Discrete B-Splines as an Approach to Computer Aided Geometric Design

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Investigations have been made of many interesting problems deriving from the applications of discrete splines to the problems of computer aided geometric design. New theory and algorithms have been developed and support mechanisms based on the Oslo algorithm have been incorporated in order to help use the geometry model directly to calculate many geometric attributes needed for the design process including surface rendering and intersections.
Discrete B-splines as an Approach to Computer Aided Geometric Design

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1. Statement of the Problem

The problems that this contract attacked are taken from the original proposal, with some additional background. At the time, the Oslo algorithm had just been developed and nonuniform, rational B-splines were not widely used since computing with them was slow and difficult. Historically the goals of good geometric models and high quality graphics had been opposing forces because high quality rendering schemes required polygonal data bases and freeform surface models required spline surfaces. One could have good graphics or good geometry, not both. Subdivision had become a technique used in design for editing control points to Bezier curves for flexibility and in computer graphics to generate smooth models for display in a divide and conquer approach. The methods used, however, could be presented as formulas which could be proved correct, but did not emanate from a theory showing why they worked or how new ones might be discovered. The theory of discrete splines gives a framework for understanding subdivision and for developing new subdivision and refinement algorithms in complicated situations.

It was proposed to bring together the geometrical design and graphical display of curved surface objects to form the basis of an integrated system for computer aided geometric design using the Oslo algorithm, based on the theory of discrete splines. It was proposed to develop, analyze, and validate practical discrete spline algorithms and support mechanisms based on the Oslo algorithm to help bring about the unification by using the geometry model to directly find attribute calculations, surface rendering, and intersections.

2. Summary of Results

A interesting variety of work has been partially supported under this contract, all in pursuit of the original goals.

Since good computer aided design requires realistic image making capabilities for the models, some work has been devoted to that area. In particular,

* Transparency of discrete spline surfaces and realistic shading models of same have been achieved.

* Dynamic shading methods for faster imaging have been implemented.

* Investigations into artificial texturing of surfaces as an aid to surfaces visualization have been made through the use of graphical operators which help convey shape information.
Theoretical and algorithmic advances in the mathematical methods have included some basic spline results such as

* Developing theory and practical algorithms, based on discrete splines, for degree raising of general spline curves. These results had immediate applications in the research performed on new 3-D modelling techniques for computer aided design.

* Initial investigations of multivariate spline theory with our Oslo colleague Professor Tom Lyche. In that context we have introduced "discrete box splines", which serve as a theoretical framework for refinement theory of continuous box splines. The proofs and algorithms developed investigating the relationships between discrete and continuous box splines open the door for investigation of other properties of continuous box splines.

* Algorithms for performing boolean operations on polyhedra, that is union, intersection, and difference have been developed. While the theory is relatively straightforward, the implementation in floating point brings this problem into the domain of fuzzy sets. Exact arithmetic was tried but found to perform unacceptably slow.

The study of methods for using this theory in the design arena involved

* Research into modelling the standard "primitives" of solid modelling using the representation proposed in this contract.

* Developing spline defined "rounded edge primitives".

* Investigating new operators which might act as the "verbs" of computer aided mechanical design.

* A modelling project for a freeform object, a helicopter was initiated as a study in tools and techniques.

Thus, investigations into many interesting problems deriving from the applications of discrete splines to the problems of computer aided geometric design. New theory and algorithms have been developed and support mechanisms based on the Oslo algorithm have been incorporated in order to help use the geometry model directly to calculate many geometric attributes needed for the design process including surface rendering and intersections.
3. Publications


"Summary of the Concepts of the Alpha_1 CAGD System", R. F. Riesenfeld, Detroit Engineer, 8–10 (April 1982).


"Spoon: A Demonstration of Shape Operators for Spline Surfaces", Elizabeth Cobb, a movie, (still incomplete).
4. Personnel Supported

Faculty and Staff:
Prof. Richard F. Riesenfeld,
Prof. Elaine Cohen,
Prof. S. Thomas (earned Ph.D. while participating on the project as a student),
Dr. Elizabeth Cobb (earned Ph.D. while participating on the project as a student),
Dr. L. Knapp,
Mr. Russell Fish,
Ms. M. Klusciewicz

Students:
P. Tinker,
J. Yen,
P. Stay (earned M.S. while participating on the project).
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