FIXED WIDTH INTERVAL ESTIMATION IN LINEAR REGRESSION

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BY

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1. INTRODUCTION

Stein (1945) describes a two-stage procedure to obtain a fixed-width confidence interval for the mean of a normal population when the variance is unknown. This is followed by works of Anscombe (1953) and Chow and Robbins (1965) who advocate sequential procedures. Hall (1981) suggests a three-stage sampling technique that combines the simplicity of Stein's procedure with the efficiency of the fully sequential method. For a linear model $Y_i = X_i \beta + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$, the corresponding problem of obtaining a fixed-width confidence interval for one of the parameters is more difficult since the variance of the usual estimate depends not only on $\sigma^2$ but also on the $X_i$. To avoid this difficulty, Stein (1945) assumes that $X_1, \ldots, X_m$ are fixed and that they are repeated as a whole, as many times as is necessary. For example, $X_1, \ldots, X_m$ may correspond to an orthogonal design which we are replicating. Bishop (1978) continues to assume that the $X_i$ are fixed.

In this paper, we consider simple linear regression $Y_i = \gamma + \beta X_i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$ and $X_i \sim N(\mu, \tau^2)$. In other words, we are sampling from a bivariate normal population. In section 2, we describe a two-stage procedure to obtain a fixed-width confidence interval for $\beta$ and prove that the specified coverage probability is attained. Essentially, we estimate $\sigma^2$ and predict $X_n$, $n > m$ on the basis of a pilot sample $(X_1, Y_1), \ldots, (X_m, Y_m)$ to determine the size of the second sample. If we sample sequentially, then there is no need to predict $X_n$, $n > m$; such a procedure is described in section 3. We show that the corresponding confidence interval attains the specified coverage probability regardless of the distribution of the $X_i$. The procedure behaves like Stein's procedure for the estimation of a normal mean. By updating the estimate of $\sigma^2$ sequentially, we arrive at another procedure. Section 4 deals with the related problem of deriving a test procedure of $H: \beta = \beta_0$ at level $\alpha_0$ which has power at least $\alpha_1$ at $\beta = \beta_0 + \Delta$ independent of the values of the other parameters. One way to construct such a test makes use of fixed-width confidence
intervals for $\theta$. A different approach which treats $X_1$ and $Y_1$ symmetrically is based on the distribution of the sample correlation coefficient. We show that the resulting test attains the specified level and power asymptotically.

2. A TWO-STAGE PROCEDURE

Suppose that $\sigma^2$ is known and the $X_1$ are known constants, then $\beta_n$ is $N(\beta, \sigma^2 P(X_1 - X_n)^2)$ where $\beta_n = \frac{P(X_1 - X_n)}{n} X_1 P(X_1 - X_n)^2$ is the least squares estimate of $\beta$ based on $(X_1, Y_1), \ldots, (X_n, Y_n)$. It follows that $P(|\beta_n - \beta| < d) \geq 1 - \alpha$ if

$$ P(X_1 - X_n)^2 \geq Z_{1-\alpha/2}^2/2 \sigma^2/d^2 = S_0 $$

where $Z_{1-\alpha/2}$ stands for the $(1 - \alpha/2)$-percentile of the standard normal distribution. Since $\sigma^2$ is unknown and the $X_1$ are stochastic, we need to estimate $\sigma^2$ and predict $X_n$, $n > m$ on the basis of the pilot sample $(X_1, Y_1), \ldots, (X_m, Y_m), m \geq 3$. An obvious estimate of $\sigma^2$ is $\hat{\sigma}^2_m = \frac{\sum_{i=1}^m (Y_{i1} - \hat{\gamma}_m X_{i1})^2}{m-2}$. To reduce the prediction problem, we note that we only need to predict $P(X_1 - X_n)^2$ for $n > m$. Since $X_1$ is $N(\mu, \tau^2)$, we make the Helmert transformation to obtain $P(X_1 - X_n)^2 = \tau^2(U_1^2 + \ldots + U_m^2)$ and $P(X_1 - X_n)^2 = \tau^2(U_1^2 + \ldots + U_m^2 + \ldots + U^2_n)$ for $n > m$ where $U_2$, $U_3$, $\ldots$ are independent standard normal variables. This allows us to make use of standard results of prediction for the gamma case. In particular, if $b_n = 1 + x_{1-c}^2(n-m)/x_{1-c}^2(n-m)$ and $x_{1-c}^2(n-m)$ and $x_{1-c}^2(n-m)$ are chi-square percentiles, then for each $n > m$, $(b_n P(X_1 - X_n)^2, \infty)$ is a $(c, g)$ guaranteed coverage interval predictor of $P(X_1 - X_n)^2$ (Aitchison & Dunsmore 1975, Ch.6). Furthermore, we can guarantee coverage simultaneously so that with probability $g$, the pilot sample $X_1, \ldots, X_m$ is such that $P(X_1 - X_n)^2 > b_n P(X_1 - X_n)^2 | X_1, \ldots, X_m \geq c$ for each $n > m$. We choose $c, g$ so that $cg > 1 - \alpha$ and define $a^* = 1 - \alpha = cg(1 - \alpha^*)$. For convenience, we let $b_m = 1$. Consider the following two-stage sampling procedure.

Procedure 1. (i) Obtain a pilot sample $(X_1, Y_1), \ldots, (X_m, Y_m)$ and calculate $\hat{\gamma}_m$, $\hat{\beta}_m$ and $\hat{\sigma}^2_m$. 

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(ii) Draw a second sample of size $N_1 - m$ where $N_1$ is the smallest $n \geq m$ such that

$$b_{n_1} \frac{m}{(X_i - X_{m})^2} > t_{1-\alpha/2}(m-2) \sigma^2_d/d^2$$

and $t_{1-\alpha/2}(m-2)$ is the $(1-\alpha/2)$-percentile of a t distribution with $m-2$ degrees of freedom.

The following theorem says that $(\hat{\beta}_{N_1} - d\hat{\beta}_{N_1} + d)$ is a $(1-\alpha)$-level confidence interval for $\beta$.

**Theorem 1.** $P(\hat{\beta}_{N_1} - \beta < d) \geq 1 - \alpha$.

Before we prove theorem 1, we first state two lemmas.

**Lemma 1.** The conditional distribution of $\hat{\beta}_{N_1}$ given $\hat{\sigma}_m$ and

$$X_1, X_2, \ldots$$

is $N(\beta, \sigma^2/S_1)$ where $S_1 = \frac{N_1}{m} (X_i - X_{m})^2$.

**Proof.** Given $X_1, X_2, \ldots, N_1$ depends only on $\sigma^2$ and $\hat{\beta}_m$ can be written as a linear combination of $Y_{m}^{*}, \hat{\beta}_m$ and $Y_{m+1}^{*}, \ldots, Y_{N_1}^{*}$, all of which are independent of $\sigma^2_m$.

**Lemma 2.** $P\left(\frac{N_1}{m} (X_i - X_{m})^2 > b_{n_1} \frac{m}{(X_i - X_{m})^2} | \hat{\sigma}_m \right) \geq c$.

**Proof.** Let $A = \{(x_1, \ldots, x_m) : \forall n > m, P\left(\frac{N_1}{m} (X_i - X_{m})^2 > b_{n_1} \frac{m}{(X_i - X_{m})^2} \mid X_1 = x_1, \ldots, X_m = x_m\right) \geq c\}$, then $P((X_1, \ldots, X_m) \in A) = g$ by our choice of $b_n$. Since $\sigma^2_m$ is independent of the $X_i$, we also have $P((X_1, \ldots, X_m) \in A | \hat{\sigma}_m) = g$. If $(X_1, \ldots, X_m) = (x_1, \ldots, x_m) \in A$ and we write

$$n_1 = N_1(\hat{\sigma}_m, x_1, \ldots, x_m),$$

then

$$P\left(\frac{N_1}{m} (X_i - X_{m})^2 > b_{n_1} \frac{m}{(X_i - X_{m})^2} \mid \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m\right)$$

$$= P\left(\frac{N_1}{m} (X_i - X_{m})^2 > b_{n_1} \frac{m}{(X_i - X_{m})^2} \mid \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m\right)$$

$$= P\left(\frac{N_1}{m} (X_i - X_{m})^2 > b_{n_1} \frac{m}{(X_i - X_{m})^2} \mid X_1 = x_1, \ldots, X_m = x_m\right)$$

$$\geq c.$$ 

Combining, we have the desired result.

**Corollary 1.** $P\left(\frac{N_1}{m} (X_i - X_{m})^2 > t_{1-\alpha/2}(m-2) \sigma^2_d/d^2 \mid \hat{\sigma}_m \right) \geq g_c$.

**Proof.** This follows from lemma 2 and the definition of $N_1$.

We now prove theorem 1.
\[ P( |\hat{\beta}_{N_1} - \beta| < d \mid \hat{\sigma}_m ) \]
\[ = E_{X_1, X_2 \ldots \mid \hat{\sigma}_m}(P( |\hat{\beta}_{N_1} - \beta| < d \mid \hat{\sigma}_m, X_1, X_2, \ldots )) \]
\[ = E(2\Phi(d/S_1/\sigma) - 1 \mid \hat{\sigma}_m) \quad \text{by lemma 1} \]
\[ \geq \Phi(2\Phi(t_{1-\alpha/2}^2/\sigma^2 - 1) \mid \hat{\sigma}_m) \quad \text{by corollary 1}. \]
Thus \[ P( |\hat{\beta}_{N_1} - \beta| < d ) \geq E(2\Phi(t_{1-\alpha/2}^2/\sigma^2 - 1) \mid \hat{\sigma}_m) \]
\[ = \Phi(1 - \alpha^2) \]
\[ = 1 - \alpha. \]

3. SEQUENTIAL PROCEDURES

If we sample sequentially, then prediction is no longer necessary.

Procedure 2. (i) Obtain a pilot sample of size \( m \). (ii) Sample sequentially until \( \frac{\text{N}}{\text{N}}(X_1 - \bar{X}_n)^2 \geq t_{1-\alpha/2}^2(\sigma^2/\sigma^2 - d^2). \)

Let \( N_2 \) be the sample size when we terminate sampling, our next theorem asserts that \((\hat{\beta}_{N_2} - d, \hat{\beta}_{N_2} + d) \) is a \((1 - \alpha)\)-level confidence interval for \( \beta \).

Theorem 2. \( P( |\hat{\beta}_{N_2} - \beta| < d ) \geq 1 - \alpha. \)

We first state a lemma.

Lemma 3. The conditional distribution of \( \hat{\beta}_{N_2} \) given \( \hat{\sigma}_m \) and \( X_1, X_2, \ldots \) is \( N(\beta, \sigma^2/S_2) \) where \( S_2 = \frac{N_2}{N}(X_1 - \bar{X}_{N_2})^2. \)

This is the analog of lemma 1 and can be proved using similar technique. We now prove theorem 2.

\[ P( |\hat{\beta}_{N_2} - \beta| < d ) \]
\[ = E(P( |\hat{\beta}_{N_2} - \beta| < d \mid \hat{\sigma}_m, X_1, X_2, \ldots )) \]
\[ = E(2\Phi(d/S_2/\sigma) - 1) \quad \text{by lemma 3} \]
\[ \geq E(2\Phi(t_{1-\alpha/2}^2/\sigma^2 - 1) \mid \hat{\sigma}_m) \]
\[ = 1 - \alpha. \]

We note that theorem 2 holds even when the \( X_1 \) are not normally
Since the estimate of \( \sigma^2 \) is not updated as we sample sequentially, procedure 2 is inefficient. It behaves like Stein's procedure for the estimation of the mean of a normal population. In fact

\[
E(S_2) = E\left( \frac{1}{N_2} (X_1 - \bar{X}_2)^2 \right)
\geq E\left( \frac{t^2_{1-\alpha/2} (m-2) \sigma^2 / d^2}{m} \right)
= S_0 \frac{t^2_{1-\alpha/2} (m-2) / Z^2_{1-\alpha/2}}{t^2_{1-\alpha/2} (m-2) / Z^2_{1-\alpha/2}}
\]

so that \( E(S_2)/S_0 = \frac{t^2_{1-\alpha/2} (m-2) / Z^2_{1-\alpha/2}}{t^2_{1-\alpha/2} (m-2) / Z^2_{1-\alpha/2}} > 1 \).

If the estimate of \( \sigma^2 \) is updated sequentially, we obtain the following procedure.

Procedure 3. (i) Obtain a pilot sample of size \( m \). (ii) Sample sequentially until \( \frac{1}{n} (X_1 - \bar{X})^2 \geq \frac{a_n \sigma^2}{d^2} \) where \( \{a_n\} \) is a sequence of constants converging to \( Z_{1-\alpha/2} \).

We expect procedure 3 to be the most efficient, but unlike procedures 2 and 3, the specified coverage probability is attained only asymptotically. Procedure 1 is least efficient since we have to deal with the additional problem of prediction, however, it has the advantage of requiring only two sampling operations.

4. A RELATED PROBLEM

A problem related to fixed-width interval estimation of \( \beta \) is that of deriving a test procedure of \( H : \beta = \beta_0 \) at level \( \alpha_0 \) which has power at least \( \alpha_1 \) at \( \beta = \beta_0 + \Delta, \Delta > 0 \). We can make use of our earlier results to solve this problem. For instance, we can use procedure 2 to obtain a \((1-\alpha)\)-level confidence interval for \( \beta \) with width \( 2d \), \( d < \Delta \) and reject \( H \) if \( \beta_0 \) lies outside that interval. The resulting test has level \( \alpha_0 \) and its power at \( \beta = \beta_0 + \Delta \) is

\[
P_{\beta_0 + \Delta}(|\hat{\beta}_{N_2} - \beta_0| > d) \geq P_{\beta_0 + \Delta}(\hat{\beta}_{N_2} > \beta_0 + d) = E(P_{\beta_0 + \Delta}(\hat{\beta}_{N_2} > \beta_0 + d | \sigma^2, X_1, \ldots))
\]

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If we choose \( d \) such that \((\Delta - d)t_{\alpha_1/2}[(m-2)/d] = t_{\alpha_1}[(m-2)]\), then the power is at least \( a_1 \). As expected, if \( d = \Delta \), then the power is at least \( \frac{1}{2} \); as \( d \to 0 \), the power increases to 1.

The technique we employ so far is to condition on the \( X_i \) and then treat them as if they are fixed. An unconditional approach treating the \( X_i \) and \( Y_i \) symmetrically is described below. Without loss of generality, the hypothesis is \( H: \beta = 0 \). Assume that we are sampling from a bivariate normal population

\[
\begin{pmatrix} X_i \\ Y_i \end{pmatrix} \sim N_2(0, \Sigma)
\]

then \( H \) is equivalent to \( \rho = 0 \) and the usual t test rejects \( H \) if \( |r| \) is too large where \( r \) is the sample correlation coefficient. Since the distribution of \( r \) depends on the parameters only through \( \rho \), we can determine the sample size such that the level - \( \alpha \) test of \( \rho = 0 \) has power \( a_1 \) at another \( \rho \) value. Bock (1977) makes use of Fisher Z-transformation to derive an approximate formula for the required sample size

\[
Z_{1-a_0/2} = (n - 3)^{1/2} \tanh^{-1} \rho = Z_{1-a_1}.
\]

Since \( \rho = \theta/(1 + \theta^2)^{1/2} \), where \( \theta = \beta \tau / \sigma \), the following procedure suggests itself.

**Procedure 4.** (i) Obtain a pilot sample of size \( m \). (ii) Sample sequentially until

\[
Z_{1-a_0/2} = (n - 3)^{1/2} \tanh^{-1} \rho_n(\Delta) < Z_{1-a_1}
\]

where

\[
\hat{\rho}_n(\Delta) = \hat{\rho}_n(\Delta)/(1 + \hat{\rho}_n(\Delta))^2, \quad \hat{\rho}_n(\Delta) = \Delta \hat{\tau}_n / \hat{\sigma}_n
\]

and

\[
\tau^2_n = \frac{P_n(0, X_n^2, Y_n^2)}{n - 1}.
\]

(iii) Perform a two-sided t test treating the final sample size \( N(\Delta) \) as if it is fixed. Thus if

\[
T_n = \hat{\beta}_n \frac{(0, X_n^2, Y_n^2)}{n - 1} \frac{\hat{\sigma}_n}{\hat{\sigma}_n(\Delta)}
\]

we reject \( H \) if \( |T_{N(\Delta)}| > t_{1-a_0/2}[N(\Delta) - 2] \).

The following theorem asserts that the test procedure attains the specified level and power asymptotically.

**Theorem 3.** \( \lim_{\Delta \to 0} P_{\beta = 0}( |T_{N(\Delta)}| < t_{1-a_0/2}[N(\Delta) - 2] ) = 1 - a_0 \).
\[ \lim_{\Delta \to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}(N(\Delta)-2)) \geq \alpha_1. \]

Proof. (i) Since \( r_n = T_n/(n-2+T^2_n)^{1/2} \) where \( r_n \) is the sample correlation coefficient computed from \((X_1,Y_1), \ldots, (X_n,Y_n),\)

\[
1 - \alpha_0 = P_{\beta=0}(|T_n| < t_{1-\alpha_0/2}(n-2))
= P_{\beta=0}((n-3)^{1/2}\tanh^{-1}r_n < C_n)
\]

where \( C_n = (n-3)^{1/2}\tanh^{-1}(t_{1-\alpha_0/2}(n-2)/(n-2+t_1^2_{1-\alpha_0/2}(n-2)))^{1/2} \). On the other hand, when \( \beta = 0 \)

\[
(n-3)^{1/2}\tanh^{-1}r_n \rightarrow N(0,1) \text{ as } n \to \infty,
\]

so we must have \( \lim C_n = Z_{1-\alpha_0/2} \). Since \( N(\Delta) \to \infty \) a.s. as \( \Delta \to 0 \),

\[
\lim C_{N(\Delta)} = Z_{1-\alpha_0/2} \text{ a.s. and it follows from a theorem of Anscombe (1952) that when } \beta = 0
\]

\[
(N(\Delta)-3)^{1/2}\tanh^{-1}r_{N(\Delta)} \rightarrow N(0,1).
\]

Thus \( \lim_{\Delta \to 0} P_{\beta=0}(|T_{N(\Delta)}| < t_{1-\alpha_0/2}(N(\Delta)-2)) \)

\[
= \lim_{\Delta \to 0} P_{\beta=0}((N(\Delta)-3)^{1/2}\tanh^{-1}r_{N(\Delta)} < C_{N(\Delta)})
= 1 - \alpha_0.
\]

(ii) Assume for the time being that under \( \beta = \Delta \)

\[
(N(\Delta)-3)^{1/2}\tanh^{-1}r_{N(\Delta)} \rightarrow N(Z_{1-\alpha_0/2} - Z_{1-\alpha_1}, 1) \text{ as } \Delta \to 0, \text{ (1)}
\]

then \( \lim_{\Delta \to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}(N(\Delta)-2)) \)

\[
\geq \lim_{\Delta \to 0} P_{\beta=\Delta}((N(\Delta)-3)^{1/2}\tanh^{-1}r_{N(\Delta)} > C_{N(\Delta)})
= \alpha_1.
\]

To prove (1), we fix \( \gamma, \sigma, \mu, \tau \) and define \( n(\Delta)-3 \) to be the least integer greater than or equal to \((Z_{1-\alpha_0/2} - Z_{1-\alpha_1})^2/(\tanh^{-1}\rho(\Delta))^2\) where \( \rho(\Delta) = \theta(\Delta)/(1+\theta(\Delta))^{1/2} \) and \( \theta(\Delta) = \Delta/\sigma \). Under \( \beta = \Delta \)

\[
(n(\Delta)-3)^{1/2}(\tanh^{-1}r_{n(\Delta)} - \tanh^{-1}\rho(\Delta)) \rightarrow N(0,1) \text{ as } \Delta \to 0,
\]

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equivalently
\[(n(\Delta)-3)\frac{1}{2}\tanh^{-1}\frac{3}{n(\Delta)} \sim N\left(Z_{1-a_0/2}-Z_{1-a_1},1\right)\text{ as } \Delta \to 0\] (2)
from which (1) follows if we can replace \(n(\Delta)\) by \(N(\Delta)\). To that end, we note that if \(X_1\) is \(N(\mu, \tau^2)\) and \(Y_1\) is \(N(\gamma, \sigma^2)\) independently of \(X_1\), then the conditional distribution of \(Y_1+BX_1\) given \(X_1\) is \(N(\gamma+BX_1, \sigma^2)\). The advantage of this representation is that it enables us to deal with a single array of random variables rather than a double array. In particular, we can show \(N(\Delta)/n(\Delta)\to 1\text{ a.s. as } \Delta \to 0\). A generalization of Anscombe's theorem enables us to replace \(n(\Delta)\) by \(N(\Delta)\) in (2), we omit the details.

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We discuss fixed-width interval estimation for the slope parameter \( \beta \) in a simple linear regression \( Y_i = \gamma + \beta X_i + \varepsilon_i \) when the \( X_i \) are also normally distributed. A two-stage procedure that combines prediction with estimation is described. In addition, we discuss two sequential procedures. The confidence intervals obtained are used to construct tests of \( H: \beta = \beta_0 \) with level \( \alpha \) and power at least \( 1 - \beta \) independent of the values of the other parameters. We also consider a sequential procedure based on the distribution of the sample correlation coefficient; the resulting test attains the specified level and power asymptotically.
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