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**TIRE-SOIL INTERACTION MODEL  
FOR TURNING (STEERED) TIRES**

**FINAL REPORT ON**

**CONTRACT DAAE07-81-C4028  
AMENDMENT P0004**



by Leslie K. Karafiath  
 Corporate Research Center  
 Grumman Corporation  
 Bethpage, New York 11714

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**U.S. ARMY TANK-AUTOMOTIVE COMMAND  
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REPORT RE-700

TIRE-SOIL INTERACTION MODEL  
FOR TURNING (STEERED) TIRES

JULY 1985

prepared by

Leslie L. Karafiath  
Materials & Structural Mechanics Directorate

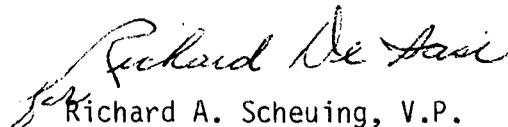
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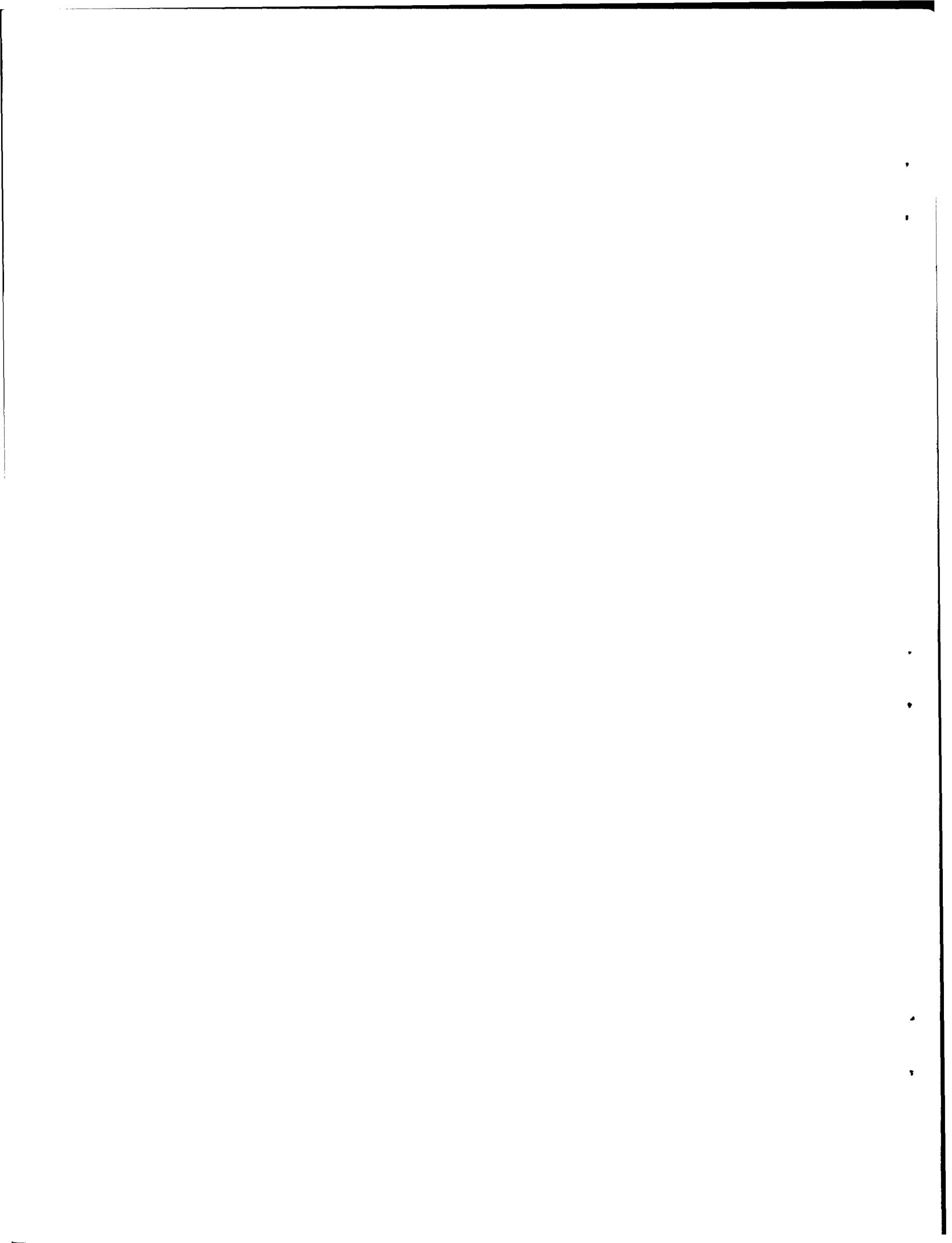
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Research and Development Center  
Warren, Michigan 48090

Approved by:

  
Richard A. Scheuing, V.P.  
Corporate Research Center

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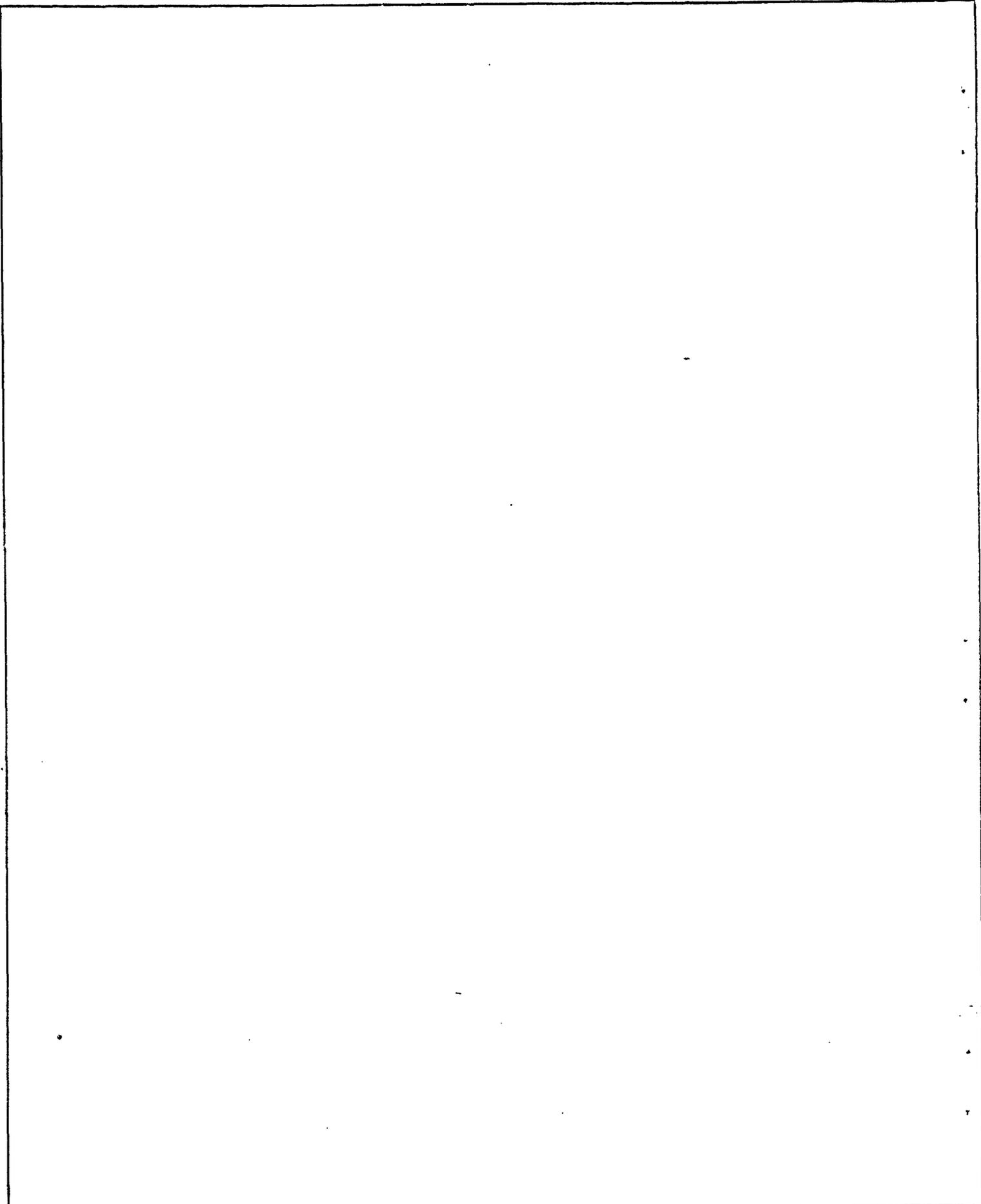


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## PREFACE

The work reported here was done for the Tank-Automotive Concepts Laboratory of the U.S. Army Tank-Automotive Command (TACOM) Research and Development Center, Warren, Michigan, under the general supervision of Mr. Donald Rees, Chief of the Survivability Research Division, Tank-Automotive Concepts Laboratory, and Mr. Zoltan J. Janosi, Chief, Applied Research Function. Mr. Janosi was also technical monitor. Their help and valuable suggestions in carrying out this work are gratefully acknowledged.

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## 1.0. INTRODUCTION

This final technical report prepared by the Corporate Research Center of the Grumman Corporation for the U.S. Army Tank-Automotive Command Research and Development Center describes the development of a tire-soil interaction model for turning (steered) tires. The computerized mathematical simulation of a wheeled vehicle moving over cross country terrain is of great interest to the military and, hence, to TACOM. Simulations are gaining an increasing role in arriving at the best design solution, in developing performance specifications, and in selecting the best bid for the procurement of specific vehicles.

Currently, TACOM uses the so called NATO Reference Mobility Model (NRMM) for this purpose. However, NRMM handles straight line motion only. This report deals with the development of a tire-soil interaction model, which is the first step toward a model that includes the turning motion of a wheeled vehicle. In contrast to straight line travel at constant speed, traveling along a curved path is generally non-steady and can only be simulated by dynamic models.

When a vehicle travels along a curved path, the travel velocity vector and the center plane of its wheels are no longer coincident. One of the consequences of this divergence is that the tractive force developed by a powered wheel may not be balanced by an external drawbar pull or inertia forces alone. Thus, a lateral force may be required to close the respective force polygon. The magnitude of the lateral force acting on the tires as a result of this condition influences not only the drawbar performance of vehicles but their steerability and stability as well. In the turning mode, there are interactions among slip angle, lateral tire deformations and soil reactions in addition to those known to exist in straight line motion. The model of tire-soil interaction for turning (steered) tires presented in this report as a result of research into these interactions is an extension of the pneumatic tire-soil interaction developed earlier for straight line travel. This model is the first step toward the development of a dynamic model of a turning tire which would consider the time rates of deformation of both tire and soil.

## 2.0 OBJECTIVES

The objectives of this work are to:

- o analyze the longitudinal and lateral forces acting on tires tracking off-road at an angle to their center planes,
- o formulate theories for the calculation of these forces from tire and soil characteristics as a function of slip angle, and

- o develop an analytical model of tire-soil interaction in the turning (steered) mode, suitable for use as a submodel in vehicle performance models.

### 3.0 CONCLUSIONS AND RECOMMENDATIONS

An analytical model of the interaction between a tire traveling at an angle to its center plane and soil has been developed to predict the lateral force acting on the tire under various loading and soil conditions. Tractive force-slip and longitudinal-lateral force relationships obtained by using the tire-soil interaction model expanded to include lateral force computations show essentially the same trend as available experimental data.

The evaluation of experimental information as well as the analytical model show that the cross-sectional geometry of the tire and its deformation due to lateral loads are important in the development of lateral forces. However, further experimental and theoretical research is needed to better characterize the cross-sectional geometry and lateral stiffness of tires traveling at an angle in soft soil to make the model responsive to the lateral behavior of tires of various shapes and structures.

The analytical model for turning (steered) tires assumes steady state motion and is only the first step toward the development of a dynamic or transient state model. In laboratory tests a tire may be forced to travel at a steady speed in one direction while it is positioned at an angle to that direction. In reality, a steer angle applied to the tire causes the vehicle to turn and, as a consequence, centrifugal forces develop. The interface shear stresses balancing this centrifugal force interact with the development of lateral shear stresses as indicated in Fig. 4-10. To simulate the maneuvering of wheeled vehicles, a dynamic vehicle model is needed with an appropriate, dynamic tire-soil interaction submodel. For a better understanding of the dynamic interactions in the turning mode, field vehicle tests, such as in the wheeled vehicle agility project, are recommended with complete instrumentation for the measurement of the deformed cross-sectional geometry of the tire and generated interface stresses.

### 4.0 DISCUSSION

#### 4.1 Review of Experimental Information on The Performance of Turning (Steered) Tires

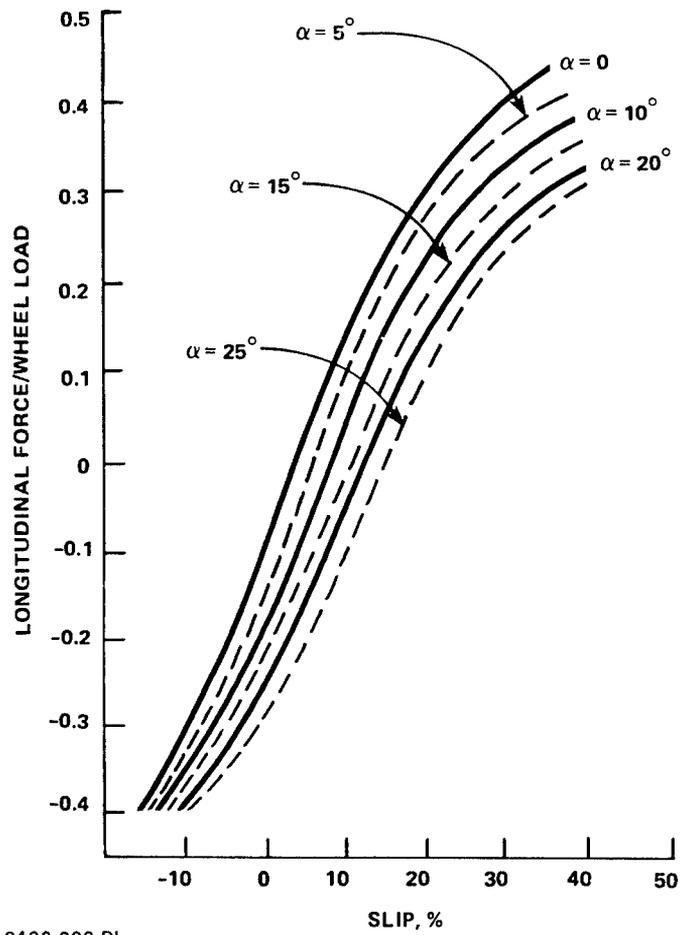
Tests with both powered and free-rolling tires positioned at an angle to the direction of travel were done in the mobility bins of the following organizations:

- o Institute for Agricultural Machinery, Munich Technical University, Germany (Ref. 1 through 4)
- o U.S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss. (Ref. 5 & 6)
- o University of Dayton, Dayton, Ohio (Ref. 6 & 7)
- o National Institute of Agricultural Engineering, Silsoe, Bedford, England (Ref. 8)
- o Deere Technical Center, Moline, Illinois (Ref. 9).

Field tests encompassing a variety of soil conditions encountered in agriculture were done in Germany with free rolling tires, using a test rig mounted rigidly on the rear of a Unimog tractor (Ref. 10). Other field tests with instrumented vehicles were done in connection with the stability problem of agricultural tractors traveling on sloping terrain. The condition critical for stability in this case is relatively firm ground with slippery grass surface. This is of little relevance to general soft soil conditions. The only field testing program involving powered wheels is that done for TACOM for the wheeled vehicle agility project (Ref. 12 & 13).

The experimental information obtained in these testing programs was reviewed from the viewpoint of their relevance to the formulation of an analytical turning (steered) tire-soil interaction model based on the application of the principles of applied mechanics. These require that the turning tire be considered to be a free body and the interface stresses satisfy both the equilibrium of forces acting on the free body and the compliance of the deformed geometry of the tire and soil. In the tire-soil interaction model for straight travel, these conditions are at least approximately satisfied. In the case of turning tires, the equilibrium of lateral forces as well as their interaction with the lateral deformation of the tire have to be considered and taken into account, at least approximately. Clearly, this requires a soil property model which allows the analytical determination of soil reactions and a tire model which allows the determination of lateral tire deformations under various lateral loading conditions. Obviously, it cannot be expected that experiments which were conducted for a purpose different from that of formulating an analytical model would be informative on specifics, but the lack of information on soil properties and lateral tire deformation makes generalization of the test results impossible and their usefulness for any purpose other than that for which they were conducted, questionable.

The experiments conducted at the Institute for Agricultural Machinery in Munich, using a six component force transducer, are the most comprehensive concerning variations of slip, slip angle and driving and braking forces. In these experiments, the general pattern of the relationship between slip and driving force for various slip angles was found as shown in Fig. 4-1. In dynamic vehicle models aimed at the analysis of the stability of



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Fig. 4-1 Trend of Change in Tractive Force-Slip Relationship with Slip Angle (After Ref. 3)

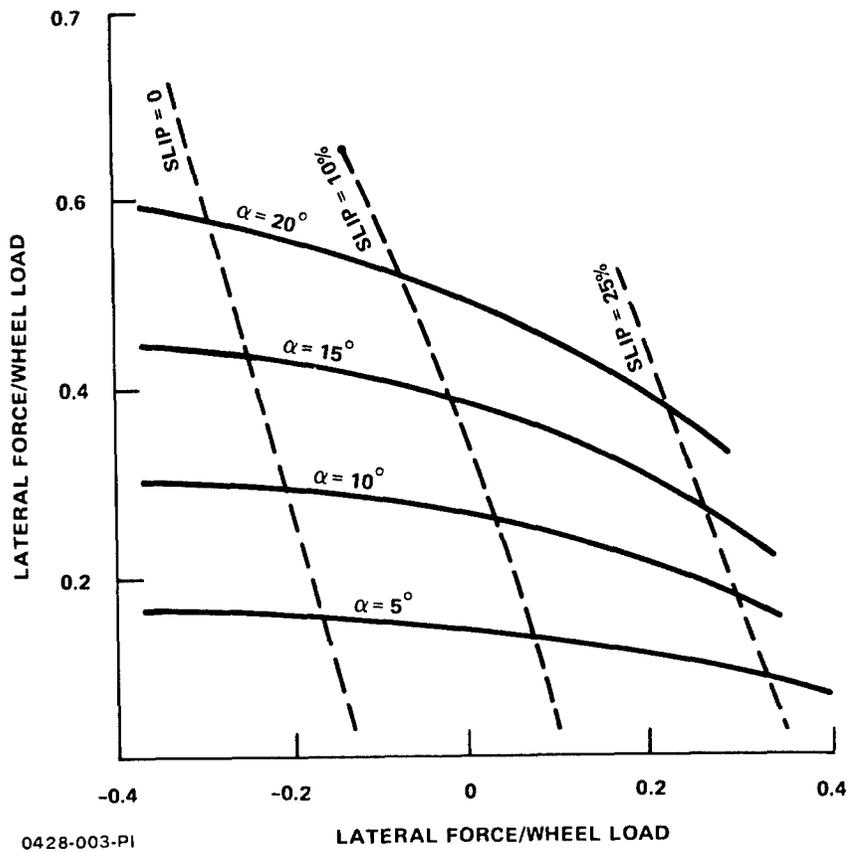
agricultural tractors, a simple relationship between the longitudinal and lateral forces acting on a turned tire is desired. For the specific conditions of the test, the relationship shown in Fig. 4-2 was established for various slip angles and approximated by a linear relationship for the vehicle dynamic model.

Unfortunately, the results of only one test series done in agricultural soil of 16% moisture content were reported (Ref. 2 & 3), and only cursory reference is made to a test series done at 21% moisture content. However, those results did not fit the patterns shown in Fig. 4-1 and 4-2. The lack of information on soil properties, vertical and lateral deformation characteristics, and cross-sectional geometry of the test tire makes the otherwise valuable testing program unsuitable as a basis for validation of an analytical turning tire model. (The tested tractor tire has a rather rounded cross-sectional profile compared to military off-road tires which have a relatively wide flat tread.)

The testing program conducted at WES in 1976 was directed toward establishing simple empirical relationships for predicting the performance of and tractive forces acting on vehicles maneuvering in off-road terrain. To this end, 49 tests were done in Yuma sand and 23 in Vicksburg clay with a 6.00-9, 4PR buffed (trailer) tire at various tire loads, deflections and slip angles. These tests encompass more soil-condition and tire-deflection variations than those done at the Institute for Agricultural Machinery in Munich. However, despite the relatively great number of tests done, there are not sufficient data points to establish empirical relationships with confidence.

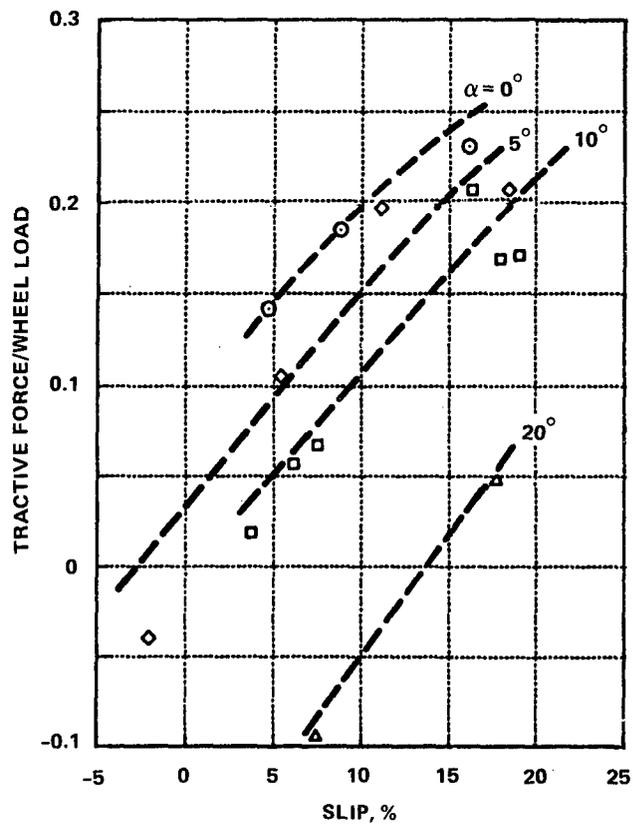
For convenient comparison, data points for test runs in sand and clay obtained for a tire load of 3,000 N and 35% tire deflection are plotted in Fig. 4-3 through 4-6 in the same way as the test results of the Institute for Agricultural Machinery. There are too few data points for any given slip angle to draw fitting curves by regression analysis. To show the general trend, fitting curves were drawn by eye exercising some judgment as to the expected form of the relationship. The slip angle effects a much sharper decrease in drawbar pull in both sand and clay (refer to Fig. 4-3 and 4-6) than one would expect based on Fig. 4-1. The small size of the tire and the low inflation pressure (4.1 psi) applied in the test series for which the results are shown in Fig. 4-2 through 4-6 make extrapolation of these test results to larger tires questionable.

The experiments reported in Ref. 6 and 7 were done with free-rolling aircraft tires in two types of soil: one a frictional soil, the other a typical cohesive soil. Since aircraft tires are different from the off-road tires used by the military, and they are operated (and have been tested) at higher deflections than customary with off-road tires, the test results, although of general interest, are not directly applicable to the problem at hand.



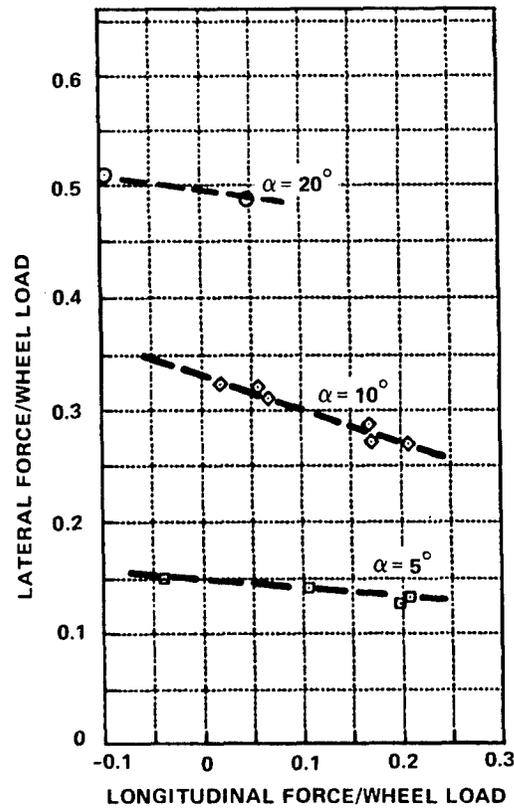
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**Fig. 4-2 Trend of Change in Relationship Between Longitudinal & Lateral Forces with Slip Angle (After Ref. 3)**



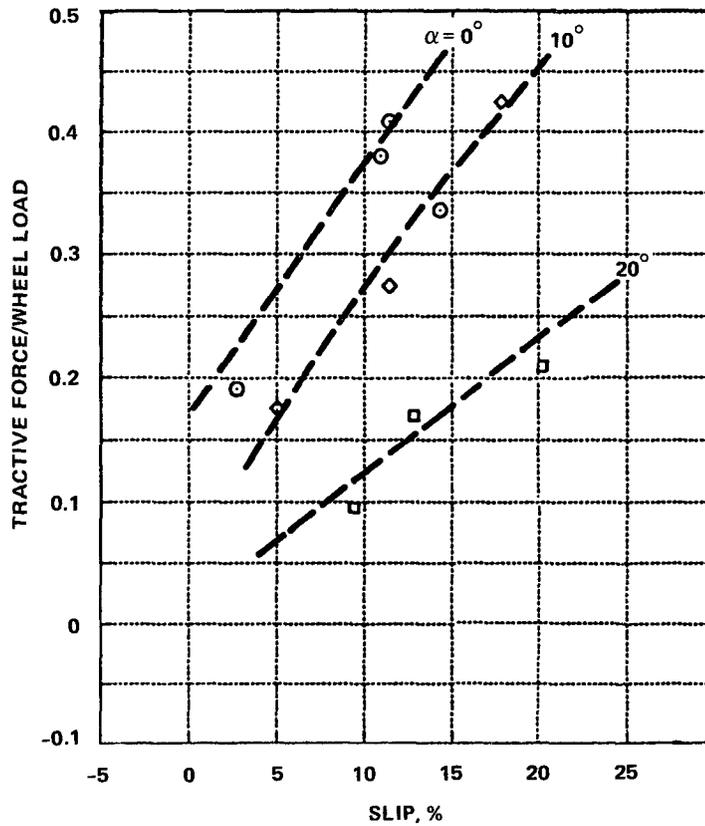
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Fig. 4-3 Tractive Force-Slip Relationship at Various Slip Angles in Sand Obtained in Experiments Done at WES (Ref. 5)



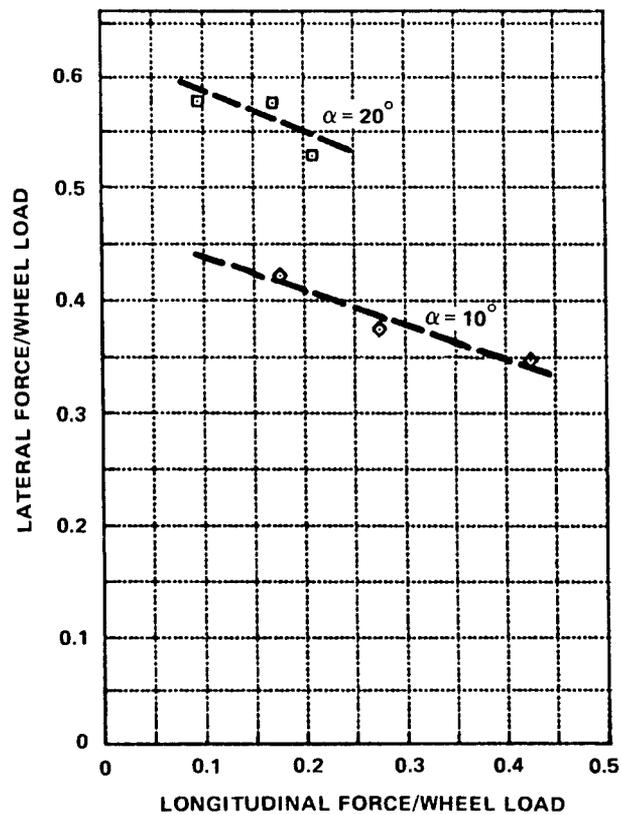
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**Fig. 4-4** Longitudinal-Lateral Force Relationship at Various Slip Angles in Sand Obtained in Experiments Done at WES (Ref. 5)



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Fig. 4-5 Tractive Force—Slip Relationship at Various Slip Angles in Clay Obtained in Experiments Performed at WES (Ref. 5)



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Fig. 4-6 Longitudinal–Lateral Force Relationship at Various Slip Angles in Clay Obtained in Experiments Performed at WES (Ref. 5)

The experiments reported in Ref. 8 and 9 refer to free-rolling agricultural tires. While the effects of tire tread and construction were significant enough to note, there was no quantification of the lateral deformation characteristics of the various types of tires believed to be responsible for the observed differences in behavior of tires while being towed at an angle. A decreased rolling resistance with increased slip angle is reported in Ref. 9. This contradicts intuition and the laboratory and field experiments reported in Ref. 1, 6, 7, 8, 10, and 11.

References 10 and 11 report the results of field tests done with various free-rolling tires in a wide variety of soil and cultivation conditions and a number of tire types. Although the soil identification does not go beyond simple description, the findings of these tests have general validity as far as the effect of tire type on rolling resistance and on the lateral force is concerned. These findings indicate that there is a significant difference (up to 60%) in the rolling resistance and side force when tires of different tread and construction are tested in otherwise similar conditions.

The field tests done for the wheeled vehicle agility project (Ref. 13) show the same pattern for the longitudinal-lateral force relationship as observed in the laboratory for each of the conditions tested (see Fig. 4-7). These include two tire sizes (9.00-20 and 11.00-20), two tire types (radial and bias ply), three loading conditions with the appropriate variation of inflation pressure and tire deflection, and two types of surface conditions (soft soil and hard surface). Because of uncertainties in the field slip measurements, drawbar pull-slip relationships have not been established.

The significance of the type of construction and associated lateral stiffness of the tire has also been observed in tests done on paved surfaces (Ref. 17 & 18), and methods have been developed to measure and characterize the lateral stiffness of tires (Ref. 17 & 19).

#### 4.2 Cross-sectional Geometry of Tires

From the preceding discussions of existing experimental information, it is evident that the cross-sectional geometry and the lateral stiffness of the tire have an important bearing on the lateral interaction of tire and soil. While the cross-sectional geometry of an unloaded tire may be easily measured or obtained from the tire manufacturer, the change in cross-sectional geometry as a function of load and inflation pressure is not readily available even for tires supported by hard surfaces. Little information is available on the cross-sectional geometry of tires traveling in soft soil. Figure 4-8, taken from Ref. 14, shows changes with inflation pressure in the cross-sectional geometry of a 12x22.5 tubeless, buffed tire, traveling in silt soil.

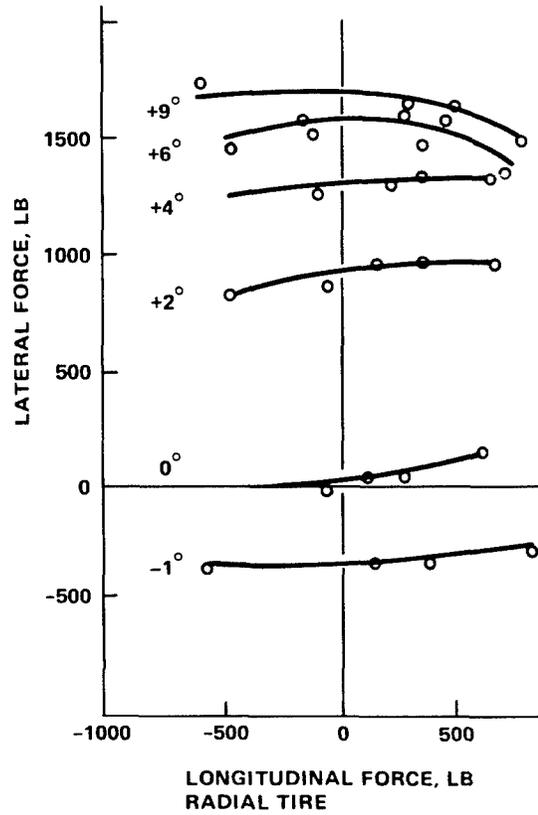
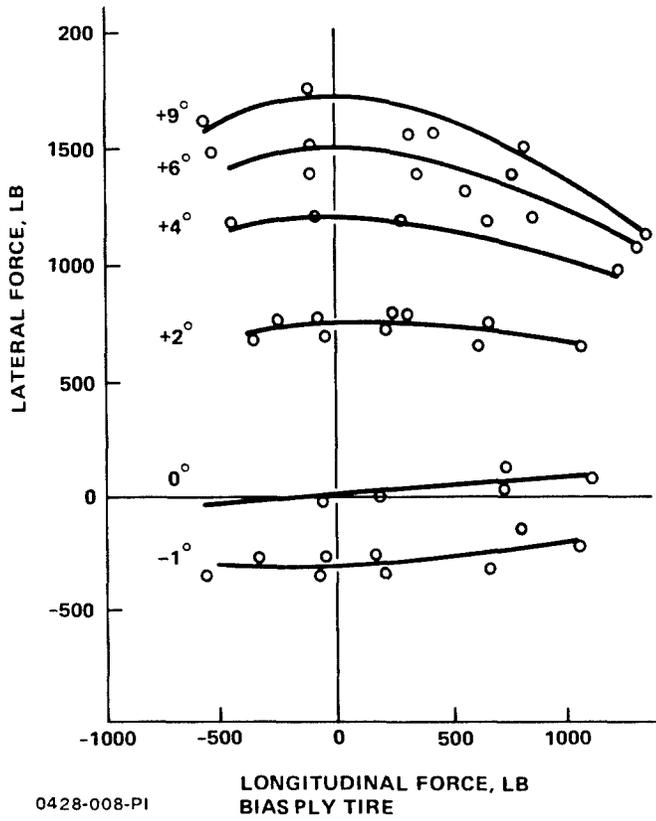
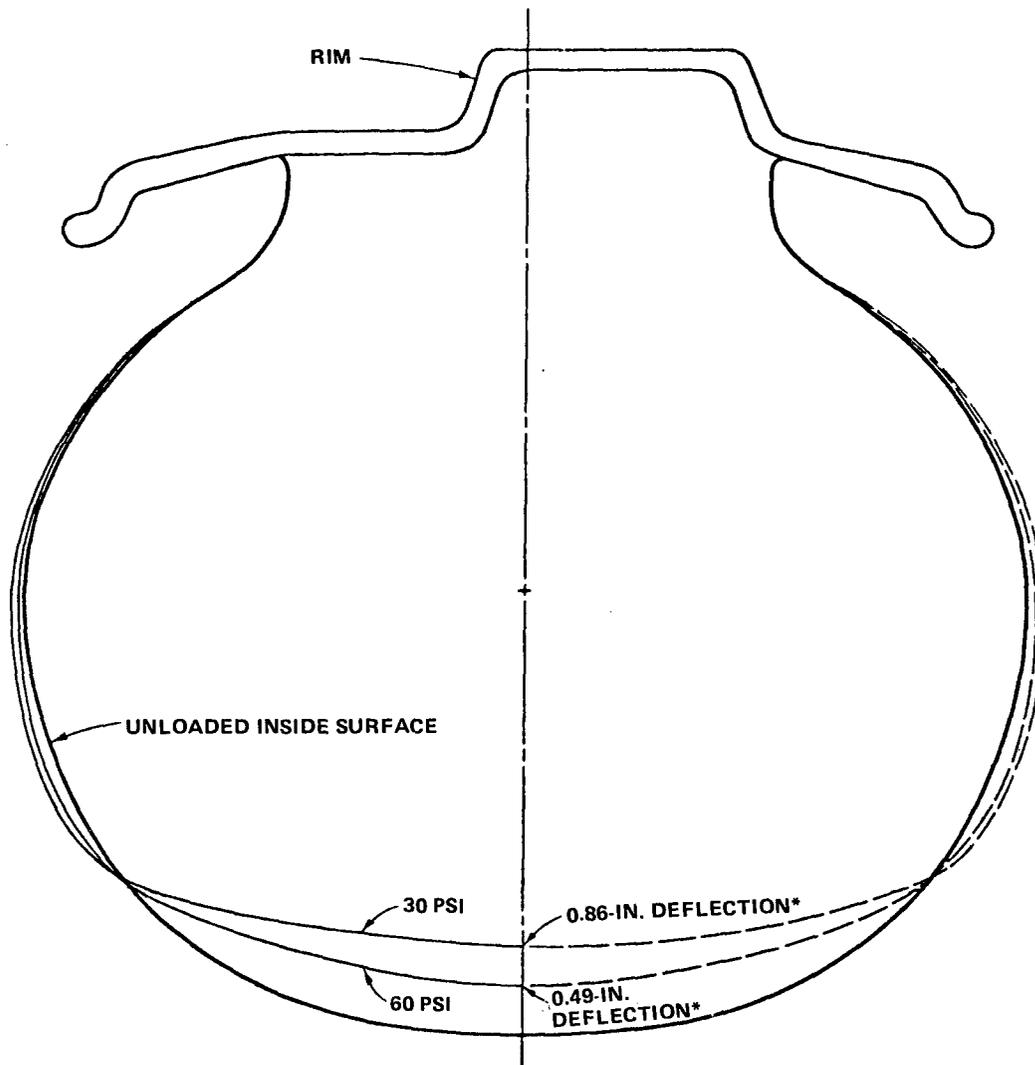


Fig. 4-7 Longitudinal-Lateral Force Relationships Obtained in Field Tests for Wheeled-Vehicle Agility Project in Loose, Dry Frictional-Cohesive Soil (Ref. 13)



| INFLATION PRESSURE<br>PSI | PASS NO. | BEFORE-TRAFFIC<br>CONE INDEX<br>0''-6'' LAYER | BEFORE-TRAFFIC<br>RATING CONE INDEX<br>0''-6'' LAYER | RUT DEPTH<br>AFTER FIRST PASS,<br>IN. |
|---------------------------|----------|---|--|---------------------------------------|
| 30                        | 1        | 127   | 79   | 0.66                                  |
| 60                        | 1        | 125   | 78   | 1.36                                  |

NOTE: 12.00-22.5, 12-PR TUBELESS TIRE.  
WHEEL LOAD 2950 LB.

\*DEFLECTIONS SHOWN ARE AVERAGES OF THE DEFLECTIONS MEASURED BY THE LINEAR POTENTIOMETER IN GAGE 1 ON ALL WHEEL REVOLUTIONS OF THE FIRST PASS.

TIRE DEFLECTION ON  
SILT SECTION 1 VS  
TWO INFLATION PRESSURES  
SPEED RANGE 2 TO 5 MPH  
SCALE IN INCHES



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Fig. 4-8 Cross-Sectional Geometry of a 12 x 22.5 Tubeless Tire in Motion on Silt (Ref. 14)

While the results of measurements shown in Fig. 4-8 and others contained in Ref. 14 are instructive, they refer to vertically loaded tires. A perusal of relevant publications brought forward by an extensive literature search indicates that experimental research into the cross-sectional geometry of tires traveling in soft soil and loaded laterally has not yet been undertaken.

Various models of the lateral deformation of tires have been proposed (Ref. 18). Of these, the model proposed by Rotta (Ref. 16) has been adopted for the turning (steered) tire-soil interaction model, essentially because it is simple and the model parameters are either available or may be readily determined.

Figure 4-9 shows the model of tire cross section and the parameters controlling its deformation as proposed by Rotta. The undeformed shape of the tire, shown by dashed lines in Fig. 4-9a, is circular. Under a vertical load, the cross section experiences a deflection "f." The deflected shape consists of a flat portion contacting the supporting hard surface and circular sidewalls. The geometry of the flattened cross section is defined by the width of the flat portion, "b," which also defines the center of the circular portions having the same radius "r." The circular portions depart at an angle  $\gamma$  from the upper flat support of width "B<sub>F</sub>." Figure 4-9 shows the shape of the tire experiencing both vertical and lateral deflection (f<sub>s</sub>). The deformed shape is asymmetrical and is characterized by the distances b<sub>1</sub> and b<sub>2</sub>, the radii r<sub>1</sub> and r<sub>2</sub>, and departure angles  $\gamma_1$  and  $\gamma_2$ .

The input parameters defining the deformed shape of the cross section are:

- H = section height
- B = width of tire
- f = vertical (radial) deflection
- f<sub>s</sub> = lateral deflection

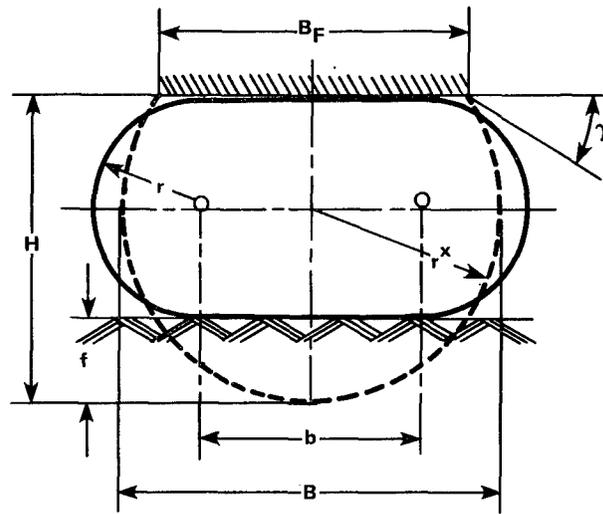
The vertical deflection of a tire on hard surface depends on the load and inflation pressure and may be obtained from the manufacturer's charts. In soft soil, the radial tire deflection is obtained from the geometry of the centerline which is routinely determined as a function of soil properties and tire stiffness in the tire-soil interaction model for straight travel.

The lateral deflection of the tire is assumed to depend linearly on the lateral load, with the coefficient of proportionality being the lateral tire stiffness coefficient C<sub>ℓ</sub>.

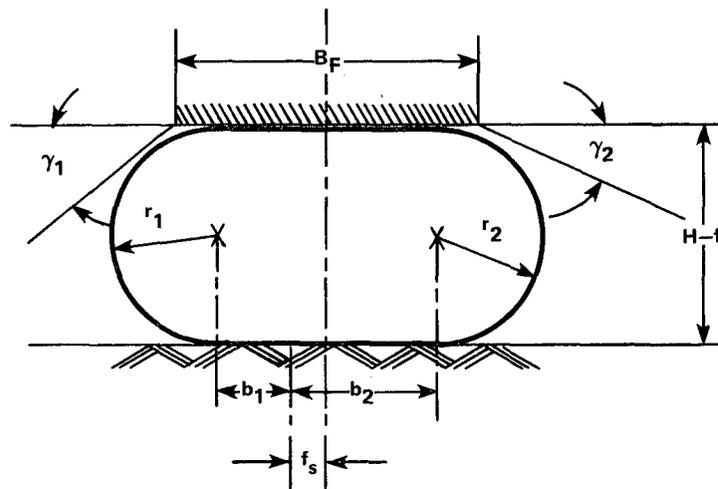
To determine the deflected shape from the input parameters, Rotta developed the following relationships:

$$B_F = \sqrt{4HB - H^2}$$

$$\frac{B_F}{B} = \psi \frac{f}{B} = \lambda \frac{f_s}{B} = \lambda_s \frac{2r}{B} = \rho \quad (1)$$



a) TIRE DEFLECTION UNDER VERTICAL LOAD



b) TIRE DEFLECTION UNDER VERTICAL & LATERAL LOAD

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Fig. 4-9 Model of Cross-Sectional Geometry of Tires Proposed by Rotta (Ref. 16)

$$\kappa_i = \frac{1 + \sqrt{1 - \psi^2} - 2\lambda}{\pi - \arcsin \psi - \psi + 2\lambda_s}$$

The values of  $\kappa_1$  and  $\kappa_2$  may be readily determined from the input data. The departure angles  $\gamma_i$  may be determined from the following relationship

$$\kappa_i = \frac{1 + \cos \gamma_i}{\pi - \gamma_i - \sin \gamma_i} \quad (2)$$

where  $i = 1, 2$ .

Equation (2) cannot be inverted for the explicit determination of  $\gamma_i$ ; however, the following relationship closely approximates Eq (2):

$$\gamma_i = 1.576 \log (1.58 \kappa_i) \quad (3)$$

Using Eq (3) for the determination of  $\lambda_i$ , the radii  $r_1$  and  $r_2$  are obtained from the following relationship:

$$r_i = \frac{B}{2} \frac{\pi - \arcsin \psi - \psi + \delta \cdot 2\lambda_s}{\pi - \gamma_i - \sin \gamma_i} \quad (4)$$

where  $\delta = -1$  for  $i = 1$   
and  $\delta = +1$  for  $i = 2$ .

#### 4.3. Lateral Forces Acting on a Tire Traveling at an Angle To Its Center Plane

The total lateral force acting on a tire traveling at an angle to its center plane consists of the following components:

1. **Lateral force to balance tractive force and drawbar pull vector.** A powered tire develops a net tractive force which is in its center plane. In steady state straight motion, this force is balanced by the drawbar pull which is assumed to act in the direction of travel. In the case of a tire traveling at an angle to its center plane, the tractive force and drawbar pull are generally not codirectional. In some instances the direction of the drawbar pull is given, as in the case of tractor-trailer units, where the dynamics of the trailer determine the instantaneous direction of the

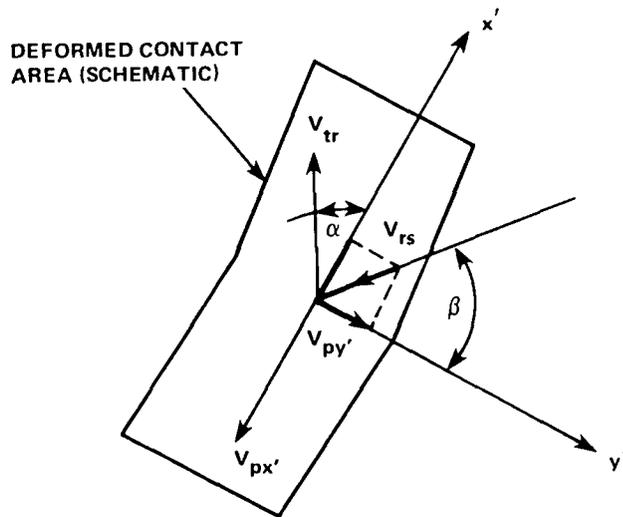
balancing force exerted at the trailer hitch. If no information on the direction of the drawbar pull is available, it is assumed to act in the direction of travel (which is also a valid assumption if the tractive force is balanced by inertial forces). Generally, the vectorial balancing of the tractive force and drawbar pull needs a lateral force vector.

**2. Lateral shear stresses in the contact area.** In the contact area of the tire, lateral shear stresses develop. These come about because of changes in the direction of the slip velocity vector. In straight travel, the slip velocity vector is in the same vertical plane as the travel velocity vector. The magnitude of the slip velocity vector is the difference between the travel velocity and the peripheral velocity of the tire. When the tire travels at an angle to its center plane, the travel velocity vector and the peripheral velocity vector are no longer codirectional. In addition, the peripheral velocity vector has two components: the longitudinal component, resulting from the rotation of the tire and the lateral component, resulting from the lateral deflection of the tire. In the case of a moving tire the lateral deflection of a peripheral point changes from a negligibly small value at the front of the contact area to its maximum value somewhere around the center of the contact area. The time rate of change of the lateral deflection of the peripheral point as it moves from the front to the center and then to the end of the contact area is the lateral peripheral velocity. The determination of the direction of the slip velocity vector and its magnitude for given vectors of travel and peripheral velocity is shown in Fig. 4-10.

In principle, the direction of the resultant of the shear stresses generated in the contact area must coincide with the direction of the slip velocity vector. Assuming that the longitudinal component of the peripheral velocity vector and the shear stresses generated by the applied torque are the same as in straight line travel and the maximum lateral deflection is known, both the direction of the slip velocity vector and the lateral component of the shear stresses may be computed.

**3. Lateral stresses acting on the curved in-ground portion of the tire.** Normal stresses develop on the curved portion of the cross section of tires which is in contact with the ground. In straight travel, the horizontal component of these normal stresses is equal and opposite at the two sides of the tire and their overall effect is cancelled out.

When the tire travels at an angle to its centerplane, the horizontal components of the normal stresses generated at the in-ground curved section of the two side walls no longer cancel each other out. In this condition, the cross-sectional geometry of the tire is no longer symmetrical but resembles that shown in Fig. 10b for the idealized model. In addition, one side of the tire faces into the direction of travel, thus encountering soil resistance, while the other side moves away from the rut, thus minimizing the lateral forces which may be generated.



$$V_{x'} = V_{tr} \cos \alpha - V_{px'}$$

$$V_{y'} = V_{tr} \sin \alpha - V_{py'}$$

$$\beta = \text{ARCTAN} \frac{V_{x'}}{V_{y'}}$$

$$\tau_{x'} = \tau_{\max} (1 - e^{S/K}) = \tau_{rs} \cos \beta$$

$$\tau_{y'} = \tau_{rs} \sin \beta = \tau_{x'} / \tan \beta$$

$\tau$  = SHEAR STRESS

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Fig. 4-10 Determination of the Slip Velocity Vector & Associated Interface Shear Stresses

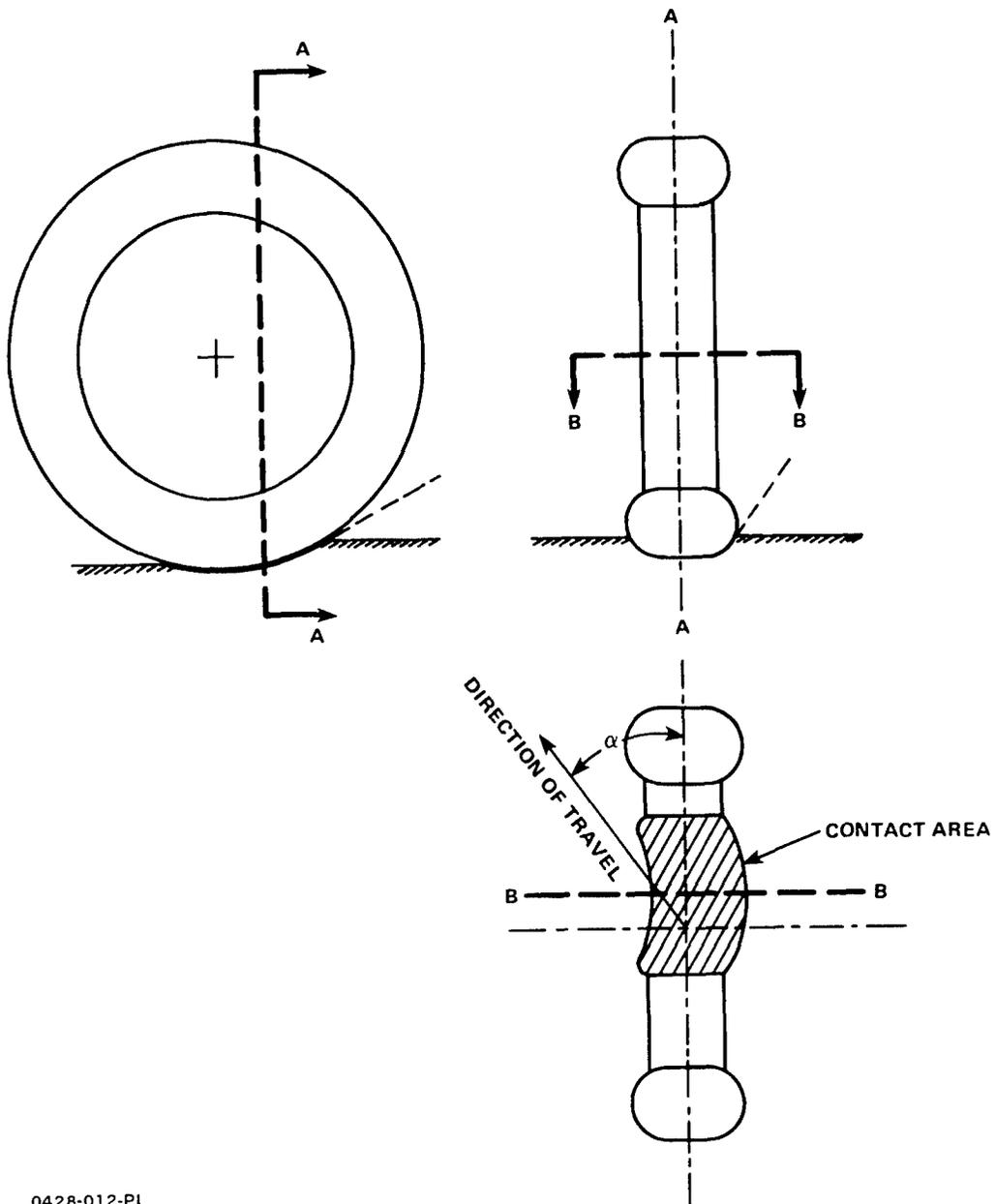
The idealized model defines the cross sectional geometry of the tire in the radial plane for given radial and lateral tire deflection. The radial deflection of the tire is obtained from the centerline geometry computed in the tire-soil interaction model for straight travel. The lateral deflection in the radial plane is assumed to vary linearly from the ends of the contact area to its center where it reaches its maximum value. This maximum value of the lateral deflection depends on the lateral force acting on the tire and the lateral stiffness of the tire. Since the lateral force and lateral deformation are interdependent, iteration is necessary to determine the maximum lateral deflection under given tire loading conditions, slip angle, and soil properties.

#### 4.4 Tire-soil Interaction Model for Turning (Steered) Tires

Tire-soil interaction is a complex contact problem which requires the determination of that geometry of the contact area which is in compliance with the displacements of both tire and soil for the given distribution of interface normal and shear stresses. Even in the relatively simple case of steady state straight-line travel, for which an interaction model was developed under earlier contract (Ref. 15), it was necessary to make simplifying assumptions (e.g., assume two-dimensional conditions) and use approximations as regards to tire centerline geometry and interface stresses to arrive at a workable model.

In the case of turning tires, the main objective of model development is the determination of lateral forces acting on the tire when traveling at an angle to its centerplane. Since lateral forces act perpendicular to the centerplane of the tire, the interaction problem becomes three-dimensional and, at least theoretically, an appropriate 3-D solution is called for. Nevertheless, as a first step toward a truly 3-D model, the problem has been approached by assuming that tire-soil interactions in the longitudinal and lateral directions are independent of each other. Using this assumption, the interface stresses, contact area dimensions, and centerline geometry for the given tire loading conditions and soil properties are determined from the tire-soil interaction model for straight travel. The drawbar pull obtained for these conditions is modified by a motion-resistance term representing the effect of passive earth resistance generated by the in-ground portion of the side of the tire when traveling at an angle to its centerplane. Aside from this addition, lateral forces are assumed to have no effect on the tire-soil interaction in the centerplane of the tire.

The model of the tire used in the tire-soil interaction model expanded to include the lateral forces in the turning (steered) mode is shown in Fig. 4-11. In the upper left corner, the elevation view of the tire is shown; the deformed centerline geometry of the tire in this plane is assumed to be the same as in straight-line travel and is determined accordingly. The cross-sectional geometry of the tire shown to the right is assumed to be governed by the lateral forces acting on the tire and is determined by appropriate algorithms based on Rotta's method discussed in detail



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Fig. 4-11 Tire Model for Tire-Soil Interaction in Turning (Steered) Mode

earlier. In the lower right corner the deformed contact area is shown; its longitudinal dimensions are determined by the centerline geometry, while its lateral excursion is determined by the lateral force acting on the tire and its lateral stiffness coefficient.

Figure 4-12 shows the tractive force-slip relationship for various slip angles computed for a tire load of 1,168 lb and 5.00-16 tire used in the experiments reported in Ref. 3. The soil was assumed to have 60 lb/sq ft cohesion and a friction angle of  $36^{\circ}$ ; the lateral stiffness coefficient of the tire was assumed as 0.00015 ft/lb. These soil property values and the lateral stiffness coefficient were estimated to represent, roughly, the conditions of the experiments. The pattern of the tractive force - slip relationship shown in Fig. 4-12 resembles those obtained in the experiments closely. The longitudinal-lateral force relationships computed in the model are shown in Fig. 4-13; these are also similar to those found in the experiments. Unfortunately, because of the lack of information on both soil properties and lateral tire deformation characteristics, the otherwise valuable experiments are unsuitable as a basis for the validation of the model. The relationships shown in Fig. 4-12 and 4-13 are in reasonable agreement with the experimental information available today.

The fundamental assumption behind both the expanded and the original model of tire-soil interaction is that the motion of the tire is steady state. In the case of straight travel this assumption is a reasonable approximation of the special but not unrealistic case of a vehicle traveling at nearly constant speed. In the case of a turning vehicle, steady state motion is a hypothetical one which is highly unlikely to occur in the field even for short periods of time and is difficult to reproduce even in tests designed for that purpose. Maneuvers of military vehicles are prime examples of non-steady motion involving linear and rotational accelerations and decelerations. These interact in several ways with the generation of lateral forces as represented in the hypothetical steady-state model. For example, the interface shear stresses resisting the centrifugal force interact with those generated in steady-state motion. The response of both the tire and soil to dynamic (impulsive) loads may be different from that assumed in the steady-state model. Further research is needed to investigate, both theoretically and experimentally, the effect of these interactions on the lateral forces generated in non-steady motion and to develop a dynamic model of tire-soil interaction for turning tires.

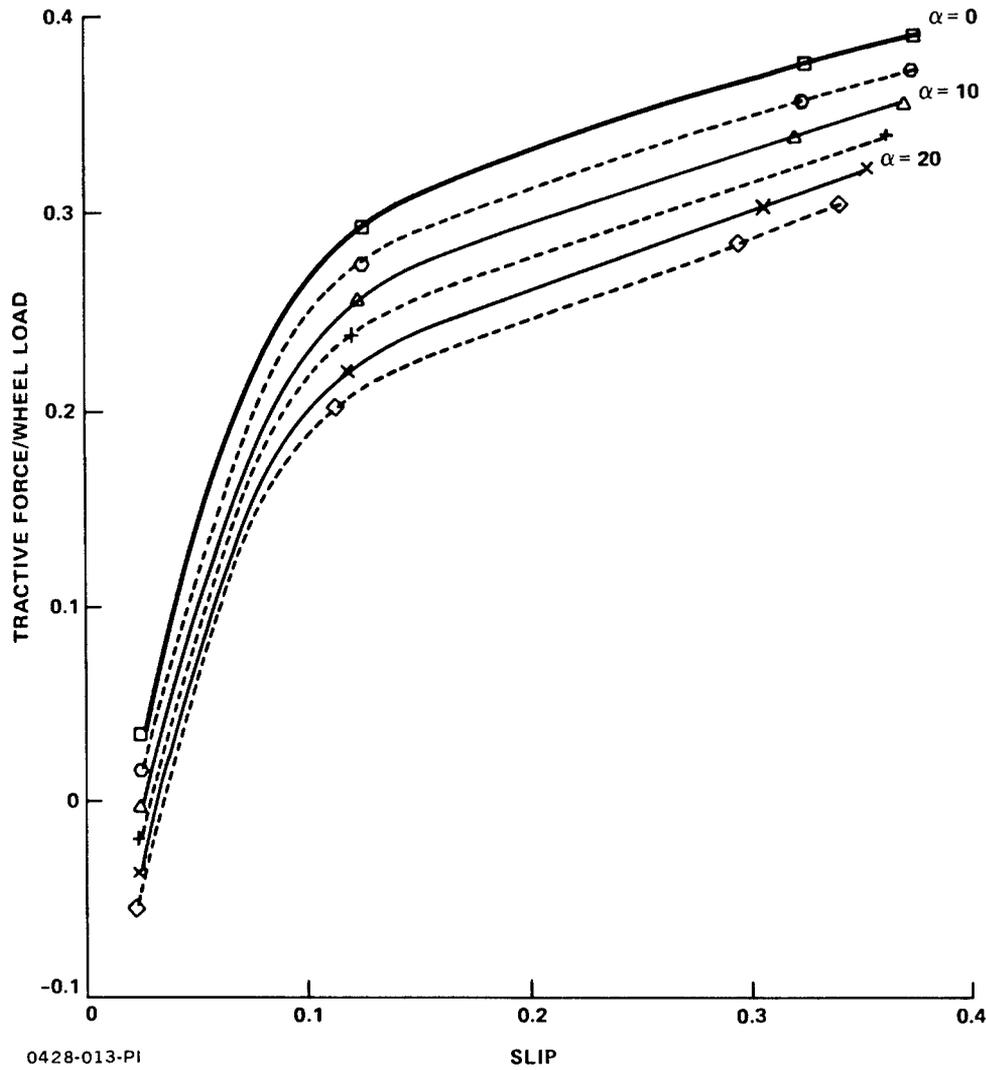
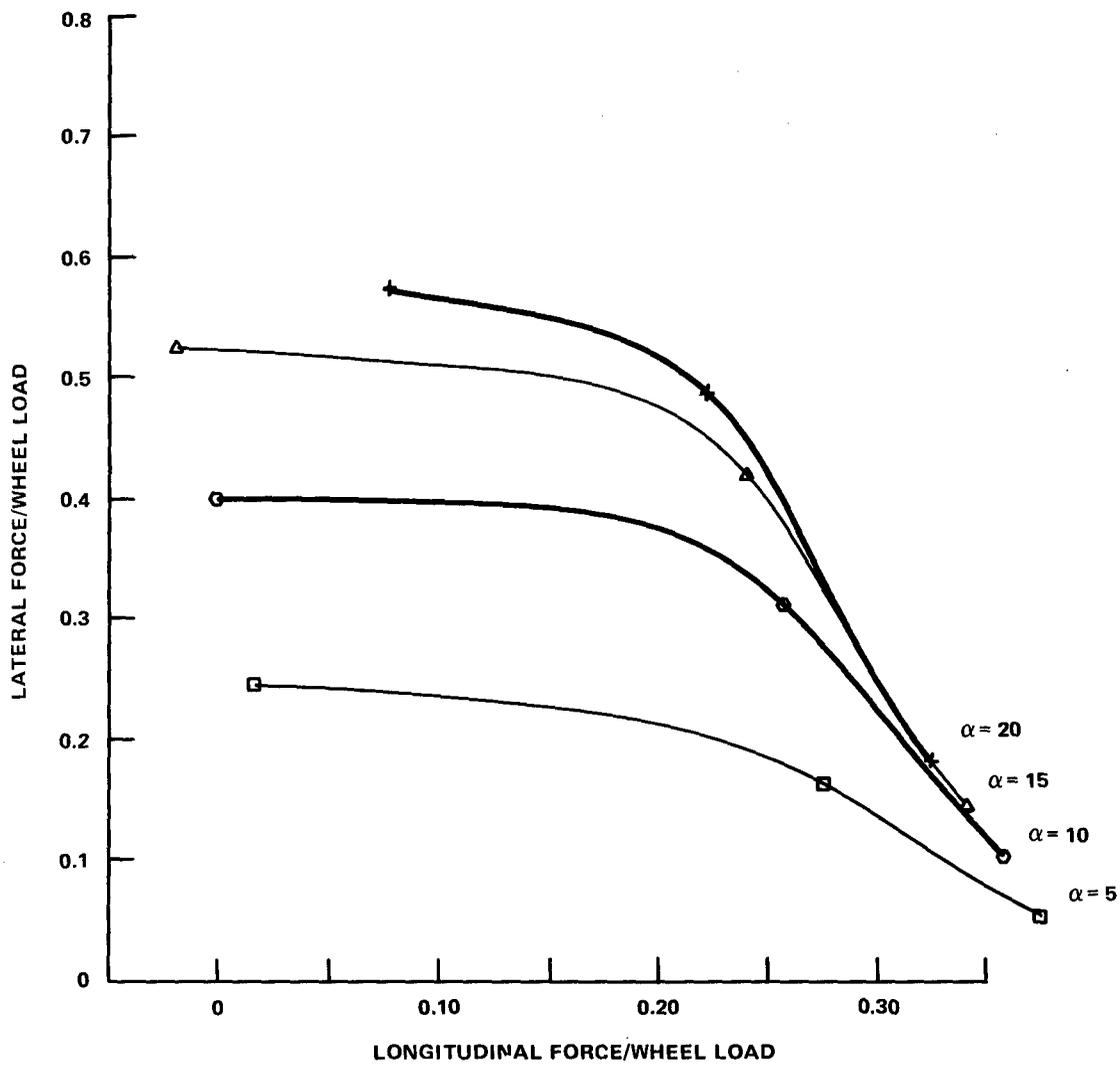


Fig. 4-12 Tractive Force–Slip Relationship Calculated Using Expanded Analytical Tire–Soil Interaction Model for Various Slip Angles



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**Fig. 4-13 Longitudinal–Lateral Force Relationships for Various Slip Angles Calculated Using Expanded Tire–Soil Interaction Model**

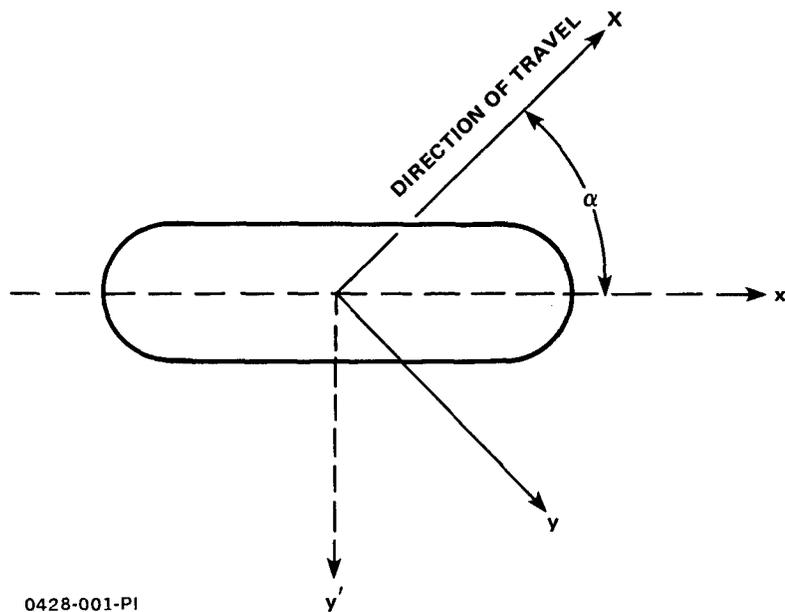
## 5.0 GLOSSARY OF TERMS

Various terms used in off-road vehicle engineering in the case of straight-line travel need to be redefined to avoid ambiguity. Figure 1 shows the plane view of a tire in the turning mode with the pertinent orientations and forces. Referring to Fig. 5.1, the following definitions apply:

- x = coordinate in the direction of travel
- y = coordinate perpendicular to the direction of travel
- x',y' = local coordinates in and perpendicular to the center plane of the tire
- a = slip angle
- v = travel velocity
- $v_{px}'$  = peripheral velocity of a point of tire in the x' direction
- $v_{py}'$  = peripheral velocity of a point of tire in the y' direction
- $v_{rs}$  = resultant slip velocity
- b = angle between resultant slip velocity vector and x'
- s = slip =  $(v_{px}' - v \cos a)/v_{px}'$
- T = tractive force acting in the center plane of tire
- R = resultant of resisting forces in the direction of the center plane of the tire
- T-R = drawbar pull in the center plane of tire
- DB = drawbar pull acting on the hitch in the direction of travel

### DESIGNATIONS WITH REFERENCE TO TIRE GEOMETRY

- B = max. width of tire in the undeflected state
- $B_F$  = flange width
- f = tire deflection under vertical load
- $f_s$  = tire deflection under lateral load
- H = section height of tire
- L = length of contact area



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**Fig. 5-1 Definition of Slip Angle & Coordinate Systems**

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