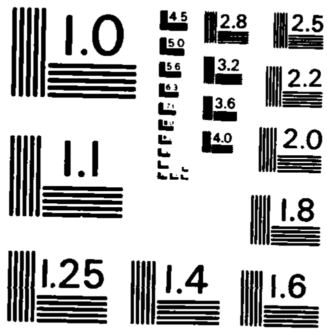
AD-A161 399   BIMODAL OPTICAL COMPUTER(U) AERODYNE RESEARCH INC   1/1
BILLERICA MA H J CAULFIELD ET AL. 06 SEP 85 ARI-RR-461
ARO-22455.1-PH-S DAAG29-84-C-0026

UNCLASSIFIED   F/G 9/2   NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

ARO 22455.1-PH-S

AD-A161 399

BIMODAL OPTICAL COMPUTER

FINAL REPORT

H.J. Caulfield and J.H. Gruninger
Aerodyne Research, Inc.

K. Steiglitz, H. Rabitz, and J. Gelfand
Princeton University

6 September 1985

U.S. Army Research Office

Contract No. DAAG29-84-C-0026

Aerodyne Research, Inc.
45 Manning Road
Billerica, MA 01821

APPROVED FOR PUBLIC RELEASE:
DISTRIBUTION UNLIMITED

11 18 - 85 228

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARO 22455.1-PH-S	2. GOVT ACCESSION NO. A161399	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Bimodal Optical Computer		5. TYPE OF REPORT & PERIOD COVERED Final Technical 10/1/84 - 9/30/85
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H.J. Caulfield and J.H. Gruninger (ARI) K. Steiglitz, H. Rabitz, J. Gelfand (Princeton U.)		8. CONTRACT OR GRANT NUMBER(s) DAAG29-84-C-0026
9. PERFORMING ORGANIZATION NAME AND ADDRESS Aerodyne Research, Inc. 45 Manning Road Billerica, MA 01821		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 6 September 1985
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

High accuracy analog, optical computer, matrix equation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Much of the current attention being given to optical computing stems from its new found ability to perform digital operations and hence to achieve high accuracy. In optics as in electronics, digital operations are slower and clumsier than analog operations. On the other hand, digital operations are more flexible and more accurate. These considerations lead us to consider whether some sorts of hybrid optical-electronic, analog-digital systems might be useful for some tasks. In this paper, we consider two closely-related

(over)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. hybrids: analog optics with digital electronics and analog optics with hybrid analog-digital electronics. Both systems will use analog optics for rapid and moderately accurate solution of complex problems and use the electronics to "bootstrap" the accuracy. That is each new optical computation should increase the accuracy of the solution. We will call such a system a Bimodal Optical Computer.

While many of these concepts appear to be new to optics, they are not new to computing in general. We will attempt to document the most useful of the prior publications, to make isomorphisms with other fields explicit, and to label those contributions which appear to be new. With all of these caveats, our primary purpose is to present a general approach for using optical computers. We will illustrate that approach with one specific problem (linear algebraic equations), but this should not obscure the generality of the concept.

ARI-RR-461

BIMODAL OPTICAL COMPUTER

FINAL REPORT

Contract No. DAAG29-84-C-0026

Submitted to:

Dr. B.D. Guenther
Department of the Army
U.S. Army Research Office
Post Office Box 12211
Research Triangle Park, NC 27709

Submitted by:

Aerodyne Research, Inc.
45 Manning Road
Billerica, MA 01821

Prepared by:

H.J. Caulfield and J.H. Gruninger
Aerodyne Research, Inc.

K. Steiglitz, H. Rabitz, and J. Gelfand
Princeton University

TABLE OF CONTENTS

1. INTRODUCTION

Much of the current attention being given to optical computing[1-4] stems from its new found[5] ability to perform digital operations and hence to achieve high accuracy. In optics as in electronics, digital operations are slower and clumsier than analog operations. On the other hand, digital operations are more flexible and more accurate. These considerations lead us to consider whether some sorts of hybrid optical-electronic, analog-digital systems might be useful for some tasks. In this paper, we consider two closely-related hybrids: analog optics with digital electronics and analog optics with hybrid analog-digital electronics. Both systems will use analog optics for rapid and moderately accurate solution of complex problems and use the electronics to "bootstrap" the accuracy. That is each new optical computation should increase the accuracy of the solution. We will call such a system a Bimodal Optical Computer.

While many of these concepts appear to be new to optics, they are not new to computing in general. We will attempt to document the most useful of the prior publications, to make isomorphisms with other fields explicit, and to label those contributions which appear to be new. With all of these caveats, our primary purpose is to present a general approach for using optical computers. We will illustrate that approach with one specific problem (linear algebraic equations), but this should not obscure the generality of the concept.

2. THE GENERIC SYSTEM

The generic system is comprised of three properly-interacting subsystems: an optical analog solver of the basic problem, a memory, and an accurate (digital or hybrid) calculator of the solution accuracy. The basic cycle is:

- Calculate an approximate solution with the optical analog processor,
- Remember that solution to high accuracy,
- Calculate the solution accuracy with the accurate computer,
- Repose the problem as an error reduction problem,
- Solve with optical analog processor,
- Using the just-calculated improvement and the stored prior solution, calculate and remember the improved solution with the accuracy computer,
- Calculate the solution accuracy with the accurate solution,
- If the solution is accurate enough, stop.
- If not, recycle.

Clearly, the convergence condition is that the error be reduced in each iteration. If this is the case, as we will show, the optical analog processor no longer limits solution accuracy.

In a purely digital system, the primary consumer of space, weight, power, time, and cost is the direct or iterative problem solver which would be replaced by the relatively small, low weight, power conservative, fast, and inexpensive optical analog processor. Thus there is the potential for significant overall system improvement using this hybrid approach.

There are two major forms the accurate processor can take. First, it can be a special purpose, fast, inexpensive digital processor. For reasons which will soon become evident, we call the hybrid system involving such a processor a "mathematical problem solver." Second, the accurate processor could be a physical system interacting with the world. The problem is then isomorphic with the control theory. We call such a processor a "physical problem solver." With a mild effort, the reader should become convinced that these two problem solvers use the same mathematics. We turn now to the mathematics.

3. ACCURACY ANALYSIS

We will examine the Bimodal Optical Computer (BOC) with specific emphasis on linear algebra as might be used, for example, for numerical solution of partial differential equations. The generic BOC method was originally proposed by Thompson[6] quite some time ago for iteratively improving the precision of mechanical devices which were used for the simultaneous solution of linear equations. This method appears to provide some considerable benefit for situations where a low accuracy, but fast device is available for providing approximate solutions to partial differential equations. This can then be linked to a higher accuracy device which is particularly well suited for forward substitution of the approximate solution into the original equation. The BOC iterative scheme, besides having been proposed by Lord Kelvin,[6] is a standard numerical approach to the iterative solution of linear systems and has been analyzed with respect to numerical round-off error by Wilkinson[7] and Stewart,[8] among others. A working model of this analog and digital bimodal electrical computer has also been constructed by Karplus.[10] This work reexplores and extends the prior work and incorporates modern linear and nonlinear optical computer techniques.

We can summarize this idea in the following way. Suppose we want to solve an n-dimensional linear system of equations.

$$\underline{\underline{A}}\,\underline{x} = \underline{b} \tag{1}$$

These problems are of great interest in their own right. In addition such systems with high dimensions arise when linear partial differential equations are solved by the finite difference method. Many other problems can be recast in this form. Suppose further that we have built a discrete optical analog processor for this problem which gives an approximate solution that can be summarized with the equation

$$\underline{\underline{\tilde{A}}}\ \underline{\tilde{x}} = \underline{\tilde{b}} \tag{2}$$

where $\tilde{A}$ and $\tilde{b}$ differ from A and b because of the limited accuracy of the analog components. We now have an approximate solution to our problem, $\tilde{x}$, which typically is accurate to a few percent. Next, we use a digital electronic computer to form the residual

$$\underline{r} = \underline{b} - \underline{\underline{A}}\ \underline{\tilde{x}} \tag{3}$$

using the actual, high precision versions of $\underline{\underline{A}}$ and $\underline{b}$. Notice that this step entails only substitution of the current solution, $\underline{x}$, in the modal equations, a relatively fast operation for even a modest digital computer. Subtracting Eq. (3) from (1) with digital electronics, we can write

$$\underline{\underline{A}}(\underline{x} - \underline{\tilde{x}}) = \underline{r} \tag{4}$$

Call the current solution error

$$\underline{x} - \underline{\tilde{x}} = \Delta\underline{x} \tag{5}$$

and write Eq. (4) as

$$\underline{\underline{A}}(\underline{\Delta x}) = \underline{r} \tag{6}$$

We now have a problem of the same form as the original with $\underline{\underline{A}}$ being the same matrix, except with the inhomogeneity term $\underline{b}$ replaced by the residual vector $\underline{r}$.

We now want to use the analog optical computer again, to estimate $\Delta \underline{x}$ and refine the solution, $\tilde{x}$, but we first can scale the equations by an appropriate large number, S, to bring the voltages and currents back to the levels in the first solution. Thus, we solve

$$\tilde{\underline{\underline{A}}}\ \underline{y} = (S\ \tilde{r}) \tag{7}$$

and then use the estimate

$$\Delta \underline{x} = \underline{y}/S \tag{8}$$

to refine the current solution to

$$\underline{x} = \tilde{\underline{x}} + \Delta \underline{x} \tag{9}$$

This process can be iterated, and under favorable conditions will converge quickly to solutions of accuracy only by the digital computer representation of $\underline{\underline{A}}$, $\underline{b}$ and the digital computation of Eq. (3). The description above for the iterative procedure was given in terms of a linear equation; however, this concept may also be applied to nonlinear systems,and would take advantage of the unique capacity of nonlinear analog circuits for the solution of the nonlinear algebraic equations of the discretized system. An analysis similar to the above treatment will again apply since the equations become quasi-linear near the true solution.

We might call this a <u>floating-point analog computation</u> where the scaling parameter, S, acts as a radix, varying from stage to stage with the size of the residuals in the equations. We note that this technique is quite similar to the very standard iterative numerical methods, such as Newton's method. In addition, we see that this technique marries analog and digital computers in a most congenial way — we take advantage of the speed and highly parallel nature of the analog system, as well as the memory and high precision of the digital system in the external loop.

We have examined the stability and convergence properties of the iteration process for this BOC. To first order we can model the error caused by solving the system on an analog computer (Step 2) by [8, Corollary 3.7]:

$$(A + E)^{-1} = (I + F)A^{-1} \tag{10}$$

where E is the error in the matrix due to the analog representation, the norm of F is bounded by

$$||F|| \leq \frac{k(A)\dfrac{||E||}{||A||}}{1 - k(A)\dfrac{||E||}{||A||}}, \tag{11}$$

$||\cdot||$ is a matrix norm, and the condition number of A is defined by

$$k(A) = ||A||\cdot||A^{-1}|| \tag{12}$$

Substituting this in Step 3 gives

$$x_{k+1} = x_k + \delta_k \tag{13}$$

$$= x_k + (I + F)A^{-1}(b - Ax_k)$$

Letting $x^* = A^{-1}b$ be the exact solution, we can rearrange this to yield

$$x_{k+1} - x^* = -F(s_k - x^*) \tag{14}$$

and taking norms of both sides,

$$||x_{k+1} - x^*|| \leq ||F||\cdot||x_k - x^*|| \tag{15}$$

We thus have a sufficient condition for geometric convergence of the process, namely $||F|| \leq 1$, which is satisfied if

$$k(A)\frac{||E||}{||A||} \leq 1/2 \tag{16}$$

where k(A) is the condition number of A and $||E||$ is the error in the analog representation of the true matrix A. Of course when convergence takes place, the errors in the digital computation may ultimately overtake the effect of the analog error that is modeled here, although the effects of analog noise may prevent that kind of ultimate accuracy.

We have performed an eigenvalue analysis of the operation of this bimodal machine for the solution of a one-dimensional partial differential equation and have performed test numerical calculations to illustrate the properties of this system. We have found that convergence depends sensitively on the coefficients of differential equations as well as the boundary conditions. The analysis which we have done so far corresponds to the case of a relaxation parameter equal to one. The use of relaxation parameters less than one should influence the range over which this convergence scheme will be stable.

The particular one-dimensional partial differential that was investigated in detail [9] was

$$\frac{d^2y(z)}{dz^2} + k[y(z)] = 0 \quad , \tag{17}$$

$$y(a_1) = b_1$$
$$y(a_2) = b_2 \tag{17a}$$

Discretizing this equation leads to the linear system $Ax = b$, where

$$A = \begin{bmatrix} 2+\delta & -1 & 0 & \cdot & 0 \\ -1 & 2+\delta & -1 & \cdot & 0 \\ 0 & -1 & 2+\delta & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 2+\delta \end{bmatrix} \tag{18}$$

and

$$\underline{b} = \begin{bmatrix} b_1 \\ 0 \\ 0 \\ \cdot \\ b_2 \end{bmatrix}$$, and the parameter $\delta = -k(\Delta z)^2$

where Δz is the grid size.

The iterative analog/digital scheme is governed by the recurrence relation

$$\underline{x}^{(i+1)} = M \cdot \underline{x}^{(i)} + \bar{A}^{-1} \cdot b \tag{20}$$

where $\bar{A}$ is a version of A perturbed because of the limited accuracy of the analog part of the system, and

$$M = I - \bar{A}^{-1}A \tag{21}$$

The stability of the method can be seen now to depend on the eigenvalues of this matrix M.

It is clear from Eq. (21) that if A is ill-conditioned, the eigenvalues of M can be unbounded. Therefore, we need to examine whether the matrix A is ill-conditioned, that is whether the matrix A^{-1} is unstable. One measure of the stability of A^{-1} is the sensitivity of det(A) to small changes in the elements of A. Since δ is a parameter, we first examine the dependence of det(A) on δ. In Figure 1 we show this dependence for $n = 30$, although the behavior is found to be the same for any value of n.

It was found that there are three distinct regions of behavior, depending on δ:

1. $|\delta| \ll 1$. Here there is, for a given accuracy of the analog part, a definite limit to how large the number of grid points n can be before the scheme becomes unstable. In particular, one may show that this upper limit is $\sim 1/\varepsilon$, where ε is the accuracy of the analog part of the system.

2. $|\delta| \gg 1$. The iteration was found to be stable in this region.

3. δ [-4,0]. Here, the stability depends critically on the eigenvalues of A, and the scheme is generally unreliable.

The analysis above, for a simple second-order linear differential equation, shows that more work is needed to put the proposed hybrid method into practice. Relaxation parameters, or more sophisticated iterative approaches, such as conjugate-gradient techniques, might be required to ensure stability for large, nonlinear systems of interest.

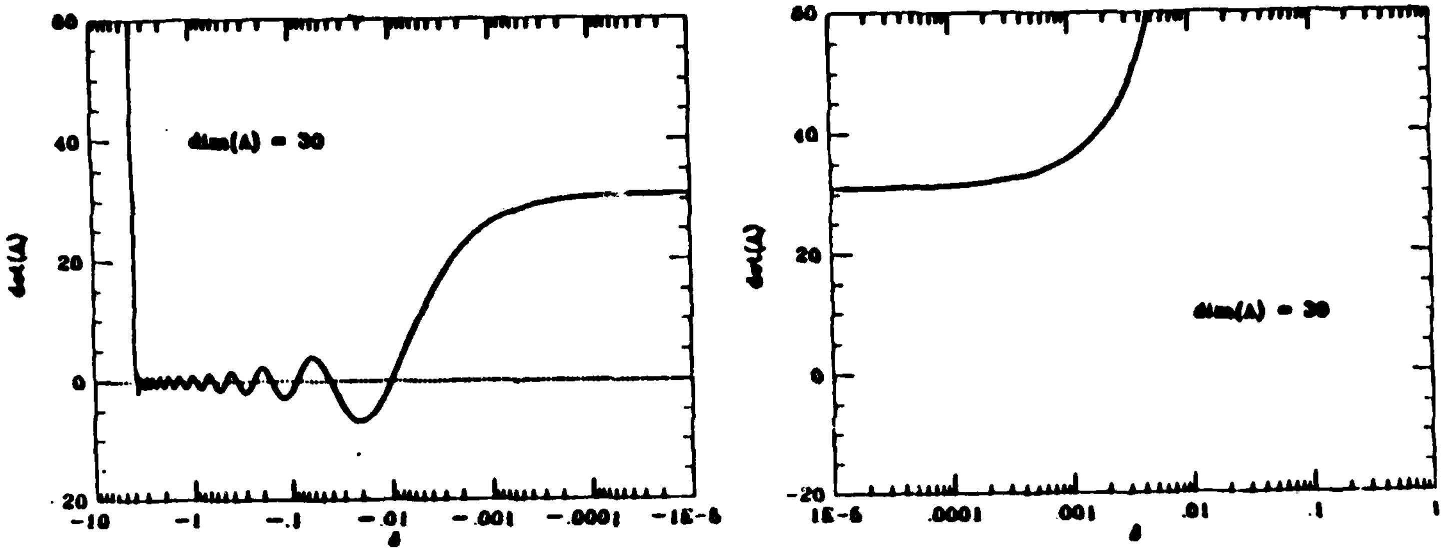


Figure 1. Determinant of the Matrix A in Eq. (18) for a Model Case of N = 30. Regions where $|\det A|$ is large are stable for solution.

4. NONLINEAR PROBLEMS

Perhaps the most important payoff with BOCs may be associated with the solution of nonlinear problems. Many physical phenomena result in nonlinear differential or ultimately algebraic equations for solution. Such problems are notoriously difficult to treat by conventional numerical methods on digital computers. This comment follows since the algorithms will involve linearization or perhaps iteration with convergence being slow or perhaps nonexistent in highly nonlinear problems. A more suitable approach would be based on directly building the nonlinear behavior into the calculation process. It appears possible to construct hybrid machines based on this logic following lines parallel to that discussed in Section 3. The key to this approach rests on the fact that nonlinear electronic or optical elements can be readily made and integrated together into an overall nonlinear computer.

As a very simple example of a nonlinear problem we may consider the search for roots of a polynomial $p(x)$ in the real variable x. It is straightforward to use optical methods to evaluate polynomials via Horner's rule. Optical polynomial evaluation can be analog [11] or digital [12]. some "tricks" which accommodate dynamic range, allow root searching by scanning, extend the range of problems addressed, etc. are given in the latter reference. Root searching for real roots simply by scanning through x and watching for $p(x) = 0$ conditions, is straightforward and fast. It is, however, not likely to be highly accurate. Suppose we form an approximate real root x_o. We can then evaluate $p(x_o)$ and

$$p_1(x) = p(x) - p(x_o)$$

digitally. Assuming we are now close to the true root, we can now change the scale of both p_1 and x to gain sensitivity. We might substitute $y = 10\ x$ and

$q_1 = 10\ p_1$, and then search $q_1(y)$ as before. This leads to a better approximation x_1 as can be verified by digital evaluation of $p(x_1)$. Accuracy is limited by the condition number of the polynomial because that limits the accuracy of the polynomial evaluation. Other similar examples can be found, and a general set of logic can be set forth as discussed below.

A nonlinear computer of the type discussed in the first paragraph would likely be of limited accuracy, but capable of achieving an extremely rapid solution without the introduction of artificial linearization or iteration algorithms. The machine could be used alone or incorporated into an overall hybrid device along the lines discussed in Section 3 and in the polynomial root searching example. This would entail introducing a high accuracy digital computer as a means of monitoring residual errors. Updated corrections to the original fully nonlinear solution could be achieved easily if the first estimate is close enough to the true answer so that the nonlinear computer effectively operates in the linear mode after the first cycle. As an alternative it would be possible to construct an additional linearized version of the machine for the accuracy updates on the solution. These approaches may be theoretically modeled as well as demonstrated in the laboratory and we plan to carry out such studies in the future.

5. CONCLUSIONS

Analog optics, when adequate for a task, is usually superior in speed, size, power consumption, and cost to all competitors. What we have suggested here is a means to extend the set of situations for which analog optics is adequate. Many studies remain to be performed on both algorithms and hardware. Nevertheless the general concept of a hybrid system appears to be extremely promising.

6. REFERENCES

1. H.J. Caulfield, J.A. Neff, and W.T. Rhodes, Laser Focus, November 100 (1983).

2. "Optical Computing," Special Issue of Opt. Eng., January (1984), D. Psaltis, Ed.

3. "Optical Computing," Special Issue of Proc. IEEE, July (1984), H.J. Caulfield, et al., Eds.

4. "Optical Computing," Special Issue of Opt. Eng., January (1985), J.A. Neff, Ed.

5. R. Athale and W.C. Collins, Appl. Opt. 21, 2089 (1982); R.P. Bocker, H.J. Caulfield, and K. Bromley, Appl. Opt. 22, 804 (1983), P.S. Guilfoyle, Opt. Eng. 23, 20 (1984).

6. W. Thompson (Lord Kelvin), Proc. Roy. Soc. (London) 28, 111 (1978).

7. J.H. Wilkinson, Rounding Errors in Algebraic Processes, Prentice-Hall, 1963.

8. G.W. Stewart, Introduction to Matrix Computations, Academic Press, New York (1973).

9. E. Tsoni, K. Steiglitz, J. Gelfand, and H. Rabitz, "Stability and Convergence Properties of Lord Kelvin's Bimodal Computer," unpublished manuscript, Princeton University, December 1984.

10. W.H. Karplut, IEEE Trans. on Computers, C-20, 831 (1971).

11. C.M. Verber, R.P. Kenan, H.J. Caulfield, J.E. Ludman, and D.P. Stilwell, Appl. Opt. 23, 817 (1984).

12. H.J. Caulfield, Appl. Opt. 23, 373 (1984).

END

FILMED

1-86

DTIC