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ASPECTS OF SAR IMAGE SEGMENTATION

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SUMMARY

An expression for the speckle structure seen on Synthetic Aperture Radar (SAR) images is derived. This model is used to investigate the applicability of the simple segmentation operators usually used for image analysis to the problem of the segmentation of SAR images. These simple operators are shown to be incapable of segmenting raw SAR images satisfactorily. However they can be applied successfully to area averaged SAR images provided certain constraints are met. Possible segmentation approaches are examined in the light of these constraints.
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1 INTRODUCTION

Synthetic Aperture Radar systems yield high resolution images which cover large areas of terrain. For operational effectiveness this large quantity of data needs to be analysed and interpreted automatically. A vital step in this process is the segmentation of the images into their constituent regions. This memo describes a theoretical study into the operation of some common segmentation operators on SAR images, carried out in the Sensor information processing section of BS1 division.

A previous study\(^1\) showed that a good candidate model for the human visual system performed poorly on an image which obeyed the speckle statistics characteristic of SAR images. Here we examine in greater detail the reasons why segmentation techniques based on applying thresholds to the image intensity or its spatial gradient perform so poorly when applied to SAR images. We shall examine both the problem of segmenting large regions which differ only slightly in their average intensity, and the problem of detecting brighter objects on uniform underlying backgrounds. In order to do this we shall first show the physical cause of the speckle phenomenon and the different statistical regime that we have to consider compared to ordinary incoherent imaging. Then we shall derive analytically the probability distributions which result from applying the simple edge detector operators to speckle images. Next we study the effect of image averaging and deduce limits on the detectability of a region as a function of both its area and the ratio of its intensity to its neighbours. Finally we discuss possible ways of segmenting SAR images in the light of these results.

2 SAR SPECKLE

One of the first things a newcomer to SAR images notices is the granular structure of all parts of the images. Whilst he may be used to seeing "noise" on the fainter parts of photographs of scenes illuminated by ordinary (ie incoherent)
visible or infra-red radiation, he is not used to seeing the brighter regions also being "noise" dominated on small scale sizes. Nevertheless this noise structure does not prevent him from extracting the details like roads, field boundaries etc from the images.

If a SAR image is studied in more detail then the observer will notice that, at any point, the magnitude of the noise is proportional to the average intensity at that point. Since this multiplicative aspect of the noise is different from the more frequently encountered additive noise it is worthwhile to explain briefly why it arises and what statistical form it takes.

Let us illuminate a scene with near-monochromatic radiation from a source such as a laser or stable radio frequency oscillator and then sample the scattered radiation. To simplify matters let the scene be a collection of \( N \) discrete, randomly distributed, scatterers.

The amplitude of the scattered field measured at the sample point is then the sum of the contribution from each scatterer.

\[
A = \sum_{k=1}^{N} a_k e^{i \phi_k}
\]

The intensity, \( I = A^* A \) where \( A^* \) is the complex conjugate of \( A \). It is shown in \([2,3]\) that for \( N \to \infty \) the resulting intensity distribution is the negative exponential

\[
P(I) dI = \frac{1}{\langle I \rangle} e^{- \frac{I}{\langle I \rangle}} dI
\]

where \( \langle I \rangle \) is the mean intensity. This distribution has the property that its variance is equal to its mean value, giving rise to the multiplicative nature of the noise. This is because the mth moment has been shown \([4]\) to be related to the 1st moment as \( \langle I^m \rangle = m! \langle I \rangle^m \) which coupled with the standard relation \( \text{VAR} = \langle I^2 \rangle - \langle I \rangle^2 \) gives \( \text{VAR} = \langle I \rangle^2 \).

If now we consider the case where the radiation has a wide range of wavelengths present, the white light situation, then the contribution to the instantaneous scattered field from each part of the spectrum is independent resulting in a mean intensity proportional to the average reflectivity. However this time the fluctuations about the mean tend to zero for an increasing wavelength range and finite observation time.

2 SEGMENTATION ON A PIXEL BY PIXEL BASIS

This difference between the statistics of the picture elements (pixels) of an image generated using near-monochromatic illumination and the statistics of white light images has important consequences for current segmentation methodologies. This is best shown by an example.

Consider Fig 1a; here there is a simple scene which consists of two regions of differing but uniform reflectivity.
Fig 1b shows a cut through the scene plotting the reflectivity as a function of distance. Fig 1c shows the underlying pixel intensity probability distribution for the scene. Clearly the two regions could be segmented perfectly using a simple threshold. If the scene is imaged using white light then the resulting pixel probability distribution will be as shown in Fig 2. Now we see that the two regions are no longer completely distinct, however it is still possible to assign pixels with little error by choosing an appropriate threshold. The theory of statistical pattern recognition shows that optimum discrimination will be had by placing the threshold at the point where the a posteriori probabilities are equal\(^5\). There is of course a region around the threshold where, because of the overlap, unambiguous region assignment is not possible. It follows that providing the underlying reflectivity changes are greater than the noise level then it is possible to segment the image.

Let us consider now imaging the same scene using near-monochromatic light. The pixel intensities are negative exponentially distributed as discussed above. Fig 3 shows the resulting probability distributions for a difference in underlying reflectivity of a factor of two. Structures having such an intensity ratio are easily segmented by eye.

We observe that there is considerable overlap of possible pixel intensities for the two regions. Indeed all possible pixel values for the weaker reflecting region are encompassed by those of the stronger region, hence it would not be possible to decide that there was a weaker region present merely on the basis of a single pixel value. By inspection of Fig 3 it is apparent that even when a threshold is placed at the optimum level there will be a large number of region mis-assignments due to the overlap.

This can be put in a quantitative manner. Let us assume initially that the a priori probability distribution for the regions is not known; ie we do not have prior knowledge of the sizes of the regions in the image. Accordingly we shall make the (usual) assumption that all regions are equally probable. Therefore we can replace the a posteriori probabilities with the conditional probabilities; remember, however, that we do know that there are only two underlying, uniform, regions present. Hence the equal probability condition for the magnitude of the threshold, \(T\), is

\[
\frac{1}{M_1} e^{-\frac{T}{M_1}} = \frac{1}{M_2} e^{-\frac{T}{M_2}}
\]

Rearranging we have

\[
T = \frac{M_1 M_2}{M_2 - M_1} \ln\left(\frac{M_2}{M_1}\right)
\]

Note that when \(M_2 = M_1\) \(T\) can take any value since the two regions are indistinguishable yet the expression above tends to \(M_1\) as \(M_2 \rightarrow M_1\). Now the probability that a pixel drawn from a region of mean intensity \(M\) will have a value greater than a threshold \(T\) is

\[
\int_T^{\infty} \frac{1}{M} e^{-\frac{I}{M}} dI = e^{-\frac{T}{M}}
\]
Hence the probability that a pixel from region 1 will be correctly assigned to region 1 is

$$1 - \exp(-\frac{T}{M_1})$$

while the probability that a pixel from region 2 will be correctly assigned to region 2 is

$$\exp(-\frac{T}{M_2})$$

It is of interest to plot the values for these expressions as a function of the underlying intensity ratio of the two regions as they show the BEST possible segmentation performance that can be achieved for our level of prior knowledge.

Curve A in Fig 4 shows the correct assignment rate for the (fainter) region 1 for the intensity ratio range 1 to 10. It can be seen that for large ratios most, ≈ 90%, of the pixels of the fainter region are correctly assigned.

Curve B shows the correct assignment rate for the brighter region. Here we notice that the correct assignment rate is lower. This means that whereas most of the fainter pixels are correctly assigned those from the brighter region are more likely to be mis-assigned.

The total mis-classification rate or error is given by the sum

$$P(\text{Error}) = \sum p(\text{error} | x) P(x)$$

where $p(\text{error} | x) = 1 - p(\text{correct classification} | x)$ and $P(x)$ is the a priori probability of occurrence of state $x$. For our case we have $P(\text{region } 1) = P(\text{region } 2) = 0.5$, $p(\text{error} | 1) = \exp(-\frac{T}{M_1})$ and $p(\text{error} | 2) = 1 - \exp(-\frac{T}{M_2})$. Fig 5 shows $P(\text{Error})$ plotted as a function of intensity ratio. We notice that for an intensity ratio of 1 there is a 50% probability of error which is of course what is to be expected when there is no distinction between the regions. However as the intensity ratio increases we note that the probability of error does NOT fall to an insignificant amount. A study of Fig 4 shows that the main contribution to the error comes from pixels from region 2, the underlyingly brighter region, being classed as coming from region 1, the fainter region. Of course this error rate depends on the values assigned to the a priori probabilities. It is instructive to ask what the error rate would be, given the same a priori assumptions, for the segmentation of a scene which actually has 1 pixel that comes from an underlying region that is 10 times brighter than its surroundings of size say 99 pixels. From before the probability of correct classification of the bright pixel is 77.4% giving an error rate of 22.6%. The probability of correct classification of a background pixel is 92.3%, giving its error rate as 7.7%. Therefore the total error rate for the classification of a single pixel is 7.8%. This means that for this scene of 100 pixels in total approx 8 will be classified as the object compared to the real 1, which is a 700% error! In practice we may know that only a few small bright regions are to be expected, in which case we would have some knowledge of the a priori probability of occurrence, however we may not know how bright they would be! Slavish use of these probabilistic methods using erroneous, or guesstimated at least, values for the various terms is a well-known trap for the unwary.
It is reasonable to enquire whether segmentation of SAR images can be achieved instead using intensity gradient information. This possibility has already been investigated for SAR images\(^{(1)}\) in the context of a theory for the human visual process where it was shown not to perform at all well. Let us examine why this should be so in more detail. A region is usually distinguished from its neighbours by a difference in its overall intensity (this is not necessarily so for texture changes). It follows that there must be a marked change in the intensity gradient along the boundary. If we consider the case of a boundary that runs parallel to one of the axes of the pixel array then this gradient is simply the difference in the intensities of the pixels divided by their spacing.

We ask therefore whether

1. the spatial gradients within the regions are such that the regions can be delineated by the gradient magnitude within the regions

or

2. the boundaries of the regions are distinguishable from the regions themselves by virtue of the gradient magnitude.

These questions can be answered by considering the shape of the probability distributions for the gradients within the regions and on the boundaries.

Let us consider again the simple image in Fig 1. Let the mean reflectivity of region 1 be \(M_1\) and the mean reflectivity of region 2 be \(M_2\).

From before the intensity distribution within region 1 is

\[
p_1(I) = \frac{1}{M_1} e^{-\frac{I}{M_1}}
\]

and within region 2

\[
p_2(I) = \frac{1}{M_2} e^{-\frac{I}{M_2}}
\]

We define the spatial gradient within a region as

\[
G = I(x) - I(x + 1)
\]

where \(x\) is an arbitrary pixel and unit spacing is assumed.

The gradient across the boundary is defined as

\[
G = I_1 - I_2
\]

where \(I_1\) and \(I_2\) are the intensities of adjacent pixels in the different regions.
Now the probability, \( p(G) \, dG \), that the gradient across the boundary lies within the interval \( G \) and \( G + dG \) is given by

\[
p(G) = \int p_1(I_1) \times p_2(I_2) \, dI_2
\]

substituting for \( I_1 \) we have

\[
p(G) = \int p_1(G + I_2) \times p_2(I_2) \, dI_2
\]

with the boundary conditions that \( p_1(G + I_2) = 0 \) if \( G + I_2 < 0 \) and \( p_2(I_2) = 0 \) if \( I_2 < 0 \).

On evaluating the integral subject to these conditions we have

\[
p(G) = \begin{cases} 
\frac{1}{M_1 + M_2} e^{-\frac{G}{M_1}} & \text{for } +ve \, G \\
\frac{1}{M_1 + M_2} e^{-\frac{G}{M_2}} & \text{for } -ve \, G 
\end{cases}
\]

Fig 6 shows the distribution. Note that the distributions for the gradients within the regions are obtained by making \( M_1 = M_2 = M \). Fig 7 shows the distribution of the gradients within two regions having a 2 to 1 difference in reflectivity.

It is apparent that the range of gradients in the brighter region encompasses that of the fainter region, just as happened for the intensity only case. Therefore it follows that spatial gradients within regions are of limited use for region segmentation.

What about the boundaries? Fig 6 shows the distribution for the 2 to 1 intensity ratio case. A comparison of Figs 6 and 7 reveals that the boundary distribution is not markedly different from the within region distribution. Hence the boundaries are indistinguishable from the interior of the regions.

We have now used all the low level, local, information in the image and have shown that the methods of segmentation which use single pixel data alone are of little use in our problem.

3 SEGMENTATION ON A LOCAL AVERAGE BASIS

The previous section showed that segmentation on a single pixel basis was not satisfactory for SAR images.

Here we consider whether it is possible to segment SAR images that have been smoothed. The rationale is that pixel averaging will alter the probability distribution function in such a way that the pdf's approach a Gaussian shape for images that have been severely smoothed so allowing the use of the simple segmentation techniques. The penalty is that region boundaries are blurred and also small regions are swamped by their larger neighbours.
Let us study first the pdf for pairwise averaging using the same simple model as before, considering the distributions within the regions and the boundary region separately.

Let the sum of the two pixels be $S$, then the probability distribution of the sum $p(s)$ is given by

$$p(S) = \int p_1(S - I_2) \times p_2(I_2) \, dI_2$$

with the boundary conditions that $p_2(I_2) = 0$ for $I_2 < 0$ and $p_1(S - I_2) = 0$ for $(S - I_2) < 0$.

Evaluating the integral we have

$$p(S) = \frac{1}{M_2 - M_1} \left( e^{-\frac{S}{M_2}} - e^{-\frac{S}{M_1}} \right) \quad \text{for } M_2 \neq M_1$$

For $M_2 = M_1$ we find

$$p(S) = \frac{S}{M_1^2} e^{-\frac{S}{M_1}}$$

by evaluating the integral for this special case.

These distributions are shown in Fig 8, again for the 2 to 1 reflectivity ratio used earlier. It can be seen that there is still considerable overlap of the within region intensity distributions, also the boundary region pdf is not markedly different from the within region pdfs.

The distribution for the average of 4 pixels at a time can be derived by considering the sum of the pairwise averaged distributions for the two cases where (i) all of the pixels are from the same region and (ii) two are from one region and two from the other.

We have for $M_1 \neq M_2$

$$p(S) = \frac{1}{(M_2 - M_1)^2} \left( e^{-\frac{S + \frac{2M_1 M_2}{M_2 - M_1}}{M_1}} - e^{-\frac{S}{M_1}} + e^{-\frac{S - \frac{2M_1 M_2}{M_2 - M_1}}{M_2}} - e^{-\frac{S}{M_2}} \right)$$

for the special case of $M_1 = M_2$ we have

$$p(S) = \frac{1}{6M_1^4} S^3 e^{-\frac{S}{M_1}}$$

These pdfs are shown in Fig 9. Although there is still overlap it can be seen that the pdfs for the within region situation are more distinct. This is of
course no more than the central limit theorem coming into play. We notice that
the boundary pdf still does not give any discrimination advantage over the within
region pdfs.

The quadruple averaging case is of special interest as it constitutes the
first stage in the "pyramid" segmentation methods described in (6,7).

In general it is possible to show that the pdf for the sum of N pixels is
the Gamma distribution

\[ P_N(S) = \frac{1}{(N-1)!} \frac{S^{(N-1)}}{M^N} e^{-\frac{S}{M}} \]

Using this equation we can study the probability of correct region assignment by
considering its dependence on both the number of pixels involved in the average
and the intensity ratio between the regions. We intend to show that it is
necessary to increase the averaging size when dealing with small intensity
ratios.

Let the two regions be as in Fig 1. We shall only consider the case of
averages that lie entirely within the regions, we do not consider the boundary
case here. As before we assume equal a priori probabilities to the regions
therefore the optimal threshold value T is given by

\[ T = \frac{M_1 M_2}{M_2 - M_1} N \ ln \left( \frac{M_2}{M_1} \right) \]

Now the probability that a sample will exceed this threshold is given by

\[ P(>T) = \int_T^\infty \frac{1}{(N-1)!} \frac{S^{(N-1)}}{M^N} e^{-\frac{S}{M}} dS \]

integrating by parts we derive the following series for the integral

\[ P(>T) = \frac{1}{(N-1)!} \frac{1}{M^N} \left\{ M e^{-\frac{T}{M}} \sum_{k=0}^{N-1} M^k \frac{d^k}{dT^k} \left( \frac{S^{(N-1)}}{M^N} \right) \right\} \]
As before we plot the optimum probabilities of correct segmentation as a function of underlying intensity ratio. Fig 10 shows the plot for the brighter region, region 2. Fig 11 is the corresponding plot for the fainter region 1. We have plotted the probability for three averaging values; 1, 4 and 16, corresponding to the original full resolution, half resolution and one quarter resolution respectively. Note: the full resolution results are the same as discussed in the single pixel section.

It is apparent from Figs 10 and 11 that pixel averaging will improve the theoretical segmentation greatly for all but the smallest intensity ratios. This is borne out in Fig 10 which is the total error rate calculated, as before, for the pixel averages. However we have improved our possible segmentation accuracy at the expense of spatial resolution; obviously if we are dealing with images that have regions that have only small intensity ratios then considerable degradation will be involved in their segmentation. It follows that the existence of small regions, like roads or hedges, could be lost if images are smoothed in a globally regular manner. This loss may be unacceptable, yet we also see that segmentation will work provided we can choose the pixels we include in the average. This is of course no more than a restatement of the power of template matching in extracting the presence of faint objects in noisy images. The penalty is having to process the images using different templates. One may consider asserting that the regions can be considered to be made up of a collection of primitive structures, for example the boundary of a region can be represented as a collection of straight line segments, in which case the computation load could be reduced. This reduction, arising from only analysing an image for these particular primitives, will still be at the expense of detecting faint regions which may need many primitives to fully represented. This is because there may not be enough pixels in some or all of these primitives to ensure their satisfactory detection if they are faint.

Let us consider finally the gradient distribution for the 4 pixel at a time case.

Using the same methodology as before we have for $M_1 = M_2$

$$p(G) = \frac{1}{96M_1^4} \left( G^3 + 6G^2M_1 + 15GM_1^2 + 15M_1^3 \right) e^{-\frac{G}{M_1}}, \quad \text{+ ve } G$$

$$p(G) = \frac{1}{96M_1^4} \left( -G^3 - 6G^2M_1 - 15GM_1^2 + 15M_1^3 \right) e^{-\frac{G}{M_1}}, \quad \text{- ve } G$$

for $M_1 \neq M_2$ we have

$$p(G) = \frac{G^3(M_1 + M_2)^3 + 12GM_1M_2(M_1 + M_2)^2 + 60GM_1^2M_2(M_1 + M_2) + 120M_1^3M_2}{6(M_1 + M_2)^3} e^{-\frac{G}{M_1}}, \quad \text{+ Ve } G$$
These last equations were derived using the algebraic manipulation programme EDUCE.

Fig 13 shows the pdfs. Again it is apparent that there is severe overlap of the distributions. Note the asymmetry of the boundary region pdf.

CONCLUSIONS

It has been shown that those segmentation operators which work on local data alone cannot segment SAR images well, even for the case of regions which are distinct enough to be easily segmented by eye. We have shown that pixel averaging results in image intensity probability distributions that are more amenable to local area segmentation. However it is also apparent that the degree of averaging necessary to achieve faint region segmentation will result in significant region overlap and loss of small but distinct regions if applied indiscriminately. We have shown that gradient segmentation operators acting on either single or averaged pixels have no advantage over straight intensity threshold operators. We have demonstrated that any satisfactory method of segmentation requires estimates of the a priori statistics in some manner. His may take the simple form of the estimate of the number of pixels we expect to be associated with the different regions. It may also take the form of structural probabilities, i.e., defining the shape of the regions in terms of correlated multivariate distributions. Certainly we see that a satisfactory method of segmentation must use data on a global, i.e. over many pixels in extent, scale. The crucial question then is how to design a method that chooses the pixels to be assigned to a given region in a way that results in the theoretical optimal correct classification rate being attained.

Three methods have possibilities. They represent a hierarchy of prior knowledge. A combination of all of them may be necessary, nevertheless we shall briefly review them separately here.

Method 1) region growing/splitting

These approaches (6,7,8) use the simplest level of a priori knowledge, the sectional form of the individual pixel probability distribution. They seek to find uniform regions by expanding or contracting the regions in area until the probability distributions of the pixel intensities within the regions do not exhibit statistically significant deviations from that expected for a uniform underlying area.

They are simple to implement but suffer from intrinsic statistical fluctuations when dealing with small region sizes. Also they cannot handle well regions that have intensity gradients across their extent.

Method 2) "primitive" analysis

This approach, of which (9) is an example, uses a higher level of prior knowledge in that the shapes of the regions being sought are assumed to be such that the regions can be described as a collection of primitive parts. In this
FIG 12

$P(\text{ERROR})$ vs $M_2/M_1$

- $P(\text{ERROR})$ decreases with increasing $M_2/M_1$
- Curves for different values of $M_2/M_1$: 1, 4, 16
FIG 7

$P(G)$

$M_1 = 1$

$M_2 = 2$
FIG 5

$P(\text{ERROR})$ vs $M_2/M_1$

$P(\text{ERROR})$ decreases as $M_2/M_1$ increases.
way the problem of segmenting an image reduces to detecting the presence of these primitive parts, for example polygons or arc segments, in the image and then using spatial relational information to decide which types of regions are present. This method should be able to extract faint regions provided each of the primitives encompasses enough pixels to allow for its detection according to the criteria mentioned in the previous section.

Method 3) Full region matching

This is the most powerful approach of all but it requires the greatest level of prior knowledge. In it the SAR images are compared with stored templates of the known possible regions until spatial agreement is found. These stored templates may be maps of the regions of interest or even prior, calibrated, SAR images of the regions. This way the largest possible number of pixels are involved and so the faintest possible regions can be detected. The penalties of this approach are the computational load coupled with the inability to work in situations where the expected region structure is unknown, eg flying over unsurveyed terrain.

It is not clear which of these approaches or combinations of approaches will provide the best overall solution to our problem, bearing in mind the trade-offs of prior knowledge, sensitivity and computing load. It seems reasonable to apply all these methods to real SAR images in order to establish more quantitative values for these trade-off parameters.

6 REFERENCES

8 Delves L M, RSRE Contract No A28B/320
**Abstract**

An expression for the speckle structure seen on Synthetic Aperture Radar (SAR) images is derived. This model is used to investigate the applicability of the simple segmentation operators usually used for image analysis to the problem of the segmentation of SAR images. These simple operators are shown to be incapable of segmenting raw SAR images satisfactorily. However, they can be applied successfully to area averaged SAR images provided certain constraints are met. Possible segmentation approaches are examined in the light of these constraints.