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GREEN’S FUNCTIONS OF THE REDUCED WAVE EQUATION BETWEEN PARALLEL PLANES

BY

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Summary

The Green’s functions in the region between planes on which either Neumann or Dirichlet boundary conditions apply are obtained in the form of double Fourier series expansions. A closed form expression for the sound field in an axisymmetric water filled channel is obtained in order to illustrate the usefulness of the expansions. Problems associated with numerical evaluation are discussed and some numerical examples are given.

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1. INTRODUCTION

The expansions of the free-space Green's functions of the
reduced wave equation in Cartesian, cylindrical and spherical
polar coordinate systems have been used extensively in the
literature to solve boundary value problems, in fluid-structure
interaction, in which at least one of the coordinate directions
is infinite. One of the Green's function expansions tabulated
by Skelton [1] appears to be relatively new to the literature (see
equation (2.2) of this memorandum). James [2] and Skelton [3]
have used it to facilitate solution of the problem of infinite
pipes and layered cylinders, respectively, that are excited by an
interior point source of sound: Fuller has used it to investigate
infinite shell vibration [4] and the sound transmission into an
idealised model of an aircraft fuselage [5]. Thus, when new forms
of Green's functions become available they are invariably
accompanied by the solution of a class of problems.

It is desirable to extend the range of Green's function
expansions to enable solution of problems involving finite
geometries. For example, the sound field inside a finite
cylindrical enclosure, and the radiation of sound from the
elastic vibrations of its surface are problems of considerable
interest in the field of noise and vibration. Solution of these
problems would be facilitated if the Green's function in the
region between planes, on which Neumann or Dirichlet boundary
conditions are applied, were available as double Fourier series
expansions in the axial and circumferential coordinates of a
cylindrical coordinate system. The Green's function expansions
when the source is on the z-axis are well known from sound
propagation work [6], but the extensions to an off-axis source
have not been found in the literature.

2. GREEN'S FUNCTIONS

(a) General

An infinite slab of acoustic fluid is located between the
planes \( z=0 \) and \( z=h \) on which either Neumann (hard) or Dirichlet
(pressure release) boundary conditions apply. A time-harmonic
point source of sound of free-field pressure \( \exp(ik|R-R_0|)/|R-R_0| \)
is located in the acoustic fluid at the cylindrical coordinates
\((r,\phi,z) = (r_o,0,z_o)\). \( k=\omega/c \) is the acoustic wavenumber in which
\( \omega \) is the angular frequency and \( c \) is the sound velocity. The
geometry is shown in Figure 1. The time-harmonic factor,
\( \exp(-i\omega t) \), is omitted from all equations.

The pressure in the fluid layer satisfies the inhomogeneous
reduced wave equation

\[
(\nabla^2 + k^2)p(R,R_0) = -4\pi \delta(R-R_0)
\]  

(2.1)

together with the selected Neumann or Dirichlet boundary
conditions at \( z=0 \) and \( z=h \). The pressure may thus be written as the sum of the free-space Green's function \( p_G(R,R_0) \) together with a scattering term \( p_S(R,R_0) \) which satisfies the homogeneous form of equation (2.1) and whose amplitude is determined by the boundary conditions.

(a) Free-Space Green's Functions

The free-space Green's function of the reduced wave equation in cylindrical polar coordinates in a form which is appropriate to the application of additional boundary conditions on cylindrical boundaries is [1]

\[
\begin{align*}
P_G(R,R_0) &= \left\{ \begin{array}{ll}
   (i/2) \sum_{n=0}^{\infty} e_n \cos(n\phi) \int J_n(\gamma r)H_n(\gamma r_0) \exp(i\alpha[z-z_0]) d\alpha & \text{for } r<r_0 \\
   \infty & \text{for } r>r_0 \\
   (i/2) \sum_{n=0}^{\infty} e_n \cos(n\phi) \int J_n(\gamma r_0)H_n(\gamma r) \exp(i\alpha[z-z_0]) d\alpha & \text{for } r>r_0
\end{array} \right.
\end{align*}
\]

where \( J_n \) and \( H_n = J_n + iY_n \) are Bessel and Hankel functions, respectively; and \( \gamma = (k^2-\alpha^2)^{1/2} \), with \( \text{Im}(\gamma) > 0 \) in order to satisfy the radiation condition.

The infinite series in equations (2.2) converge only in a generalised function sense. They converge numerically provided that dissipation is included in the system by setting, for example, the sound velocity \( c \) to be the complex value \( c(1-i\eta) \) where \( \eta \) is a very small hysteretic loss factor. However, when \( r=r_0 \) the series cannot be evaluated numerically even when dissipation is included.

An alternative form of equation (2.2) which is required later is determined from an application of a variant of Graf's addition theorem for Bessel functions [7], viz.,

\[
H_0(\gamma\sqrt{r^2+r_0^2-2rr_0\cos\phi}) = \sum_{n=0}^{\infty} e_n H_n(\gamma r_0) J_n(\gamma r) \cos(n\phi) \quad r \geq r_0
\]

in which, if \( r_0 < r \) it is necessary only to interchange \( r_0 \) and \( r \). The required form is obtained, by substituting equation (2.3) into equation (2.2), as

\[
P_G(R,R_0) = (i/2) \int H_0(\gamma\sqrt{r^2+r_0^2-2rr_0\cos\phi}) \exp(i\alpha[z-z_0]) d\alpha \quad (2.4)
\]
(c) **Application of the Method of Images**

It is well known [6] that the scattered pressure due to the presence of a single rigid plane boundary is equivalent to the pressure of a second, or 'image', point source which is located at an equal distance on the opposite side of the boundary to the original source, both sources having equal magnitude and phase. Similarly, the pressure due to the presence of a single pressure release plane boundary is equivalent to the pressure of an image source of equal magnitude to, but out of phase with the original source. For the case to be considered here in which the source is located between two parallel plane boundaries the pressure in the fluid may be calculated by summing an infinite series of image sources obtained as a result of successive mirror reflections of the source in the boundaries. The locations of the image sources (see Figure 1) on the vertical axis are straightforwardly obtained as the four groups of coordinates

\[ z = 2(m+1)h + z_0 \]
\[ z = 2(m+1)h - z_0 \]
\[ z = -2mh + z_0 \]
\[ z = -2mh - z_0, \quad m=0,1,2,... \]

The three combinations of boundary conditions to be considered are (i) both upper and lower boundaries are rigid, (ii) both upper and lower boundaries are pressure release, and (iii) the upper boundary is pressure release and the lower boundary is rigid.

(d) **Both Boundaries are Rigid**

When both of the plane boundaries are rigid it is clear that all the image sources have the same phase as the real source. The radial and azimuthal coordinates of the image sources are also identical to those of the real source. In order to calculate the Green's function in this case it is sufficient therefore to replace the \( \exp(-i\alpha z_0) \) term in equations (2.2) and (2.4) by the infinite sum \( I(\alpha, z_0) \), where

\[
I(\alpha, z_0) = \sum_{m=0}^{\infty} \{ \exp(-i\alpha[z_0 - 2mh]) + \exp(-i\alpha[-z_0 - 2mh]) + \exp(-i\alpha[2(m+1)h + z_0]) + \exp(-i\alpha[2(m+1)h - z_0]) \} \quad (2.5)
\]
This expression may be simplified to give

\[ I(\alpha, z_0) = 2\cos(\alpha z_0) \sum_{m=-\infty}^{\infty} \exp(2iamh) \]  
(2.6)

The generalised function identity obtained from the Poisson summation formula [8]

\[ \sum_{m=-\infty}^{\infty} \exp(2\pi ima/d) = d \sum_{m=-\infty}^{\infty} \delta(\alpha-md) \]  
(2.7)

which is valid for integration purposes, allows equation (2.6) to be rewritten more usefully as

\[ I(\alpha, z_0) = (2\pi/h)\cos(\alpha z_0) \sum_{m=-\infty}^{\infty} \delta(\alpha-m\pi/h) \]  
(2.8)

When the \( \exp(-i\alpha z_0) \) term in equations (2.2) is replaced by this expression for \( I(\alpha, z_0) \) the \( \alpha \)-integration is straightforward and the Green's function of the reduced wave equation between two parallel rigid planes takes the required form

\[
P_{G}(R, R_0) = \begin{cases} 
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} e_n \cos n\phi J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(m\pi z_0/h) \exp(im\pi z/h) & r < r_0 \\
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} e_n \cos n\phi J_n(\gamma_m r_0) H_n(\gamma_m r) \cos(m\pi z_0/h) \exp(im\pi z/h) & r > r_0 
\end{cases}
\]  
(2.9)

in which \( \gamma_m = (k^2-m^2\pi^2/h^2)^{1/2} \). This may be simplified to

\[
P_{G}(R, R_0) = \begin{cases} 
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e_n \cos n\phi J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(m\pi z_0/h) \cos(m\pi z/h) & r < r_0 \\
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e_n \cos n\phi J_n(\gamma_m r_0) H_n(\gamma_m r) \cos(m\pi z_0/h) \cos(m\pi z/h) & r > r_0 
\end{cases}
\]  
(2.10)

Similarly, replacing the \( \exp(-i\alpha z_0) \) term in equation (2.4) by the expression for \( I(\alpha, z_0) \) in equation (2.8), integrating with respect to \( \alpha \) and rearranging the resulting infinite sum gives
an alternative form of the Green's function of the reduced wave equation between two parallel rigid planes, viz.,

\[
P_G(R, R_0) = \frac{i\pi}{h} \sum_{m=0}^{\infty} e^m H_0(\gamma_m) [r^2 + r_0^2 - 2rr_0 \cos \theta] \cos(m\pi z_0/h) \cos(m\pi z/h)
\]  
(2.11)

(e) Both Boundaries are Pressure Release

When both of the plane boundaries are pressure release it is clear that a phase change occurs at each reflection. In order to calculate the Green's function it is again sufficient to replace the \(\exp(-i\alpha z_0)\) term in equations (2.2) and (2.4) by an infinite sum \(I(\alpha, z_0)\) which, in this case, is

\[
I(\alpha, z_0) = \sum_{m=0}^{\infty} \left[ \exp(-i\alpha[z_0 - 2mh]) - \exp(-i\alpha[z_0 - 2mh]) \right] + \exp(-i\alpha[2(m+1)h+z_0]) - \exp(-i\alpha[2(m+1)h-z_0])
\]  
(2.12)

As in the previous case this may be simplified considerably, viz.,

\[
I(\alpha, z_0) = -2i \sin(\alpha z_0) \sum_{m=-\infty}^{\infty} \exp(2i\alpha mh)
\]  
(2.13)

which, after applying the generalised function identity, equation (2.7), may be written as

\[
I(\alpha, z_0) = -(2i\pi/h) \sin(\alpha z_0) \sum_{m=-\infty}^{\infty} \delta(\alpha - \pi m/h)
\]  
(2.14)

When the \(\exp(-i\alpha z_0)\) term in equations (2.2) is replaced by this expression for \(I(\alpha, z_0)\) the \(\alpha\)-integration is again straightforward, and the Green's function of the reduced wave equation between two parallel pressure release planes is

\[
P_G(R, R_0) = \begin{cases} 
(\pi/h) \sum_{n=0}^{\infty} \cos \phi \sum_{m=-\infty}^{\infty} n \gamma_m H_n(\gamma_m) R H_n(\gamma_m) \sin(m\pi z_0/h) \sin(m\pi z/h) & r \leq r_0 \\
(\pi/h) \sum_{n=0}^{\infty} \cos \phi \sum_{m=-\infty}^{\infty} n \gamma_m H_n(\gamma_m) R H_n(\gamma_m) \sin(m\pi z_0/h) \sin(m\pi z/h) & r > r_0 
\end{cases}
\]  
(2.15)
in which \( \gamma_m = (k^2 - m^2 \pi^2 / h^2)^{1/2} \). This may be rewritten as

\[
P_G(R, R_0) = \begin{cases} 
\sum_{n=0}^{\infty} \sum_{m=1}^{\infty} (2\pi i / h) e \cos n \phi \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \sin(m \pi Z_0 / h) \sin(m \pi Z / h) & r < r_0 \\
\sum_{n=0}^{\infty} \sum_{m=1}^{\infty} (2\pi i / h) e \cos n \phi \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \sin(m \pi Z_0 / h) \sin(m \pi Z / h) & r > r_0 
\end{cases}
\]

(2.16)

Similarly, replacing the \( \exp(-i\alpha Z_0) \) term in equation (2.4) by the expression for \( I(\alpha, Z_0) \) in equation (2.14), integrating with respect to \( \alpha \) and rearranging the resulting infinite sum gives an alternative form of the Green's function of the reduced wave equation between two parallel pressure release planes, viz.,

\[
P_G(R, R_0) = (2\pi i / h) \sum_{m=0}^{\infty} H_0(\gamma_m \sqrt{r^2 + r_0^2 - 2rr_0 \cos \phi}) \sin(m \pi Z_0 / h) \sin(m \pi Z / h)
\]

(2.17)

(f) One Rigid Boundary and One Pressure Release Boundary

The case in which the boundary at \( Z=0 \) is rigid and the boundary at \( Z=h \) is pressure release will now be considered. When one boundary is rigid and the other is pressure release, phase changes occur only at reflections at the pressure release boundary. In order to calculate the Green's function it is again sufficient to replace the \( \exp(-i\alpha Z_0) \) term in equations (2.2) and (2.4) by an infinite sum \( I(\alpha, Z_0) \) which, in this case, is

\[
I(\alpha, Z_0) = \sum_{m=0}^{\infty} (-1)^m \left[ \exp(-i\alpha [Z_0 - 2mh]) + \exp(-i\alpha [Z_0 - 2mh]) 
- \exp(-i\alpha [2(m+1)h + Z_0]) - \exp(-i\alpha [2(m+1)h - Z_0]) \right]
\]

(2.18)

As in the previous cases this may be simplified considerably, viz.,

\[
I(\alpha, Z_0) = 2\cos(\alpha Z_0) \sum_{m=-\infty}^{\infty} (-1)^m \exp(2i\alpha mh)
\]

(2.19)

This is, however, a form which is not suitable for a direct application of the generalised function identity, and it is necessary to rewrite equation (2.19) as
\[ I(\alpha, z_0) = 2 \cos(\alpha z_0) \left( 2 \sum_{m=-\infty}^{\infty} \exp(4i\alpha m h) - \sum_{m=-\infty}^{\infty} \exp(2i\alpha m h) \right) \]  

(2.20)

The generalised function identity may now be applied to each of the infinite sums in this equation, allowing it to be written as

\[ I(\alpha, z_0) = (2\pi/h) \cos(\alpha z_0) \left( \sum_{m=-\infty}^{\infty} \delta(\alpha-m\pi/2h) - \sum_{m=-\infty}^{\infty} \delta(\alpha-m\pi/h) \right) \]  

(2.21)

which may itself be simplified to

\[ I(\alpha, z_0) = (2\pi/h) \cos(\alpha z_0) \sum_{m=-\infty}^{\infty} \delta(\alpha-(2m+1)\pi/2h) \]  

(2.22)

When the \( \exp(-i\alpha z_0) \) term in equations (2.2) is replaced by this expression for \( I(\alpha, z_0) \), the \( \alpha \)-integration is straightforward and the Green's function of the reduced wave equation between a pressure release plane and a parallel rigid plane is

\[
\begin{align*}
P_G(R, R_0) &= \left\{ \begin{array}{c}
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(\alpha m z_0) \exp(\alpha m z) \\
(i\pi/h) \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(\alpha m z_0) \exp(\alpha m z)
\end{array} \right\} \\
\end{align*}
\]

(2.23)

in which \( \alpha_m = (2m+1)\pi/2h \), and \( \gamma_m = (k^2 - \alpha_m^2)^{1/2} \). This may be rewritten as

\[
\begin{align*}
P_G(R, R_0) &= \left\{ \begin{array}{c}
(2i\pi/h) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(\alpha m z_0) \cos(\alpha_m z) \\
(2i\pi/h) \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} J_n(\gamma_m r) H_n(\gamma_m r_0) \cos(\alpha m z_0) \cos(\alpha_m z)
\end{array} \right\} \\
\end{align*}
\]

(2.24)

Similarly, replacing the \( \exp(-i\alpha z_0) \) term in equation (2.4) by the expression for \( I(\alpha, z_0) \) in equation (2.22), integrating with respect to \( \alpha \) and rearranging the resulting infinite sum gives an alternative form of this Green's function, viz.,
3. **SOUND FIELD IN CHANNEL**

(a) **Mathematics**

The use of the Green's functions derived in Section 2 is illustrated by considering the problem of finding the pressure due to a time-harmonic source which is located in an axisymmetric channel containing water. Figure 2 shows the geometry. On the free surface at \( z=0 \) a Dirichlet boundary condition applies, and on the other boundaries defined by the surfaces \( z=0, r=a \) and \( r=b \), Neumann boundary conditions apply. The pressure in the channel may be written as the sum of the Green's function of Section 2(f), together with a scattering term which satisfies the homogeneous form of the wave equation (2.1) and the boundary conditions on the planes \( z=0 \) and \( z=h \). Thus

\[
p(r,\theta,z) = p_G(r,\theta,z) + p_S(r,\theta,z)
\]

in which \( p_G(r,\theta,z) \) is given by equation (2.24), and the scattering term is given by

\[
p_S(r,\theta,z) = \sum_{n=0}^{\infty} e_n \cos n \theta \sum_{m=0}^{\infty} \left\{ A_{mn} J_n(\gamma_m r) + B_{mn} H_n(\gamma_m r) \right\} \cos(\alpha_m z)
\]

The unknown coefficients \( A_{mn} \) and \( B_{mn} \) may be determined by imposing the Neumann boundary conditions \( \partial p(r,\theta,z) / \partial r = 0 \) on the cylindrical surfaces \( r=a \) and \( r=b \), to give the matrix relation

\[
\begin{bmatrix}
A_{mn} \\
B_{mn}
\end{bmatrix} = \left( -2\pi \cos(\alpha_m z_0) / hD_{mn} \right) \begin{bmatrix}
H'_n(\gamma_m b) & -H'_n(\gamma_m a) \\
-J'_n(\gamma_m b) & J'_n(\gamma_m a)
\end{bmatrix} \begin{bmatrix}
J_n(\gamma_m a)H_n(\gamma_m r_0) \\
J_n(\gamma_m b)H_n(\gamma_m r_0)
\end{bmatrix}
\]

\[
\begin{bmatrix}
2\times1 \\
2\times2 \\
2\times1
\end{bmatrix}
\]

where

\[
D_{mn} = J'_n(\gamma_m a)H'_n(\gamma_m b) - H'_n(\gamma_m a)J'_n(\gamma_m b)
\]

is the dispersion relation whose zeros \( \omega=\omega_{mn} \) are the natural frequencies of the system.

From a computational point of view it is shown below that the alternative form of the Green's function given by equation (2.25) is better suited to the numerical evaluation of equation (3.1).
(b) Convergence of Series

In order to investigate the convergence of the series in equations (2.24) and (3.2) it is necessary to consider the behaviour of the terms as \( m \to \infty \). In this case \( y_m \) is a monotonically increasing positive imaginary number \( ix_m \), say. The asymptotic forms of the Bessel functions

\[
J_n(ix_m) \sim \exp[x_m + i(n\pi/2 + \pi/4)](2i\pi x_m)^{1/2}
\]

(3.5)

\[
H_n(ix_m) \sim 2\exp[-x_m - i(n\pi/2 + \pi/4)]/(2i\pi x_m)^{1/2}
\]

are then used to obtain the asymptotic forms

\[
A_{mn} J_n(y_m r) \propto \exp[x_m (r-b)+x_m (r_0-b)]
\]

(3.6)

\[
B_{mn} H_n(y_m r) \propto \exp[x_m (a-r)+x_m (a-r_0)]
\]

(3.7)

of the coefficients in the series expansions of the scattered pressure, \( p_0(r, \phi, z) \). Clearly, unless \( r=r_0=b \), equation (3.6) decreases exponentially as \( x_m \to \infty \). Additionally, unless \( r=r_0=a \), equation (3.7) decreases exponentially as \( x_m \to \infty \). Thus, unless both source and receiver are situated on the inner or the outer cylindrical boundary, rapid convergence of the scattered pressure summation is assured.

If the same reasoning is applied to the terms in the Green's function expansion given by equation (2.24) it becomes evident that

\[
J_n(y_m r) H_n(y_m r_0) \propto \exp[x_m (r-r_0)]
\]

(3.8)

\[
J_n(y_m r_0) H_n(y_m r) \propto \exp[x_m (r_0-r)]
\]

(3.9)

Thus unless \( r=r_0 \) convergence is assured but at a rate which is less than that of the scattered term. When \( r=r_0 \) the series does not converge numerically - it exists only as a generalised function. The terms in the alternative form of the Green's function decay exponentially as \( x_m \to \infty \) unless both \( r=r_0 \) and \( \phi=0 \). Thus the alternative form of the Green's function is suitable for numerical evaluation everywhere except when the receiver is on a vertical line through the source.

The results of this Section are applicable to other problems of this kind. The main point that arises is that the alternative forms of the Green's functions should be used, where practical, when numerical computations are required.
4. NUMERICAL RESULTS

A Fortran program has been written to calculate the sound field due to a time harmonic source located in an axisymmetric channel. The first version of the program computed the Green’s function using equation (2.24). Severe numerical problems were encountered. When the alternative form of the Green’s function equation (2.25) was used in the computations rapid convergence was achieved, thus helping to validate the analysis of the previous Section.

In the plots of Figure 3 the following constants in SI units were used:

\[
\begin{align*}
  h &= 0.5 \\
  a &= 1.0 \\
  r_0 &= 1.5 \\
  z_0 &= 0.25
\end{align*}
\]
\[
\begin{align*}
  c &= 1500.0 \\
  b &= 2.0 \\
  r &= 1.5 \\
  z &= 0.25
\end{align*}
\]

Thus both source and receiver are located at midwater. The sound levels are in dB ref. 1 micropascal, and are to be referred to a free field source level of 120dB at 1m.

In Figure 3A the angular separation between source and receiver is \(\phi=90^\circ\). The sound level spectrum consists of numerical values at 8Hz spacing, which are joined by straight lines. Below the cut-on frequency of the first mode at 750Hz the sound pressure is negligible. Above this cut-on frequency the spectrum is dominated by very sharp resonant peaks whose levels would increase without bound if the resolution in frequency were increased. This is because of the absence of dissipation in the system. In Figure 3B the angular separation between source and receiver is the maximum of 180\(^\circ\). The feature which distinguishes this plot from the previous one is the increased density (factor of 2) of resonant peaks, simply due to the factor \(\cos(n\phi)\) in the summation which vanishes for odd values of \(n\) when \(\phi=90^\circ\).

5. CONCLUDING REMARKS

The Green’s functions in the region between parallel planes on which either Neumann or Dirichlet boundary conditions apply have been presented in a form which facilitates:

(i) Theoretical analysis of certain boundary value problems in finite axisymmetric regions.

(ii) Numerical evaluation of the sound fields without incurring problems of severe ill-conditioning.

A closed-form expression for the sound field in an axisymmetric channel and some numerical results have been obtained as an example. The availability of Green’s functions result in their application to practical problems. For example, James [9] has used the Green’s functions of equation (2.10) to investigate the
sound radiation from a simply supported elastic shell which is excited by an interior point source.

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REFERENCES


FIG. 1 GEOMETRY AND IMAGE SOURCE APPROXIMATION
FIG. 2 GEOMETRY OF AXISYMMETRIC CHANNEL
A. Source and receiver separated by 90°

B. Source and receiver separated by 180°

Fig. 3  Sound field in axisymmetric channel