DEVELOPMENT OF COMPUTER PROGRAMS USING THE METHOD OF IMAGES TO PREDICT THE SOUND FIELD IN A WEDGE OVERLAYING A FAST FLUID AND COMPARISON WITH LABORATORY EXPERIMENTS

by

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**Abstract**

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Development of Computer Programs Using the Method of Images to Predict the Sound Field in a Wedge Overlaying a Fast Fluid and Comparison with Laboratory Experiments

by

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1. **INTRODUCTION**

Several developments based on the method of images have been used to predict the sound field in a wedge shaped fluid medium. In 1978, with the aid of computer models, Coppens, Sanders, Ioannou, and Kawamura (Ref. 1) predicted the pressure amplitude and phase in the upslope direction along the bottom of a wedge shaped fluid layer overlaying a fast fluid bottom. One of the models that they used employed a source at infinity, while the other allowed the source to be placed a finite distance from the wedge apex. Both models allowed the source depth to vary.

In a further development, Baek (Ref. 2) in 1984 expanded on this approach by using a source at infinity to predict pressure amplitude and phase everywhere within the wedge in the upslope direction. As with the previous models, the source position could be varied in depth. The geometry is shown in Figure 1. Baek's program, named WEDGE, was validated for several simple cases, such as a pressure release bottom and a rigid bottom. Further exercise of the model was recommended.

To date, models have been limited to predictions directly upslope. That is, the source has to be further from the apex than the receiver and, as shown in Figure 1, both the source and receiver have to be in a plane.
perpendicular to the shoreline and perpendicular to the surface.

A computer program overcoming these shortcomings was obtained [Ref. 3]. The computer model, named CROSS SLOPE, predicts pressure amplitude and phase anywhere within the wedge. A typical geometry for an upslope prediction is shown in Figure 2. The following definitions apply and will be used throughout:

- \( \theta \) = wedge angle
- \( R_1 \) = normalized source distance
- \( \gamma \) = source angle measured upward from the interface
- \( R_2 \) = normalized receiver distance from the shoreline
- \( \delta \) = receiver angle measured upward from the interface
- \( Y_0 \) = normalized distance along the shore between the receiver and the source
- \( D_1 \) = the ratio of the medium density to the bottom density (density ratio)
- \( C_C \) = the ratio of the speed of sound in the medium to the speed of sound in the bottom (speed of sound ratio)
- \( X_L \) = the wave number in the wedge medium divided into the absorption in the bottom
All distances are normalized in units of the dump distance. A dump distance $X$ is the distance measured perpendicular to the shore from the apex to the point where the depth is equal to the minimum uniform depth that can support propagation of the lowest normal mode. This distance is a function of the wavelength $\lambda$ in the wedge, wedge angle $\beta$, and the critical angle of the bottom $\theta_c$.

$$X = \frac{\lambda}{4} \sin(\theta_c) \tan(\beta)$$

A detailed development of this concept is outlined in Reference 2. In addition, the source amplitude is taken to be proportional to the source distance so that the predicted pressure would not become vanishingly small for large source distances.

The purpose of this research was to:

1. implement, test and evaluate the CROSS SLOPE program in a high speed computer language,

2. test the WEDGE program in the same conditions as the CROSS SLOPE program and compare the two programs where appropriate,

provide output in a graphic form for ease of interpretation,
position of maxima is not effected (note: the dump distance automatically scales the change in true distance in these graphs). Calculations for a six degree wedge and a three degree wedge are shown (Figure 16 through 19).

D. SOURCE POSITION

Changes in the source position had a noticeable effect on the position of the relative maxima of pressure. As the source was moved outward, a corresponding proportional movement of the receiver parallel to the apex produced the same relative pattern. That is, if the source was moved outward to twice its original position, a plot of receiver position as a function of twice $Y_0$ produced the same graphic presentation (Figure 20). This is consistent with mode theory which predicts that the position of the maxima parallel to the apex is inversely proportional to the source distance.

The wedge was also evaluated for changes in receiver angle at a constant source distance in the upalope direction. This produced varied patterns depending on the modal interaction present. Comparison of calculated versus experimental values is the subject of Chapter IV.
When all three of these values were changed the position and number of relative maxima remained the same, however higher modes were depressed. Figures 8 and 9 show the pressure amplitude and the relative maxima respectively, for this test condition.

B. RECEIVER ANGLE

The model was next tested for response to changes in the receiver angle. Theory predictions that the pressure maxima should be independent of the receiver angle (see Appendix A). The three dimensional plots of the relative maxima for different receiver angles show such a relationship. The pressure field retains its characteristic shape at a given receiver angle as shown in Figures 10, 11, and 12. Figures 13 through 15 show that the relative maxima occur in the same locations regardless of angle of the receiver.

C. WEDGE ANGLE

As the wedge angle decreases, the dump distance increases. This can easily be seen from the definition of the dump distance. The increase in dump distance allows the modes to interact over a greater distance. As a result, more modal interference is expected as the wedge angle is decreased. The CROSS SLOPE model shows such an effect. The increase in modal interaction is indicated by the increase in relative maxima and decrease in pressure amplitude. The
The model was tested by varying one of the base parameters and holding the others constant. Comparison of plots for relative maxima as well as three-dimensional plots were then used to determine the effect of each parameter. After each case the model was returned to the base case and another parameter was varied. Finally the model was evaluated for the parameters of an experimental model. The result of those tests are discussed in Chapter IV.

A. SPEED OF SOUND RATIO, DENSITY RATIO, ATTENUATION

The base case was first examined for the effects of changing the density ratio. The positions and number of maxima exhibited little change as this ratio was decreased to 0.5.

The speed of sound ratio was then varied down to a value of 0.5. Again the positions and number of relative maxima showed only minor variations. This suggests that the major features of the sound field in the wedge are relatively insensitive to minor variations in the characteristics of the bottom.

Finally the value of XL was increased from 0.0001 to 0.01. The pattern did not change significantly, but as expected the amplitude of the maxima showed a sharp decrease as the attenuation increased (XL increased).
investigation, it was determined that the pattern differed by a factor of one over the speed of sound ratio. This scaling was traced to a difference in the definition of the path length as it related to the dump distance. The error was determined to exist in the CROSS SLOPE program. After correction, the two patterns coincided at six hundred dump distances. A graph depicting the asymptotic approach of the finite source value for a pressure amplitude was made by taking a prominent pressure amplitude peak and recording the distance from the shore at which it occurs as the source was moved outward (Figure 5).

The effect of changing the properties of the wedge and source on the predictions by CROSS SLOPE were evaluated. As a base case, the following parameters were used (see page 10 for definitions):

- $\theta = 10.0^\circ$
- $\gamma = 5.0^\circ$
- $\delta = 0.0^\circ$
- $R_1 = 40.$
- $D_1 = 0.9$
- $CC = 0.9$
- $XL = 0.0001$

A three dimensional depiction of the pressure field for these conditions as a function of $R_2$ and $Y_0$ is shown in Figure 6. The corresponding relative maximum plot is shown in Figure 7.
III. EVALUATION

WEDGE had previously been tested for some known conditions (Ref. 3). These conditions were also tested for when the program was rewritten in Basic. The Basic version produced the results to the accuracy previously started. CROSS SLOPE was also tested for these same parameters with similar results. Additionally, CROSS SLOPE was tested along the apex. The program produced a pressure amplitude of 0.000000 except in a few cases for which the program produced a value of 0.000001. This is attributed to an accumulation of roundoff error and could not be overcome.

CROSS SLOPE was next compared to WEDGE in an upslope configuration. Both models were set to predict the pressure field along the bottom from the apex outward to the twelfth dump distance. The source in CROSS SLOPE was moved outward exponentially until the pattern asymptotically approached the pattern predicted by WEDGE for an infinitely distant source. Initially it was thought that this might occur at approximately eighty (80) dump distances. However, the pattern showed a systematic continuing development as the source exceeded one hundred (100) dump distances. After approximately six hundred (600) dump distances the pattern stabilized. At this value the pattern was very similar to the WEDGE prediction only elongated. Upon further
necessary to initialize the size of the data file array before the graphics program was run.

For three dimensional graphs, pressure amplitude was plotted as a function of two of the three variables in receiver position. This produced a three dimensional contour map. The clearest presentation was a series of two dimensional graphs stacked one after another (see Figure 3).

As the evaluation progressed it became apparent that another type of two dimensional graph was necessary. This form displayed the positions at which the maximum pressure amplitude occurred as a function of two variables in receiver position (see Figure 4). The hyperbolic lines shown are the loci of the turning points calculated from the WKB approximation applied to adiabatic normal modes [Ref. 3 and 4]. For the source position considered, only the odd numbered modes exist. The details of these predictions are presented in Appendix A. The hyperbolae are only dependent on the wedge angle, source distance, and receiver distances (R2 and Y0). The calculations predict that the patterns are independent of the other six parameters used in the wedge problem.

With these forms of output and the speed of the Fortran versions of both WEDGE and CROSS SLOPE, both programs could easily be used to study the effect of changing parameters in the sound field in the wedge.
CROSS SLOPE, or to be written as separate programs. The data used to compute the graphs can therefore be generated within the Fortran program or read in from a data file. The latter method was employed since it offered more versatility. The size of the data file array or the number of points used did not prove to be a limitation on the graphs that were generated.

Two-dimensional graphs were initially used. By holding all but one parameter constant, a plot of pressure amplitude versus the variable parameter was obtained. The receiver position was usually varied to map the pressure field for a given source location. The amount of incremental change of the variable parameter was adjusted to produce smooth graphs with the minimum number of data points.

With CROSS SLOPE's ability to predict pressure amplitude in three dimensions, it followed that three-dimensional graphic should be developed. DISSPLA supports this although several problems had to be overcome. The manual describing this option was incomplete and very confusing. In this mode of graphics, a matrix could be used to define data points but the format had to be in a definite fashion. The most limiting of these requirements was that the array containing the data points must be initially dimensionalized to the exact size of the data file array. The data file normally passed this information during the program. This made it
CROSS SLOPE was rewritten in Fortran with the use of double precision variables and function calls. Although Fortran normally supports double precision, the IBM 3033 at the NAVAL POSTGRADUATE SCHOOL runs faster and more efficiently when using the Extended Fortran compiler. The translation from Basic to Fortran required some changes in the format of function calls as well as the use of some different variable names. A typical calculation of pressure amplitude was reduced to 0.01 seconds of execution time for a ten degree wedge. In addition, the computer values of pressure amplitude and phase were the same for the Fortran version as those for the double precision version in Basic from the WANG microcomputer.

C. DEVELOPMENT OF GRAPHIC PROGRAMS

At this stage it became necessary to consider alternate forms of data output. Due to the large volume of computed values, the numerical form of output had become ineffective in recognizing details. The graphic package DISSPLA available on the IBM 3033 computer was used in conjunction with these programs.

The DISSPLA package can be used to produce both two-dimensional and three-dimensional graphs. DISSPLA is a option similar to a compiler which uses function calls from a Fortran program to generate graphs. DISSPLA's use of Fortran allows function calls to be written into WEDGE or
for a ten degree wedge when the source was at twenty dump distances. Since no approximations were used for the finite source, this variation is caused by the difference in precision on the two different microcomputers. As the source gets closer to the receiver, differences in phase between sequential reflections become more pronounced. When this occurs the differences between computer versions should be greater. For this reason, double precision offers an advantage for phase determination when dealing with small receiver - source distances.

Several method of reducing the execution time of CROSS SLOPE were examined. The IBM personal computer supports the use of compilers although they are not integral to the machine. To use this method would reduce the execution time by a factor of ten to twenty and support the use of double precision. The cost of this option was small. The disadvantages are that it would require modifications to the program which make the program more hardware dependent. Another alternative would be to use a math coprocessor in the IBM personal computer. This option would allow speeds comparable to a mainframe, but would require the use of a compiler making the program very hardware dependent. The last alternative examined was to program in Fortran and execute the program on the IBM 3033. This proved to be the most desirable method since it kept the program relatively hardware independent while allowing increased speed.
B. DEVELOPMENT OF THE CROSS SLOPE PROGRAMS

CROSS SLOPE had initially been developed on a WANG microcomputer. This machine uses Intrepretive Basic in double precision which allows calculations to fourteen place accuracy. Translation of the program into an IBM version of Interpretive Basic required only very minor changes. Certain variable name changes were the only necessary modification. Because neither version of Interpretive Basic supported the use of complex arithmetic this was not a problem as in WEDGE.

The time to execute CROSS SLOPE was dramatically increased over the execution time for WEDGE. With a finite source distance, computation for each image becomes more involved although the geometry to determine the path length remains straightforward. A typical computation time for a single value of pressure amplitude in a ten degree wedge was 105 seconds on either machine. Because smaller wedge angles have a greater number of images, the time of computation went up almost inversely with a decrease in wedge angle. In the IBM version, certain subroutines and function calls were later incorporated into the main program in an attempt to reduce the execution time. These changes reduced the computation time for a ten degree wedge to approximately 45 seconds per pressure amplitude prediction.

The differences in pressure amplitude predictions between the two versions differed by a maximum of 0.0003%
differences between a compiled program and an interpreted one. A compiled program translates the program as a whole into machine language and then runs it. After it has run the entire program, the program is then translated back. In an interpreted program a line at a time is translated into machine language and then worked on. The answer is then translated back and the program goes to the next line. This cycle is repeated until the entire program is completed.

Despite the time difference, a vertical pressure amplitude profile for a constant receiver distance was only limited by the speed of the output device when run in Interpretive Basic. Considering the time to compile, load, and execute the Fortran version, and the fact that this was not necessary for the Basic version, the overall operating time was equal for the two versions.

Both versions of WEDGE were in single precision which calculates to seven place accuracy. The computed pressure amplitudes outputed by the two programs were consistent within this accuracy. This precision was maintained for small angles (ie. 1.5 degrees) where differences between the two programs should be more severe due to the greater number of images. Since the initial cost and the continuing investment is small, and the time difference is small, the use of the personal computer for this program appears preferable.
Several minor problems arising from this application had to be overcome.

Because Interpretive Basic does not support complex variable arithmetic some subroutines had to be rewritten. Cosine and sine functions were used to reproduce complex numbers, overcoming this minor inconvenience. Another problem which occurred was setting up a database for future graphic output. Fortran, as used on the IBM 3033 computer, readily supports writing output to data files while the IBM personal computer normally writes output to devices (i.e., CRT screen and/or printer) but not to data files. To overcome this, data files were setup within the Basic version of the program and saved after execution. The Basic program uses the same file name each time, which necessitated renaming each data file after each program run. If the data file is not renamed, the program would write over the data the next time the program is run. No other major modifications had to be made to the Basic version of this program.

The approximations used by WEDGE for an infinite source distance made it compute very fast irrespective of the wedge angle. The calculation of pressure amplitude for 250 receiver positions took approximately 2.0 seconds for a ten degree wedge angle. In actual computing time, the Fortran version ran approximately ten (10) times faster than the Basic version. The vast majority of the time difference between the programs could be accounted for in the
II. DEVELOPMENT

Although much of the work of the various forms of these two programs occurred simultaneously, they will be discussed here separately. Each program was written in several computer languages and compared with the other versions. The main requirement for a language selection was a wide range of common use. Certain applications were employed for speed and accuracy while others were employed for ease of use and low cost. To meet these criteria, Fortran and Basic were the primary computer languages evaluated.

A. DEVELOPMENT OF THE WEDGE MODEL

As received, WEDGE was programmed in Fortran and designed to run on the BM 3033 computer at the NAVAL POSTGRADUATE SCHOOL. Fortran was selected for its versatility, ease of use of complex variables and wide acceptance. As a first step, WEDGE was translated into a code which could run on a personal computer. Interpretive Basic was chosen because it is widely accepted and because it is integrated into most microcomputer hardware systems. An IBM personal computer was chosen as the hardware system because it has become an industry standard, and because it could easily be interfaced with the IBM 3033 computer.
4. compare existing experimental values with the CROSS SLOPE predictions,

5. test feasibility of using both programs on a personal computer.
IV. EXPERIMENTAL RESULTS

Data were taken in a ten degree wedge shaped model with water overlaying a sand bottom. The model tank had been prepared by Lt. M. Kosnik, and the experiments, which were conducted jointly with him, are reported in reference 4. All measurements were taken directly upslope from a directional source at half depth located approximately forty dump distances from the apex. A frequency of 100 kHz was chosen to fit the physical limitations of the model. A speed of sound ratio of 0.89982, a density ratio of 0.5051, and an absorption of 0.01 were previously determined for the apparatus (Ref. 3). A LC-5 receiver mounted on a vertical traveler was used to record pressure amplitude versus receiver angle. The receiver was moved outward from the apex in intervals of 1.04 dump distances. Since the receiver was kept on axis with the source, no cross slope measurements are available. The dump distance was calculated to be 4.85 cm.

Repeated measurements of pressure amplitude as a function of depth for a given distance were very consistent. Measurements at what was assumed to be the same distance from the apex were less consistent. The position of the apex was hard to determine because of small variations in the water line on the sand (shoreline). This lead to
variations of ± 0.4 dump distances in the initial starting position. Additionally, to keep the sand moist, the slope was exposed and then covered each day. This caused a variation in the initial configuration of the apparatus including source location depending on how much water was removed. The combination of these effects made reproducibility of $R_2$ very difficult.

The receiver was moved in intervals of 5.0 ± 0.1 cm. in the horizontal direction. This corresponds to 1.04 ± 0.02 dump distances for a ten degree wedge. The finite size of the receiver presented a physical limitation on how close the acoustical center could get to the bottom. The receiver averaged readings over the size of the probe. Close to the apex, the water is shallow and the receiver averaged over a larger percentage of the depth than at further distances. This also means that a greater percentage of received signal at the bottom was not accurately surveyed. In fact, at the first two dump distance the probe was not fully immersed. The extent of the influence of receiver size is shown in table 1. These limitations make the first few measurement questionable. Measurements made within the first few dump distances, although consistent, were not given much weight.

The bottom profile of the tank was also subject to variations. The slope was sculptured underwater after the sand had settled for some time, and details of how this was done are provided in reference 4. As a result of the
procedure employed, minor variations in slope could be seen throughout the tank. This is depicted in Figure 21. The effect of these irregularities is to produce local variations in the wedge angle making the wedge less than ideal. In addition to changing the wedge geometry, these variations change the calculated dump distance.

Figures 22 and 23 show predicted three-dimensional pressure profiles of the area surveyed in the experiment. As shown, the pressure amplitude changes greatly over a short distance (less than 1/2 a dump distance in most cases). Through trial and error, theoretical predictions based on the CROSS SLOPE model were fitted to the experimental data: The wedge angle was varied between 9.8° and 10.0° to provide the best fit to experimental data. For a ten degree wedge this produces a corresponding change in the receiver distance of ±0.04 dump distances from the calculated interval. The predictions and the experimental data are shown in Figures 24 through 29. The rapidly changing characteristics of the pressure profile made it impossible to fit data past the eighth dump distance.
V. RECOMMENDATIONS AND CONCLUSIONS

A. RECOMMENDATIONS

The following recommendations and areas of further investigation are suggested:

It is recommended that an experiment be conducted to measure pressure amplitude across the slope. This poses some problems in model geometry since the cross slope patterns are very large. As discussed previously, the pattern is rather independent of all variables except source distance, making it easy to verify.

Comparison of pressure amplitude between CROSS SLOPE and the I.F.D. model as used by Kosnik [Ref. 4] show similar results. The versatility of CROSS SLOPE and speed of execution clearly make it preferential to the I.F.D. model developed by Jeager [Ref. 5]. The speed of WEDGE in those situations where an infinitely distant source is applicable make it the method of choice.

The experiment uses a large angle making it close to the limits of application of the I.F.D. model used by Kosnik. Unfortunately, the graphic capabilities had not been as extensively developed at the time of that work. The two models produced results consistent with each other. It is recommended the a smaller wedge angle be used in further
experiments to minimize the difference between these programs.

CROSS SLOPE and WEDGE are limited to angular geometries. The parabolic equation approach works well in different situations. It is recommended that a program to mate the two for complex geometries such as a flat bottom leading to a sloped beach be considered.

The personal computer offers several advantages over the mainframe. Unfortunately, to date there are no defacto standards for operating system and programming language. The effect of this is to make a program dependent on hardware unless it is in its simplest form. The cost and effort to develop these programs in a higher-level computer language capable of operating on a microcomputer seem to outweigh the hardware dependence.

B. CONCLUSIONS

The following conclusions are offered:

The approximation for an infinitely distant source can be used for source distances over six hundred dump distance from the apex.

The CROSS SLOPE program successfully predicts the on-axis upslope sound profile in a wedge shaped medium.

The CROSS SLOPE program predicts a pressure profile across the slope which is not inconsistent with adiabatic normal mode theory.
Minor variations in the wedge angle have a major effect on the sound profile.
The turning points can be calculated using the results of Reference 4. The following definitions will be used in this development:

- $z =$ local depth of the wedge
- $B =$ wedge angle
- $X =$ dump distance
- $h =$ $z/X$
- $n =$ mode number
- $R_1 =$ normalized source distance
- $R_2 =$ normalized distance from the shore at which the ray turns around
- $Y_0 =$ normalized distance along the shore at which the ray turns around
- $\theta =$ grazing angle the ray makes with the bottom when it turns around.

It has been shown (Ref. 4) that the grazing angle $\theta$ at turn-around is also the angle between the asymptotes of the hyperbolic ray path and the shoreline. Furthermore, the quantity

$$h \sin \theta/c = \text{a ray invariant}$$
For direct upslope propagation, we have $\phi = \pi/2$. Since this ray must turn around at the dump distance, the depth at which the ray turns around must be

$$ht = n = \beta$$

for small $\beta$. Combining these two equations, we have

$$ht = n = \beta/\sin \phi$$

Now, let us find the locus of turning points for each mode. If we let the receiver be at the turning point, then we want the receiver distance $R_2$ as a function of $R_1$ and $Y_0$.

Since $R_2$ is measured from apex to turning point,

$$ht/R_2 = \beta$$

From the diagram, we see that

$$R_1 / (R_1^2 + Y_0^2) = \frac{ht}{R_2} = \beta$$

Now, $ht = R_2 = \beta$, so that

$$R_2 = \beta = n = \beta / (R_1 / (R_1^2 + Y_0^2))$$

and finally,

$$R_2 = n = (R_1^2 + Y_0^2) \beta / R_1.$$ 

$R_2$ is thus a hyperbola with asymptotes

$$\lim R_2 = \pm n = Y_0 / R_1$$

and has its minimum value when $Y_0 = 0$,

$$(R_2)_{\text{min}} = n$$
Figure 1. Up slope Wedge Geometry
Figure 2. Three-Dimensional Wedge Geometry
Figure 5. Position of Peak Pressure as Source Distance Increases
Figure 6. Pressure on Bottom of Wedge as Calculated
Calculated by Image Model
Figure 17. Loci of Pressure Maxima on Bottom of Wedge
(o image mode, - adiabatic normal mode model)
Figure 14. Loci of Pressure Maxima on Bottom of Wedge
(o image mode, - adiabatic normal mode model)
Figure 12. Pressure on Bottom of Wedge as Calculated
Calculated by Image Model
Figure 10. Pressure on Bottom of Wedge as Calculated by Image Model
Figure 8. Pressure on Bottom of Wedge as Calculated
Calculated by Image Model
Figure 22. Pressure Within the Wedge in the Upslope Direction
Figure 23. Pressure Within the Wedge in the Upslope Direction
Figure 24. Pressure Amplitude as a Function of Depth at a Fixed Distance (o experiment, - image theory)
Figure 25. Pressure Amplitude as a Function of Depth at a Fixed Distance (o experiment, - image theory)
Figure 26. Pressure Amplitude as a Function of Depth at a Fixed Distance (0 experiment, - image theory)
Figure 27. Pressure Amplitude as a Function of Depth at a Fixed Distance (o experiment, - image theory)

8 - 9.90  G - 5.10  R1 - 40.0000  R2 - 6.7800  d1 - 0.5051  c2 - 0.8990  XL - 0.0100
Figure 28. Pressure Amplitude as a Function of Depth at a Fixed Distance (o experiment, - image theory)
LIST OF REFERENCES


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