USE OF THE WAVENUMBER TECHNIQUE WITH THE LLOYDS MIRROR FOR AN ACOUSTIC DOUBLET(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA P B KING MAR 85
# Use of the Wavenumber Technique With the Lloyds Mirror For an Acoustic Doublet

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**Abstract:**

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Use of the Wavenumber Technique
With the Lloyds Mirror
For an Acoustic Doublet

by

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ABSTRACT

This thesis examines a method to determine the depth of a point source in an isospeed ocean environment. Using the Fourier Transform on the acoustic pressure field in the range domain results in the attainment of the acoustic pressure spectrum in the wavenumber domain and a characteristic nodal spacing unique to the source-receiver depths. Quantitative examination of a magnitude plot of the spectrum and use of simple mathematical formulae yield the source depth. The debilitative effects of narrowband noise and surface roughness on the pressure spectrum are also examined. The pressure spectrum is recognizable in noise after the pressure field in the range domain has been lost in the noise field. The effect of surface gravity waves on the pressure spectrum is similar to that on the pressure field in the range domain: the characteristic nodal spacing is suppressed as the height of the surface waves increases.
# TABLE OF CONTENTS

I. HISTORY AND INTRODUCTION .................................. 14

II. THEORY ......................................................... 18
   A. THE LLOYDS MIRROR PHENOMENON .......................... 18
   B. THE RELATIONSHIP BETWEEN $K$, $\gamma$, AND $\beta$ ........ 20
   C. THE WAVENUMBER TECHNIQUE AND THE LLOYDS MIRROR .... 20
   D. THE EFFECTS OF SURFACE ROUGHNESS ...................... 27
   E. THE EFFECT OF ADDING NOISE .............................. 28

III. RESULTS AND CONCLUSIONS .............................. 33
   A. THE FFT ALGORITHM ........................................ 33
   B. SOURCE DEPTH DETERMINATION ............................ 38
   C. THE EFFECT OF SURFACE ROUGHNESS ....................... 42
   D. THE EFFECT OF NOISE ..................................... 43
   E. SUMMATION ................................................ 44

APPENDIX A: LLOYDS MIRROR PRESSURE FIELD SOURCE CODE .... 93

LIST OF REFERENCES ........................................ 109

BIBLIOGRAPHY .................................................. 110

INITIAL DISTRIBUTION LIST .................................. 111
LIST OF TABLES

I  Critical Values Used in the Research ........ 34
II  Results of Source Depth Determination Runs .... 39
LIST OF FIGURES

2.1 The Geometry of the Lloyds Mirror Effect ...... 30
2.2 A Classic $|P(R)|$ vs. R Curve .......... 31
2.3 The Relationship Between K, $\gamma$ and $\beta$ ........ 32
3.1 Pressure Spectrum Using FFT2C ........... 46
3.2 Pressure Spectrum Using FFTCC .......... 47
3.3 Pressure Spectrum Using Cooley-Tukey FFT .......... 48
3.4 Non-smoothed Pressure Field ........... 49
3.5 Pressure Spectrum Showing Gibbs Phenomenon .......... 50
3.6 Pressure Field Combined With a Hanning Window .... 51
3.7 Pressure Spectrum Combined With a Hanning Window ........ 52
3.8 Theoretical Source Depth Determination Curve .......... 53
3.9 Graph of Pressure Spectrum, Source at 22.0 Meters ........ 54
3.10 Graph of Pressure Spectrum, Source at 31.4 Meters .......... 55
3.11 Graph of Pressure Spectrum, Source at 15.7 Meters .......... 56
3.12 Graph of Pressure Spectrum, Receiver at 22.0 Meters .......... 57
3.13 Graph of Pressure Spectrum, Receiver at 31.4 Meters .......... 58
3.14 Graph of Pressure Spectrum, Receiver at 15.7 Meters .......... 59
3.15 Pressure Spectrum, Range Window Set at 47.1 Meters .......... 60
3.16 Pressure Spectrum, Range Window Set at 50.3 Meters .......... 61
3.17 Pressure Spectrum, Range Window Set at 62.8 Meters ........................................... 62
3.18 Pressure Spectrum, Sea State 0 ................................................................. 63
3.19 Pressure Spectrum, Sea State 2 ................................................................. 64
3.20 Pressure Spectrum, Sea State 3 ................................................................. 65
3.21 Pressure Spectrum, Sea State 5 ................................................................. 66
3.22 Pressure Spectrum, Sea State 0, $K = 1.0$ .............................................. 67
3.23 Pressure Spectrum, Sea State 3, $K = 1.0$ .............................................. 68
3.24 Pressure Spectrum, Sea State 0, $K = 2.0$ .............................................. 69
3.25 Pressure Spectrum, Sea State 3, $K = 2.0$ .............................................. 70
3.26 Pressure Spectrum, SS 2, Range Window Set at 50 Meters ........................................... 71
3.27 Pressure Spectrum, SS 2, Range Window Set at 100 Meters ........................................... 72
3.28 Pressure Spectrum, SS 2, Range Window Set at 200 Meters ........................................... 73
3.29 Pressure Spectrum, SS 2, Range Window Set at 300 Meters ........................................... 74
3.30 Pressure Field, $K = 1.0$, $\mu = 0.005$ .................................................. 75
3.31 Pressure Spectrum vs. Gamma ................................................................. 76
3.32 Pressure Spectrum vs. Beta ................................................................. 77
3.33 Pressure Field, $K = 1.0$, $\mu = 0.01$ .................................................. 78
3.34 Pressure Spectrum vs. Gamma ................................................................. 79
3.35 Pressure Spectrum vs. Beta ................................................................. 80
3.36 Pressure Field, $K = 2.0$, $\mu = 0.0001$ ............................................... 81
3.37 Pressure Spectrum vs. Gamma ................................................................. 82
3.38 Pressure Spectrum vs. Beta ................................................................. 83
3.39 Pressure Field, $K = 2.0$, $\mu = 0.001$ ............................................... 84
3.40 Pressure Spectrum vs. Gamma ................................................................. 85
3.41 Pressure Spectrum vs. Beta ................................................................. 86
3.42 Pressure Field, $K = 2.0$, $\mu = 0.005$ ............................................... 87
3.43 Pressure Spectrum vs. Gamma ................................................................. 88
3.44 Pressure Spectrum vs. Beta ................................................................. 89
3.45 Pressure Field, $K = 2.0$, $\mu = 0.01$ ............... 90
3.46 Pressure Spectrum vs. Gamma .................. 91
3.47 Pressure Spectrum vs. Beta .................... 92
LIST OF SYMBOLS

A  Amplitude

c  Sound Speed

e  2.718281828...

FFT  Fast Fourier Transform

FFT⁻¹  Inverse Fast Fourier Transform

f  Frequency

f(r)  Generic Function of Range

\mathcal{J}(\gamma)  Spectral Density of \mathcal{f}(r)

Hν  Hankel Function

i  Square Root of -1

Jν  Bessel Function of the First Kind

k  Wavenumber

M  Surface Roughness Factor

m  Index of Calculations or Null Number in the Range Domain

N  Number of Points in the Wavenumber Spectrum
n  Index of Calculations or Null Number in the Wavenumber Spectrum

\mathbb{N}  Complex Narrowband Noise Field

\mathbb{P}  Time Independent Factor of Complex Pressure

\mathbb{P}_s  Complex Pressure of the Source

\mathbb{P}_r  Complex Pressure of the Reflection

\mathbb{P}_n  Complex Pressure in the Presence of Noise

\mathbb{P}(k)  Pressure Field in the Wavenumber Spectrum

\mathbb{R}  Time Dependent Factor of Complex Pressure

\mathbb{R}_1  Distance of the Direct Wave Path

\mathbb{R}_2  Distance to the Image (Reflected Path)

RCVR  Receiver

r  Range Between Source and Receiver

SRC  Source

t  time factor

\mathbb{W}  Hanning Window

\mathbb{Y}_0  Bessel Function of the Second Kind

\mathbb{Z}_s  Source Depth
$z$  
Receiver Depth

$\Delta r$  
Range Increment

$\beta$  
Vertical Component of the Wavenumber

$\Delta \beta$  
Vertical Wavenumber Increment

$\gamma$  
Horizontal Component of the Wavenumber

$\Delta \gamma$  
Horizontal Wavenumber Increment

$\theta$  
Angle of Grazing Incidence at the Surface

$\lambda$  
Wavelength

$\pi$  
3.14159265...

$\phi$  
Phase Angle

$\psi_R$  
Surface Reflection Coefficient

$\omega$  
Angular Frequency

$\sqrt{}$  
Square Root Operator

$\int$  
Integration Operator
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I. HISTORY AND INTRODUCTION

This thesis is the third in a series of investigations into a proposal put forth by R. Lauer of Naval Underwater Systems Center, New London, CT, in 1979. In his memorandum [Ref. 1], Lauer describes a "new" way to analyze sound propagation in the ocean and two applications of the method, which he named the "Wavenumber Technique," or W.T. In his proposal, Lauer stated that a source of sound in the ocean could be pinpointed in both depth and range by a single omnidirectional hydrophone provided the source is generating a continuous wave tone.

The W.T. was initially described by F. DiNapoli [Ref. 2] as an intermediary step in his development of a speedier computer algorithm used in analyzing sound propagation as a function of range. DiNapoli and Lauer proposed converting the pressure as a function of range $P(r)$ to the pressure spectrum as a function of the wavenumber $P(k)$. The conversion is accomplished by taking a weighted Fourier transform of the pressure field $P(r)$. Once the pressure spectrum is obtained, analysis of the pressure field can be accomplished in a manner analogous to that presently used in signal processing.

Lauer proposed two uses for his "Wavenumber Technique:"  
1. determination of the depth of the acoustic source, an ability which has obvious tactical applications;  
2. use of the W.T. to evaluate the accuracy of existing, and future, acoustic models such as P.E. and P.A.C.T., by breaking the generated pressure field down into ray path "families," such as bottom-bounce, refracted-surface-reflected, and surface-ducted families, thus giving a quantitative read-out of what
proportion of the total acoustic energy is being channelled through the various ray paths.
For both uses, a knowledge of the acoustic environment is necessary.

It is the intended purpose of this thesis to investigate the validity of the first application of Lauer's wavenumber technique by using it in conjunction with an acoustic phenomenon that is well-known and for which accurate results can be calculated with precision; this phenomenon is the Lloyds Mirror for an Acoustic Doublet. Using several source/receiver combinations, this investigator intends to compare actual source depth with that predicted by the W.T. If Lauer's technique is cogent, the source depths calculated via the two methods should be equal, or very close within a statistically acceptable degree.

As mentioned in the beginning of this thesis, two prior investigators, B. Stamey and J. Blanchard, looked into applying the W.T. to determining source depth. Stamey's investigation [Ref. 3] utilized a parabolic equation computer model developed by R.K. Brock, the Split Step Fast Fourier Transform, or SSFFT, to generate the pressure fields. He qualitatively compared the model to a Normal Mode model and a P. E. Finite Difference model, both range dependent, and to DiNapoli's Fast Field Program model, a range independent model. Stamey concluded that the W.T. showed promise as an analysis tool and that further investigations were warranted.

J. Blanchard carried Stamey's researches one step further [Ref. 4]. Using two Parabolic Equation computer algorithms, Brock's Split Step Fast Fourier Transform and Jaeger's Implicit Finite Difference, as his pressure field generators, he examined the use of the spacing between nulls of the pressure spectrum to determine source depth. His results are interesting and support the need for further investigation.
The author was not satisfied with Stamey's and Blanchard's findings. Both individuals acknowledged shortcomings in their respective studies, especially with regard to the mutual presence of the "U-shaped phenomenon" encountered in the pressure spectrum. One should remember, however, that their theses were preliminary studies only and were produced within a highly restricted time frame. Also, they used existing computer algorithms specifically designed to approximate acoustic propagation in a velocity-variant medium. However, the environmental settings used by both gentlemen were that of the Lloyds Mirror for an Acoustic Doublet which requires a constant sound speed throughout the water column; thus, the approximations, assumptions and "fudge factors" used by these models make their results for an isospeed medium highly artificial and somewhat suspect. Fortunately, however, there exists for the Lloyds Mirror a simple, geometric solution specifically designed for an isospeed environment [Ref. 5]. Since the pressure fields for varying source and receiver depths and, therefore, the corresponding pressure spectra, can be precisely calculated, it was thought this model would be a good check on the operational applicability of Lauer's proposed technique.

Secondary considerations of this study were to examine what effect, if any, the introductions of, firstly, surface waves and, secondly, noise would have on the pressure spectrum. The first objective was simulated by use of mathematical formulae given in reference 5; for the second objective, it was thought that complex noise, random in both amplitude and phase and similar to a Rician distribution [Ref. 6: p. 189], would provide a simple but reasonable approximation of an ocean noise field. This investigator intends to gradually intensify the surface waves and the noise field, separately, until the original pressure spectrum is no longer recognizable. In this manner, it should
be possible to make a qualitative assessment of their respective debilitating effects under the carefully controlled conditions found in the Lloyds Mirror phenomenon.

Section II sets forth in more detail the theoretical development of each point described in this introduction. Section III summarizes the investigator's results and the conclusions drawn from those results. Section IV contains a listing of the computer algorithms utilized.
II. THEORY

This chapter presents the mathematical basis of the wavenumber technique.

A. THE LLOYS MIRROR PHENOMENON

A diverging monofrequency spherical pressure wave [Ref. 5: p. 112] can be written in complex form as

\[ p(R, t) = P(R)e^{-i\omega t} = \frac{A}{R} e^{i(kR - \omega t)} \quad \text{(eqn 2.1)} \]

where \( P(R) \) is the spatial factor. Referring to Figure 2.1, \( R_1 \) is the range from the source to the receiver and \( R_2 \) is the range from the image to the receiver. For convenience in all that follows, we shall set \( A \) to unit magnitude. The following equations apply to the acoustic waves propagating directly out from the source \( P_s \) and from the apparent image \( P_i \) (which is actually the wave reflected from the surface):

\[ P_s(R, t) = P_s(R)e^{-i\omega t} = \frac{1}{R_1} e^{i(kR_1 - \omega t)} \]
\[ P_i(R, t) = P_i(R)e^{-i\omega t} = \frac{-1}{R_2} e^{i(kR_2 - \omega t)} \quad \text{(eqn 2.2)} \]

where \( P_s \), \( P_s \), and \( P_i \) are complex functions of horizontal range \( r \), vertical depth \( z \) and time \( t \); the minus sign in the equation for \( P_i \) is derived from the surface reflection coefficient, \( \psi_R = -1 \), for a smooth surface. Inspection of Figure 2.1 reveals that

\[ R_1 = \sqrt{(z_r - z_s)^2 + r^2}, \quad R_2 = \sqrt{(z_r + z_s)^2 + r^2} \]
and \( \Delta r = \left| R - R_2 \right| \)

And so the total field can be written as

\[
\hat{P} = \hat{P}(R)e^{-i\omega t} = \frac{e^{i(kR_1 - \omega t)}}{R_1} - \frac{e^{i(kR_2 - \omega t)}}{R_2}
\]  (eqn 2.3)

Since \( \hat{P}_1 \) and \( \hat{P}_2 \) both have the same time factor, \( \exp(-i\omega t) \), we can retain just the spatial factors and equations 2.3 reduce to

\[
\hat{P}(R) = \left[ \frac{e^{i k R_1}}{R_1} - \frac{e^{i k R_2}}{R_2} \right]
\]  (eqn 2.4)

Equation 2.4 is a form of the complex pressure as a function of range used in the computer algorithm shown in Appendix I.

Inspection of Figure 2.1 reveals that, for \( R \gg Z_s \) and \( \Theta \) very small,

\[
r \approx R
\]
or

\[
\sin \Theta \approx \frac{z_f}{R}
\]
or

\[
\Delta r \approx \frac{z_f z_s}{R}
\]  (eqn 2.5)

and the pressure amplitude can be approximated by

\[
\hat{P}(R) \approx \frac{2}{R} \left| \sin \left( \frac{kz_f z_s}{R} \right) \right|
\]  (eqn 2.6)

Looking at just the formula for the pressure amplitude, we can see that as
\[
\left(\frac{k z z_s}{R}\right) \rightarrow n \pi, \quad n = 0, 1, 2, 3, \ldots 
\]  
\text{(eqn 2.7)}

the pressure amplitude goes to zero, producing the classic \(|P(R)| \text{ vs } R\) curve shown in Figure 2.2

B. THE RELATIONSHIP BETWEEN K, \(\gamma\), AND \(\beta\)

In his description of the Wavenumber Technique [Ref. 1: p. 5-6], Lauer utilizes the horizontal and vertical components of the wavenumber, \(k\). The general relationship among these three terms is illustrated in Figure 2.3 and can be written mathematically as

\[
k = \text{the wavenumber} = \frac{2\pi f}{c}
\]

\[
\gamma = \text{the horizontal component of } k = k \cos \phi
\]

\[
\beta = \text{the vertical component of } k = k \sin \phi
\]

or

\[
k = \sqrt{\gamma^2 + \beta^2} 
\]  
\text{(eqn 2.8)}

C. THE WAVENUMBER TECHNIQUE AND THE LLOYDS MIRROR

Given a point source in free space, the monofrequency pressure field at the receiver, \(P(R)\), can be expressed as a spherical wave (time factor suppressed),
where, from Figure 2.1,

\[ R^2 = r^2 + Z_r^2 \quad \text{for} \quad Z_r > Z_s \]

or

\[ R^2 = r^2 + Z_s^2 \quad \text{for} \quad Z_r < Z_s \]  

(egn 2.10)

In integral form, \( \mathcal{P}(R) \) can be written as [Ref. 7: p. 127]

\[
\mathcal{P}(R) = \frac{e^{ikR}}{R} = \int_{0}^{\infty} \frac{J_0(\gamma r) e^{\pm i|Z_r-Z_s|}}{i\beta} \gamma d\gamma \tag{eqn 2.11}
\]

which is taken from the Fourier-Bessel Transform Pair [Ref. 7: p. 126],

\[
\mathcal{L}(r) = \int_{0}^{\infty} \mathcal{A}(\gamma) J_0(\gamma r) \gamma d\gamma
\]

\[
\mathcal{A}(\gamma) = \int_{0}^{\infty} f(r) J_0(\gamma r) r dr
\]  

(egn 2.12)

where \( \mathcal{L}(r) \) represents the acoustic pressure function in the range domain, and \( \mathcal{A}(\gamma) \) is the acoustic pressure spectrum in the wavenumber domain. The sign of the exponential function in the integral is based on which of \( Z_s \) and \( Z_r \) is greater. For the case of receiver depth being greater than source depth so that waves from both image and source are travelling downward at the receiver depth, the total pressure at the receiver can be expressed as

\[
\mathcal{L}(R) = \int_{0}^{\infty} \frac{e^{i\beta (Z_r-Z_s)} - e^{i\beta (Z_r+Z_s)}}{i\beta} J_0(\gamma r) \gamma d\gamma \tag{eqn 2.13}
\]

Use of Euler's Identity reduces Equation 2.13 to
\[ P(R) = -2 \int_0^\infty \frac{e^{it^2}}{t^2} \sin(tz) J_0(\gamma r) \gamma d\gamma \quad \text{(eqn 2.14)} \]

Using the relationship between the Hankel functions \[ \text{[Ref. 5: p. 449]}, \]

\[ H_n^{(1)}(\gamma r) = J_n(\gamma r) + iY_n(\gamma r) \]

and

\[ H_n^{(2)}(\gamma r) = J_n(\gamma r) - Y_n(\gamma r) \quad \text{(eqn 2.15)} \]

one can re-write the Bessel function in equation 2.14 as

\[ J_n(\gamma r) = \frac{1}{2} [H_n^{(1)}(\gamma r) + H_n^{(2)}(\gamma r)] \quad \text{(eqn 2.16)} \]

so that

\[ 2q(\gamma) = \int_0^\infty f(r) H_n^{(1)}(\gamma r) r \, dr + \int_0^\infty f(r) H_n^{(2)}(\gamma r) r \, dr \quad \text{(eqn 2.17)} \]

Letting \( r' = -r \) and looking at the second term on the right hand side of equation 2.17,

\[ \int_{-\infty}^0 f(-r') H_n^{(2)}(-\gamma r') r' \, dr' = \int_0^\infty f(r) H_n^{(1)}(\gamma r) r \, dr \quad \text{(eqn 2.18)} \]

Now, assuming that \( f(r) = f(-r) \), then

\[ 2q(\gamma) = \int_0^\infty f(r) H_n^{(1)}(\gamma r) r \, dr \quad \text{(eqn 2.19)} \]

Therefore,
for $\gamma r > 2\pi$, the asymptotic approximation of the Hankel function for large argument can be used:

$$H_\nu^{(2)}(\gamma r) \approx \sqrt{\frac{2}{\pi\gamma r}} e^{i(\gamma r - \frac{\pi}{4})}$$ (eqn 2.21)

assuming $|\gamma \xi(r)|$ goes to zero faster than $(\ln r)$,

$$L(R) = -\sqrt{\frac{2}{\pi\gamma r}} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} \left[ \sqrt{\frac{\gamma}{\beta}} e^{i\beta z r \sin(\beta z_s)} \right] e^{i\gamma r} d\gamma$$ (eqn 2.22)

Let the terms inside the brackets in equation 2.22 be defined as $g(\gamma)$, then

$$L(R) = -\sqrt{\frac{2}{\pi\gamma r}} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} g(\gamma) e^{i\gamma r} d\gamma$$

and

$$\int_{-\infty}^{\infty} g(\gamma) e^{i\gamma r} d\gamma = -\sqrt{\frac{\pi r}{2}} \sqrt{r} L(R)$$ (eqn 2.23)

or, using equation 2.12,

$$\xi(\gamma) = -\left[ e^{i\frac{\pi}{4}} \right] \sqrt{\frac{\pi}{2\pi}} \int_{-\infty}^{\infty} \sqrt{r} L(R) e^{-i\gamma r} dr$$ (eqn 2.24)

Note that equation 2.24 has the form of the Fourier transform of the pressure function if we define

$$f(r) \equiv \sqrt{r} L(R)$$ (eqn 2.25)
so that \( \mathcal{Z}(R) \) and \( g(\gamma) \) are Fourier Transform pairs.

This equation can be easily evaluated [Ref. 8],

\[
q(\gamma) = -\left[ i \frac{\pi}{4} e^{i \frac{\pi}{2} \frac{1}{\sqrt{2} \pi}} \right] \frac{1}{\sqrt{\beta}} e^{i \beta z_r \sin(\beta z_s)}
\]

(\text{eqn 2.26})

for \( z_r > z_s \). And the magnitude of \( q(\gamma) \) is

\[
\left| q(\gamma) \right| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_s)
\]

(\text{eqn 2.27})

For the case where \( z_s > z_r \), it can be shown that

\[
q(\gamma) = -\frac{\pi}{\sqrt{2} \pi} e^{i \frac{\pi}{4}} e^{i \beta z_s \sin(\beta z_r)}
\]

(\text{eqn 2.28})

and the magnitude of \( q(\gamma) \) is

\[
\left| q(\gamma) \right| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_r)
\]

(\text{eqn 2.29})

A close look at the magnitude of the pressure spectrum function in equation 2.27 reveals that nodes occur for values of

\[
\beta z_s = n\pi, \quad n = 0, 1, 2, 3, \ldots
\]

(\text{eqn 2.30})
If the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber then the spacing between nodes $\Delta \beta$ is uniform and the depth of the source can be derived from the relationship given by

$$z_s = \frac{\pi}{\Delta \beta} \quad \text{(eqn 2.31)}$$

In the event the source is deeper than the receiver, looking at equation 2.29, we can see that nodes now occur for values of

$$8z_r = n\pi, \ n = 0, 1, 2, 3, \ldots \quad \text{(eqn 2.32)}$$

Plotting equation 2.29 as a function of the vertical wavenumber, the spacing between nodes will now reveal the receiver depth based on

$$z_r = \frac{\pi}{\Delta \beta} \quad \text{(eqn 2.33)}$$

Notice in equations 2.31 and 2.33 the exchange of source and receiver depths. By placing the receiver at a shallower depth than the source, no new information (namely the source depth) is to be found.

In summary, there is a way to calculate the complex pressure amplitude as a function of range using the complex pressure spectrum. Conversely, if the complex pressure $z(r)$ has been measured at the receiver, then the complex pressure spectrum $z(k)$ can be derived by making use of the relationship between the Fourier-Bessel Transform Pairs (see equation 2.12), and the depth of the source can be found from the magnitude of that spectrum (see equation 2.31).
The foregoing is the mathematics involved in deriving the theoretical pressure spectrum. One purpose of this thesis was to compare the theoretical value of the pressure spectrum as derived with the Fourier-Bessel Transform with the pressure spectrum obtained by use of the computerized Discrete Fourier Transform (DFT), otherwise known as the Fast Fourier Transform, or FFT. At this point, it will be helpful to review what happens in the FFT.

Starting with the Fourier-Bessel transform pair shown in equation 2.12, \( f(r) \) and \( g(\gamma) \) can be re-written in terms of a discrete sum as is done in the DFT:

\[
\text{FFT}^{-1} \left\{ g(\gamma) \right\} = \sum_{n=0}^{N-1} g(n \Delta \gamma) e^{\frac{2 \pi i n n}{N}} = f(m \Delta r) \quad (\text{eqn 2.34})
\]

where \( N = 1, 2, 3, \ldots \), up to some large positive integer, and represents the number of transform points, and

\[
\gamma r = (n \Delta \gamma)(m \Delta r) = \frac{2 \pi m n}{N} \quad (\text{eqn 2.35})
\]

based on the relationship between \( \Delta r \) and \( \Delta \gamma \) as described in sampling theory [Ref. 2: p. 2],

\[
\Delta \gamma \Delta r = \frac{2 \pi}{N}
\]

or

\[
\Delta r = \frac{2 \pi}{N \Delta r} \quad (\text{eqn 2.36})
\]

By convention, the sign of the exponential function in equation 2.34 is taken as negative when performing the "forward" transform,

\[
\text{FFT} \left\{ f(m \Delta r) \right\} = g(n \Delta \gamma) \quad (\text{eqn 2.37})
\]
and is taken as positive when performing the "inverse" transform,

\[ \text{FFT}^{-1}\left\{ q(n\Delta\gamma) \right\} = f(m\Delta r) \]  

(\text{eqn 2.38})

Then,

\[ \int_{-\infty}^{\infty} q(\gamma) e^{i\gamma' r} d\gamma = \left[ \sum_{n=-N}^{N} q(n\Delta\gamma) e^{i\frac{2\pi mn}{N}} \right] \Delta\gamma \]

where

\[ q(n\Delta\gamma) = \frac{\sqrt{\gamma}}{\beta} e^{i\beta z_s} \sin(\beta z_s) \]

Therefore

\[ P(m\Delta r) = -\sqrt{\frac{2}{\pi r}} e^{-\frac{i\pi}{4}} \left[ \Delta\gamma \text{FFT}^{-1}\left\{ q(n\Delta\gamma) \right\} \right] \]

and

\[ |q(n\Delta\gamma)| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_s) \]  

(\text{eqn 2.39})

D. THE EFFECTS OF SURFACE ROUGHNESS

Surface roughness \[\text{Ref. 5: p.409}\] reduces the Lloyds Mirror effect. The rougher the surface, the greater the effect for angles increasingly closer to grazing incidence. If \( M \) represents the surface roughness factor, then equation 2.3 can be re-written as

\[ P(R) = \frac{e^{ikR}}{R} - e^{-M^2} e^{ikR} \]

where

\[ M = \frac{4H \sin\phi}{\lambda} \]  

(\text{eqn 2.40})
\( \phi \) is the grazing angle of incidence, \( H \) is the average height of the surface gravity waves, and \( \lambda \) is the wavelength of the narrowband continuous wave acoustic signal. When \( M \) is less than 1, the surface is said to be smooth; when \( M \) is greater than 1, the surface is said to be rough.

Looking at Figure 2.1, we can see that the overall effect of increasing surface roughness is to reduce the contribution of the surface reflection or "image" to the interference pattern at the receiver. In other words, as \( H \) increases, the energy detected by the receiver is increasingly only the energy coming directly from the source. When the effect of surface roughness is a maximum such that the contribution to the pressure field of the "image" is completely suppressed, then

\[
q_s(\gamma) = \int_0^\infty \frac{e^{ikR}}{R} \cdot J_0(\gamma R) dR
\]

\[
= \frac{e^{i\beta|z_s-z_r|}}{i\beta}
\]  

(eqns 2.41)

and the magnitude of the spectrum is

\[
|g(\gamma)| = \frac{1}{b}
\]  

(eqns 2.42)

E. THE EFFECT OF ADDING NOISE

In an effort to simulate a realistic ocean environment, a normally distributed narrowband noise field based on a Rician distribution [Ref. 6: p. 189] was adapted. The resulting acoustic pressure field can be expressed as
\[ \mathcal{P}_n(R) = \mathcal{P}(R) + n \]  

where \( n \) is a normally-distributed random function with a mean of zero, a standard deviation of one and possesses both amplitude and phase. The noise operates independently on both the amplitude and phase components of the pressure field. Since \( n \) is independent of range and wavenumber, it is treated as a constant by the Fourier Transform.

A fourth intention of this thesis was to discover the degree to which noise degrades the wavenumber spectrum compared with the corresponding degradation of the range domain. To produce the random noise fields used in this research, the routine listed as "GGNML" in the IMSL library [Ref. 9] was called upon twice to generate independent, pseudo-random functions which interact separately and simultaneously with both the amplitude and phase components of \( \mathcal{P}(R) \). While not an elaborate scheme, owing to time and resource constraints, it was felt that this modest simulacrum of narrowband noise would give a fairly accurate, first cut "feel" for the effects of noise on the wavenumber spectrum. The reader should be aware, however, that no direct consideration of coherency along the range path was taken into account by this method.
Figure 2.1 The Geometry of the Lloyds Mirror Effect
Figure 2.2 A Classic $|P(R)|$ vs. R Curve
Figure 2.3  The Relationship Between K, γ and β
III. RESULTS AND CONCLUSIONS

A. THE FFT ALGORITHM

The research for this thesis was conducted entirely with computer algorithms to model the Lloyds Mirror, the Fast Fourier Transform, the range windows, waves, and the narrow-band noise. This was necessary since the Lloyds Mirror is an idealized representation of a situation that occurs only rarely in nature, and at that is limited to isospeed surface ducts. Also, time and money constraints were such that the use of computer models was an absolute necessity. For these reasons and because the FFT is the heart of the Wavenumber Technique, finding a reliable, easy-to-use computer algorithm to perform the FFT on the complex pressure function was deemed very important.

Initially, the IMSL library routines, FFT2C and FFTCC [Ref. 9: p. 232], were used to generate the pressure spectra. Although the author was unable to ascertain just how Stamey performed the FFT on his data, it is highly probable that he used an IMSL library routine. While in his thesis, Blanchard stated that he used the FFTCC routine. The magnitude of a typical spectrum generated by the routine FFT2C is shown in Figure 3.1.

The same scenario used to generate Figure 3.1 was used to produce the spectrum shown in Figure 3.2, but the routine FFTCC was utilized. As can be seen, the resulting graphs of the pressure spectrum magnitude are practically identical. Indeed, the set-up of the pressure fields for insertion into each routine differ only in that FFT2C requires a data set consisting of an integral power of two number of points, whereas FFTCC will handle any number of points. And, when
the Cooley-Tukey FFT listed in Appendix I was used to transform the same pressure field as was utilized to produce Figures 3.1 and 3.2, Figure 3.3 was the result. This algorithm differs somewhat from the IMSL routines by the manner in which the pressure field data is treated. Both FFT2C and FFTCC are designed to handle a complex array. The Cooley-Tukey FFT, on the other hand, is designed to transform real data; but the pressure field is complex. To get around this apparent conundrum, the pressure data is defined as either real or imaginary by its placement in the array. As can be seen, the resultant graph of the spectrum magnitude is almost indistinguishable from that produced by either IMSL routine. In conclusion, it does not matter which FFT routine is used to generate the pressure spectrum, as long as the pressure field in the range domain is "fitted" into the array in a manner suitable to the particular algorithm used.

### TABLE I

<table>
<thead>
<tr>
<th>$K$ (m$^{-1}$)</th>
<th>$N$</th>
<th>$r$ (m)</th>
<th>$(m^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>1024</td>
<td>0.524</td>
<td>$1.172 \times 10^{-2}$</td>
</tr>
<tr>
<td>2.0</td>
<td>1024</td>
<td>0.785</td>
<td>$7.816 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.0</td>
<td>1024</td>
<td>1.571</td>
<td>$3.906 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

It is important to note four things about the FFT at this point.

1. Since the FFT is given only a finite sample of a function while the theoretical Fourier Transform
looks at the complete function, the computer algorithm is designed to overcome this necessary shortcoming by assuming the submitted finite sample repeats its pattern an infinite number of times. This can present problems. If the right hand side of one pattern does not flow smoothly into the left hand side of the next repetition, as is shown in Figure 3.4, then high frequency oscillations known as the Gibbs Phenomenon can be introduced into the spectrum. (See Figure 3.5) This is a very real danger where acoustic pressure fields are concerned. For example: if the sample size does not include the entire pattern (i.e., the pressure magnitude has not been completely attenuated prior to the endpoints of the sample), this unwanted "jitter" in the spectrum can result. To avoid this situation, not only was a pressure field symmetric about the origin (the source) used, but also a Hanning Window was constructed to reduce both the amplitude and the phase components of the pressure field to zero at the endpoints, thus avoiding the Gibbs Phenomenon. Figure 3.6 shows the same pressure field depicted in Figure 3.4 combined with a Hanning Window. Its spectrum is shown in Figure 3.7. The Gibbs Phenomenon is absent. The form of the window is given below. The reader's attention is directed to Appendix I for a view of the manner in which it is combined with the pressure field.

The Hanning Window:

\[ W = 0.5 \left[ 1 + \cos \left( \frac{\pi}{NPTS} (t-1) \right) \right] \]  

(eqn 3.1)
where "NPTS" is the number of data points, and $I = 1, 2, 3, ..., NPTS$.

The Hanning window does not affect the acoustic pressure field in any other way; the interference pattern in the range domain characteristic of the Lloyd's Mirror for an acoustic doublet remains the same except for its amplitude near the end points. As shall be shown later in this chapter, it is the pattern of the entire pressure field, not simply the amplitude of the pattern, on which the W.T. performs its legerdemain.

2. Because the vertical wavenumber beta is real and has physical meaning for values of the horizontal wavenumber less than $k$, only those values of the magnitude of the acoustic pressure spectrum corresponding to $\gamma < k$ were retained.

3. In the Wavenumber Technique, the relationship between the horizontal range step size $\Delta r$ and the horizontal wavenumber step size $\Delta \gamma$ is extremely important. Recalling equation 2.36,

$$\Delta \gamma \Delta r = \frac{2 \pi}{N}$$  \hspace{1cm} (eqn 3.2)

for a given "N," the term on the right hand side, $2 \pi / N$, is a constant. Consequently,

$$\Delta r \propto (\Delta \gamma)^{-1}$$ \hspace{1cm} (eqn 3.3)

Simultaneously, in accordance with sampling theory, in order to prevent aliasing in the pressure spectrum, there is an upper limit on the size of $\Delta r$: 

36
\[ \Delta r \leq \frac{\lambda}{2} \quad \text{(eqn 3.4)} \]

Normally, \( \Delta r \) is chosen such that it is much less than half a wavelength; specifically, \( \Delta r < \frac{\lambda}{5} \) or \( \frac{\lambda}{6} \), thus a good sample of the range domain is ensured. However, from equation 2.36, the smaller \( \Delta r \) becomes, the larger \( \Delta \gamma \) becomes and the coarser the sample sizes of the spectrum become. Also, to ensure adequate samples in the spectrum, examination of equations 2.25 and 2.36 and [Ref. 10] reveals that

\[ \Delta \gamma \leq \frac{1}{2k} \left( \frac{\pi}{z} \right)^2 \quad \text{(eqn 3.5)} \]

where \( z \) is the deeper of the two depths, whether source or receiver. So there is a delicate balance to maintain between the sizes of \( \Delta r \) and \( \Delta \gamma \). In the spirit of compromise, the investigator chose to use a range step size equal to a quarter of a wavelength where

\[ \lambda \equiv \frac{2\pi}{k} \quad \text{(eqn 3.6)} \]

For most cases investigated, values for \( k \), \( N \), \( \Delta r \) and \( \Delta \gamma \) are summarized in Table I. As can be seen, using these values and equation 3.5, for a "\( z \)" value of 25 wavelengths, the maximum size \( \Delta \gamma \) can assume and still ensure an adequate sample size is \( \Delta \gamma = 0.0004 \text{ /m} \). Clearly the sizes of \( \Delta \gamma \) shown in Table I are too coarse by a factor of approximately 20. Only by increasing \( N \), the number of sample points, can this discrepancy be corrected.
However, as mentioned earlier, computer resources were limited. The effect on the research results is not significant; nevertheless, it is recommended the reader bear this limitation in mind during the succeeding pages.

4. Looking again at Figures 3.1 through 3.3, the FFT is the curve marked by the solid line, while the theoretical Fourier Transform is the curve marked by the dotted line. Notice the large discrepancy between the theoretical values and the actual values in the region corresponding to small vertical wavenumbers. This happens to correspond to a region of the spectrum characterized by more rapid fluctuations in the spectrum. However, the size of beta is constant. Consequently, fewer samples of this region of the spectrum are available as compared with that region corresponding to high beta values. Since the plotting routine utilized is interpolating between sampled points of the spectrum, the graph at this end is less smooth and results in a marked difference between the curves. Where the curves are determined by more points and are smoother, the discrepancy between theory and computerized reality is much less noticeable.

B. SOURCE DEPTH DETERMINATION

Theory predicts that, for a source shallower than the receiver, when the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber, the resultant nodal spacing can be used to determine source depth by either consulting a chart (see Figure 3.8) or by performing a simple calculation (see equation 2.31). To test this prediction, several scenarios were designed. Figures 3.9
through 3.11 are typical of the many cases run. In each, source depth is less than receiver depth. For each case, careful measurement of the $\Delta \beta$ spacing coupled with use of equation 2.31 yielded the (already known) source depth with less than a two per cent error, which can be attributed to human error in the measurement of $\Delta \beta$.

**TABLE II**

Results of Source Depth Determination Runs

<table>
<thead>
<tr>
<th>Figure Nr.</th>
<th>ZS (m)</th>
<th>ZR (m)</th>
<th>(Meas'd) Z(Calc'd)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>21.998</td>
<td>47.124</td>
<td>0.14 m$^{-1}$</td>
</tr>
<tr>
<td>3.10</td>
<td>31.416</td>
<td>47.124</td>
<td>0.10 &quot;</td>
</tr>
<tr>
<td>3.11</td>
<td>15.708</td>
<td>47.124</td>
<td>0.20 &quot;</td>
</tr>
<tr>
<td>3.12</td>
<td>47.124</td>
<td>21.998</td>
<td>0.14 &quot;</td>
</tr>
<tr>
<td>3.13</td>
<td>47.124</td>
<td>31.416</td>
<td>0.10 &quot;</td>
</tr>
<tr>
<td>3.14</td>
<td>47.124</td>
<td>15.708</td>
<td>0.20 &quot;</td>
</tr>
</tbody>
</table>

To test what happens when the receiver depth is shallower than the source depth, the cases depicted in Figures 3.12 through 3.14 were run. The scenario is the same as for Figures 3.9 through 3.11; only the source and receiver depths have been exchanged. As can be seen, the $\Delta \beta$ spacing corresponds with the receiver depth. This is again as predicted by theory. Table II summarizes the findings of Figures 3.9 through 3.14.
The scenarios shown in Figures 3.9 through 3.14 all start with a pressure field measured horizontally from the source; i.e., the exact range to the source is known. What if the range to the source is not known, or only minimal information concerning the range is known? In this case, only a portion of the pressure field starting at some initial range $P_o$ can be sampled. What is the effect on the spectrum?

To answer these questions, a range window was constructed and placed at varying initial ranges from the source. As Figure 2.2 illustrates, as range from the source increases, the nodes of the Lloyd's Mirror interference pattern in the range domain become more widely spaced. Given fewer nodes to sample, will the spectrum be the same? In accordance with theory, the spectrum is independent of range (see equation 2.39) and so should be the same wherever the window is placed.

Figures 3.15 through 3.17 illustrate that the spectrum is not entirely independent of range. The figures utilize the same scenario as was used above, differing only in the range from the source at which the sampling of the pressure field begins. As can be seen, beginning at the right hand side which corresponds to high vertical wavenumber values, or the low horizontal wavenumber values (see equation 2.8), a "washing out" of the spectrum occurs, increasing as the range window is moved further from the source. Again, the theoretical curve is marked by the dotted line.

When the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber, the useful portion of the resulting graph is the last two-thirds of the beta range. For $K = 2.0/m$, this is the range $0.67/m < \beta < 2.0/m$. This phenomenon is explained more fully in Section III A 4. With this caveat in mind and based upon multiple source, receiver, range window
combinations run through the model, the following criterion was established: the nodal spacing of the spectral plots was no longer determinable once the initial range \( R_0 \) was increased to approximately three times the source depth. In other words, minimum knowledge of the range from receiver to source must be available to ensure adequate sampling of the acoustic pressure field in the range domain. This, in turn, will produce a pressure spectrum of such a quality that source depth determination can be made.

What causes this not entirely unexpected result? As the range window is moved further from the source, for \( Z_s \neq Z_r \), those samples of the pressure field corresponding to low gamma values are lost to the spectrum first. Low gamma values correspond to waves arriving at high angles with respect to the horizontal. At greater receiver ranges, \( Z \) more closely approximates \( r \) and the arrival angles become closer to the horizontal (see Figure 2.1).

As the window is moved further from the source and fewer nodes of the interference pattern are encountered, if a wider window were used, could this "washing out" effect in the spectrum be minimized or even eliminated? In the author's opinion, it would be minimized; because of the reasoning in the preceding paragraph, it probably would not be eliminated. However, a wider window would require more sample points; because of equation 2.36, one cannot merely make \( \Delta r \) larger. Time and the computer resources available to this investigator were limited, and the test cases run were already using the maximum available array sizes, thus precluding a quantitative illustration of this hypothesis. Succeeding research into the W.T. might include ways to test this point.

In conclusion, the Wavenumber Technique has the potential for being a valuable operational tool in determining the depth of an acoustic source provided the receiver is at
a greater depth than the source. It is strongly recommended that further research into use of the W.T. in a realistic ocean environment be done.

C. THE EFFECT OF SURFACE ROUGHNESS

This and the succeeding section were purely qualitative portions of the research. Therefore, in the figures the spectra were plotted as functions of the horizontal wavenumber for ease of discussion except where a particular point about the vertical wavenumber was being illustrated, and the theoretical curve was omitted.

As described in Chapter II, the effect of surface roughness is to suppress the contribution of the image to the dipole interference pattern. Moreover, by increasing \( M \), the effect is to suppress the image's contribution to the pressure field in the range domain and reduce the pressure spectrum to the contribution of the direct path wave only (see equation 2.41); the resultant magnitude of the spectrum is inversely proportional to the vertical wavenumber (see equation 2.42). As Figures 3.18 through 3.21 illustrate, this is exactly what happens in the FFT. In Figure 3.21, an inverse beta curve has been manually superimposed onto the actual curve to reflect this point.

Looking at the pressure field in the range domain \( \mathcal{P}(F) \) the effect of surface roughness is inversely proportional to the wavelength of the signal. For longer wavelengths, the Lloyds Mirror effect is more tolerant of surface roughness than it is for shorter wavelengths. Does the same relationship hold for the pressure spectrum? Comparison of the respective sea states in Figures 3.22 and 3.23 with Figures 3.24 and 3.25 illustrate that it does.

This investigator also looked at the effect of varying the range window in the presence of waves. This is similar
to what was looked at in Section B of this chapter. Results should be similar and for the same reasons. For a given sea state, source and receiver geometry and wavelength, the range window was moved successively further from the source (see Figures 3.26 through 3.29). As can be seen, the "washing out" effect of the spectral nodes begins to effect the lower horizontal wavenumber values first.

D. THE EFFECT OF NOISE

Random noise was simulated in this research. Being independent of range and frequency, it was treated as a constant by both the Fourier Transform and the FFT. The investigator wished to compare qualitatively the effects of equal amounts of noise on the pressure field in the range domain and in the wavenumber domain. To do this, successively larger values of the "noise factor" were used in computing $\hat{p}$ (see equation 2.43). Values of the noise factor used in the research were limited to the maximum amplitude of the pressure field $P(R)$. Since dipole radiation is predicated, as range from the source increases, the maxima of the interference pattern decrease as the inverse square of the range (i.e., $R^{-2}$). Hence, even small noise factors can have a devastating effect on the pressure field as the receiver is stepped out in range. The destructive effect on the spectrum of successively more intense amounts of noise at varying wavelengths is illustrated in Figures 3.30 through 3.47.

Several conclusions concerning the pressure spectra can be drawn from these results:

1. For equal amounts of noise, longer wavelengths are affected less than shorter wavelengths. Compare Figures 3.31 and 3.32 with Figures 3.43 and 3.44, and Figures 3.34 and 3.35 with Figures 3.46 and 3.47.
This was not a surprising result since the same principle holds for the range domain.

2. The spectrum is affected less by noise than is the range domain. Compare Figures 3.30 through 3.35 and Figures 3.36 through 3.47 for an illustration of this point at two different wavelengths and various noise levels. This was a rather welcome surprise.

3. If the magnitude of the pressure spectrum is plotted as a function of beta, the $\Delta \beta$ spacing can still be detected even after the pressure function in the range domain has been "swallowed up" by the noise field. Compare Figures 3.33 through 3.35 and Figures 3.42 through 3.44. This was not surprising in view of 2. above.

E. SUMMATION

All of the foregoing results are based specifically on the Lloyds Mirror for an Acoustic Doublet. This is a highly idealized and artificial environment. Any thought of immediately applying conclusions reached in this paper to a real world situation is premature. However, the results are still significant if only for supporting the statement made by each preceding investigator that the Wavenumber Technique should be examined very closely for a future operational role, especially in view of the current trends in source levels and ambient noise levels.

Certain fundamental conclusions regarding the author's research into the Wavenumber Technique can now be made within the confines of the above restriction:

1. Source depth can be determined quickly and easily from the acoustic pressure spectrum provided
   a) the receiver is deeper than the source,
b) Some knowledge of the range from receiver to source is available, and

c) The magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber.

2. Source depth determination in even a noisy environment is possible. While the introduction of increasing amounts of narrow band noise adversely affects both the range domain and the wavenumber domain, the pressure spectrum seems able to withstand the chaotic destruction longer than does the pressure field in the range domain.

3. The ability to determine source depth is adversely affected by the height of surface gravity waves since surface roughness reduces the Lloyds Mirror as the sea state increases.
Figure 3.1  Pressure Spectrum Using FFT2C
PRESSURE SPECTRUM VS KZ

SOURCE DEPTH = 20 m, K = 2.0/m,
RECEIVER DEPTH = 50 m, N = 1024

Figure 3.2 Pressure Spectrum Using FFTCC
Figure 3.3 Pressure Spectrum Using Cooley-Tukey FFT
Figure 3.4  Non-smoothed Pressure Field

49
Figure 3.5  Pressure Spectrum Showing Gibbs Phenomenon
Figure 3.6  Pressure Field Combined With a Hanning Window
Figure 3.7  Pressure Spectrum Combined With a Manning Window

52
Figure 3.8 Theoretical Source Depth Determination Curve
Figure 3.9  Graph of Pressure Spectrum, Source at 22.0 Meters
PRESSURE SPECTRUM VS KZ

RECEIVER DEPTH = 47.124 M, K = 2.0/M, N = 1024

Figure 3.10  Graph of Pressure Spectrum, Source at 31.4 Meters
PRESSURE SPECTRUM VS KZ
RECEIVER DEPTH = 47.124 M, K = 2.0/M, N = 1024

Figure 3.11 Graph of Pressure Spectrum, Source at 15.7 Meters
PRESSURE SPECTRUM VS KZ

SOURCE DEPTH = 47.124 M, K = 2.0/M.
N = 1024

Figure 3.12  Graph of Pressure Spectrum,
Receiver at 22.0 Meters
PRESSURE SPECTRUM VS KZ

SOURCE DEPTH = 47.124 M, K = 2.0/M,
N = 1024

Figure 3.13 Graph of Pressure Spectrum, Receiver at 31.4 Meters
PRESSURE SPECTRUM VS KZ

SOURCE DEPTH = 47.124 M, K = 2.0/M,
N = 1024

Figure 3.14 Graph of Pressure Spectrum,
Receiver at 15.7 Meters
PRESSURE SPECTRUM VS KZ
SOURCE DEPTH = 21.998 M, K = 2.0/M
RECEIVER DEPTH = 47.124 M, N = 1024

Figure 3.15 Pressure Spectrum, Range Window Set at 47.1 Meters
Figure 3.16 Pressure Spectrum, Range Window Set at 50.3 Meters
Figure 3.17  Pressure Spectrum, Range Window Set at 62.8 Meters
PRESSURE SPECTRUM VS. KZ

SOURCE DEPTH = 21.998 M, \( k = 2.0 \)/M
RECEIVER DEPTH = 47.124 M, \( h = 0.0 \) M

Figure 3.18  Pressure Spectrum, Sea State 0
Figure 3.19  Pressure Spectrum, Sea State 2
Figure 3.20 Pressure Spectrum, Sea State 3
Figure 3.21  Pressure Spectrum, Sea State 5
Figure 3.22 Pressure Spectrum, Sea State 0, $K = 1.0$
Figure 3.23  Pressure Spectrum, Sea State 3, $K = 1.0$
Figure 3.24  Pressure Spectrum, Sea State 0, \( K = 2.0 \)
Figure 3.25  Pressure Spectrum, Sea State 3, $K = 2.0$
PRESSURE SPECTRUM VS KH

SOURCE DEPTH = 21.998 M, N = 1024
RECEIVER DEPTH = 47.124 M

Figure 3.26  Pressure Spectrum,
SS 2, Range Window Set at 50 Meters
Figure 3.27  Pressure Spectrum, SS 2, Range Window Set at 100 Meters
Figure 3.28 Pressure Spectrum, SS 2, Range Window Set at 200 Meters
SOURCE DEPTH = 21.998 M, N = 1024
RECEIVER DEPTH = 47.124 M

Figure 3.29  Pressure Spectrum, SS 2, Range Window Set at 300 Meters
Figure 3.30 Pressure Field, $K = 1.0$, $\mu = 0.005$
Figure 3.31 Pressure Spectrum vs. Gamma
Figure 3.32  Pressure Spectrum vs. Beta
MAGNITUDE OF PRESSURE AS A FN OF RANGE

SOURCE DEPTH = 21.998 M, RECEIVER DEPTH = 47.124 M.
RANGE STEP SIZE = 1.57 M, N = 1024

Figure 3.33  Pressure Field, $K = 1.0$, $\mu = 0.01$
Figure 3.34 Pressure Spectrum vs. Gamma
Figure 3.35  Pressure Spectrum vs. Beta
Figure 3.36  Pressure Field, $K = 2.0$, $\mu = 0.0001$
Figure 3.37  Pressure Spectrum vs. Gamma
Figure 3.38 Pressure Spectrum vs. Beta
MAGNITUDE OF PRESSURE AS A FN OF RANGE

SOURCE DEPTH = 21.998 M, RECEIVER DEPTH = 47.124 M,
RANGE STEP SIZE = 0.785 M, N = 1024

Figure 3.39  Pressure Field, K = 2.0, \( \mu = 0.001 \)
Figure 3.40  Pressure Spectrum vs. Gamma
Figure 3.41  Pressure Spectrum vs. Beta
MAGNITUDE OF PRESSURE AS A FUCN OF RANGE

SOURCE DEPTH = 21.998 M, RECEIVER DEPTH = 47.124 M,
RANGE STEP SIZE = 0.785 M, N = 1024

Figure 3.42  Pressure Field, \( K = 2.0, \mu = 0.005 \)
Figure 3.43 Pressure Spectrum vs. Gamma
Figure 3.44  Pressure Spectrum vs. Beta
Figure 3.45  Pressure Field, $K = 2.0$, $\mu = 0.01$
Figure 3.46 Pressure Spectrum vs. Gamma
Figure 3.47  Pressure Spectrum vs. Beta
1. In the inverted coordinate system, do you want to make any changes? (YES/NO) * 
   - FAC (1, 2) E 0.01
   - GAX (1, 2) E 10, 21
   - (GAX, NC) E 10, 21

2. Select the surface reflection case. Do you want TESDC/70?
   - TESDC/70
   - TESDC/71

3. Calculate the complex pressure field
   (Continued from the previous page)
DESCRIPTION OF WORK ON PAGE 139 OF HIS "PRINCIPLES ...", 3RD ED.

1.0 DO YOU DESIRE TO ADD NOISE? (YES/NO) *  

1.1 FORMAT (/A2, A) WHAT IS THE NOISE FACTOR? (A POSITIVE REAL NUMBER <= 1.0) *  

1.2 READ (M, 11)  

1.3 C = (C, 11)  

1.4 FORMAT (/A2, A) WHAT IS DSER1? *  

1.5 READ (M, 12)  

1.6 C = (C, 12)  

1.7 FORMAT (/A2, A) WHAT IS DSER2? *  

1.8 READ (M, 13)  

1.9 C = (C, 13)  

1.10 CALL GENT (DSER1, LMAX, M)  

1.11 CALL GENT (DSER2, LMAX, M)  

1.12 GENT (L, 11) (2MAX) ; M (2MAX)  

2.0 X = 0.0, L = 0.0, F = 2.0, P10 = 0.0  

2.1 C = 1.0, LMAX = 0.0, ALNO1 = 0.0  

2.2 CONTINUE  

2.3 X = X + 0.1, L = L + 0.1, F = F + 0.1, P10 = P10 + 0.1  

2.4 CONTINUE  

2.5 END
USE OF THE WAVENUMBER TECHNIQUE WITH THE LLOYDS MIRROR FOR AN ACOUSTIC DOUBLET(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA P B KING MAR 85
DADA(2*I) = DATA(I.IMAX + (2*I))

CONTINUE

CALL PPLOT(DADA,DE,INAX,IMAX,RO)

CALL PCH2(DATA,IMAX,-1,1)

WRITE(6,352)

FORMAT(14*I,4X,'SPEC DATA REAL',3X,'SPEC DATA IMAG')/

DO 13 I = 1,IMAX

WRITE(6,351) I,DATA(2*I - 1),DATA(2*I)

CONTINUE

FORMAT(2X,14,2(4X,F10.7))

DG = 2.)* PI* (F*LIMAX* DB)

WRITE(6,201)

FORMAT(16X,2X,14X,'CK',10X,10X,10X,'KZ')/

DO 20 I = 1,IMAX4

FLY = FL041(I)

G4(I) = FLY * DG

IF (G4(I,GE,AK)) GO TO 400

GO TO 420

CONTINUE

******************************************************************************

IF G4(I) < AK, THEN G4(I) IS REAL, AND IS REDUCED TO A FORM OF THE

C SIN(X)/X FUNCTION, APPROXIMATELY. ACTUAL FORM IS GIVEN BELOW.

******************************************************************************

G4(I) = SQRT(AK**2 - G4(I)**2) + EPS

Z(I) = AK* SIN(G4(I) - G4(I))/3D(I)

GO TO 450

CONTINUE

******************************************************************************

IF G4(I) < AK, THEN G4(I) = SQRT OF A NEGATIVE NUMBER, AND THE

THEORETICAL FUNCTION, P(I), REDUCED TO THE HYPERBOLIC SIN (SINH).

******************************************************************************
BEGIN = SQRT(GM(1)**2 - AK**2)
Q1 = EXP(-DER * (ZP - Z3)) * EXP(-(ZM * (ZP + Z3)))
Q2 = ABS(CQ * SQRT(GM(1))/(2.0 * DER))
GO TO 50

CONTINUE

C
CKRE = DATA(Z*1 - 1)
CKIM = DATA(Z*1)
CKM(I) = SQRT(CKRE**2 + CKIM**2)
CONTINUE

I.MAX5 = I.MAX4 - 1
CALL WHELP(1,MAX5,SM,CKM,Q,AK,BE)

CALL ATWPLT(I.MAX5,SM,CKM,AK,BE)

431
CALL DGNEFL
STOP
END

SUBROUTINE FOUR2(DATA, N, ISIGN, IFORM)

C
COOLEY-TUYKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN.
MULTI-DIMENSIONAL TRANSFORM, EACH DIMENSION A POWER OF 2.
COMPLEX OR REAL DATA.

TRANSFM((K1,K2,...) = SUM(DATA(J1,J2,...) * EXP(ISIGN*2*PI*SQR(-1) * 
* (J1-1)**K1 + (J2-1)**K2 + ...)) * SUMMED FOR ALL
J1 AND K1 FROM 1 TO N(1), J2 AND K2 FROM 1 TO N(2).

ETC. FOR ALL NOIM SUBSCRIPTS. NOIM MUST BE POSITIVE AND
EACH N(IDIM) JUST BE A POWER OF 2. ISIGN IS +1 OR -1.

LET N(1) = N(1)**2 IF NOIM = 0 AND NOIM = 1.

RETURNED BY A**1 (ONE, VICE VERSA) RETURNS NIOT

TIMES THE ORIGINAL DATA. IFORM = 1, 0 OR -1, AS DATA IS
COMPLEX IF THE FIRST HALF OF A COMPLEX ARRAY TRANSFORM
VALUES ARE RETURNED TO ARRAY DATA. THEY ARE COMPLEX, REAL OR
THE FIRST HALF OF A REAL ARRAY, AS IFORM = 1, -1 OR 0.

THE TRANSFORM OF A REAL ARRAY (IFORM = 0) DIMENSIONED N(1) BY N(2)
BY ... WILL BE RETURNED IN THE SAME ARRAY, NOW CONSIDERED TO
BE COMPLEX OF DIMENSIONS N(1)/2+1 BY N(2) BY ... WITH THE FOLLOWING
NOTE THAT IF
IFORM = 0 OR IFORM = 1, NUMERICAL VALUE OF THE TRANSFORM IS LIKELY TO BE
ZERO AND THE MISSING VALUES MAY BE OBTAINED BY COMPLEX CONJUGATION.
THE INVERSE TRANSFORMATION, OF A HALF COMPLEX ARRAY DIMEN-
SIZED N(1)/2+1 BY N(2) BY ..., IS ACCOMPLISHED BY SETTING IFORM TO -1.
IN THE J ARRAY, N(1) MUST BE THE TRUE N(1), NOT N(1)/2+1.
THE TRANSFORM WILL BE REAL AND RETURNED TO THE INPUT ARRAY.
FUNDAMENTAL PROPORTIONAL TO N Orbit(NOM) RATHER THAN
THE VALUE NORM**2. FURTHERMORE, LESS ERROR IS BUILT UP.
SUBROUTINE BITREV (DATA, NDREV, NREMN)
SHUFFLE THE DATA BY BIT REVERSAL.
DIMENSION DATA(NREV, NREMN)
COMPLEX DATA
EXCHANGE DATA (J1, J4REV, J3) WITH DATA(J1, J4, J5) FOR ALL J1 FROM 1
TO NDREV, ALL J4 FROM J1 TO J4REV - 1 (WHICH MUST BE A POWER OF TWO), AND ALL J5 FROM J4REV TO NREMN. J4REV - 1 DESIGNS THE BIT REVERSAL OF J4 - 1. E.G. 3241 BECOMES 11001, ETC.
DIMENSION DATA(J1)
J1 = 2
IP1=IP0*NF/EV
IP4=IP1*\n
IP5=IP4*NREV
I4REV=1
I4REV = 1*(I4REV-1)*IP1
DO 60 I4=1,IP4,IP1
DO 20 I3=1,I4-1,1
10 I1MAX=I4*IP1-IP0
DO 20 I1=1,I1MAX,1
DO 20 I2=1,J4-1,1
DC 20 J4=1,IP4
J5=1
J5=1*(J4-1)*IP5+(J4-1)*IP1+(J5-1)*IP4
I5REV=I4REV*15-I4
I5REV = 1*(J4-1)*IP5+(J4REV-1)*IP1+(J5-1)*IP4
TEMP=DATA(I5)
TEMP1=DATA(I5+1)
DATA(I5-I5REV) = DATA(I5REV+1)
DATA(I5REV) = TEMP
DATA(I5REV+1) = TEMP1
C ADD ONE WITH DOWNWARD CARRY TO THE HIGH ORDER BIT OF J4REV-1.
J4REV=I4REV-1
IP4=124/2
IP2=IP4*2
IF (I4REV-IP2) 60,63,50
50 I4REV=I4REV-IP2
IP2=IP2/2
IF (IP2-IP1) 60,40,40
00 I4REV=I4REV+IP2
RETURN
END

SUBROUTINE COOL2 (DATA,NPREV,N,NREV,ISIGN)

DISCRETE FOURIER TRANSFORM OF LENGTH N. IN-PLACE COOLEY-TUKEY
ALGORITHM. BIT-REVERSED IC NORMAL ORDER, SAME-TUKEY PHASE SHIFTS.
DIMENSION DATA(NPREV+1,NPREV+1)
COMPLEX DATA
DATA(J1,J2,J3,J4,J5) = SUM(DATA(J1,J2,J3,J4,J5) * EXP(ISIGN*2*PI*I*(J4-1)*
(J44-1)/N)) SUMMED OVER J4 = 1 TO N FOR ALL J1 FROM 1 TO NPREV.
J4 FROM 1 TO N AND J5 FROM 1 TO NPREV. N MUST BE A POWER OF TWO.
METHOD LET IPREV TAKE THE VALUES 1, 2 OR 4, 8 OR 16, ..., N/16.
THE CHOICE BETWEEN 2 OR 4, ETC., DEPENDS ON WHETHER N IS
A POWER OF FOUR. DEFINE IFAC = 2 OR 4, THE NEXT FACTOR THAT
IPREV MUST TAKE. AND INEX = N/(IFAC*IPREV). THEN--
01 MENSIGN DATA(NPREV,IPREV,IFAC,IPREV,NREV)
COMPLEX DATA
DATA(J1,J2,J3,J4,J5) = SUM(DATA(J1,J2,J3,J4,J5) * EXP(ISIGN*2*PI*I*
(J3-1)*IFAC*(J2-1)/(IFAC*IPREV)) SUMMED OVER J3 = 1
1 TO IFACT, J4 FROM 1 TO I4M AND J5 FROM 1 TO J5M. THIS IS
A PHASE-SHIFTED DISCRETE FOURIER TRANSFORM OF LENGTH IFACT.
FACTORIZATION N BY FOURS SAVES ABOUT TWENTY FIVE PERCENT OVER FACTOR-
ING BY TWOS. DATA MUST BE BIT-REVERSED INITIALLY.
IT IS NOT NECESSARY TO REWRITE THIS SUBROUTINE INTO COMPLEX
NOTATION SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
IMAGINARY PARTS IN ADJACENT STORAGE LOCATIONS. IT MUST ALSO
STORE ARRAYS WITH THE FIRST SUBSCRIPT INCREASING FASTEST.
DIMENSION DATA(1)
TMOPI=6.283195972*FLOAT(ISIGN)
IPQ=2
IP1=IP0*IPRFN
IP4=IP1*N
IP5=IP4*IPRFN
IP2=IP1
IP=IP1*IPROD
NPA2=N
10 IF (NPA2-2) 30,39,20
20 NPA2=NFACT/4
30 GO TO 10
40 IF (IP2-IP4) 40,169,160
40 12=IP2+2
12=IP2*IFACT
DO 50 I1=1,IP1,IP0
50 DC 50 I1=1,IP1,IP0
15 = I1*J1-1)*IP2+(J4-1)*IP3+(J5-1)*IP4
59 TEMPL=DATA(I3A)+TEMPR
DATA(I3J)=DATA(I3A)-TEMPR
DATA(I3J+1)=DATA(I3J+1)-TEMP
DATA(I3A)=DATA(I3J)+TEMPR
DATA(I3J+1)=DATA(I3J+1)+TEMP
59 IP2=IP2
C DO A FOURIER TRANSFORM OF LENGTH FOUR (FROM ETI REVERSED ORDER)
60 IF (IP2-IP4) 60,169,160
60 12=IP2+2
12=IP2*IFACT
C CONVITE THIS TETFIF TPEF WE AND WE IN DOUBLE PRECISION, IF AVAILABLE.
12=DATA(I3A)+IP2
DATA(I3J)=DATA(I3A)-IP2
DATA(I3J+1)=DATA(I3J+1)-IP2
DATA(I3A)=DATA(I3J)+IP2
DATA(I3J+1)=DATA(I3J+1)+IP2
60 IP=IP2+2
70 IF (IP2-IP4) 70,169,160
70 12=IP2+2
12=IP2*IFACT
C
SUBROUTINE FIXRL (DATA, N, NORM, SIGN, IFORM)

FOR IFORM = 0, CONVERT THE TRANSFORM OF A DOUBLED-UP REAL ARRAY,

CONSIDERED COMPLEX, INTO ITS TRUE TRANSFORM. SUPPLY ONLY THE
FIRST HALF OF THE COMPLEX TRANSFORM, AS THE SECOND HALF HAS
CONJUGATE SYMMETRY. FOR IFORM = -1, CONVERT THE FIRST HALF
OF THE TRUE TRANSFORM INTO THE TRANSFORM OF A DOUBLED-UP REAL
ARRAY. N MUST BE EVEN.

USING COMPLEX NOTATION AND SUBSCRIPTS STARTING AT ZERO, THE
TRANSFORMATION IS

DIMENSION DATA(N, NORM)
ZSIP = EXP(ISIGN*2*PI*1/N)
DO 10 I=0, NORM-1
DATA (J*2) = CONJ(DATA (J*2)) *(1+1)
10 CONTINUE
Z = (1+ (2*IFORM+1) * ZSIP**11) / 2
1ICNJ = N/2-I
DIP = DATA(I, I2) - CONJ (DATA (1+ICNJ, I2))
TEMP = Z*DIP
DAPA(I, I2) = (DATA(I, I2) - TEMP) *(1-IFORM)
DAPA(I+ICNJ, I2) = DATA(I+ICNJ, I2) + CONJ(TEMP) *(1-IFORM)

IF IT = ICNJ, THE CALCULATION FOR THAT VALUE COLLAPSES INTO
A SIMPLE CONJUGATION OF DATA (I, I2).

DIMENSION DATA(1)
THEPL = (2.3) ** 0.5
IJO = 2
IP0 = IJP0 *(N/2)
IP1 = IP0 *(N/2)
IF (IFORM) EQ 19, 70, 70
PACK THE REAL INPUT VALUES (TWO PER COLUMN)
10 J1=IP1+I
DATA(2) = DATA (J1)
IF (N/2 - 1) EQ 73, 73, 23
20 J1=J1+IPE
IPE=IP1+1
DATA(J1) = DATA (J1)
IF (N/2 - 1) EQ 24, 23, 21
DATA(J1) = DATA (J1)
J1=J1+1
IF (I-2) EQ 30, 30, 1
DO 30 J2=IP1+1
11 I1=I1*10
30 I1=I1+1 I2=I2-10
RETURN
END
DATA(11) = DATA(J1)
DATA(11+1) = DATA(J1+1)
J1 = J1 + 1:
DATA(I2+1) = DATA(J1)
I2 = I2 + 1, IP2, IP1
DATA(I2) = DATA(I2) + DATA(I2+1)
DATA(I2+1) = DATA(12*PR-DATA(I2+1))
IF (N=2) 200, 200, 90
THETA = TWOPI/FLOAT(N)
SINH = SIN(THETA/2.),
ZSTFR = 2.*SINH*SINH
ZSTPI = SIN(THETA)
ZP = (1.-ZSTET)/2.
ZI = (1.-ZSTFR)/2.
IF (IPORM) 190, 110, 11C
ZB = 1.-ZF
ZI = ZI
I10 = I20+1
I1MAX = I10*(N/4)+1
DC 190 I1 = I1MIN,11MAX,IP0
DO 180 L2 = 1, IP2, IP1
I2CNJ = I20*(N/2+1)-2*I10+I2
IF (I2-I2CNJ) 150, 120, 120
12J DATA(I2+1) = DATA(I2+1)
130 IF (IPORM) 170, 140, 180
190 DIFR = DATA(I2) - DATA(I2CNJ)
DIF = DATA(I2+1) + DATA(I2CNJ+1)
TEMPER = DIFR*ZP - DIFI*ZI
TEMP = DIFI*Z1 + DIFI*ZR
DATA(I2) = DATA(I2) - TEMF
DATA(I2+1) = DATA(I2+1) - TEMF
DATA(I2CNJ) = DATA(I2CNJ) + TEMF
DATA(I2CNJ+1) = DATA(I2CNJ+1) - TEMF
IF (IPORM) 160, 180, 180
16C DATA(I2CNJ) = DATA(I2CNJ) + DATA(I2CNJ)
DATA(I2CNJ+1) = DATA(I2CNJ+1) + DATA(I2CNJ+1)
170 DATA(I2) = DATA(I2)+DATA(I2)
DATA(I2+1) = DATA(I2+1)+DATA(I2+1)
180 CONTINUE
190 ZF = ZSTFR*TEMFR*ZSTPI*Z1*ZF
Z1 = ZSTFR*Z1, ZSTPI*TEMFR*Z1
C PRECISION SAVES 11 digits, AT A SLIGHT LOSS IN ACCURACY. IF AVAILABLE, TESS05750
C USE OCTALEL PRECISION TO COMPUTE Z2 AND ZI.
200 IF (I.FORM) 270, 210, 210
210 I2=IP2+1
I1=I2
J1=IPO*(N/2)*IPM1
SC TC 250
220 DATA(J1)=DATA(J1)
DATA(J1+1)=DATA(J1+1)
I1=I1-IE0
J1=J1-IE0
230 IF (L2-31) 220, 240, 240
240 DATA(J1)=DATA(J1)
DATA(J1+1)=0.
250 I2=I2-IE1
J1=J1-IE1
DATA(J1)=DATA(J1)
DATA(J1)=DATA(J1+1)
I1=I1-IE0
J1=J1-IE0
260 IF (L2-31 260, 260, 230)
270 RETURN
END

SUBROUTINE PLOTJ (DATA, DR, IAX, JMAX, HO)

******************************************************************************

THIS SUBROUTINE PLOTS THE [2(X)] VS A CURVE, STARTING AT THE SOURCE AND MOVING IN A POSITIVE DIRECTION OF RANGE.

******************************************************************************

DIMENSION DATA(4096), PJ0D(2048), R(2048)

LMAX = I1I*(JMAX/5.) + 1
MAX = JMAX - 1
JMAX = JMAX - 2

DO 30 J = 1, JMAX
FLY = PJ0D(I)
I{1} = HO + FLY * DR
P(JMAM-1) = -F(JMAX+1)
CONTINUE 30

PMIN = F0
PIMAX = F(JMAX)
WALL = (6, 100)
100 FORMAT (/3X,'I1,6X,'RANGE',7X,'REALP',7X,'IMAGP',5X,'MODULUS OF F')

DC 10 I = 1, JMAX

P(M1) = SORT((DATA(2*I-1))**2 + (DATA(2*I))**2)

WRITE(6,200) F(I), DATA(2*I-1), DATA(2*I), PHAD(I)

CONTINUE

120 FORMAT (2X,14,2X,F10.2,3(2X,F10.7))

P/MAX = -1.0

DO 20 I = 1, JMAX

IF (P(M1(I)) .GE. P/MAX) P/MAX = P(M1(I))

CONTINUE

20 CALL COMRES

CALL AKRZA2D(4.0, 0.0)

CALL XNAME('RANGE', 4) / 100

CALL INAME('MAGNITUDE OF PRESSURE', 2, 'P', 0, 100)

CALL LINESE(0.5)

CALL HEADIR('MAGNITUDE OF PRESSURE AS A FUNCTION OF RANGE $x$, 100, 1, 0, 1)

CALL HCCHK

CALL GRAF(RMIN, 'SCALE', RMAX, 0.0, 'SCALE', P/MAX)

CALL CURVE(F, PHAD, JMAX, IMARK)

CALL EHFL(1)

RETURN

END

SUBROUTINE STHELI(IMAX,GM,CRN,W,AK,BE)

******************************************************************************

***** THIS SUBROUTINE PLOTS THE ACOUSTIC PRESSURE SPECTRUM AS A FUNCTION

OF THE HORIZONTAL GAGE NUMBER

******************************************************************************

DIMENSION GM(4096), CRN(4096), W(4096), BE(4096)

IMARK = INT(IMAX/5.) + 1

GMIN = GM(1)

GMAX = AK

P/MAX = -1.0

W/MAX = 1.0

DO 100 I = 1, IMAX

IF (CRN(I) .GE. P/MAX) P/MAX = CRN(I)

CONTINUE

100 DO 15 I = 1, IMAX

CONTINUE

END
IF (C(I).GT.0.0) QMAX = Q(I)
       CONTINUE
C
IF (Pimax.GE.Qmax) QMAX = Pmax
      SMAX = Qmax

******************************************************************************

GMAX IS THE MAXIMUM VALUE KH CAN TAKE ON BASED ON THE EQUATION

((DELTA K) * (DELTA R)) = (2 * PI/IMAX)

GMAX IS THE MAXIMUM VALUE KH CAN TAKE ON AND STILL PRODUCE "REAL"
THEORETICAL Q AND ETA VALUES.  KI < SQRT(AK**2 + KH**2) => Q REAL.

SCALE THE HORIZONTAL AXIS IN KH (GM)

******************************************************************************

CALL COMPRS
CALL AREA2D(4.0,6.0)
CALL XNAME('HORIZONTAL WAVE NUMBER, 1/MI',100)
CALL YNAME('PRESSURE SPECTRUM MAGNITUDES',100)
CALL LINESP(0.5)
CALL HEADIN('PRESSURE SPECTRUM VS KHs',100,1.0,1)
CALL NCHECK
CALL GRAPH(GMIN,'SCALE',GMAX,0.0,'SCALE',SMAX)
CALL CURVE(GMIN,CKM,IMAX,IMARK)
CALL POOL
CALL CURVE(GA,0,IMAX,IMARK)
CALL ERROR('POOL')
CALL ENDPL(2)
       LEAVE
       END

SUBROUTINE WVLPL(IMAX,CKM,C,AK,EE)
******************************************************************************

THIS SUBROUTINE PICKS THE PRESSURE SPECTRUM AS A FUNCTION OF THE
VERTICAL WAVE NUMBER (EE).
******************************************************************************

DIMENSION CKM(4096),C(4096),EF(4096)

IMAX = IMAX/5.) + 1
QMAX = EF(1)
QMIN = EF(IMAX)
PMAX = -1.0
QMAX = -1.0
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