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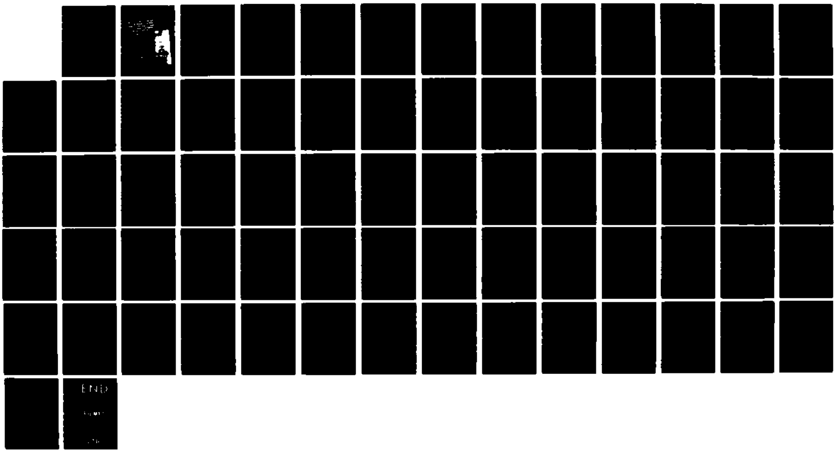

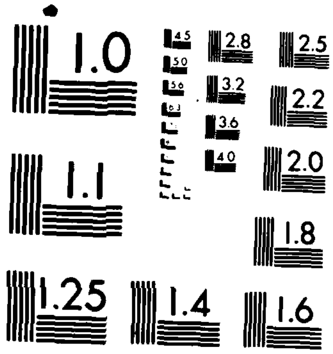

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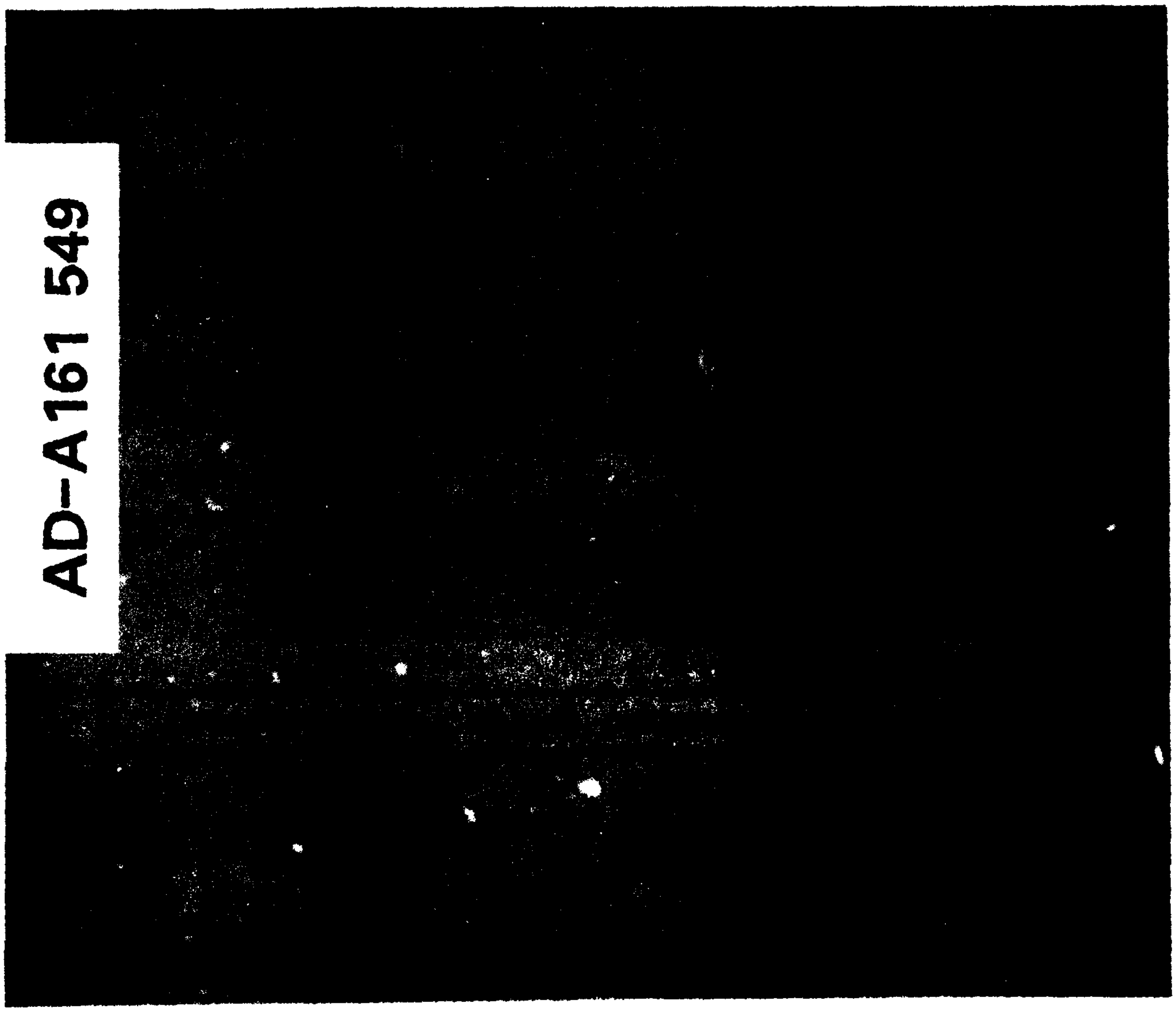
MULTIPLE-USER INTERFERENCE IN A FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATION SYSTEM USING DIFFERENTIAL PHASE-SHIFT KEYING



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MULTIPLE-USER INTERFERENCE IN A FREQUENCY-HOPPED SPREAD-SPECTRUM COMMUNICATION SYSTEM USING DIFFERENTIAL PHASE-SHIFT KEYING

BY

ALEX WAIHO LAM

B.S., University of Illinois, 1982

THESIS

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ABSTRACT

A design of short Reed-Solomon hopping sequences which have excellent correlation properties is proposed. Within the design, any two sequences will have at most one partial hit or one full hit when used asynchronously. The behavior of a near-optimum differential phase-shift keying (DPSK) receiver with multiple-user interference over a nonfading channel is discussed. Bit error rates for the receiver with single interference in one chip and the receiver with single interference in J distinct chips are found. Finally, generalization of the multiple-user interference model is considered.

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CHAPTER I

INTRODUCTION

In 1977, Cooper and Nettleton proposed a frequency-hopping multiple-access (FHMA) system for cellular mobile radio communications [1], [2]. The details and advantages of this system have been discussed extensively [3], [4] by previous authors and shall not be repeated here. Basically, half of the allotted mobile radio spectrum is used for base-to-mobile transmission and the other half for mobile-to-base. The data stream is partitioned into groups of n bits, and each group is encoded into $2^n = N$ channel bits using an (N,n) orthogonal code. The codewords are taken to be the rows of an $N \times N$ Hadamard matrix [5] with elements ± 1. Each of the N bits is transmitted using a different carrier frequency, according to the user's unique frequency-hopping pattern, or address. A differentially coherent phase-shift keying (DPSK) technique is used to modulate each bit onto its particular carrier frequency. The basic time-frequency encoded waveform can be written as [3]

$$s(t) = \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-kT_c)\cos(2\pi(f_0+a_k\Delta f)t+\theta_k) \tag{1.1}$$

where P denotes signal power, p_{T_c} is the rectangular pulse function:

$$p_{T_c}(t) = \begin{cases} 1 \ , & 0 \leq t < T_c \\ 0 \ , & \text{otherwise} \end{cases} \tag{1.2}$$

T_c is the time-chip duration, θ_k denotes the phase of the k-th radio-frequency (RF) pulse and contains the binary DPSK information, f_0 is the nominal carrier frequency, Δf is the minimum frequency shift, and a_k is the element of the address that specifies the k-th carrier frequency of the hopping pattern. If we let the waveform duration be T, we have that $T = NT_c$, where N is the number of chips per waveform. Each time-chip's phase is compared to the corresponding time-chip's phase in the previous waveform to obtain the DPSK information.

The model for the DPSK receiver used in [1] and [2] is shown in Fig. 1.1. Cooper and Nettleton studied the performance of this receiver via Monte Carlo computer simulation. Martersteck [3] has recently derived an optimum receiver for the signaling format just discussed (Eq. (1.1)) and has also studied a simplified version of the receiver which is nearly optimal for small signal-to-noise ratio (SNR). He has shown that the bit error rate performance of the near-optimum receiver is better than that of the Cooper-Nettleton receiver, in both nonfading and fading environments with no interference. An outline of the aforementioned derivation is briefly given below.

A receiver for the DPSK Cooper-Nettleton system (assuming a nonfading channel) must decide on the transmitted codeword based on the received waveform

$$r(t) = \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-kT_c)\cos(\omega_k t + \theta_k)$$

$$+ \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-T-kT_c)h_{ik}\cos(\omega_k t + \theta_k) + \eta(t) \qquad (1.3)$$

where $\omega_k \triangleq 2\pi(f_0+a_k\Delta f)$, h_{ik} is the ±1 element in the i-th row and k-th column of the Hadamard matrix, $\eta(t)$ is the additive white Gaussian noise (AWGN) due to receiver thermal noise, θ_k is a random phase variable, and $\underline{s}_i$ (the i-th row

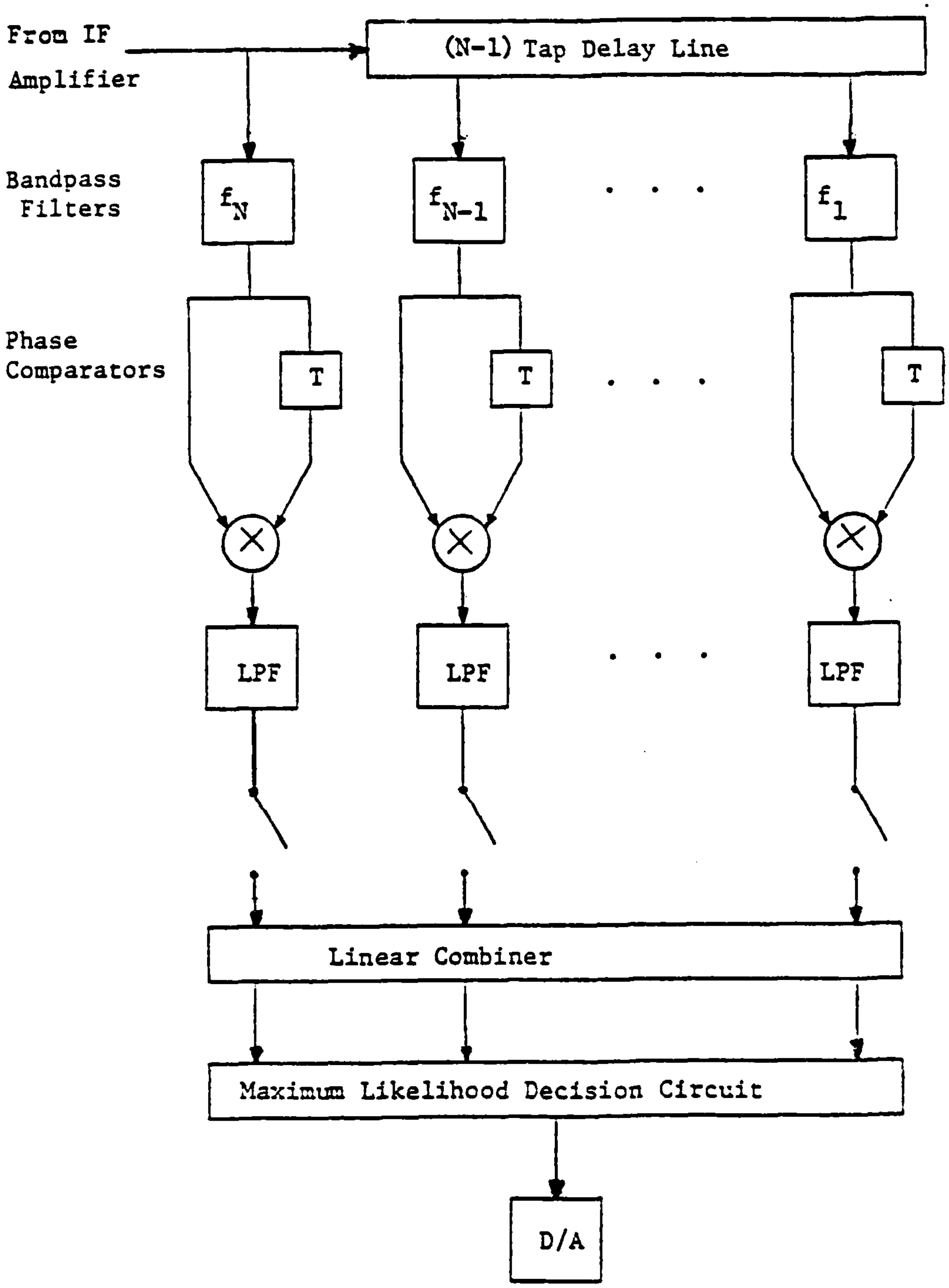

Figure 1.1. Cooper-Nettleton receiver analytical model [1].

of the Hadamard matrix) is the transmitted codeword. Let H_i denote the hypothesis that the i-th row ($\underline{s}_i$) of the Hadamard matrix is being transmitted. Defining

$$X_k = \int_{kT_c}^{(k+1)T_c} r(t)\cos\,\omega_k t\,dt, \quad Y_k = \int_{kT_c}^{(k+1)T_c} r(t)\sin\omega_k t\,dt$$

$$X_k' = \int_{T+kT_c}^{T+(k+1)T_c} r(t)\cos\omega_k t\,dt, \quad Y_k' = \int_{T+kT_c}^{T+(k+1)T_c} r(t)\sin\omega_k t\,dt \qquad (1.4)$$

and using optimum receiver theory, the optimum (nonfading) receiver will base its decision on the statistic

$$\Lambda(H_i) = \sum_{k=0}^{N-1} \ell n\{I_0(\{8PN_0^{-2}[(X_k+h_{ik}X_k')^2 + (Y_k+h_{ik}Y_k')^2]\}^{\frac{1}{2}})\},\ i=0,1,\ldots,N-1 \qquad (1.5)$$

where $I_0(\cdot)$ is the zero order modified Bessel function of the first kind. The optimum receiver must then calculate $\Lambda(H_i)$ for $i = 0,1,\ldots,N-1$ and decide $\underline{s}_i$ was transmitted whenever $\Lambda(H_i) > \Lambda(H_j)$ for all $j = 0,1,\ldots,N-1$ and $j \neq i$. In other words, the optimum receiver employs a maximum likelihood decision rule.

The optimum receiver structure can be simplified by approximating the function $\ell nI_0(x)$ in Eq. (1.5). In particular, $\ell nI_0(x) \approx x^2/4$ for $x << 1$, and $\ell nI_0(x) \approx x$ for $x >> 1$. Thus, for small bit SNRs, Eq. (1.5) becomes

$$\Lambda^*(H_i) = \sum_{k=0}^{N-1} [(X_k+h_{ik}X_k')^2 + (Y_k+h_{ik}Y_k')^2] \quad ,\ i = 0,1,\ldots,N-1 \qquad (1.6)$$

which further reduces to

$$\Lambda^*(H_i) = \sum_{k=0}^{N-1} (X_kX_k' + Y_kY_k')h_{ik} \quad ,\ i = 0,1,\ldots,N-1 \qquad (1.7)$$

Either of these two expressions determines the structure of the so-called small-SNR receiver. The former expression for $\Lambda^*(H_i)$ is easiest to work with analytically, whereas the latter expression is easiest to implement since fast Hadamard transforms may be used [4]. Curiously enough, the small-SNR receiver can also be shown to be optimum in a Rayleigh-fading environment provided that X_k and Y_k, $k = 0,1,\ldots,N-1$ are uncorrelated. For the scheme considered by Martersteck, the X_k and Y_k are nearly uncorrelated. Thus, the small-SNR receiver is nearly optimum over a fading channel. We shall adopt the convention of [3] and refer to the small-SNR receiver as the near-optimum receiver. A block diagram of this receiver is shown in Fig. 1.2.

For large chip SNRs, Eq. (1.5) yields

$$\Lambda^{**}(H_i) = \sum_{k=0}^{N-1} [(X_k+h_{ik}X_k')^2 + (Y_k+h_{ik}Y_k')^2]^{\frac{1}{2}} , \quad i = 0,1,\ldots,N-1 \qquad (1.8)$$

and we call the receiver induced by this expression the large-SNR receiver.

The upper bound (union bound) for the bit error rate performance was obtained analytically by Martersteck for the near-optimum receiver over a fading and nonfading channel with no interference. Recently, McClatchey [4] has found that the bit error probability for the large-SNR receiver and the near-optimum receiver (for a nonfading channel) are both upper bounded by the same function. Since the union bound is known to be tight for large SNRs, the two receivers thus have nearly the same performance at large SNRs. Furthermore, we would expect the near-optimum receiver performance to be better

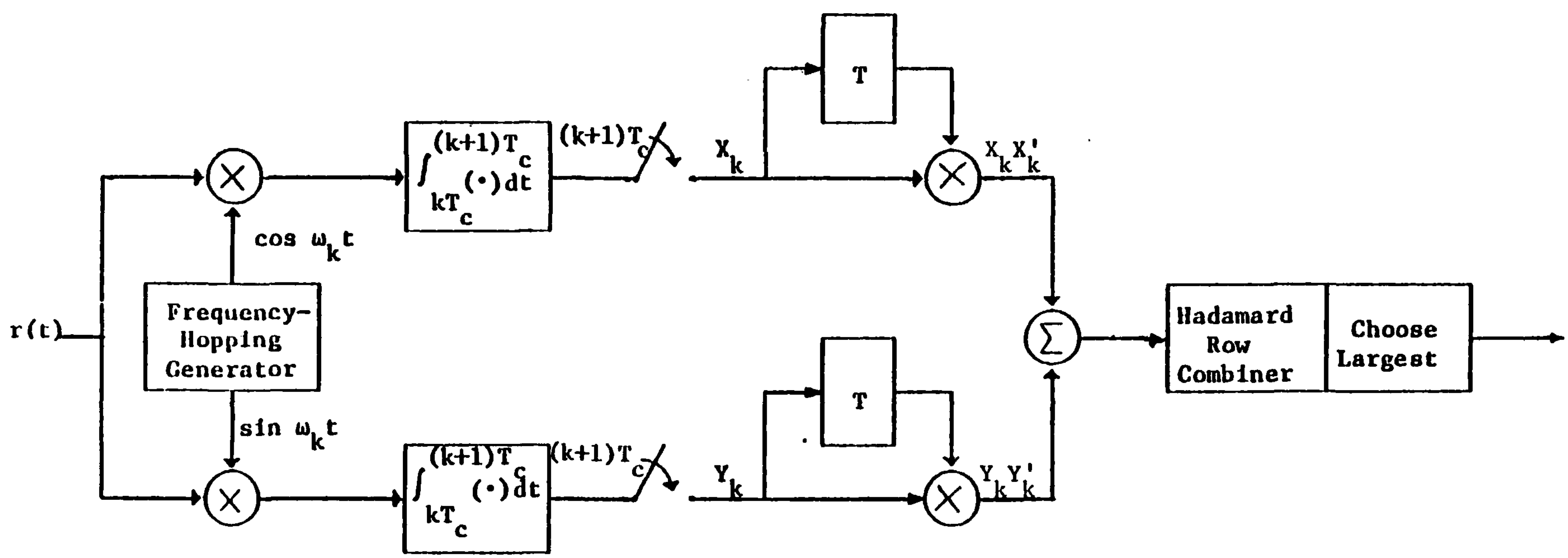

Figure 1.2. Block diagram of a near-optimum receiver model (from [3]).

than the large-SNR receiver performance (for a nonfading channel) at small SNRs, since the former receiver is based on a small-SNR approximation whereas the latter is based on a large-SNR approximation. Finally, we know that the near-optimum receiver is nearly optimum in a Rayleigh-fading channel. It follows that no further consideration of the large-SNR receiver is necessary.

Figure 1.3 (from [4]) shows the simulation data (via Monte Carlo simulations) for the optimum and near-optimum receiver for N = 32. Upper-bound performance for the near-optimum receiver and simulation performance for the Cooper-Nettleton receiver are also shown in Fig. 1.3. Note that the near-optimum receiver performance (simulated) is better than the Cooper-Nettleton receiver performance for SNRs above 3 dB in a nonfading no-interference environment. In a digital voice communication system, a bit error rate of about 10^{-3} is adequate for maintaining reliable communications. At N = 32, this corresponds to a bit SNR of about 8 to 10 dB. Since the union bound is tight for large bit SNRs, the upper-bound performance of the near-optimum receiver for normal usable ranges of bit SNR is thus expected to represent the receiver's true performance. This fact may be verified by Fig. 1.3 for N = 32 with a nonfading no-interference environment. In this thesis, it is assumed that the upper-bound bit error rate is a true performance measure for the near-optimum receiver when a sufficiently large bit SNR is used.

From all the facts that we have seen, the near-optimum receiver is by far the best receiver known for a Cooper-Nettleton system: it is relatively simple to implement, easy to synthesize, is analytically tractable, and has performance that is superior to the Cooper-Nettleton receiver for large bit SNRs in a no-interference environment.

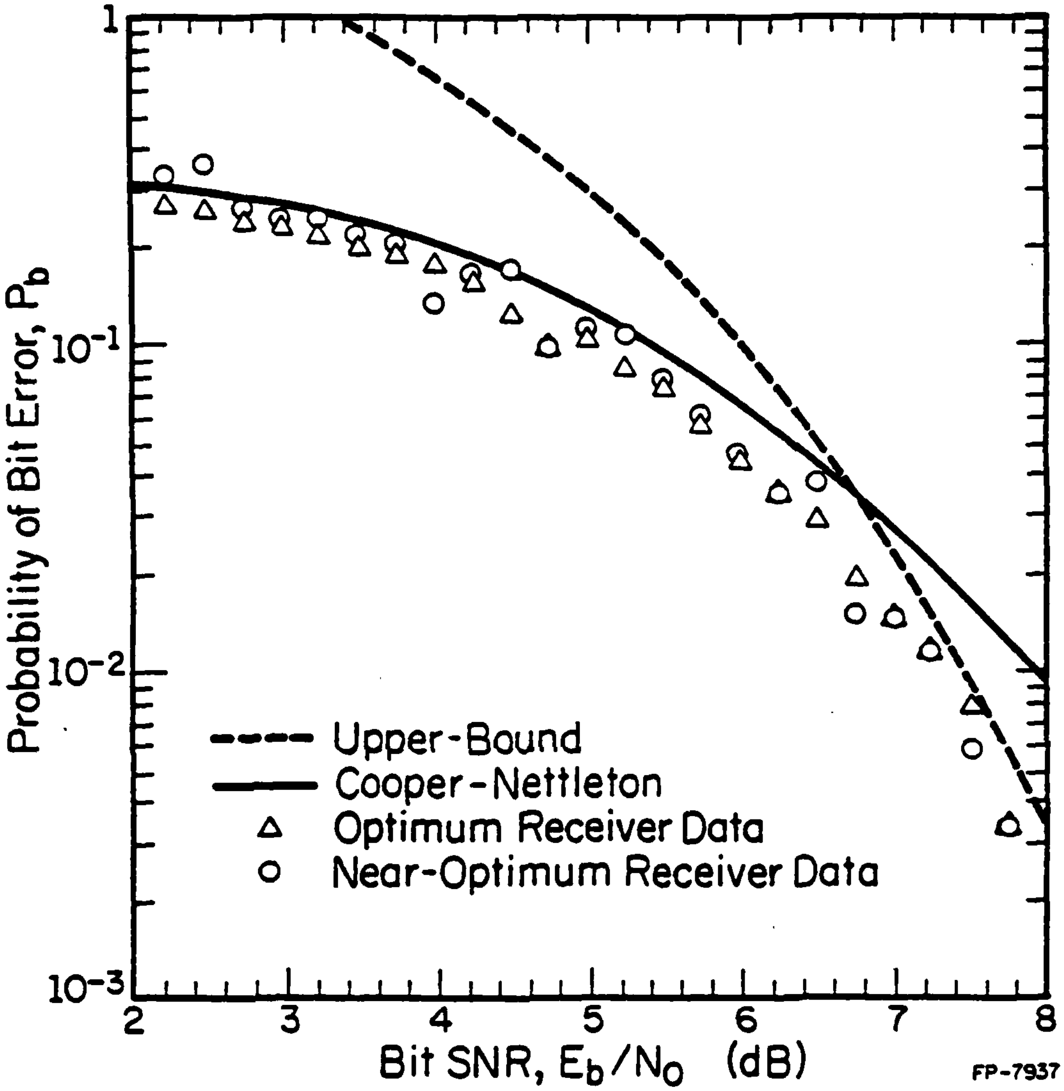

Figure 1.3. Comparison of bit error probability simulation data for the optimum and near-optimum receivers for a single-user, a nonfading channel, and N = 32. Also, corresponding upper-bound near-optimum receiver performance and Cooper-Nettleton receiver performance are shown (from [4]).

This thesis investigates one important system aspect which has not yet been considered: the multiple-user interference problem. In a FHMA system, users usually share the same frequency band, but their hopping sequences are not necessarily disjoint. Consequently, cross-user interference is bound to arise if large numbers of mobiles are transmitted simultaneously in the same cell. This multiple-user interference can degrade FHMA system performance rather seriously. Indeed, Yue [6] has estimated that the mobile-to-base number of users per cell for a DPSK/FHMA system with Cooper-Nettleton receivers can be as low as 26, assuming random addressing and bit error rates less than 10^{-3}. The number 26 is so low maybe because users are hopping randomly on the spectrum band. In this thesis, instead of random hopping, we assume that each user has its own specific nonrandom frequency-hopping sequence.

If the mobiles can transmit in a fashion such that the probability of having two or more mobiles transmitting the same frequency at the same time is small, then we may be able to keep the multiple-user interference below a tolerable level. In the analysis of [1], the multiple-user interference was modeled as a random white Gaussian process. We do not make the same assumption in this thesis. However, we assume that signals from all the mobile transmitters received by the receiver do have the same power.

From the foregoing content of this chapter, we know that each mobile is transmitting ± 1 channel bits (elements of the rows of a $N \times N$ Hadamard matrix) using frequencies according to its own unique frequency-hopping pattern. For $N = 4$, the Hadamard matrix is simply

$$\tilde{H} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} . \tag{1.9}$$

Suppose there are only two mobiles, say A and B, transmitting in the system. Without loss of generality, we may assume that A is transmitting $\underline{s}_0$, the 0-th row of $\tilde{H}$ (all ones), in the time slot [0,T]. If a receiver A^* at the base is decoding the message sent by A, A^* has to tune at A's transmitting frequencies from time to time using A's unique hopping sequence. The receiver A^* then computes $\Lambda^*(H_i)$ and chooses the hypothesis for which $\Lambda^*(H_i)$ is largest. However, in order to compare $\Lambda^*(H_0)$ and $\Lambda^*(H_1)$, it need only compare $(X_1X_1' + Y_1Y_1') + (X_3X_3' + Y_3Y_3')$ with $-(X_1X_1' + Y_1Y_1') - (X_3X_3' + Y_3Y_3')$ because $h_{00} = h_{10}$ and $h_{02} = h_{12}$. Now consider the signal of B. Suppose that B's signal is such that A and B are using the same frequency in the 2nd time-chip. However, note that in deciding whether $\Lambda^*(H_0)$ or $\Lambda^*(H_1)$ is larger, the quantities X_2X_2', Y_2Y_2' are not even considered. Thus, the presence of the interferer makes no difference in this decision. As a second case, consider if the interference from B occurred in the 1st (or 3rd) time-chip, then the comparison between $\Lambda^*(H_1)$ and $\Lambda^*(H_0)$ will inevitably be affected; but B may be transmitting a +1 or a -1 bit. If B is sending a +1 bit, the interference will be constructive. In other words, interference from B is actually helping A^* to decide correctly that A is transmitting a +1 bit, so the error probability decreases. On the other hand, if B is sending a -1 bit, the interference will be destructive, and the error probability increases. As a summary, in computing the probability $P(\Lambda^*(H_1) > \Lambda^*(H_0))$, an interference at the 0-th or 2nd time chips will have no effect on the probability, while an interference at the 1st or 3rd time-chips will be equally likely to be constructive or destructive. It is interesting to note that the system has a sort of 'built-in' anti-interference capability. This capability is a direct consequence of the system's orthogonal channel coding scheme, and is expected to increase with the codeword length N.

Likewise, we can compute $P(\Lambda^*(H_j) > \Lambda^*(H_i))$ for all i and j. It turns out that this probability does not depend on j or i. This is intuitively satisfactory because every codeword is equally likely to be transmitted, and the interference signal is randomly distributed among the time-chips. The word error probability can then be upper bounded by the union bound, which is simply $3 \cdot P(\Lambda^*(H_j) > \Lambda^*(H_i))$ in this case.

For multiple interference at multiple time-chip periods, the analysis is more involved, but it still follows a similar line of reasoning. In most cases, part of the interference signals will be rejected. Among the rest of the interferences which are considered by the receiver, some of them will be constructive, while the others will be destructive. We have shown that the word error probability is bounded by the union bound: $(N-1)P(\Lambda^*(H_j) > \Lambda^*(H_i))$. We will also show that the bit error probability is bounded by $\frac{N}{2} P(\Lambda^*(H_j) > \Lambda^*(H_i))$. The detailed development of this function is largely algebraic and manipulative in nature, and is therefore relegated to the Appendix.

The remainder of this thesis is organized as follows. In Chapter II we discuss a design of good frequency hopping sequences in which any two sequences will have very little interference over all time shifts asynchronously. In Chapter III we study the performance of the near-optimum receiver with only one interfering signal at a particular time-chip period. In Chapter IV we present an upper bound of the bit error rate for the near-optimum receiver with multiple-user interference assuming that each time-chip has at most one interferer. We also discuss possible generalizations of this special multiple-user interference model. Finally, in Appendix A, we present a derivation of a special probability function $P_e(u,x)$ which plays an essential role in this thesis.

CHAPTER II

HOPPING PATTERNS FOR THE FREQUENCY-HOPPED MULTIPLE ACCESS COMMUNICATION SYSTEM

In a frequency-hopped (FH) communication system, the spreading of the spectrum is achieved by hopping the frequency of the carrier signal at regular intervals. The hopping pattern is generated by applying a random or pseudorandom sequence of inputs to a frequency synthesizer. Typically, the available RF bandwidth is partitioned into q nonoverlapping frequency intervals called slots, and the q different frequencies generated by the frequency hopper are the center frequencies for these slots.

For the mobile communication system described in [1], some spectral division is made so that the mobile-to-base (upstream) band does not overlap the base-to-mobile (downstream) band. We may assume that the base is able to transmit signals to the mobiles synchronously. This will add a time delay to some of the speech communications, although the delay is usually negligible. We also assume that the separation between two center frequencies, Δf, is greater than $1/T_c$, T_c being the chip-time, so that there will be no interference between two adjacent frequency slots, and the FH-DPSK system will perform like a frequency division multiplexing system with the added advantage of being a spread-spectrum system.

For our purposes, a full-hit occurs whenever two transmitters are transmitting in the same frequency slot over a whole chip-time period, and a partial-hit occurs whenever two transmitters are transmitting in the same frequency slot over part of a chip-time period. Our objective in this chapter is to produce large numbers of frequency-hopping patterns for the upstream and downstream communications such that the number of hits between two hopping patterns is also small.

2.1 A Hopping Pattern Design for the Asynchronous Upstream Communications

In this case, the mobiles are free to transmit signals to the base asynchronously so the number of hits between two patterns (with periods N) over all possible time shifts must be considered. We may assume for the moment that the system is pseudoasynchronous (asynchronous but with chip-synchronization) so that we can consider the number of full-hits between two patterns. If $\underline{x} = (x_0, x_1, \ldots, x_{N-1})$ is a vector of length N, we define the left-shift operator T^i as follows:

$$T^i(\underline{x}) = (x_i, x_{i+1}, x_{i+2}, \ldots, x_{N-1}, x_0, \ldots, x_{i-1}), \quad 0 \leq i \leq N-1 \qquad (2.1)$$

If $\underline{y}$ is also a vector of length N, then the Hamming distance $d_H(\underline{x},\underline{y})$ between $\underline{x}$ and $\underline{y}$ is defined as

$$d_H(\underline{x},\underline{y}) = \sum_{i=0}^{N-1} x(x_i, y_i) \qquad (2.2)$$

where $x(j,k) = 1$ if $j \neq k$ or 0 if $j = k$ is an indicator function. It is clear that if $d_H(\underline{x},T^i(\underline{y}))$ is equal to N for all i, then there will be no full-hits between $\underline{x}$ and $\underline{y}$ over all time shifts. There are no partial hits either. On the other hand, if $d_H(\underline{x},T^i(\underline{y})) = N$ and $d_H(\underline{x},T^{i+1}(\underline{y})) = N-1$ (or vice versa) then there is a full hit for one time delay, no hits for another time delay, and a partial hit for any intermediate time delay. If $d_H(\underline{x},T^i(\underline{y})) = d_H(\underline{x},T^{i+1}(\underline{y})) = N-1$ for a fixed i, then it is possible to have two partial-hits between $\underline{x}$ and $\underline{y}$ when the chip-synchronization condition is lost. We shall present a construction method of designing a large class of hopping patterns in which any two patterns will have at most one partial (or full)-hit when used asynchronously; that is, if $d_H(\underline{x},T^i(\underline{y})) = N-1$, then $d_H(\underline{x},T^{i-1}(\underline{y})) = d_H(\underline{x},T^{i+1}(\underline{y})) = N$. First, however, consider the following.

Suppose GF(q) is a finite field ([7]) with α as a primitive element. Solomon ([8],[9]) had found a set of q Reed-Solomon (RS) sequences that can be considered as hopping patterns. These sequences are of the form $\underline{x} = a_x(1,1,\ldots,1) + (1,\alpha,\alpha^2,\ldots,\alpha^{N-1})$ with $N \leq q-1$, $a_x \in GF(q)$, α a primitive element of GF(q). Thus, the ℓ-th coordinate (entry), $0 \leq \ell \leq N-1$, of $\underline{x}$ is $x_\ell = a_x + \alpha^\ell$. The coordinates of $\underline{x}$ are not repeating simply because if $\ell \neq m$ $(0 \leq \ell,m \leq N-1 \leq q-2)$ then $a_x + \alpha^\ell \neq a_x + \alpha^m$ thus $x_\ell \neq x_m$. Note also that if $\underline{x}$ belongs to the set of sequences, then $T^i\underline{x}$ is not a member of the set. Consider now two distinct sequences $\underline{x}$ and $\underline{y}$ with $x_\ell = a_x + \alpha^\ell$, $y_\ell = a_y + \alpha^\ell$, $a_x \neq a_y$. Obviously $x_\ell \neq y_\ell$ for all ℓ, thus we have no hits in the synchronous case. This is equivalent to saying that $d_H(\underline{x},\underline{y}) = N$. But in the <u>pseudoasynchronous</u> (asynchronous system with chip-synchronization) case, $\underline{x}$ and $T^i(\underline{y})$ $(i \neq 0)$ can have at most one full-hit because the equation $a_x + \alpha^\ell = a_y + \alpha^{i+\ell}$ or $(1-\alpha^i)\alpha^\ell = a_x - a_y$ has exactly one solution in GF(q), and the solution corresponds to a hit only if $\ell \in \{0,1,\ldots,N-1\}$. Thus when $N < q-1$, $d_H(\underline{x},T^i(\underline{y})) = N-1$ for some i, $0 < i \leq N-1$; and when $N = q-1$, $d_H(\underline{x},T^i(\underline{y})) = N-1$ for all i, $0 < i \leq N-1$. Therefore, when $N = q-1$, any two sequences from the set will almost always have two partial-hits when used asynchronously (with no chip-synchronization). This is, of course, highly undesirable.

<u>Example</u>. Consider GF(11) so that $q = 11$, $N = 10$. Take $\alpha = 2$ $(2^{10} \equiv 1 \pmod{11})$, $a_x = 0$, $a_y = 1$. Hence $\underline{x} = (1,2,4,8,5,10,9,7,3,6)$, $\underline{y} = (2,3,5,9,6,0,10,8,4,7)$. It is easy to see that $d_H(\underline{x},\underline{y})) = N$ and $d_H(\underline{x},T^i(\underline{y})) = N-1 \;\forall\; i,\ 1 \leq i \leq 9$.

Although in the example we chose the length N to be equal to q-1, in general the sequences with length N < q-1 are more attractive because by suitable search among the set of the q sequences we may be able to find a few with no hits for some shifts. However, this method of generating RS sequences suffers a major disadvantage: only q patterns can be generated.

In order to generate a large class of hopping patterns with the desired property (maximum one hit between any two patterns asynchronously), we may consider sequences of length N < q-1. In particular, we assume N is a divisor of q-1. To this end, suppose a sequence $\underline{x}$ of length N is to be generated:

$$\underline{x} = a_x(1,1,1,\ldots,1) + b(1,\alpha^{W},\alpha^{2W},\ldots,\alpha^{(N-1)W}) \tag{2.3}$$

where $W = (q-1)/N$, $a_x \in GF(q)$ and $b \in GF(q)$. For $0 \leq j \leq W-1$, define a collection of sequences $G(\alpha^j)$ as

$$G(\alpha^j) = \{\underline{x} : a_x \in GF(q), \quad b = \alpha^j\} \tag{2.4}$$

then the set D of sequences,

$$D = \{\underline{x} : \underline{x} \in G(\alpha^j), \qquad 0 \leq j \leq W-1\} = \bigcup_{j=0}^{W-1} G(\alpha^j) \tag{2.5}$$

is a collection of nonconstant sequences since $b \neq 0$. From our previous discussion, it is obvious that each group, say $G(\alpha^j)$, contains q sequences of nonrepeating coordinates. Moreover, two sequences in the same group will have no full-hit when used synchronously, have at most one full-hit when used pseudoasynchronously, and are not cyclic shifts of each other. Furthermore, all the sequences in D are distinct and are not cyclic shifts of each other. To see this, let $\underline{x} \in G(\alpha^j)$ and $\underline{y} \in G(\alpha^k)$, $0 \leq j, k \leq W-1$. Suppose $\underline{x} = T^i(\underline{y})$ for some i, it means that for fixed a_x and j, we can find fixed

a_y, k, and i such that $a_x + \alpha^j\alpha^{\ell W} = a_y + \alpha^k\alpha^{(i+\ell)W}$ or $a_x - a_y = \alpha^{\ell W}\alpha^j[\alpha^{k+iW-j}-1]$ holds for all ℓ ($a_x \neq a_y$, $k \neq j$ so $\alpha^{k+iW-j} \neq 1$). But this is impossible. Actually since we take b (Eq. (2.3)) to be nonzero and $b = \alpha^j$ for $0 \le j \le W-1$ we automatically exclude all constant and cyclic sequences from D because $G(\alpha^{j+iW}) = T^i\{G(\alpha^j)\}$ for $0 \le j \le W-1$, $0 \le i \le N-1$, and $W = (q-1)/N$. Note that there can be at most one solution of ℓ to the equation $a_x - a_y = \alpha^{\ell W}\alpha^j[\alpha^{k+iW-j}-1]$ for fixed a_x, a_y ($a_x \neq a_y$), k,j ($k \neq j$), and i, so we can conclude that the maximum number of full-hits between any two sequences in D is 1 (if k = j, $\underline{x}$ and $\underline{y}$ are in the same group so we are done; if $a_x = a_y$, since $\alpha^{k+iW-j}-1 \neq 0$ so ℓ does not exist). Hence $d_H(\underline{x},T^i(\underline{y})) \ge N-1$ for $\underline{x},\underline{y} \in D$, $0 \le i \le N-1$.

Suppose $\underline{x},\underline{y} \in D$. Suppose also that $\underline{x} \in G(\alpha^j)$, $\underline{y} \in G(\alpha^k)$ and $d_H(\underline{x},T^i(\underline{y})) = N-1$ for some fixed i, $0 \le i \le N-1$. This means that exactly one coordinate, say the ℓ-th coordinate of $\underline{x}$ and $T^i(\underline{y})$ is the same. Hence if we let

$$\underline{x} = a_x(1,1,1,\ldots,1) + \alpha^j(1,\alpha^W,\alpha^{2W},\ldots,\alpha^{(N-1)W})$$

$$\underline{y} = a_y(1,1,1,\ldots,1) + \alpha^k(1,\alpha^W,\alpha^{2W},\ldots,\alpha^{(N-1)W}) \tag{2.6}$$

then we must have

$$a_x + \alpha^j\alpha^{\ell W} = a_y + \alpha^k\alpha^{(i+\ell)W} \tag{2.7}$$

Now if $\underline{r} \in G(\alpha^j)$, $\underline{r} = a_r(1,1,\ldots,1) + \alpha^j(1,\alpha^W,\ldots)$, then we claim that there is $\underline{s} \in G(\alpha^k)$ such that $d_H(\underline{r},T^i\underline{s})$ is also equal to N-1. In particular, $\underline{s}$ can be chosen with $a_s = a_r + (a_y - a_x)$ such that $\underline{r}$ and $T^i\underline{s}$ will collide at exactly the same ℓ-th coordinate. To see this, let

$$\underline{s} = a_s(1,1,\ldots,1) + \alpha^k(1,\alpha^W,\alpha^{2W},\ldots)$$

Then we must have

$$a_r + \alpha^j\alpha^{\ell W} = a_s + \alpha^k\alpha^{(i+\ell)W} \tag{2.8}$$

Combining Eqs. (2.7) and (2.8), we can solve for a_s. Thus, we have proved the following proposition.

Proposition 2.1. If $\underline{x} \in G(\alpha^j)$, $\underline{y} \in G(\alpha^k)$ and $d_H(\underline{x},T^i(\underline{y})) = N-1$ for some i, $0 \le i \le N-1$, then for every $\underline{r} \in G(\alpha^j)$, there is $\underline{s}$ in $G(\alpha^k)$ such that $d_H(\underline{r},T^i(\underline{s}))$ is also N-1.

In a second case, suppose $\underline{x},\underline{y}$ are defined as in Eq. (2.6) with $0 \le j$, $k \le W-2$. Then if $\underline{r} \in G(\alpha^{j+1})$, we claim that there is $\underline{s}$ in $G(\alpha^{k+1})$ such that $d_H(\underline{r},T^i(\underline{s}))$ is equal to N-1. Similarly, $\underline{s}$ can also be chosen with $a_s = a_r + \alpha(a_y-a_x)$ so that $\underline{r}$ and $T^i(\underline{s})$ will also collide at exactly the same ℓ-th coordinate. This is obvious once we note that

$$a_r + \alpha^{j+1}\alpha^{\ell W} = a_s + \alpha^{k+1}\alpha^{(i+\ell)W} \tag{2.9}$$

and solve for a_s together with Eq. (2.7). Observe that $G(\alpha^W) = T^1\{G(\alpha^0)\}$. Hence if k = W-1, everything that we have just stated applies except $\underline{s}$ will be in $G(\alpha^0)$ and $d_H(\underline{r},T^{i+1}\underline{s})$ is N-1. If j = W-1, $\underline{r}$ will be in $G(\alpha^0)$ and $d_H(T^{-1}\underline{r},T^i(\underline{s})) = N-1$.

Corollary 2.1. If $\underline{x} \in G(\alpha^j)$, $\underline{y} \in G(\alpha^k)$ and $d_H(\underline{x},T^i(\underline{y})) = N-1$ for some i, $0 \le i \le N-1$, then for every $\underline{r} \in G(\alpha^{j+1})$ there is $\underline{s}$ in $G(\alpha^{k+1})$ such that $d_H(\underline{r},T^i(s)) = N-1$. Note that $0 \le j$, $k \le W-1$, and $G(\alpha^W) = T^1\{G(\alpha^0)\}$.

Suppose now we want to know when $d_H(\underline{x},T^i(\underline{y}))$ and $d_H(\underline{x},T^{i+1}(\underline{y}))$ will both equal N-1. Intuitively, for a given field GF(q), this condition should depend on the pattern length N and hence on W. As usual, we assume that $\underline{x}$ and $T^i(\underline{y})$ collide exactly once at the ℓ-th coordinate. For $d_H(\underline{x},T^{i+1}(\underline{y}))$ to equal N-1, $\underline{x}$ and $T^{i+1}(\underline{y})$ must then collide exactly once at some other coordinate, say the m-th coordinate, where m is necessarily different from ℓ. Thus we have the simultaneous equations:

$$\begin{aligned} a_x + \alpha^j \alpha^{\ell W} &= a_y + \alpha^k \alpha^{(i+\ell)W} \\ a_y + \alpha^j \alpha^{mW} &= a_y + \alpha^k \alpha^{(i+1+m)W} \end{aligned} \tag{2.10}$$

which reduce to

$$\alpha^{mW}\alpha^j(\alpha^{(i+1)W+k-j}-1) = \alpha^{\ell W}\alpha^j(\alpha^{iW+k-j}-1) \tag{2.11}$$

Let $\alpha^{k_1} = \alpha^{iW+k-j}-1$, $\alpha^{k_2} = \alpha^{(i+1)W+k-j}-1$, and let α^{∞} denote 0. Then we may solve for m:

$$m = \ell + \frac{k_1-k_2}{W} \tag{2.12}$$

Since $\alpha^{q-1} = 1$ and m must be an integer, Eq. (2.12) makes sense only when (k_1-k_2) mod(q-1) is a multiple of W. Equivalently, $W|(k_1-k_2)$ mod(q-1). Note that in Eq. (2.11) $\alpha^{k_1} = 0$ only when k = j and i = 0; $\alpha^{k_2} = 0$ only when k = j and i = N-1. But then $\underline{x},\underline{y}$ will be in the same group and so $d_H(\underline{x},\underline{y}) = N$ and hence there is no need to consider $d_H(\underline{x},T^{i+1}(\underline{y}))$ for i = 0 and i = N-1. Thus we assume that we do not consider the case $k_1 = \infty$, or $k_2 = \infty$.

Proposition 2.2. Suppose $\underline{x} \in G(\alpha^j)$, $\underline{y} \in G(\alpha^k)$. For each i, $0 \leq i \leq N-1$, let k_1, k_2 be such that

$$\alpha^{k_1} = \alpha^{iW+k-j}-1 \neq \alpha^{\infty}$$

$$\alpha^{k_2} = \alpha^{(i+1)W+k-j}-1 \neq \alpha^{\infty}$$

If $W|(k_1-k_2)\bmod(q-1)$ for some i, then $d_H(\underline{x},T^i(\underline{y})) = d_H(\underline{x},T^{i+1}(\underline{y})) = N-1$ for these i. Otherwise, $d_H(\underline{x},T^i(\underline{y})) = d_H(\underline{x},T^{i+1}(\underline{y})) = N-1$ can never happen $\forall$ i.

Our problem now reduces to the extraction from D of a subset D_g of sequences which satisfy our constraints. For a fixed q, there are altogether $W \cdot q = (q-1)q/N$ sequences in D. To pick D_g from D, we do not have to generate all the sequences and compare them. Indeed, Proposition 2.2 provides a reasonably fast and easy algorithm to find all the good sequences. Meanwhile, let us introduce a few more definitions. We say a group $G(\alpha^s)$ is a good group if the maximum number of partial-hits between any two sequences within $G(\alpha^s)$ is less than or equal to one, otherwise, $G(\alpha^s)$ is a bad group. Likewise, a group $G(\alpha^r)$ is a relatively good group with respect to $G(\alpha^s)$ if the maximum number of partial-hits between any two sequences from the two groups is less than or equal to one. A set D_g of mutually good groups is such that all the groups inside D_g are relatively good groups with respect to one another.

Suppose now we have a given field GF(q), a primitive element α of the GF(q), and a fixed sequence length N, N divides q-1. Thus $W = (q-1)/N$ is determined.

We first set $j = 0$, $k = 0$ in Proposition 2.2. If $W|(k_1-k_2)\bmod(q-1)$ for some i, then there exists $\underline{x},\underline{y} \in G(\alpha^0)$ such that $d_H(\underline{x},T^i(\underline{y})) = D_h(\underline{x},T^{i+1}(\underline{y})) = N-1$ for these i. Hence there will be two partial-hits between $\underline{x}$ and $\underline{y}$ when they are used asynchronously. Thus $G(\alpha^0)$ is a <u>bad group</u>. Then Corollary 2.1 implies that $G(\alpha^1),G(\alpha^2),\ldots,G(\alpha^{W-1})$ are all bad groups, hence the whole set D is bad. Therefore, we have to start over with a larger q or a smaller N. Suppose $W|(k_1-k_2)\bmod(q-1)$ for no i, then $G(\alpha^0)$ is a <u>good group</u>. Hence Dg consists of at least a single group $G(\alpha^0)$ (or $G(\alpha^j)$, $0 \le j \le W-1$, but we shall always pick $G(\alpha^0)$). Note that checking the conditions in Proposition 2.2 is equivalent to comparing all the sequences (since $\underline{x},\underline{y}$ are arbitrary) and their time shifts from groups $G(\alpha^j)$ and $G(\alpha^k)$, but in a much simpler manner.

Next we set $j = 0, k = 1$ in Proposition 2.2 and check the stated conditions. Similarly, if $W|(k_1-k_2)\bmod(q-1)$ for some i, then $G(\alpha^1)$ is a <u>relatively bad</u> group with respect to $G(\alpha^0)$. Otherwise, $G(\alpha^1)$ is a <u>relatively good</u> group with respect to $G(\alpha^0)$. Proceeding with $j = 0$, $k = 2,3,\ldots,W-1$, we can then find all the <u>relatively bad</u> groups with respect to $G(\alpha^0)$, i.e.,the set of all relatively bad groups to $G(\alpha^0)$ is $\underline{RBG(\alpha^0)} = \{G(\alpha^{b_1}),G(\alpha^{b_2}),\ldots,G(\alpha^{b_n})\}$ for some b_n, $1 \le b_n \le W-1$. For convenience we shall write $RBG(\alpha^0) = \{b_1,b_2,\ldots,b_n\}$. Then Corollary 2.1 implies that the set of <u>relatively bad</u> groups with respect to $G(\alpha^1)$ is $\underline{RBG(\alpha^1)} = \{b_1+1,b_2+1,\ldots,b_n+1\}$. Inductively, all the $RBG(\alpha^j)$'s can be found without difficulties. Observe that Corollary 2.1 also implies that if $G(\alpha^r)$ is relatively good with respect to $G(\alpha^s)$, then $G(\alpha^{r+1})$ must be relatively good with respect to $G(\alpha^{s+1})$. With all this information, it is now extremely easy to construct D_g, the set of <u>mutually good</u> groups, by a simple deletion process.

<u>Example</u>. Consider GF(281) and let $\alpha = 3$ be a primitive element of GF(281).

The largest possible N is 8, so W = 35. D consists of $35 \times 281 = 9835$ codewords. Each codeword $\underline{x}$ is of the form $\underline{x} = a_x(1,1,\ldots,1) + b(1,60,228,192,280,221,53,89)$, where $a_x = 0,1,2,\ldots,280$, $b = 1,3,9,27,81,\ldots,194,20$, i.e., $b = \alpha^0, \alpha^1, \ldots \alpha^{34}$. Proposition 2.2 and Corollary 2.1 imply

$$RBG(\alpha^0) = \{7,10,13,22,25,28\}$$

$$RBG(\alpha^1) = \{8,11,14,23,26,29\}$$

$$RBG(\alpha^2) = \{9,12,15,24,27,30\}$$

$$RBG(\alpha^3) = \{10,13,16,25,28,31\}$$

$$\vdots$$

$$RBG(\alpha^{34}) = \{6,9,12,21,24,27\}$$

To find D_g, we first find the sets

$$D_0 = D - RBG(\alpha^0)$$

$$= \{0,1,2,3,4,5,6,8,9,11,12,14,16,17,18,19,20,21,23,24,26, 27,29,30,21,32,33,34\}$$

$$D_1 = D_0 - RBG(\alpha^1)$$

$$= \{0,1,2,3,4,5,6,9,12,15,16,17,18,19,20,21,24,27,30,31,32,33,34\}$$

$$D_2 = D_1 - RBG(\alpha^2)$$

$$\vdots$$

Then D_g is just the last set D_n at the end of this process. In this case, the process ends at the 8-th step, hence $n = |D_g| = 9$ (we always start with the 0-th step). Thus $D_g = \{G(\alpha^0), G(\alpha^1), G(\alpha^2), G(\alpha^3), G(\alpha^4), G(\alpha^5), G(\alpha^6), G(\alpha^{20}) G(\alpha^{21})\}$. Notice that the choice of D_g is not unique, but the number of sequences in D_g is fixed $(9 \times 281 = 2529)$.

Some good designs of D_g's with various q and N are exhibited in Table 2.1. Notice that for most cases $N_{max}/q < 0.1$. It is conjectured that there exists a stronger version of Proposition 2.2 which can provide a deeper understanding of the structural properties on D, hence on D_g. For $N/q < 0.03$, the codewords in the D_g's of Table 2.1 appear to be excellent, i.e., $d_H(\underline{x}, T^T(\underline{y})) = N-1$ for a very few number of i's when compared with N. In fact, the complete Hamming distance distributions of each D_g may be evaluated through the help of Proposition 2.1 and Corollary 2.1.

We have then succeeded in constructing a design D_g of good hopping patterns in which any two patterns will have at most one hit (partial or full) when used asynchronously.

2.2 A Hopping Pattern Design for the Synchronous Downstream Communication

In this case we assume that the base is able to transmit signals to the mobiles synchronously. This will add a time delay to some of the speech communications, although the delay is usually negligible.

Suppose we have $t \cdot q$ available frequencies in the downstream band (t an integer, q a prime or prime power). Partition the frequencies into t disjoint groups. For each group, map the q elements of the finite field GF(q) isomorphically to the q distinct frequencies. Then we apply Reed-Solomon (RS) sequences (with $N = q-1$) discussed in Section 2.1 to each group. Hence, for each group, we get q sequences of length $N = q-1$, and there will be no hits between any two sequences when used synchronously. Note that the method we mention here is just a special case of what we have discussed in Section 2.1.

TABLE 2.1

Some good designs D_g of GF(q). $G(\alpha^{b_n}) = \{\underline{x} = \underline{x} = a_x(1,1,\ldots,1) + \alpha^{b_n}(1, \alpha^W, \alpha^{2W}, \ldots, \alpha^{(N-1)W})\}$, where $W = (q-1)/N$.

q-1	α	$\{b_n : D_g = \{G(\alpha^{b_n})\}\}$		
		N=8	N=16	N=32
$256=2^8$	5	0,2,4,6,8,10,12,14,16,18,20 22,24,26,28,30	x (not possible)	x
$280=2^3\cdot 5\cdot 7$	3	0,1,2,3,4,5,6,20,21	x	x
$336=2^4\cdot 3\cdot 7$	10	0,1,2,3,16,17,18,19 32,33,34,35	0,1,2,3	x
$352=2^5\cdot 11$	3	0,1,2,3,4,5,6	0,1,7,8,15	0
$448=2^6\cdot 7$	3	- (possible, but not calculated)	0,1,4,8,21,25	0,2
$576=2^6\cdot 3^2$	5	-	0,1,2,3,8,9,10	0,1
$672=2^5\cdot 3\cdot 7$	5	-	0,1,6,7,10, 11,14,20	0,2
$768=2^8\cdot 3$	11	-	-	0,7
$1152=2^7\cdot 3^2$	3	-	-	0,16,20

In practice, the pattern length N is usually a power of 2. We know that 17 is a prime, so patterns of length N = 16 are obtainable. However, 33 is not a prime, hence patterns of length 32 may be constructed by annexing two patterns of length 16 together. This simple annexing procedure can certainly be carried out for larger N's. One other possibility is to use extended Reed-Solomon sequences. For example, when q = 33, we can first generate 32 sequences of length 31, and then to the end of each of these sequences we add a parity check element. These 32 extended sequences (of length 32) have no hits when used synchronously.

CHAPTER III

PROBABILITY OF ERROR ANALYSIS OF A NEAR-OPTIMUM FH-DPSK RECEIVER WITH ONE INTERFERENCE AND A NONFADING CHANNEL

In a multiple-access communication system, cross-user interference must be considered in order to gain a true measure of the system performance. Since a bit error rate performance analysis for a single-user and a non-fading channel has already been given in [3], we now describe a similar analysis of the near-optimum FH-DPSK receiver with at most one interference at the n-th time-chip over a nonfading channel.

Suppose that the system is chip-synchronous. If the i-th codeword is being transmitted, the DPSK receiver must consider two frames of data in order to make a decision. Moreover, if the n-th time-chip is interfered (a <u>full-hit</u>) by another user transmitting the ℓ-th codeword with the same power, then the receiver input over the 2T-second interval may be represented as

$$r(t) = \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-kT_c)[\cos(\omega_k t+\theta_k) + \delta_{kn}\cos(\omega_k t + \phi_m)]$$

$$+ \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-T-kT_c)[h_{ik}\cos(\omega_k t + \theta_k) + \delta_{kn} h_{\ell m}\cos(\omega_k t + \phi_m)]$$

$$+ n(t) \tag{3.1}$$

where h_{ik} is the ±1 element of the i-th row and k-th column of the Hadamard matrix, $n(t)$ is white Gaussian noise (WGN) with two-sided spectral density

$\frac{1}{2}N_0$ that is thermally generated in the receiver, and δ_{kn} is the Kronecker delta function. The Θ_k's and Φ_m are modelled as independent identically distributed (i.i.d.) random variables, with uniform densities over $(0,2\pi]$ since we are employing noncoherent phase signaling.

Recall (Eq. (1.6)) that the near-optimum receiver computes

$$\Lambda(H_j) = \sum_{k=0}^{N-1} [(X_k+h_{jk}X_k')^2 + (Y_k+h_{jk}Y_k')^2] \ , \quad j = 0,1,\ldots,N-1 \tag{3.2}$$

and decides the hypothesis H_i whenever $\Lambda(H_i) > \Lambda(H_j)$ for $j = 0,1,\ldots,N-1$ and $j \neq i$. Then given the i-th codeword was transmitted so that H_i is true, the conditional word probability that the receiver makes an incorrect decision is

$$P[\epsilon|H_i] = P\left[\left\{\bigcup_{\substack{j=0\\ j\neq i}}^{N-1} [\Lambda(H_j) > \Lambda(H_i)]\right\} \Big| H_i\right] \tag{3.3}$$

Equation (3.3) is bounded from above by the union bound, thus

$$P[\epsilon|H_i] \leq \sum_{\substack{j=0\\ j\neq i}}^{N-1} P[\Lambda(H_j) > \Lambda(H_i)|H_i] \tag{3.4}$$

Conditioning on H_ℓ, $\underline{\Theta}$, and Φ_m, and by using Eq. (3.2) we obtain

$$\begin{aligned} &P[\Lambda(H_j) > \Lambda(H_i)|H_i,H_\ell,\underline{\Theta},\Phi_m] \\ &= P\Big[\sum_{k \in \mathcal{J}_j} \{(X_k-h_{ik}X_k')^2+(Y_k-h_{ik}Y_k')^2\} \\ &> \sum_{k \in \mathcal{J}_j} \{(X_k+h_{ik}X_k')^2+(Y_k+h_{ik}Y_k')^2\}|H_i,H_\ell,\underline{\Theta},\Phi_m\Big] \end{aligned} \tag{3.5}$$

where $\mathcal{J}_j = \{k: h_{ik} = -h_{jk}\}$. Here the terms for which $h_{ik} = h_{jk}$ are disregarded since they do not affect the comparison. Also, $\mathcal{J}_j$ will contain exactly N/2 elements by the orthogonal property between the rows of the Hadamard matrix.

From [3], we know that all the random variables appeared in Eq. (3.5) are mutually independent Gaussian r.v.'s. By definitions (Eq. (1.4)), we calculate

$$
\begin{aligned}
E[X_k|H_i,H_\ell,\underline{\Theta},\phi_m] &= \sqrt{P/2}\; T_c[\cos\theta_k + \delta_{kn}\cos\phi_m] \\
E[Y_k|H_i,H_\ell,\underline{\Theta},\phi_m] &= -\sqrt{P/2}\; T_c[\sin\theta_k + \delta_{kn}\sin\phi_m] \\
E[X_k|H_i,H_\ell,\underline{\Theta},\phi_m] &= \sqrt{P/2}\; T_c[h_{ik}\cos\theta_k + \delta_{kn}h_{\ell m}\cos\phi_m] \\
E[Y_k|H_i,H_\ell,\underline{\Theta},\phi_m] &= -\sqrt{P/2}\; T_c[h_{ik}\sin\theta_k + \delta_{kn}h_{\ell m}\sin\phi_m]
\end{aligned}
\tag{3.6}
$$

and obtain that each r.v. has common conditional variance $N_oT_c/4$.

We then further define four normalized independent Gaussian r.v.'s as

$$
A_k \triangleq \frac{X_k - h_{ik}X'_k}{\sqrt{N_0T_c/2}}\,, \quad B_k \triangleq \frac{Y_k - h_{ik}Y'_k}{\sqrt{N_0T_c/2}}
$$

$$
C_k \triangleq \frac{X_k + h_{ik}X'_k}{\sqrt{N_0T_c/2}}\,, \quad D_k \triangleq \frac{Y_k + h_{ik}Y'_k}{\sqrt{N_0T_c/2}}
\tag{3.7}
$$

so that their respective distributions are

$$
A_k \sim \begin{cases} \eta(0,1) & k \neq n \\ \eta(\sqrt{PT_c/N_0}[1-h_{in}h_{\ell m}]\cos\phi_m, 1) & k = n \end{cases}
$$

$$
B_k \sim \begin{cases} \eta(0,1) & k \neq n \\ \eta(-\sqrt{PT_c/N_0}[1-h_{in}h_{\ell m}]\sin\phi_m, 1) & k = n \end{cases}
$$

$$
C_k \sim \begin{cases} \eta(2\sqrt{PT_c/N_0}\cos\theta_k, 1) & k \neq n \\ \eta\left(2\sqrt{PT_c/N_0}\left[\cos\theta_n + \frac{(1+h_{in}h_{\ell m})}{2}\cos\phi_m\right], 1\right) & k = n \end{cases}
$$

$$D_k \sim \begin{cases} \eta(-2\sqrt{PT_c/N_0}\,\sin\theta_k, 1) & k \neq n \\ \eta(-2\sqrt{PT_c/N_0}[\sin\theta_n + \frac{(1+h_{in}h_{\ell m})}{2}\sin\phi_m], 1) & k = n \end{cases} \tag{3.8}$$

where $\eta(\mu,1)$ denotes the standard Gaussian r.v. with mean μ and unit variance. Consequently, our problem in solving Eq. (3.5) is thus reduced to determining

$$P[A > B | H_i, H_\ell, \underline{\Theta}, \Phi_m] = \int_{\beta=0}^{\infty} \int_{\alpha=\beta}^{\infty} f_{AB}(\alpha,\beta)d\alpha d\beta = \int_0^{\infty} f_B(\beta) \int_{\beta}^{\infty} f_A(\alpha)d\alpha d\beta \tag{3.9}$$

where $A = \sum_{k \in \mathscr{J}_j} [A_k^2 + B_k^2]$ and $B = \sum_{k \in \mathscr{J}_j} [C_k^2 + D_k^2]$ are independent. A general solution to this complex probability function can be found in Appendix A in which the results are stated in terms of the parameters

$$s = \sum_{k \in \mathscr{J}_j} \{(E[A_k]^2 + (E[B_k])^2\} \tag{3.10}$$

and

$$s' = \sum_{k \in \mathscr{J}_j} \{(E[C_k])^2 + (E[D_k])^2\} \tag{3.11}$$

Since the single interfnece occurs at the n-th time-chip (n arbitrary), and the h_{ik}'s are equally likely to be +1 or -1, we have to consider the following three cases separately.

<u>Case (i)</u>. $n \notin \mathscr{J}_j = \{k: h_{ik} = -h_{jk}\}$.

Obviously this case happens with probability ½ since $|\mathscr{J}_j| = N/2$. It also implies that the interference signal will not appear in the comparison of Eq. (3.5). Hence $s = 0$, and $s' = 2PT/N_0$. The analysis is thus reduced to the simpler no-interference case discussed in [3]. In [3] (or see Appendix A) it was shown that

$$P[\Lambda(H_j) > \Lambda(H_i) | H_i, H_\ell, \underline{\Theta}, \Phi_m] = P[\Lambda(H_j) > \Lambda(H_i) | H_i, \underline{\Theta}] \triangleq p$$

$$= 2^{-N/2} e^{-PT/2N_0} \sum_{k=0}^{N/2-1} 2^{-k} \sum_{\ell=0}^{k} \frac{(PT/2N_0)^\ell}{\ell!} \binom{N/2+k-1}{k-\ell} \tag{3.12}$$

Since the result does not depend on $\underline{\Theta}$ (or j or i, as might be intuitively expected because the rows of the Hadamard matrix are all orthogonal and have equal probabilities of being transmitted), the upper bound on the conditional (on Case (i)) word error probability are obtained directly:

$$P[\epsilon | (i)] \leq (N-1)p \tag{3.13}$$

The relationship between the conditional (on Case (i)) bit error probability ($\triangleq P_b(i)$) and the conditional word error probability is given in [5] as

$$P_b(i) = \frac{2^{n-1}}{2^n-1} P[\epsilon | (i)] = \frac{N/2}{N-1} P[\epsilon | (i)] \tag{3.14}$$

Substituting Eq. (3.13) into Eq. (3.14) we obtain

$$P_b(i) \leq \frac{N}{2} p = P_e(0, PT/2N_0) \tag{3.15}$$

where $P_e(u,x)$ is defined in Eq. (A.15).

Case (ii). $n \in \mathcal{J}_j$, and $h_{in} h_{\ell m} = -1$.

Note that $P[h_{in} h_{\ell m} = \pm 1] = \frac{1}{2}$. This case thus occurs with probability $\frac{1}{4}$. From Eq. (3.10) and Eq. (3.11) we get

$$s = \frac{4PT_c}{N_0} [\cos^2\phi_m + \sin^2\phi_m] = \frac{4PT_c}{N_0} \text{ and } s' = \frac{2PT}{N_0} \tag{3.16}$$

It then follows easily from Appendix A that the conditional (on Case (ii)) bit error probability is given by

$$P_b(ii) \leq P_e\left(\frac{PT_c}{N_0}, \frac{PT}{2N_0}\right) \tag{3.17}$$

(Case (iii). $n \in \mathcal{J}_j$, and $h_{in} h_{\ell m} = +1$.

Similarly this case also occurs with probability ¼. Although s is found to be zero, the analysis here is still more involved since

$$s' = \frac{2PT}{N_0} + \frac{4PT_c}{N_0}\left[2\cos\theta_n\cos\phi_m + \cos^2\phi_m + 2\sin\theta_n\sin\phi_m + \sin^2\phi_m\right]$$

$$= \frac{2PT}{N_0} + \frac{4PT_c}{N_0}[1 + 2\cos\psi_{nm}] \tag{3.18}$$

where $\psi_{nm} = \theta_n - \phi_m$, modulo 2π, is not a constant. Observe that θ_n and ϕ_m are i.i.d. r. v.'s with uniform distribution over $(0,2\pi]$, we can therefore show that Ψ_{nm} is also uniform in $(0,2\pi]$. With this information in mind, and by Fact A.2 in Appendix A, we thus obtain the conditional (on Case (iii)) bit error probability by removing the conditioning on Ψ_{nm}:

$$P_b(iii) = E\left\{P_e\left(0, X = \frac{PT}{2N_0} + \frac{PT_c}{N_0} + \frac{2PT_c}{N_0}\cos\Psi_{nm}\right)\right\}$$

$$\leq \tfrac{1}{2} P_e\left(0, \frac{PT}{2N_0} + \frac{3PT_c}{N_0}\right) + \tfrac{1}{2} P_e\left(0, \frac{PT}{2N_0} - \frac{PT_c}{N_0}\right) \tag{3.19}$$

Combining all the results from these three cases, we thus successfully provide an upper bound of the bit-error probability for the chip-synchronous system with single interference as

$$P_b \leq \tfrac{1}{2} P_b(i) + \tfrac{1}{4} P_b(ii) + \tfrac{1}{4} P_b(iii) \tag{3.20}$$

where $P_b(i)$, $P_b(ii)$, and $P_b(iii)$ are given by Eqs. (3.15), (3.17), and (3.19), respectively.

Although it took some effort to have arrived at Eq. (3.20), the upper bound is still not general. In fact, in a realistic system, chip-synchronization between the transmitters is almost unrealizable. As a consequence, the receiver will experience a single partial-hit (we avoid the possibility that two or more partial-hits may actually happen by time shifting, also see Chapter II) rather than a single full-hit at the n-th time-chip. This problem may be remedied by annexing an independent r.v.,Σ, with uniform distribution over (0,1) to the interference signal. Thus Eq. (3.8) becomes

$$A_k \sim \begin{cases} \eta(0,1) & k \neq n \\ \eta(\sigma\sqrt{PT_c/N_0}\,[1-h_{in}h_{\ell m}]\cos\phi_m,1) & k=n \end{cases}$$

$$B_k \sim \begin{cases} \eta(0,1) & k \neq n \\ \eta(-\sigma\sqrt{PT_c/N_0}\,[1-h_{in}h_{\ell m}]\sin\phi_m,1) & k=n \end{cases}$$

$$C_k \sim \begin{cases} \eta(2\sqrt{PT_c/N_0}\,\cos\theta_k,1) & k \neq n \\ \eta(2\sqrt{PT_c/N_0}\,[\cos\theta_n + \sigma\frac{(1+h_{in}h_{\ell m})}{2}\cos\phi_m],1) & k=n \end{cases}$$

$$D_k \sim \begin{cases} \eta(-2\sqrt{PT_c/N_0}\,\sin\theta_k,1) & k \neq n \\ \eta(-2\sqrt{PT_c/N_0}\,[\sin\theta_n + \sigma\,\frac{(1+h_{in}h_{\ell m})}{2}\sin\phi_m],1) & k=n \end{cases} \tag{3.21}$$

where $\sigma \in (0,1)$.

Conditioning on Σ, the conditional bit-error probabilities for a partial-hit are easily found to be

$$\tilde{P}_b(i|\Sigma = \sigma) \leq P_e(0,PT/2N_0) \tag{3.22}$$

$$\tilde{P}_b(ii|\Sigma = \sigma) \leq P_e(PT_c\sigma^2/N_0,PT/2N_0) \tag{3.23}$$

$$\tilde{P}_b(iii|\Sigma = \sigma) \leq \tfrac{1}{2} P_e\left(0,\frac{PT}{2N_0}+\frac{PT_c}{N_0}(\sigma^2+2\sigma)\right) + \tfrac{1}{2} P_e\left(0,\frac{PT}{2N_0}+\frac{PT_c}{N_0}(\sigma^2-2\sigma)\right) \tag{3.24}$$

which are similar to Eqs. (3.15), (3.17), and (3.19). Note that $P_e(PT_c\sigma^2/N_0,PT/2N_0) \leq P_e(PT_c\sigma/N_0,PT/2N_0)$ which is convex in σ, $P_e(0,PT/2N_0 + PT_c/N_0(\sigma^2+2\sigma))$ is convex and monotone decreasing in σ, and $P_e(0,PT/2N_0 + PT_c/N_0(\sigma^2-2\sigma))$ is monotone increasing in σ (see Appendix A, Fact A.3) $\forall\ \sigma \in (0,1)$, we thus obtain

$$\tilde{P}_b(i) \leq P_e(0,PT/2N_0) \tag{3.25}$$

$$\tilde{P}_b(ii) \leq \tfrac{1}{2} P_e(0,PT/2N_0) + \tfrac{1}{2} P_e(PT_c/N_0,PT/2N_0) \tag{3.26}$$

$$\tilde{P}_b(iii) \leq \tfrac{1}{4} P_e(0,PT/2N_0) + \tfrac{1}{4} P_e(0,PT/2N_0 + 3PT_c/N_0) + \tfrac{1}{2} P_e(0,PT/2N_0-PT_c/N_0) \tag{3.27}$$

by removing the conditioning on Σ. Finally, the average bit-error probability for the chip-asynchronous system with single partial interference is bounded by Eq. (3.20)

$$\begin{aligned}\tilde{P}_b &\leq \tfrac{1}{2}\tilde{P}_b(i) + \tfrac{1}{4}\tilde{P}_b(ii) + \tfrac{1}{4}\tilde{P}_b(iii) \\ &\leq \frac{11}{16} P_e\left(0,\frac{PT}{2N_0}\right) + \frac{1}{8} P_e\left(\frac{PT_c}{N_0},\frac{PT}{2N_0}\right) + \frac{1}{16} P_e\left(0,\frac{PT}{2N_0}+\frac{3PT_c}{N_0}\right) + \frac{1}{8} P_e\left(0,\frac{PT}{2N_0} - \frac{PT_c}{N_0}\right)\end{aligned} \tag{3.28}$$

For the chip-synchronous system with single full interference, the upper bound (Eq. (3.20)) is

$$P_b \leq \tfrac{1}{2} P_b(i) + \tfrac{1}{4} P_b(ii) + \tfrac{1}{4} P_b(iii)$$

$$\leq \tfrac{1}{2} P_e(0,PT/2N_0) + \tfrac{1}{4} P_e(PT_c/N_0,PT/2N_0) + 1/8\, P_e(0,PT/2N_0 + 3PT_c/N_0) + 1/8\, P_e(0,PT/2N_0 - PT_c/N_0) \quad (3.29)$$

On the other hand, if the receiver encounters no interference, the upper bound [2] is just

$$P_b \leq P_e(0,PT/2N_0) \quad (3.30)$$

These bounds were evaluated in terms of the bit SNR, E_b/N_0, for several values of N, and are shown graphically in Figures 3.1-3.7. Note that $PT/N_0 = N\, PT_c/N_0 = n\, E_b/N_0$.

As one might expect, the upper bound is largest in case of a full-hit for all N's, since the receiver encounters the most interference energy. Figures 3.3 and 3.4 show that the anti-interference capability of the receiver increases drastically for larger N's. This is also intuitively expected since more signal energy is present relative to the interference signal energy, for larger N's. However, there is a trade-off between the code lengths and the receiver performance because receiver complexity also increases significantly with N.

At a bit error rate of 10^{-3}, Figures 3.5, 3.6, and 3.7 indicate that a code length of N = 32 is optimum for the receiver with no-hit, single partial-hit, and single full-hit, respectively. Moreover, for N = 32, bit SNR's of about 8.2 dB, 8.3 dB, and 8.4 dB are adequate for a bit-error rate of 10^{-3} for the three cases, respectively.

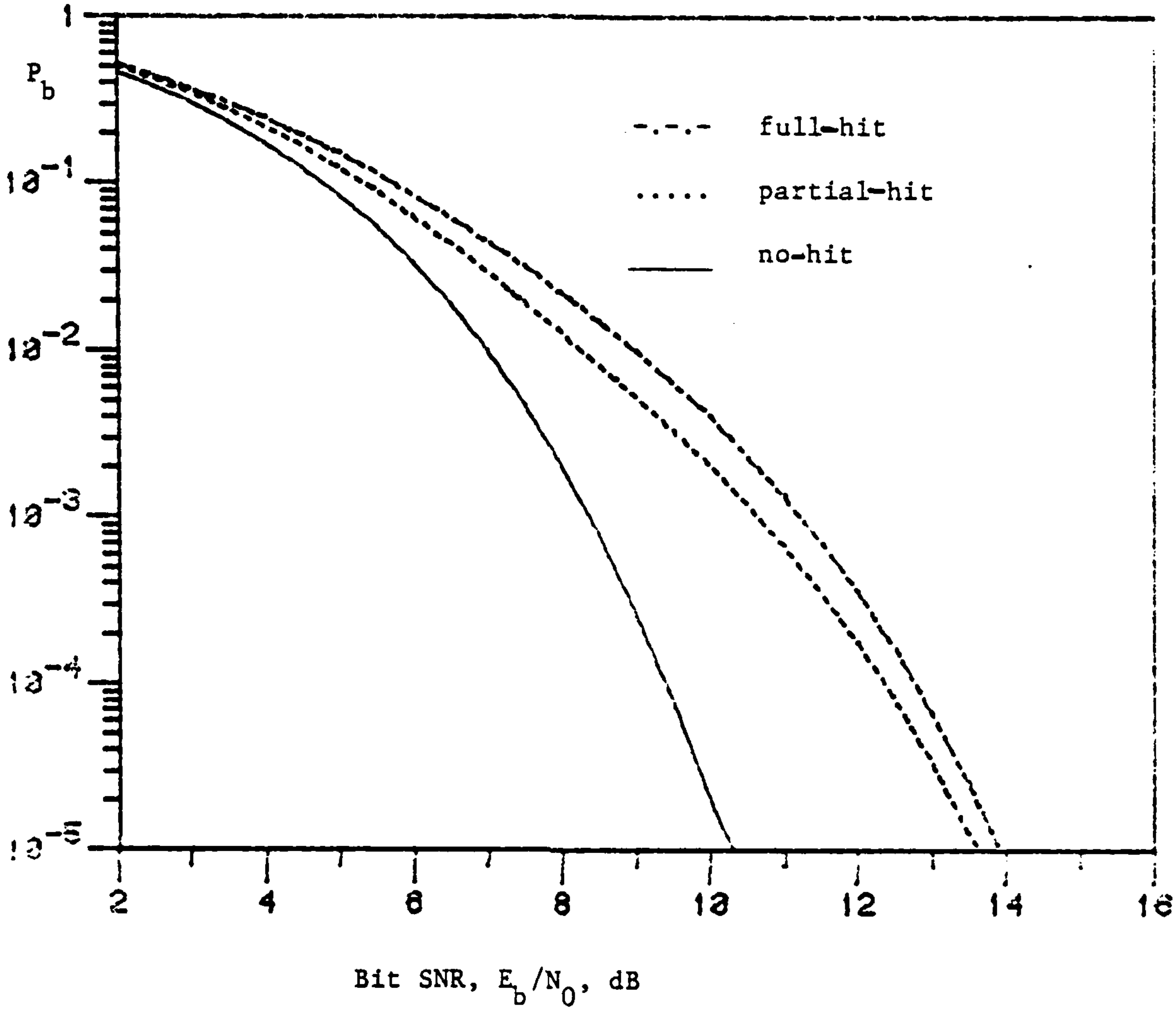

Figure 3.1. Upper bounds of average bit-error probabilities for the near-optimum receiver with a single interference and nonfading channel. N = 8.

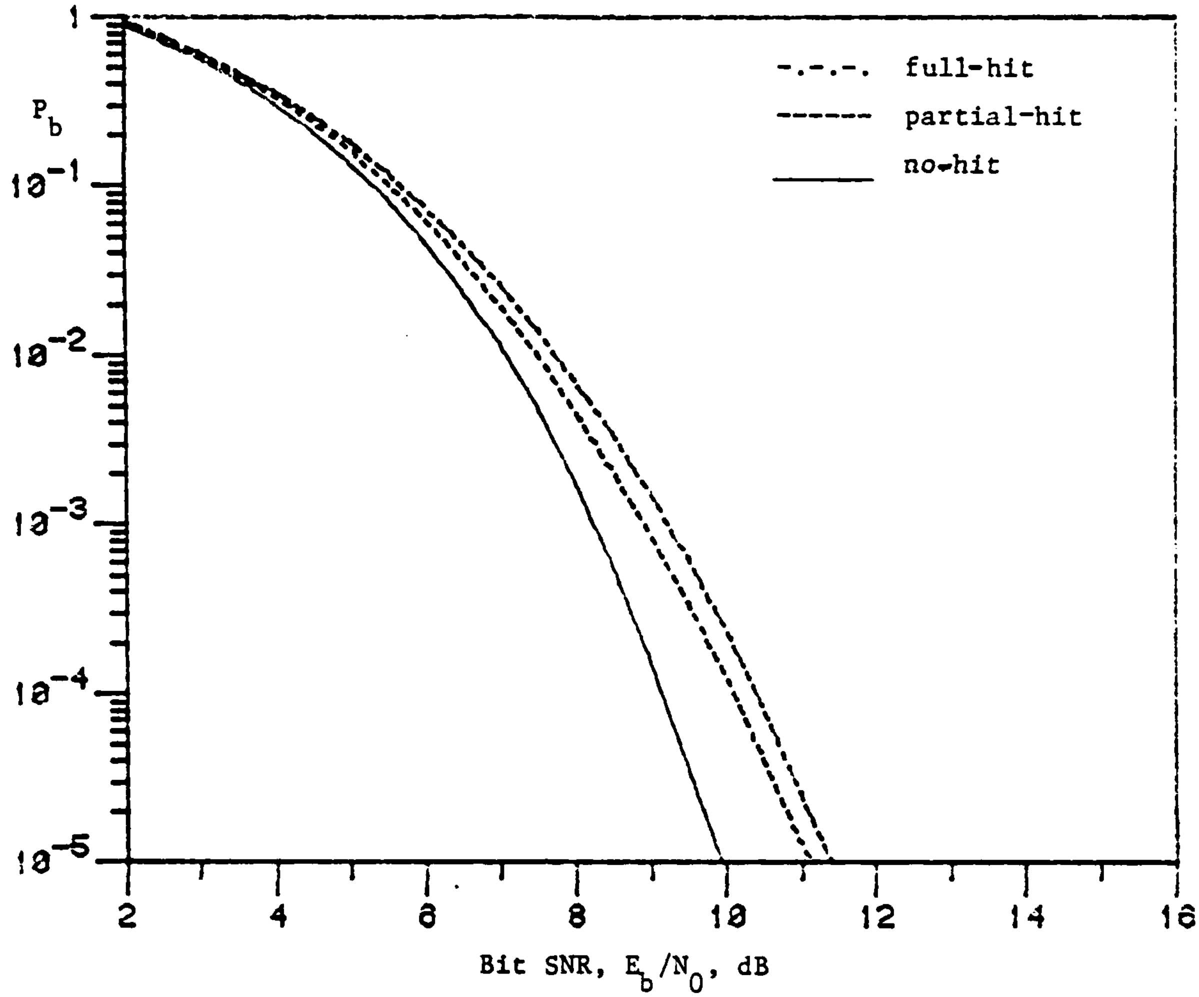

Figure 3.2. Upper bounds of average bit-error probabilities for the near-optimum receiver with a single interference and nonfading channel. N = 16.

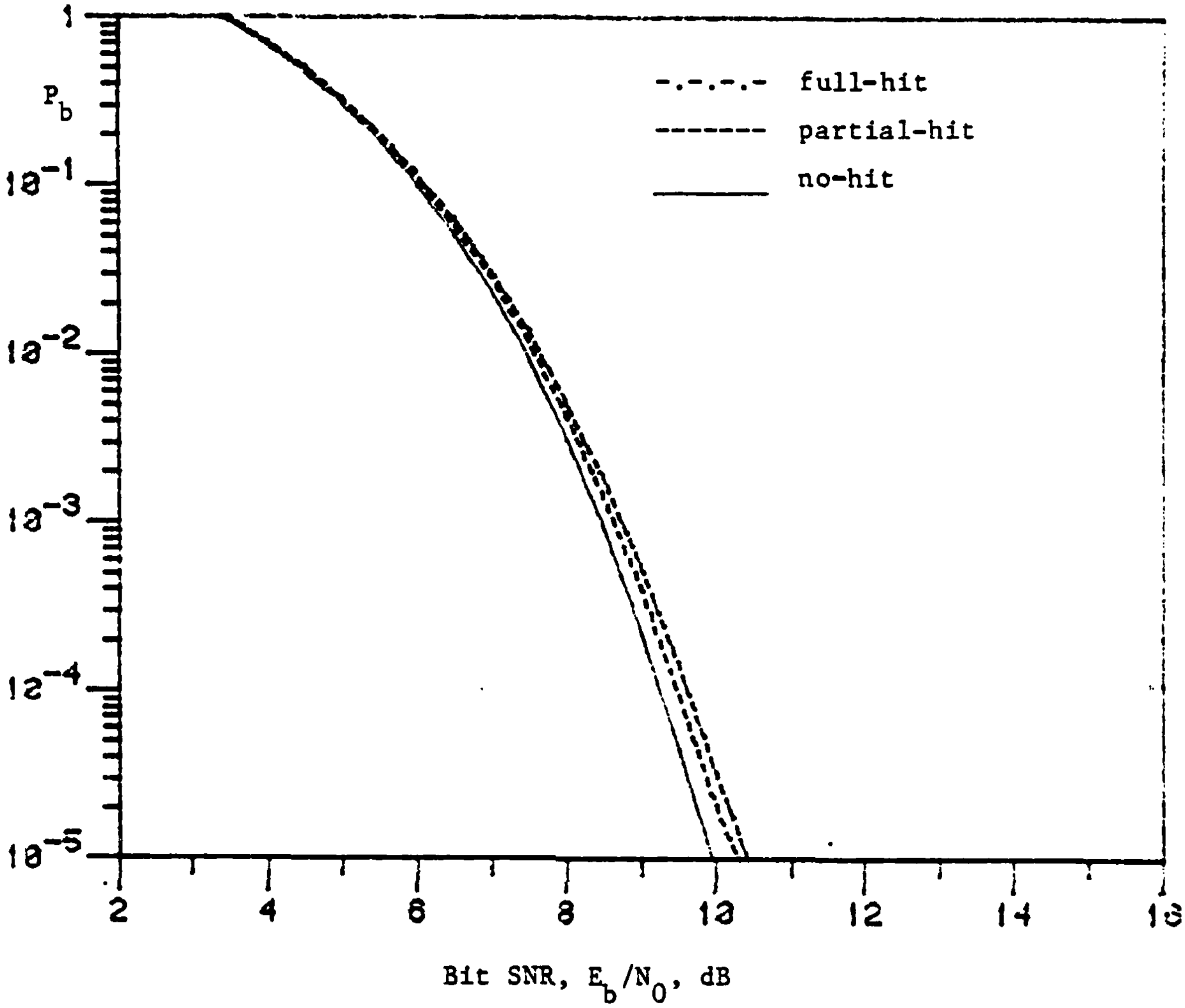

Figure 3.3. Upper bounds of average bit-error probabilities for the near-optimum receiver with a single interference and nonfading channel. N = 32.

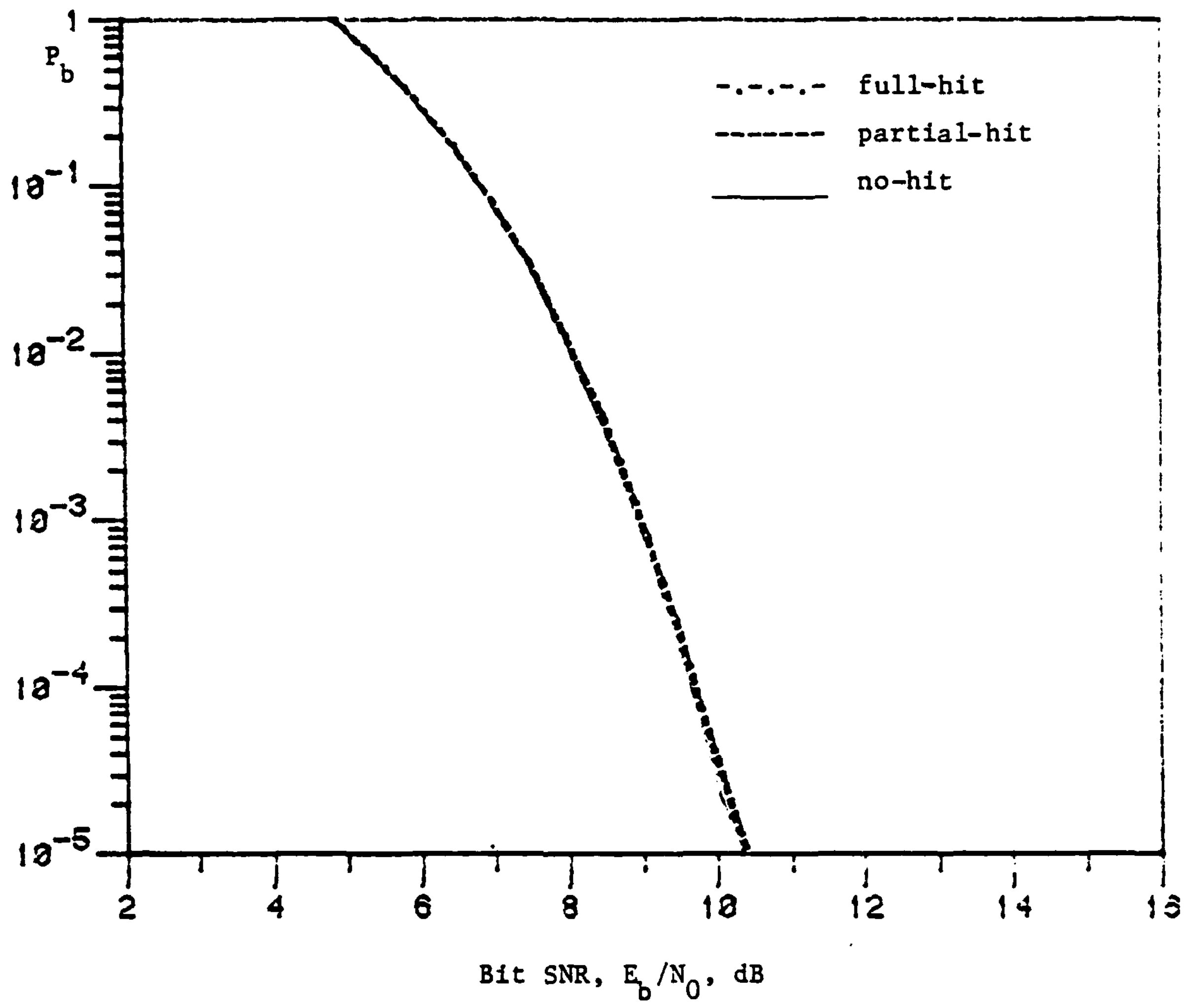

Figure 3.4. Upper bounds of average bit-error probabilities for the near-optimum receiver with a single interference and nonfading channel. N = 64.

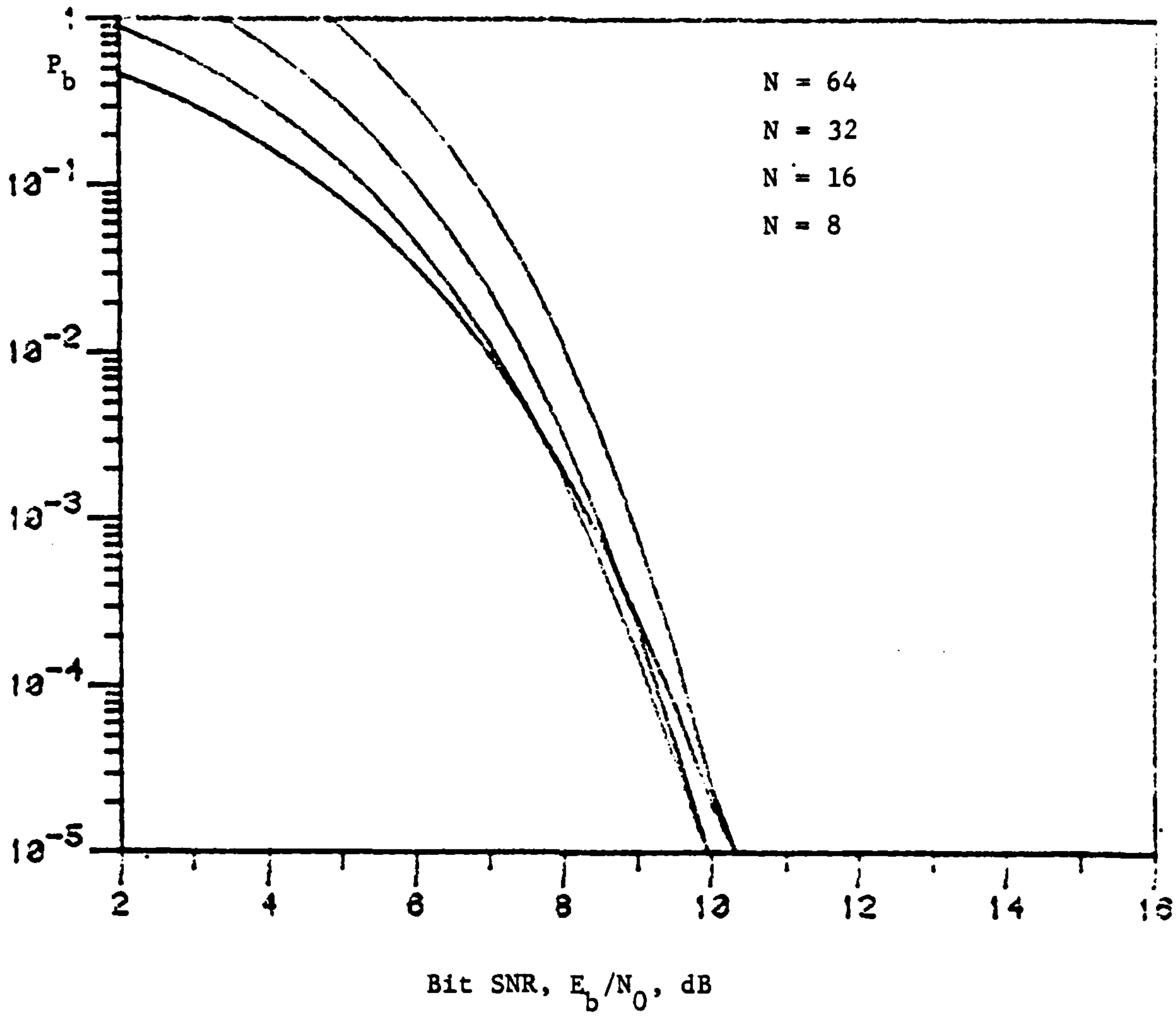

Figure 3.5. Upper bounds of average bit-error probabilities for the near-optimum receiver with no interference and nonfading channel.

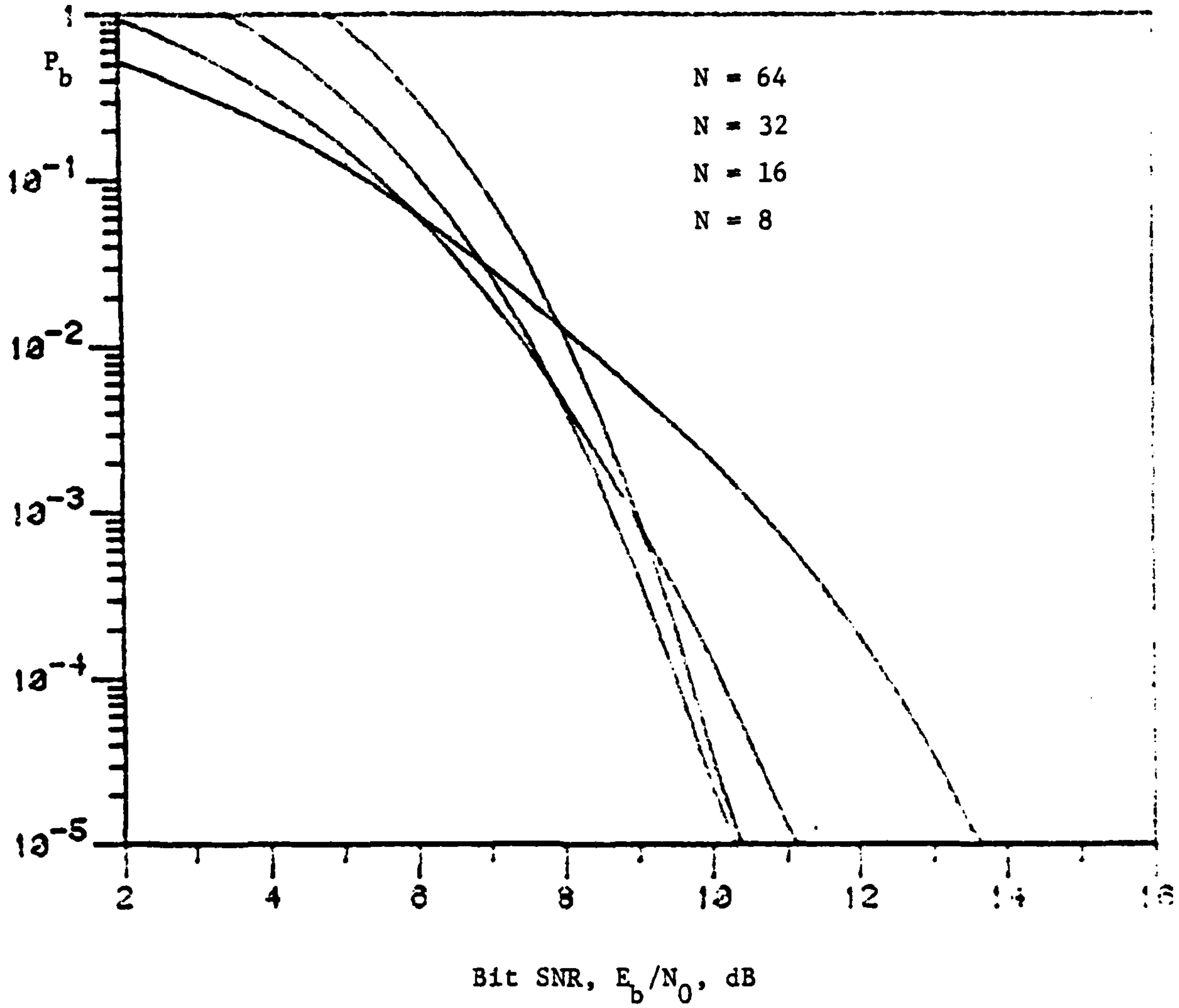

Figure 3.6. Upper bounds of average bit-error probabilities for the near-optimum receiver with single partial-interference and nonfading channel.

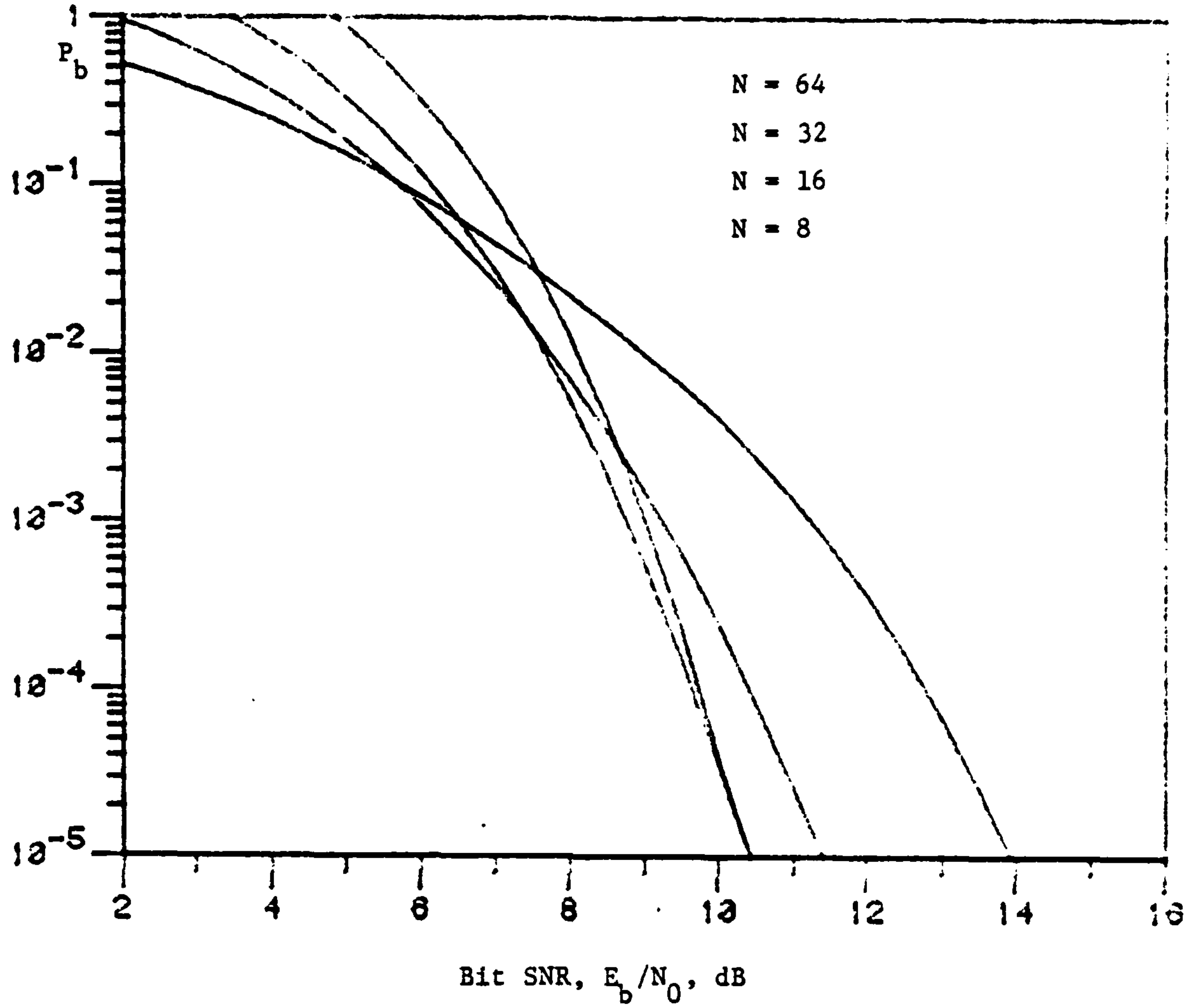

Figure 3.7. Upper bounds of bit-error probabilities for the near-optimum receover with single full-interference and nonfading channel.

CHAPTER IV

PROBABILITY OF ERROR ANALYSIS OF A NEAR-OPTIMUM FH-DPSK RECEIVER WITH MULTIPLE-USER INTERFERENCE AND A NONFADING CHANNEL

We are now in a position to evaluate the performance of the FH-DPSK receiver in the presence of multiple-user interference over a nonfading channel. In Chapter III we have shown that the effect of a single full-hit on the error probability is always more severe than that of a single partial-hit. Therefore, for the sake of simplicity, we assume that the system is chip-synchronous. Under this assumption, the upper bound on the error probability will be looser than the one for a realistic asynchronous system. However, the simpler upper bound, which is a union bound on the worst-case error probability, will also be applicable to the more realistic system.

Suppose the i-th codeword is being transmitted. Suppose also that the $n_1, n_2, \ldots, n_J$-th $(0 \leq J \leq N)$ time-chips have interference, each from a single distinct interferer, and the J distinct interferers are transmitting the $\ell_1, \ell_2, \ldots, \ell_J$-th (the ℓ_i's may not be distinct) codewords respectively with the same power. Then the receiver input over the 2T-second interval is (Eq. (3.1))

$$r(t) = \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-kT_c)\left[\cos(\omega_k t+\theta_k) + \sum_{\tau=1}^{J} \delta_{kn_\tau} \cos(\omega_k t+\phi_{m_\tau})\right]$$

$$+ \sqrt{2P} \sum_{k=0}^{N-1} p_{T_c}(t-T-kT_c)\left[h_{ik}\cos(\omega_k t+\theta_k) + \sum_{\tau=1}^{J} \delta_{kn_\tau} h_{\ell_\tau m_\tau} \cos(\omega_k t+\phi_{m_\tau})\right]$$

$$+ \eta(t) \tag{4.1}$$

where $h_{\ell_\tau m_\tau}$ is the ± element of the ℓ_τ-th row and the m_τ-th column of the Hadamard matrix, $\Theta_k \sim U(0,2\pi]$ for all k, $\Phi_{m_\tau} \sim U(0,2\pi]$ for all m_τ.

Thus, given H_i, $\underline{\Theta}$, $\{n_\tau\}_{\tau=1}^{J}$, $\underline{H}_\ell$, $\underline{\Phi}_m$, the conditional expectations (Eq. (4.1)) are

$$E[X_k|H_i,\underline{\Theta},\{n_\tau\}_{\tau=1}^{J},\underline{H}_\ell,\underline{\Phi}_m] = \begin{cases} \sqrt{P/2}\,T_c\cos\theta_k & ,\ k \neq n_1,n_2,\dots,n_J \\ \sqrt{P/2}\,T_c[\cos\theta_{n_1}+\cos\phi_{m_1}] & ,\ k = n_1 \\ \vdots & \\ \sqrt{P/2}\,T_c[\cos\theta_{n_J}+\cos\phi_{m_J}] & ,\ k = n_J \end{cases}$$

$$= \sqrt{P/2}\,T_c[\cos\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}\cos\phi_{m_\tau}]$$

$$E[Y_k|H_i,\underline{\Theta},\{n_\tau\}_{\tau=1}^{J},\underline{H}_\ell,\underline{\Phi}_m] = -\sqrt{P/2}\,T_c[\sin\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}\sin\phi_{m_\tau}]$$

$$E[X'_k|H_i,\underline{\Theta},\{n_\tau\}_{\tau=1}^{J},\underline{H}_\ell,\underline{\Phi}_m] = \sqrt{P/2}\,T_c[h_{ik}\cos\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}h_{\ell_\tau m_\tau}\cos\phi_{m_\tau}]$$

$$E[Y'_k|H_i,\underline{\Theta},\{n_\tau\}_{\tau=1}^{J},\underline{H}_\ell,\underline{\Phi}_m] = -\sqrt{P/2}\,T_c[h_{ik}\sin\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}h_{\ell_\tau m_\tau}\sin\phi_{m_\tau}] \quad (4.2)$$

and the conditional variances are all equal to $\sigma^2 \equiv N_0T_c/4$. After normalization, it follows from definitions (Eq. (3.7)) that

$$A_k = \frac{X_k-h_{ik}X'_k}{\sqrt{2\sigma^2}} \sim \eta\left(\sqrt{\frac{PT_c}{N_0}}\sum_{\tau=1}^{J}\delta_{kn_\tau}(1-h_{ik}h_{\ell_\tau m_\tau})\cos\phi_{m_\tau},1\right)$$

$$B_k = \frac{Y_k-h_{ik}Y'_k}{\sqrt{2\sigma^2}} \sim \eta\left(-\sqrt{\frac{PT_c}{N_0}}\sum_{\tau=1}^{J}\delta_{kn_\tau}(1-h_{ik}h_{\ell_\tau m_\tau})\sin\phi_{m_\tau},1\right)$$

$$C_k = \frac{X_k+h_{ik}X'_k}{\sqrt{2\sigma^2}} \sim \eta\left(\sqrt{\frac{PT_c}{N_0}}\left\{2\cos\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}(1+h_{ik}h_{\ell_\tau m_\tau})\cos\phi_{m_\tau}\right\},1\right)$$

$$D_k = \frac{Y_k+h_{ik}Y'_k}{\sqrt{2\sigma^2}} \sim \eta\left(-\sqrt{\frac{PT_c}{N_0}}\left\{2\sin\theta_k+\sum_{\tau=1}^{J}\delta_{kn_\tau}(1+h_{ik}h_{\ell_\tau m_\tau})\sin\phi_{m_\tau}\right\},1\right) \quad (4.3)$$

The set $\mathscr{J}_j$ is defined to be $\{k:h_{ik} = -h_{jk}\}$. Using Eqs. (3.9), (3.12), and (3.15) we see that the conditional bit error probability is bounded by

$$\frac{N}{2}\cdot P(A > B|H_i,\underline{\Theta},\{n_\tau\}_\tau^J,\underline{H}\ ,\underline{\Phi}_m) = \frac{N}{2}\cdot P\left(\sum_{k\in\mathscr{J}_j}(A_k)^2+(B_k)^2 > \sum_{k\in\mathscr{J}_j}(C_k)^2+(D_k)^2|H_i,\ldots,\underline{\Phi}_m\right) \tag{4.4}$$

If $n_{\tau 0} \notin \mathscr{J}_j$, the interference signal at the $n_{\tau 0}$-th time-chip will not affect the overall comparison. Suppose now that ε of the J n_τ's lie inside $\mathscr{J}_j$. Since $|\mathscr{J}_j| = N/2$, and because we have restricted the number of interferers at each time-chip period to be at most one, it is clear that $(J-N/2)\,u(J-N/2) \le \varepsilon \le \min(J,N/2)$ where $u(n) = 1$ if $n \ge 0$, 0 if $n < 0$ is the unit step function. Thus, the probability that ε of the J n 's lie inside $\mathscr{J}_j$ is

$$P(E = \varepsilon) = \frac{\binom{J}{\varepsilon}}{\sum_\beta \binom{J}{\beta}} \tag{4.5}$$

In this case, Eq. (4.4) suggests that

$$A \equiv 2(N/2 - \varepsilon) \text{ random variables (r.v.'s)} \sim \eta^2(0,1)$$

$$+\ \varepsilon \text{ r.v.'s} \qquad \sim \eta^2\left(\sqrt{\frac{PT_c}{N_0}}\,(1-h_{in_\tau}h_{\ell_\tau m_\tau})\cos\phi_{m_\tau},1\right)$$

$$+\ \varepsilon \text{ r.v.'s} \sim \eta^2\left(-\sqrt{\frac{PT_c}{N_0}}\,(1-h_{in_\tau}h_{\ell_\tau m_\tau})\sin\phi_{m_\tau},1\right), \text{ for some } \varepsilon \text{ of } \tau\text{'s}$$

$$B \equiv (N/2 - \varepsilon) \text{ r.v.'s} \sim \eta^2\left(\sqrt{\frac{PT_c}{N_0}}\ 2\cos\theta_k,1\right)$$

$$+\ (N/2 - \varepsilon) \text{ r.v.'s} \sim \eta^2\left(-\sqrt{\frac{PT_c}{N_0}}\ 2\sin\theta_k,1\right)$$

$$+\ \varepsilon \text{ r.v.'s} \sim \eta^2\left(\sqrt{\frac{PT_c}{N_0}}(2\cos\theta_{n_\tau}+(1+h_{in_\tau}h_{\ell_\tau m_\tau})\cos\phi_{m_\tau}),1\right)$$

$$+\ \varepsilon \text{ r.v.'s} \sim \eta^2\left(-\sqrt{\frac{PT_c}{N_0}}\,(2\sin\theta_n + (1+h_{in_\tau}h_{\ell_\tau m_\tau})\sin\phi_{m_\tau}),1\right)$$

for some $(N/2-\varepsilon)$ of k's and some ε of τ's (4.6)

where $\eta^2(\mu,1)$ denotes the square of a Gaussian r.v. which has mean μ and variance 1. Rearranging the indexes on h_{in_τ}, $h_{\ell_\tau m_\tau}$, and ϕ_{m_τ}, we have from Eqs. (3.10) and (3.11) the parameters

$$s = 2\,\frac{PT_c}{N_0}\sum_{\tau=1}^{\varepsilon}(1-h_{in_\tau}h_{\ell_\tau m_\tau}) \tag{4.7}$$

$$s' = 4\,\frac{PT_c}{N_0}\cdot\frac{N}{2} + 2\,\frac{PT_c}{N_0}\sum_{\tau=1}^{\varepsilon}(1+h_{in_\tau}h_{\ell_\tau m_\tau}) + 4\,\frac{PT_c}{N_0}\sum_{\tau=1}^{\varepsilon}(1+h_{in_\tau}h_{\ell_\tau m_\tau})\cdot\cos(\theta_{n_\tau}-\phi_{m_\tau})$$

Note that $(1-h_{in_\tau}h_{\ell_\tau m_\tau})$ can either be 0 or 2. Thus, assuming ζ of the ε $(1-h_{in_\tau}h_{\ell_\tau m_\tau})$ terms in Eq. (4.7) are equal to zero, we obtain

$$s = 4\,\frac{PT_c}{N_0}(\varepsilon-\zeta)$$

$$s' = 2\,\frac{PT}{N_0} + 4\,\frac{PT_c}{N_0}\zeta + 8\,\frac{PT_c}{N_0}\sum_{\tau=1}^{\zeta}\cos\psi_\tau \tag{4.8}$$

where $\Psi_\tau \triangleq \theta_{n_\tau} - \phi_{m_\tau}$ (mod 2π) is a r.v. $\sim U(0,2\pi]$. The solution to Eq. (4.4) is then given by Eq. (A.15) in Appendix A to be $P_e(u,x)$ with $u = s/4$ and $x = s'/4$.

Fortunately, all the random variables in Eq. (4.4) are mutually independent. This fact enables us to remove the conditionings one by one via taking independent expectations with respect to each individual random variable. Using Fact A.2 in Appendix A successively ε times, we can remove the conditionings on Ψ_τ's and obtain an upper bound of the bit-error probability to be

$$\sum_{\alpha=0}^{\zeta} \left(\frac{1}{2}\right)^{\zeta} \binom{\zeta}{\alpha} P_e\left(\frac{PT_c}{N_0}(\varepsilon - \zeta), \frac{PT}{2N_0} + \frac{PT_c}{N_0}(3\zeta - 4\alpha)\right) \tag{4.9}$$

But $P(Z = \zeta)$ is binomially distributed, hence, removing the conditioning on Z, the upper bound is

$$\sum_{\zeta=0}^{\varepsilon} \binom{\varepsilon}{\zeta}\left(\frac{1}{2}\right)^{\zeta}\left(\frac{1}{2}\right)^{\varepsilon-\zeta} \sum_{\alpha=0}^{\varepsilon} \left(\frac{1}{2}\right)^{\zeta} \binom{\zeta}{\alpha} P_e\left(\frac{PT_c}{N_0}(\varepsilon - \zeta), \frac{PT}{2N_0} + \frac{PT_c}{N_0}(3\zeta - 4\alpha)\right) \tag{4.10}$$

Finally, removing the conditioning on E (the number of n_τ's that lie inside $\mathcal{J}_j$) yields

$$P_b \leq \sum_{\varepsilon} P(E=\varepsilon) \sum_{\zeta=0}^{\varepsilon} \binom{\varepsilon}{\zeta}\left(\frac{1}{2}\right)^{\varepsilon} \sum_{\alpha=0}^{\zeta} \left(\frac{1}{2}\right)^{\zeta} \binom{\zeta}{\alpha} P_e\left(\frac{PT_c}{N_0}(\varepsilon - \zeta), \frac{PT}{2N_0} + \frac{PT_c}{N_0}(3\zeta - 4\alpha)\right) \tag{4.11}$$

where $P(E=\varepsilon)$ is given by Eq. (4.5). Thus, given that J interferers are present and each of the interferers is interfering a distinct time-chip period, the bit-error probability is bounded by Eq. (4.11). It is not difficult to see that Eq. (4.11) agrees with Eq. (3.30) and Eq. (3.29) for the J = 0 and J = 1 cases, respectively .

The expression in Eq. (4.11) can be determined using a digital computer as a function of the bit SNR, E_b/N_0 for various values of N and J. Note that $E_b/N_0 = \frac{N}{n}\frac{PT_c}{N_0} = \frac{PT}{nN_0}$, and $N = 2^n$. Several upper bounds are displayed graphically in Figures 4.1 to 4.4. For N = 8 and N = 16, the receiver apparently fails to operate within practical limits of bit SNRs if each of the time-chips is interfered by a different interferer. For larger N's, the receiver's anti-interference capability is seen to be quite satisfactory. In fact, at

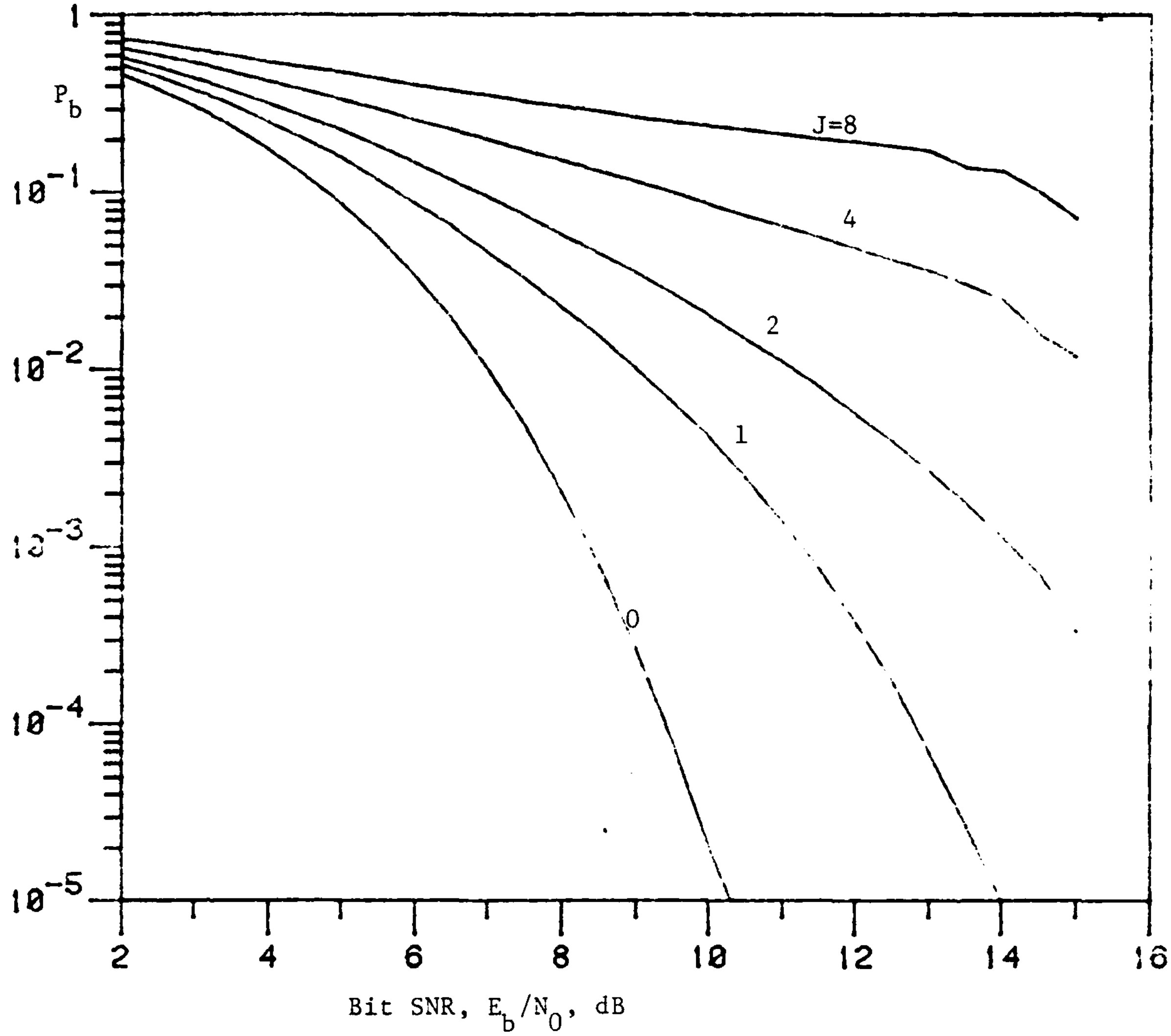

Figure 4.1. Upper bounds of average bit-error probabilities for the near-optimum receiver with single interference at J distinct time-chips and nonfading channel. N = 8.

BP1

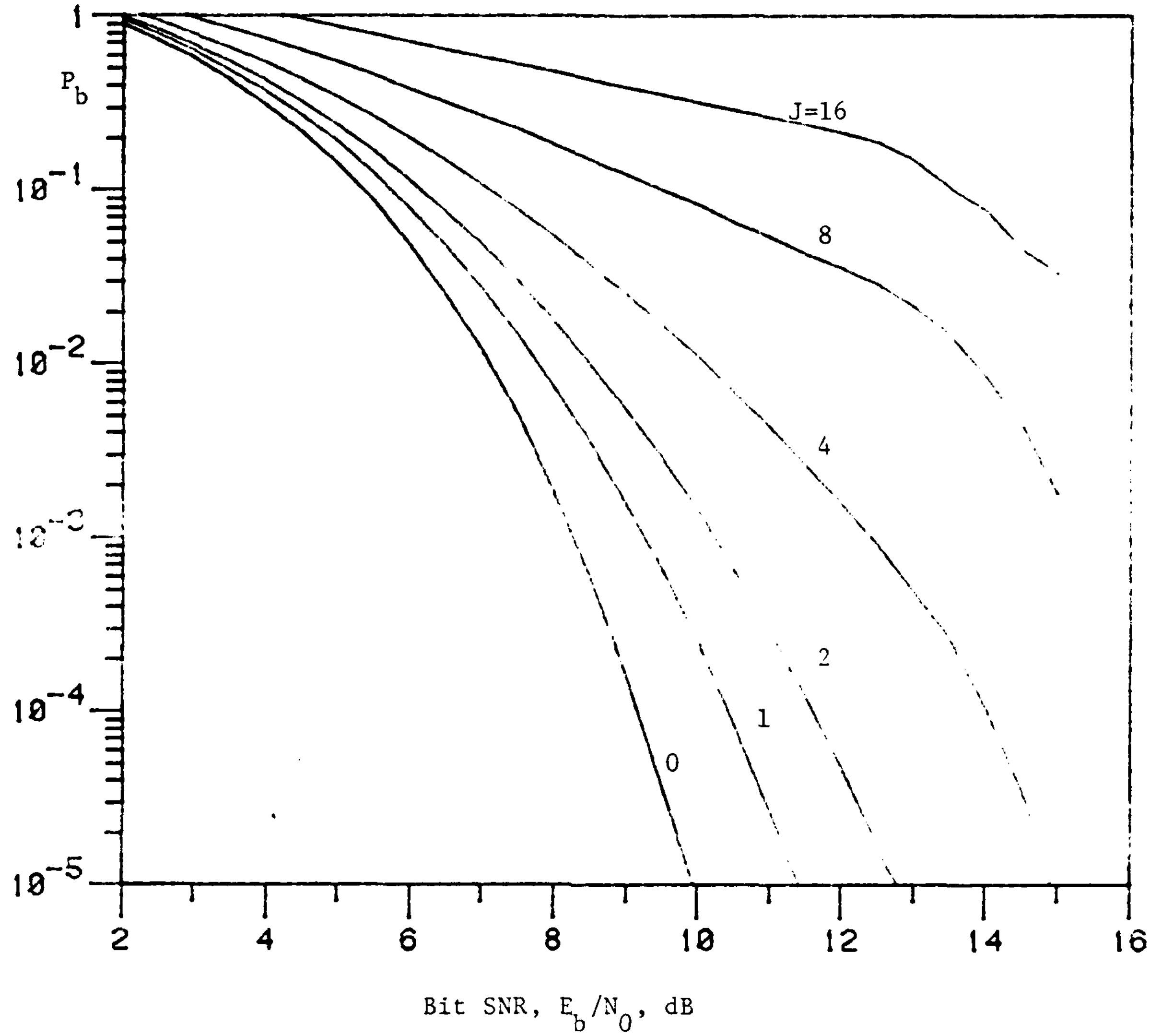

Figure 4.2. Upper bounds of average bit-error probabilities for the near-optimum receiver with single interference at J distinct time-chips and nonfading channel. N = 16.

BP1

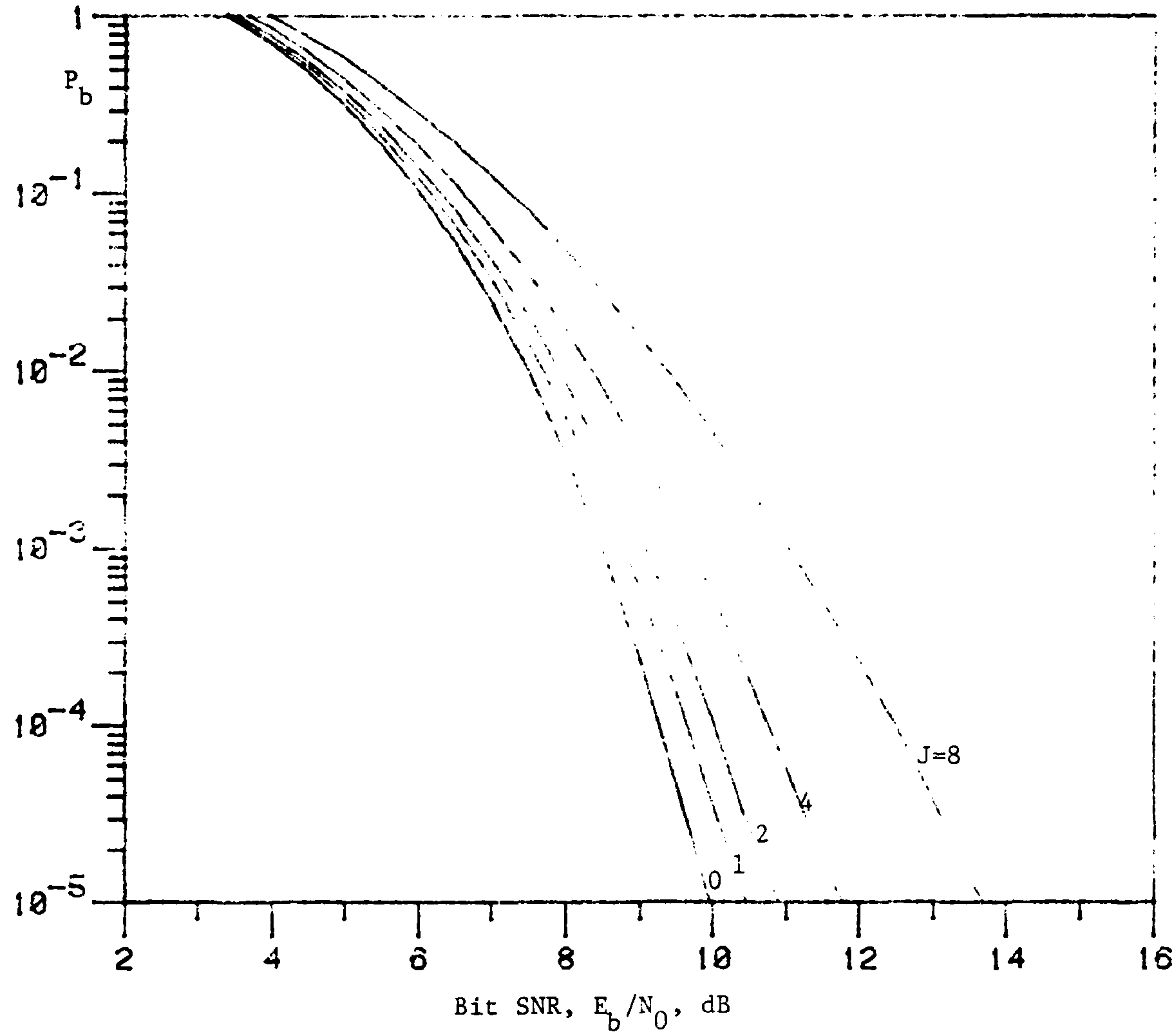

Figure 4.3. Upper bounds of average bit-error probabilities for the near-optimum receiver with single interference at J distinct time-chips and nonfading channel. N = 32

BP1

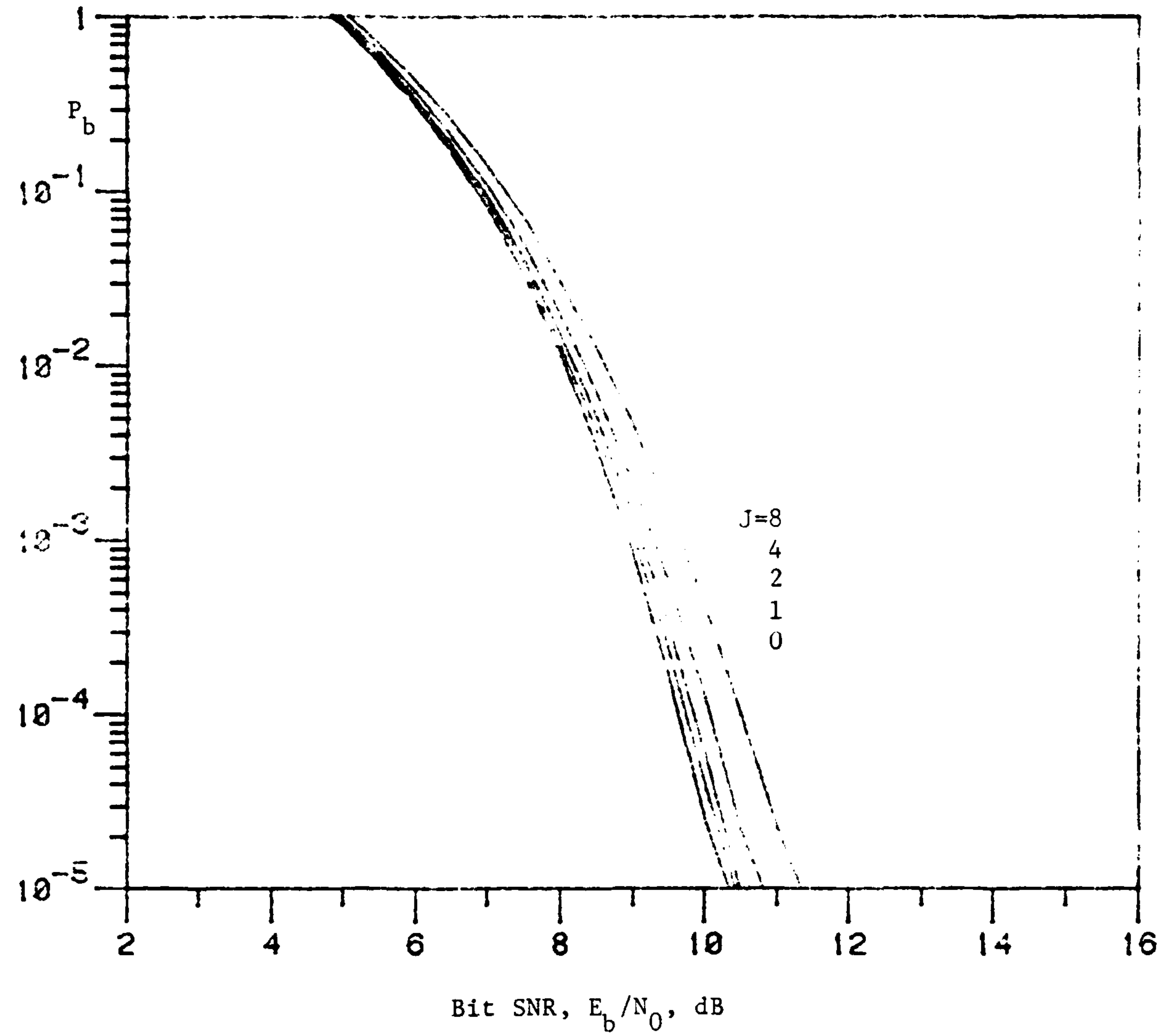

Figure 4.4. Upper bounds of average bit-error probabilities for the near-optimum receiver with single interference at J distinct time-chips and nonfading channel. N = 64.

$P_b = 10^{-3}$ and assuming that 1/8 of the N time-chips are each interfered by a distinct interferer, the degradation in bit SNR is just 1.25 dB for N = 32 and 0.8 dB for N = 64. By using the frequency-hopping patterns that we developed in Chapter II, it is believed that the number J of interferers (to the receiver) will be small if the number M of simultaneous users in the system is not too large. Although multiple interference at the same time-chip period are possible in a realistic system, it will happen with a small probability since any two sequences have at most one hit asynchronously and each sequence is hopping on a small portion of the available frequency spectrum (for example $N/q \approx 0.03$ in Chapter II).

For a fixed number M of simultaneous users, we may assume that each group in a good design D_g (Table 3.1) accommodates approximately an equal number of users. With the help of Corollary 2.1, it should not be difficult to compute the entire hitting characteristic of the system with M users. Then we can deduce the number of interferers (to the receiver) and their distributions so that the bit-error probability rates can be calculated. Typically, N is 32 and P_b is 10^{-3}. Thus, within practical limits of the bit SNRs (10 ~ 20 dB), the maximum number of simultaneous users in the system can be found.

As a preliminary investigation, we may assume that about 1/10 of the total $|D_g|$ potential users are actually transmitting signals. Note that q distinct frequencies are available (see Chapter II) and the sequence length is N. Hence, any frequency f_k will appear in some $|D_g| \cdot N/10q$ distinct sequences and the maximum number of interferers can be as large as $(|D_g| \cdot N/10q-1)N$. But among the $|D_g|/10$ users' sequences, some of them may never have hits with the receiver's sequence because two sequences' frequency slots can be disjoint. On the other hand, some of the sequences may have more than one frequency in common with the receiver's sequence, so that the number of interferers can be much smaller. Furthermore, there are N possible time shifts of a sequence so it is very unlikely that all the potential interferers will actually interfere with the received signals. It is then reasonable to assume that the number of interferers J is about $|D_g| \cdot N/10q-1$. For example, from Table 2.1, we may take the design D_g of GF(577) with N = 32. $|D_g|$ is given to be 2 × 577 = 1154. Assuming that $M \approx 115$ simultaneous users are present in the system, then J is about 5. We shall allow multiple interferences at some time-chip periods. Consequently, the ways that J interferers will distribute in N time-chips will have a Bose-Einstein distribution. The bit-error rates can then be estimated easily by using the function $P_e(u,x)$. Although this kind of interference analysis will inevitably expend a substantial amount of computer CPU time, it is certainly a necessary follow-up to this thesis.

CHAPTER V

CONCLUSION

In this thesis we have discussed a design of short Reed-Solomon sequences in which any two sequences will have at most one full-hit or one partial-hit when used asynchronously. The result showed that these RS sequences have excellent correlation properties and therefore are very suitable for frequency-hopping application, since they can extend the number of multiple-access users. We also discussed the behavior of the FH-DPSK near-optimum receiver with multiple-user interference in a nonfading channel, assuming that each time-chip has at most one interferer of the same power. The receiver performance was found to degrade gradually with an increase in the number of interfering signals. Furthermore, a longer codeword length was shown to be more preferable. Finally, generalization of the multiple-user interference model was considered.

APPENDIX A

THE ERROR PROBABILITY $P_e(u,x)$

Our point of departure is the probability expression

$$P[A > B] = \int_{\beta=0}^{\infty} \int_{\alpha=\beta}^{\infty} f_{A,B}(\alpha,\beta)d\alpha d\beta = \int_{0}^{\infty} f_B(\beta) \int_{\beta}^{\infty} f_A(\alpha)d\alpha d\beta \tag{A.1}$$

where A is independent of B. Now suppose that A and B are both being a sum of N independent, squared Gaussian random variables with nonzero means and unit variances. Then A and B will have the noncentral chi-squared density with N degrees of freedom given in [10] as

$$f_A(\alpha) = \frac{1}{2} \left(\frac{\alpha}{s}\right)^{\frac{N-2}{4}} e^{-\frac{(\alpha+s)}{2}} I_{\frac{N-2}{2}}(\sqrt{\alpha s}), \qquad \alpha > 0 \tag{A.2}$$

and

$$f_B(\beta) = \frac{1}{2} \left(\frac{\beta}{s'}\right)^{\frac{N-2}{4}} e^{\frac{-(\beta+s')}{2}} I_{\frac{N-2}{2}}(\sqrt{\beta s'}), \qquad \beta > 0 \tag{A.3}$$

where s and s' are sums of the squares of the means of the Gaussian r.v.'s in A and B, respectively. The inner integral in Eq. (A.1) is then found [10] to be

$$\int_{\beta}^{\infty} f_A(\alpha)d\alpha = Q_{N/2}(\sqrt{s},\sqrt{\beta}) \tag{A.4}$$

where

$$Q_{N/2}(\sqrt{s},\sqrt{\beta}) = Q(\sqrt{s},\sqrt{\beta}) + e^{\frac{-(s+\beta)}{2}} \sum_{r=1}^{\frac{N-2}{2}} \left(\sqrt{\frac{\beta}{s}}\right)^r I_r(\sqrt{s\beta}) \tag{A.5}$$

is a generalization of the Marcum Q-function. Observing that $Q(\sqrt{s},\sqrt{\beta})$ is given by [10] as

$$Q(\sqrt{s},\sqrt{\beta}) = 1 - e^{\frac{-(s+\beta)}{2}} \sum_{r=1}^{\infty} \left(\sqrt{\frac{\beta}{s}}\right)^r I_r(\sqrt{s\beta}) \tag{A.6}$$

where $I_r(\sqrt{s\beta})$ is the r-th order modified Bessel function of the first kind given by

$$I_r(\sqrt{s\beta}) = \sum_{k=0}^{\infty} \frac{\left(\frac{\sqrt{s\beta}}{2}\right)^{r+2k}}{k!(r+k)!} \tag{A.7}$$

we obtain

$$Q_{N/2}(\sqrt{s},\sqrt{\beta}) = 1 - e^{\frac{-(s+\beta)}{2}} \sum_{r=N/2}^{\infty} \sum_{k=0}^{\infty} \frac{s^k \beta^{r+k}}{2^{r+2k} k!(r+k)!} \tag{A.8}$$

Upon substitution for Eq. (A.8) and Eq. (A.3) into Eq. (A.1) and by using the fact that $f_B(\beta)$ is a probability density function we get

$$\int_0^{\infty} f_B(\beta) Q_{N/2}(\sqrt{s},\sqrt{\beta}) d\beta = 1 - \int_0^{\infty} e^{\frac{-(s+\beta)}{2}} \sum_{r=N/2}^{\infty} \sum_{k=0}^{\infty} \frac{s^k \beta^{r+k}}{2^{r+2k} k!(r+k)!}$$

$$\cdot \frac{1}{2} \left(\frac{\beta}{s'}\right)^{\frac{N-2}{4}} e^{\frac{-(\beta+s')}{2}} I_{\frac{N-2}{2}}(\sqrt{\beta s'}) d\beta$$

$$= 1 - 2^{-\frac{N}{2}} e^{-\frac{s}{2}} e^{-\frac{\nu}{2}} \sum_{r=N/2}^{\infty} \sum_{k=0}^{\infty} \frac{s^k}{2^{2r+3k} k!} \int_0^{\infty} \frac{y^{r+k}}{(r+k)!} \left[\frac{1}{2}\left(\frac{y}{\nu}\right)^{\frac{N-2}{4}} e^{-\frac{(y+\nu)}{2}} I_{\frac{N-2}{2}}(\sqrt{y\nu})\right] dy \tag{A.9}$$

where $\nu = s'/2$, and $y = 2\beta$. The integral in the right-hand side of this expression may be solved with the least amount of difficulty through indirect computation. We first notice that the characteristic function of a positive random variable Y may be put into the following form by using the power series expansion of the exponential function:

$$\Phi_Y(j\omega) = E\{e^{j\omega Y}\} = \sum_{k=0}^{\infty} (j\omega)^k \left[\int_0^{\infty} \frac{y^k}{k!} f_Y(y) dy\right] \tag{A.10}$$

The integral in brackets is observed to be the integral of interest, where $f_Y(y)$ is the noncentral chi-square density encountered earlier in Eq. (A.2). Thus, the value of the integral is simply the value of the coefficient of $(j\omega)^{k+r}$ in the series expansion of the characteristic function corresponding to the noncentral chi-square density with N degrees of freedom. That characteristic function is given in [10] as $\Phi_Y(j\omega) = (1-2j\omega)^{-N/2}\exp[\nu j\omega/(1-2j\omega)]$. This function may be expanded by first using the power series expansion for the exponential function, indexed by ℓ. Then by using the binomial expansion for each $(1-2j\omega)^{-(N/2+\ell)}$ term, indexed by m, we obtain

$$\Phi_Y(j\omega) = \sum_{\ell=0}^{\infty} \frac{(j\omega\nu)^{\ell}}{\ell!} \sum_{k=0}^{\infty} \binom{N/2+\ell+m-1}{m} (2j\omega)^{m} \tag{A.11}$$

where $\binom{a}{b}$ denotes the usual binomial coefficient $\frac{a!}{b!(a-b)!}$. Finally, by collecting all the contributions to the coefficient of $(j\omega)^{k+r}$, the value of the integral is found to be

$$2^{k+r} \sum_{\ell=0}^{k+r} \frac{(\nu/2)^{\ell}}{\ell!} \binom{N/2+k+r-1}{k+r-\ell} \tag{A.12}$$

Thus,

$$P[A > B] = 1-2^{-\frac{N}{2}} e^{-\frac{s}{2}} e^{-\frac{\nu}{2}} \sum_{r=N/2}^{\infty} \sum_{k=0}^{\infty} \frac{s^k}{2^{2r+3k}k!} 2^{k+r} \sum_{\ell=0}^{k+r} \frac{(\nu/2)^{\ell}}{\ell!} \binom{N/2+k+r-1}{k+r-\ell}$$

$$= 1-2^{-\frac{N}{2}} e^{-2u} e^{-x} \sum_{r=N/2}^{\infty} \sum_{k=0}^{\infty} \frac{2^{-r}u^k}{k!} \sum_{\ell=0}^{k+r} \frac{x^{\ell}}{\ell!} \binom{N/2+k+r-1}{k+r-\ell} \tag{A.13}$$

where $u = s/4$ and $x = \nu/2 = s'/4$. Hence Eq. (A.1) is solved. Notice that if $s = 0$, then Eq. (A.13) simplifies to

$$2^{-N/2} e^{-x} \sum_{r=0}^{N/2-1} 2^{-r} \sum_{\ell=0}^{r} \frac{x^{\ell}}{\ell!} \binom{N/2+r-1}{r-\ell} \tag{A.14}$$

which is exactly the special case error probability expression p given in [3]. We then define

$$P_e(u,x) = \frac{N}{2} P[A > B] \tag{A.15}$$

where $P[A > B]$ is given in (A.13). It is obvious that $P_e(u,x)$ is continuous and differentiable in u and x. Furthermore, it can be shown that $P_e(u,x)$ is convex [11] in both u and x, monotone increasing in u, and monotone decreasing in x by observing that $\frac{\partial^2}{\partial u^2} P_e(u,x) \geq 0$, $\frac{\partial^2}{\partial x^2} P_e(u,x) \geq 0$, $\frac{\partial}{\partial u} P_e(u,x) > 0$, and $\frac{\partial}{\partial x} P_e(u,x) < 0 \ \forall\ u, x \geq 0$, respectively. Figure A.1 and Figure A.2 show graphs of $P_e(u,x)$ for interesting values of u and x. In particular, $P_e\left(u, x = \frac{PT}{2N_0} = \frac{n}{2}\frac{E_b}{N_0}\right)$ and $P_e\left(u = \frac{PT_c}{N_0} = \frac{n}{N}\frac{E_b}{N_0}, x\right)$ are exhibited in the figure with $N = 2^n = 32$.

Finally, we present some useful facts which follow readily from the convexity of $P_e(u,x)$.

Fact A.1 Given $a > 0$, $b > 0$, and $\Sigma \sim U(0,1)$,

$$E\{P_e(u, X = a + b\Sigma)\} \leq \frac{1}{2}\{P_e(u,a+b) + P_e(u,a)\} \tag{A.16}$$

Fact A.2 Given $b > 0$, $a-b > 0$, and $\Phi \sim U(0,2\pi]$,

$$E\{P_e(u, X = a + b\cos\Phi)\} \leq \frac{1}{2}\{P_e(u,a+b) + P_e(u,a-b)\} \tag{A.17}$$

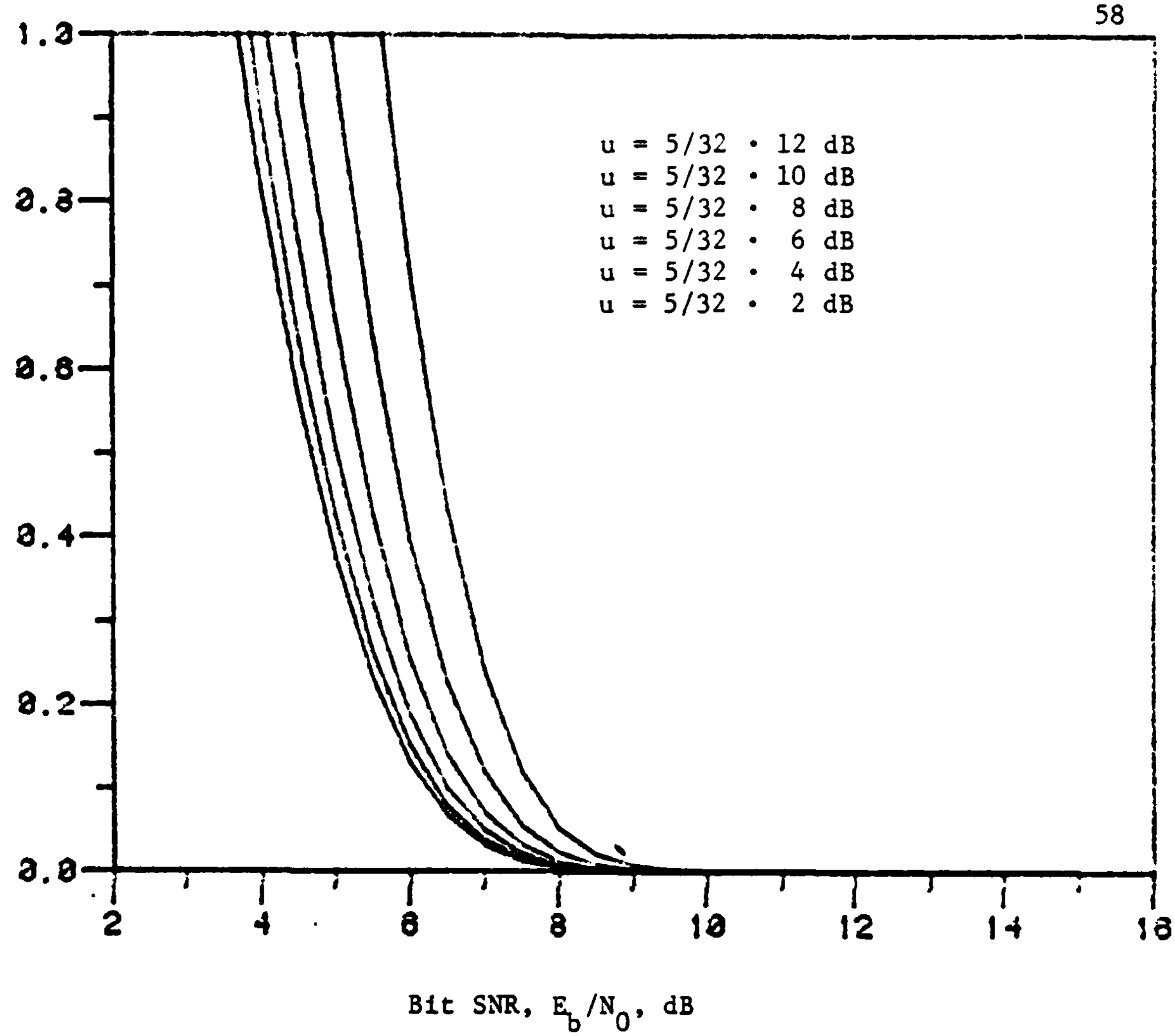

Figure A.1. Graphs of $P_e(u, \frac{n}{2}\frac{E_b}{N_0})$ with n = 5, N = 32. Six fixed values of $u(\triangleq \frac{n}{N}\frac{E_b}{N_0})$ are used.

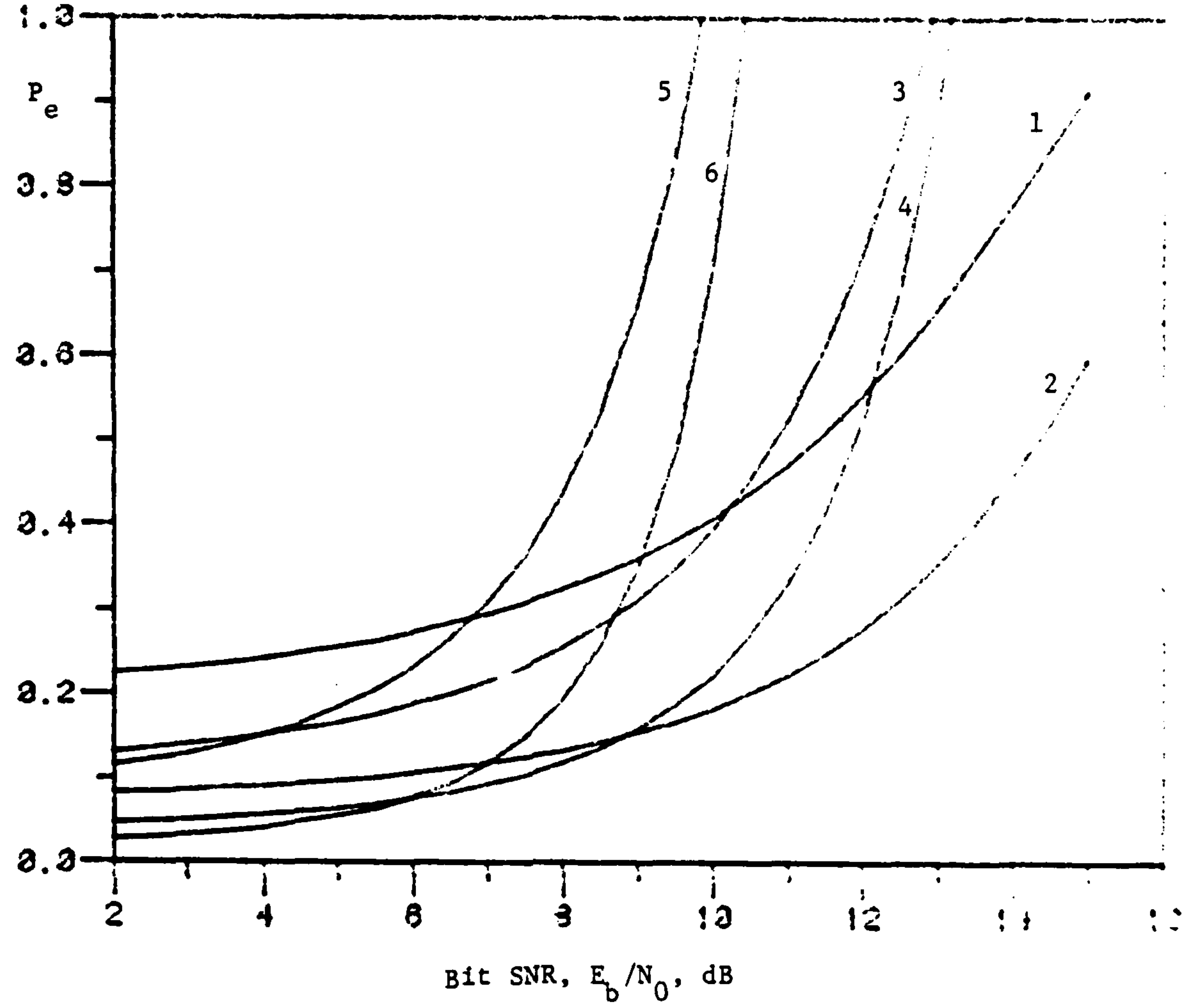

Figure A.2. Weighted graphs of $c \cdot P_e(\frac{n}{N}\frac{E_b}{N_0},x)$ with $n = 5$, $N = 32$. Six fixed values of $x(\triangleq \frac{n}{2}\frac{E_b}{N_0})$ are used.

Graph 1	$x = 5/2 \cdot 2$ dB,	$c = 10^{-1}$
2	$x = 5/2 \cdot 4$ dB,	$c = 10^{-1}$
3	$x = 5/2 \cdot 6$ dB,	$c = 1$
4	$x = 5/2 \cdot 8$ dB,	$c = 10^{1}$
5	$x = 5/2 \cdot 10$ dB,	$c = 10^{4}$
6	$x = 5/2 \cdot 12$ dB,	$c = 10^{8}$

Fact A.3 Given $\sigma \in (0,1)$, $P_e(0, \frac{PT}{2N_0} + \frac{PT_c}{N_0}(\sigma^2 + 2\sigma))$ is convex and monotone decreasing in σ, and $P_e(0, \frac{PT}{2N_0} + \frac{PT_c}{N_0}(\sigma^2 - 2\sigma))$ is monotone increasing in σ.

Fact A.2 is trivial once we observe that $g(\psi) = P_e(u, a+b\cos\psi)$ is convex and increasing in $(0,\pi]$, and convex and decreasing in $(\pi,2\pi]$. Since Φ is uniform, $E\{g(\Phi)\}$ will simply be equal to $1/2\pi$ times the area under $g(\psi)$. The result then follows from convexity after approximating $g(\psi)$ by two straight lines. Fact A.3 may be verified by Figure A.3 and Figure A.4.

This concludes our discussion on $P_e(u,x)$.

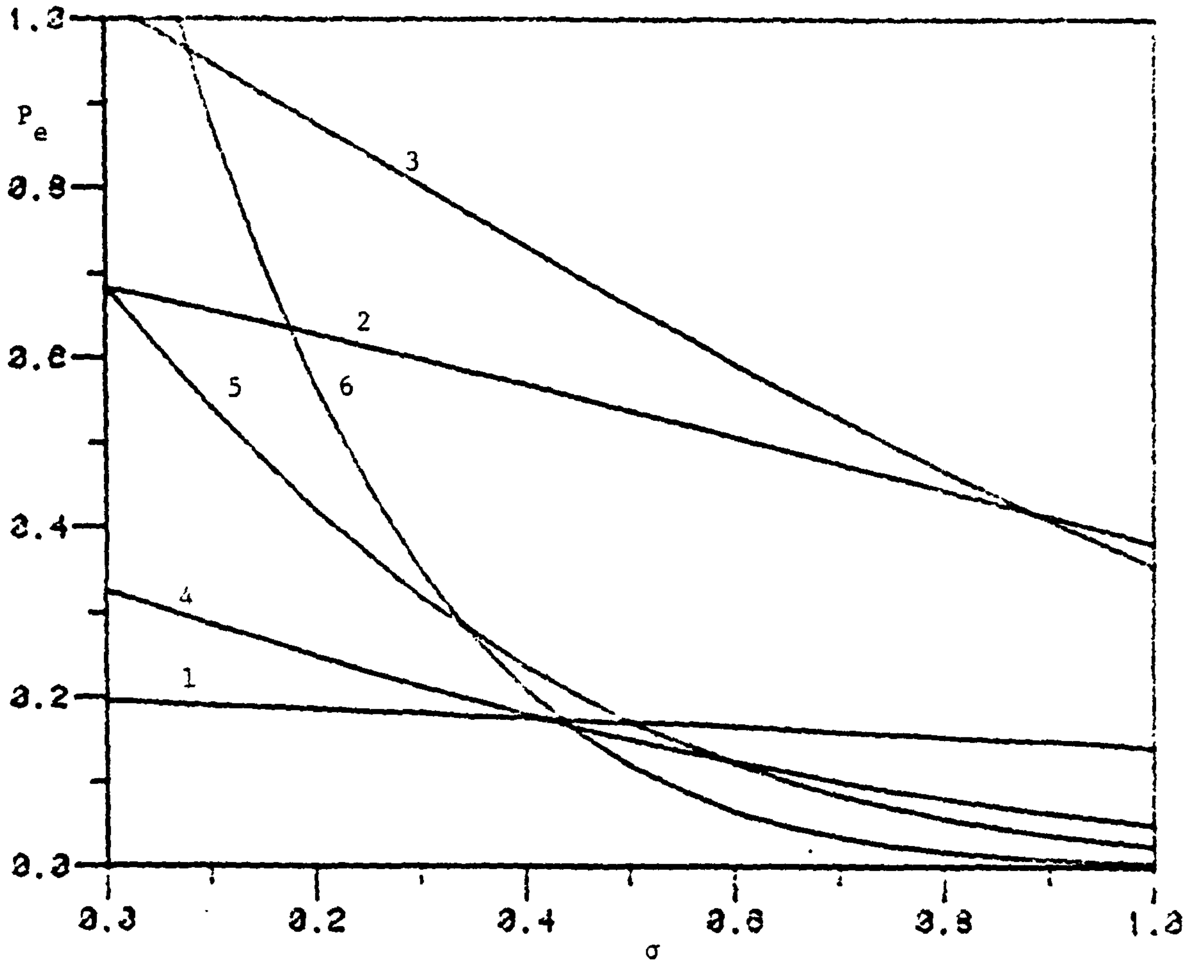

Figure A.3. Weighted graphs of $c \cdot P_e(0, \frac{PT}{2N_0} + \frac{PT_c}{N_0}(\sigma^2+2\sigma))$ with

$PT/N_0 = n\,\frac{E_b}{N_0}, \frac{PT_c}{N_0} = \frac{n}{N}\,\frac{E_b}{N_0}$, $n = 5$, $N = 32$.

Graph 1 $E_b/N_0 = 2$ dB, $c = 10^{-1}$

2 $E_b/N_0 = 4$ dB, $c = 1$

3 $E_b/N_0 = 6$ dB, $c = 10^{1}$

4 $E_b/N_0 = 8$ dB, $c = 10^{2}$

5 $E_b/N_0 = 10$ dB, $c = 10^{5}$

6 $E_b/N_0 = 12$ dB, $c = 10^{10}$

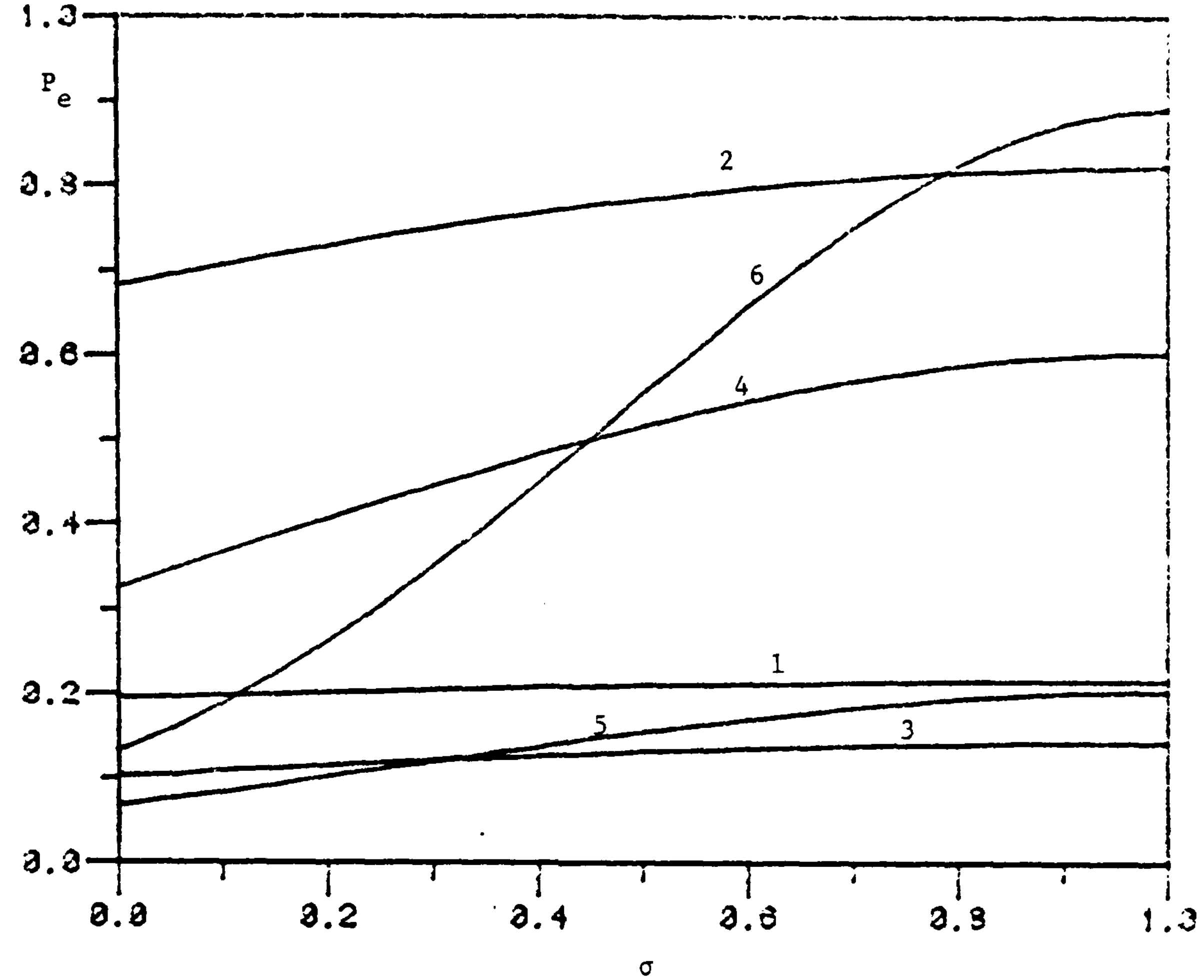

Figure A.4. Weighted graphs of $c \cdot P_e(0, \frac{PT}{2N_0} + \frac{PT_c}{N_0}(\sigma^2 - 2\sigma))$ with

$PT/N_0 = n\frac{E_b}{N_0}$, $\frac{PT_c}{N_0} = \frac{n}{N}\frac{E_b}{N_0}$, $n = 5$, $N = 32$.

Graph 1 $E_b/N_0 = 2$ dB, $c = 10^{-1}$

2 $E_b/N_0 = 4$ dB, $c = 1$

3 $E_b/N_0 = 6$ dB, $c = 1$

4 $E_b/N_0 = 8$ dB, $c = 10^{2}$

5 $E_b/N_0 = 10$ dB, $c = 10^{4}$

6 $E_b/N_0 = 12$ dB, $c = 10^{9}$

REFERENCES

1. G. R. Cooper and R. W. Nettleton, "Spectral efficiency in cellular land-mobile communications: A spread-spectrum approach," Final Report, NSF Grant ENG 76-80536, 1976.

2. R. W. Nettleton and G. R. Cooper, "Performance of a frequency-hopped differentially modulated spread-spectrum receiver in a Rayleigh fading channel," IEEE Trans. Veh. Technol., vol. VT-30, pp. 14-29, Feb. 1981.

3. J. T. Martersteck, "A near-optimum receiver for a Cooper-Nettleton, frequency-hopped, land-mobile communication system," M.S. thesis, University of Illinois, Urbana-Champaign, 1983 Ch. IV.

4. S. A. McClatchey, "Performance analysis and simulation results for a Cooper-Nettleton mobile radio communication system," M.S. thesis, University of Illinois, Urbana-Champaign, 1984.

5. A. J. Viterbi, Principles of Coherent Communication. New York: McGraw-Hill, 1966, Ch. 8.

6. O. Yue, "Frequency-hopping, multiple-access phase-shift-keying system performance in a Rayleigh fading environment," Bell Syst. Tech. J., vol. 59, pp. 861-879, Jy. 1980.

7. F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes, New York: North-Holland, 1978.

8. G. Solomon, "Optimal frequency hopping sequences for multiple-access," Proceedings of the 1973 Symposium on Spread Spectrum Communications, vol. 1, AD-915 852, pp. 33-35.

9. E. A. Geraniotis, "Analysis of binary frequency-hopped spread-spectrum multiple-access communications systems," M.S. Thesis, University of Illinois, Urbana, 1978, Ch. 3.

10. C. W. Helstrom, Statistical Theory of Signal Detection, 2nd ed., New York: Pergamon, 1968.

11. R. J. McEliece, The Theory of Information and Coding, Reading: Addison-Wesley, 1977.

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