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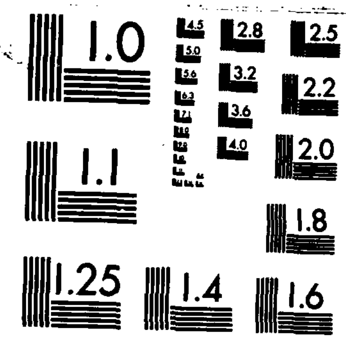
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USE OF MMLE3 TO DETERMINE

LOWER ORDER EQUIVALENT SYSTEMS

IN THE TIME DOMAIN

THESIS

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Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
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Master of Science

by

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PREFACE

In testing new aircraft for flying qualities, determination of lower order equivalent systems (LOES) has become important. This thesis presents a technique for determining lower order equivalent systems in the time domain. This data reduction technique is also useable for determining basic flying qualities data, such as short period frequency and damping ratio.

This thesis was accomplished under the auspices of the joint Air Force Institute of Technology/USAF Test Pilot School Master's program. The data reduction technique developed by this thesis should deprive TPS students of the joy of spending long hours staring at strip charts by substituting as a more efficient data reduction technique. Additionally, it will be useful in different USAFTPS projects for determination of lower order equivalent systems.

I would like to express my gratitude to Major (Dr.) James T. Silverthorn for the guidance, assistance, and time he gave me during the course of this thesis.

Richard A. Schroeder

Table of Contents

List of Figures

Figure | Page

List of Tables

ABSTRACT

This ~~report~~ thesis explains the data reduction technique for determining lower order equivalent system (LOES) parameters using flight test maneuver time history data. This technique was developed under the auspices of the joint AFIT/USAF Test Pilot School Master's Program.

This technique uses MMLE3, a modified NASA-developed general program for maximum likelihood parameter estimation. MMLE3 requires a state-space model relating input to output and a time history of the input and the output. From this information, accurate estimates of the components of the state-space model can be made. The LOES parameters determined by MMLE3 would be aircraft short period frequency, damping ratio, and time delay.

Based on simulation testing, it was determined that this data reduction technique was feasible and accurate. The state-space model for the testing was a second order state-space model incorporating the Pade approximation for time delay. From simulated data using a pure time delay, frequency estimates and damping ratio estimates were within 2.25% of the true values, while the time delay estimate was within 18% of the true value. These results were based on a sampling rate of 8 samples per second, which matches the USAF TPS in-flight instrumentation system's sampling rate. The effect of different sampling rates and signal noise on estimate accuracy was also investigated. The result, as expected, was that a higher sampling rate increased the accuracy of all the estimates. For any given sampling rate, an increase in the noise decreased the accuracy of all of the estimates.

The algorithm was further tested by using flight test data gathered in a T-38. The state-space model was modified to a modified second order model, to determine basic aircraft short period frequency and damping ratio. The flight test results effectively matched the published values of frequency and short period for the T-38.

Based on the results of this testing, this technique of determining LOES parameters from time history data is effective. It should be incorporated into the USAF TPS curriculum for data reduction.

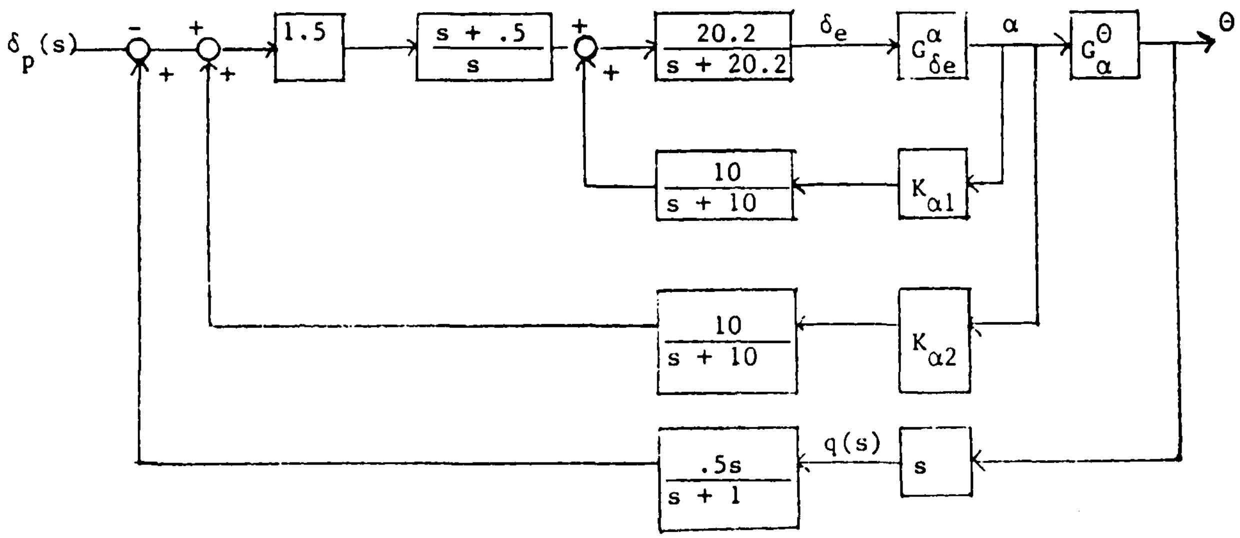
USE OF MMLE3 TO DETERMINE LOWER ORDER EQUIVALENT SYSTEMS IN THE TIME DOMAIN

1.1 BACKGROUND

The requirement to test military aircraft flying qualities exists. The criteria defining acceptable flying qualities are contained in MIL-F-8785C, Military Specification: Flying Qualities of Piloted Airplanes.

MIL-F-8785C essentially defines flying qualities in terms of two decoupled fourth order dynamic responses. One fourth order response defines aircraft longitudinal motion in terms of short period frequency and damping ratio and phugoid frequency and damping ratio. The other fourth order response defines aircraft lateral-directional motion in terms of dutch roll frequency and damping ratio, roll mode time constant, and spiral stability. MIL-F-8785C is based on fitting aircraft motion to these two decoupled fourth order responses.

Frequently, new aircraft cannot easily be evaluated against MIL-F-8785C because of complex aircraft dynamics. The complex aircraft dynamics are a result of improvements in flight control system design, which allow the designer to optimize aircraft response to control input for any given flight condition. This optimization can introduce additional aircraft dynamics, producing an aircraft/flight control system which no longer fits the defined flying qualities of MIL-F-8785C. As an example of how complex the aircraft dynamics can become, Figure 1 shows a simplified block diagram for the F-16 longitudinal control system in the landing configuration, as well as the overall transfer function relating pitch rate (q) and the pilot's longitudinal control input (δ_p) (Ref 9).



$$\frac{q(s)}{\delta_p(s)} = \frac{86.26(s+0.07)(s+0.5)(s+0.61)(s+1)(s+10)}{(s^2+0.039s+0.045)(s^2+1.866s+0.525)(s+0.37)(s+1.91)(s+10.97)(s+17.35)}$$

Pitch Rate Response Relation to Pilot Input
(F-16 Powered Approach Configuration)

Figure 1

The closed loop transfer function denominator, which characterizes the longitudinal dynamic response, is eighth order, a much higher order of dynamic response than the fourth order response expected by MIL-F-8785C. The problem arising from this is testing the flying qualities of this type of aircraft against MIL-F-8785C.

MIL-F-8785C deals with this problem by defining the flying qualities of this type of aircraft in terms of lower order equivalent system (LOES). For longitudinal motion, the LOES would be the apparent aircraft short period handling qualities. This LOES would be as shown in figure 2.

$$\delta_p \longrightarrow \boxed{\frac{K_q(s+1/T_{\theta e})e^{-\tau s}}{s^2+2\zeta_e\omega_e s+\omega_e^2}} \longrightarrow q$$

Pitch Rate Response Relation to Pilot Input
(Lower Order Equivalent System)

Figure 2

The terms in this closed loop transfer function contain an aircraft response equivalent to the eight order response shown in Figure 1. The parameters: equivalent frequency, ω_e , equivalent damping ratio, ζ_e , and the equivalent time delay, τ_e , effectively duplicate the higher order response, and define the LOES. MIL-F-8785C uses the LOES parameters to define acceptable flying qualities.

The most common technique for determining LOES parameters uses frequency domain data. Determining LOES parameters in the frequency domain involves gathering the time history of both the input and the output, converting them to the frequency domain via Fast Fourier Transform algorithm, calculating the experimental Bode plot, and then determining the LOES parameters that minimize a hted least squares curve fit to this frequency response (Ref

The method of gathering time domain data to convert into frequency domain data is computationally intensive. By use of maximum likelihood parameter estimation, it should be possible to determine LOES parameters directly, in the time domain.

The time interval, T, used for the majority of the testing was 0.125 seconds. This interval was used because the in-flight instrumentation system samples at this interval.

There were two purposes of this phase of testing MMLE3. First, this testing would verify that MMLE3 worked for a problem other than the sample problem provided with the program. Second, this testing could be used to analyze what flight test techniques (FTTs) should or could be used in flight to determine a LOES.

For longitudinal motion, a LOES is defined by the aircraft short period response (ref 1,2). Aircraft pitch rate response is most commonly used as the aircraft response which defines the LOES (ref 2). To determine the LOES, the aircraft short period dynamics have to be excited. The response has to be large enough to be detected, yet remain within the linear range of aircraft motion (ref 6,8). There are FTTs currently used which accomplish this (ref 9).

One FTT for exciting an aircraft short period is the elevator doublet. In this maneuver, the pilot makes a cyclic control input, first nose up, and then nose down, of equal magnitude in both directions, and then returns the control to the neutral position. The control input frequency matches the aircraft's short period frequency. The damping ratio can be estimated by counting the number of visible overshoots during the aircraft's free response. An estimate of the damping ratio equals seven minus the number of overshoots divided by ten. An elevator singlet can also be used with similar results.

The other common FTT used to evaluate the aircraft short period is the n/α sweep. In this maneuver, the pilot attempts to used for the simulation was F-89 aircraft data obtained from Aircraft Dymamics and Automatic Control (ref 7).

The simulation model used was:

$$\underline{\dot{x}}(t) = A\underline{x}(t) + B\underline{u}(t)$$
$$\underline{z}(t_i) = \underline{x}(t_i) \tag{19}$$

The A and B matrices were:

$$A = \begin{bmatrix} -0.0097 & 0.0016 & -0.061 & 0.0485 \\ -0.0955 & -1.43 & 0.9962 & 0.003 \\ 0.0 & -15.51 & -2.776 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} \quad B = \begin{bmatrix} 0.0052 \\ -0.0314 \\ -4.90 \\ 0.0 \end{bmatrix}$$

corresponding to the state vector $\underline{x}(t) = (V \ \alpha \ q \ \theta)^T$.

The eigenvalues of the A-matrix are:

$\lambda_{1,2}=-2.1003\pm j3.8700$

$\lambda_{3,4}=-0.0080\pm j0.0605$

producing these frequencies and damping ratios:

$\omega_{nsp}=4.403$ rad/sec, $\zeta_{sp}=0.477$

$\omega_p=0.060$ rad/sec, $\zeta_p=0.133$

The data was generated using the same propagation scheme as MMLE3 uses to generate its estimates. The control input which MMLE3 uses to determine the calculated states $\underline{x}(t_{i+1})$ uses the average control input over t_i and t_{i+1}. This propagation is used to compensate for control motion between samplings of the control position. To ensure that good parameter estimates result from the simulated data, the same propagation scheme was used to generate the data.

The formula used to generate the data was:

$$\tilde{x}_\xi(t_{i+1}) = e^{AT}\,\tilde{x}_\xi(t_i) + A^{-1}(e^{AT}-I)B[\tfrac{1}{2}u(t_i)+\tfrac{1}{2}u(t_{i+1})] \tag{20}$$

Chapter 3

3.1 MMLE3 Installation

The NASA provided MMLE3 program was incompatible with the USAF Test Pilot School PDP-11/34 computer. It required more memory capability then was available on the computer. In order to be able to use the basic algorithm, the NASA provided program had to be modified. The modifications involved were reduction in the matrix sizes used in MMLE3, removal of an option to generate estimates in the presence of state noise, removal of the program's plotting capability, and the development of a software overlay structure. The many modifications made to the program made it possible that the basic algorithm would somehow be affected and rendered useless. To verify that the program was functioning normally after it had been adapted, it was necessary to test it against a known case.

When the program was obtained from NASA, a sample data time history, the associated math-model, and the resulting estimates were also obtained. For the initial testing of MMLE3, this was used to verify that the program was functioning normally. The sample data used was test case 4 in reference 4. This installation of MMLE3 yielded the proper estimates from this data.

3.2 Testing MMLE3

To increase confidence that the program was working properly, and to determine flight test maneuvers for later use, simulated data was generated for processing by MMLE3. The data generated was based on aircraft longitudinal motion. Information

With an adequate state-space model and a good initial estimate at the answer, then MMLE3 only needs a time history relating control input and system output to yield estimates of state parameters.

$$\left\{\begin{matrix}\dot{x}_5(t)\\ \dot{x}_6(t)\end{matrix}\right\} = \begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}\left\{\begin{matrix}x_5(t)\\ x_6(t)\end{matrix}\right\} + \begin{bmatrix}1\\ 1\end{bmatrix} u(t)$$

$$z(t_i) = [f_1(\lambda_1,\lambda_2) \;\; f_2(\lambda_1,\lambda_2)]\left\{\begin{matrix}x_5(t_i)\\ x_6(t_i)\end{matrix}\right\} \qquad (18)$$

Here, the diagonal state matrix consists of the system eigenvalues, λ_1, and λ_2. Once again, though, the states x_5 and x_6 are dummy states which do not necessarily have physical significance.

In the math-models shown above, various matrix components are independent of the system response. These matrix components are not allowed to vary while MMLE3 estimates the system parameters (which consist of the other matrix elements). MMLE3 input conventions allow the user to define which matrix elements are allowed to vary and any multiplicative relationships between matrix elements. (See Appendix B)

For use in determining LOES parameters, the standard controllable state-space model will be used as a basis. This state-space model will be used because it is the least complicated, with response to interrelationships of different matrix elements.

2.4 Summary

MMLE3 is the basis for determining LOES parameters from aircraft control input and pitch rate time histories. MMLE3 can determine various system parameters, based on what state-space model is used to define the system. In order to use MMLE3, the state-space model needs to be defined, and an initial estimate of the answer needs to be available. In order for MMLE3 to converge to a solution, this initial estimate needs to be reasonable, or MMLE3 will not converge.

$$\frac{z(s)}{u(s)} = \frac{\tau s+1}{s^2+2\zeta\omega_n s+\omega_n^2} \tag{15}$$

Several state-space models which relate the input u to the output z can be constructed. Each model is a valid math-model which yields slightly different information about the system.

The standard controllable state-space model is:

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$z(t_i) = \begin{bmatrix} 1 & \tau \end{bmatrix} \begin{Bmatrix} x_1(t_i) \\ x_2(t_i) \end{Bmatrix} \tag{16}$$

The states x_1 and x_2 are dummy variables which serve only to generate the output z. These states need not have any physical significance, but only need to be properly related to $z(t_i)$.

The standard observable state-space model is:

$$\begin{Bmatrix} \dot{x}_3(t) \\ \dot{x}_4(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{Bmatrix} x_3(t) \\ x_4(t) \end{Bmatrix} + \begin{bmatrix} \tau \\ (1-2\zeta\omega_n\tau) \end{bmatrix} u(t)$$

$$z(t_i) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{Bmatrix} x_3(t_i) \\ x_4(t_i) \end{Bmatrix} \tag{17}$$

In this model, the state x_3 is identical to the output z. The state x_4 is a dummy state used to complete the math model. The control input is slightly more complicated to generate a math model which properly relates input to output.

The canonical state-space model is:

$$\text{Covariance}(\underline{\xi}) \geq \sum_{i=1}^{N} [(\nabla_{\xi}\tilde{z}_{\xi}(t_i))^T R^{-1} (\nabla_{\xi}\tilde{z}_{\xi}(t_i))] \qquad (14)$$

The covariance matrix is directly analagous to the variance in a scaler gaussian distribution. The smaller the values of the elements of this matrix, the higher the confidence of the estimate of the corresponding unknown value in the vector $\underline{\xi}$. As more samples are taken, (14) approaches an equality (ref 3, 5). Although relatively few samples may be taken, the covariance matrix (sometimes called the Fisher information matrix) can still be taken as an indication of confidence in the estimate. If the value s relatively large, then the corresponding estimate is not as good an estimate as if the corresponding value in the covariance matrix were relatively small. The covariance matrix is computed by MMLE3 to give a measure of confidence in each estimate.

2.3 Suitable Math Models for MMLE3

In order for MMLE3 to determine the best fit to a math model, it requires a time history of the input and the output and a math model relating input and output. The math model needs only to be a valid state-space model which relates the input to the output. This allows use of a state-space model which yields the information which the user wants from the model.

In control theory, there are many different state-space models which can describe the same system. As a brief example, a simple second order system will be examined (ref 5).

Consider the classical second order dynamic response described by the transfer function:

$$\underline{\xi}_{i+1} = \underline{\xi}_i - [\nabla_\xi^2 J(\underline{\xi}_i)]^{-1} \nabla_\xi J(\underline{\xi}_i) \tag{11}$$

$$\nabla_\xi J(\underline{\xi}_i) = \sum_{i=1}^{N} [(\tilde{z}_\xi(t_i) - \underline{z}(t_i))^T R^{-1} (\nabla_\xi \tilde{z}(t_i))] \tag{12}$$

$$\nabla_\xi^2 J(\underline{\xi}_i) = \sum_{i=1}^{N} [(\nabla_\xi \tilde{z}_\xi(t_i))^T R^{-1} (\nabla_\xi \tilde{z}_\xi(t_i))] + \sum_{i=1}^{N} [(\tilde{z}_\xi(t_i) - \underline{z}(t_i))^T R^{-1} (\nabla_\xi^2 \tilde{z}_\xi(t_i))] \tag{13}$$

Computation of the second term of (13) is significantly more complex than either the first term of (13) of all of (12). If $\underline{\xi}$ is close to $\underline{\xi}_{true}$, this second term approaches the value zero. the modified Newton-Raphson algorithm assumes that the second term is zero, and ignores it. This places a restriction on the algorithm that the initial guess be close to the true value. This means that some estimate of the answer must be available, and it must be reasonably close to the true answer. To allow a better probability of convergence when the initial guess may not be good, an option exists in the program to perform the first several iterations using a gradient algorithm rather than this modified Newton-Raphson algorithm.

The answer yielded by this maximum likelihood estimator can be shown to be (Ref 4, 5, 6):

1) Consistent
2) Asymptotically unbiased
3) Asymptotically efficient
4) Asymptotically gaussian

Because the estimator has these characteristics, it can be shown that the minimum covariance in the estimate is given by: (Ref 3, 6) measurements taken. Given the value Z, the intent is to determine $\underline{\xi}$. Mathematically, we are looking for the probability density function $p(\underline{\xi}|Z)$.

By using Baye's rule, this value can be determined by using other known values. Baye's rule says:

$$p(Z|\underline{\xi}) = \frac{p(\xi, Z)}{p(\underline{\xi})} \tag{7}$$

$$p(\underline{\xi}|Z) = p(Z|\xi)\,\frac{p(\xi)}{p(Z)} \tag{8}$$

Z is a set of measured values. p(Z) is, then, constant. $p(\underline{\xi})$ is also a constant because no restrictions are placed on its possible values. The probability density function $p(Z|\underline{\xi})$ can be written based on the statistics of the noise $n(t_i)$. (Ref 3, 5)

$$p(Z|\underline{\xi}) = [(2\pi)^m|R|]^{-N/2} \quad \exp\{-\tfrac{1}{2}\sum_{i=1}^{N}(\tilde{z}_\xi(t_i)-\underline{z}(t_i))^T R^{-1}(\tilde{z}_\xi(t_i)-\underline{z}(t_i))\} \tag{9}$$

Maximizing this probability is the same as maximizing the probability $p(\underline{\xi}|Z)$ because the two are related by a constant.

A simpler, but identical, problem can be yielded by taking the natural logarithm of (9), disregarding the constant terms, and minimizing the negative. The cost functional then becomes:

$$J(\underline{\xi}) = \tfrac{1}{2}\sum_{i=1}^{N} [(\tilde{z}_\xi(t_i)-\underline{z}(t_i))^T \; R^{-1} \; (\tilde{z}_\xi(t_i))] \tag{10}$$

Minimizing this functional yelds the vector $\underline{\xi}$ which best fits the math model representing the aircraft dynamics.

MMLE3 maximizes this functional by using a modified Newton-Raphson algorithm. The algorithm is:

where R is the measurement noise covariance matrix.

The math model is a theoretical duplicate of the aircraft dynamics. It is:

$$\dot{x}_{\xi}(t) = A(\underline{\xi})\tilde{x}(t) + B(\underline{\xi})\underline{u}(t) \tag{5}$$

$$\tilde{z}_{\xi}(t_i) = C(\underline{\xi})\tilde{x}(t_i) + D(\underline{\xi})\underline{u}(t_i) \tag{6}$$

The vector $x_{\xi}(t)$ is the calculated value of the state vector, based on the known control inputs and the estimated values for $\underline{\xi}$. The vector $z_{\xi}(t)$ is the calulated value of the measurement vector based on $x_{\xi}(t)$. The vector $\underline{\xi}$ contains the unknown parameters which define the math model through the matrices A, B, C, and D.

The actual aircraft dynamics are assumed to be deterministic. By measuring the error between the actual aircraft dynamics and the output of the math model, and then adjusting the math model based on the error, MMLE3 determines the maximum likelihood values for the components of the matrices which define the math model. The noise which corrupts the measurements of the aircraft dynamics is assumed to be white noise of known statistics. Because of this, determining the vector $\underline{\xi}$ becomes a problem in stochastic estimation.

The known value is the vector measurement $\underline{z}(t_i)$. Here, it will be considered that the vector Z incorporates all measurements taken up to time t_i. N is the total number of calculating the total difference between the system measurement $\underline{z}(t)$ and the calculated output $\tilde{z}(t)$ for the entire time history, and using that error to determine how to change the vector $\underline{\xi}$. The total squared error at the end of each iteration in MMLE3 is compared with the previous iteration's total squared error. When these two values are within 0.1% of each other, MMLE3 assumes that it has converged to a solution.

2.2 Theory of MMLE3

MMLE3 is based on the assumption that there is some functional relationship between the control input $\underline{u}(t)$ and the output $\underline{z}(t)$. This relationship is expressed by:

$$\underline{x}(t) = A(\underline{\xi}_{true})\underline{x}(t) + B(\underline{\xi}_{true})\underline{u}(t) \quad (1)$$

$$\underline{z}(t_i) = C(\underline{\xi}_{true})\underline{x}(t_i) + D(\underline{\xi}_{true})\underline{u}(t_i) + \underline{n}(t_i) \quad (2)$$

The vector $\underline{x}(t)$ consists of "n" state variables, the input vector $\underline{u}(t)$ has "m" components, and the output vector $\underline{z}(t_i)$ has "p" components. For the version of MMLE3 on the USAF TPS computer, the maximum value of n, m, or p is 5. The noise vector $\underline{n}(t)$ corrupts the measurement vector $\underline{z}(t_i)$. The statistics of n(t) are assumed to be:

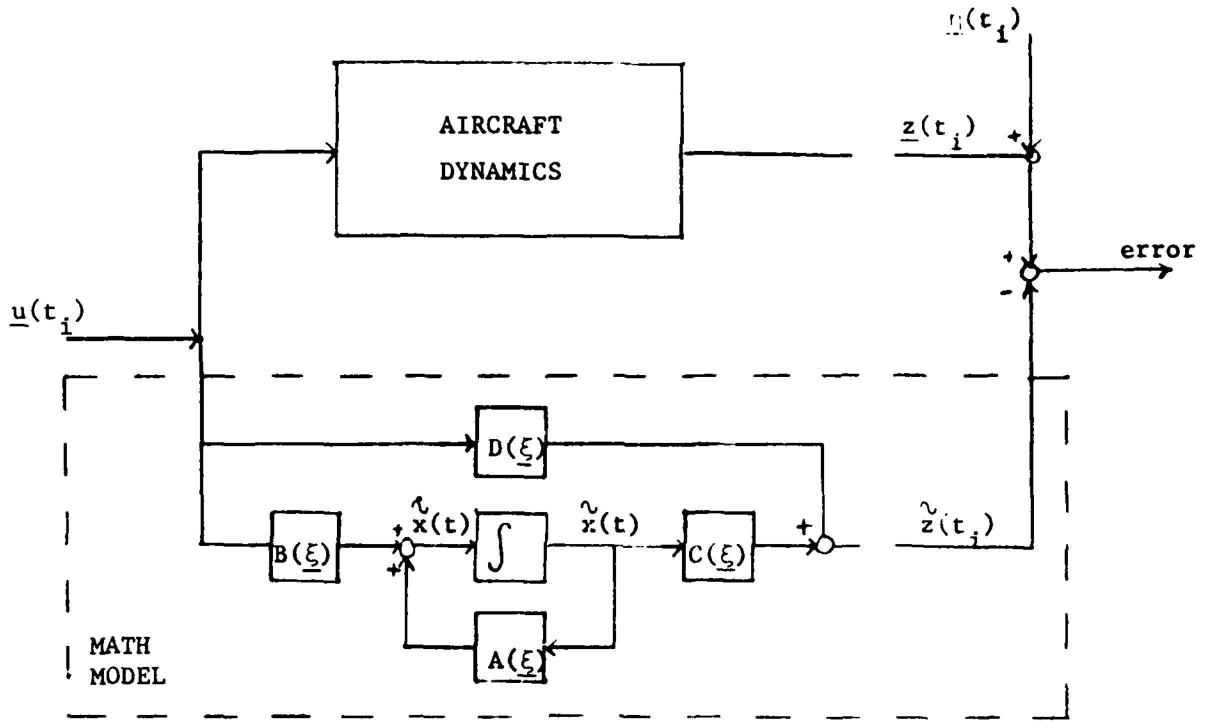
$$E\{\underline{n}(t_i)\} = \underline{0} \quad (3)$$

$$E\{\underline{n}(t_i)^T \underline{n}(t_j)\} = \begin{cases} R & t_i = t_j \\ 0 & t_i \neq t_j \end{cases} \quad (4)$$

Chapter 2

2.1 Introduction to MMLE3

MMLE3, the program used to determine the LOES parameters, is a general program for maximum likelihood parameter estimation. (Ref 3, 4) MMLE3 has been implemented on the USAF TPS PDP-11/34 computer to solve the problem shown in Figure 3.



Block Diagram of MMLE3 Math Model

Figure 3

The math model used in MMLE3 is a discrete time representation of a continuous system reacting to small perturbations from a steady state condition. The model involves a vector of unknown parameters, $\underline{\xi}$, to be estimated by the MMLE3 algorithm. The measurements of both the math model and the real system are at discrete times. The math model relates the control input $\underline{u}(t)$ to the calculated output $\tilde{z}(t)$. MMLE3 adjusts the math model by data was used to determine suitable flight test techniques for exciting aircraft short period response to generate good time histories for MMLE3. Through simulated data, basic flying qualities data reduction on MMLE3 is evaluated. With a pure time delay introduced into the simulation, MMLE3 is used to identify system parameters with a math model incorporating a time delay. Finally, a lag filter is introduced into the simulation model to model a simple flight control system. MMLE3 is used to determine LOES parameters based on this data.

Flight test data is also processed by MMLE3 to show the capability of MMLE3 to successfuly generate flying qualities data from the USAF TPS Data Acquisition System. The results of this limited flight test are presented in Chapter 4.

Conclusions and recommendations are made in the final chapter of this thesis.

1.2 Objectives

This thesis will present an approach for determining LOES parameters based on data gathered in the time domain. This thesis was sponsored by the USAF Test Pilot School (USAF TPS), and had specific objectives. The objectives were:

1. Provide a useable maximum likelihood estimation algorithm for use on the USAF TPS computer system.

2. Determine the feasibility of using maximum likelihood estimation to determine LOES parameters in the time domain.

3. Determine whether or not the current USAF TPS in-flight instrumentation system is adequate for this application.

4. Assuming that this technique is feasible and that the instrumentation is adequate, provide the USAF TPS a flying qualities data reduction system using the maximum likelihood estimation algorithm.

1.3 Overview

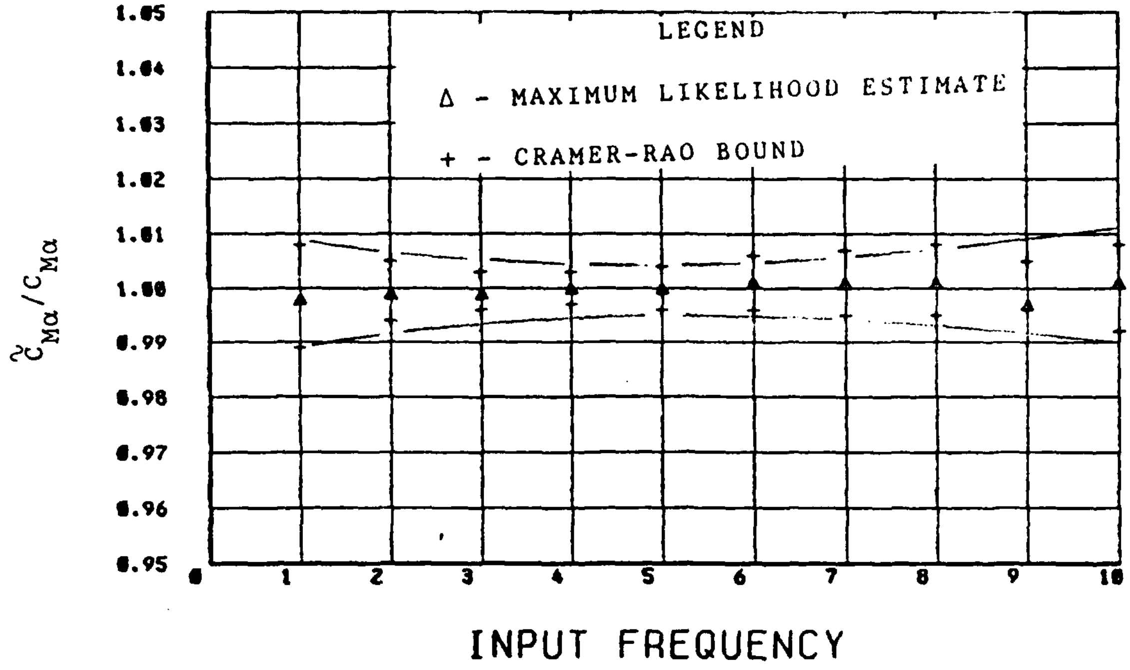
This thesis follows the organization explained in the following paragraphs.

Chapter 2 discusses the theory of the maximum likelihood estimation program (MMLE3). MMLE3 is a general program for maximum likelihood parameter estimation developed by the National Aeronautics and Space Administration to extract stability derivatives from flight test data. Chapter two explains in detail the algorithm used in the program, along with its limitations on the program. Chapter 2 also discusses general requirements for the state-space model and the input data.

Verification testing of the installation of the MMLE3 program on the computer is discussed in Chapter 3. Simulated put in a sinusoidal input to the elevator. The ideal input will have the pilot moving the control stick forward as the aircraft nose is rising, and moving the control stick backwards as the aircraft nose is falling. If the maneuver is done well, the pilot's input frequency equals the aircraft's damped short period frequency. By doing both maneuvers at one trim point, a pilot can determine an approximation to both the short period frequency and damping ratio.

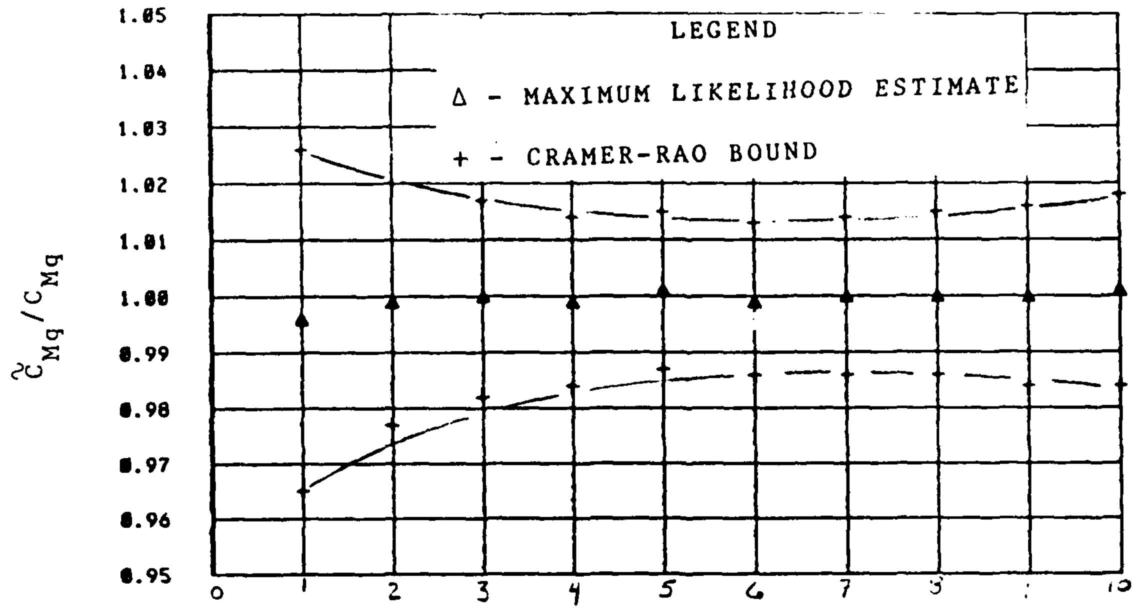
Since the above FTTs are already used to determine approximations to an aircraft's short period, the same FTTs should be good for generating data for MMLE3 to determine short period frequency, damping ratio, and time delay. Both types of control inputs were used in the simulation program to generate data.

Initially, MMLE3 was used to detemine the parameters of the A-matrix most closely related to the short period response. These parameters are pitch stiffness, $C_{m\alpha}$, A93,2), pitch damping, C_{mq} , A(3,3), and elevator power, $C_{m\delta e}$, B(1,3). These are the elements in row three of both the A-matrix and the B-matrix. All of the other parameters in the matrices were assumed known and therefore held constant by MMLE3. Ten seconds of simulated data from both FTTs were processed by MMLE3 to determine these parameters. Aircraft pitch rate output q was fitted to control input to determine thes parameters. Results from MMLE3 are shown in Figures 4, 5, and 6.



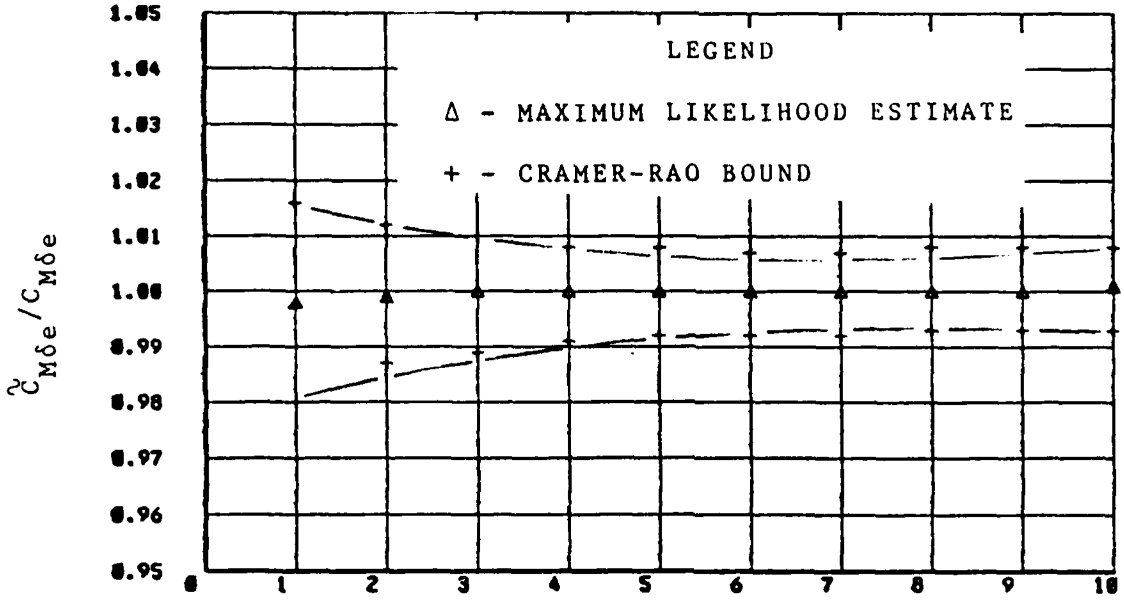
Variation of Accuracy in $C_{m\alpha}$ Estimate
with Different Sinusoidal Input Frequencies

Figure 4



Variation of Accuracy in C_{mq} Estimate
with Different Sinusoidal Input Frequencies

Figure 5



Variation of Accuracy in $C_{m\delta e}$ Estimate with Different Sinusoidal Input Frequencies

Figure 6

With an elevator doublet or singlet as the input, MMLE3 converges to the solution which matched the values used to generate the simulated data. With sinusoidal input frequencies between 1 and 10 radians per second, MMLE3 also converges to the same values as the ones used to generate the simulated data. If the sinusoidal input frequencies are close to the natural frequency, there is virtually no difference between the Cramer-Rao bounds of the estimates yielded by either FTT, as shown by table 1.

Input Technique	$\tilde{C}_{m\alpha}/C_{m\alpha}$	$\tilde{C}_{mq}/C_{mq}$	$\tilde{C}_{m\delta e}/C_{m\delta e}$
Step	1.001 (0.005)	1.000 (0.018)	1.000 (0.009)
Doublet	1.000 (0.004)	1.000 (0.014)	1.000 (0.010)
n/α sweep 4.0 rad/sec	1.001 (0.003)	0.999 (0.013)	1.000 (0.009)

(*.***) denotes the estimate's Cramer-Rao bound

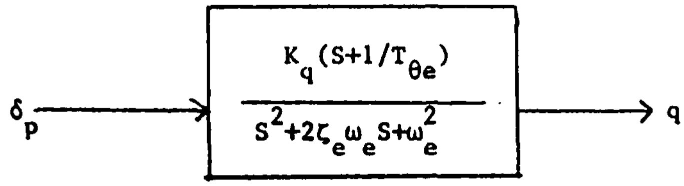
Comparison of Estimate Accuracies
Using Different Flight Test Techniques

Table 1

MMLE3 yields these results as long as the initial guess is close to the final solution. With an initial guess of the correct answer, the solution converged in approximately four iterations. With the initial guess up to 50% off, it takes no more than 10 iterations to converge. The final estimate, though, was always the same. Because of the excellent results of this test of MMLE3, it was apparent that it was functioning properly. It was also apparent that classical FTTs would be suitable for exciting short period dynamics for MMLE3 parameter estimation.

3.3 MMLE3 Parameter Estimation for a Second Order Response

Further testing of MMLE3 was conducted using the same simulated data. Short period frequency and damping ratio were determined by fitting the measured pitch rate response and control input to a second order response, as shown in Fig 7.



Pitch Rate Response Relation to Pilot Input

Figure 7

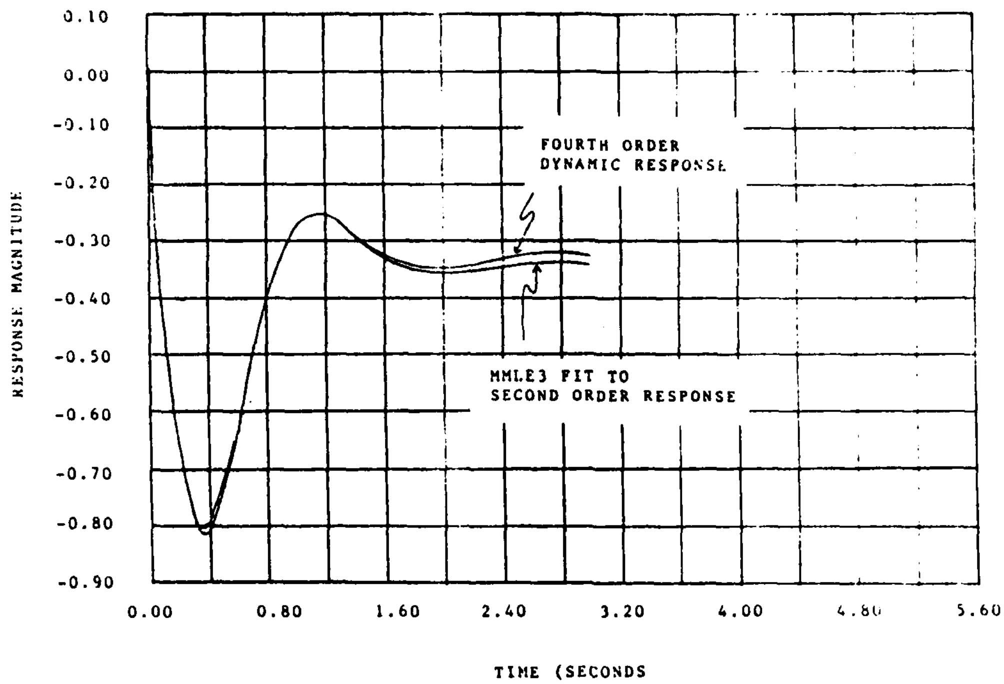
The transfer function is easily convertible into the standard controllable state-space model. The model is:

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} + \begin{Bmatrix} 0 \\ K_q \end{Bmatrix} \delta_p(t)$$

$$q(t_i) = \begin{bmatrix} 1/T_{\Theta e} & 1 \end{bmatrix} \begin{Bmatrix} x_1(t_i) \\ x_2(t_i) \end{Bmatrix} \tag{21}$$

There are three unknowns associated with this model: ω_e, ζ_e, and K. The value $1/T_{\Theta e}$ is assumed to be identical to the value $1/T_{\Theta 2}$, 1.372, as given for the F-89 aircraft (ref 7).

As discussed previously, an elevator doublet, a sinusoidal elevator input or a simple step input are all good for exciting short period dynamics for parameter estimation. No significant differences were noted between estimates yielded by the three control inputs. Figure 8 compares the simulated time history of pitch rate response to a unit step input to the time history of a second order transfer function with the parameters estimated by MMLE3. As can be seen, there are no significant differences between the two time histories.



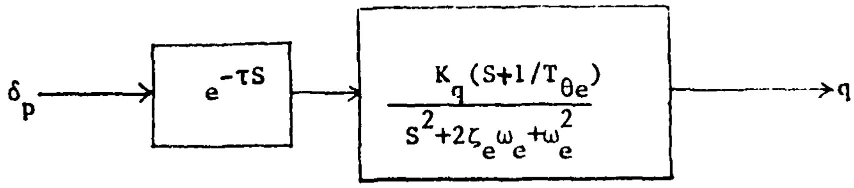
Time History Comparison

Figure 8

For a system with no time delay, the MMLE3 algorithm would be extremely useful for determining short period frequency and damping ratio. This data reduction technique would use a forced response, and fit the response to a second order state-space model, using all measured data points. This would be a significant improvement to the log-decrement method of data reduction currently used at the USAF TPS (ref 9).

3.4 Incorporating A Time Delay

Actual aircraft do not respond instantaneously to a pilot's control input. After a control input, there may be some time delay, associated with system lags, before the aircraft reacts. MMLE3 should be able to estimate that delay, if given an adequate state-space model. To determine if MMLE3 could identify a time delay, the simulation model was modified to incorporate a pure time delay between control input and aircraft reaction. A simple block diagram of a system incorporating a time delay is shown in Fig 9.



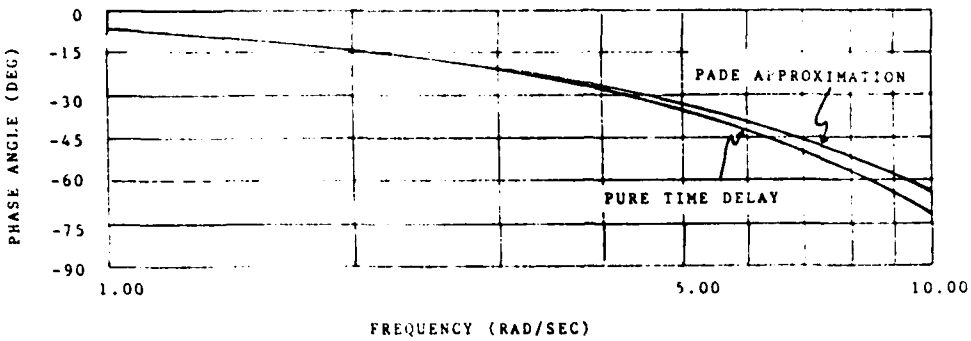
Pitch Rate Response Relation to Pilot Input
(Time Delay Incorporated)

Figure 9

The time delay in the system is introduced by the term $e^{-\tau s}$. The time delay is τ. In order to transform this transfer function into a state-space model of the same form as (19), the Pade approximation to a time delay is used. The Padé approximation is:

$$e^{-\tau s} \cong \frac{-(\tau s-2)}{(\tau s+2)} \tag{22}$$

In the frequency domain, this approximation is fairly good for low frequency inputs, as shown by the Bode phase plot in Fig 10. The Bode magnitude plots match exactly. Based on this, the frequency sweeps used to excite the short period dynamics should be low frequency inputs, to preserve the approximation accuracy. Since both the elevator doublet and the n/α sweep are low frequency inputs, the Padé approximation should be adequate for the time delay model for these inputs.



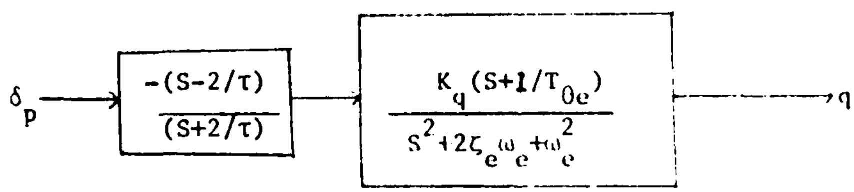
Phase Diagram of Padé Approximation
τ = 125 milliseconds
Figure 10

In the time domain, the Padé approximation response to a step input is:

$$L^{-1}\left\{\frac{-(s-2/\tau)}{(s+2/\tau)}\right\} = 1-2e^{-2t/\tau} \tag{23}$$

The net effect of this is to shift time response of a system by some amount of time slightly less than τ. A slight initial response opposite the overall response is induced because of the exponential form, but this effect should be small.

The time delay exists between the pilot's input and the aircraft reaction. In block diagram format, it is:



Pitch Rate Response Relation to Pilot Input
(Padé Time Delay Approximation Incorporated)

Figure 11

The input to the aircraft is x_3, a dummy variable which represents the pilot input and the time delay before the aircraft reacts. The math model for x_3 is:

$$\dot{x}_4(t) = -(2/\tau)x_4(t) + (2/\tau)\delta_p(t)$$
$$x_3(t) = 2x_4(t)+\delta_p(t) \tag{24}$$

Incorporating the time delay into the standard controllable model in (21) yields the new model, which has a time delay.

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_4(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_e^2 & -2\zeta_e\omega_e & 2K_q \\ 0 & 0 & -2/\tau \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \\ x_4(t) \end{Bmatrix} + \begin{bmatrix} 0 \\ -K_q \\ 2/\tau \end{bmatrix} \delta_p(t)$$

$$q(t_i)=[1/T_{\Theta e} \quad 1 \quad 0]\{x_1(t_i) \quad x_2(t_i) \quad x_4(t_i)\}^T \tag{25}$$

The model above uses the three parameters which define a LOES, time delay, equivalent frequency, and equivalent damping ratio. By adjusting these parameters so that the model output best matches a time history, a LOES is defined.

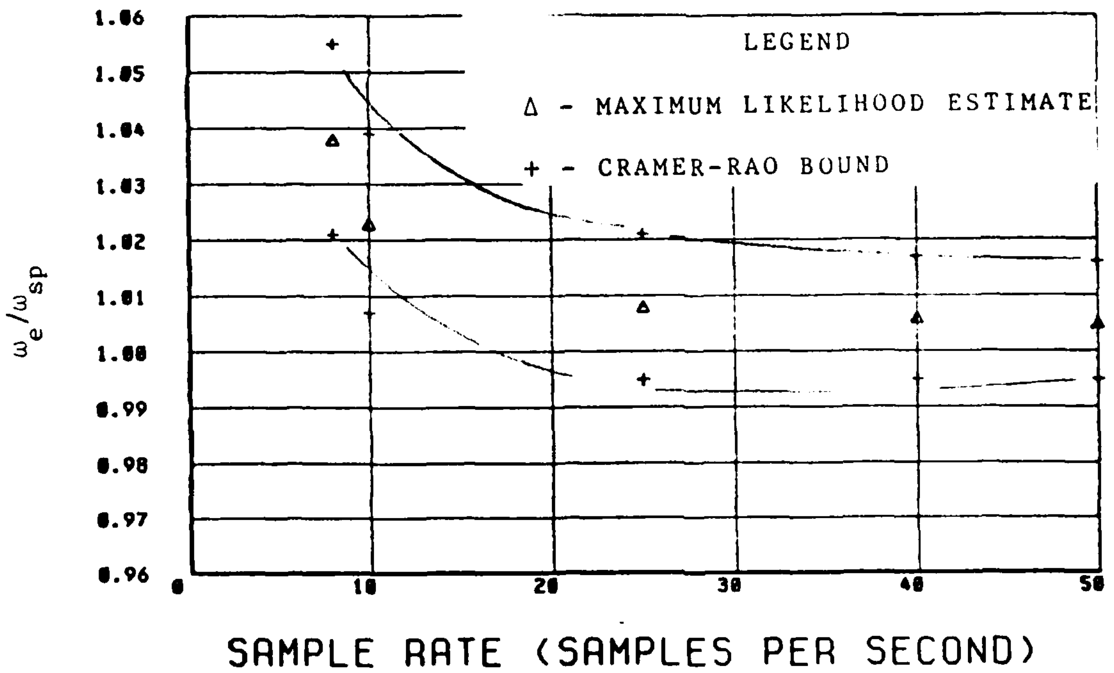
To test the model, simulated data incorporating a 125 millisecond time delay was generated. The simulated data was generated using both low frequency sinusoidal control inputs and step inputs. Table 2 summarizes the results:

	ω_{nsp} $(\frac{rad}{sec})$	ζ_{sp}	τ (sec)
True Value	4.403	0.477	0.125
Estimated Value	4.501 (0.036)	.470 (0.016)	.147 (0.003)

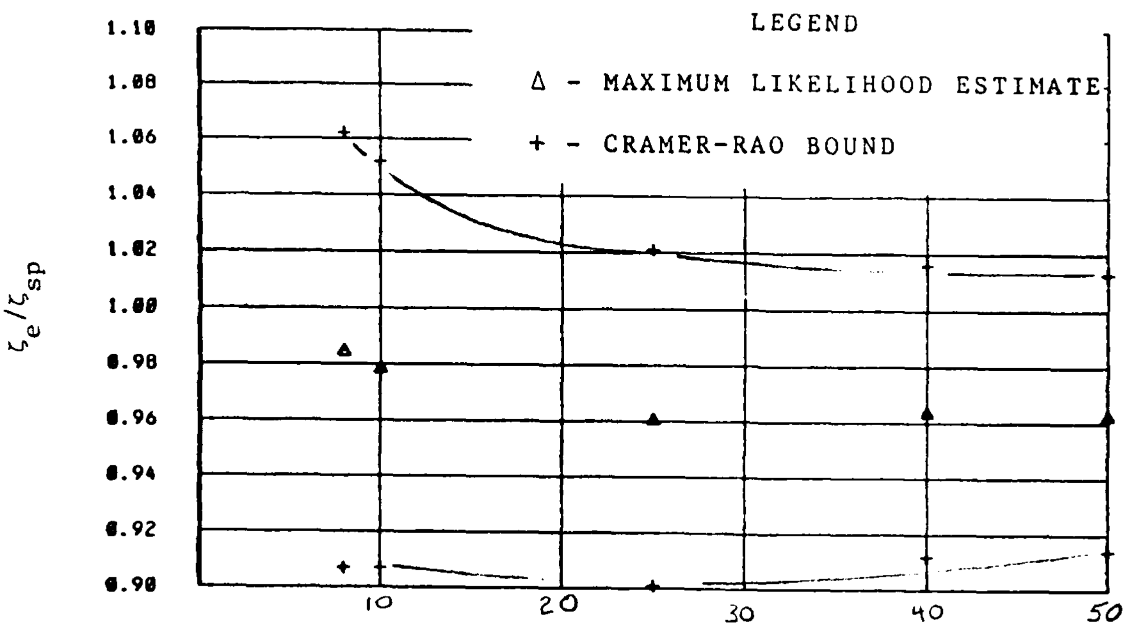
(*.***) denotes the estimate's Cramer-Rao bound
Maximum Likelihood Estimates
Based on Time Delay Model
Table 2

3.5 Effect of Data Sampling Rate on Estimate

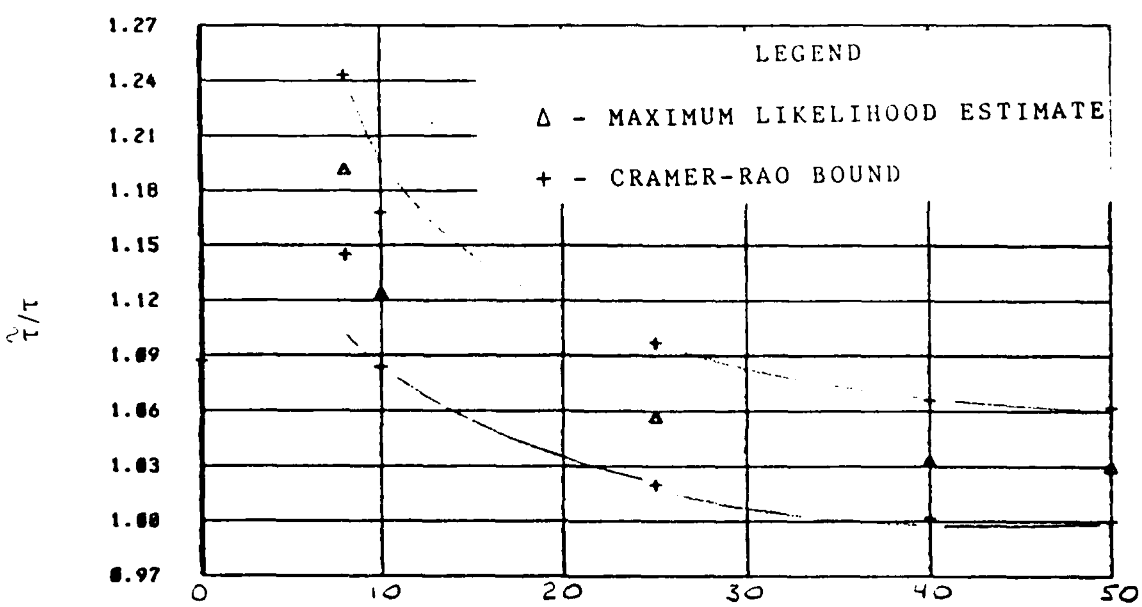
Since all data being analyzed was computer generated, no physical limits existed for the data sampling rate. To analyze the effect of data sampling rate on estimate accuracy, data was generated at different sampling rates, and then processed by MMLE3. The only difference in the data generated was the sampling rate (inverse of the time interval T) and the actual time delay. The time delay used at the 8 samples per second (sps) and 40 sps points was 125 milliseconds. At the 25 sps and 50 sps points, the time delay was 120 milliseconds. At the 10 sps point, the time delay was 100 milliseconds. The data is presented in Figures 12, 13, and 14.



Variation in ω_e Estimate Accuracy with Sampling Rate
Figure 12



Variation in ζ_e Estimate Accuracy With Sampling Rate
Figure 13



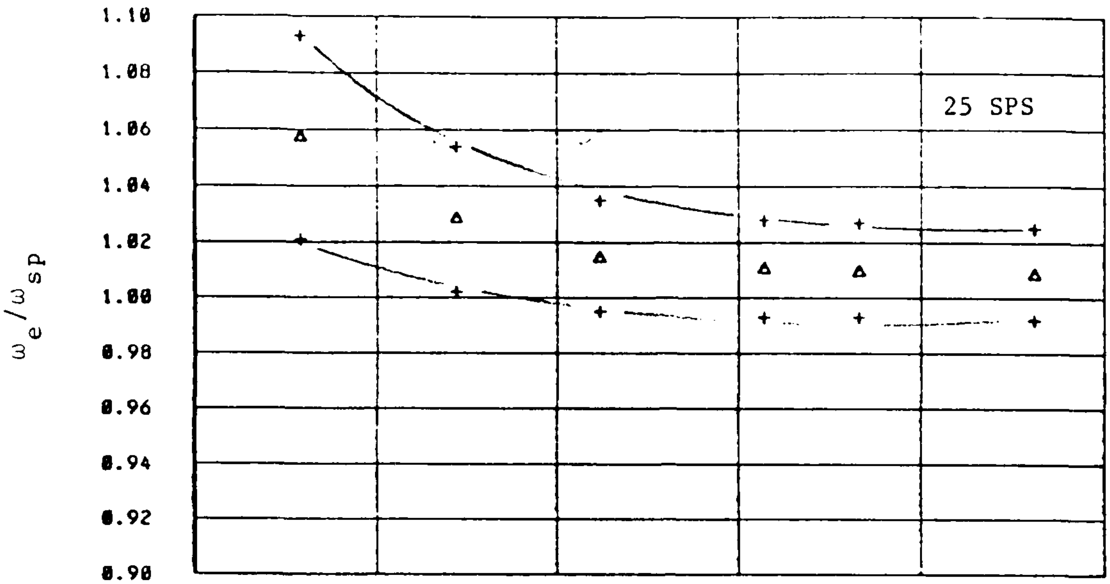
Variation in $\tilde{\tau}$ Estimate Accuracy With Sampling Rate
Figure 14

As can be seen from the preceding graphs, the accuracy of the estimate increases as the sampling rate increases. With a higher sample rate, a more accurate estimate of all parameters is obtained. Of particular interest, the estimate of the time delay, $\tilde{\tau}$, increases in accuracy with a higher sample rate. From these results, it appears that a data rate of 20 to 25 samples per second will yield good estimates through MMLE3. It also appears that a data rate of 8 samples per second will yield marginally acceptable estimates.

3.6 Effect of Measurement Noise on Estimate Accuracy

Every instrumentation system will introduce some noise and inaccuracies. These inaccuracies and the noise in the measurements complicate the estimation problem by "hiding" the signal. The overall effect of the noise, at best, is to increase the Cramer-Rao bounds of the estimate. At worst, it will completely "hide" the signal so that no parameters can be identified. A suitable measurement of the amount of noise present in the data was necessary. Since simulated data was used, a form of signal-to-noise ratio (SNR) was used. For the graphs which follow, the SNR is the ratio of the magnitude of the largest observed response to the root mean square (RMS) of the noise covariance.

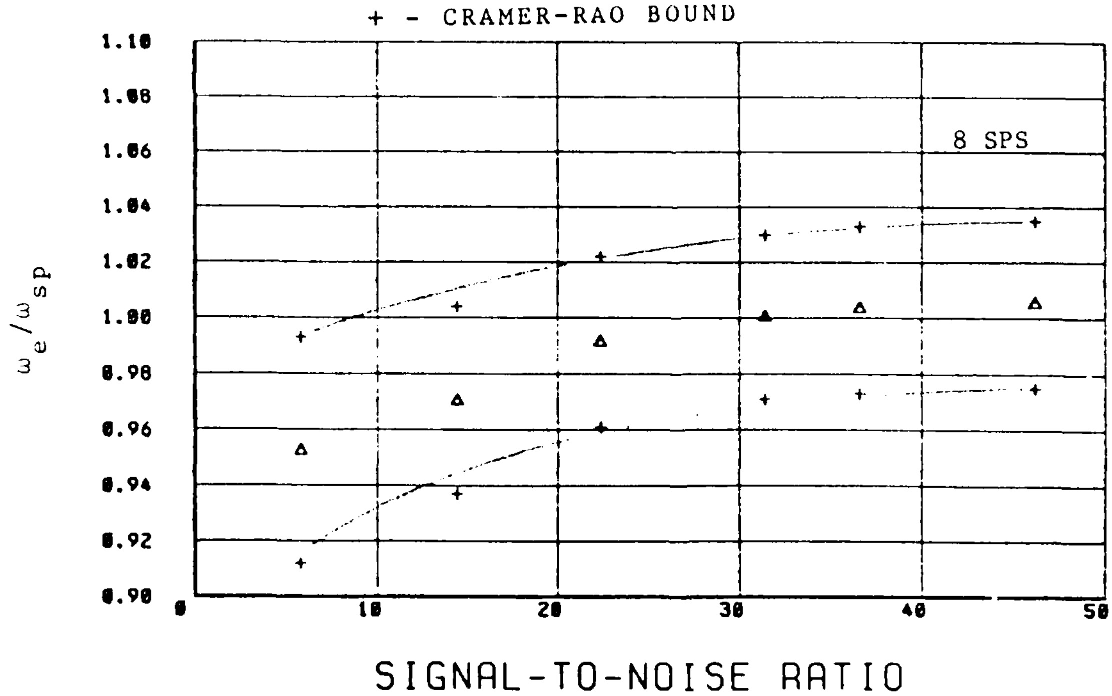
To generate noise corrupted data, a subroutine which generated zero-mean, white noise was incorporated into the simulation. The RMS level was adjusted to match the $\pm$ 1 degree/second accuracy of the USAFTPS instrumentation systems. The simulated noise-corrupted data was generated using the basic simulation program. After noise-free data was generated, the white-noise subroutine was called to introduce noise into the pitch rate respone. The SNR was adjusted by varying the size of the control input while holding the noise RMS constant. Analysis of the effect of noise on estimate accuracy was done at a data rate of 8 sps and 25 sps. Figures 15, 16, and 17 display the results of the effects of different signal-to-noise ratios on estimate accuracy.



LEGEND

Δ - MAXIMUM LIKELIHOOD ESTIMATE

+ - CRAMER-RAO BOUND



Effect of Noise on ω_e Estimate Accuracy
Figure 15

lues as well. The MMLE3 results are slightly higher than the cepted values. The difference in aircraft motion, though, tween a 0.30 damping ratio and a 0.36 damping ratio would be discernable to a pilot. The accepted values for the damping tio, and the MMLE3 results, are both essentially constant, rresponding to the theoretical prediction for short period mping ratio at a constant altitude. Overall, the MMLE3 results r damping ratio are accurate.

The MMLE3 results for T-38 short period motion, both equency and damping ratio, match the accepted values. This ows that the USAF TPS in-flight instrumentation system is equate for use with MMLE3. This also demonstrates a basic ying qualities data reduction capability using MMLE3. ocessing the test flight's data with MMLE3 showed two main strictions on the type of useable data.

The primary restriction on the data is that the time history the maneuver included a stable point immediately prior to the neuver. The MMLE3 results for elevator doublet inputs at 240 AS/0.48 Mach and 400 KIAS/0.78 Mach are absent from Table 3. is is because the data time history for those FTTs were useable by MMLE3. At these test points, the elevator doublet s flown immediately after the $n/_{\alpha}$ sweep, without pausing to trim the aircraft for stable flight. The overall time history d not have a good stable point prior to the FTT time history. thout a stable point against which to measure perturbations, e MMLE3 results failed to converge. As shown by Table 4, the tual time history length of the stable point is unimportant.

.3 Test Results and Analysis

Table 3 summarizes the MMLE3 results and shows the accepted alues for T-38 short period frequency and damping ratio for the est points.

The MMLE3 results for short period frequency correspond losely to the accepted values. The low speed point, 200 IAS/0.48 Mach, and the high speed point, 400 KIAS/0.78 Mach, ielded MMLE3 results which almost exactly matched the accepted alues. The accepted values were extracted from the results ublished in AFFTC-TR-61-15, T-38A Category II Stability and ontrol Tests. Figure A-1, in Appendix A, is a copy of the graph n AFFTC-TR-61-15 which summarizes aircraft short period haracteristics. The curves in Figure A-1 are faired to fit both heory and the original flight test data. The original data oints are near 0.48 Mach and 0.78 Mach. This accounts for the lose match between the MMLE3 results and the accepted values at hese test points. The MMLE3 results for the mid-speed point, 00 KIAS/0.61 Mach, don't match the accepted value quite as well. s can be seen in Figure A-1, the accepted value is an estimate, ased on the curve faired to fit the flight test data. The 15% ifference between the accepted value and the MMLE3 result is mall enough to be attributed to the way the curve in Figure A-1 s faired. However, the results from the second order fit of itch rate output to elevator position input support the other IMLE3 results for this point. Overall, it is apparent that the IMLE3 results for short period frequency are accurate, based on he accepted values.

The MMLE3 results for damping ratio don't match the accepted

FLIGHT TEST RESULTS
T-38 SHORT PERIOD FREQUENCY AND DAMPING RATIO
15,000 FEET PRESSURE ALTITUDE

TEST POINT	FTT	MMLE3 RESULTS $\omega_{e_{sp}}$ $(\frac{rad}{sec})$		MMLE3 RESULTS $\zeta_{e_{sp}}$		ACCEPTED VALUES[1] $\omega_{n_{sp}}$	ACCEPTED VALUES[1] ζ_{sp}
248 KIAS/ .48 Mach	n/α Sweep	2.39	(0.08)	0.35	(0.06)	2.36	0.30
300 KIAS/ .61 Mach	Elevator Doublet	2.68	(0.08)	0.36	(0.04)	3.25	0.30
	Low Frequency Sinusoidal Control Input	2.77	(0.07)	0.36	(0.05)		
	n/α Sweep	2.77	(0.06)	0.36	(0.04)		
400 KIAS/ .78 Mach	n/α Sweep	4.14	(0.09)	0.36	(0.04)	4.28	0.30
300 KIAS/ .61 Mach	Elevator Doublet[2]	2.75	(0.05)	0.30	(0.04)	3.25	0.30
	Low Frequency Sinusoidal Control Input[2]	2.77	(0.04)	0.28	(0.03)		
	n/α Sweep[2]	2.80	(0.05)	0.23	(0.03)		

[1] Values extracted from AFFTC-TR-61-15, <u>T-38A Category II Stability and Control Tests</u>

[2] Values determined using second order state-space model with elevator position input and pitch rate output.

(*.**) indicates the estimate's Cramer-Rao bounds

Flight Test Results

Table 3

for hands-off flight between each FTT. In addition to the n/α sweep and elevator doublet, a low frequency sinusoidal control input was flown. The control input frequency was low relative the aircraft's damped frequency. The additional data gathered at this test point was intended for use as a comparison to the results of the simulation study, as reported in the previous chapter.

$$
\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_e^2 & -2\zeta_e\omega_e & K_q \\ 0 & 0 & LAG \end{bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ LAG \end{Bmatrix} \delta_p(t)
$$

$$
q(t_i) = \begin{bmatrix} 1/T_{\Theta 2} & 1 & 0 \end{bmatrix} \begin{Bmatrix} x_1(t_i) \\ x_2(t_i) \\ x_3(t_i) \end{Bmatrix} \tag{27}
$$

The value $1/T_{\Theta 2}$ was experimentally determined to be 1.00 using the relationship (ref 7).

$$
1/T_{\Theta 2} \cong (n_z/\alpha)(g/V_{true}) \tag{28}
$$

The value LAG was allowed to vary.

Allowing the value LAG to vary opens up different data analysis questions. To avoid these questions, and to serve as a check of the frequency and damping ratio results, some data was also processed using the second order state-space model of equation (21), with pitch rate as the output and elevator position as the input.

4.3 Test Methods and Conditions

Only one data flight was flown. Table 3 summarizes the test points and conditions.

At each test point, the aircraft was trimmed at test conditions for stable hands off flight. An n/α sweep, as described in Chapter 3, followed by an elevator doublet were then performed. These FTTs were used to generate the time history data for MMLE3.

The 300 KIAS/0.61 Mach test point was the primary test point for this flight. At this test point, the aircraft was trimmed

Chapter 4

4.1 Flight Test Objectives

A limited amount of flight test data was collected to:

1. Determine whether or not the current USAF TPS in-flight instrumentation system is adequate for use with MMLE3, and

2. Assuming that the instrumentation is adequate, provide the USAFTPS a flying qualities data reduction system using MMLE3.

Both of these objectives could be met by successfully using MMLE3 to reduce flight test data.

The primary USAF TPS aircraft are the NA-37, T-38, and the RF-4C. These aircraft are simple aircraft, dynamically, with longitudinal dynamic responses which are adequately described by fourth order equations. The dynamic responses of these aircraft are known, and can be compared to MMLE3 results as a check on reasonability. The T-38 was used for this test because the author was qualified in that aircraft.

4.2 Data Reduction

The flight test was intended to yield aircraft short period frequency and damping ratio data. The aircraft pitch rate, q, was the output, and the longitudinal elevator control position, δ_p, was the input. The in-flight instrumentation system samples these parameters at 8 samples per second. The data was not pre-processed to filter any noise or to remove any control hysteresis, such as friction and breakout forces. Because of this, the state-space model used in MMLE3 included a second order short period response augmented with a first period lag. The lag was used to account for any control surface servo dynamics or hysteresis. Equation 27 shows the model.

Since only simulated data was used, a qualitative analysis of the effect of sampling rate on estimate accuracy was made. As expected, with higher sampling rates, more accurate estimation can be expected for all parameters.

The effect of noise on estimate accuracy was also qualitatively analyzed. With a SNR of 20 or better, reasonable estimates (compared to no noise present) could be obtained. With higher sampling rates, the noise had slightly less effect, resulting in more accurate estimates.

The final test of MMLE3 was determining lower order equivalent systems. Simulated data was generated, incorporating a lag filter into the simulation model. The data was then processed by MMLE3 to yield a lower order equivalent system. As expected: ω_e was slightly less than ω_{sp}, ζ_e was slightly higher than ζ_{sp}, and the estimated time delay, τ_e, was close to the time to one-half amplitude for the lag filter used in the simulated data.

Lag Filter Constant	$T_{1/2}$ (sec)	8 SPS (sec)	25 SPS (sec)
5.0	0.139	0.117	0.106
10.0	0.069	0.078	0.068
15.0	0.046	0.063	0.050
20.0	0.035	0.058	0.039

Equivalent Time Delay Estimates

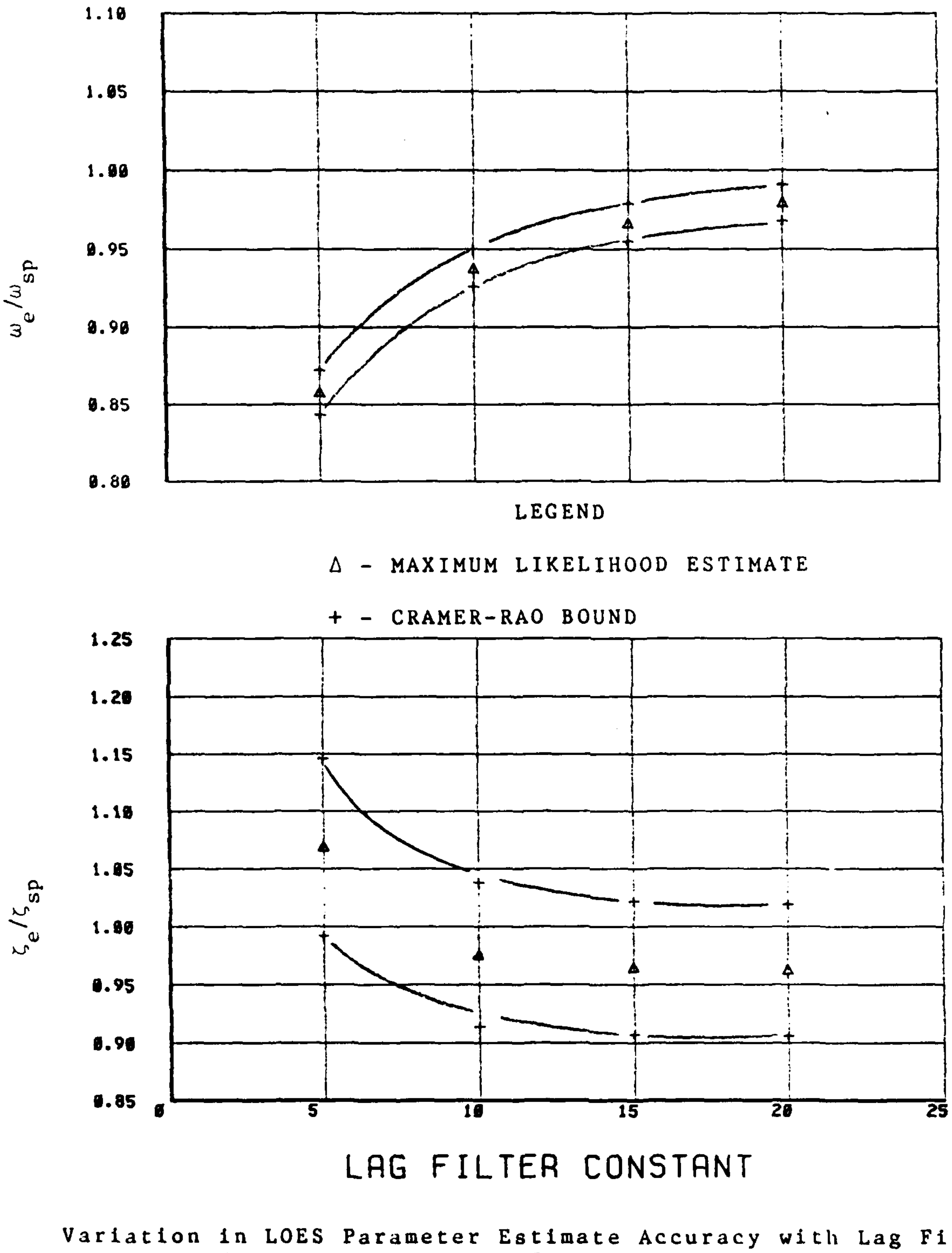
Table 2

As can be seen from Figures 19 and 20 and Table 2, the actual results were essentially what was expected. This test completed the testing of MMLE3 with simulated data, and verified that MMLE3 can be used to determine LOES parameters in the time domain.

3.6 Summary of Chapter 3

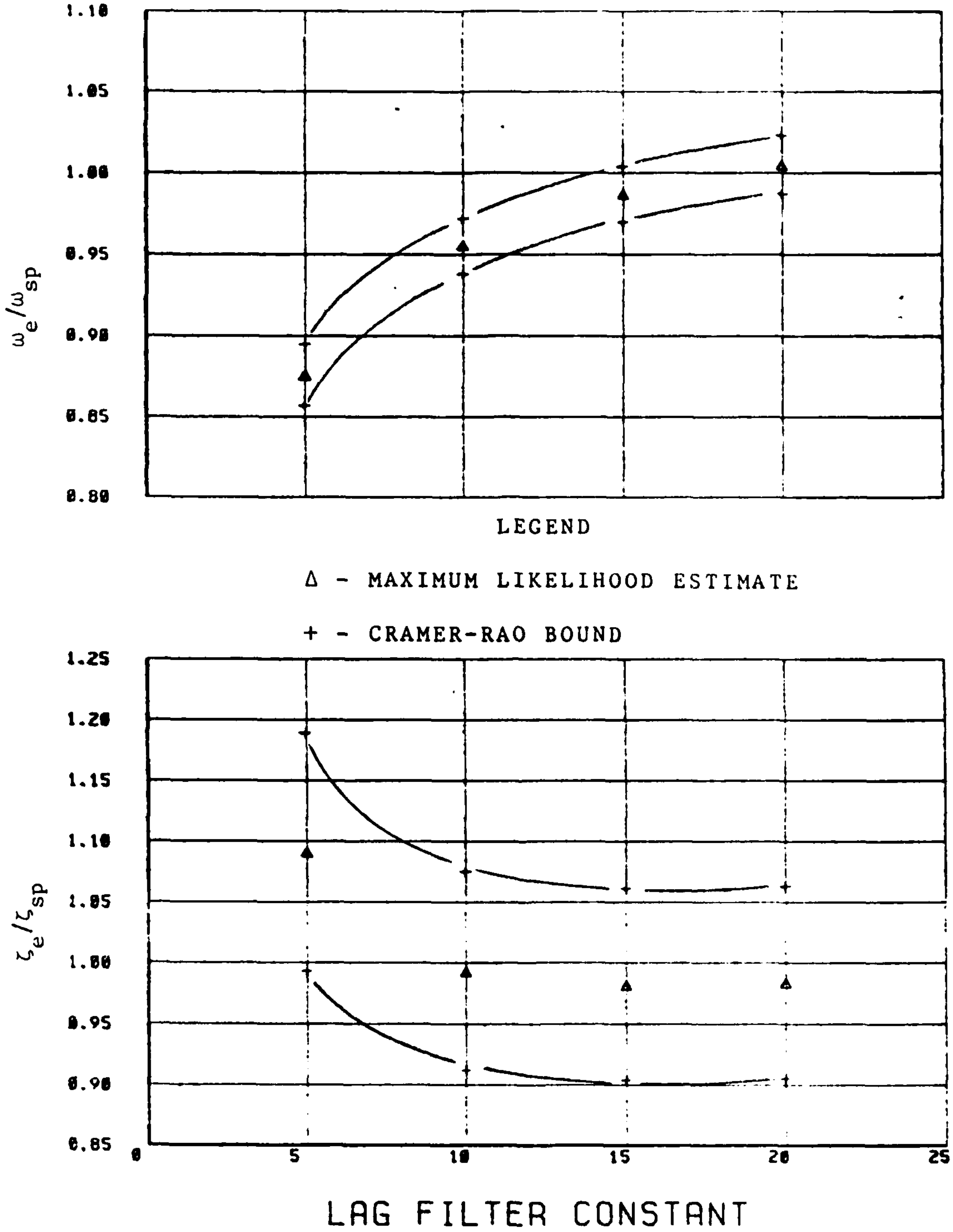
Based on the results of testing MMLE3 on simulated data, it was apparent that MMLE3 was properly installed on the USAF TPS computer. Analysis of simulated data indicated that classical FTTs should be good techniques of exciting the aircraft short period response for parameter identification.

The data system which would be available for flight test was limited to 8 samples per second. For that reason, simulated data was generated at 8 samples per second to match the data system. Results for parameter estimation at a data rate of 8 samples per second were good, resulting in estimates of frequency and damping ratio within 2.5% of the true value.The time delay estimate was 20% high, which did not seem unreasonable when compared to other LOES determination techniques (ref 2).



Variation in LOES Parameter Estimate Accuracy with Lag Filter
(Data generated at 25 samples per second)

Figure 20



Variation of LOES Parameter Estimate Accuracy with Lag Filter
(Data generated at 8 samples per second)

Figure 19

$$\left\{\frac{\dot{\underline{x}}(t)}{\delta_e(t)}\right\} = \left[\begin{array}{c|c} A & B \\ \hline \underline{0}^T & -K \end{array}\right] \left\{\frac{\underline{x}(t)}{\delta_e(t)}\right\} + \left[\frac{0}{K}\right] \delta_p(t) \tag{26}$$

$$\underline{z}(t_i) = \underline{x}(t_i)$$

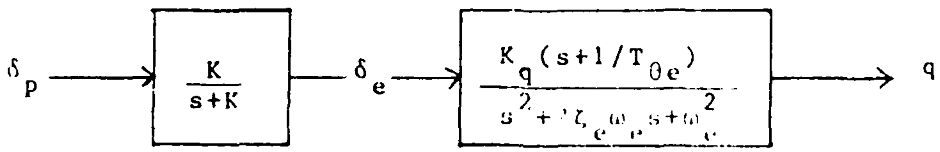
The data was propagated using the same propagation scheme as MMLE3, as explained in section 3.2 of this thesis. This data was used to test MMLE3 for identification of the defining parameters of an LOES, τ_e, ω_e and ζ_e .

It was expected that MMLE3 would successfully identify LOES parameters, with the following differences from the true characteristics of the system (ref 10). With a small lag filter constant, K, the equivalent frequency, ω_e , would tend to be smaller than the actual short period frequency. As K increases in value, ω_e should approach the true value ω_{sp} , but stay slightly low. The equivalent damping ratio, ζ_e , with a small value of K, should be large relative to the true damping ratio, ζ_{sp} . As K increases, ζ_e should approach ζ_{sp} , but stay slightly high. the equivalent time delay should be roughly the same value as the lag filter's time to one-half amplitude $T_{1/2}$. With higher sampling rates, ω_e and ζ_e should be essentially the same, but with smaller Cramer-Rao bounds. τ_e , the equivalent time delay, should be smaller with a higher sampling rate, but still close to the lag filter $T_{1/2}$. Figures 19 and 20 show the estimates for ω_e and ζ_e for 8 SPS and 25 SPS, respectively. Table 3 shows the equivalent time delays.

As can be seen, a signal-to-noise ratio of approximately 20 begins to yield estimates comparable to estimates from uncorrupted data. The effect of higher sampling rates on noise corrupted data is essentially the same as the effect of a higher sampling rate on uncorrupted data.

3.7 Identifying an Equivalent Time Delay

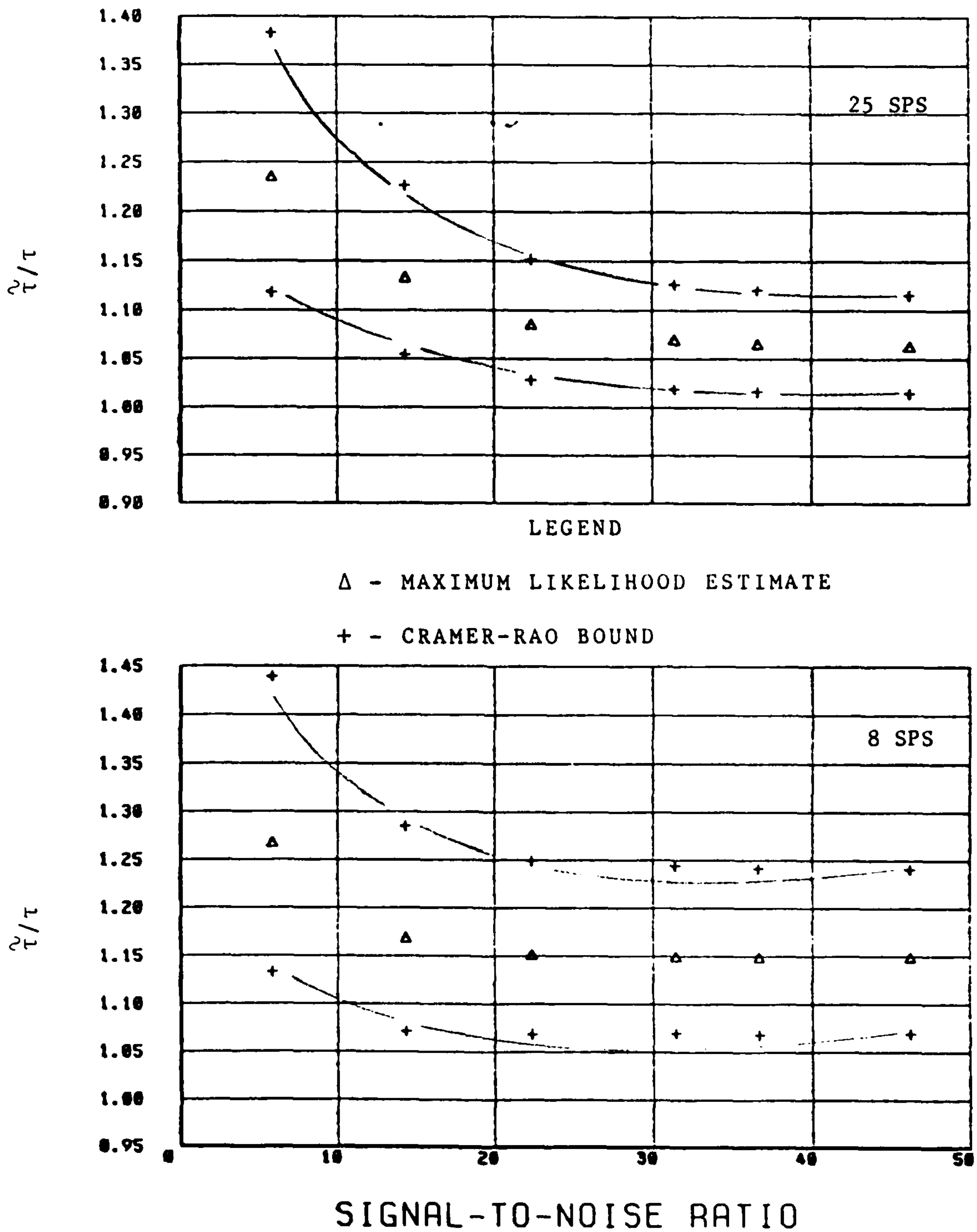
The model used in the simulation, up to this point, has directly linked the pilot's control input to the aircraft response. The time delay in the simulation has been a pure time delay, rather than one introduced by the flight control system. A more accurate simulation model would introduce a lag filter between the pilot's control input and the aircraft response to model delays caused by the flight control system (ref 2, 9, 10). The lag filter introduced into the simulation will effectively change the apparent short period frequency and damping ratio, as well as introducing an equivalent time delay, τ . Figure 18 shows the simplified block diagram for using a lag filter between pilot control input and pitch rate response.



Pitch Rate Response Relation to Pilot Input
(Lag Filter Incorporated)

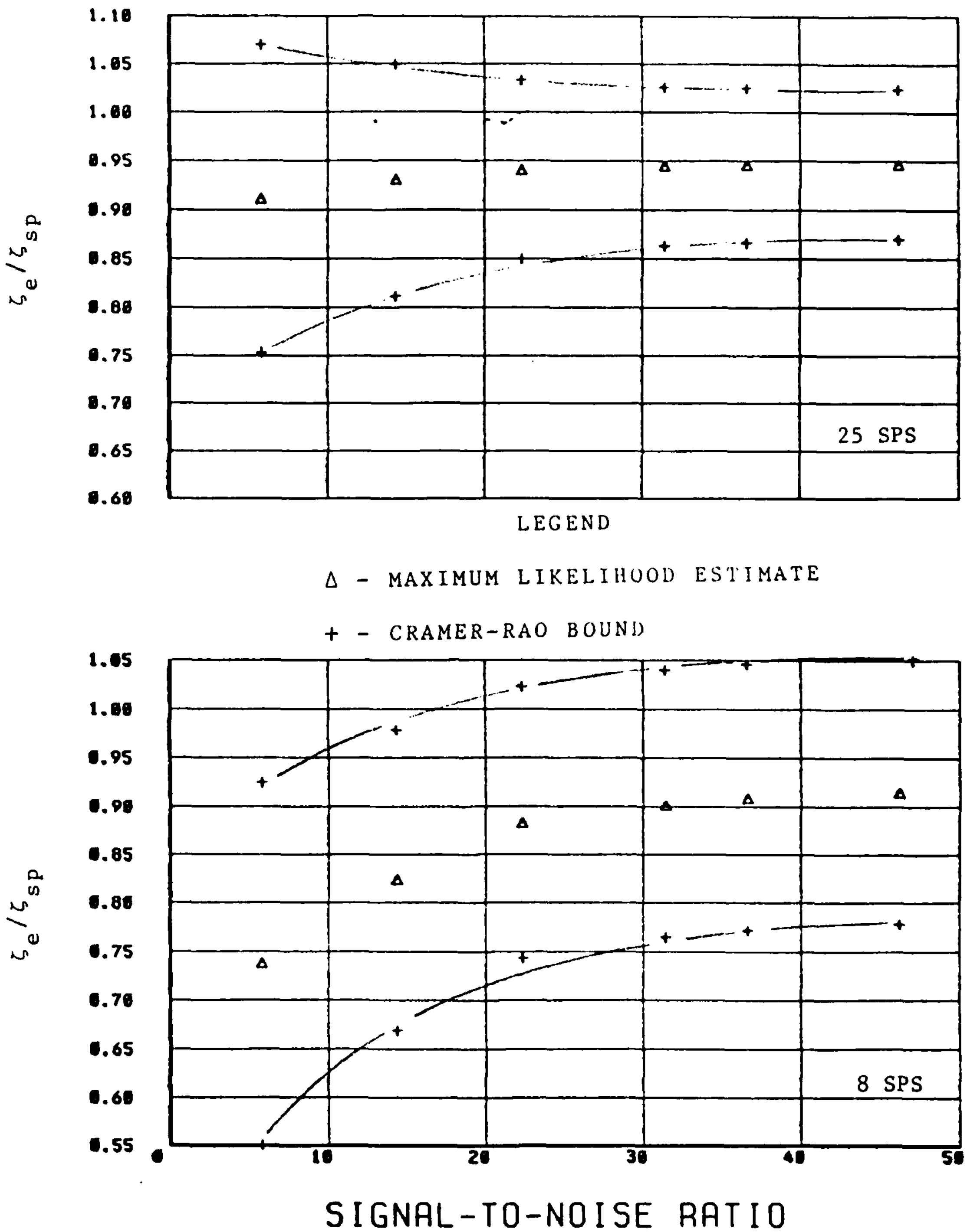
Figure 18

To generate data with a lag filter incorporated, the simulation model was modified to incorporate a lag filter. This new simulation model was:



Effect of Noise on $\tilde{\tau}$ Estimate Accuracy

Figure 17



Effect of Noise on ζ_e Estimate Accuracy

Figure 16

It is important, though, that the aircraft be trimmed for hands-off flight immediately prior to any FTT, and that some of the stable point be recorded along with the FTT time history.

Type FTT: n/α sweep
FTT Time History Length: 26.125 seconds

Stable Point Time History Length	$\omega_{e_{sp}}(\frac{rad}{sec})$	$\zeta_{e_{sp}}$
2.250 Seconds	2.77 (0.06)	0.36 (0.04)
0.125 Seconds	2.77 (0.06)	0.36 (0.04)

(*.**) indicates the estimate's Cramer-Rao bounds

Effect of Stable Point Time History Length
on MMLE3 Frequency and Damping Ratio Estimates

Table 4

The secondary restriction on the data is on the length of the time history. If an elevator doublet time history records from the stable point through the entire doublet, approximately 5 seconds, then the results will match the results shown in table 3 for the elevator doublet. For an n/α sweep, though, the time history length is more critical. The longer the time history, the more ccurate the estimates. Table 5 shows the effect of the FTT time history length on the estimates.

Type FTT: n/α sweep
Stable Point Time History Length: 2.25 seconds

Time History Length	$\omega_{e_{sp}} (\frac{rad}{sec})$	$\zeta_{e_{sp}}$
26.125 Seconds	2.77 (0.06)	0.36 (0.04)
12.000 Seconds	2.82 (0.07)	0.37 (0.07)
6.000 Seconds	2.91 (0.11)	0.40 (0.07)

(*.**) indicates the estimate's Cramer-Rao bounds

Effect of Time History Length
on MMLE3 Frequency and Damping Ratio Estimates

Table 5

Based on the results shown in table 5, it seems reasonable that a minimum of a 15 second time history, from the stable point and initiation of the n/α sweep, would be necessary to yield a good estimate of frequency and damping ratio.

4.4 Summary of Chapter 4

The purpose of the flight test was to determine if the current USAFTPS in-flight instrumentation system could provide data of high enough quality for reduction by MMLE3. If the data was good enough, the flight test data would be used to demonstrate a flying qualities data reduction system. The output of this data reduction system would be the aircraft short period natural frequency and damping ratio.

As shown by the results, the current USAFTPS in-flight instrumentation system is adequate to provide MMLE3 quality data. The single test flight was used to gather T-38 short period motion data, and to process that dat through MMLE3. The MMLE3 results for T-38 short period frequency and damping ratio closely matched the accepted values for T-38 short period frequency and damping ratio. The data processing showed two restrictions on what type of data was useable for MMLE3. The time history of the FTT must be immediately preceded by a stable point, where the aircraft is trimmed for hands-off flight. This stable point must be included in the data time history for MMLE3. Additionally, if the maneuver being analyzed is an n/α sweep, the total time history (including stable point) should be at least 15 seconds to ensure good results from MMLE3.

Chapter 5

5.1 Conclusions

This thesis accomplished all objectives. The USAF TPS currently has a working version of MMLE3, a general maximum likelihood parameter estimation program installed and operating on its computer system.

Based on simulation testing, lower order equivalent systems can be determined from time domain data using MMLE3. With high data sampling rates, highly accurate estimates can be obtained. With low data sampling rates, marginally acceptable estimates are obtained.

Based on both simulation testing and flight test data, MMLE3 can be used for basic flying qualities data reduction. Flight test data obtained from a USAF TPS T-38 using the USAF TPS in-flight instrumentation system showed that this data was of sufficient quality to yield good estimates of frequencies and damping ratios. To get good results, the maneuver in flight has to start from a stable point so that MMLE3 has good initial conditions to work from.

5.2 Recommendation

Based on the results of testing, MMLE3 can be used to determine LOES parameters using time history data. It can be also be used to determine basic flying qualities data. The following recommendations are made:

1. MMLE3 should be incorporated into the USAFTPS curriculum for basic flying qualities data reduction.

2. MMLE3 should be incorporated into the systems phase of the curriculum for any associated LOES testing.

List of References

1. MIL-F-8785C, Military Specification: Flying Qualities of Piloted Airplanes., 5 November 1980.

2. Hoh, Roger, Lecture on "Highly Augmented Airplane Flying Qualities", USAF Test Pilot School, March 1984.

3. Maine, Richard E. and Iliff, Kenneth W., User's Manual for MMLE3, a General FORTRAN Program for Maximum Likelihood Parameter Estimation., NASA Technical Paper 1563, November 1980.

4. Maine, Richard E., Programmer's Manual for MMLE3, a General FORTAN Program for Maximum Likelihood Parameter Estimation., NASA Technical Paper 1690, June 1981.

5. Maybeck, Peter S., Stochastic Models, Estimation, and Control Academic Press, Inc., New York, 1979.

6. Eykhoff, Pieter, System Identification, John Wiley and Sons, New York, January 1979.

7. McRuer, Duane, Ashkenas, Irving, and Graham, Dunstan, Aircraft Dynamics and Automatic Control., Princeton University Press, Princeton, New Jersey, 1973.

8. Iliff, Kenneth W. and Maine, Richard E., Practical Aspects of Using a Maximum Likelihood Estimation Method to Extract Stability and Control Derivatives From Flight Data, NASA Technical Notes D-8209, 1976.

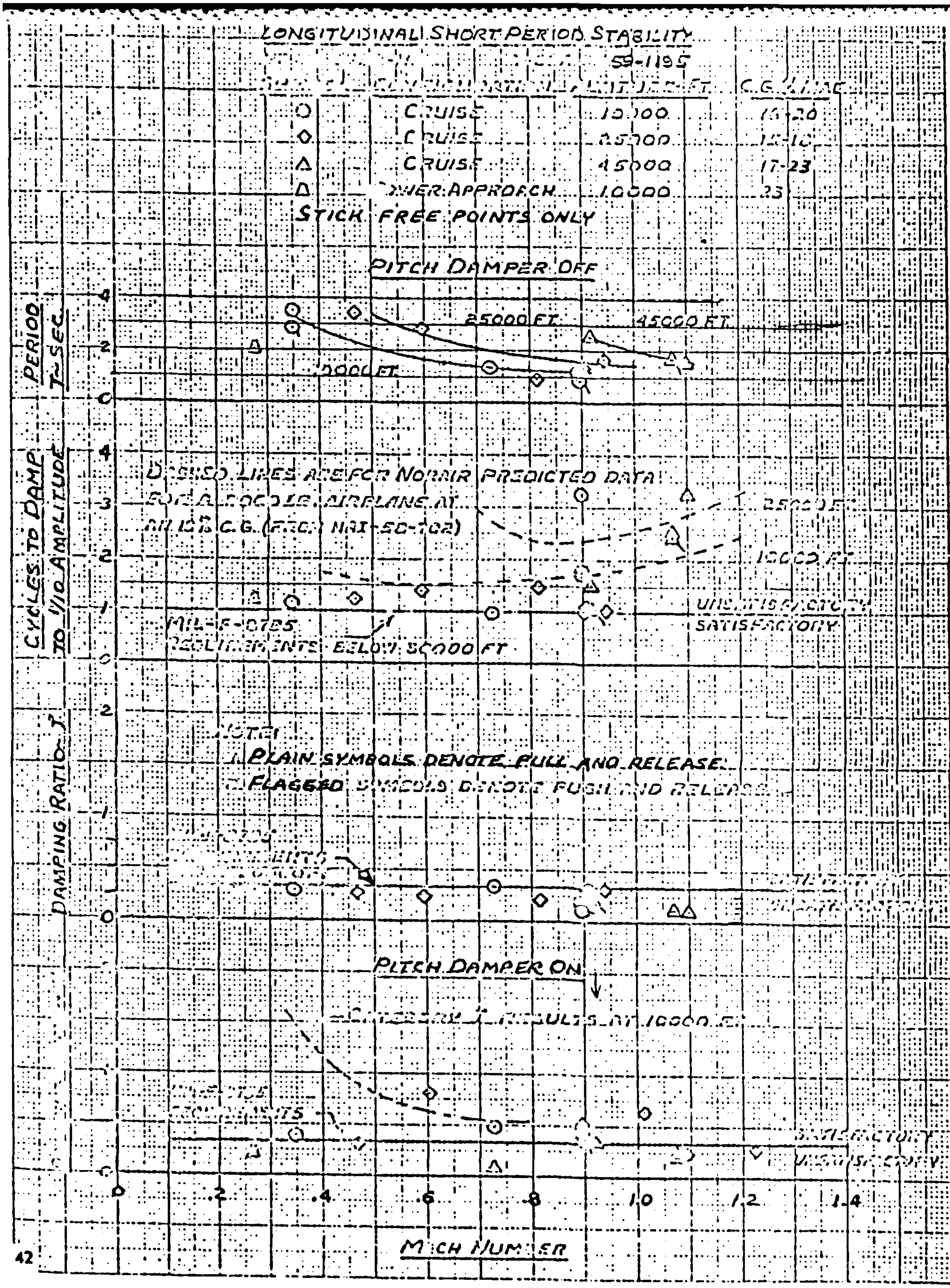
9. FTC-TIH-79-2, Flying Qualities and Flight Test Techniques, USAF Test Pilot School, Edwards AFB, CA, August 1979.

10. Ball, J., Berthe, C., Knotts, L., Parrag, M., Smith, R. Flight Syllabus and Background Material for the U.S. Air Force/U.S. Navy Test Pilot School Variable Stability Programs., Advanced Technology Center, Applied Technology Group of Arvin Industries, Inc., January 1983.

11. Lusby, William A. Jr., et al, AFFTC-TR-61-15, T-38A Category II Stability and Control Tests, Air Force Flight Test Center, Edwards AFB, Ca, August 1961.

Appendix A

T-38 Longitudinal Short Period Data



Longitudinal Short Period Stability
Figure A-1

The log-decrement method of data reduction, as performed here, is explained in reference 9, on pages 7.59 through 7.64. The strip chart record is the basis for this data reduction method. The free response, after an elevator doublet, is used to determine short period frequency and damping ratio.

The control input used to define the end of the forced response and the beginning of the free response, was longitudinal control position, as marked on the strip chart in figure A-2. Pitch rate response was used as the aircraft free response.

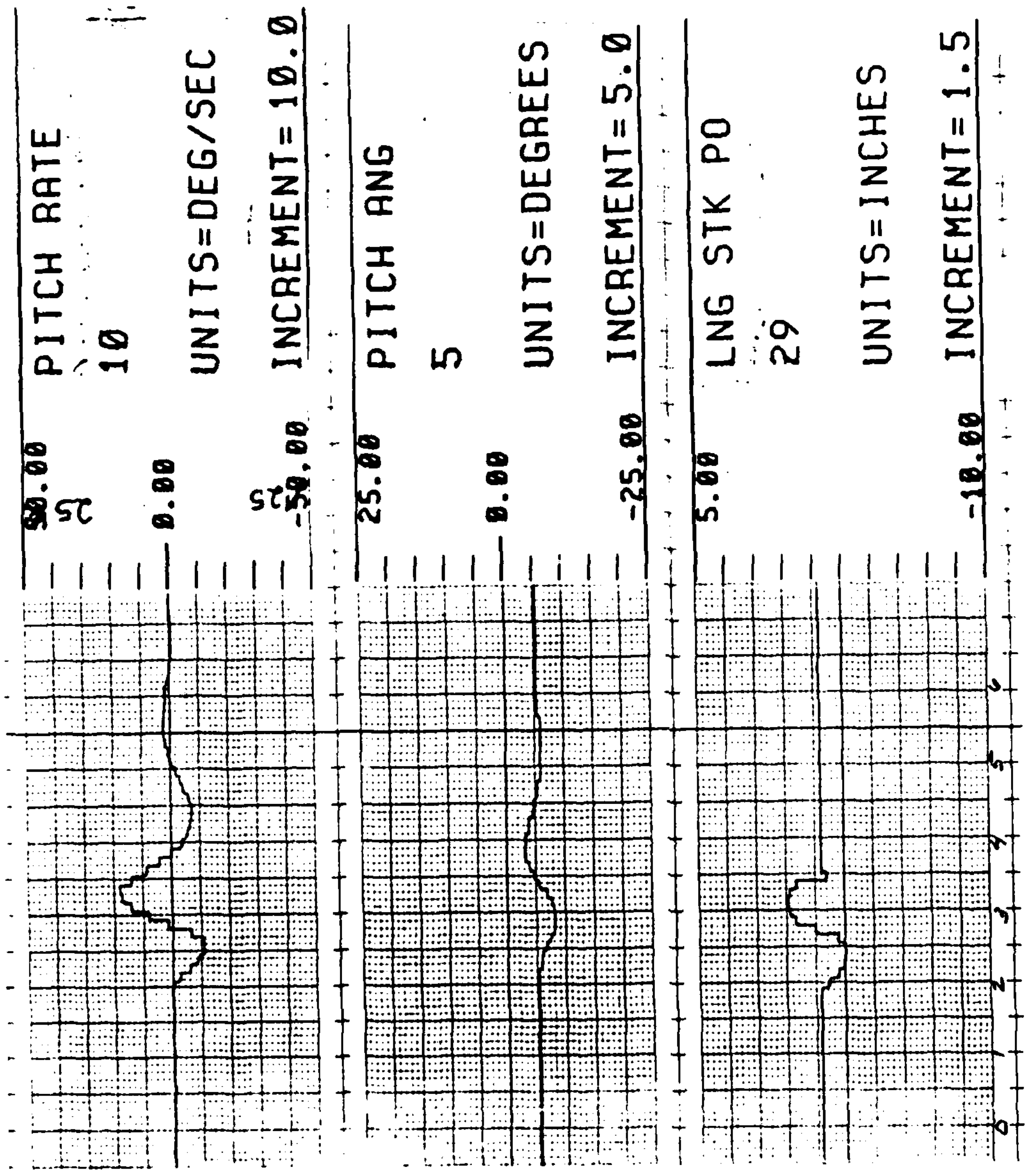
Three free response "peaks" can be seen. These peaks are the magnitude of the overshoot from the steady state condition. Using a computer printout of the response, it is easier to determine the values of these peaks, although not necessarily more accurate. The peaks were:

$X_0 = 3.4102$	$X_1/X_0 = 0.2857$
$X_1 = 0.9751$	$X_2/X_1 = 0.6662$
$X_2 = 0.6496$	$X_2/X_0 = 0.1905$

The time between the peaks were 1.229 seconds and 1.293 seconds. Treating that as the damped short period, the damped short period frequency is 2.49 rad/sec.

Using the subsidence ratios and using the chart in reference 9 on page 7.62, the damping ratio is approximately .240.

These results are from fitting only three points to a second order response model, as defined by the charts used. This data reduction technique is very subjective, and prone to interpretive errors.



Sample Flight Test Time History

Figure A-1

Appendix B

Preparation of Time History for MMLE3

As stated in section 2, MMLE3 only needs an adequate state-space model and a time history relating input to output to generate good estimates. At the beginning of the time history data file, the state-space model is defined by the user. For this appendix, the input for the time delay state-space model (equation 25) will be shown and explained. The beginning of the data file is:

```
NO USE OF STANDARD AIRCRAFT ROUTINES
000000125 000010000
AN       3          3

  -19.60     -4.214        -9.772
                          -16.0
BN       3          1
    0.0
    4.886
   16.0
AV
    0.        0.           0.
    1.        1.           1.
    0.        0.           1.
CN       1          3
    1.372     1.           0.
GGI      1          1
40000.
HARD
BN(02,01)=AN(02,02)*-.5
BN(03,01)=AN(03,03)*-1.
END
```

The first lie indicates to MMLE3 that no aircraft stability derivatives are being determined. Rather than using a predetermined state-space model, the state-space model defined by the input matrices will be used. The next line defines the start and stop times for the data to be processed. The time history data for an entire flight can be in one data file, but the different test points of that flight can be analyzed separately by MMLE3 if the start and stop times for the different FTTs are known. These times are in hours, minutes, seconds, and illiseconds. The Fortran format for this date line is 2(3I2,I3,IX)).

The 'N' suffixed matrices define the state-space model. The tate-space model which MMLE3 uses is

$$\underline{x}(t)=A\underline{x}(t)+B\underline{u}(t)$$
$$\underline{z}(t_i)=C\underline{x}(t_i) \qquad (29)$$

he 'N' suffixed matrices define the matrices define the matrices f the above equations; and give the initial estimate of what the nswer should be.

The 'V' suffixed matrix define exactly which elements of the atrix are allowed to vary. If the value 1. appears in the AV atrix, then the corresponding value in the AN matrix can be hangedby MMLE3. The 'V' suffixed matrices define thevector .

The inverse noise covariance matrix GGI is identical to the atrix R used in Section 2.2 of the thesis. It is not articularly for determination of lower order equivalent system arameters. The GGI (R) matrix acts as a weighting matrix for he different output states being estimated. Since only one utput state, pitch rate, is used in this model, GGI becomes nimportant. Varying GGI will not vary the maximum likelihood stimates or the Cramer-Rao bounds.

The 'HARD' line following the GGI input defines restraints n the solution. For the time delay model, both the transfer unction gain and the time delay appear in both the AN matrix and he BN matrix. This constraint forces MMLE3 to treat these alues properly when varying the value in one of the matrices.

The line containing 'END' signals the end of user input atrices and the beginning of the time history data.

Only the matrices which are necessary to the state-space del need to be input. This adaptation of MMLE3 can accept up 5 state model for either input or output.

For the purposes of USAF TPS use of MMLE3, the time history ta files are generated automatically. However, if a user wants use MMLE3 in a different manner, then a different state-space del will have to be set up. For more information on MMLE3, and tting up data files, see reference 3.

VITA

Richard A. Schroeder was born on 27 January 1955 in waukee, Wisconsin. He graduated from high school in Lake est, Illinois in 1972 and then attended the USAF Academy. er graduation from the USAF Academy in 1976 with a degree in ineering Mechanics, he attended pilot training. Upon pletion of pilot training, he was assigned as a C-141 pilot to 14 Military Airlift Squadron at Norton AFB, California. This ignment lasted from January 1978 until July 1981, when he was ssigned to the 4953 Test Squadron at at Wright-Patterson AFB, o, as a C-141 instructor pilot and research pilot. In June 2, he began the joint Air Force Institute of Technology/USAF t Pilot School Master's Program. In June, he completed the demics portion of the program at AFIT, and entered USAF Test ot School class 83B at Edwards AFB, California. In June 1984, n completion of the Test Pilot School, he was reassigned to 6512 Test Squadron at Edwards AFB as an experimental test ot.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

MMLE3, a general maximum likelihood estimator, is used for determination of lower order equivalent systems for aircraft longitudinal motion. The lower order equivalent systems are defined by time delay, equivalent frequency, and equivalent damping ratio for short period motion. Simulation test results showed determination of equivalent frequency and damping ratio to within ±2.5% of true value and determination of time delay to within 18% of the true value. Flight test results showed similar results. Overall, the technique of determining lower order equivalent systems using time-domaindata and a maximum likelihood estimator is a technically feasible and accurate data reduction method.

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