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Investigation of Rolling-up and Interaction of Leading-edge 
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Abstract

The objective of this paper is to establish a simple two-
dimensional theoretical model in an attempt to use a computer to 
numerically simulate the experimental results of Hummel regarding 
the rolling-up and interaction of the leading-edge and trailing-
edge vortex sheets on a delta wing. It was found experimentally 
that when the leading vortex is present the trailing-edge vortex 
sheet will roll up another vortex downstream from the trailing-
edge. Furthermore, the circulation of the leading-edge vortex is 
opposite in direction to that of the trailing-edge vortex. The 
numerical results are in good agreement with the experimental 
pictures.

I. Introduction

One of the problems of major concern for researchers in 
aerodynamics and aircraft designers is the non-linear aerodynamic 
characteristics caused by the separation of body and wing 
vortices of the aircraft at large angles of attack. Effective 
utilization of the additional lift generated by body and wing 
vortices can improve the aerodynamic properties of the aircraft 
and increase the maneuverability. To study the mechanism of
formation of body and wing vortices and the complicated interaction between various vortex systems as well as between vortices and the aircraft in detail is the key to the accurate estimation of various non-linear force and torque terms on the aircraft. Therefore, the study of vortex motion has important practical values.

The study of the leading-edge vortex of a slender delta wing began in the forties and fifties. There are significant advances in recent years. In addition to measuring force and pressure, recent experimental studies focused on the application of display technique to the flow field as well as on the detection of fine details of the spatial flow field. Based on the "contours" of total pressure, static pressure and dynamic pressure measured, as well as on the spatial distribution of the flow direction, we can have a more direct and profound understanding of the vortex flow field.

In the early stage, the theoretical study of leading-edge vortex was based on the conic flow assumption which simplified a three-dimensional flow problem to a two-dimensional problem on a transverse plane, including the work done by C.E. Brown and W.H. Michael\cite{1}, K.W. Mangler and J.H.B. Smith\cite{2}, and the later improvement made by J.H.B. Smith\cite{3}. The Smith model divides the vortex layer into two points. The outer part uses a broken line section to replace the vortex layer. The inner part uses a concentrated vortex to represent the core and a "vortex transport line" to connect inner and outer regions. The model could be used to obtain the shape of the vortex layer, and the strength
and intensity of the core. However, the accuracy is not desirable. After computers are extensively used, a vortex lattice method with leading-edge separation vortex, introduced by C.M. Belotserkovskiy [5] and O.A. Kandil, D.T. Mook and A.H. Nayfeh [6], is a representative method. The leading-edge vortex layer is replaced by several discrete vortex threads. Through iterations, the positions of free vortex threads are determined. The boundary conditions on the wing surface are also simultaneously satisfied. In order to accurately calculate the load distribution on the wing, P.E. Rubbert et al [7] introduced the "free vortex layer" method by using higher order surface elements. It can be used to calculate the shape of non-conical flow fields and vortex layers, as well as the load distribution on the entire wing.

All the experimental and theoretical studies discussed above are focused on the rolling-up of the leading-edge vortex layer, the force and torque characteristics, and the calculation of load distribution. It seems that there is little work done on the development of leading-edge vortex downstream from the trailing-edge and the interaction between leading-edge and trailing-edge vortices. We are very much intrigued by the work done by D. Hummel [8]. Hummel performed a series of fine manuscripts. In particular, he did an experimental study of the interaction between leading-edge and trailing-edge vortices. From his measured total pressure, static pressure and spatial flow direction distribution, we can see that
two spiral vortices are gradually formed downstream from the trailing-edge. One is the leading-edge vortex and the other is the vortex rolled-up by the trailing vortex layer. The circulations of these two vortices are opposite in direction. A schematic diagram of the flow pattern is shown in Figure 1.

Figure 1. Schematic Diagram for the Formation of Downstream Vortices of a Slender Delta Wing

Inspired by Hummel's experiment results, we attempted to establish a simple theoretical model to simulate Hummel's results numerically on a computer. This study will benefit the understanding of the structure of a down wash flow field.

II. Theoretical Analysis

In order to study the interaction between leading-edge and trailing-edge vortices, we must first obtain the shape, position and strength of the rolling-up of the leading-edge at the trailing-edge. In addition, we must also have the intensity distribution of the trailing-edge vortex, i.e., the vortex
intensity, or spanwise circulation, distribution on the wing. Figures 2 and 3 show the pressure distribution on the wing surface and the vortex line shape measured by Hummel. From the figures, one can see that the surface pressure distribution and the vortex line are essentially different from those obtained based on the linearized slender wing theory of Jones due to the presence of the leading-edge vortex. However, as compared to Smith's\(^{[3]}\) theory, the shape of the pressure distribution, the position of the suction peak and the shape of the vortex are qualitatively similar. However, there are some differences quantitatively. In other words, as a preliminary theoretical investigation, a two-dimensional model can reflect the major characteristics of the flow field. But, we did not choose Smith's vortex layer model. Instead, a simpler two-dimensional unsteady flow analogy was used. Our theoretical model was built based on a discrete vortex method, which does not require iterations to solve a set of non-linear equations.
Figure 2. Pressure Distribution on Delta Wing $A=1$, $a=20.5^\circ$

1. experimental

Figure 3. Adhered Vortex Vector (right) and Adhered Vortex Line (left)
It is an initial value problem of a series of normal differential equations. It is not limited to the "conic flow" assumption which will facilitate the extension to more complicated airfoils. It also facilitates the further consideration of "secondary vortex" separation problems.

Based on the two-dimensional unsteady flow analogy, the three-dimensional flow of a delta wing with an attack angle can be considered as an unsteady flow around a two-dimensional plate in the x plane. The width of the plate at any time corresponds to the wing span in the x position. When the flow passes the edge of the plate, the boundary layer is separated. A free shear layer is formed due to the velocity difference between the upper and lower surface. Based on existing studies, it is known that, as long as it does not involve the mechanism of separation (which is a viscosity effect), the free shear layer after separation can be assumed as an inviscid vortex layer. As a next step, the vortex layer is replaced by several discrete point vortices. Therefore, the flow around the plate satisfies Laplace equation.

\[
\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0
\]  \hspace{1cm} (1)

From this point on, the plate can be transformed into a circle by using a complex function method. Hence, the mathematics of the problem becomes a flow around a cylinder with a finite number of point vortices outside the circle. In this case, the complex potential expression of the flow is:
Fig. 4: Conformal mapping definition
\[ W(\zeta) = \frac{-i}{2} \left| V_\infty \right| \sin \alpha \left( \zeta - \frac{\zeta^2}{2} \right) - \sum_{i=1}^{n} \frac{i \sqrt{1 + 2 \pi \ln \left( \frac{\zeta - \xi_i}{\zeta + \xi_i} \right)}}{(\zeta + \xi_i)(\zeta - \xi_i)} \]  

where the first term is the complex potential of a uniform incident flow around a cylinder and the second term is the complex potential generated by the finite number of point vortices and their image vortices outside the circle. The boundary conditions are automatically satisfied on the circle. The conformal mapping is

\[ x = \frac{1}{2} \left( \zeta + \frac{\zeta^2}{\zeta} \right) \] (3a)

\[ \zeta = x + \sqrt{x - \frac{r^2}{2}} \] (3b)

It should be noted that in this transformation there is a magnifying factor \( \frac{dX}{d\zeta} \bigg|_\infty = k \), at infinity. Therefore, \( U'_\infty = \frac{1}{2} V_\infty \sin \alpha \).

The diameter of the circle is the width of the plate.

In addition to surface boundary conditions, the Kutta condition must also be satisfied.

\[ \frac{dW}{d\zeta} \bigg|_{\zeta = \infty} = \frac{dW}{dX} \bigg|_{X = \infty} \] (4)

Because \( d\zeta/dX \rightarrow \infty \) at the edge of the wing, therefore, we must have \( dW(\zeta)/d\zeta = 0 \). Hence, we get

\[ \sum_{i=1}^{n} \frac{p_i}{2\pi} \left( \frac{\xi_i}{\xi_i - r} + \frac{\xi_i}{\xi_i + r} \right) = \frac{1}{2} \left| V_\infty \right| \sin \alpha \] (5)

The point vortex moves downstream at the local velocity. Therefore, we must also find the velocity of every point vortex.
If the velocity of the kth vortex is expressed as:

\[ v_k = -iw_k - \frac{dW(X)}{dX} \bigg|_{x=x_k} + \frac{d}{dX} \left[ \frac{iT_k}{2\pi} \ln(X - X_k) \right] \bigg|_{x=x_k}, \quad k = 1, 2, \ldots, N. \quad (6) \]

where the second term on the right indicates that the inducing velocity of the kth vortex itself should be subtracted from the calculation of the velocity at the kth vortex because it is a velocity singular point. After using equations (2) and (3) to calculate, equation (6) becomes

\[ v_k = -iw_k - (G_i + G_r + G_o) \frac{dX}{dx} \bigg|_{x_k} + G_o \quad (7) \]

where

\[ G_i = -\frac{i}{2} |V_m| \sin \left(1 + \frac{r}{\zeta_i}\right), \]

\[ G_r = -\sum_{j=1}^{N} \frac{iT_j}{2\pi} \frac{1}{\zeta_k - \zeta_j}, \]

\[ G_o = -\sum_{j=1}^{N} \frac{iT_j}{2\pi} \left( \frac{1}{\zeta_k + \zeta_j} - \frac{1}{\zeta_k - \zeta_j} - \frac{1}{\zeta_k - \zeta_j} \right), \]

\[ G_o = -\frac{is\sqrt{k}}{2\pi} \frac{i}{\zeta_i} \left(1 - \frac{r}{\zeta_i}\right)^2 \frac{dX}{dx} \bigg|_{x_k} = \frac{2}{1 - \frac{r}{\zeta_i}}. \]

Intensity and Position of Newly Generated Leading-edge Vortex.

In the unsteady flow analogy, the leading-edge vortex is approximated by many discrete vortices. Newly generated vortices continue to be separated from the edge of the wing into the flow field with time. Therefore, the number of vortices continues to increase in the flow field. The intensity of newly generated vortex has a great effect on the shape and position of the rolling-up of the leading-edge vortex and the surface pressure distribution. Many authors have investigated this problem.
The variation of the vortex flow in the shear layer near a leading-edge separation point with time is:

\[ \frac{\partial \gamma}{\partial t} = \frac{1}{2} (V_{\text{down}} - V_{\text{up}}) \]  

(8)

where \( V_{\text{up}} \) and \( V_{\text{down}} \) represent the velocities at the upper and lower surface of the shear layer near a separation point.

According to Sacks\(^4\) method, let \( V_s = \frac{1}{2} (V_{\text{up}} + V_{\text{down}}) \), i.e., the average velocity of the shear layer which is also the velocity at which the shear layer is dragged out the edge of the wing. \( \gamma_x = V_{\text{down}} - V_{\text{up}} \) is the vortex intensity on a unit length. Based on these, equation (8) can be re-written as:

\[ \Delta \gamma = \Delta t \cdot (V_s \gamma_x) = (V_s \Delta t) \cdot \gamma_x = \Delta s \cdot \gamma_x \]  

(9)

\( \Delta s \) is the length of the shear layer dragged out in \( \Delta t \) time. The expression for \( V_s \) is found to be

\[ V_s = \frac{1}{2} \sum_{i=1}^{n} \frac{\epsilon_i}{2\pi} \left[ \frac{x_i}{(\xi_i - x)^2} - \frac{x_i}{(\xi_i - x)^4} \right] \]  

(10)

Because of symmetry, \( w_s \) is equal to zero in practice. There is only a \( y \)-direction velocity which shows that the shear layer is dragged out tangentially from the leading-edge. In our computation, \( V_s \) was found with equation (10) in order to determine the vortex layer length \( \Delta s \). Then, the simplified point vortex is centered in this section of vortex layer so that the intensity of the newly generated vortex could be determined by using the Kutta condition.

Surface Pressure Distribution and Attached Vortex Line.

From the definition of the pressure coefficient \( c_p = \frac{P - P_\infty}{\frac{1}{2} \rho U_\infty^2} \), we get
where $\varphi_y = v$, $\varphi_x = w$, $\varphi = R.P.\{W\}$. The calculation of $\varphi_x$ must take two aspects into account. One is that the semi-wing span $s(x)$ is a function of $x$. The variation caused by $s(x)$ is $\varphi_x = ds(x)/dx. R.P.\delta W(X)/\delta s$. The other is $\varphi_x$ caused by the variation of the point vortices with $x$. The derivation of the entire formula is tedious. It is omitted here. On the wing surface, $\varphi_x = 0$. The attached vortex line on the wing surface can be determined based on the following

$$\gamma = n_x (V_{up} - V_{down})$$

(12)

$\gamma$ is the attached vortex vector on the wing surface. Let $\gamma_x$ and $\gamma_y$ be the components of $\gamma$ in the $x$ and $y$ directions, respectively. $\gamma = \gamma_x i + \gamma_y j$. Then, $\gamma_x = -(\varphi_{yup} - \varphi_{ydown})$, $\gamma_y = \varphi_{xup} - \varphi_{xdown}$, where $\varphi_{xup}$, $\varphi_{xdown}$, $\varphi_{yup}$, and $\varphi_{ydown}$ are the velocity components on the upper and lower wing surface.

After the attached vortex line on the wing surface is found, it is very easy to obtain the intensity of the trailing-edge tail vortex. Due to the fact that $\gamma_x = -\partial \gamma / \partial y$, the intensity of the $i$th tail vortex as a result of trailing-edge discretization is

$$(\Delta \gamma_i)_{\text{trailing-edge}} = \int_{y_i}^{y_{i+1}} (\gamma_x)_{\text{trailing-edge}} dy$$

(13)

III. Brief Description of Results of Computation

Mathematically, this computation is to solve a variable number of first order normal differential equations. We can use
the Runge-Kutta method to calculate gradually from the apex of the wing downstream. With each increasing step, a new vortex is generated,

Figure 5. Rolling-up of Leading-edge Vortex on Top of Wing Surface

Figure 6. Surface Pressure Distribution and Attached Vortex Distribution (Trailing-edge of the Wing)
resulting in two additional equations. When calculating the trailing-edge, the intensity of the trailing-edge vortex is found and combined into the original equation to be moved downstream.

In order to compare with the experiment, we calculated a delta wing whose aspect ratio \( A = 1.0 \) and attack angle \( \alpha = 20.5^\circ \). Figure 5 shows the gradual rolling-up of the leading-edge vortex over the upper wing surface. Figure 6 shows the pressure distribution and attached vortex intensity distribution at the trailing-edge. The pressure distribution is very close to that calculated by Smith. However, it is different from the experimental result (see Figure 2). From the experimental result one can see that the flow in the front part of the delta wing approaches the conical flow assumption. However, the rear part, especially near the trailing-edge, is no longer a conical flow. The suction peak decreases with increasing \( x \). But, this tendency cannot be calculated using a two-dimensional model. This is because the two-dimensional model does not meet the trailing-edge Kutta condition. Although the load distribution on the wing surface can be more accurately calculated based on a three-dimensional flow model using a higher order surface element "free vortex layer" method currently under development, yet it takes too much computing time. As a qualitative analysis, we chose the two-dimensional model.

From the distribution of attached vortex intensity along the span \( 
\gamma_x \) one can see that \( \gamma_x \) is negative over most of the wing span. It is positive near the edge of the wing. In addition,
under the leading-edge vortex, $\gamma_x$ has a negative maximum. According to the calculated result reported in reference [9], a vortex will be rolled-up at the extremum $|\gamma_x|$. When $\gamma_x$ is negative, the vortex rolled-up is clockwise.

Figure 7 shows the rolling-up of leading-edge and trailing-edge vortices and their interaction. For comparison, Hummel's experimental results are again shown in Figure 8. From the figures, the two situations are quite similar. In Figure 7(a), the trailing-edge vortex layer already begins to fluctuate. It bulges slightly at the extremum $|\gamma_x|$ and develops downstream. On one hand, it continues to bulge and enlarge and gradually rolls up into a clockwise vortex. On the other hand, because of the side wash velocity effect induced by the leading-edge vortex, the trailing-edge vortex layer extends in the direction of the wing edge. The vortex rolled up by the trailing-edge moves outward. It is initially on the right lower side of the leading-edge vortex and then gradually rises. From the figure one can also see that the trailing-edge vortex begins to roll up at approximately 1/4 of a wing span ($x/c_r=1.10$) from the trailing-edge. At 1/2 wing span ($x/c_r=1.20$), it has already developed well.
In order to study the effect of the leading-edge vortex on the trailing-edge vortex, we also did two interesting numerical experiments. Figure 9 shows the effect of the leading-edge vortex layer. We artificially neglected the vortex layer and consolidated the leading-edge as a point vortex. The consolidation is based on the conservation of vortex moment and circulation. In the Figure, the symbol ▲ represents the consolidated leading-edge vortex. We found in the Figure that the trailing-edge vortex could also roll-up a vortex. However, the shape and position are quite different from those shown in Figures 7 and 8. Thus, the effect of the leading-edge vortex layer cannot be neglected.
Figure 8. Hummel's Experimental Results [3]

Figure 9. Rolling-up of Trailing-edge Vortex When Leading-edge Vortex is Consolidated into a Point Vortex
Figure 10 shows the rolling-up of the trailing-edge vortex when neglecting leading-edge vortex. Just as expected, a clockwise vortex is rolled-up at the extremum $|\gamma_x|$. A counterclockwise vortex is rolled-up at the wing tip. Because the $\gamma_x$ value is very small at the wing tip, only a small vortex is rolled-up. In addition, the number of point vortices is not sufficient to see clearly. Because of the absence of the leading-edge vortex side wash velocity effect, the trailing-edge vortex layer extends slowly. The clockwise vortex is on the inside of the wing tip vortex.
IV. Conclusions

In this work, a two-dimensional discrete vortex model was established based on the two-dimensional unsteady analogy. Numerical simulation of Hummel's experimental results was realized on a computer. The rolling-up of the leading-edge and trailing-edge vortices and the results of their interaction thus obtained are very similar to Hummel's experimental results. It proves that it is basically feasible to study the mechanism using a two-dimensional model. Major physical pictures of the flow field can be obtained.

1. In addition to the vortex rolled up by the leading-edge vortex downstream, the trailing-edge vortex will roll up another vortex. The circulations of these two vortices are opposite.

2. Under the influence of the side wash velocity induced by the leading-edge vortex, the trailing-edge vortex layer extends toward the edge of the wing. Initially, a vortex is rolled up on the lower right side of the leading-edge vortex. Then, as the circulation gradually increases, it rises comparatively. The presence of the leading-edge vortex accelerates the rolling-up process of the trailing-edge vortex and also pulls it outward.

3. The wash flow field is complicated where there are leading-edge and trailing-edge vortices present. There is a need to understand it better. This study has helped the understanding of the physical picture of the wash flow field. However, because of the characteristic deficiencies of the two-dimensional model, there are discrepancies in the quantitative determination of the
pressure distribution on the wing surface. A more complex three-dimensional vortex layer model must be used to more accurately calculate the pressure distribution on the wing surface. This work is just a preliminary investigation.

After this paper was sent for review, we discovered two similar studies done abroad. Kandil[10] used a vortex lattice method to calculate a three-dimensional flow field. But, the structure of the rolled-up vortex layer is not clear. The work done by Hoeijmakers[1,2] et al is similar to ours. They also used a two-dimensional vortex layer model to obtain similar results. However, the methodology is not quite the same. On the wing surface and in the vortex layer, they used dipole distribution, vortex layer shape and wing surface dipole strength distribution and solved them by iteration. We established a series of point vortex equations through conformal mapping to convert it to a problem of solving for initial values.

References
Abstract

Hummel's experiment on the rolling-up and interaction of the leading-edge and trailing-edge vortex sheets at slender delta wing is modeled numerically by a simple two-dimensional theory. The numerical results show that the trailing-edge vortex sheet will roll-up at the downstream of the wing, in the presence of the leading-edge vortex, and the direction of the circulation of the leading-edge vortex is opposite to the trailing-edge vortex. The numerical results are in good agreement with the experiment. This work is important to further understand of the downstream flow-field of a wing.
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