Power Density and Threshold Control Strategies for Radar Track Initiation

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This report describes a strategy for controlling the power density and thresholds on a pulse-to-pulse basis on long range surveillance radars having fixed power aperture products for the purpose of improving a target track initiation range. By using a mathematical model and Monte-Carlo simulations, it was found that the track initiation range could be significantly increased for the specific cases studied. The improvements varied from about 20% to 50% depending on the strategies and conditions. An argument was constructed that showed that these improved track initiation ranges were obtained without significant degradation in radar performance for other targets as long as the target density was not too large.
Power Density and Threshold Control Strategies for Radar Track Initiation

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POWER DENSITY AND THRESHOLD CONTROL STRATEGIES FOR RADAR TRACK INITIATION

INTRODUCTION

In the past, many radars operated in fixed modes or modes selected from a small menu by the operator. The radar's operation did not depend, to a large extent, on the measured environment. However, over the last few years studies have been directed at dynamically improving the radar operation based on radar measurements, as for example:

(a) by controlling the radars pulse bursts and filters based on measured clutter characteristics [1],
and
(b) by controlling the selected frequency based on signal-to-noise ratios of prior pulses and jamming spectral characteristics [2].

Furthermore, electronic agile radars have managed energy based on dwell time and have used look back for quick track initiation. In all these cases, the desired result is to enhance the radar capability using the same resources by better matching it to the dynamically changing environment. The objective of this study was to improve the track initiation range of a long range mechanically azimuth scanned surveillance radar with fixed power aperture product. Similar to the previous studies discussed, the means of achieving this goal depended on measurements and modification of the radar characteristics. Specifically, the power density and thresholds are dynamically controlled based on prior pulse echoes while the antenna size and transmitter power remain fixed.

The basic system concept is described first, then a mathematical model of the system is developed, and a detailed description of the control strategies is defined within this model. Finally, the system performance is evaluated by using a Monte Carlo simulation, and the results are presented and discussed.

BASIC CONCEPTS

The basic system concept is illustrated in Fig. 1. During normal operation, the radar transmits a fan beam or divides power equally among multiple beams on a single pulse and then receives signals on multiple narrow beams. The radar is also constructed to allow, upon designation, all the energy to be transmitted through one of the narrow beams. Multiple beam frequency scanned radars can achieve this by merely transmitting all the pulse energy on one frequency channel while other radars may require more complex distribution networks. The net effect is that, upon designation, the power density can be increased significantly on a pulse in one region of space at the expense of other regions of space. Since there are multiple pulses during the time the fan beam sweeps over a point in space, the spatial power density can be varied considerably. One object is to control the power density by choosing either a fan or pencil transmit beam on a pulse-by-pulse basis. The other controlled radar parameter is the threshold used in the detector. At least in a thermal noise environment, the threshold can be lowered over small regions of space without significantly altering the overall false alarm count per scan. The second object is to choose one of two thresholds over small regions of space through strategy and prior radar measurements.

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The track initiation criteria is simple, one which has been used for a long time. Simply, if \( m \) uncorrelated detections occur over \( n \) scans in a nearly straight line, a track is initiated. The objective of our control strategy is to control the radar’s power density and threshold to quickly obtain the necessary \( m \) out of \( n \) detection track confirmation without significantly altering the radar’s overall performance such as sensitivity and false track generation. In this study \( m \) and \( n \) were set to 3 and 5 respectively.

One control strategy under consideration is called "Single Scan Track Initiation Strategy," while the other is "Multiple Scan Track Initiation Strategy." The first uses the same detections per scan as the track initiation system previously described, while the latter uses detections obtained from integrating the signals over multiple scans and accounts for target motion [3-5]. The remaining procedure described is the same for both strategies. A potential track is initiated from any new uncorrelated detection. After each scan, the number of detections which are associated with a potential track over the last 5 scans are counted. If there are one or more detections, the presence of a potential track is defined for the next scan whose beam location is the same as the one of the last detection. Again if any new uncorrelated detection is obtained, a potential track is initiated and the same procedure is followed. As the radar beam sweeps over a potential target on the next scan, for some of the pulses available all of the power is transmitted in the beam containing the potential target and/or the threshold is lowered in a range interval containing the potential target. This process continues on a potential target until either a track is initiated, which removes the potential target, or there is no longer a potential target present, which is a zero out of five criteria.

Undoubtedly there are many different objectives and associated strategies that could be used in operating a flexible radar as described. The reason we chose the objective of improving the track initiation range and strategies of controlling the power density and thresholds is as follows. Whenever a target enters a radar air space, its signal is usually quite weak. Even though the probability of detection is low, occasionally it can be detected but not often enough to be confirmed as a target. However, at least
in a low target density environment, a weak target can be tracked after it is confirmed as a real target. Consequently, our objective is to confirm the presence of a new weak target by using power density and threshold control and place it on track. Specifically, after a weak target is detected, the potential target is quickly confirmed by controlling the radar. If the target density is not very high (as usually is the case), and if the radar is only modified over small angular sectors and range intervals per scan, the overall radar performance should be approximately the same as it is when no modification occurred. Furthermore, the location of the modifications changes every few scans.

SYSTEM MODEL

Since an experimental system was unavailable, the system’s performance is evaluated by using a simulation. Before describing the mathematical model of the system, a number of simplifying assumptions are required to keep the simulation easy to construct and evaluate, but at the same time to yield reasonably reliable results. First, the uncorrelated target density is assumed to be low enough so that the probability of two or more weak targets appearing over the same small angular sector and over a small window in time is very low. Consequently, only a single target is used. Secondly, the false alarm rate is set low enough so that we can virtually ignore an occasional false potential target which will occur in a small region over a short time. Furthermore, even though the threshold is being lowered surrounding a false potential target, the probability of confirming the false target is still low. The effect of lowering the threshold over a range interval surrounding a real potential target is to yield a slightly higher probability of confirming a track than if only the range cell containing the target is used. Consequently, with little error, only the range cell containing the single real target needs to be considered.

Of course under locally heavy target densities, the strategies presented are not adequate, since the above simplifying assumptions are not valid. The complexities required for controlling and evaluating the system performance, in this case, are beyond the scope of this study. However, there are many cases, which are thought to be the most probable ones, where the controlling strategies and simplifying assumptions are valid.

Signal Generation

Because of the simplifying assumptions, only the echo signal at the range sample containing a single target needs to be generated for the analysis. A single target flies a radial course toward the radar. The complex baseband signal received, \( R_{ij} \), sampled at the range of the target while the beam is over the target, is given by

\[
R_{ij} = S_{ij} + N_{ij},
\]

for

\[
i = 1, 2, \ldots, p
\]

\[
j = 1, 2, \ldots
\]

where \( S_{ij} \) and \( N_{ij} \) are the complex numbers representing signal and noise for the \( i \)th pulse and \( j \)th scan. There are \( p \) pulses while the beam is over the target. Both the noise and signal samples are assumed to be zero mean Gaussian distributed independent complex samples with standard deviations of \( \sigma_{N} \) and \( \sigma_{S} \) respectively in both the real and the imaginary parts which are uncorrelated. This standard signal model provides a Rayleigh fluctuating target from pulse to pulse (Swerling Case 2) embedded in additive thermal noise. The signal to noise ratio \( (S/N)_{ij} \) on the \( i \)th pulse and \( j \)th scan is then given by

\[
(S/N)_{ij} = \frac{\sigma_{S}^2}{\sigma_{N}^2}.
\]
The signal level changes as the target flies the radial course, and it is given by

\[ \sigma_{ij}^2 = k_i / r_j^4 \]

where \( r_j \) is the range of target from the radar. The proportionality constant \( k_i \) is

\[ k_i = K \quad \text{for } i = 1, 2, \ldots, q \]

and

\[ k_i = L K \quad \text{for } i = q + 1, \ldots, p \]

where \( q \) is the number of pulses that occur when the energy is transmitted through the fan beam, \( p - q \) is the number of pulses that occur when the energy is transmitted through the designated narrow beam, and \( L \) is the number of beams. In this study \( K \) is set equal to

\[ K = 5 \times (35 \text{ nm})^4. \]

and \( L \) is set to six.

The value of \( K \) corresponds to a long range surveillance radar that will detect a \( 10^{-3} \text{ m}^2 \) target at about 35 nm. We set \( p = 4 \) and \( q = 2 \) in cases where the power density strategy is applied and \( p = 4 \) and \( q = 4 \) in other cases which include the normal operating cases. The target range \( r_j \) on the \( j \)th scan is given by

\[ r_j = r_0 - v t (j - 1) \]

where \( r_0 \) is the initial range of 80 nm, \( v \) is the radial velocity in knots, and \( t \) is the time between scans measured in hours.

Integration and Threshold

Two integrators are used in this study. The first integrates \( p \) pulses across a beam and is given by

\[ y_j = \sum_{l=1}^{p} |R_{ij}|^2. \]

The second integrates pulses across a beam followed by integrating over \( g \) scans while compensating for the range motion. This is written as,

\[ z_j = \sum_{l=1}^{g} y_{j-l+1} \text{ or } \]

or by

\[ z_j = \sum_{l=1}^{g} \sum_{l=1}^{p} |R_{i(l-l+1)}|^2. \]

In this study, \( g \) is set to be equal to 8. There are three thresholds; high and low thresholds denoted by \( T_H \) and \( T_L \) to which the signal \( y_j \) is compared, and a threshold \( T_M \) to which the signal \( z_j \) is compared. The thresholds \( T_H \) and \( T_M \) are both set to yield a probability of false alarm of \( 10^{-6} \). By using the tables in reference [6], \( T_H \) is set to 42.7 \( \sigma_i^2 \) based on \( p = 4 \) pulses and \( T_M \) is set to 132.8 \( \sigma_i^2 \) based on \( p \times g = 4 \times 8 = 32 \) pulses. The threshold \( T_L \) is discussed under the control strategy section.

Track Initiation Criteria

For all strategies, a new track is said to be initiated if

\[ d = \sum_{k=1}^{n} \delta_{j-k+1} \geq m. \]
otherwise no track is initiated. In our study a \( m = 3 \) out of \( n = 5 \) track initiation criteria is used. The quantity \( \delta_j \) is defined by

\[
\delta_j = \begin{cases} 
1 & \text{if } y_j > T \\
0 & \text{if } y_j < T 
\end{cases}
\]

where \( T \) is either \( T_H \) or \( T_L \) depending on the control strategy for the \( j \)th scan. The lower threshold is set so that the probability of \( d \geq m \) is \( 10^{-6} \) in thermal noise. This requires a probability of \( \delta_j \) exceeding the threshold on one scan of .0046. By using the tables in Ref. 6, the lower threshold \( T_L \) is equal to \( 27.1 \sigma_{\bar{T}} \). Even though the threshold is lowered in the vicinity of a potential target, the false track initiation would be quite small. This threshold is slightly low if the velocity is not known.

**Control Strategy**

There are two basic control strategies based on single scan detections and long term multiple scan integration. Furthermore, either the power density, the threshold, or both are controlled for each strategy. The normal operation, however, does not modify the power density or threshold.

**Single Scan**

If a new potential target is generated by a new detection, then

\[
\sum_{k=1}^{n} \delta_{j-k+1} = 1
\]

and the controls are applied on the next scan. The single scan control strategy applies the increased power density, lower threshold, or both on the next scan on the potential target if

\[
\sum_{k=1}^{n} \delta_{j-k+1} > 0
\]

on the current scan. If

\[
\sum_{k=1}^{n} \delta_{j-k+1} = 0
\]

then the potential target is dropped and the radar returns to its normal operation until a new potential target is generated later.

**Multiple Scan**

If a new potential target is generated by a new detection, then

\[
\sum_{k=1}^{n} \psi_{j-k+1} = 1
\]

and the controls are applied on the next scan. The quantity \( \psi_j \) is computed by

\[
\psi_j = \begin{cases} 
1 & \text{if } z_j > T_M \\
0 & \text{if } z_j < T_M 
\end{cases}
\]

The multiple scan control strategy applies the increased power density, lower threshold or both on the next scan on the potential target if

\[
\sum_{k=1}^{n} \psi_{j-k+1} > 0
\]
on the current scan. If

\[ \sum_{k=1}^{n} \psi_{j-k+1} = 0 \]

then the potential target is dropped and the radar returns to its normal operation.

Observe that the single scan and multiple scan track initiation strategy are almost identical except the first uses only the detections from a single scan integration while the latter uses detections from a single scan integration followed by a multiple scan integration.

**SIMULATION RESULTS**

A Monte Carlo simulation was conducted by using the system model described. A target was moved along a radial path starting from a distance of \( r_0 = 80 \text{ nm} \). The simulation continued until a track was initiated. At this point, the target was placed again at \( r_0 = 80 \text{ nm} \), and the process was repeated 1000 times. A probability density or normalized histogram of the track initiation ranges was computed. In Fig. 2 is shown a typical example of a target with a radial velocity of 450 knots. The multiple scan control strategy was used, and both the power density and threshold were controlled. By observing Fig. 2 is noted that a significant improvement in track initiation range was obtained with the controlled system over normal operation. In addition the controlled system exhibited more spread in the track initiations ranges.

![Fig. 2 - Probability density of detection range for the radar using normal operations and using one of the control strategies](image)

The mean and standard deviations for a number of situations were computed from the probability densities of track initiation range. Figure 3 contains plots of the mean, and Table 1 contains both the mean and standard deviations. Using three velocities of 112.5 knots, 450 knots, and 900 knots, both the single scan and multiple scan track initiation strategies were evaluated while controlling either
Fig. 3 — Average track initiation ranges versus radial velocity for a number of control strategies

Table 1 — Mean and Standard Deviations of Track Initiation Range as a Function of Radial Velocity and Control Strategy

<table>
<thead>
<tr>
<th>Velocity</th>
<th>( \mu_R = +112.5 ) knots</th>
<th>( \mu_R = +450 ) knots</th>
<th>( \mu_R = +900 ) knots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m_R ) (nm)</td>
<td>( \sigma_R ) (nm)</td>
<td>( m_R ) (nm)</td>
</tr>
<tr>
<td>Normal Operation</td>
<td>41.0</td>
<td>2.3</td>
<td>37.8</td>
</tr>
<tr>
<td>Single Scan Strategy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Threshold Control</td>
<td>48.6</td>
<td>2.9</td>
<td>43.8</td>
</tr>
<tr>
<td>• Power Density Control</td>
<td>49.4</td>
<td>3.2</td>
<td>44.4</td>
</tr>
<tr>
<td>• Both</td>
<td>51.0</td>
<td>4.4</td>
<td>45.2</td>
</tr>
<tr>
<td>Multiple Scan Strategy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Threshold Control</td>
<td>52.4</td>
<td>2.8</td>
<td>47.5</td>
</tr>
<tr>
<td>• Power Density Control</td>
<td>56.1</td>
<td>2.6</td>
<td>50.5</td>
</tr>
<tr>
<td>• Both</td>
<td>59.7</td>
<td>4.4</td>
<td>54.0</td>
</tr>
</tbody>
</table>
power density and thresholds or both. For comparison, a normal operating system was used as reference. The results show:

(a) lower speed targets have largest initiation ranges,
(b) the multiple scan strategy was better than the single scan strategy,
(c) the power density control was better than lowering the threshold,
(d) using power density control and lower thresholds together was better than using each individually, and
(e) all the control strategies were better than the normal operation.

Before discussing the results further, two other statistics were computed from the same Monte Carlo trials and are presented in Figs. 4 and 5. The average number of potential tracks per target vs target radial velocity for all control strategies is shown in Fig. 4. For example, if there were two potential tracks for one target; it would mean that one potential target was started but dropped, then a second one was started and then confirmed as a track. Figure 5 shows the average number of scans required to confirm the final or last potential target as a true target by the track initiation criteria vs target radial velocities for all the control strategies. Using the previous example on the second potential target that was confirmed as a real target, the number of scans from the time the second potential target was started to the time it was confirmed was counted to yield the desired result for one trial. By observing Figs. 4 and 5, it was found that tracks were initiated very rapidly after the first initial detection by using all cases for the single scan strategy and for both power density and threshold control for the multiple scan strategy. The rapid track initiation is shown in Figs. 4 and 5 by noting that most of the time only one potential track was generated, and that it was confirmed as a real track under the track initiation criteria in a small number of scans. Under the multiple scan strategy and using the lower threshold only, it was found that a significant number of potential tracks were generated at the lower velocities, but the last potential track was confirmed in a few scans. Also under the multiple scan strategy, but using only the power density control, it was found that few potential targets were generated at slow speeds, but it took a significant number of scans to confirm a real track.

![Fig. 4 - Average number of potential tracks per target versus radial velocity for a number of control strategies](image-url)
From these results, some general conclusions can be derived. First, the tracks could be initiated further out in range than normally, because detections on weak targets far out in range occasionally occur, but do not occur often enough under normal operations to initiate a track. Using control strategies for the same situation, the track usually can be initiated rather quickly after an initial detection, thus increasing the track initiation range. The improvement in track initiation range is limited to the range of the first detection, when the single scan strategy is used, because after it is received, the track is usually quickly confirmed. This initial detection range can be extended farther in range by using multiple scan signal integration to enhance the signals. Consequently by using the multiple scan initiation strategy, the track initiation range is increased over the one that uses the single scan strategy. When the range of the first detection is significantly increased, as it was in the multiple scan strategy, the effective signal-to-noise ratio under the control strategy must be increased enough so that the track can be confirmed quickly. If the signal-to-noise ratio is not increased sufficiently, a significant amount of time is consumed in the new mode before the track can be confirmed. By observing the results, it was found that not enough effective signal-to-noise ratio was available from either the power density control or from the threshold control to provide a quick confirmation, but if used together there was enough effective signal-to-noise ratio to provide quick confirmation. In addition, the average track initiation range is higher for the slower velocity targets because there are more opportunities to make an initial detection.

The degradation that occurs for targets at the same azimuth but at different elevation angles, as the target being operated on in the system described is 3 dB. This occurred because half of the pulses were transmitted normally and the other half through only one of the multiple beams. This should have little effect on those targets already in track because often the targets of interest (inbound) have extra signal-to-noise ratio available. Even if there are several weak targets at the same azimuth but at different elevation, all should be placed in track sequentially, because once an initial detection occurs, usually doesn’t take long to confirm it as a track. Most of the space does not contain a new target at just about the initial detection range, consequently the control strategy has no effect over large regions.
of space. Therefore at least in a low target density environment, the control strategies should have little effect on targets other than the designated targets to which controls are being applied.

SUMMARY

A means of improving the radar track initiation range of fixed power aperture product long range surveillance radar was investigated. Specifically, the power density and threshold were controlled over small regions of space based on events which occurred on previous scans. To study the system, a number of simplifying assumptions were made; a mathematical model was developed and a Monte-Carlo simulation was conducted. A set of specific control strategies and system parameters were defined for this mathematically modeled system. Finally several statistics, which indicated system performance, were obtained from the simulation and were presented.

The simulation results indicated that the track initiation ranges that used the control strategies could be significantly improved for normal operations. The important factors were:

a. Obtaining an initial detection on a weak target far out in range and
b. Applying enough resources to confirm the track in a short time.

If this is accomplished, it appears that track initiation ranges can significantly be improved in low target density regions without significantly degrading the overall system performance.

REFERENCES


