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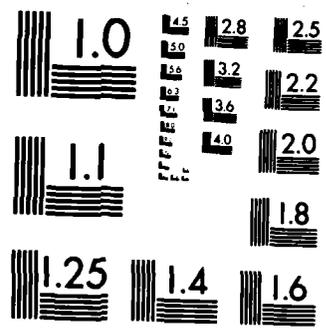
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Dr. Tom Spence  
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October 29, 1984

Ref No: N00014-82-K-0371

Dear Tom:

I am sending you an annual report mostly in pictures. I chose the topics which could be more easily explained by the figures and that summarize the approach we are taking. George is in Australia and the secretaries are on strike. I had to cut and paste because I cannot find the originals. I hope it will serve the purpose.

There are many subtle points that I could elaborate on, but it seemed to me that you would prefer more pictures than text. I kept the text to a necessary minimum to make the figures understandable. The published papers give all the details, so it would be unnecessary duplication. I hope the message is understandable.

Thank a lot for your help

*Manuel*  
Manuel Fiadeiro,  
Research Scientist,  
Yale University.

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## A) THE SEARCH FOR THE LEVEL OF NO MOTION

The inverse method applied to the mass conservation equations is a powerful method to obtain, from a set of oceanographic stations, a velocity field in geostrophic equilibrium obeying the conservation constraints imposed. The solution is a barotropic field that represents the absolute velocity at a given reference level. If there are less constraints than station-pairs, the problem is underdetermined and there are an infinite number of solutions. In these circumstances one has to choose which solution to adopt.

It has been the practise, for the purpose of calculating the absolute velocity field, to assume a level of no motion within the water column. This heuristic assumption has been implemented in many ways. The inverse method provides a mathematical formulation and a quantitative way to assess this assumption.

If a level of no motion exists and the mass constraints are obeyed it is reasonable to assume that the transport imbalance relative to that level should vanish. Due to errors in the data we can expect instead a small variance from zero. We proposed a procedure in which the reference level is changed from the surface to the bottom, to search for the minimum magnitude of the transport imbalance vector. This procedure gives a good estimate of the mean level of no motion and in most cases, acceptable residues for the imbalances.

We can also use the projection matrix of the parameter space (the "resolution" matrix) to look for the level that gives the smallest of the minimum norm solutions. These two procedures give a "best" reference level no more than 100m apart. In most cases the residual error and the variance of the solution are within acceptable limits and no correction is necessary. Examples are shown for the Coral Sea (Gascoyne 2/60, 1962) and Bermuda triangle (Atlantis 215, 1955) datasets.

The Bermuda triangle needs special mention. Here we divided the water mass in five layers. The upper two layers contained shallow stations at which

we assumed zero velocity at the bottom (OV stations). Some of these stations are across the Gulf Stream where this assumption is most likely to be false. Thus, we could expect large imbalances for those two layers. We then calculated the residual transport for the bottom three layers including only those stations with a maximum depth near or below the reference level. The search procedure gave us a best reference level at 3500m. The imbalance of the upper two layers was then attributed to the OV stations and an inverse solution was found for them. The resulting Gulf Stream transport across the Ft. Pierce section became comparable with the measured flow.

As a final note we restate that these results are not a proof that a level of no motion exists. They only show that there are solutions compatible with a level of no motion. While the problem is underdetermined there will be an infinite number of solutions. The inverse solutions are projections of the true solution into a subspace of the parameter space. We have access only to a limited representation of the solution, but at least, this representation should vanish if the assumption is true. So far, we have always found a solution compatible with a level of no motion and have presented the baroclinic field relative to that level as the most probable absolute velocity field.

#### Publications:

On the determination of absolute velocities in the ocean by Manuel E. Fiadeiro and George Veronis. J. Mar., Res. 40(sup.), 159-182, 1982.

see also: Comments on "On the determination of absolute velocities in the ocean" by L.L. Fu and Reply to L.L. Fu J. Mar. Res., 42, 259-262, 1984.

Circulation and heat flux in the Bermuda Triangle by M.E. Fiadeiro and George Veronis. J. Phys. Ocean., 13, 1158-1169, 1983.

CORAL SEA

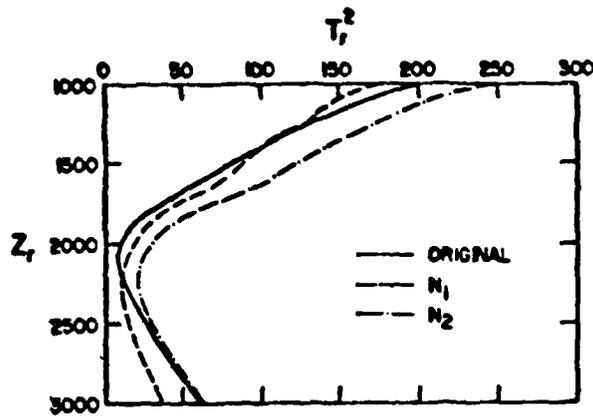


Figure 3. Mean square transport imbalances,  $T_r^2$ , (in  $Sv^2$ ) vs reference depth,  $z_r$ , (in m), for original, noisy 1 and noisy 2 data sets (shown by solid, dash and dash-dot curves respectively).

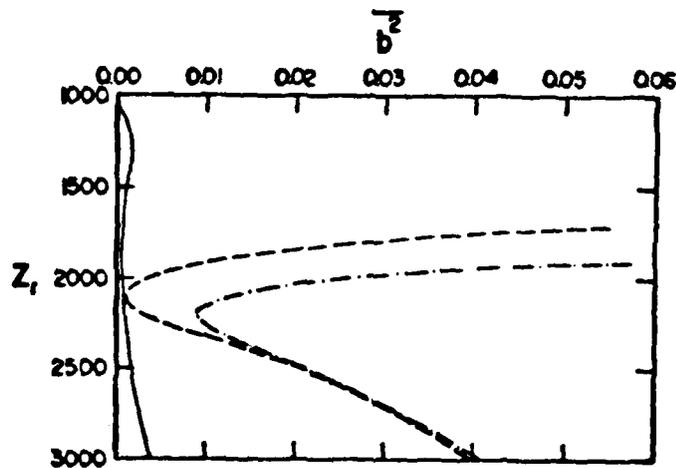


Figure 6. Horizontally averaged mean square barotropic correction,  $\bar{b}^2$  (in  $cm^2 s^{-2}$ ), vs reference depth,  $z_r$ , for original data. Solid, dash and dash-dot curves are for correction with one, two and three eigenvectors respectively.

The minimum residual transport and the smallest minimum norm solution indicate a possible level of no motion at about  $2200 \pm 100m$ .



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CORAL SEA

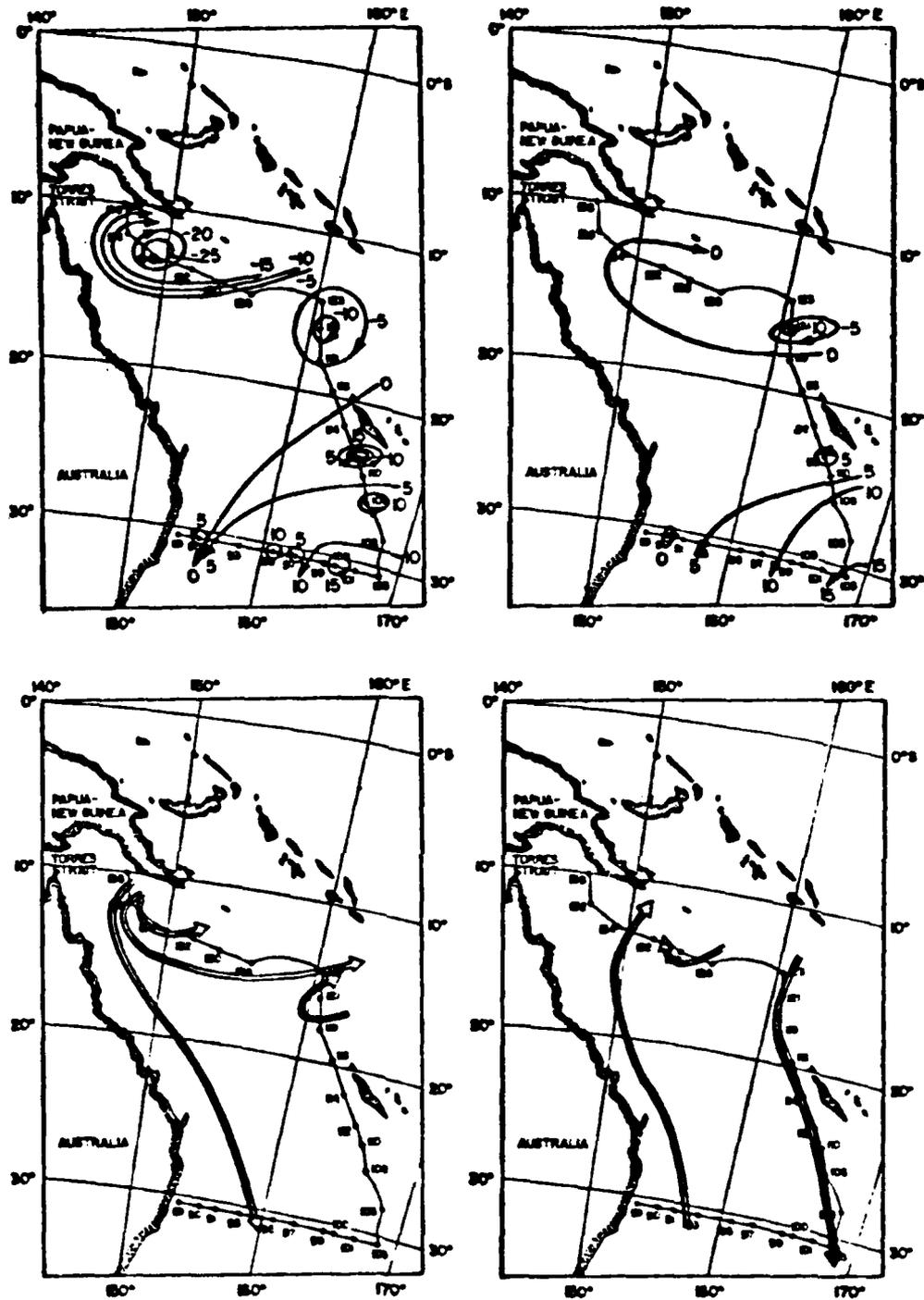


Figure 9. (a) Contours of transports for layer with  $\sigma_t < 40$  along cruise track. Each contour represents total transport from St. 89. Contour curves not unique; (b) Contours for  $40 < \sigma_t < 41$ ; (c) Broad arrows show direction of transport in oxygen minimum layer,  $41 < \sigma_t < 41.4$ ; (d) Same as (c) for bottom layer,  $\sigma_t > 41.4$ .

The mass transport of the 4 layers based on a level of no motion at 2140m.

BERMUDA TRIANGLE

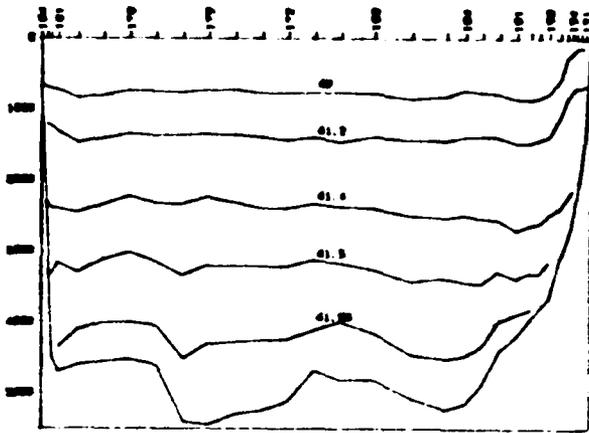


FIG. 3. Surfaces with  $\sigma_t = 40, 41.2, 41.4, 41.5, 41.55$  along cruise track from station 184 to 150 used to differentiate layers with characteristic  $O_2$  and/or  $S$  values. Surface with  $\sigma_t = 40$  is so close to upper surface near Cape Henry that air-sea exchange processes may cause transfer of water between two upper layers, hence, these two layers combined to form single, thick surface layer likely to be conserved.

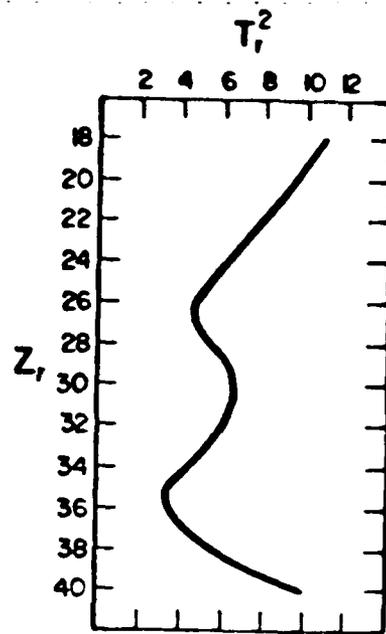


FIG. 5.  $T_i^2 (= \sum T_{ij}^2)$  versus  $z_i$  for bottom three layers for composite data set with linear interpolation and  $\partial z_i/\partial z = 0$  for vertical extrapolation. Minimum  $T_i^2$  achieved for  $z_i = 3500$  m and secondary minimum for  $z_i = 2600$  m.

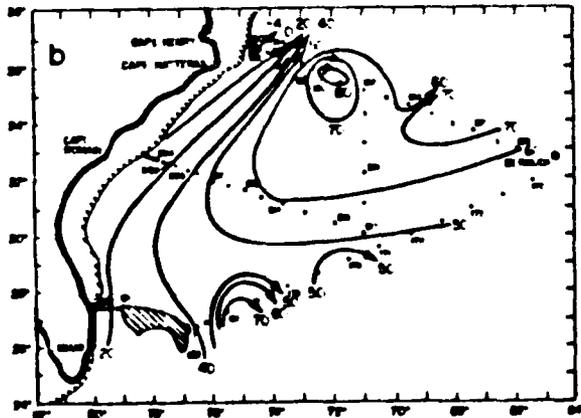
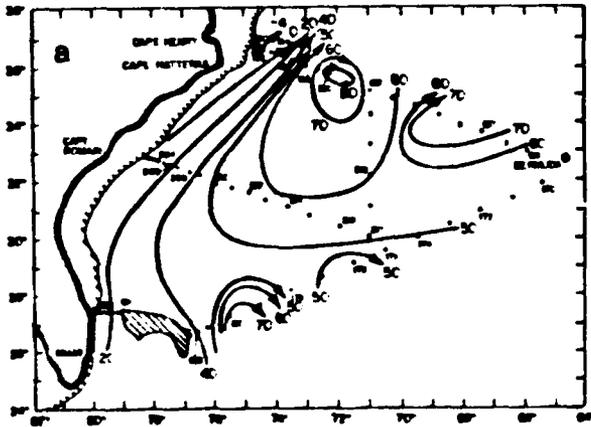


FIG. 7. (a) Transport contours indicating magnitude and direction of flow for top layer ( $\sigma_t < 41.2$ ) relative to station 200. Maximum transports relative to coast at stations 180, 213, and 161. (b) Alternative construction showing transport contours for 60 Sv closed near northern section.

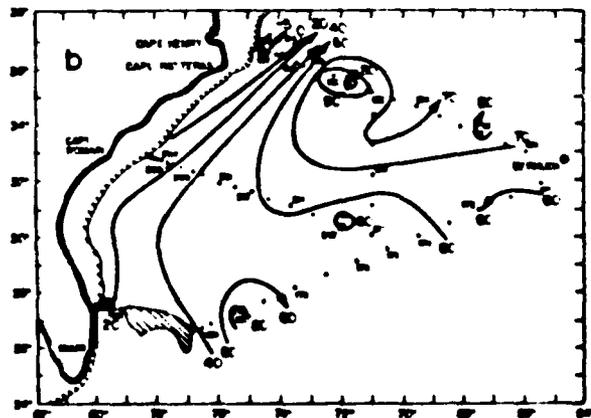
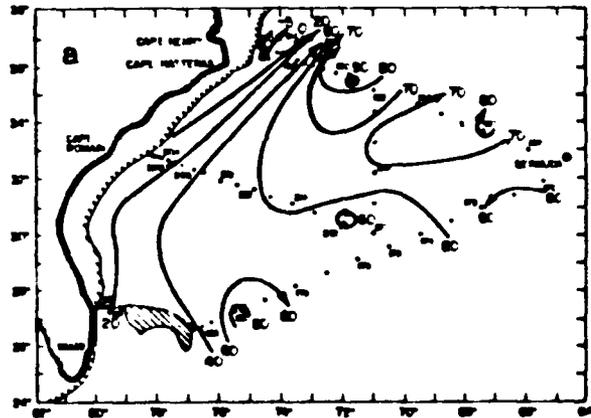


FIG. 8. (a) As in Fig. 7 but for top two layers. (b) Alternative construction for contours with 70 Sv near northern section.

## B) THE INVERSE PROBLEM WITH TRACER MIXING

Figure 1 shows the  $\sigma_3$  and PV ( $f \frac{\partial \rho}{\partial z}$ ) vertical sections along the meridian 50°W, Atlantis cruise ATL 229. To the left, St.5416 is off the Grand Banks and to the right, St.5471 is off the French Guiana. The section closes a large section of the Atlantic Ocean, the Caribbean and the Gulf of Mexico.

The assumption that properties are conserved by layers cannot hold for such large bodies of water. To the south (right), the water masses present a characteristic combination of  $\sigma_3$  and PV found nowhere else. To the north, the deep water between 1000 and 3000m has an homogeneous PV= $10^{-8}$ . Between Sts. 5436 and 5449 a minimum PV at  $\sigma_3 < 40.7$  marks the body of 18° water that has annual contact with the surface further east and to the north. In such circumstances one would be hard pressed to choose conservative layers. It is obvious the different water masses are being mixed inside the region closed by the section.

This example points out the need to include mixing effects in the formulation of the inverse problem if we hope to obtain any meaningful results. To that purpose we undertook a theoretical study of the mixing effects on the inverse procedures.

We started our study with two-dimensional tracer distributions generated by a model. We used an extremely simple flow field (a constant flow from left to right) and a constant eddy diffusion coefficient. The direct problem was solved in a grid of 16x64 points. For the inverse problem a small region of 6x6 points is sampled and treated as observed data.

In figures 2 to 7 the original flow field would be represented by equidistant streamlines with a value of 0 at the bottom and 4 at the top. We show the result of different procedures.

Fig.2 shows solutions of underdetermined systems, using the equations for only one tracer;  $c_1$  and  $c_2$  are different tracers and (5,7) and (23,6) refer to the sampling region of the direct solution. The solutions are entirely different from the input flow field. The patterns change with the tracer used and region sampled. These are minimum norm solutions; we can retrieve the original flow field by using two tracers or one tracer and additional correct constraints to make the problem well or overdetermined.

Fig.3 represents an interesting case. We added incorrect information ( $u=0$ ) to the system; a) with a weight of  $10^{-1}$ , b) with a weight of  $10^{-4}$ . The

streamlines change with the weight of the velocity constraints. As the weight decreases the pattern tends to the minimum norm solution shown in fig.2c. This is in contrast with the cases where we added correct information; the correct streamfunction was retrieved independently of the weight of the velocity constraints. This can prove to be a very useful technique in practise: "orthogonal" information compatible with the tracer is accepted by the system but incorrect information that conflicts with the tracer is rejected by the system.

In practise, data have errors and the parameters are mean properties of the field for the scale of the observations. We added random noise and sampled the model data at every other grid point. In these circumstances the model parameters give residuals to the equations and the inverse procedure finds a solution with even smaller residues. The question is to determine how far is the solution from the expected parameters of the model. The underdetermined cases were hopeless, the residues can be made to vanish but the solution has no physical significance. Fig.4 shows solutions of strict well-determined systems. We added to the tracer equations the condition  $v=0$  at the bottom and top of the sampling region. Fig.5 shows solutions when we impose  $v=0$  all over the sampling region (the system is overdetermined). In fig.6 we show solutions with a no vorticity condition for the 9 interior points ( $\frac{\partial^2 \psi}{\partial x^2}=0, \frac{\partial^2 \psi}{\partial y^2}=0$ ) and in fig.7 solutions with two tracers. As can be seen, under certain conditions, overdetermined system (least-squares solutions) can give reasonable approximations of the expected flow field. As a rule of thumb, we found out that if the inverse of the condition number of the coefficient matrix is smaller than the mean residue square of the equations, the solutions are very good approximations of the expected velocity field; otherwise, one should be extremely careful to interpret to results.

Regarding the application to oceanographic data, we can know very precisely the condition number of the inversion matrix, once the problem is set up; but we lack a good estimation of the errors in the data. We are presently working on this problem.

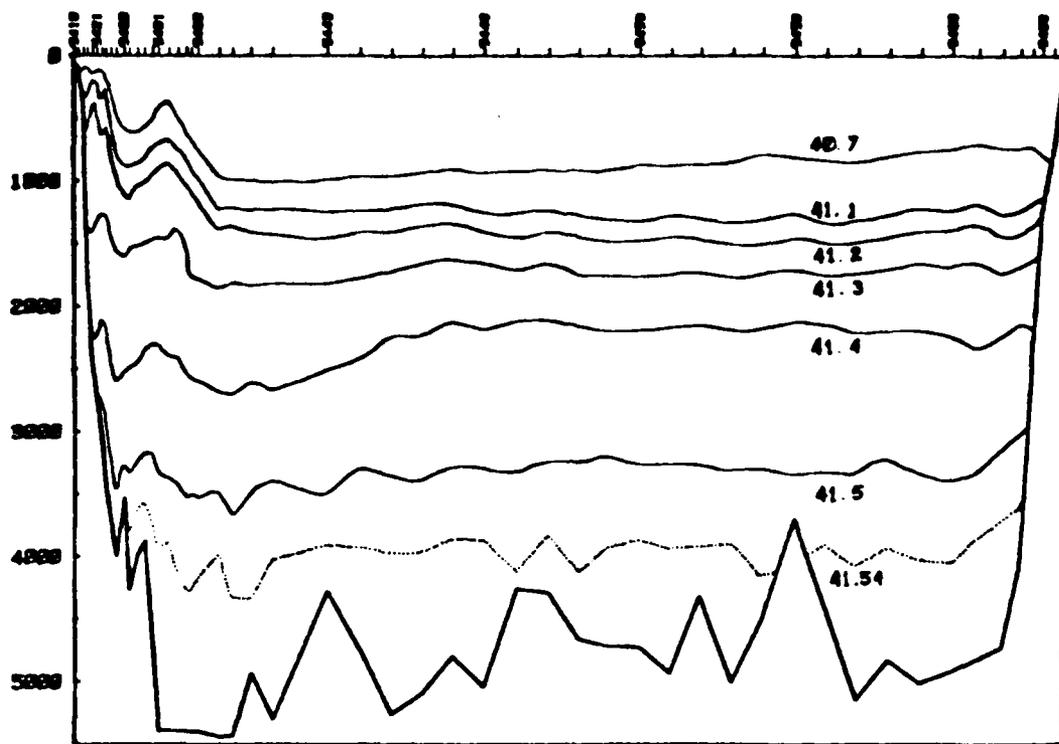
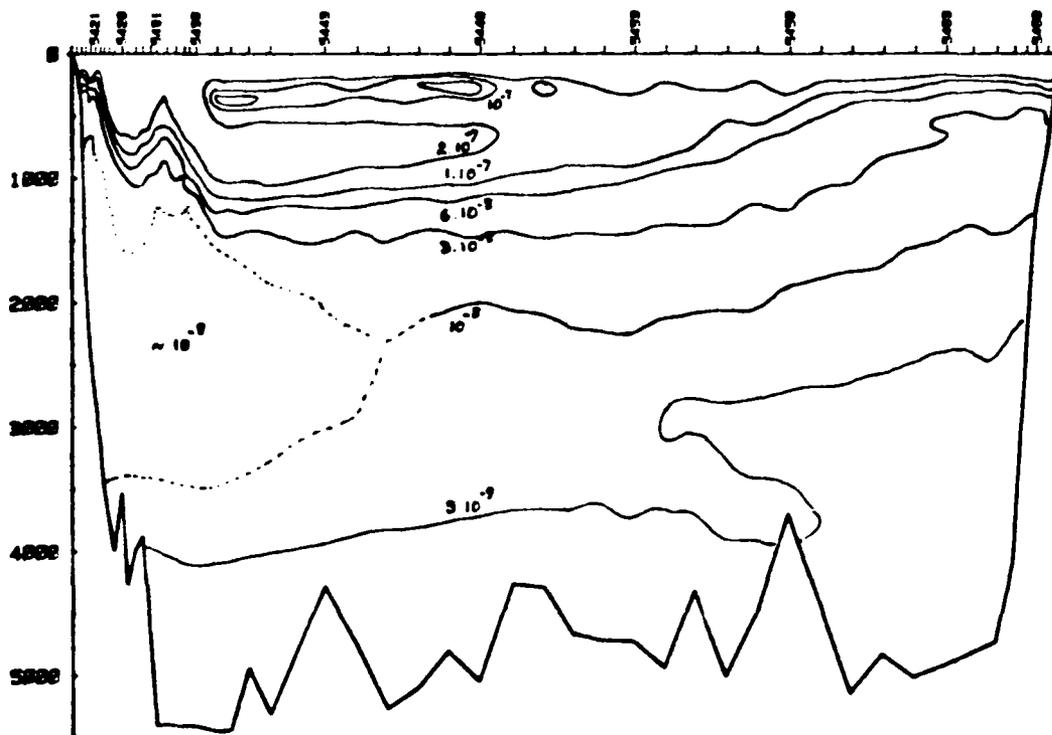


Fig.1.  $\sigma_3$  (above) and PV (below) for the 50°W section of ATL229.



Unique combinations of  $\sigma_3$  and PV indicate mixing effects in the portion of the ocean closed by the section.

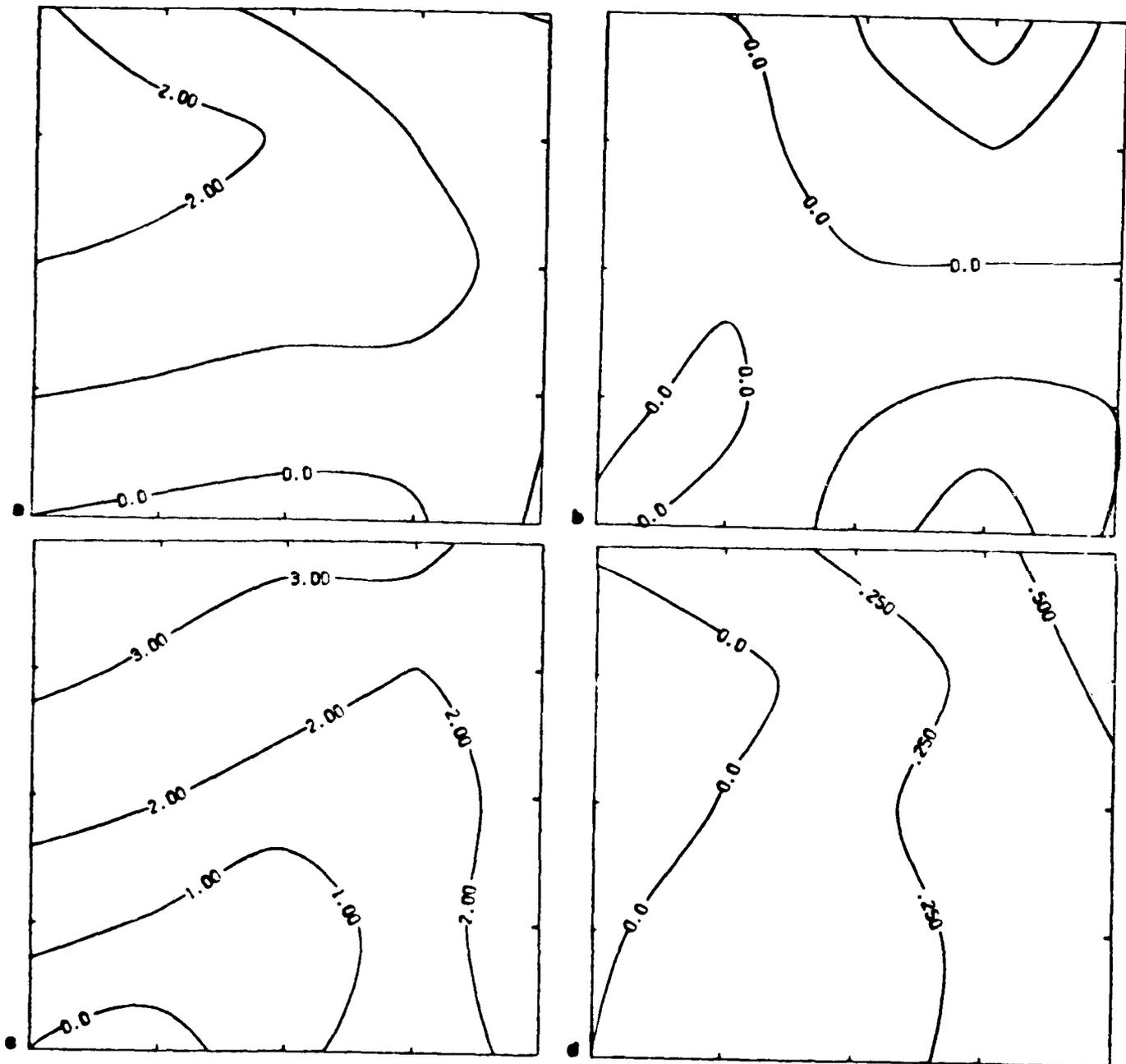


Fig.2. Streamlines obtained from inverse calculation for underdetermined system with single tracer in indicated sampling regions. a)  $c_1$  in (5,7); b)  $c_1$  in (23,6); c)  $c_2$  in (5,7); d)  $c_2$  in (23,6).

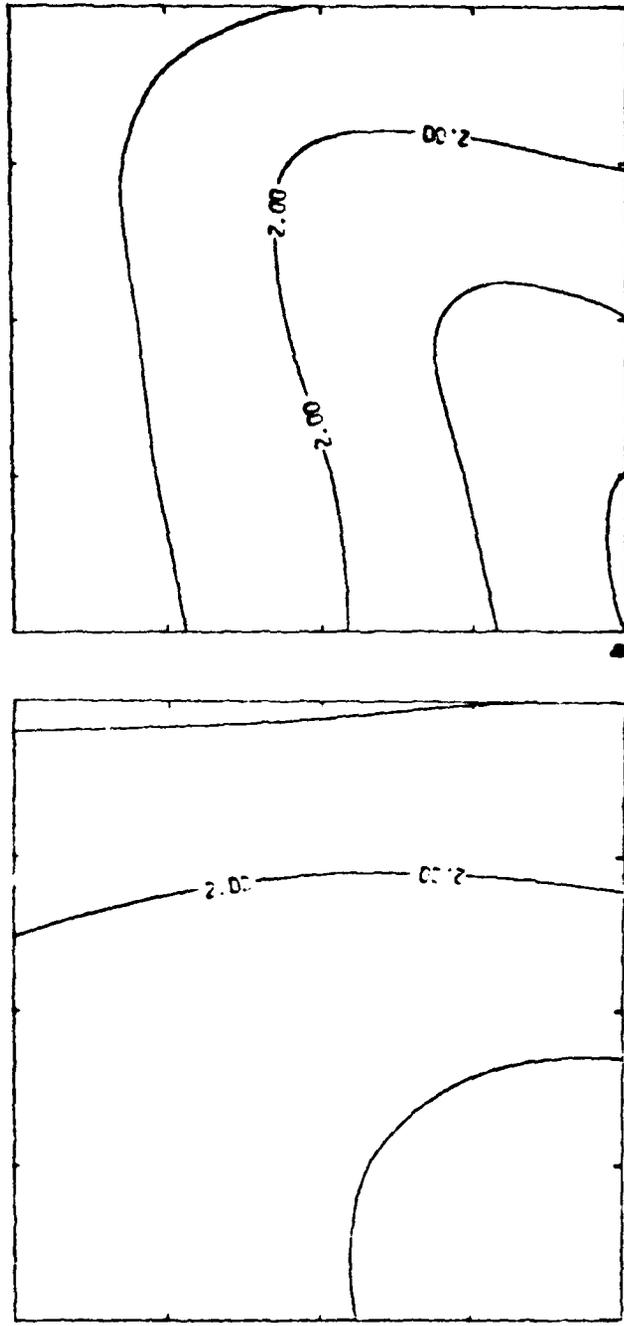


Fig.3. Streamlines obtained for 1x1 grid by imposing incorrect velocity constraints ( $u=0$ ) at all points. a)  $c_2$  in (5,7) with weights  $10^{-4}$  on  $u=0$ ; b)  $c_2$  in (5,7) with weights  $10^{-4}$  on  $u=0$ . Case b) resembles fig.2c for which no velocity information is available.

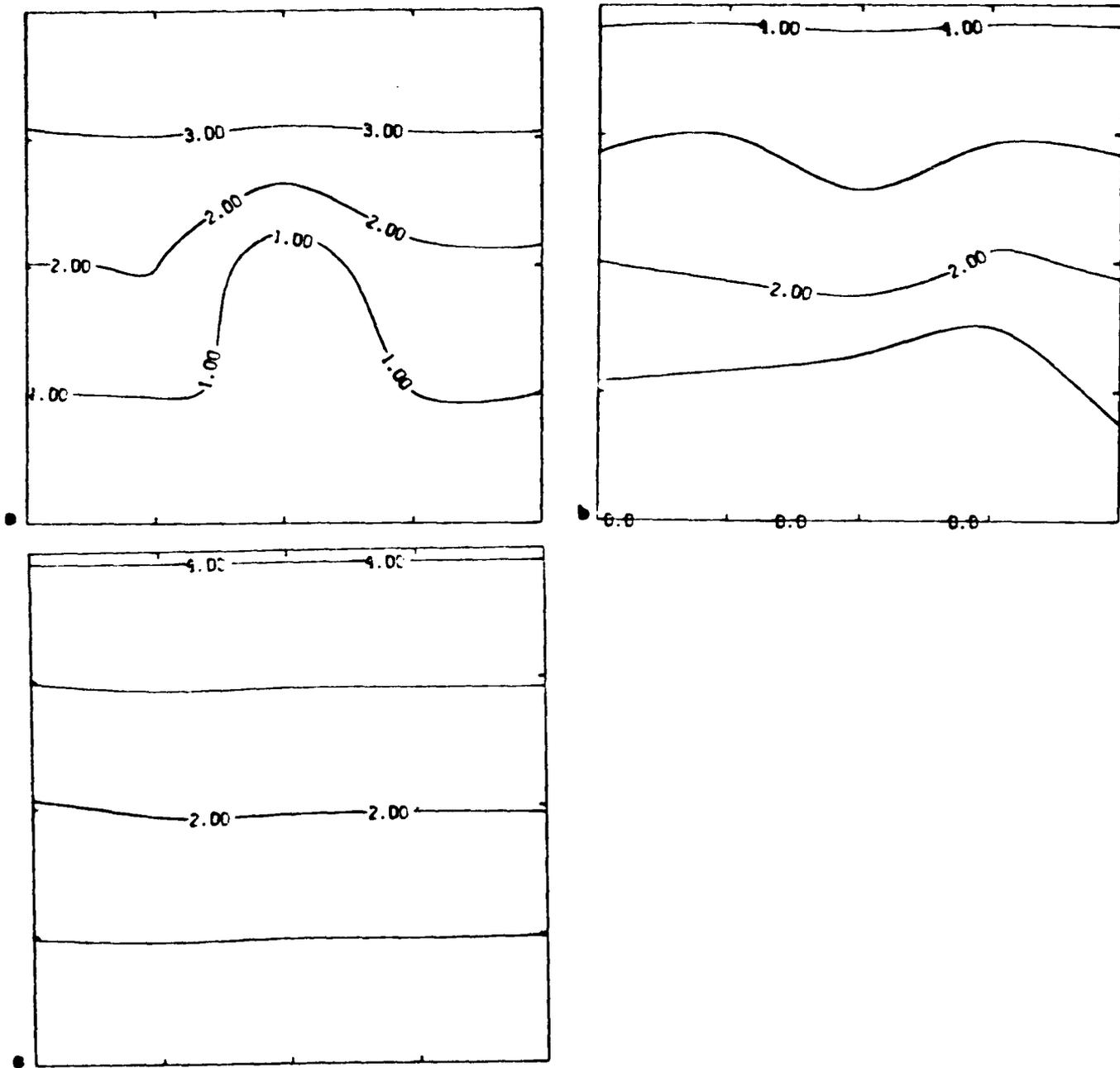


Fig.4. Streamlines using 2x2 grid and with added constraint  $v=0$  along top and bottom boundary of sampling region. System is well-determined. a)  $c_2$  in (5,7); b)  $c_1$  in (5,7); c)  $c_1$  in (23,6).

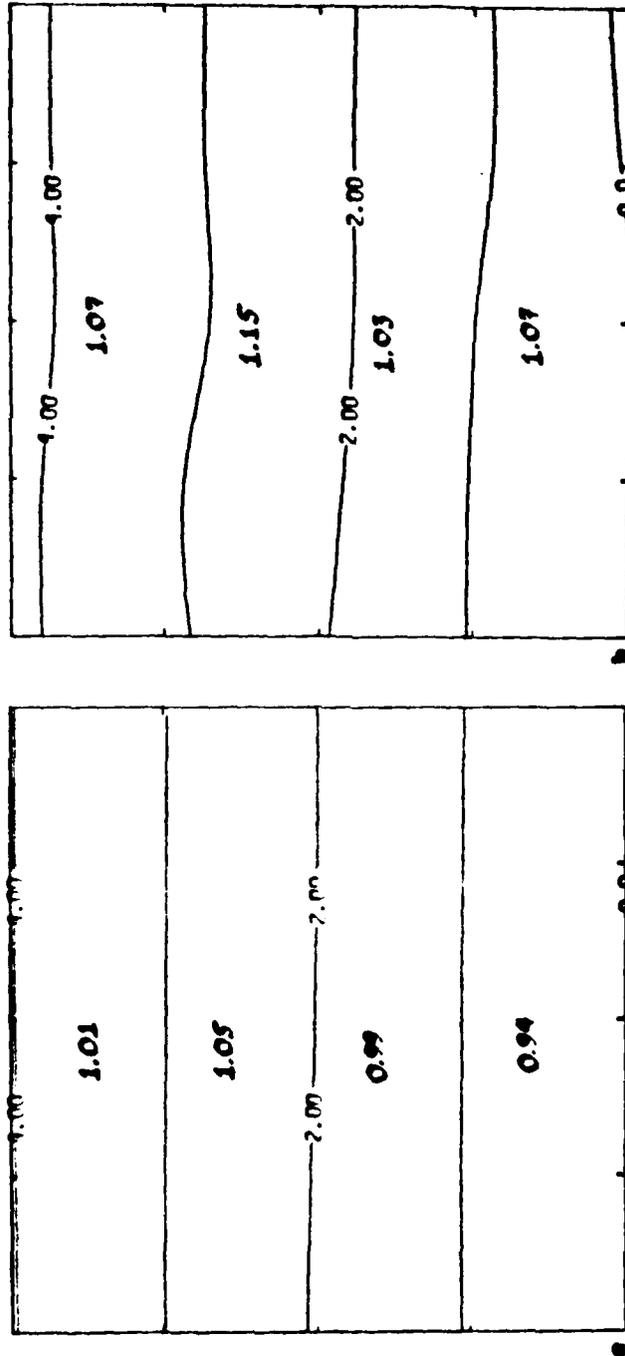


Fig. 5. Streamlines using  $2 \times 2$  grid and based on  $c_1$  and imposed condition that  $v=0$  everywhere. a) in (5,7) with weight  $10^{-1}$  on  $v=0$ ; b) in (5,7) with weight  $10^{-4}$  on  $v=0$ . Case b) close to actual flow in spite of small weights on velocity constraints. Numbers between streamlines are average velocities between rows of gridpoints. Actual velocity=1.

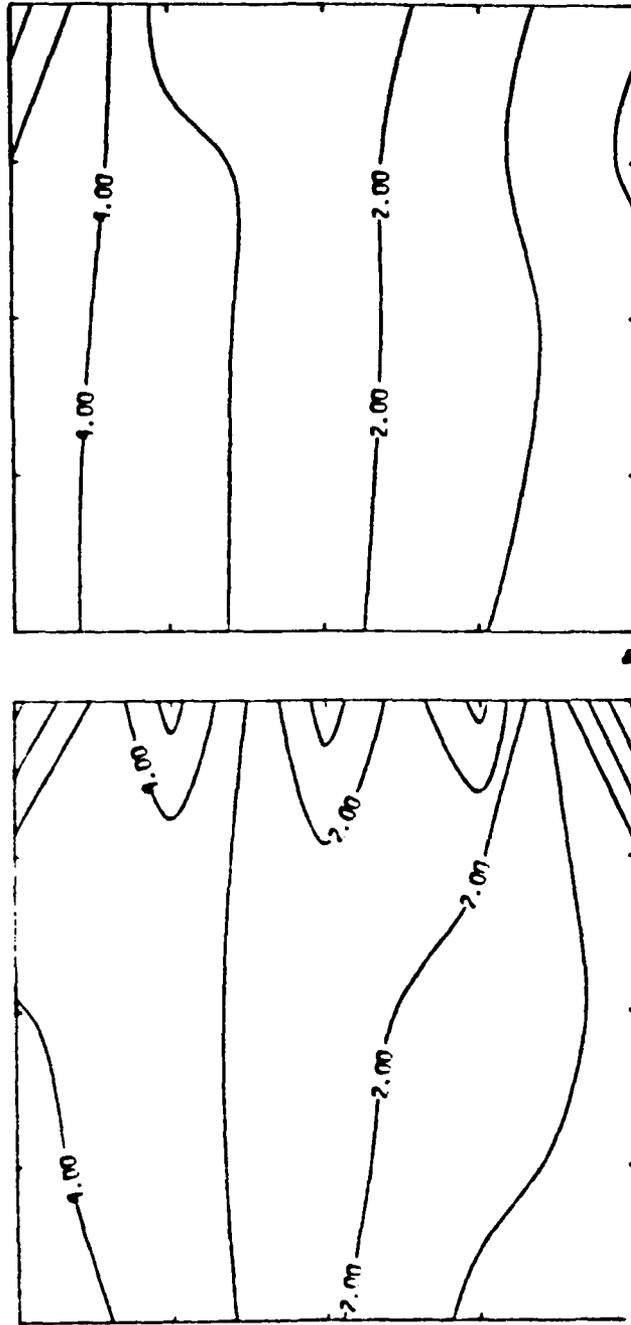


Fig. 6. Streamlines using 2x2 grid and with added constraint of zero vorticity at 9 interior grid points. a)  $c_2$  in (5,7); b)  $c_1$  in (5,7).

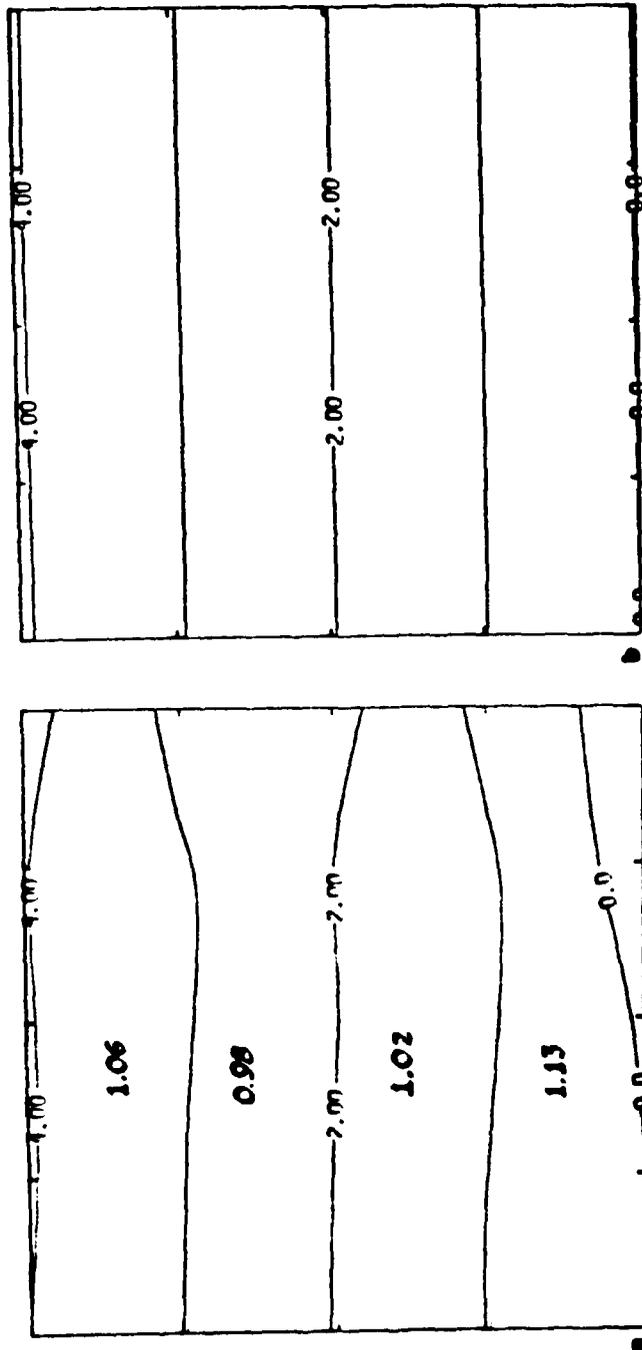


Fig. 7. Streamlines obtained using  $c_1$  and  $c_2$  on  $2 \times 2$  grid. a) in (5,7); b) in (23,6). Numbers between streamlines are average velocities between rows of gridpoints. Actual velocity=1.

Publications.

Obtaining velocities from tracer distributions by M.E. Fiadeiro and George Veronis. submitted to J. Phys. Ocean.

Inverse methods for ocean circulation by George Veronis. Advanced Study Institute Lectures, Scripps Institution of Oceanography. In press

An overview of the inverse method in oceanography by M. Fiadeiro (for publication).

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Contract no.: N00014-82-K-0371

Title: Expanded study of the inverse problem for ocean circulation.

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