINUE
      RETURN
      END
C )))))))))))))))))))))))))))))))))))))))))))))))))))))
C
C                VADD
C    ADDS VECTORS
C )))))))))))))))))))))))))))))))))))))))))))))     )))
C
      SUBROUTINE VADD(N,C1,A,B)
      DIMENSION A(1),B(1)
      DO 1 I=1,N
 1    A(I)=A(I)+C1*B(I)
      RETURN
      END
C
C )))))))))))))))))))))))))))))))))))))))))))))))))))))))
C
C   XNORM1
C
C ))))))))))))))))))))))))))))))))))))))))))))))))))))))))
C
      FUNCTION XNORM1(N,A)
C     COMPUTES AN APPROXIMATION TO NORM OF A-- NOT A B   D
      DIMENSION A(1)
      COMMON/MAINA/ NDA,NDA1
      NN=N*NDA
      C1=0.
      TR=A(1)
      IF (N.EQ.1) GO TO 20
      I=2
      DO 10 II=NDA1,NN,NDA
```

# Appendix B

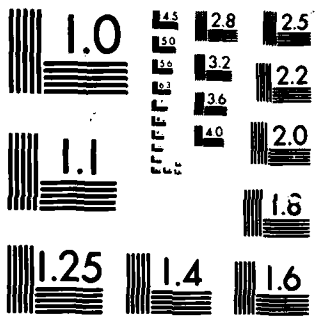
## Test Structure Mode Shapes

AD-A163 960 CONTROL OF A LARGE SPACE STRUCTURE USING DIRECT OUTPUT FEEDBACK AND MODAL..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. E E KELLER 2/2

UNCLASSIFIED DEC 85 AFIT/GAE/AA/85D-10 F/G 22/2 NL

END

FILMED




MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

Generalized Degrees of Freedom Associated With the Finite Element Nodes (Node points are as shown in figure 4)

| Node | $x_i$ | $y_i$ | $z_i$ | $Theta_{xi}$ | $Theta_{yi}$ | $Theta_{zi}$ |
|---|---|---|---|---|---|---|
| 1 | 46 $A_8$ | 47 $A_9$ | – | – | – | 1$A_1$ |
| 2 | 2 | 3 | 4 | 5 | 6 $A_2(-)$ | 7 |
| 3 | 2 | 3 | 4 | 5 | 8 $A_2(+)$ | 7 |
| 4 | 9 | 10 | 11 | 12 $A_4(-)$ | 13 | 14 |
| 5 | 9 | 10 | 11 | 15 $A_4(+)$ | 13 | 14 |
| 6 | 16 | 17 | 18 | 19 $S_1$ | 20 $S_2$ | 21 $S_3$ |
| 7 | 22 | 23 | 24 | 25 | 26 | 27 |
| 8 | 28 $A_3$ $S_4$ | 29 $A_5$ $S_5$ | 30 | 31 | 32 | 33 |
| 9 | 34 | 35 | 36 | 37 | 38 | 39 |
| 10 | 40 $A_6$ $S_6$ | 41 $A_7$ $S_7$ | 42 | 43 | 44 | 45 |
| 11 | 48 $S_8$ | 49 $S_9$ | 50 $S_{10}$ | 51 $S_{11}$ | 52 $S_{12}$ | 53 $S_{13}$ |
| 12 | 54 | 55 | 56 | 57 | 58 | 59 |
| 13 | 60 | 61 | 62 | 63 | 64 | 65 |
| 14 | 66 $S_{14}$ | 67 $S_{15}$ | 68 $S_{16}$ | 69 | 70 | 71 |
| 15 | 72 | 73 | 74 | 75 | 76 | 77 |
| 16 | 78 | 79 | 80 | 81 | 82 | 83 |
| 17 | 84 | 85 | 86 | 87 | 88 | 89 |
| 18 | 84 | 85 | 86 | 87 | 90 | 89 |
| 19 | 91 | 92 | 93 | 94 | 95 | 96 |
| 20 | 91 | 92 | 93 | 97 | 95 | 96 |
| 21 | 98 | 99 | 100 | 101 | 102 | 103 |
| 22 | 104 | 105 | 106 | | | |
| 23 | 107 | 108 | 109 | | | |
| 24 | 110 | 111 | 112 | – | – | – |
| 25 | 113 | 114 | 115 | – | – | – |
| 26 | 116 | 117 | 118 | 119 | 120 | 121 |
| 27 | 122 | 123 | 124 | 125 | 126 | 127 |
| 28 | 128 | 129 | 130 | 131 | 132 | 133 |

$S_i$ – Sensors

$A_i$ – Actuators

```
p*
 THIS FILE CONTAINS THE INFORMATION NEEDED TO RUN ANY ACOSS DERIVITIV
PROGRAM FOR THE NASA GROUND TEST FACILITY APPARATUS
THIS INFO IS FOR THE ANTENNA CONFIGURATION WITH THE VCOSS
MODS.  THIS INFO WAS NOT EXPERIMENTALLY VERIFIED, SO
IT REALLY IS BORDERLINE UTILITY
 DOF 1.
 .1016E-02 -.6448E-01 .3656E-06 -.7749E-01 -.1615E-05 -.3119E-03
  .3251E-06  .1852E-02  .7944E-.7  -.6870E-03
 -.6454E-12  -.1675E-11  .1342E-01  .3390E-05  .3294E-02  .1118E-08
 -.3998E-04  -.9612E-07  -.2636E-17  -.2228E-16  -.1828E-05
 -.2506E-05  -.5085E-01  .4494E-06  .4301E-01  -.5183E-05  -.1282E-04
 1.580E-01  .1531E-08  -.3329E-03
 DOF 6
 .1699E-14  .1566E-18  -.6380E-21  .1144E-12  .5347E-15  -.4609E-11
 -.97.3E-15  .6421E-11  .1848E-15  -.1808E-11  -.2995E-20  -.5036E-20
 .7151E-10  .1931E-13  .8705E-10  .3480E-16  -.1245E-11  -.5580E-15
 -.1146E-24  .4360E-25  -.1615E-13  -.2519E-13  -.5606E-09  .4938E-14
 .3717E-09  -.3884E-13  .4273E-13  -.8683E-09  .3861E-17  .3233E-11
 DOF 8
 -.2247E-06  .2981E-10  -.2384E-11  .1424E-03  .7999E-06  -.6712E-02
 -.2098E-05  .1034E-01  .3080E-06  -.2744E-02  -.4055E-11  -.7591E-11
 .7300E-01  .1735E-04  .1635E-01  .5688E-08  -.2034E-03  -.3455E-06
 -.3287E-16  .6579E-17  -.5952E-05  -.7406E-05  -.1122E-00  .9482E-06
 .4906E-01  -.4678E-05  .4796E-05  -.8493E-01  .1945E-09  .2725E-03
 DOF 12
 -.1808E-20  -.4804E-20  -.3866E-14  .3321E-16  -.9801E-11 -.1212E-14
 -.2525E-10  -.2345E-14  .1308E-10  .2312E-13  .9799E-21  -.1033E-19
 -.6745E-14  .3007E-10  .3314E-14  -.7663E-14  -.7220E-16  -.2210E-09
 .1975E-23  .2239E-24  -.5212E-10  -.5835E-10  -.1079E-13  -.3227E-12
 -.1815E-12  -.8602E-09  -.2563E-08  -.2009E-12  .2229E-11  .2203E-13
 DOF 15
 .2022E-12  .1231E-10  .1634E-08  -.2393E-06  -.7914E-02  -.1077E-05
 -.4556E-01  -.4058E-05  .2073E-01  .3745E-04  .1449E-11  -.1656E-10
 -.1056E-04  .4852E-01  -.3022E-06  .7033E-06  .5079E-08  -.2940E-01
 .2589E-15  .3551E-16  -.1526E-01  .6049E-02  -.5828E-06  -.1801E-04
 -.9389E-05  -.3541E-01  -.1258E-00  -.9432E-05  .9617E-04  .9649E-06
  DOF 19
 .2022E-12   .1231E-10   .1634E-08   -.2393E-06
 -.7914E-02   -.1077E-05   -.4556E-01   -.4058E-05
 .2073E-01   .3745E-04   01449E-11   -.1656E-10
 -.1045E-04   -.4852E-01   -.3022E-06   .7033E-6
 .5079E-08   -.2940E-01   .2589E-15   .3551E-16
 -.1516E-01   .6049E-02   -.5828E-06   -.1801E-04
  -.9389E-05   -.3541E-01   -.1258E-00   -.9432E-05
 .9617E-04   .9649E-06
 DOF 20
 -.2247E-6   .2981E-10   -.2384E-11   .1424E-03
 .7999E-06   -.6712E-02   -.2078E-05   .1034E-01
 .3080E-06   -.2744E-02   --.4055E-11   -.7591E-11
 .7300E-01   .1735E-04   .1635E-01   .5688E-08
 -.2034E-03   -.3455E-06   -.3287E-16   .6579E-17
 -.5952E-05   -.7406E-05   -.1122E-00   .9482E-06
 .4906E-01   -.4678E-05   .4796E-05   -.8493E-01
 .1945E-09   .2725E-03
 DOF 21
```

```
.1016E-02   -.6448E-01   .3656E-06   -.7749E-01
-.1615E-05   -.6712E-02   .3251E-06   .1852E-02
.7944E-7   -.6870E-03   -.6454E-12   -.1675E-11
.1342E-01   .3390E-5   .3294E-02   .1118E-08
-.3998E-04   -.9612E-07   -.2636E-17   -.2228E-16
-.1828E-05   -.2506E-5   -.5085E-01   .4494E-06
.4301E-01   -.5183E-05   -.1282E-04   .1580E-00
.1531E-08   -.3329E-03
DOF 46
.36E-1 .2325E-6 .2068E-7 -.1611E-2 -.145E-5 .1192E-1
.3964E-6 -.1650E-2 -.4489E-7 .4108E-3 .6283E-12 .1054E-11
-1.005E-2 -.2314E-5 -.3805E-2 -.1492E-8 .5338E-4 .1632E-7
.6533E-17 -.1601E-15 .3843E-6 .5165E-6 .8403E-2 -.7136E-7
-.3871E-2 .3759E-6 -.2788E-6 .5337E-2 -.2162E-10 -.1811E-4
DOF 47
.1917E-7 -.189E-6 -.3339E-1 .1741E-6 -.9893E-2 -.1167E-5
-.3841E-2 -.2245E-6 .1184E-2 .1956E-5 .7655E-13 -.8048E-12
-.3531E-6 .1342E-2 .5394E-7 -.1224E-6 -.1153E-8 -.2406E-2
-.6852E-16 .3606E-17 -.4618E-3 -.4456E-3 -.6023E-7 -.1737E-5
-.7039E-6 -.31E-2 -.6225E-2 -.4599E-6 .4648E-5 .4593E-7
DOF 51
-.1854E-10   .2788E-10   .3413E-06   .1086E-5
-.1195E-01   -.1097E-05   .5159E-01   .2952E-05
-.9038E-02   -.1604E-04   -.1000E-12   .5021E-11
-.5376E-05   .4583E-01   -.3997E-05   .2267E-04
 .8119E-07   -.1709E-02   -.1142E-15   .3965E-17
-.1608E-01   -.1193E-01   -.3346E-05   -1022E-03
-.2721E-04   -.1929E-02   -.7346E-02   .8200E-05
-.3337E-04   -.2360E-05
DOF 52
-.4120E-06   -.9032E-10   -.4681E-11   .1949E-04
.1241E-05   -.1203E-01   .4866E-5   -.1845E-01
-.6721E-06   .4940E-02   .5317E-11   .1600E-10
-.6583E-01   -.1177E-04   .7074E-01   .2965E-07
-.1063E-02   .8967E-07   -.6228E-16   -.3497E-16
-.2876E-05   -.5357E-05   -.7859E-01   .6143E-06
-.2684E-01   .5270E-05   .9389E-05   -.3153E-01
-.1569E-07   -.4686E-03
DOF 53
.1012E-02   -.6448E-01   .3653E-06   .5343E-01
.1256E-04   .4235E-02   -.4414E-04   -.3935E-02
-.2193E-05   -.2405E-01   -.9792E-11   .4148E-10
.1775E-02   -.4566E-04   .1242E-02   -.8209E-08
-.5978E-04   .4524E-06   .4623E-18   .4010E-15
.8301E-06   -.1948E-05   -.1186E-01   .1826E-07
-.3052E-01   .2045E-05   -.1330E-04   .6635E-02
-.1298E-06   -.8707E-03
THESE ARE THE COORDINATES THAT DESCRIBE THE
POSITION OF THE ANTENNA. 66 67 68 ARE X,Y,Z RESPECTIVELY.
69,70,71 ARE THETA X,Y,Z RESPECTIVELY.
DOF 66
.3805E-1 -.1309 .7622E-6 .1078E-0 .3916E-4 -.1131
-.3989E-4 -.227E-1 -.1426E-5 .4804E-1 -.1692E-10 .1045E-9
-.2586E-1 -.1113E-3 .2508E-1 -.2259E-7 -.5130E-3 .3181E-5
-.5697E-17 .3675E-15 .2777E-4 .1719E-4 -.5221E-1 .5317E-5
-.9199E-1 .1738E-4 .2704E-4 .1144E-1 -.3594E-7 -.2648E-2
```

```
DOF 67
.1926E-7 -.1893E-6 -.334E-1 -.2819E-5 .1206 .1397E-4
-.1646E-1 -.3627E-5 -.1954E-2 .1262E-4 -.1 68E-11 -     -11
-.3168E-5 .932E-2 .1257E-5 -.511E-4 -.304E-7 -.1042E-1
.1251E-14 .4518E-17 .8581E-2 -.1044E-1 .2998E-6 .7962E-5
.4535E-5 .1807E-2 .4251E-2 -.1401E-5 -.5678E-3 .4871E-6
DOF 68
.3782E-10 -.5756E-10 -.6916E-6 -.2208E-5 .2437E-1 .2235E-5
-.1101E-0 -.7387E-5 .1322E-1 .3262E-4 -.2534E-12 -.5092E-11
.1205E-4 -.1055E-0 .1249E-4 .6555E-5 -.2466E-6 .4769E-2
.1404E-15 -.8592E-17 .1173E-0 .1309 -.6560E-4 .5274E-3
-.9317E-4 -.5476E-_ -.9462E-2 .9553E-5 -.2762E-3 -.2358E-5
DOF 69
-.1869E-10 .2869E-10 .3401E-6 .1088E-5 -.1202E-1 -.1102E-5
.5583E-1 .4209E-5 -.3784E-2 -.1554E-4 -.2534E-12 -.5092E-11
-.6158E-5 .5467E-1 -.7269E-5 -.27E-4 .1411E-6 -.3544E-2
-.4586E-16 .5539E-17 -.7373E-1 -.8675E-1 .4843E-4 -.324E-3
.7978E-4 .4847E-2 .1031E-1 -.108E-4 .7156E-4 .2833E-5
DOF 70
-.4157E-6 -.7492E-10 -.4597E-11 .906E-5 .1222E-5 -.1199E-1
.5768E-5 -.2587E-1 -.5155E-6 .6809E-2 .6809E-11 .2244E-10
-.6363E-1 -.1255E-4 .7001E-1 .2128E-7 -.9899E-3 .1089E-6
-.4974E-16 -.3158E-16 -.2698E-5 -.5253E-5 -.7951E-1 .6228E-6
-.2793E-1 .5497E-5 .9984E-5 -.3302E-1 .1134E-7 .184E-3
DOF 71
.1012E-2 -.6448E-1 .3653E-6 .5354E-1 .1276E-4 .3734E-2
-.4863E-4 -.5127E-2 .1833E-5 .1761E-1 -.6738E-11 .3379E-10
-.8287E-3 -.5328E-4 .7845E-2 -.1776E-7 -.1508E-3 .1634E-5
-.5555E-17 .4103E-15 .1363E-4 .7324E-5 -.4993E-1 .4237E-6
-.1256 .2155E-4 .3231E-4 .2617E-1 .5812E-7 -.178E-2
DOF 28
 .036        .232E-6     .2066E-7   -.9668E-3   .5146E-5  -.4457E-1
-.1103E-4    .5136E-1    .1459E-5   -.1311E-1  -.1942E-10 -.3401E-10
 .2892E0     .6104E-4   -.2286E0    -.1008E-6   .3603E-2  -.2310E-5
 .2174E-15  -.2280E-15  -.2037E-4   -.1446E-4   .5333E-1  -.6990E-6
-.1624E0     .1923E-4    .6092E-5    .4065E-1  -.5818E-8  -.5655E-3
DOF 29
 .1918E-7   -.1891E-6   -.3339E-1    .1175E-5   .4681E-8   .6250E-5
 .1770E00    .1166E-4   -.5844E-1   -.9661E-4  -35422E-11  .3831E-10
 .6400E-5    .1759E-1   -.1446E-4    .3749E-4   .2987E-6   .2029E00
-.1331E-14  -.2163E-15  -.1338E00    .2784E00   .1057E-4   .2738E-3
 .3071E-4   -.4109E-1   -.8299E-1   -.1220E-4   .1902E-3   .2101E-5
DOF 40
 .3600E-1    .2318E-6    .2063E-7   -.7745E-3   .1264E-4  -.1137E00
 .2235E-5   -.2481E-2   -.6148E-6    .2058E-2  -.3102E-11  .1785F 10
 .1425E-1   -.1943E-6   -.2965E-1   -.1089E-7   .3817E-3   .5887E-6
 .4087E-16  -.4293E-15   .1034E-4    .1175E-4   .7184E-1  -.4337E-6
 .1161E0    -.1657E-4   -.1372E-4   -.9338E-2  -.1668E-7   .6678E-3
DOF 41
 .1925E-7   -.1893E-6   -.3340E-1   -.2156E-5   .1133E00   .1330E-4
 .1301E-1   -.1827E-5   -.7468E-2    .2840E-5  -.1129E-11 -.3088E-11
-.6446E-5    .3726E-1   -.1174E-5   -.3728E-4   .1898E-7  -.1144E-1
 .1131E-14   .6912E-17  -.1240E-2   -.1768E-1  -.1788E-5  -.5432E-4
-.1218E-4    .6229E-3   -.2494E-3    .3622E-5  -.5890E-3  -.9580E-6
DOF 48
.3600E-01  .2318E-06  .2063E-07  -.7626E-03  .1339E-04  -1.211E-01
```

```
.5203E-05  -.1373E-01  -.102E-05  .5071E-02  .1408E-12  .2761E-10
-.2591E-01  -.7373E-05  .1350E-01  .7201E-08  -.2666E-03  .6434E-06
.2874E-17  -.4506E-15  .8582E-05  .8487E-05  .2391E-01  -.5902E-07
.9976E-01  -.1336E-04  -.7999E-05  -.2857E-01  -.2626E-07  .3819E-03
 DOF 49
.1926E-07  -.1893E-06  -.3340E-01  -.2819E-05  1.206E-01  .1397E-04
-.1646E-01  -.3627E-05  -.1954E-02  .1262E-04  -.1068E-11 -.6152E-11
-.3168E-05  .9320E-02  .1257E-05  -.5111E-04  -.3040E-07  -.1042E-01
.1251E-14  .4517E-17  .8580E-02  -.1044E-01  .2998E-06  .7960E-05
.4534E-05  .1806E-02  .4250E-02  -.1400E-05  -.5684E-03  .4870E-06
 DOF 50
.2810E-14  -.1726E-13  .4308E-14  -.1202E-10  .6696E-06  .6404E-10
-.1837E-04  -.3464E-08  -.1030E-04  .1989E-08  -.5700E-14  .2524E-13
.1548E-08  -.8275E-05  -.8377E-07  .1585E-05  .2010E-08  -.1212E-02
  .4604E-17  .1389E-17  .6411E-02  .5109E-02  -.2248E-05  .1886E-04
-.3723E-05  -.2301E-03  -.5525E-03  .5904E-06  -.3228E-04  -.1595E-06
NATURAL FREQUENCIES RAD/SEC
    0   0   0   .3601389   .8209141   .8404761   2.11471   2.665896
    2.74208   2.835489   2.950763   2.954657   3.604164   3.903844
  6.464519   6.526102   6.526102   7.903797   8.15414   8.16517
    8.761278   9.438749   11.04083   11.24278   13.24458   13.7368
   16.7332   17.23949   18.06101   18.06101
NATURAL FREQUENCIES SQUARED.
0.0 0.0 0.0  .1297  .6739  .7064  4.472  7.107  7.519  8.040   8.707
8.730  12.99  15.24  41.79  42.59  42.59  62.47  66.49  66.67  76.76
89.09  121.9  126.4  175.4188.7  280.0  297.2  326.2  326.2
.1016E-02 -.6448E-01 .3656E-06 -.7749E-01 -.1615E-05 -.3119E-03
-.1615E-05  -.3119E-03  .3251E-06  .1852E-02  .7944E-.7  -.6870E-03
-.6454E-12  -.1675E-11  .1342E-01  .3390E-05  .3294E-02  .1118E-08
-.3998E-04  -.9612E-07  -.2636E-17  -.2228E-16  -.1828E-05
-.2506E-05  -.5085E-01  .4494E-06
```

## Bibliography


1. Aldrige, 2nd Lt Edward S. Decentralized Control of a Large Space Structure as Applied to the CSDL 2 Model. MS Thesis, GA/AA/82D-1. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, December 1982 (AD-A124702).

2. Calico, Robert A. Jr. and W.T. Miller. "Decentralized Control of a Flexible Spacecraft," AIAA/AAS Astrodynamics Conference. AIAA-82-1414. San Diego, Ca August 9-11,1982.

3. Daniel, J. W., and Noble, B. Applied Linear Algebra. Englewood Cliffs, N.J: Prentice-Hall, Inc., 1977.

4. Laub, Alan J. "A Shur Method for Solving Algebraic Riccati Equations," IEEE Transactions on Automatic Control, Vol. AC-24, No. 6, p 913.

5. Maybeck, Peter S. Stochastic Models, Estimation, and Control, Vols. 1-3. New York: Academic Press, 1979, 1982.

6. Mierovich, L. Analytical Methods In Vibrations. New York: The Macmillan Company, 1967.

7. Mierovich, L. et al. "A Comparison of Control Techniques for Large Flexible Systems," Journal of Guidance, Control and Dynamics, Vol. 6, No. 4, July-August 1983.

8. Mierovich, L. and H. Oz. "Computational Aspects of the Control of Large Flexible Structures," Proceedings of 18th IEEE Conference on Decision and Control. December 12-14,1979.

9. Miller, William T. Decentralized Control of Large Space Structures. MS Thesis, GA/AA/83D-8. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, December 1983 (AD-A136781).

10. Thyfault, Capt David V. Decentralized Control of a Large Space Structure Using Direct Output Feedback. MS Thesis, GA/AA/83D-8. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, December 1983 (AD-A136781).

11. Varhola, 2Lt. Dennis W. Model Reduction Techniques Applied to the Control of Large Space Structures. MS Thesis, GA/AA/83D-8. School of

Engineering, Air Force Institute of Technology (AU), Wright–Patterson AFB OH, December 1983 (AD–A136781).

12. AFWAL–TR–83–3075. Vibration Control of Space Structures VCOSS B: Momentum Exchange and Truss Dampening. Study conducted by TRW for Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, July 1983.

# Vita

Eric Keller is a native of Iowa. He interrupted his studies in 1978 to pursue the art of custom bicycle frame building in Waterloo, Wisconsin. After finding this a satisfying but unhealthy activity, he started racing bicycles full time. Circumstances and economics put a stop to this career before it had a chance to develop. Thus forced to face reality, he re–enrolled in college. This resulted in a Bachelor of Science in Mechanical Engineering with a Minor in Mathmatics from Virginia Tech in 1982. After a brief stay in San Antonio Texas, he received his commission from Officer Training School, USAF. Before entering the AFIT Aeronautical Engineering program in June 1984, he served as Registriar for the AFIT School of Civil Engineering from January to June 1984.

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## Abstract

Direct output feedback control using one or two controllers is applied to the NASA Ground Test Facility offset antenna model. This is a test structure designed to have the vibration characteristics associated with large space structures.

The control problem is transformed from physical variables into modal variables and reformulated into a first order system. This system is truncated to a reduced order model with residual modes used only in performance evaluation. Optimal linear quadratic regulator techniques are used to design the gain matrices, and full state feedback is approximated by use of generalized inverses of the observation matrices. Spillover is eliminated through the use of transformation matrices.

The structure is shown to be controllable with this method. Alternative sensor placement is explored, and found to cause improvement in performance. The torsion modes are found to be particularly important to the performance of the structure, but need more sensing and actuation to be adequately controlled. Two controller systems require more sensors and actuators than available to achieve acceptable performance with this structure.

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