THE ORIENTATION DISTRIBUTION OF NONSPHERICAL AEROSOL PARTICLES WITHIN A CLOUD (U) HEBREW UNIV JERUSALEM (ISRAEL) DEPT OF ATMOSPHERIC SCIENCES I GALLILY DEC 84
The Orientation Distribution of Nonspherical Aerosol Particles within a Cloud

Second Technical Interim Report

by

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The models

Having performed a study on the orientation density function (o.d.f.) of small spheroidal aerosol particles in a general field of an arbitrary strength, (1,2) it became possible to treat the orientation problem in a turbulent medium. To this end, two interconnected physical models were applied. In the first, "The Realizations Model," it was assumed that the turbulent particle field constituted an ensemble of an infinite number of realizations, j, each one of which is characterized by one set of the o.d.f., \( F(j)(x,t) \), values. The latter was taken in that model to essentially coincide with the previously found solution of the Fokker-Planck equation (1,2) in the field of the realization.

\[
\partial F(j)/\partial t - \nabla \cdot [\omega (W_{ik}^{(j)}, R) - \alpha_e^{(j)}(\nabla F)] = 0 \tag{1}
\]

where \( \omega \) is the rotational velocity of the particles and \( \alpha_e^{(j)} \) is their rotational Peclet number defined by \( \omega_c / D_e \), \( \omega_c \) being a typical component of the (fluid) gradient tensor, \( D_e \) an effective rotational diffusion coefficient and \( R \) the particle aspect ratio, \( W_{ik} \) is a gradient component \( \partial u_i / \partial x_k \) (a component of the fluid velocity). The use of the solution is based on the estimate that, even for the highest (Kolmogoroff) frequency component of \( u_i \), the rotational Reynolds number for the studied particles is small enough to render their motion (quasi) stationary.
In the second, "The Micro-turbulence Model," a relation between the so-called turbulent rotational diffusion coefficient \( D_z \) of the particles and the physical characteristics of the fluid field was supplied. This coefficient and the Brownian rotational diffusion one, \( D_b \), compose the effective diffusion coefficient.

\[
D_z = D_t + D_b.
\]  

Expressing the stochastic quantities of the system as \( F = \overline{F} + F' \), \( \omega_i = \overline{\omega}_i + \omega'_i \), \( \omega'_i = \overline{\omega}'_i \) and \( u_i = \overline{u}_i + u'_i \), it could be shown that the realizations' average of \( F \), \( F'(\overline{W}, D_z, D_b, R) \), for a space-time point is actually \( F(\overline{W}, D_z, R) \) for that point and that

\[
\omega' F' = -D_t \nabla F.
\]

The particles considered were taken to be much smaller than the Kolmogoroff scale; so, the turbulent rotational diffusion coefficient itself, \( D_t \), was assumed in the second model to depend on the randomizing action of the turbulent pressure fluctuations at the particles' surface which arise from the (Kolmogoroff) micro-turbulence.

From dimensional analysis it was obtained that

\[
D_t = \left( \frac{\epsilon}{\alpha} \right)^{1/2}
\]  

where \( \epsilon \), the turbulent dissipation energy, is given for an homogeneous steady field by (3)

\[
\epsilon = \alpha \sum_{i,k} \overline{W}_{ik}^2.
\]
Applications

As the turbulent gradient tensor $W'_{ik}$ is not completely known in many fluid systems, it was necessary in the present study either to supplement the missing data by the results of numerical simulations or (reasonable) guesses or both. In cases of interest, there was conducted a parametric investigation in which the effect of structure changes in the gradient tensor on the o.d.f. was tested.

The numerical simulations of the turbulent field were carried out according to the method of Wang and Frost (4) where $u'_i = u'_i(t)$ is found. In this method, however, only the three components $W'_{i1}$ ($i = 1, 2, 3$) could be acquired due to the applicability of the relationship known as the Taylor hypothesis, viz.

$$\frac{\partial u'_i}{\partial t} = -\overline{u}_i \frac{\partial u'_i}{\partial x},$$

(6)

to those components alone.

The rest of the gradient components were extracted either from the experimental findings of Klebanoff (5) for the studied situations of a turbulent flow over a flat surface as in the atmospheric boundary layer or from those of Wignansky and Fiedler (6) for the studied situation of a free round turbulent jet, or from parametric checks as mentioned above. The experimentally acquired gradient components are given in the form of

$$\Delta W'_{ik} = \left( \frac{W'_{ik}}{\Delta} \right)^{1/2}$$

and so were taken the rest of the components. The (turbulent) fluid was assumed to be incompressible, viz. $W_{ii} = 0$, and it was set that

$$\Delta W'_{zz} = \Delta W'_{ss} = \frac{1}{2} \Delta W'_{ii},$$

(7)

or with reversed signs.

Obviously, since in addition to the average value of $F_{ik}$, or its maximal value $F_{mn}$, some measure of the spread of the o.d.f. values is desired, a procedure was adopted in which this ($F$) function was calculated for the following three realization fields:

$$w^{(1)}_{ik} = \overline{W}_{ik},$$

(8a)
(2) \( \bar{w}_{ik} = \bar{w}_{ik} - \Delta w_{ik} \) \hspace{1cm} (8b)

and

(3) \( \bar{w}_{ik} = \bar{w}_{ik} + \Delta w_{ik} \) \hspace{1cm} (8c)

Finally, to account for any uncertainties in the turbulent field data, the (determining) effective Peclet number was parameterized by a numerical factor to be

\[ \alpha_e \text{ or } \alpha_0 = \frac{w_0}{D_e} \] \hspace{1cm} (9)

where \( w_0, w_0', w_0'' \), \( w_0'' = (\Delta w_{ik})_0 \) being a typical turbulent component.

Results

Two physical situations were studied:

1. The near-ground atmospheric boundary layer, in which only

\[ u_i(z) = \left( \frac{u^*}{k_0} \right) \ell n \left( 1 + \frac{z}{z_o} \right) \] \hspace{1cm} (10)

and

\[ \bar{w}_{i3} = \partial \bar{u}_i / \partial z = \frac{u^*}{k_0(z-z_o)} \] \hspace{1cm} (11)

essentially exist, \( z \) being the elevation above ground, \( \bar{u}_i \) the mean horizontal velocity, \( u^* \) the friction velocity, \( k_0 \) von Karman Constant and \( z \) the roughness height.
2. The turbulent round free jet, for which the Schlichting's solution for the average velocity components (7) was used. In this solution, the (molecular) kinematic viscosity $\nu$ is replaced by a virtual (turbulent) kinematic viscosity $\nu_\infty$ given in the equation

$$\nu_\infty = 0.16 \sqrt{J'/\rho}$$

(12)

where $J'$ ($=J/\rho$) = 1.59 $b \sqrt{u_0}$, $\bar{u}_0$ is the kinematic momentum of the jet, $b$ is its half-width and $u_0$ is the average fluid velocity along its axis ($b = cx_\text{in which c is 0.63 to 0.79, as experimentally found}$).

In the boundary layer situation, both the case of a "weak turbulence", where $\nu_\infty \approx \nu_0$, and a "strong turbulence", where $\nu_\infty >> \nu_0$, were investigated.

For the first case, numerical simulations were employed while for the second one Klebanoff's experimental data (5) supplemented by simulations was applied. Also, in the latter case, the version

$$\overline{w^2_{32}} = \overline{w^2_{23}} = \overline{w^2_{13}}$$

(13a)

or

$$\overline{w'^2_{32}} = \overline{w'^2_{23}} = \overline{w'^2_{13}}$$

(13b)

was taken. The calculations in both cases were carried out with $\alpha = 0.15 \text{ cm}^2/\text{sec.}$, $\delta$ (thickness of the atmospheric boundary layer) = 10$^3$ m, $z_0 = 0.1$ m, $k_0 = 0.4$, $r_{DB} = 1 \text{ sec}^{-1}$, and $\epsilon$ deduced from Eq. (5). The calculation time for $F$ was greater than the relaxation time of the particles, and $\Delta \theta = \Delta \varphi = \pi/12$ or $\pi/24$.

In the turbulent jet situation, the ratio between the various values of was obtained (6) through
where \( c_2 = 1 + \exp(-200r^2) \), \( r \) being the radial distance within the jet.

For both situations, the absolute values of \( \Delta \bar{W}_{i\kappa} \) were deduced by normalization according to Eq. (5). Values of \( \bar{F} \), and \( F_m \), together with the deviations of \( F \) for fields (2) & (3) (above), were calculated as a function of height above ground (and hence \( \varepsilon \) ) in the boundary layer situation, the aspect ratio of the particles, \( R \) (\( R > 1 \) for fibers and \( R < 1 \) for platelets), and the parameterization factor \( \alpha_0 \). However, only the following typical figures are presented here:

![Diagram](image)

Figure 1. The average o.d.f., \( \bar{F} \), vs. \( \Theta \) for the atmospheric boundary layer, "weak turbulence", with:

- \( z = 2.5 \) to \( 20 \)m (1) 2.5m, (2) 5m, (3) 10m, (4) 20m,
- \( R = 10 \), \( \Phi_m = \pi / 2 \),
- \( \alpha_0 = 1 \), \( \bar{W}_{13} (= \bar{W}_o) \) according to Eq. (11), \( \bar{W}_o = \Delta \bar{W}_{i3} \).

Solid lines are simulation results (254 realizations, usually); points are values calculated from the \( \bar{W}_{i\kappa} \) field.
Figure 2. The maximal o.d.f., $F_m$, vs. height for the atmospheric boundary layer, "strong turbulence". $R = 50$; $\epsilon$ decreases with height according to Ball (8).

Equation (7)

$(1) \; W_n \; , \; (2) \; W_s \; , \; (3) \; W_t \; ;$ dashed lines are cases where terms of Eq. (7) are taken with opposite signs; points relate to combination of Eq. (13b).

(Line 1 essentially coincides with the random distribution one.)
Figure 3. The maximal o.d.f., $F_m$, vs. particle aspect ratio $R$ for the atmospheric boundary layer, "strong turbulence".

$z = 5m$, $\varepsilon = 230 \text{ cm}^2/\text{sec}^{-3}$

(1) $W_{ik}^{(1)}$, (2) $W_{ik}^{(2)}$, (3) $W_{ik}^{(3)}$

(Line 1 essentially coincides with the random distribution one)
Figure 4. The maximal o.d.f. $F_m$ vs. the parameter $\alpha_0$ for the atmospheric boundary layer; "strong turbulence".

$R = 50$ ———— $R = .02$ ———— $z = 2.5m; \epsilon = 5 \text{ cm}^2/\text{sec}^3$.

(1) $W_{ik}^{(1)}$, (2) $W_{ik}^{(2)}$, (3) $W_{ik}^{(3)}$
Figure 5. The maximal o.d.f., $F_m$, vs. particle aspect ratio $R$ for a free turbulent jet.

Location: $x_1 = 2$, $x_2 = x_3 = 0.05$ (see ref. 1),

$u = 5$ sec$^{-1}$, $\epsilon = 250$ cm$^2$/sec$^3$

$(1),(1'): W_{i_k}^{(1)};(2),(2'): W_{i_k}^{(2)};(3),(3'): W_{i_k}^{(3)}$; for $(1'), (2'), (3')$ terms of Eq. (7) are taken with opposite signs.

(Line 1 essentially coincides with the random distribution one)
Figure 6. The maximal o.d.f., $F_m$ vs. the parameter $\alpha_o$ for a free turbulent jet.

$\bar{W}_0 = 5 \text{ sec}^{-1}$, $x_1 = 2$, $x_2 = x_3 = 0.05$ (see ref. 1)

$\epsilon = 125 \text{ cm}^2/\text{sec}^3$, $R = 50$ , $R = 0.02$
Conclusions

Atmosphere boundary layer (up to 20 m height):

1. The average o.d.f. in a weak turbulent field shows structured (preferred) orientation.
2. The maximal (and average) o.d.f. in a strong, commonly occurring, turbulent field of the average realization $W_{ik}^{(1)}$ essentially coincide with the random distribution while the spread of the values of the function between fields $W_{ik}^{(2)}$ and $W_{ik}^{(3)}$ is quite significant. This spread of values may have practical connotations.
3. The values of $F_m$ and its deviations increase with the parameter as expected.

Free turbulent jet:
Conclusions 2 and 3 of the former situation apply here too.

References
