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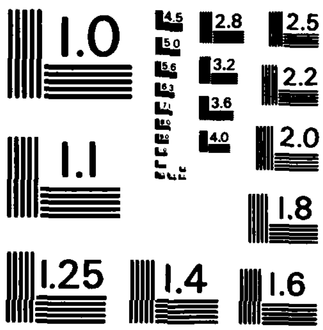

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REPORT SD-TR-85-77

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AD-A162 542

Control of Missile Dispersion Via Roll Rate Modulation

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15 November 1985

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED

Prepared for

U.S. AIR FORCE BALLISTIC MISSILE OFFICE
Norton Air Force Base, CA 92409
and
DEFENSE NUCLEAR AGENCY
Washington, DC 20305

SPACE DIVISION
AIR FORCE SYSTEMS COMMAND
Los Angeles Air Force Station
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This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-85-C-0086 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by W. P. Thompson, Director, Aerophysics Laboratory. Capt David Thyfault/YNSA was the project officer.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A162 542

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SD-TR-85-77	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CONTROL OF MISSILE DISPERSION VIA ROLL RATE MODULATION		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER TR-0086(6550)-1
7. AUTHOR(s) Daniel H. Platus and Jennifer Shandling		8. CONTRACT OR GRANT NUMBER(s) F04701-85-C-0086
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, Calif. 90245		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Ballistic Missile Office Norton Air Force Base, CA 92409 Defense Nuclear Agency Washington, DC 20305		12. REPORT DATE 15 November 1985 13. NUMBER OF PAGES 48
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Space Division Los Angeles Air Force Station Los Angeles, Calif. 90009-2960		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Missile Aerodynamics
Missile Control
Missile Dispersion
Roll Control
Roll Modulation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Feedback control via roll rate modulation to limit cross-range dispersion of spinning missiles is investigated. The dispersion to be controlled is assumed to result from lift variations that do not average out over a roll cycle when the missile roll rate remains constant. By modulating the roll rate in a prescribed manner through feedback control, lift nonaveraging and the resulting cross-range dispersion are minimized. Two different approaches to the feedback control are investigated. In one approach, the

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

roll rate is modulated by feedback of the state variables to prevent the occurrence of lateral velocity increments that produce dispersion. In the other approach, the roll rate is modulated harmonically about a steady roll rate, and the amplitude of the harmonic variation is controlled to limit dispersion. [illegible] angle of [illegible] equations [illegible] linear and nonlinear differential equations; Time [illegible]; [illegible]; charts;

PREFACE

The work reported herein by the principal author, J. Shandling, was performed at The Aerospace Corporation under the Massachusetts Institute of Technology (MIT) Engineering Internship Program. This work was in partial fulfillment to meet her requirements for the master of science degree in mechanical engineering at MIT. The authors are grateful to N. Thyson of Avco Systems Division for suggesting the possibility of dispersion control by roll rate modulation and to Prof. W. Vander Velde of MIT for suggesting the harmonic roll modulation approach. This work was jointly sponsored by the U.S. Air Force Ballistic Missile Office (BMO) and the Defense Nuclear Agency (DNA). Capt. S. W. Opel, BMO/MYES, was technical monitor for the Ballistic Missile Office and B. Gillis, DNA/SPAS, was the technical monitor for the Defense Nuclear Agency.

CONTENTS

FIGURES

FIGURES (Continued)

TABLES

I. INTRODUCTION

Roll-lift dispersion of spinning missiles is caused by nonaveraging of the lift vector in the presence of moment disturbances that perturb the angle of attack.[1] Lift nonaveraging can occur either from angle-of-attack variations at constant lift precession rate or from roll rate variations for a trimmed missile with constant lift.[2] Control of lift nonaveraging should therefore be possible by controlling either the lift variations, the lift vector precession rate, or a combination of both. The latter method was investigated to control the dispersion of an untrimmed missile in which both angle-of-attack and lift precession rate feedback were used in the control law.[3] In another investigation to control dispersion of a rolling trimmed missile, angle of attack and angle-of-attack rate feedbacks were applied to pitch and yaw control moments at constant roll rate.[4]

In this report we consider only roll rate modulation about a steady roll rate to control lift nonaveraging. Roll modulation complicates the analysis because the missile equations for complex angle-of-attack motion contain roll rate terms in the coefficients. The equations are linear for constant roll rate but become nonlinear when the roll rate is time dependent. The magnitude of the roll coupling depends on the coordinate system used to describe the complex angle-of-attack motion. Both body-fixed and aeroballistic coordinates are used to investigate different feedback controls. The advantages of the different coordinate systems are described.

II. ANALYSIS

A. FORMULATION

The equations of motion of a spinning missile can be expressed in different coordinate systems. Shown herein are the differential equations for the complex angle-of-attack motion in body fixed coordinates, Eq. (1), and aeroballistic coordinates, Eq. (2),

$$\ddot{\delta} + [\nu + ip\,(2 - \mu)]\dot{\delta} + \left[\omega^2 - p^2(1 - \mu) + i\left(\dot{p} + p(\nu - \nu_m)\right)\right]\delta = im_t \tag{1}$$

$$\ddot{\xi} + [\nu - i\mu p]\dot{\xi} + \left[\omega^2 - ip\nu_m\right] = im_t e^{i\int pdt} \tag{2}$$

for small angle motion. The complex angles of attack in these two coordinate systems are related by

$$\xi = \delta\, e^{i\Phi} \tag{3}$$

where Φ is the roll angle in inertial space, defined by

$$\Phi = \int pdt \tag{4}$$

and m_t represents pitch and yaw moment disturbances about the y and z axes divided by the pitch or yaw moment of inertia,

$$m_t = m_y + im_z = \frac{M_y + iM_z}{I} \tag{5}$$

Disturbance moments can occur from various sources. During the reentry phase of a ballistic missile, asymmetries occur from ablation shape change, which can cause lift variations leading to dispersion errors. The lift forces do not average out for each roll revolution giving rise to a net force perpendicular to the flight path. The sideways motion produced is called the transverse velocity and can be calculated by integrating the missile acceleration in the cross plane. By assuming the missile to be a point mass in this plane under the action of a rotating lift force, the transverse velocity, $V = v + iw$, is given by

$$V = V(0) - \frac{L_\theta}{m} \int_0^t \delta e^{i\int p dt} dt \tag{6}$$

$$V = V(0) - \frac{L_\theta}{m} \int_0^t \xi dt \tag{7}$$

where Eq. (6) is in body fixed coordinates and Eq. (7) is in aeroballistic coordinates. The lift force derivative L_θ is in general slowly varying and assumed constant over the time duration of the disturbances.

The nonaveraging lift effects can be compensated for by implementing roll control. The roll acceleration is defined by

$$\dot{p} = \frac{\ell_x + \ell_c}{I_x} \tag{8}$$

where ℓ_x is the roll disturbance moment and ℓ_c is the control moment.

The transverse velocity is a measurement of missile dispersion and is used as the cost function. There are two additional quantities which are desirable to minimize the magnitude of the applied control moment and the deviation of the roll rate from a specific value. The complete cost function is

$$I + |V_{\substack{steady\\state}}|^2 + W_c \int_0^t \ell_c^2 dt + W_p \int_0^t (p - p_o)^2 dt \tag{9}$$

where W_c and W_p are suitable weighting functions.

The second-order differential equation for the complex angle of attack, Eqs. (1) or (2) involves coefficients that are functions of the roll rate p. A nonconstant roll rate gives rise to nonlinear terms.

B. STATE VARIABLE FEEDBACK

In this approach, the control moment ℓ_c is assumed to be a linear function of the state variables of the form

$$\frac{\ell_c}{I_x} = A\Delta\alpha + \beta\Delta\beta + C\dot{\alpha} + D\dot{\beta} + E\int\alpha + F\int\beta \tag{10}$$

The equations of motion are linearized in the body fixed coordinates by taking small perturbations δ_+, p_+ about quasisteady values δ_o, p_o

$$\delta = \delta_o + \delta_+ \tag{11}$$

$$p = p_o + p_+ \tag{12}$$

and neglecting higher order terms.

The resulting linear control equations in terms of the perturbations are

$$\ddot{\delta}_+ + [\nu + ip_o(2 - \mu)]\dot{\delta}_+ + [\omega^2 - p_o{}^2(1 - \mu)$$

$$+ ip_o(\nu - \nu_m)]\delta_+ + \left[2p_o p_+(1 - \mu)\right.$$

$$\left. + i\left(\dot{p}_+ + p_+(\nu - \nu_m)\right)\right]\delta_o = im_t \tag{13}$$

$$\dot{p} = A\alpha_+ + B\beta_+ + C\dot{\alpha}_+ + D\dot{\beta}_+ + E\int\alpha_+ + F\int\beta_+ \tag{14}$$

Roll moment disturbances are assumed zero. The transverse velocity is expressed as

$$V(t) = V(0) - \frac{L_\theta}{p_o m}\int_o^t (\delta_o + \delta_+)e^{i\tau}\, e^{i\lambda}d\tau \tag{15}$$

where we have defined λ

$$\lambda = \int_0^\tau \frac{p}{p_o} d\tau \tag{16}$$

and changed to a nondimensional roll angle variable τ defined by

$$d\tau = p_o dt \tag{17}$$

We assume a steady-state condition defined by the initial conditions $p = p_o$ and $\delta = \delta_o$ at $t = 0$. Assuming $\mu = \nu = 0$ and neglecting smaller terms, the equation for the angle of attack is, with respect to τ, where $d/dt = p_o\ d/d\tau$,

$$p_o^2 \frac{d^2(\delta_+)}{d\tau^2} + 2\ ip_o^2 \frac{d(\delta_+)}{d\tau} + \omega^2\delta_+ = im_t \tag{18}$$

The Laplace transformation of Eqs. (14) and (18) results in

$$\delta_+(s) = \beta(s) + i\alpha(s)$$

$$= \frac{(im_\tau)\ \left[\left(s^2 + \omega^2/p_o^2\right) - 2\ is\right]}{p_o^2\left[\left(s^2 + \omega^2/p_o^2\right)^2 + 4\ s^2\right]} \tag{19}$$

$$p(s) = \frac{p_o sA + p_o^2 s^2 C + E}{p_o^2 s^2}\ \alpha(s)$$

$$+ \frac{p_o sB + p_o^2 s^2 D + F}{p_o^2 s^2}\ \beta(s) \tag{20}$$

Also in the Laplace transform domain is

$$\lambda(s) = \frac{p(s)}{p_o s} \tag{21}$$

The transverse velocity can be evaluated in terms of $\delta(s)$ and $p(s)$ by the following procedure: The upper limit of the integral in Eq. (15) is taken to be sufficiently large to include the time duration of the perturbations. Without loss of generality this limit can be taken as ∞, and Eq. (15) can be written in the form

$$V = V(0) - \frac{L_\theta}{p_o m} \int_o^\infty F(\tau)\, e^{i\tau} d\tau \tag{22}$$

where

$$F(\tau) = (\delta_o + \delta_+)\, e^{i\lambda(\tau)} \tag{23}$$

From the definition of the Laplace transform

$$F(s) = \mathscr{L}[F(\tau)] \equiv \int_o^\infty F(\tau)\, e^{-s\tau} d\tau \tag{24}$$

the integral in Eq. (22) is observed to be the Laplace transform of $F(\tau)$ with $s = -i$, or

$$V = V(0) - \frac{L_\theta}{p_o m} [F(s)]_{s=-i} \tag{25}$$

where $F(s)$ is obtained in terms of $\delta(s)$ and $\lambda(s)$. By expanding the exponential in Eq. (23), $F(\tau)$ can be approximated by

$$\begin{aligned} F(\tau) &= \delta_o + \delta_+(\tau) + i\delta_o(t) \\ &\quad + i\delta_+(\tau)\lambda(\tau) - \frac{\delta_o \lambda^2(\tau)}{2} + \dots \end{aligned} \tag{26}$$

which has the Laplace transform with respect to τ

$$\begin{aligned} F(s) &= \frac{\delta_o}{s} + \delta(s) + i\delta_o\lambda(s) + i\mathscr{L}\{\delta(\tau)\lambda(\tau)\} \\ &\quad - \frac{\delta_o}{2}\mathscr{L}\{\lambda^2(\tau)\} + \dots \end{aligned} \tag{27}$$

Eq. (22) can now be written as

$$V = V(0) - \frac{L_\theta}{p_o m} [\frac{\delta_o}{s} + \delta(s) + i\delta_o\lambda(s) + i\mathscr{L}\{\delta(\tau)\lambda(\tau)\}$$

$$- \frac{\delta_o}{2} \mathscr{L}\{\lambda^2(\tau)\} + \ldots]_{s=-i} \tag{28}$$

The velocity caused by the initial angle of attack averages out for each cycle and is expressed by

$$V = \frac{-L_\theta}{p_o m} [\frac{\delta_o}{s}]_{s=-i} \tag{29}$$

The velocity which causes dispersion is then the increment

$$\Delta V = \frac{-L_\theta}{p_o m} [\delta(s) + i\delta_o\lambda(s) + i\,\mathscr{L}\{\delta(\tau)\lambda(\tau)\}$$

$$- \frac{\delta_o}{2} \mathscr{L}\{\lambda^2(\tau)\} + \ldots]_{s=-i} \tag{30}$$

1. SOLUTION FOR CONTROL

In order to compensate for the lift force asymmetry caused by pitch and yaw moment disturbances, the roll rate is adjusted throughout the cycle. For example, the roll rate is decreased when the lift force is in the direction opposing that of the dispersion. The necessary roll motion is found from Eq. (30) by setting ΔV equal to zero

$$\Delta V = \frac{L_\theta}{p_o m} [F(s)]_{s=-i} = 0 \tag{31}$$

where

$$F(s) = \delta(s) + i\delta_o\lambda(s) + i\,\mathscr{L}\{\delta(\tau)\lambda(\tau)\}$$

$$- \frac{\delta_o}{2} \mathscr{L}\{\lambda^2(\tau)\} + \ldots$$

The solution is the feedbacks that cause the real and imaginary components of $F(s)$ to vanish when $s = -i$

$$F(-i) = 0 \tag{32}$$

Examined first is the case where there is an initial nonzero angle of attack, δ_o. This is caused by an existing asymmetry, either built-in or previously caused during flight. The lift vector produced from the initial angle of attack is of constant magnitude and precesses in space at the roll rate frequency. With a constant body-fixed angle of attack, the lift force averages and the missile executes a spiral motion about the mean flight path.

A first-order approximation to $F(s)$ is

$$F_1(s) = \delta(s) + i\delta_o\lambda(s) \tag{33}$$

The complete expression to determine the gains for an arbitrary moment disturbance, m_t, is obtained by substituting Eqs. (19) and (21) into Eq. (33) and setting $F_1(-i) = 0$ which yields

$$\frac{m_t}{\delta_o} = [M]\ \{[G] + i[H]\}[K] \tag{34}$$

where

$$[M] = [m_y \quad m_z]$$

$$[G] = \frac{(\sigma + 1)}{p_o^3\{(\sigma - 1)^2 - 4\}} \begin{bmatrix} p_o(\sigma-1) & 0 & 0 & -2p_o^2 & 0 & 2 \\ 0 & -p_o(\sigma-1) & -2p_o^2 & 0 & 2 & 0 \end{bmatrix}$$

$$[H] = \frac{(\sigma + 1)}{p_o^3\{(\sigma - 1)^2 - 4\}} \begin{bmatrix} 0 & -2p & -p_o^2(\sigma-1) & 0 & (\sigma-1) & 0 \\ -2p_o & 0 & 0 & p_o^2(\sigma-1) & 0 & -(\sigma-1) \end{bmatrix}$$

$$[K]^T = [A\ B\ C\ D\ E\ F]$$

The magnitude of the control moment must be minimized. Examination of the solution reveals that the gains are inversely proportional to the initial angle of attack. The lift force resulting from the angle of attack provides the muscle needed to control dispersion. For a very small angle of attack the control moment might be too large and cause large roll rates. The assumption, $p_o \gg p_+$ must be upheld. Also, roll rates of great magnitudes are not practical.

Three types of control are examined: derivative, proportional, integral. The expressions for the gains are shown in Table 1 for the case where $\delta_o = i\alpha_o$. The magnitude of the control moment is the product of the gain, the feedback variable, and the roll moment of inertia.

By neglecting the higher order terms in Eq. (30), we find the feedbacks to be independent of the type of disturbance (i.e., impulse, step). However, the direction of the disturbance is important because linear superposition principles do not hold. For an arbitrary moment disturbance with components in both the y and z directions, two nonzero feedbacks are required to satisfy Eq. (34). They are dependent on the ratio of the directional components of the disturbance. For a disturbance about one axis, only a single feedback is required. The feedback variable is the same for either case, m_y or m_z. The gain depends on the direction of the disturbance but is independent of its magnitude.

An initial angle of attack α_o requires the nonzero gain to be B, C, or E for a disturbance either in the y or z direction. If the disturbance has components in both directions, A, D, or F must also be nonzero. These results are summarized for proportional control with $\delta_o = i\alpha_o$

Case I - $m_t = m_y$

$$\dot{p}_1 = B_1 \beta$$

$$B_1 = \frac{p_o^{\,2}[(\sigma - 1)^2 - 4]}{2\alpha_o(\sigma + 1)}$$

Table 1. Gains for $\delta_o = i\alpha_o$

Moment Direction	Control Type: Derivative $\dot{p} = C\dot{\alpha}$	Proportional $\dot{p} = B\beta_+$	Integral $\dot{p} = E\int\alpha_+$
m_y	1	$\frac{p_o(\sigma - 1)}{2}$	$-p_o^2$
m_z	$\frac{-(\sigma - 1)}{2}$	$-p_o$	$\frac{p_o^2(\sigma - 1)}{2}$

Gains = k × (expression from table)

$$k = \frac{p_o[(\sigma - 1)^2 - 4]}{\alpha_o(\sigma^2 - 1)}$$

Case II - $m_t = im_z$

$$\dot{p}_2 = B_2\beta$$

$$B_2 = \frac{-p_o^{\,2}[(\sigma - 1)^2 - 4]}{\alpha_o(\sigma + 1)(\sigma - 1)}$$

Case III - $m_t = m_y + im_z$

$$\dot{p}_3 = A_3\alpha + B_3\beta$$

$$A_3 = \frac{\gamma}{1 + \gamma^2}(B_2 - B_1)$$

$$B_3 = \frac{\gamma^2}{1 + \gamma^2} B_1 + \frac{1}{1 + \gamma^2} B_2$$

where

$$\gamma = -m_y/m_z$$

If the initial angle of attack is $\delta_o = \beta_o$, the feedback variable in Cases I and II is α. The single feedback has always zero initial value. The perturbation from the nonzero initial angle of attack does not have to be calculated.

For the case of zero initial angle of attack, $\delta_o = 0$, the expression for F(s), Eq. (27) reduces to

$$F(s) = \delta_+(s) + i\,\mathscr{L}[\delta_+(\tau)\lambda(\tau)] - \frac{1}{2}\mathscr{L}[\delta_+(\tau)\lambda^2(\tau)] + \ldots \qquad (35)$$

The first-order approximation will no longer hold because the feedbacks do not appear in any linear terms. A second-order approximation of F(s) is required

$$F(s) = \delta_+(s) + i\,\mathscr{L}[\delta_+(\tau)\lambda(\tau)] \tag{36}$$

The solution for zero dispersion is found from Eq. (31) by setting $F(-i) = 0$. The first term in Eq. (36) is easily evaluated. With $s = -i$ Eq. (19) becomes

$$\delta_+(-i) = \frac{\mathscr{L}(im_t)}{p_o^{\,2}(\sigma + 1)} \tag{37}$$

The higher order term in Eq. (36) must be evaluated using a Laplace transform multiplication theorem:[5] "If $f_1(t)$ and $f_2(t)$ are $\mathscr{L}$-transformable functions having the $\mathscr{L}$-transforms $F_1(s)$ and $F_2(s)$, respectively, and if $F_1(s) \triangleq A_1(s)/B_1(s)$ is a rational algebraic fraction having q first-order poles and no others, then

$$\mathscr{L}[f_1(t)f_2(t)] = \sum_{k=1}^{q} \frac{A_1(s_k)}{B_1'(s_k)} F_2(s - s_k) \tag{38}$$

By substituting $f_1(t) = \delta(\tau)$ and $f_2(t) = \lambda(\tau)$ into Eq. (38) and setting $s = -i$ the second-order term can be obtained.

For an impulsive moment disturbance the solution required for $m_t = m_y$,

$$m_y\left[2p_oB - p_o^{\,2}C + \frac{E}{\sigma + 1}\right] + ip_o^{\,4}[4(\sigma + 1)-1] = 0 \tag{39}$$

and for $m_t = im_z$,

$$m_z\left[2p_oA + p_o^{\,2}D - \frac{F}{\sigma + 1}\right] + ip_o^{\,4}[4(\sigma + 1)] = 0 \tag{40}$$

No solution exists. Addition of the third-order term in F(s) gives the same results. An impulsive moment is uncontrollable by this approach. There is no angle of attack to provide the lift force required for control.

For a step moment disturbance about the y- or z-axis, the solution requires one nonzero feedback. The expressions for the gains in the case $m_t = im_z$ are shown for the three types of control.

2. PROPORTIONAL

$$A = \frac{1}{2p_o m_z(\omega/Q_1 + \omega/Q_2 + 1/Q_3)} \tag{41}$$

3. DERIVATIVE

$$D = \frac{-Q_3}{p_o^2(\sigma - 1)m_z} \tag{42}$$

4. INTEGRAL

$$F = \frac{1}{m_z[\omega/Q_1 + \omega/Q_2 + (\sigma - 1)/Q_3]} \tag{43}$$

where

$$Q_1 = \frac{4p_o^4\sigma^5(2\omega^2 + 2p_o\omega - p_o^2)}{\sigma + 1}$$

$$Q_2 = \frac{4p_o^4\sigma^5(2\omega^2 - 2p_o\omega - p_o^2)}{\sigma + 1}$$

$$Q_3 = \frac{\omega^2 p_o^3[(\sigma - 1)^2 - 4]}{\sigma + 1}$$

The gains are inversely proportional to the magnitude of the disturbance. The controlling force is the angle-of-attack lift force caused by the disturbance. An examination of Eqs. (41), (42), and (43) shows that the gains are very large. With the values $\sigma = 24$ and $p_o = 10\pi$ rad/s the gains are calculated as $A = -4.78 \times 10^6$, $D = 1.64 \times 10^4$, and $F = 1.62 \times 10^7$. Roll rates with these feedbacks are abnormally large and impractical. The initial assumption of $|p_o| >> |p_+|$ no longer holds, which suggests that the control is ineffective.

5. NUMERICAL EXAMPLES

The equations of motion, Eqs. (1) and (6), can be expressed by equating the real and imaginary components

$$\ddot{\alpha} + \nu\dot{\alpha} + p(2 - \mu)\dot{\beta} + [\omega^2 - p^2(1 - \mu)]\alpha + [\dot{p} + p(\nu - \nu_m)]\beta = m_y \tag{44}$$

$$\ddot{\beta} + \nu\dot{p} - p(2 - \mu)\dot{\alpha} + [\omega^2 - p^2(1 - \mu)]\beta - [\dot{p} + p(\nu - \nu_m)]\alpha = -m_z \tag{45}$$

$$V_y = V_y(0) - \frac{L_\theta}{m}\int (\beta\cos\Phi - \alpha\sin\Phi)dt \tag{46}$$

$$V_z = V_z(0) - \frac{L_\theta}{m}\int (\alpha\cos\Phi + \beta\sin\Phi)dt \tag{47}$$

These were integrated numerically for both open- and closed-loop responses to impulse and step disturbances in the y-z plane. Cases of a nonzero initial angle of attack are shown. The moment producing the initial angle of attack is found from the equations of motion in steady state.

The following examples show undamped behavior. The first-order feedback gains were used in the closed-loop cases. Inputs and system parameters are included in Table 2. The gains are summarized in Table 3.

Open- and closed-loop responses to an impulsive moment about the y-axis are shown in Figs. 1 and 2. The impulse was approximated by a rectangular pulse of 0.0001 s duration. The magnitude of the disturbance was such that the open-loop oscillation was roughly 0.1-deg amplitude. The closed-loop example is a case with derivative control where $\dot{p} = C\dot{\alpha}$. The dispersion velocity crossplots are shown for both the open- and closed-loop in Figs. 1a and 2a, respectively. Fig. 1b is the open-loop response of α, the angle of attack component directly affected by the m_y disturbance. The closed-loop

Table 2. Inputs and System Parameters

m	$= 22.68$ kg
L_θ	$= 3.275 \times 10^5$ N/rad
p_o	$= 10\,\pi$ rad/s
σ	$= 24$
μ	$= 0$
ν	$= 0$
$\Phi(0)$	$= 0$
$v\ (0)$	$= -40.109$ m/s
$w\ (0)$	$= 0$
$\alpha(0)$	$= 5$ deg
$\beta(0)$	$= 0$ deg
m_y (steady state)	$= 1980.96\ s^{-2}$
m_z (steady state)	$= 0$

Table 3. Gains for $\alpha_o = 5$ deg

	A	B	C	D	E	F
m_y	0	118,755	330	0	− 324,410	0
m_z	0	− 10,325	-3780	0	3,730,730	0

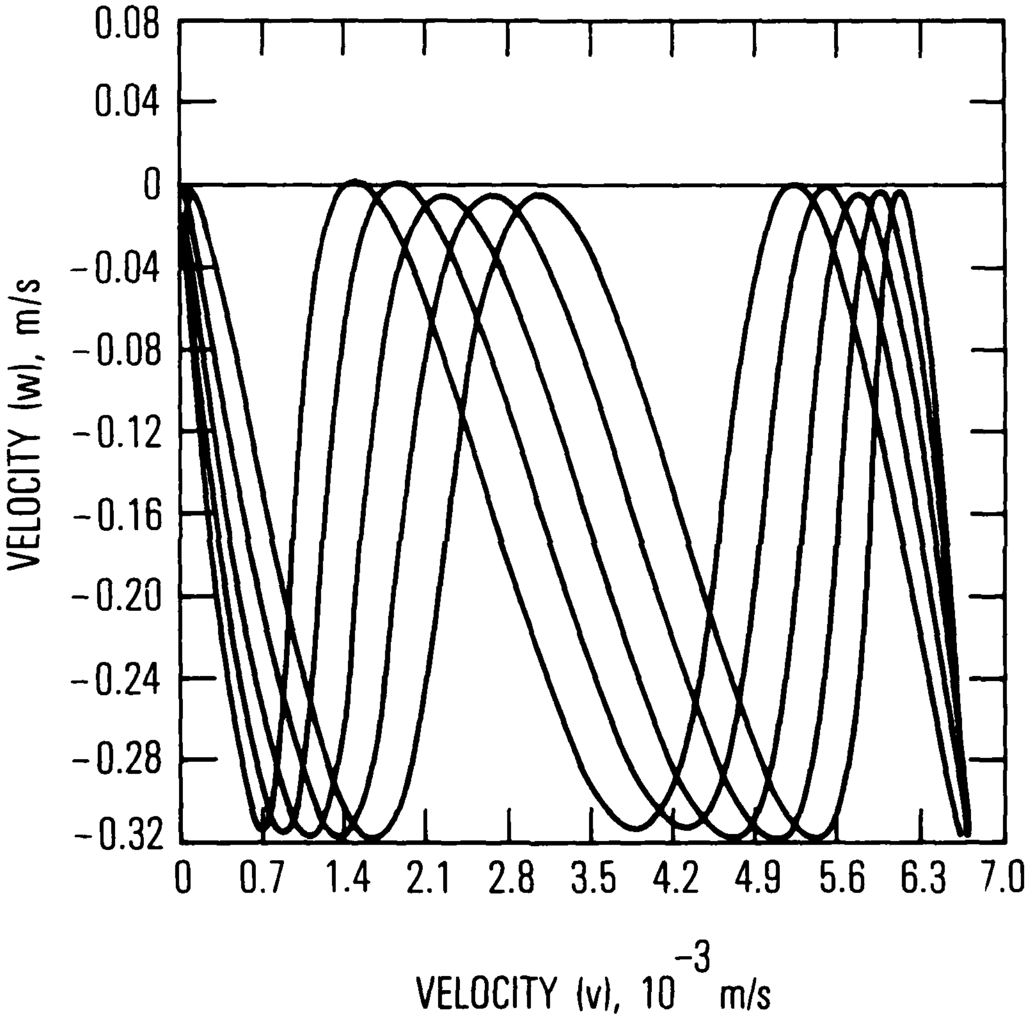

Fig. 1a. Open-Loop Response to Moment Impulse. Dispersion velocity crossplot.

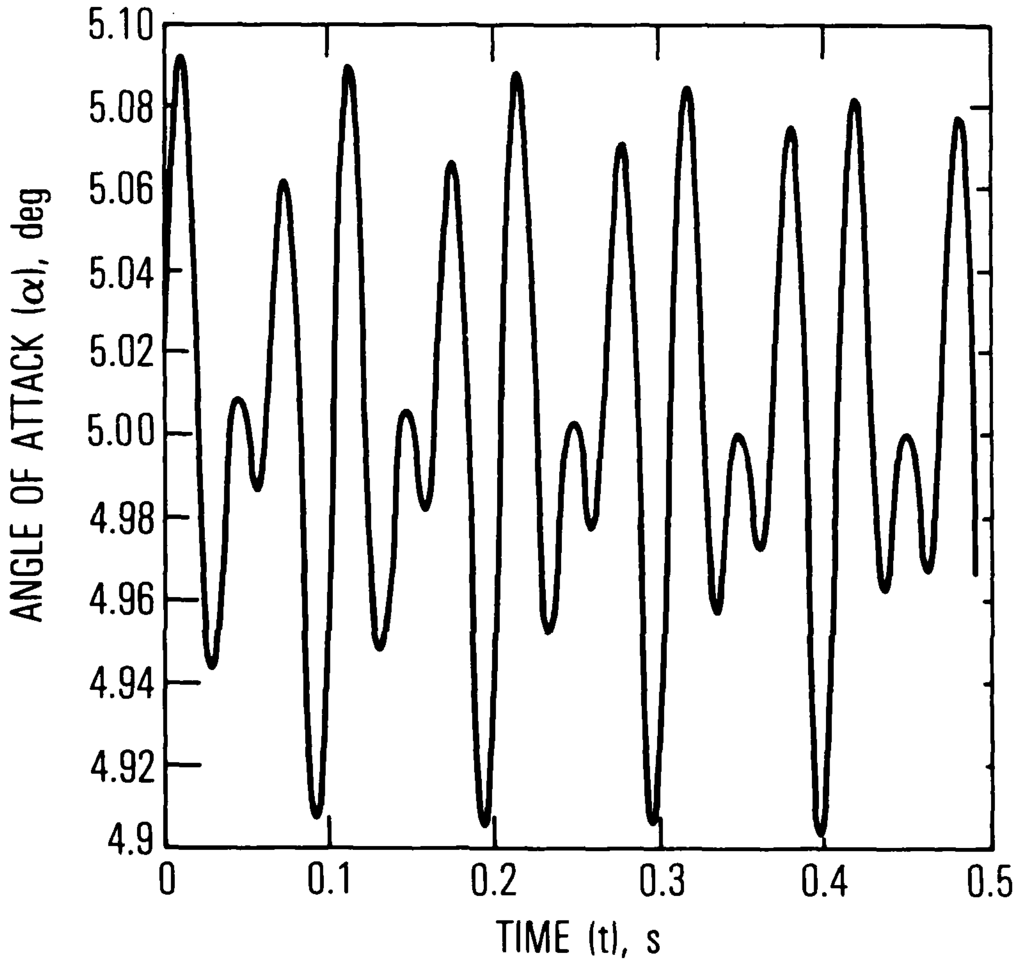

Fig. 1b. Open-Loop Response to Moment Impulse. α-component of angle of attack.

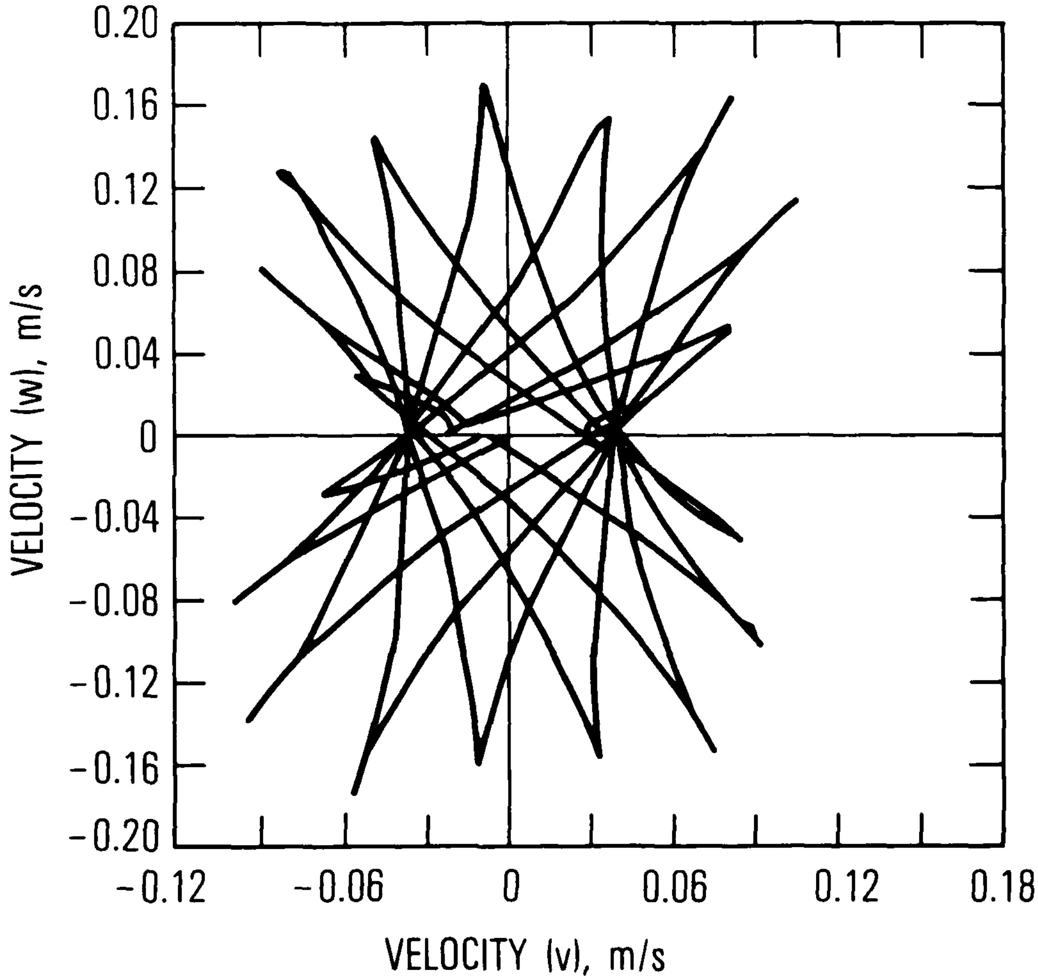

Fig. 2a. Closed-Loop Response to Moment Impulse. Dispersion velocity crossplot.

behaviors of the roll rate and α are shown in Figs. 2b and 2c. The open-loop mean value of the dispersion velocity w is -0.16 m/s. With the closed-loop, the roll rate oscillates with only a 0.6 rad/s amplitude, and the mean value of the dispersion velocity becomes virtually zero.

Figures 3 and 4 show the response to a step moment resulting in a 0.1-deg step in β. The open-loop behavior of α is shown in Fig. 3b. The closed-loop example demonstrates proportional control of the form $\dot{p} = B\beta$. The open and closed-loop crossplots of the transverse velocities are shown in Figs. 3a and 4a. In addition the behavior of the roll rate, p, and the feedback, β, are shown in Figs. 4b and 4c for the closed-loop case. The mean value of the dispersion velocity is reduced from an open-loop value of 0.8 m/s to a closed-loop value of zero. The roll rate amplitude is approximately 1.3 rad/s.

C. SINUSOIDAL FEEDBACK

The missile response to a sinusoidal roll rate modulation of the form

$$p = p_o + \Delta \sin qt \tag{48}$$

is examined in order to determine the parameters Δ and q required for effective dispersion control. It is convenient for this analysis to use the aeroballistic equations of motion for the complex angle of attack, Eq. (2), which with the roll rate behavior of Eq. (48) can be written

$$\begin{aligned} &\ddot{\xi} + (\nu - i\mu p)\dot{\xi} + (\omega^2 - ip\nu_m)\xi \\ &\quad = im_t \exp i[p_o t + \hat{z}(1 - \cos qt)] \end{aligned} \tag{49}$$

where $\hat{z}$ is the nondimensional ratio Δ/q. For Δ small relative to p_o the roll coupling on the left side of Eq. (49) is very weak because μ and ν_m are, in general, small terms. For a first-order approximation to the roll modulation we assume zero damping $\nu = \nu_m = 0$ and $\mu = 0$, which reduces Eq. (49) to

$$\ddot{\xi} + \omega^2\xi = im_t \exp i[p_o t + \hat{z}(1 - \cos qt)] \tag{50}$$

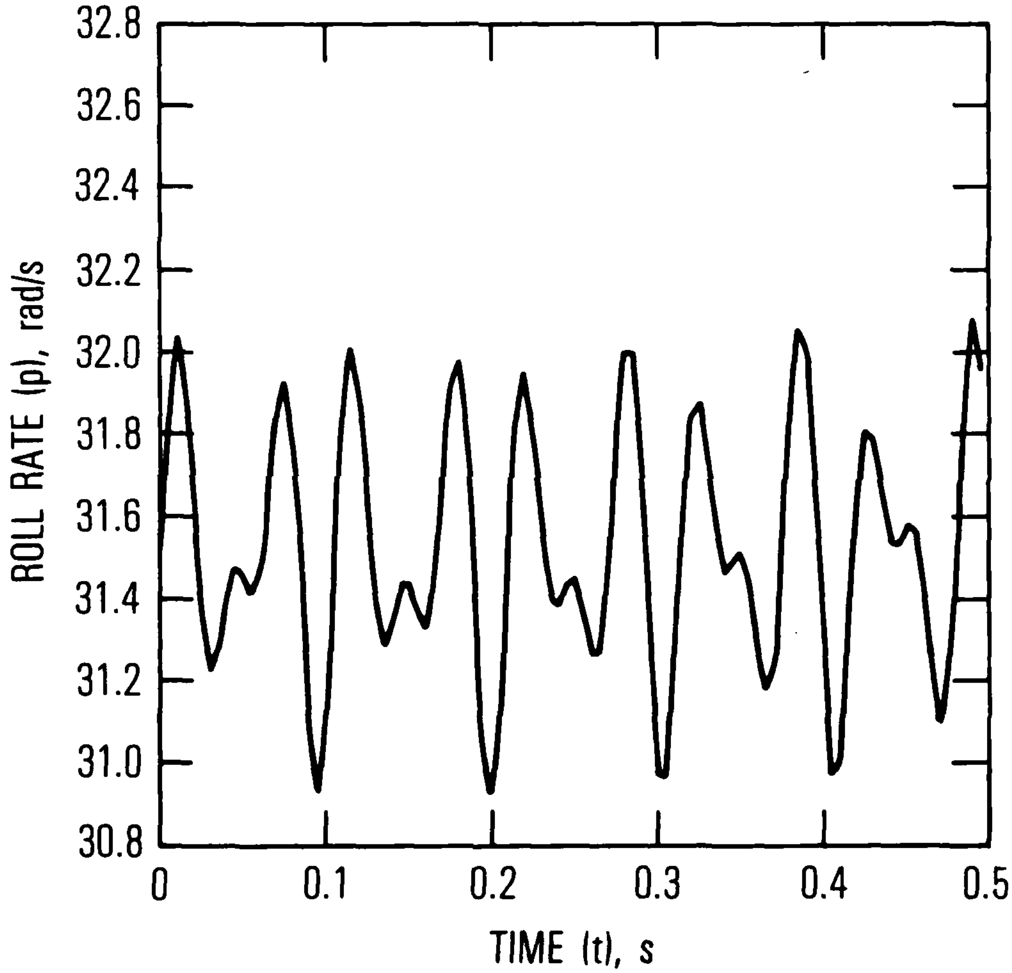

Fig. 2b. Closed-Loop Response to Moment Impulse. Roll rate.

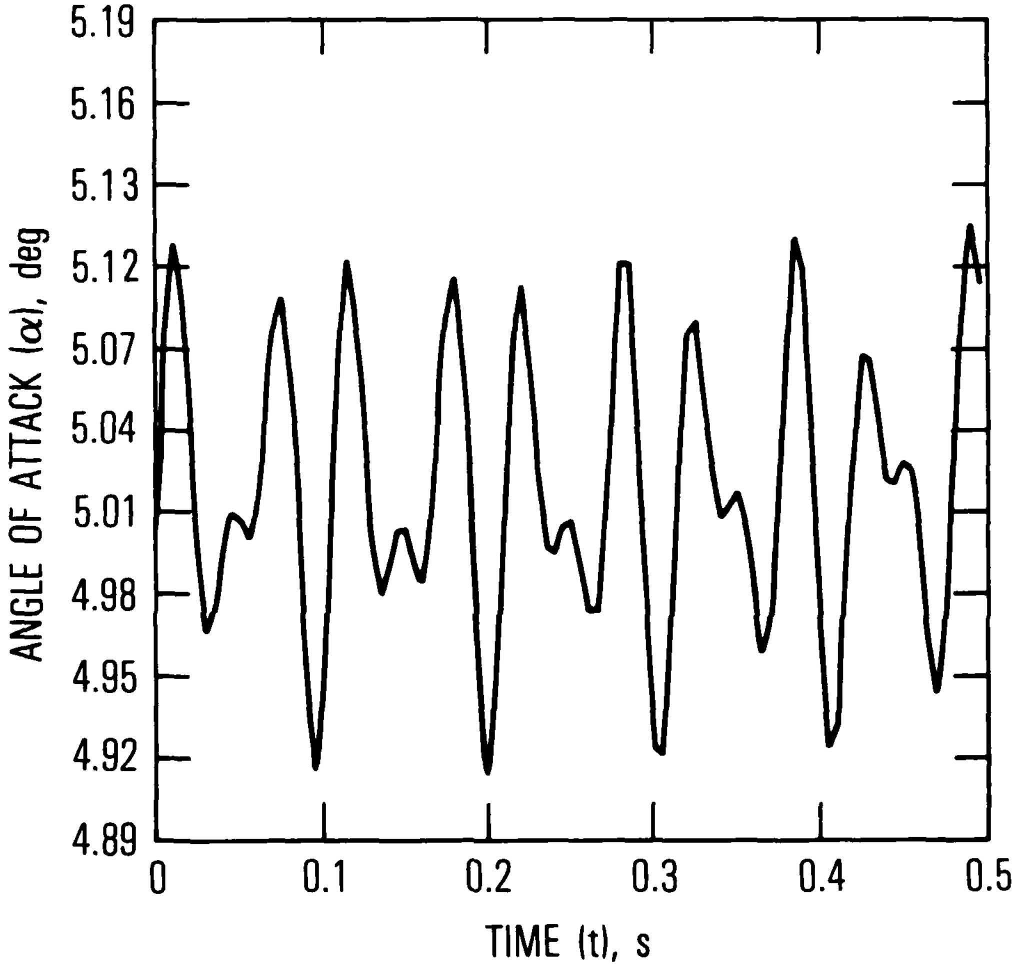

Fig. 2c. Closed-Loop Response to Moment Impulse.
α-component of angle of attack.

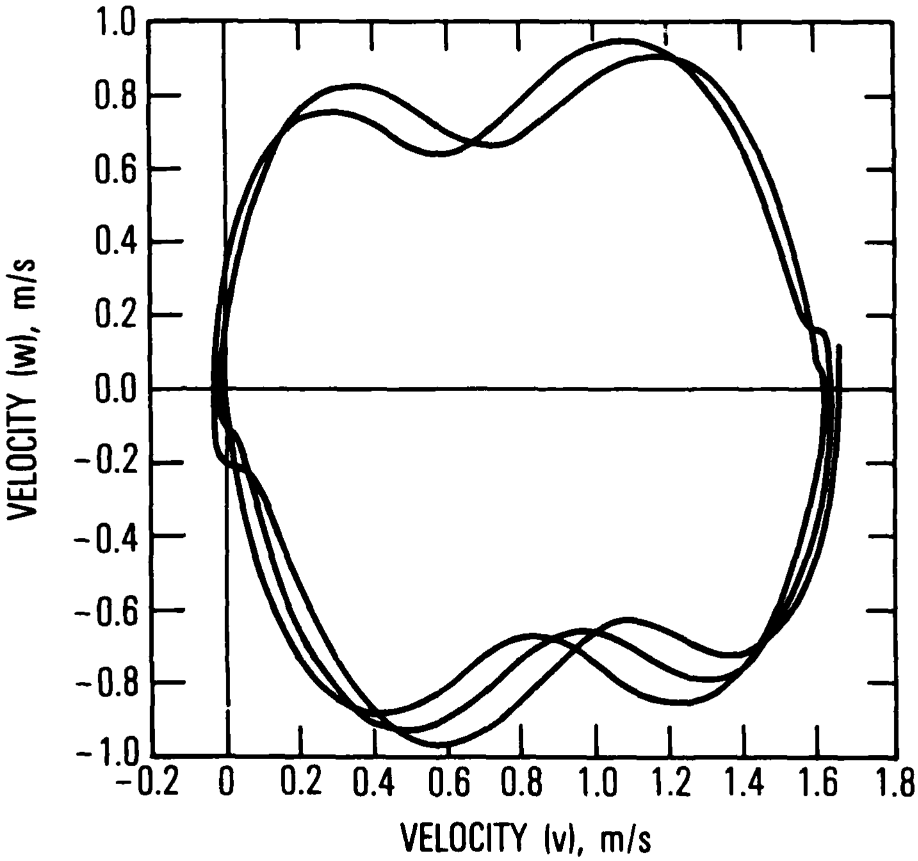

Fig. 3a. Open-Loop Response to Moment Step. Dispersion velocity crossplot.

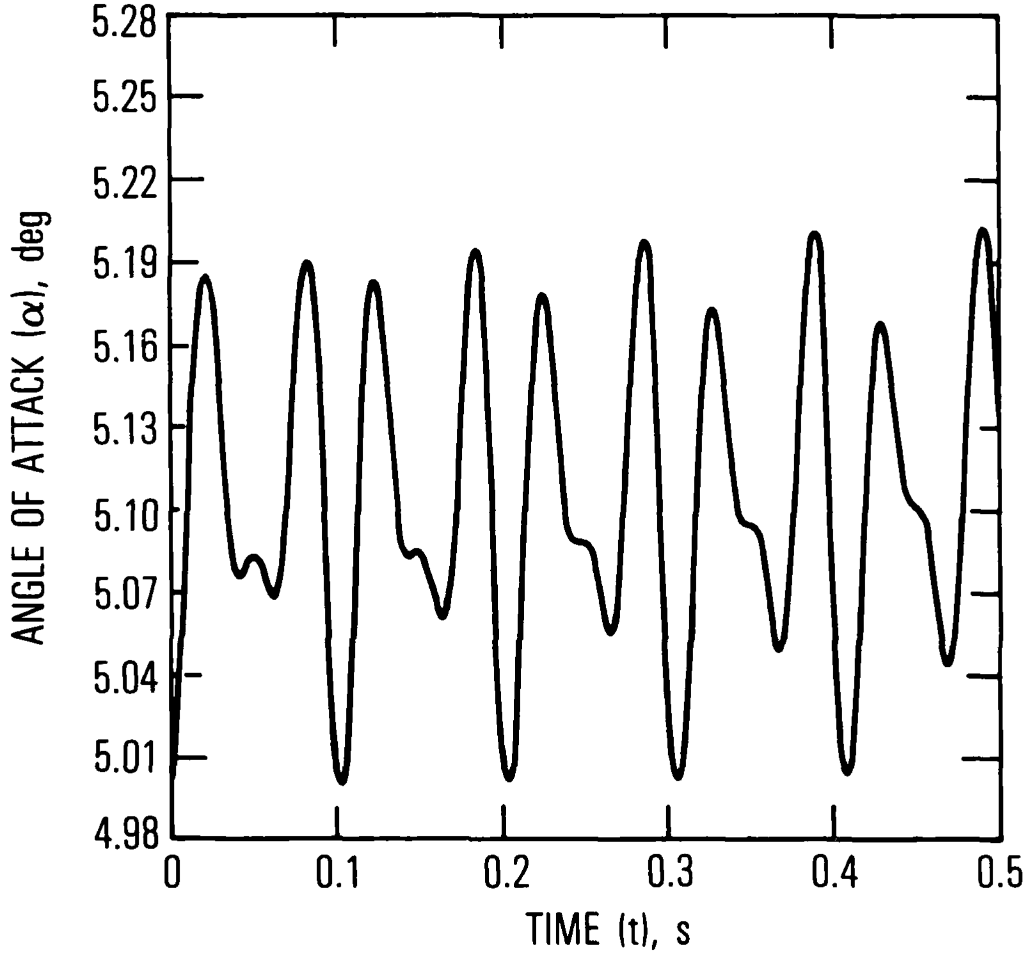

Fig. 3b. Open-Loop Response to Moment Step. α-component of angle of attack.

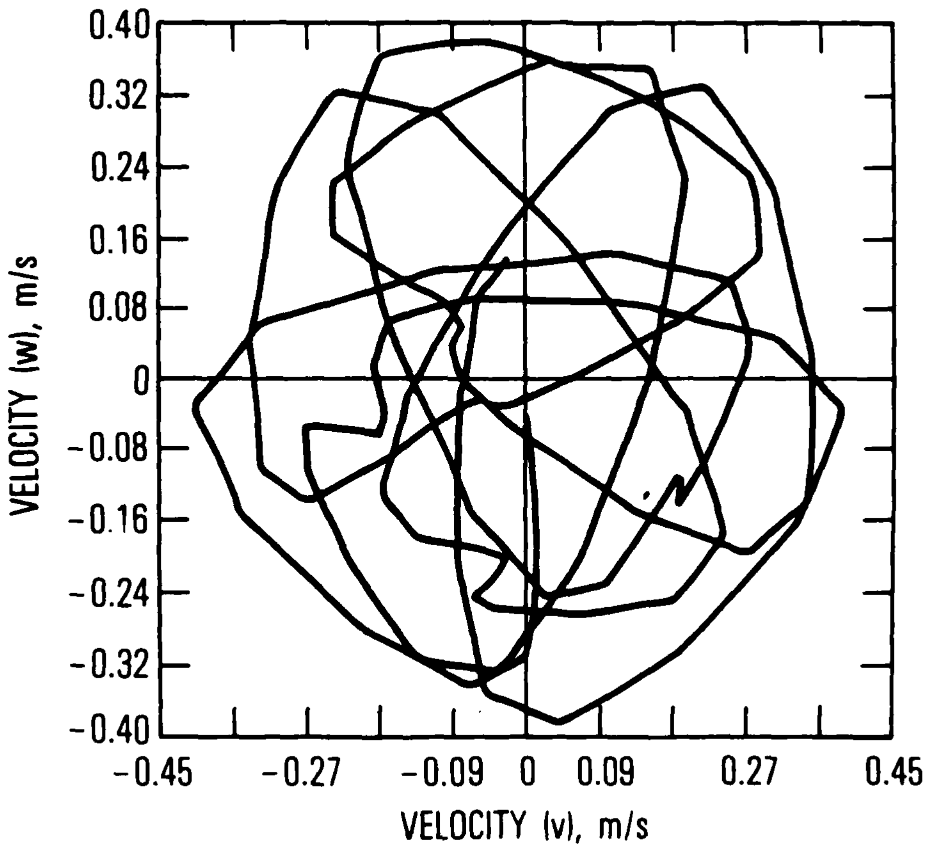

Fig. 4a. Closed-Loop Response to Moment Step. Dispersion velocity crossplot.

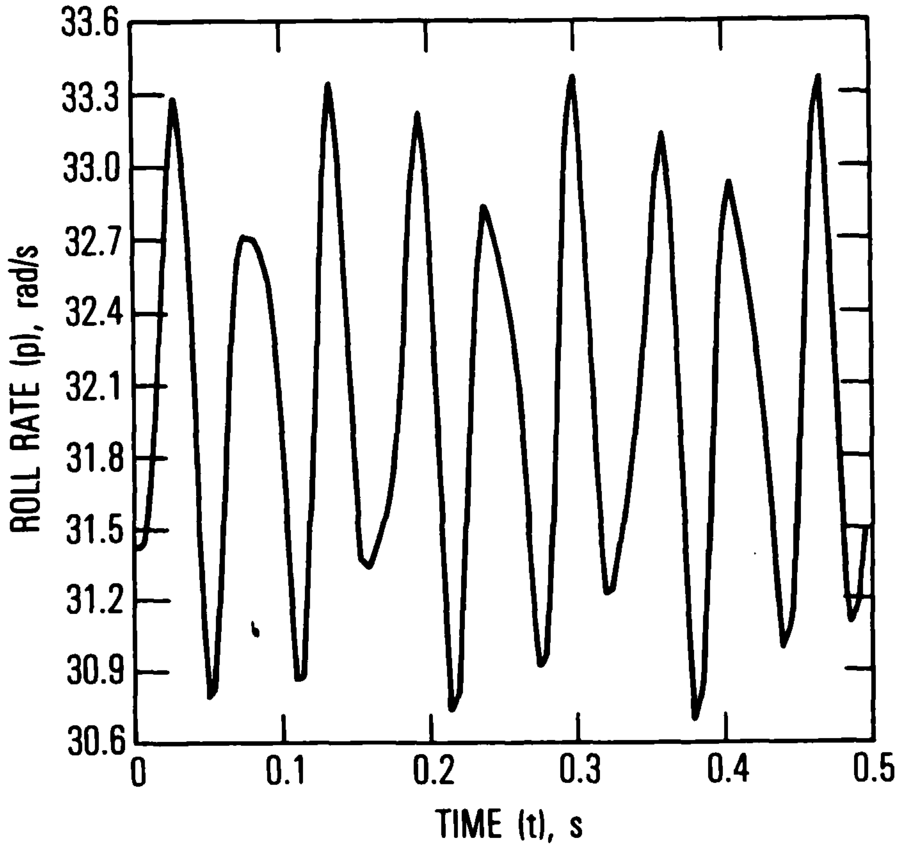

Fig. 4b. Closed-Loop Response to Moment Step. Roll rate.

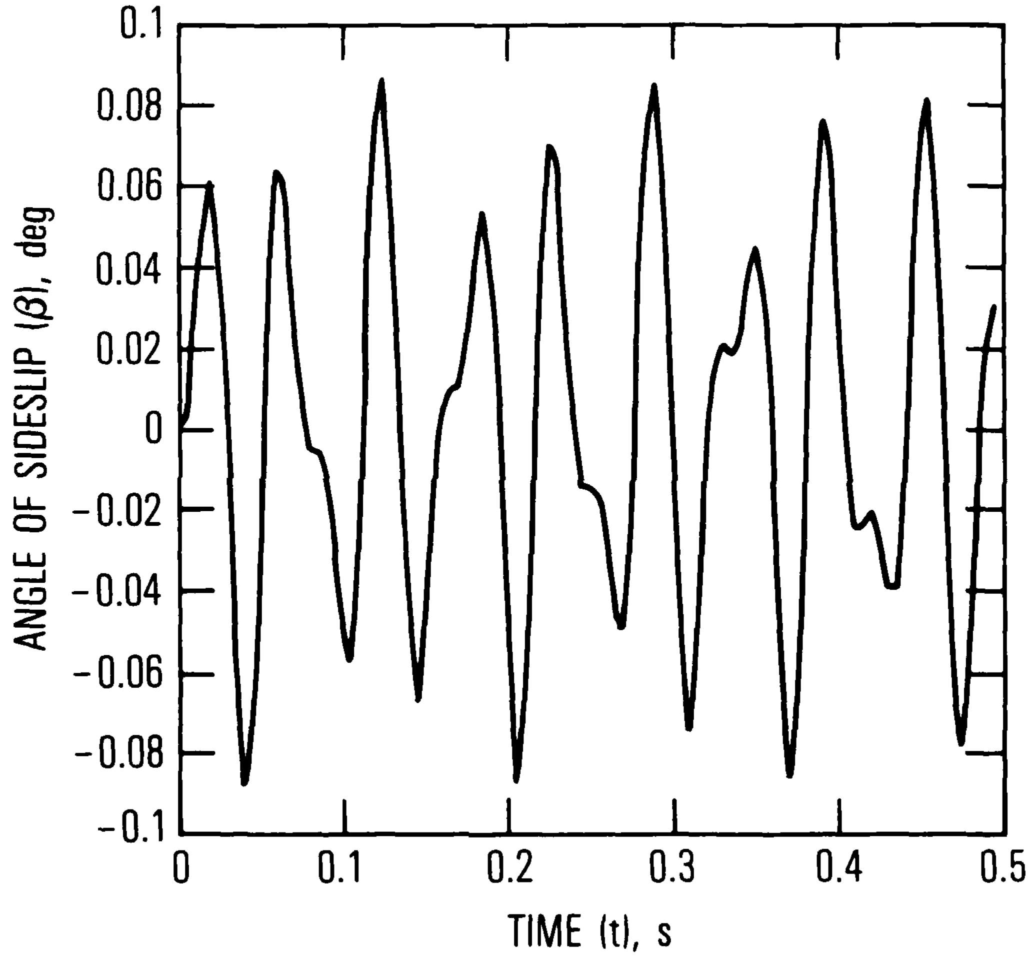

Fig. 4c. Closed-Loop Response to Moment Step. β-component of angle of attack.

With these approximations it is apparent that the response to an impulsive moment m_t with zero initial angle of attack is not controllable because the right side of Eq. (50) is defined only at $t = 0$ and is therefore independent of roll rate. Consider the response to a moment step $m_t = -im_z = \bar{m}\,H(t)$. Eq. (50) can be written

$$\ddot{\xi} + \omega^2\xi = \bar{m}H(t)e^{i(\hat{z}+p_o t)}[\cos(\hat{z}\cos qt)$$

$$- i\sin(\hat{z}\cos qt)] \tag{51}$$

where the sinusoidal terms can be expanded in terms of Bessel functions with the relations[6]

$$\cos(\hat{z}\cos\theta) = J_o(\hat{z}) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(\hat{z})\cos(2k\theta) \tag{52}$$

$$\sin(\hat{z}\cos\theta) = 2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(\hat{z})\cos[(2k+1)\theta] \tag{53}$$

Eq. (51) becomes

$$\ddot{\xi} + \omega^2\xi = \bar{m}H(t)e^{i(\hat{z}+p_o t)}\{J_o(\hat{z}) - 2J_2(\hat{z})\cos 2qt + 2J_4(\hat{z})\cos 4qt - \ldots$$

$$-i[2J_1(\hat{z})\cos qt - 2J_3(\hat{z})\cos 3qt + \ldots]\}$$

$$= \bar{m}H(t)e^{i(\hat{z}+p_o t)}[J_o(\hat{z}) + \ldots + 2(-i)^n J_n(\hat{z})\cos nqt + \ldots] \tag{54}$$

The Laplace transform of Eq. (54) gives for $\xi(s)$

$$\xi(s) = \frac{\overline{m}e^{i\hat{z}}}{s^2 + \omega^2}\left[\frac{J_o(\hat{z})}{s - ip_o} + \ldots \right.$$

$$\left. + \frac{2(-i)\ J_n^n(\hat{z})(s - ip_o)}{(s - ip_o)^2 + (nq)^2} + \ldots\right] \tag{55}$$

The inverse of which is

$$\xi(t) = \frac{\overline{m}e^{i\hat{z}}J_o(\hat{z})}{\omega^2 - p_o^2}\left[e^{ip_o t} - \cos\omega t - \frac{ip_o}{\omega}\sin\omega t\right] + \ldots + \overline{m}\ e^{i\hat{z}}J_n(\hat{z})(-i)^n$$

$$\left\{\frac{1}{\omega^2 - (p_o + nq)^2}\left[e^{i(p_o+nq)t} - \cos\omega t - \frac{i(p_o+nq)}{\omega}\sin\omega t\right] + \frac{1}{\omega^2 - (p_o - nq)^2}\right.$$

$$\left.\left[e^{i(p_o-nq)t} - \cos\omega t - \frac{i(p_o - nq)}{\omega}\sin\omega t\right]\right\} \tag{56}$$

The transverse velocity, Eq. (7), is obtained by integrating $\xi(t)$ in Eq. (56). The result is

$$V(t) = -\frac{L_\theta\overline{m}e^{i\hat{z}}J_o(\hat{z})}{m(\omega^2 - p_o^2)}\left[\frac{e^{ip_o t} - 1}{ip_o} - \frac{\sin\omega t}{\omega} - \frac{ip_o}{\omega^2}(1 - \cos\omega t)\right] + \ldots + \frac{L_\theta\overline{m}\ e^{i\hat{z}}J_n(\hat{z})(-i)^n}{m}$$

$$\left\{\frac{1}{\omega^2 - (p_o + nq)^2}\left[\frac{e^{i(p_o+nq)t} - 1}{i(p_o + nq)} - \frac{\sin\omega t}{\omega} - \frac{i(p_o + nq)}{\omega^2}(1 - \cos\omega t)\right.\right.$$

$$\left.\left. + \frac{1}{\omega^2 - (p_o - nq)^2}\left[\frac{e^{i(p_o-nq)t} - 1}{i(p_o - nq)} - \frac{\sin\omega t}{\omega} - \frac{i(p_o - nq)}{\omega^2}(1 - \cos\omega t)\right]\right\} \tag{57}$$

For p_o equal to integer multiples of q, a singularity occurs at $n = n_s$ where $p_o - n_s q = 0$. If we let $p_o - n_s q = \varepsilon$, we can obtain the value of the singularity as $\varepsilon \to 0$. The singular term in Eq. (57) having p_o - nq in the denominator can be written

$$\lim_{\varepsilon \to o} \frac{e^{i\varepsilon t} - 1}{i\varepsilon} = \lim_{\varepsilon \to o} \frac{\cos \varepsilon t + i\sin \varepsilon t - 1}{i\varepsilon} \approx t \tag{58}$$

We are interested in the steady-state or d.c. component of V(t), which is found to be

$$V_{\substack{\text{steady}\\\text{state}}} = -\frac{iL_\theta \theta_T e^{i\hat{z}}}{mp_o}\left[J_o(\hat{z}) + \ldots + \frac{2(-i)^n J_n(\hat{z})}{1 - (n^2 q^2/p_o^2)}\right.$$

$$\left. + \ldots + (-i)^{n_s} J_{n_s}(\hat{z}) \left(\frac{1}{2} - itp_o\right)\right] \tag{59}$$

where $\overline{m}$ has been replaced by $\omega^2\theta_t$, θ_t being the magnitude of the trim step. In the limit as Δ and $\hat{z} \to 0$, $J_o(\hat{z}) \to 1$ and $J_n(\hat{z}) \to 0$, and Eq. (59) reduces to the open-loop value for the transverse velocity increment due to a trim step.

1. NUMERICAL EXAMPLES

Undamped responses to a step moment with and without the roll modulation were obtained from numerical integration of the equations of motion. The open-loop response of the velocity increments is shown in Fig. 5. The mean value, calculated from Eq. (59), with $\hat{z} = 0$ is w = 0.802 m/s. The two examples with roll modulation are the cases where $q = p_o$ (Fig. 6) and $q = p_o/2$ (Fig. 7). For $q = p_o$ the roll control is

$$\dot{p} = 50\pi \cos 10\pi t \tag{60}$$

The amplitude is such that the Δ of Eq. (48) is roughly equal to 5. The steady-state response of the transverse velocity components is predicted from Eq. (59) with n = 0,1 and $n_s = (p_o/q) = 1$.

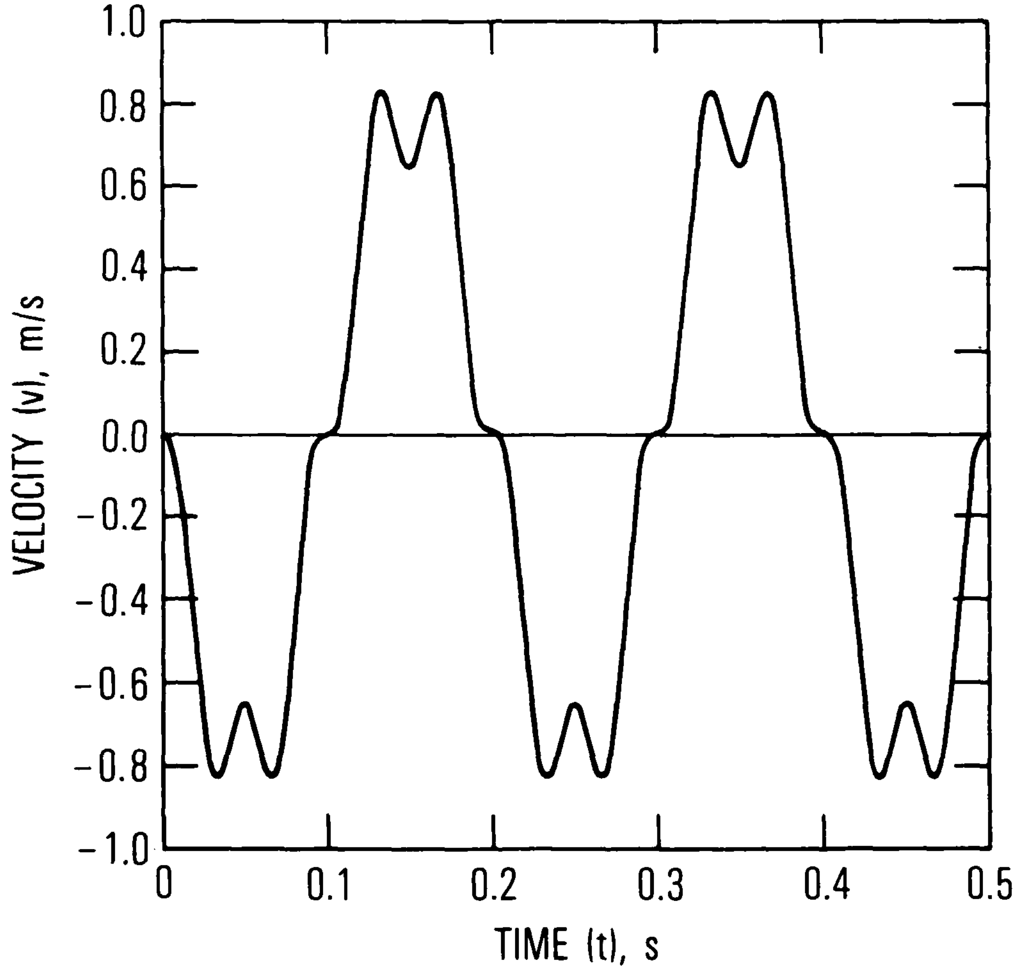

Fig. 5a. Open-Loop Response to Moment Step. v-component of dispersion velocity.

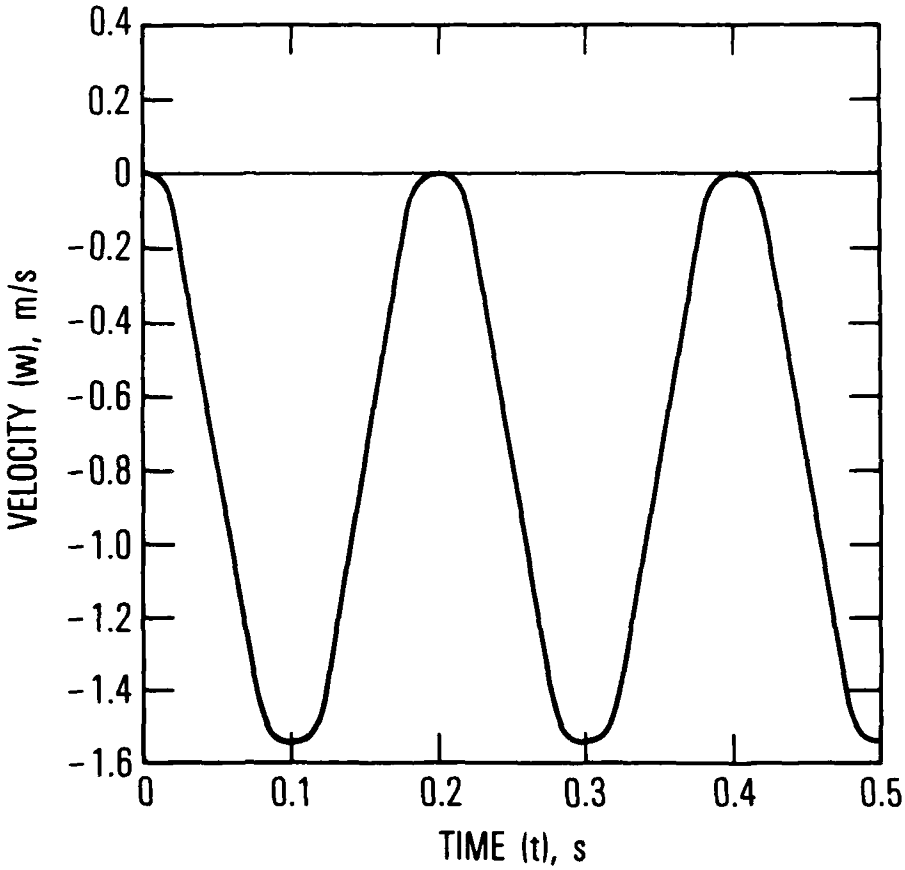

Fig. 5b. Open-Loop Response to Moment Step. w-component of dispersion velocity.

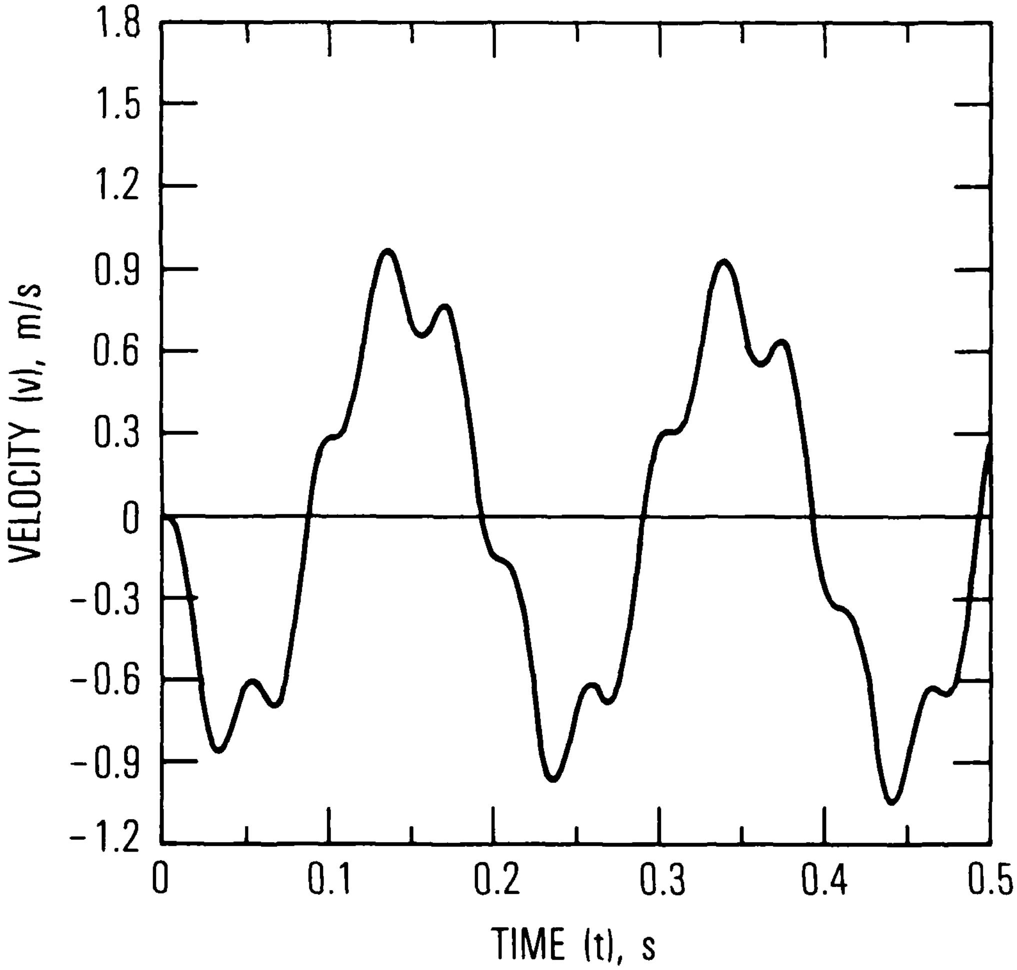

Fig. 6a. Roll Modulation Response to Moment Step. v-component of dispersion velocity.

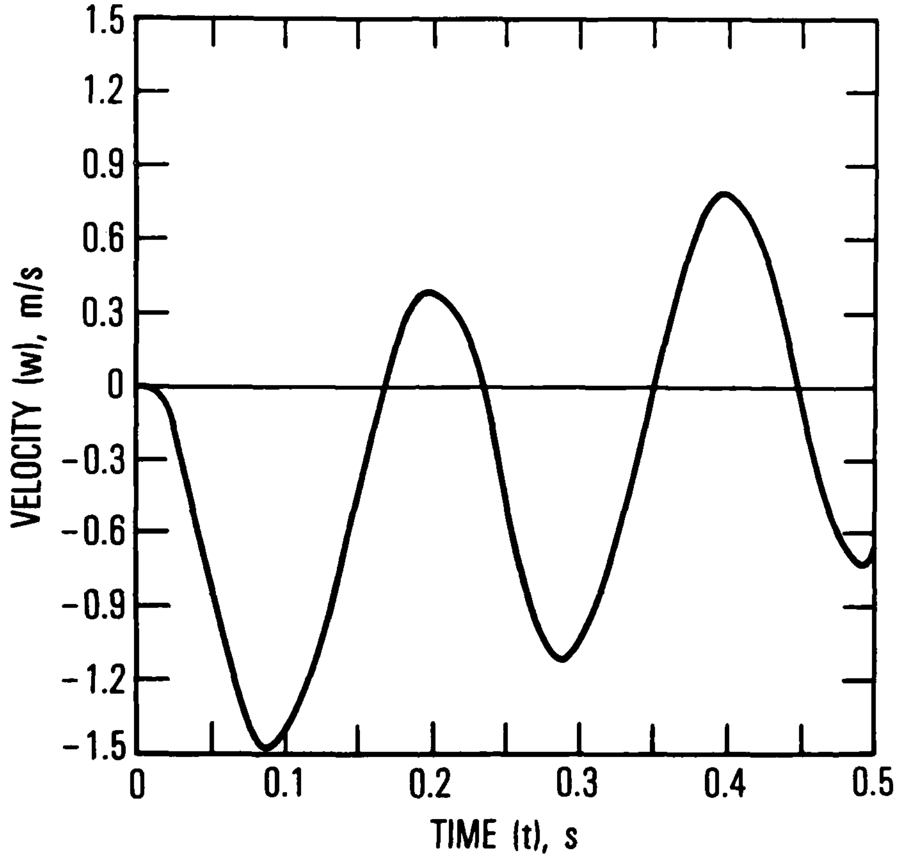

Fig. 6b. Roll Modulation Response to Moment Step. w-component of dispersion velocity.

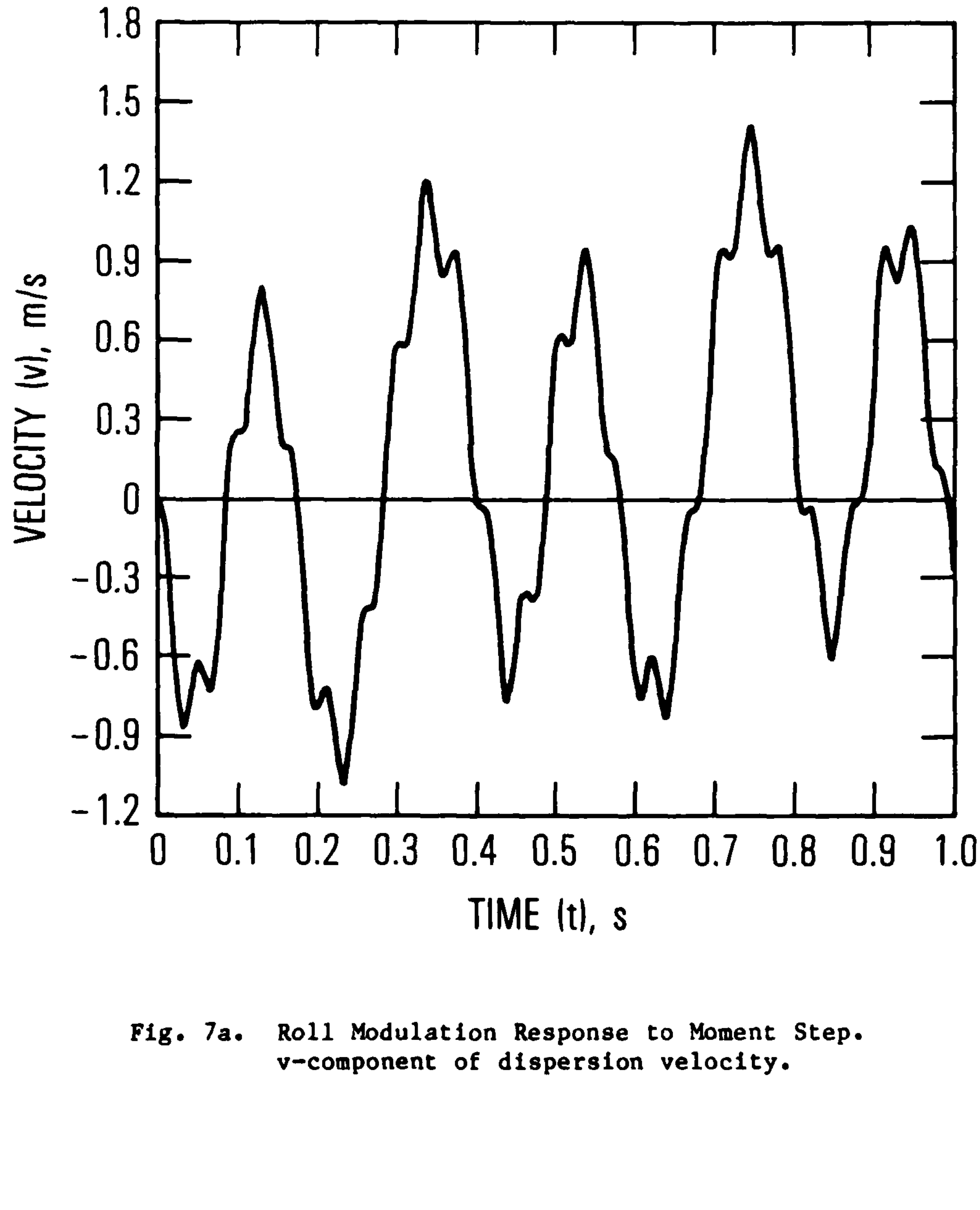

Fig. 7a. Roll Modulation Response to Moment Step. v-component of dispersion velocity.

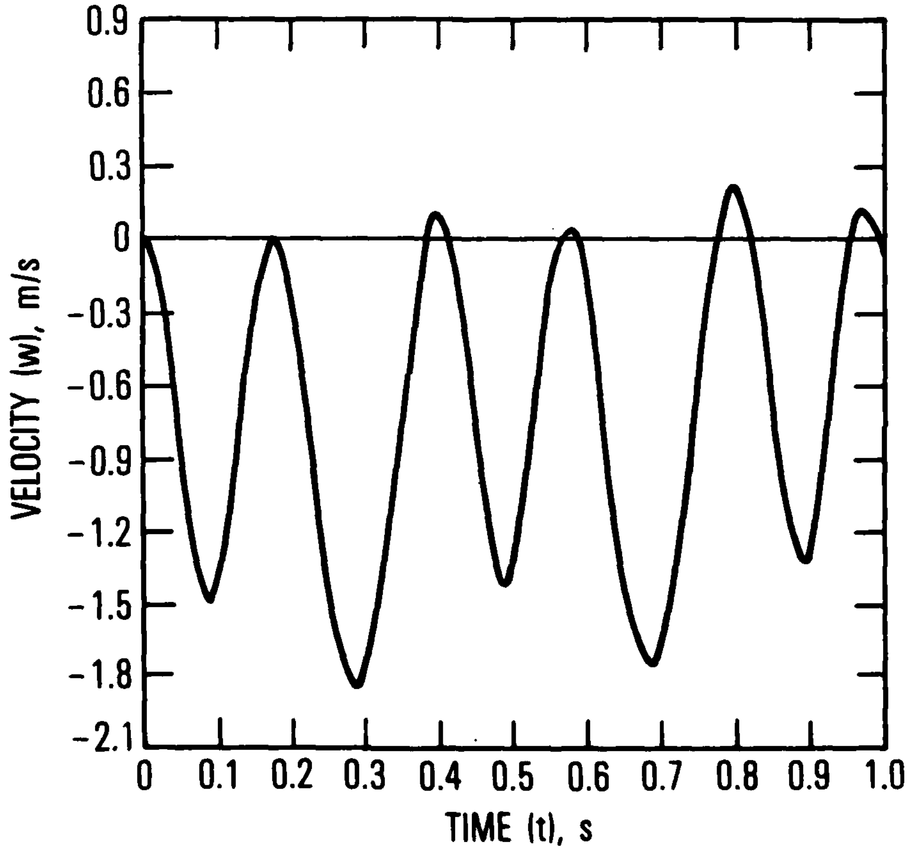

Fig. 7b. Roll Modulation Response to Moment Step.
w-component of dispersion velocity

$$v = -0.317\ t + 0.094 \tag{61}$$

$$w = 1.980\ t - 0.791 \tag{62}$$

For $q = p_o/2$ the roll control is

$$\dot{p} = 10\pi^2 \cos 5\pi t \tag{63}$$

Using Eq. (59) and taking $n = 0, 1, 2$ where $n_s = (p_o/q) = 2$, the response is calculated to be

$$v = 0.458\ t - 0.090 \tag{64}$$

$$w = 0.193\ t - 0.865 \tag{65}$$

The mean values of v and w approximated by Eqs. (61), (62), (64), and (65) agree well with the values obtained numerically and demonstrate a capability to compensate for lift nonaveraging dispersion. If we consider the amplitude Δ of the roll modulation term to be an equivalent control deflection, we can write a control law of the form

$$\Delta = -Ar - B\dot{r} - C \int r dt \tag{66}$$

where r is the missile cross-range dispersion. The control loop is shown in Fig. 8. A first-order approximation to the feedback gains can be obtained from the steady-state response to a moment step, Eq. (59). For small values of $\hat{z} = \Delta/q$, the Bessel functions $J_n(\hat{z})$ for $n \geq 1$ are small relative to $J_o(\hat{z}) \approx 1$ and the singularity term dominates the response. For the case $q = p_o$, the steady-state transverse velocity is approximately

$$v + iw = -\frac{iL_\theta \theta_T e^{i\hat{z}}}{mp_o} \left[J_o(\hat{z}) - iJ_1(\hat{z})\left(\frac{1}{2} - ip_o t\right)\right]$$

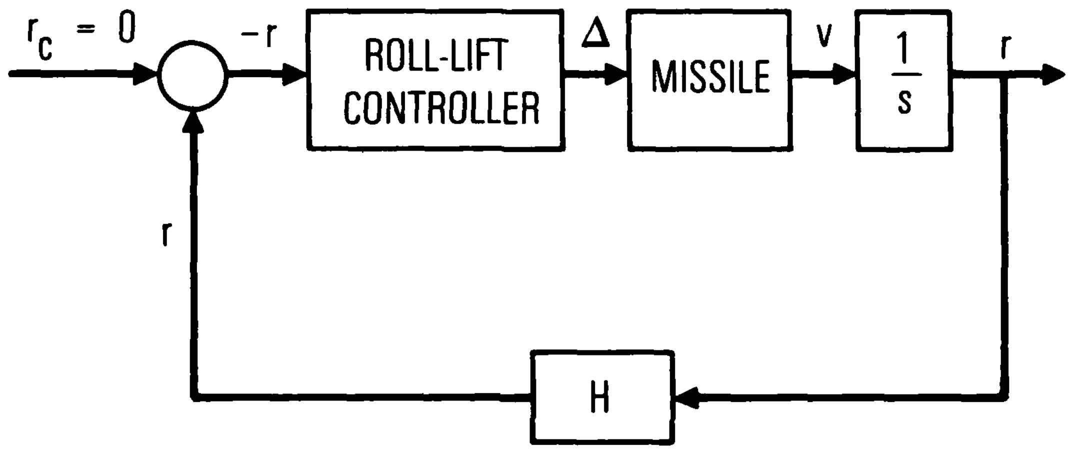

Fig. 8. Roll Modulation Control Loop.

With $e^{i\hat{z}} \approx 1$, $J_o(\hat{z}) \approx 1$ and $J_1(\hat{z}) \approx \hat{z}/2$, the component w is

$$w = -w_o(1 - t\Delta/2) \tag{67}$$

where $w_o = L_\theta \theta_T / mp_o$ is the open-loop response. If we let r = z and consider proportional control only, the dispersion z from Eqs. (66) and (67) is described by

$$\dot{z} + \frac{w_o A t z}{2} = -w_o \tag{68}$$

The solution to Eq. (68) is

$$z(t) = e^{-\frac{Aw_o t^2}{4}} \left[-w_o \int_o^t e^{\frac{Aw_o t^2}{4}} dt + z(o)\right] \tag{69}$$

which with the change of variable $\tau = t\sqrt{Aw_o}/2$ can be written

$$z(\tau) = -\sqrt{\frac{w_o}{A}}\, e^{-\tau^2} \int_o^\tau e^{\tau^2} d\tau \tag{70}$$

where

$$D(\tau) = e^{-\tau^2} \int_o^\tau e^{\tau^2} d\tau \tag{71}$$

The function $D(\tau)$, known as Dawson's integral, is tabulated.[6] We can select a value for the gain A to limit $z(\tau)$ to some prescribed fraction of the open-loop dispersion $w_o t$. Shown in Fig. 9 is a comparison of the open- and closed-loop response for a gain of A = 100, which from Eq. (70) should give a closed-loop response approximately 15% of the open-loop value at t = 0.45 sec.

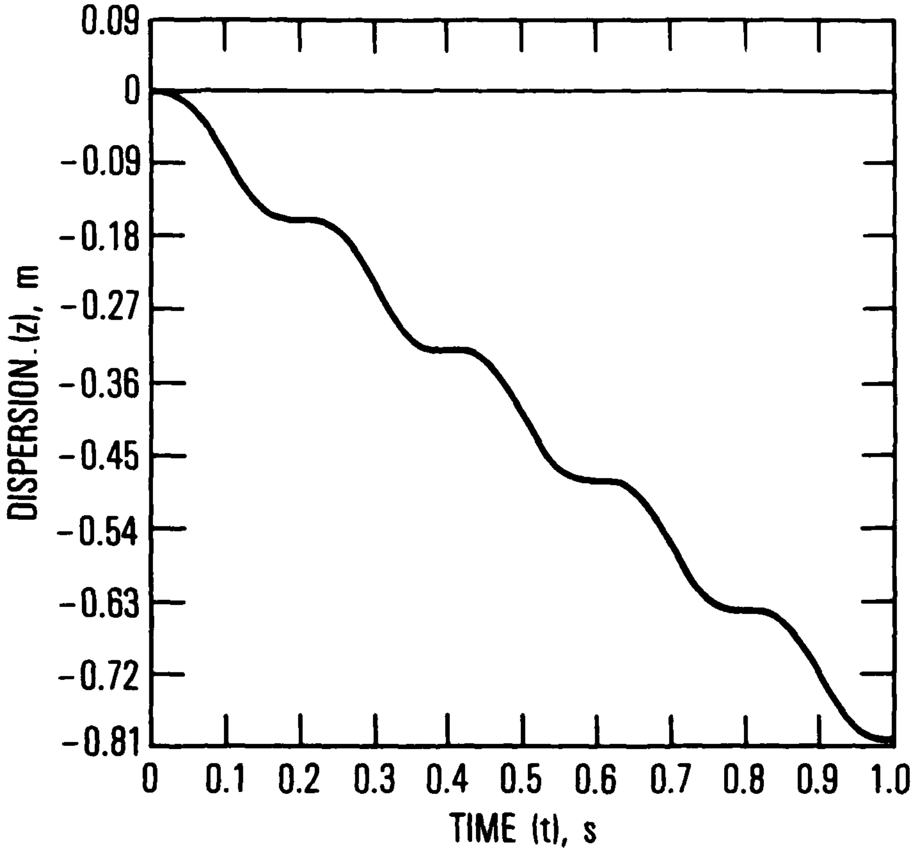

Fig. 9a. Open-Loop Response to Moment Step.
z-component of dispersion.

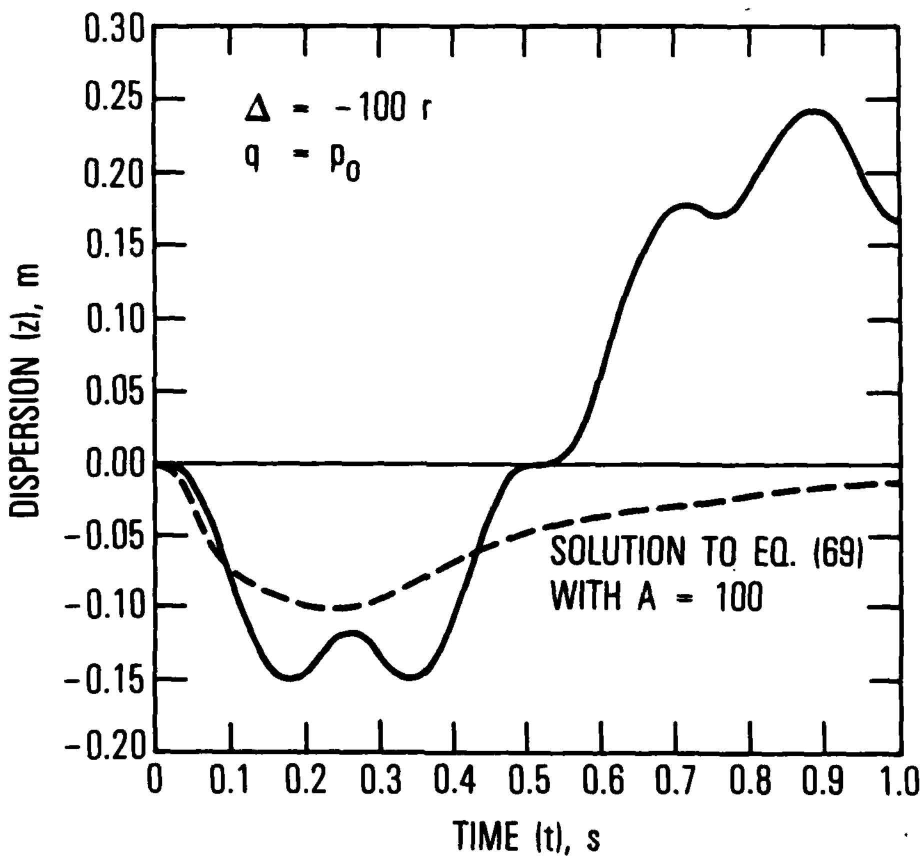

Fig. 9b. Closed-Loop Response to Moment Step. z-component of dispersion.

III. CONCLUSIONS

Transverse velocity resulting from moment disturbances that perturb the angle of attack can be controlled by modulating the roll rate with state variable feedback. Without an initial trim angle of attack, the dispersion velocity is uncontrollable. Feedbacks that cause the transverse velocity to average zero can be calculated by linearizing the equations of motion in body-fixed coordinates and obtaining a solution in the Laplace transform domain without the need to invert the transformed equations. A first-order solution is sufficient and shows the gains to be independent of the type of moment disturbance. Numerical examples show effective control with only small deviations from the steady-state roll rate, indicating small roll moment requirements. An alternative feedback control adjusts the amplitude of a small harmonic modulation of the roll rate about a steady value. The roll modulation frequency required for effective control is derived in the aeroballistic coordinate system, which results in minimal roll coupling with the complex angle-of-attack motion. A quasisteady solution for dispersion velocity with constant amplitude roll modulation is then used with a slowly varying amplitude to derive a control law to limit dispersion. A first-order approximation for the feedback gain required produces effective control that agrees well with predicted values for the magnitude of dispersion relative to the open-loop response to a moment step.

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ABBREVIATIONS AND SYMBOLS

A,B,C,D,E,F	feedback gains
H(t)	unit step function
i	$\sqrt{-1}$
I	pitch or yaw moment of inertia
I_x	roll moment of inertia
J_n	n^{th} order Bessel function
ℓ_c	roll control moment
ℓ_x	roll moment disturbance
L	lift force
L_θ	lift force derivative
m	missile mass
m_t	complex disturbance moment, $m_y + im_z$
Δm_t	magnitude of disturbance moment
$\bar{m}$	moment step
M_y, M_z	pitch and yaw disturbances
p	roll rate
p_o	initial roll rate
q	roll modulation rate
r	y + iz
s	Laplace transform variable
S	aerodynamic reference area
t	time
v	y component of transverse velocity
V	transverse velocity in cross plane, v + iw
ΔV	transverse velocity increment, $\Delta v + i\Delta w$
w	z component of transverse velocity
z	nondimensional ratio Δ/q; coordinate
α	angle of attack
α_o	initial angle of attack

α_+	angle-of-attack perturbation
β	angle of sideslip
β_o	initial angle of sideslip
β_+	angle of sideslip perturbation
γ	$-m_y/m_z$
δ	complex angle of attack, $\beta + i\alpha$
δ_o	initial complex angle of attack, $\beta_o + i\alpha_o$
δ_+	complex angle of attack perturbation
$\delta^*(t)$	unit impulse function
$\Delta\delta$	complex angle-of-attack increment
Δ	sinusoidal control parameter
θ	angle of attack (Euler angle)
τ	nondimensional roll angle $p_o t$; $t\sqrt{Aw_o}/2$, Eq. (70)
μ	moment of inertia ratio, I_x/I
ν	aerodynamic damping parameter
ν_m	yaw moment damping parameter
ξ	complex angle of attack, $\beta + i\alpha$
σ	$\omega^2/p_o{}^2$
Φ_i	roll angle relative to inertial reference
ω	undamped natural pitch frequency

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The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military space systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

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