EXPERIMENTAL GENERATION OF STRATIFIED SHEAR FLOWS

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August 1985

SBIR Phase I Final Report

Prepared for
Air Force Office of Scientific Research
Boiling AFB, DC 20332

Attn: Lt. Col. Gerald J. Dittberner
Chemistry and Atmospheric Sciences
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**Abstract:**

An annular, stratified flow tank was designed and built to study gravity wave, mean flow (critical layer) interactions. The tank contained stratified salt water, and an initial shear profile was generated by blowing air over the water surface. Internal gravity waves were generated by displacing the bottom floor of the tank in a known way.

A preliminary critical layer experiment was performed to prove the feasibility of studying critical layer interactions in the experimental facility. Quantitative measurements of mean flow velocities, velocity perturbations, and vertical wavelengths were obtained.

**Subject Terms:**

- Critical layers
- Stratified
- Shear flow
- Gravity waves
This report describes the results of a Small Business Innovation Research (SBIR) Phase I contract awarded to Physical Dynamics, Inc. by the Air Force Office of Scientific Research. The objectives of the Phase I effort were (1) to design and build a stratified shear flow facility and (2) to demonstrate the feasibility of generating stratified shear flows in the facility. In Phase II, these stratified shear flows would be used to study gravity wave, mean flow (critical layer) interactions.

This report documents the Phase I study. In addition to the objectives outlined above, we also present the results of a preliminary critical layer experiment.
ACKNOWLEDGEMENTS

Timothy J. Dunkerton and David C. Fritts participated in the Phase I study with theoretical and numerical modeling. Their participation in the design of the tank and in the interpretation of the experimental results is gratefully acknowledged. J. Francis Smith and Bruce J. Higgins assisted in the design, construction, and running of the experimental facility. The interest and support of Lt. Col. Gerald J. Dittberner, of AFOSR, is greatly appreciated.
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1. Identification and Significance of the Problem

Gravity waves are a ubiquitous feature of the earth’s atmosphere. These waves exist over a broad range of frequencies and wavelengths, having periods of several minutes to several days, and spatial scales from a few kilometers to the circumference of the earth. The vertical group velocity, as well as the complexity and significance of these waves, changes markedly from the smallest to the largest scales. The small-scale waves are essentially buoyancy waves which, if flow conditions permit, may propagate from the lower to the upper atmosphere in a matter of hours. The large-scale waves, on the other hand, are affected by rotation and radiative damping, and are essentially inertia-gravity and/or equatorial waves which may take several days or weeks to propagate through the middle atmosphere.

Despite the wide range of gravity wave scales, these waves have two important features at all scales which make them very significant for the earth’s general circulation. First and foremost is their ability to transport momentum upward over many scale heights. The small-scale waves are believed to provide an essential drag acting to oppose Coriolis torques in the winter mesosphere (Lindzen, 1981; Matsumo, 1982; Holton, 1982). The large-scale equatorial waves are responsible for quasi-biennial and semiannual oscillations in the tropical middle atmosphere (Holton and Lindzen, 1972; Dunkerton, 1979, 1982b). Second, gravity waves mix tracers and photochemical species in the region of their unstable breakdown. In the atmosphere, wave amplitude growth leading to unstable breakdown is caused by mean wind shear as a wave approaches its critical level (where its phase speed equals the mean flow speed; Booker and Bretherton, 1967) and by the reduction of density with height (Fritts, 1984).

Observationalists have attempted to isolate and study gravity wave, critical-level interactions (Merrill and Grant, 1979) as well as equatorial wave, mean-flow interactions (Wallace and Kousky, 1968). However, it has generally been difficult to measure phase speeds accurately and, hence, to locate gravity wave critical levels and to observe the interactions quanti-
tatively. Much of our present understanding is derived from theoretical and numerical research in this area, which is reviewed below. These studies, however, suffer from their inability to make objective comparison to observations of gravity waves and their mean flow interactions.

To help solve this dilemma, a few researchers have attempted gravity wave, critical-level experiments under controlled, laboratory conditions. These studies will also be reviewed below. These experimenters have performed some pioneering research, yet their experiments have had some major shortcomings, namely:

1. The flow facilities have all been restrictive in terms of the time each flow facility maintained constant flow conditions. In the studies to date, the duration of the experiments under constant flow conditions has ranged from several seconds to several minutes. In no studies have the interactions been observed for more than several minutes.

2. Only qualitative, flow visualization studies have been performed to date, and no quantitative measurements have been reported. In this regard, some of the measurements shown below, made during our feasibility study, represent the first quantitative measurements of their kind.

We will show below that the new experimental facility built in the Phase I study overcomes the shortcomings of the facilities used in previous gravity wave, critical-level experiments. We will also show that quantitative measurements of a gravity wave, critical-level flow can be made in this new facility.
2. Related Research

A propagating gravity wave encounters a "critical level" when the mean flow equals the horizontal phase speed of the wave. The linear theory for the interaction of a gravity wave with a critical level was first developed by Bretherton (1966) and Booker and Bretherton (1967). Subsequent research has been performed by Benney and Bergeron (1969), Maslowe (1973, 1977), Grimshaw (1975), Fritts (1978, 1979, 1982a, 1982b), Brown and Stewartson (1980), Dunkerton (1980, 1981, 1982a, 1982b), Lindzen (1981), Dunkerton and Fritts (1984), and many others.

As discussed in Dunkerton and Fritts (1984), researchers have uncovered three possible shortcomings of the linear theory. First, because the critical layer is a barrier to wave propagation, wave action "piles up" beneath the critical layer. If the wave action density increases sufficiently, convective breaking of the wave will occur, leading to irreversible mixing. Second, because of irreversible wavebreaking, there may be permanent mean flow accelerations of the fluid below the critical layer. Third, nonlinear terms may become important in the critical layer after a sufficiently long time. Once these terms become important, wave reflection from the critical layer and, possibly, wave transmission through the critical layer may occur.

Most of the linear and/or nonlinear studies of gravity wave, critical layer interactions have never been experimentally verified. The importance of experimental verification lies in the fact that the various theoretical solutions of gravity wave, critical-layer interaction involve hydrodynamic instability at some point in their temporal evolution, at least at high Reynolds numbers characteristic of the atmosphere. These instabilities may be thought to include convection, Kelvin-Helmholtz instability, and wave-wave interactions. The relevant question, then, is to what extent will any of these instabilities act to modify the critical layer evolution under circumstances applicable to the atmosphere? Specifically, how will they modify critical layer reflection, absorption, mean flow accelerations, and transmission? Are nonlinear steady-state critical layers attainable? How does the critical layer evolution depend on the relative importance of viscosity and turbulence?
These questions have motivated our experimental study of critical layer interactions. To our knowledge, there have been only four experimental, laboratory, gravity wave, critical-level interaction studies (Bretherton et al, 1967; Thorpe, 1973 and 1981; and Koop, 1981). The first three of these experiments (Bretherton et al, 1967; Thorpe, 1973 and 1981) were performed in a tilting tank. In this facility, a long horizontal tube of rectangular cross section is filled with a stably stratified salt solution. When the tube is tilted through a small angle, the heavier fluid runs to the bottom due to gravity, and the lighter fluid on top, because it is constrained in the closed tank, runs in the opposite direction. Thus, a vertical velocity shear is created. If the tube remains tilted, the fluids continue to accelerate, causing an ever increasing shear and forcing the Richardson number lower and lower. If the tube is returned to the horizontal position after some prescribed time, the Richardson number will remain constant. The flow in the tank is eventually disrupted at the center either by the growth of Kelvin-Helmholtz billows, if the Richardson number is below one-quarter, or by the arrival of surges which develop and propagate from the ends of the tube. Typically, steady state conditions are maintained in the tank for no more than 20 - 30 seconds, this time depending on the aspect ratio of the tank as well as the stratification profile.

The first two experiments performed in the tilting tank (Bretherton et al, 1967 and Thorpe, 1973) looked at gravity waves which were generated in the lee of a rectangular-shaped obstacle which was attached to the floor of the tank. For these experiments, the density gradient was constant. Because the lee waves were stationary with respect to the obstacle, their critical level was at the center of the tank, where the velocity was zero. These experiments were somewhat cursory in nature and were reported either in an appendix to a main paper or as a small part of a much larger investigation. Qualitative observations of dye layers and shadowgraph photographs showed that the critical layers were efficient absorbers of gravity waves with no detectable energy transmitting through the critical layer to the upper half of the fluid. The Richardson number range for these studies was 0.5 to 200.
In a later, more extensive study, Thorpe (1981) used a tilting tank with a corrugated floor to generate the gravity waves. The corrugated floor generated well defined, sinusoidal waves of known wavelength. Again, these experiments consisted of qualitative observations of dye layers. The running time of this tank was too short (13 - 15 seconds) to allow for steady-state critical layer flow. A theoretical analysis was also performed in this study, and comparisons were made between the theory and the dye photographs. Large amplitude internal waves could not be generated due to dense fluid lying in the troughs of the corrugated floor. (As an aside, we have no inherent difficulty in generating large amplitude waves in our facility since we are forcing the waves directly and are not relying on fluid moving past an obstacle to generate the waves.) As in the earlier studies, no waves were observed to propagate beyond the critical layers. There is, however, in one experiment, a hint of wave reflection from the critical layer and a hint of wave transmission through the critical layer (Thorpe, 1981, Figure 3).

In a different experiment, Koop (1981) used a closed-return, stratified shear facility to study a variety of interactions of internal gravity waves with shear, critical layers being one of the interactions studied. In this facility, a modified "Kovasznay" pump (modified from the original design of Odell and Kovasznay (1971)) was used to create the velocity profile. In the Kovasznay pump, a series of circular plates accelerate the incoming flow. The plates are horizontal, to minimize the mixing of the fluid, and, hence, to minimize destroying the ambient stratification. Koop modified the original Kovasznay design by adding impellers to the circular plates, thereby improving the efficiency of the pump. In Koop's experiment, the top half of the pump was rotated at a different speed than the bottom half of the pump. This created essentially a two-layer system; in Koop's experiment the bottom layer was quiescent while the top layer moved at a nearly constant speed. The transition region between these two layers was the region where the velocity varied and was the region of interest for the critical layer studies. The vertical thickness of this region was ~5 cm. The velocity profile in this transition region was not always smooth nor did it always decrease monotonically, and it varied with time. Due to mixing downstream of the pump, the
Initially linear density profile develops a kink after ~7 minutes of running. This kink and the change in the velocity profile essentially limit the steady-state running time of the facility.

Internal gravity waves were produced by Koop by towing either a sinusoidally corrugated plate on the floor of the tank (similar to Thorpe (1981)) or a cylinder. The plate produced a nearly monochromatic wave (except for end effects), while the cylinder produced a broad-banded but spatially compact signal. The measurements were qualitative and consisted of shadowgraph photographs of the resulting flowfields. In both cases, no wave motions were observed above the critical layer. Similar to Thorpe (1981), Koop also notes the difficulty of generating large amplitude waves due to flow recirculation in the troughs of the corrugated bottom. The Richardson numbers in Koop's critical layer experiments were $319$ and $150$.

In summary, four experimental studies of gravity wave, critical-level interactions have been reported in the literature. They have been pioneering in their efforts, but have not been totally satisfying. Either (with the tilting tank) the run time of the flow has been too short to study the flow field adequately, or (with the closed return facility) the region of the velocity shear was limited in depth, the velocity profile was not "clean" and varied with time, and the velocity profile could not be easily modified or accurately controlled. Finally, and most importantly, no quantitative measurements have been reported with which to compare to theoretical or numerical models. Thus, we believe that our study fills an obvious gap in our understanding of this phenomenon.
3. The Experimental Facility

The experimental facility we designed and built in Phase I is the annular tank shown schematically in Figure 1. The outer diameter of the tank is 1.8 m (6 ft), the inner diameter is 1.2 m (4 ft), and the depth of the tank is 40 cm (16 in). The sides of the tank are made of clear acrylic to allow us to observe the flow inside the tank.

The bottom of the tank is composed of a 0.6 cm (0.25 in) thick rubber sheet overlaying sixteen 1.0 cm (3/8 in) thick acrylic sheets. The ends of each acrylic sheet rest on top of vertical pistons which are each driven by a stepper motor. The stepper motors are individually controlled by a North Star computer to allow us to move the bottom floor of the tank in a known way. The acrylic sheets provide structural rigidity between pistons, and the rubber sheet acts both as a water seal at the bottom of the tank and as a stretching membrane as the pistons drive the bottom floor of the tank up and down.

The wave tank was filled using two 300-liter storage tanks. The filling method is similar to that of Delisi and Orlanski (1975). As water is fed from the fresh water storage tank to the wave tank, salt water enters the fresh water storage tank and is mixed with a stirrer. This saltier water is then fed to the wave tank, and the process continues. The resulting density profile in the wave tank is nearly exactly linear. For the experiments discussed below, the Brunt-Vaisala frequency,

\[ N = \sqrt{-g \frac{\partial \rho}{\partial z}} = 1.24 \text{ s}^{-1} \]  

(1)

where \( g \) is the acceleration due to gravity, \( \rho \) is density, and \( z \) is the vertical coordinate.

Vertical shear was generated by blowing air on the top surface of the water, as described in the next section. Once a steady state velocity profile was achieved, we used the stepping motors and pistons at the bottom of the tank to generate gravity waves. The gravity waves we generated were monochromatic, of wavenumber two, and had a phase speed such that there was a critical layer near the top of the tank. Once started, the wave forcing was continuous.
Figure 1. Schematic drawing of the experimental facility.
Neutrally buoyant particles were added to the storage tanks before and during the filling process. These particles were composed of crushed Pliolite, a Goodyear Rubber compound. The particles were illuminated from the top of the wave tank using a 1000-watt theatrical spotlight. The spotlight was located at the side of the tank, and the light was reflected into the tank by a mirror. A 2.5-cm wide slit at the top of the viewing section allowed just the particles in the center part of the tank (in the radial direction) to be illuminated.

The experiments were documented using both a video camera and a 35-mm camera. The video camera documented each run, while the 35-mm camera was used to record streak motions of the neutrally buoyant particles. To obtain particle streaks, the film was exposed for either 1 or 5 seconds. These streak photographs were hand digitized on a HIPAD digitizing tablet and processed on a Prime 550 computer to yield instantaneous velocity profiles.
4. Generation of the Stratified Shear Flow

To generate the shear flow in the stratified water in the tank, we used a blower to circulate air at the top of the tank. The air moved the surface water and, eventually, the water below the surface. Care was taken to increase the air speed slowly, so as to minimize the amount of mixing at the air-water interface. Once a suitable steady-state velocity profile was reached, we generated internal waves at the bottom of the tank.

The velocity profile measured 30 minutes before the generation of the waves began is shown in Figure 2. This figure shows velocity vs nondimensional depth, the total depth of the tank, \( z_m \), being 40 cm. In this figure, the circles, squares, and triangles represent three independent digitizations from the same 5-second exposure photograph. The scatter in these points is small, indicating that the velocity profile is reproducible on successive digitizations of the same photograph. The stars and diamonds are digitizations of 1-second exposure photographs taken several seconds apart. The increased scatter in these points is due to the fact that the streaks are smaller in the 1-second exposures, resulting in larger percentage errors in determining the velocities. The irregular spacing of the data points is a reflection of the irregular spacing of the neutrally buoyant particles in the flow field. Also shown in Figure 2 is a subjective, hand-drawn curve through the data points.

The velocity profile at the start of the experiment, minutes before starting the waves, is shown in Figure 3. The dashed line in Figure 3 is the curve from the velocity profile in Figure 2, taken 30 minutes earlier. To within the experimental accuracy, these two curves are identical, indicating that the velocity profile is in steady state equilibrium.

We also measured the radial variation of the velocity profile at the start of the experiment. These measurements show that the velocity in the center half of the tank at the level of the critical layer (\( U = 2.6 \text{ cm/s} \)) varies by approximately 10%. With the velocity profile shown, this means that the depth of the critical layer varies by approximately 1.5 cm across the center half of the tank (a distance of 15.2 cm).
Figure 2. Velocity profile 30 min. before the start of the experiment. $z$ is depth from the bottom of the tank; $z_m$ is the total water depth (40 cm).
Figure 3. Velocity profile 30 min. before the start of the experiment (dash line) and at the start of the experiment (solid line).
5. Our Preliminary Critical Layer Experiment

The velocity profiles shown in Figures 2 and 3 are suitable for gravity wave, critical-layer studies in that the profiles are steady-state and the range of possible wave phase speeds is large (up to \( \sim 4 \) cm/s). In order to prove the feasibility of performing this type of study, we performed one preliminary critical layer experiment with the velocity profile shown in Figure 3.

At \( t = 0 \), we started the stepping motors and pistons at the bottom of the tank to generate internal gravity waves. These waves propagated into the tank and, eventually, reached a critical layer. The waves we used were wavenumber two, had an amplitude of 0.95 cm (1.9 cm peak-peak), and a phase velocity (on the tank centerline) of 2.6 cm/s. From Figure 3, the critical layer for these waves was at \( z/z_m \sim 0.75 \). Waves were generated continuously throughout the experiment. For these waves in this flow, the dissipation scale height, \( d \), was 64 cm, where

\[
d = \frac{kc^4}{\nu N^3}
\]

\( k \) is zonal wavenumber, \( c \) is the phase speed of the forced wave (2.6 cm/s), \( \nu \) is kinematic viscosity, and \( N \) is the Brunt-Vaisala frequency from equation (1). The percentage of saturated momentum flux at the lower boundary, \( \varepsilon \), was 0.20, where

\[
\varepsilon = \left( \frac{NA}{c} \right)^2
\]

and \( A \) is the wave amplitude.

At selected times during the experiment, photographs were taken at four equally spaced times during a wave cycle. Figure 4 shows the velocity field from one of these photographs at a time of 6 minutes and 41 seconds (06:41) after the waves were started. The solid line is a subjective curve fit through the data points. Clearly observable in this figure is the decrease in vertical wavelength as the wave approaches its critical level.

Figure 5 shows the results for the four photographs taken during one complete wave cycle from 06:18 to 07:28 (we will call this \( t = 7 \) min). The
Figure 4. Particle velocities taken from a time-lapse photograph. Time is 06:41.
Figure 5. Particle velocities for four equally spaced times during one wave cycle. Times are 06:18 to 07:28. The heavy, solid line is the mean velocity profile.
heavy, solid line on this figure is the mean velocity profile, taken as the average of the four, raw measurements at a given depth. We see from this figure that the velocity perturbations decrease as the wave approaches the critical level at \( z/z_m \sim 0.75 \).

From the data which went into Figure 5, we can estimate the vertical wavelength of the wave as a function of depth. To make these estimates, we measure the vertical distance over half the vertical wavelength (minimum velocity to minimum velocity and maximum velocity to maximum velocity) and place the estimate of total vertical wavelength at the average depth. These results are shown in Figure 6. In this figure, the circles indicate the estimates for the minimum to minimum measurements while the plus signs indicate the estimates for the maximum to maximum measurements. From the plot, there does not appear to be any difference between the two methods of estimating the vertical wavelength. This plot clearly shows the decrease in vertical wavelength as the wave approaches the critical level. The solid line in Figure 6 is the theoretical prediction for the vertical wavelength, given by

\[
\lambda_z = \frac{2\pi(c - \overline{U})}{N}
\]

where \( \overline{U}(z) \) is the current profile. To determine \( \lambda_z \), the value of \( \overline{U} \) was estimated from the solid curve in Figure 5, and \( c - \overline{U} \) was estimated with \( c = 2.6 \text{ cm/s} \). There is good agreement between the predicted and measured values of vertical wavelength for \( z/z_m \leq 0.5 \). Above that value, the measured wavelengths are substantially larger than the predicted values. This discrepancy, however, may be due to our method of measuring or plotting the wavelength. Additional analysis may shed some light on this issue.

Figure 7 shows data from the four time-lapse photographs taken between 69:44 and 70:54 (we will call this \( t = 70 \text{ min} \)). Comparing this figure to Figure 5, we see that at \( t = 70 \text{ min} \), the mean flow has substantially increased and the velocity perturbations have substantially decreased (note the shift in horizontal axis in Figure 7 relative to Figure 5). Again, the heavy solid line in Figure 7 is the mean velocity profile which is taken as the average of the four photographs.
Figure 6. Estimates of the vertical wavelength from the laboratory experiment (symbols) and theory (solid line) for $t = 7$ min.
Figure 7. Same as Figure 5 for times 69:44 to 70:54.
Figure 8 shows the initial velocity profile and the mean velocity profile after \( t = 7, 21, 38, 70, \) and 92 minutes. A steady state condition appears to have been reached by \( t = 70 \) minutes.

Figure 9 shows the maximum velocity perturbations as a function of depth for the times displayed in Figure 8. As observed in Figures 5 and 7, the velocity perturbations decrease with increasing time and reach steady state conditions around \( t = 70 \) minutes.

In this experiment, no overturning or turbulence was observed at or below the critical level. We knew \textit{a priori} from a gravity wave, critical-layer model by Dunkerton (part of the Phase I feasibility study) that the waves we generated at the bottom of the tank would not saturate and break at the critical level. The reason for using these waves, as opposed to using waves that would break, was due to limitations of the computer we used to run the stepping motors. (In the Phase I study, the problem was one of computer memory size limitations and was not a problem inherent in the facility. In the Phase II study being proposed, we will be changing the computer we will use to run the stepper motors, and we will no longer have this limitation.)

It should be emphasized that the results shown here are preliminary, and that no other flow conditions have, as yet, been run. Still, we believe that the results shown here represent the first quantitative measurements of gravity wave, critical-level interactions. The objectives of the Phase II study are to repeat these measurements under a wide range of experimental conditions, to extend the conditions to include convective wavebreaking and transient forcing, to extend our measurements and flow visualization techniques, and to compare the laboratory observations to theoretical and numerical predictions.
Figure 8. Evolution of the mean velocity profile. Times are minutes after the start of the experiment.
Figure 9. Maximum velocity perturbation profiles at t = 7 (left), 21, 38, 70, and 92 (right) min.
REFERENCES


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