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QUASI-OPTIC STUDY OF
DIELECTRIC RADOMES AND LENSES

FINAL REPORT
L. B. Felsen
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I. Background

Transmission of electromagnetic fields from a large distributed aperture through a curved and tapered dielectric shell poses one of the difficult problems in the analysis of radome covered antennas. The research summarized here has been concerned with approximate methods for addressing this problem area.

Under the predecessor contract (Grant No. DAAG-20-78-G-0168), attention had been given separately to the modeling (a) of distributed fields with tapered amplitude, and (b) of line source excited transmission through a curved or tapered dielectric shell. For amplitude tapers resembling Gaussian profiles, the radiated fields in a) were shown to be representable compactly by complex ray fields emitted from a source at a complex coordinate location. These complex ray fields could then be interpreted physically in terms of evanescent waves. The feasibility of tracking these complex ray fields continuously from the source through the near zone to the far zone was demonstrated for the case of a parabolic reflector antenna excited by a Gaussian beam feed.\(^1\) The complex ray tracing requires analytic extension of ray paths and boundaries into a complex coordinate space, the only real coordinate point being at the location where the complex ray meets the observer. Although more time-consuming than real ray tracing, it was shown that a complex ray code could readily be constructed and implemented.

The investigation under b) had shown that conventional ray theory can be employed to provide the fields radiated from a line source through a curved or tapered dielectric layer. Here, it was seen to be important to include multiple inside reflections between the layer boundaries, with inclusion of the appropriate

ate ray divergence coefficients. It was shown to be possible to define a new "collective ray" field which accounts in compact form, via a curvature and taper corrected slab transmission coefficient, for the multiply reflected rays. Keeping some ordinary rays before invoking the collective treatment furnished a new hybrid representation with broad flexibility. Various numerical tests established the effectiveness of the hybrid format [1,2].

Against this background, the follow-up study described herein was concerned with combining the techniques employed in a) and b) to attack the problem of transmission through the radome of Gaussian-like distributed aperture fields. This was to be modeled by tracing complex rays from a complex source point through the analytic extension of the radome to an exterior observer in the near or far zone. In the real physical space, this represents transmission of a Gaussian-type beam, due account being taken of multiple internal reflections (either individually or collectively), and allowing for beamwidths large enough to illuminate sections of the radome with appreciable curvature or tapering. The advantage of the complex ray approach is the avoidance of integrations over an interposed equivalent aperture outside the radome, which has been customary in affecting the near zone-far zone transition.

The strategy outlined above was tested on a carefully selected sequence of two-dimensional models, and the results are described in the next section. Although there was insufficient time to deal with the three-dimensional case, the conclusions from the present investigation indicate the feasibility of extending the method to three-dimensional configurations.
II. Accomplishments

A. Complex Ray Tracing Through a Circular Cylindrical and Wedge-Shaped Layer

The first phase of the investigation was concerned with extending the techniques of ordinary and collective rays from real rays to complex rays. This was done by the above-noted analytic continuation to a complex coordinate space, with major attention given to the numerical implementation. A complex ray code was constructed successfully, and was employed to provide the real space fields in the near zone and the far zone along direct and multiply reflected paths. Summing these rays fields individually was taken as the reference solution. A sufficient number of rays was retained to make the omitted ray fields smaller than a specified percentage (<0.1%) of the total. Collective summation was then invoked and compared with the reference solution. It was found that over broad ranges of radome parameters and incident beam widths, a single collective ray, or a hybrid format with one ordinary and one collective ray, was adequate. This established the validity of the collective ray scheme also for beam-type fields modeled via complex rays. The calculations included not only single complex sources generating a Gaussian-type beam but also two separate in-phase and anti-phase sources generating aperture profiles with a broad maximum and a central null, respectively. For the latter, the effect of the radome on the location of the null (boresight error) was studied in detail [3].

B. Paraxial Approximations

The most time-consuming part in the numerical implementation of complex ray tracing is the search for the complex ray parameter (for example, the complex departure angle at the complex source point) that ensures that the ray reaches a specified observer via a complex trajectory satisfying the analytically extended ray reflection and refraction laws. No comparable difficulties are encountered when tracing real rays. Therefore, it is relevant to ask whether
paraxial approximations are applicable, whereby the off-axis fields are expressed in terms of the on-axis values, recognizing that the beam axis trajectory proceeds entirely in real space like a real ray. A comprehensive study was performed, dealing first with a single plane boundary between two dielectric half spaces, then with a single circular cylindrically curved boundary, and finally with a circular cylindrical layer. In all cases, the paraxial approximations for ordinary and collective complex ray fields were compared with the full complex ray reference solution. It was found that there exist substantial ranges of parameters wherein the paraxial approximations are adequate. In these approximations, there were included corrections of the on-axis fields not only in the phase but also in the ray reflection, transmission and divergence coefficients. The effects of these corrections were studied systematically, with the conclusion that the phase correction is the most important [4,5].

Although not relevant for the radome problem, the validity of complex ray tracing and of paraxial approximations was explored also for the single plane interface when the source is located in the medium with higher dielectric constant. This gives rise to phenomena of total reflection and generation of a lateral wave, with a transition region surrounding the critically reflected ray. While incident beam fields modeled by complex rays give rise to a more diffuse critical reflection transition than conventional ray fields, non-uniformized complex ray theory may be inadequate when the beam axis lies along or near the critical ray. A uniform complex ray transition function was employed to study this effect, and criteria were developed to establish the validity or not of non-uniform complex ray fields in this transitional domain pertaining to the onset of total reflection.
C. Transition Function Near Critical Refraction on a Curved Boundary

Related to the planar interface investigation was a study of the behavior of critically refracted fields on a curved boundary. Here, the geometrical ray hierarchy is enriched by the presence of caustics and shadow zones not encountered on a plane interface. The various ray-optical domains were classified and the corresponding ray and transitional fields expressed quantitatively by GTD and new transition function formulas [6]. Special attention was given also to the validity of the so-called "tunneling hypothesis," which seeks to incorporate a leakage term (due to surface curvature) in the reflection coefficient of a totally reflected ray [7].

D. Complex Ray Modeling of Non-Gaussian Aperture Distributions

A study was undertaken to assess whether aperture distributions with non-Gaussian amplitude taper could be modeled effectively by the complex ray method. The envelope function was assumed in the form exp(-kl), where k is the wavenumber and I is a polynomial function of the aperture coordinate, x. For a Gaussian, \( I \propto x^2 \), whereas for a more flattened profile, one may choose \( I \propto x^{2n} \), \( n > 1 \). A detailed analytical and numerical study, employing steepest descent analysis of the complex spectral field integral representation, was undertaken, with the conclusion that complex ray tracing for profiles with \( n > 2 \) becomes rather complicated due to the appearance of complex ray transition regions that invalidate the tracing of the ordinary complex ray fields. Although a unique set of rules was given to perform the complex ray analysis, it is not regarded as practical for \( n > 2 \) [8].

III. Conclusions

The results from this study confirm the utility of complex ray tracing (via ordinary complex rays, collective complex rays or hybrid forms) for Gaussian
beam type fields transmitted through tapered or curved two-dimensional shell radomes. The numerical algorithm, which avoids the need for integrations over an equivalent aperture when passing from the near zone to the far zone, can be simplified substantially for a range of applications by recourse to paraxial approximations. These conclusions are expected to remain valid also for more general three-dimensional configurations. Although non-Gaussian aperture distributions are not well modeled by complex ray fields per se, recent studies indicate that general aperture fields can be expressed as discrete superpositions of Gaussian fields. This makes the results for single Gaussians reported here directly relevant for the tracking, via obstacles and interfaces, of this more general class of incident fields. It is intended to pursue this aspect in the future.

IV. Personnel

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V. Publications


* P. D. Einziger received the R. W. P. King Award for these papers from the IEEE Antennas and Propagation Society.


