A number of results were obtained pertaining to signal detection and data compression for image processing. These results led to improved performance over previous approaches, with special attention given to methods which required less statistical knowledge and which were easier to implement. In particular, robustness and nonparametric techniques were employed to allow the exploitation of whatever knowledge was available, while retaining insensitivity to the remaining inexactness in knowledge. In addition, because the presence of dependency in the underlying random processes often complicates detector design, investigations into when weak dependency could be ignored were undertaken; moreover, results were obtained pertaining to the general subject of the extent of variation (induced by incomplete knowledge of the dependency) in the form of the detector data processor. Finally, some results were obtained which allowed relaxing stationarity assumptions which were placed on the signal in earlier work.
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PERSONNEL

PRINCIPLE INVESTIGATOR: Don R. Halverson

GRADUATE RESEARCH ASSISTANT: Michael W. Thompson


12. D.R. Halverson and G.L. Wise, "Approximately optimal memoryless detection of ran-


SUMMARY OF RESEARCH

The research supported by Grant AFOSR-82-0033 was primarily concerned with signal detection, however some effort was also directed toward the area of block truncation coding for image compression. Results obtained pertaining to the latter will be summarized first.

Block truncation coding (BTC) is a technique for image compression which is easy to implement and often possesses good performance characteristics, relative to other approaches, even in the presence of many channel errors. Furthermore, the BTC algorithm does not depend on knowledge of the specific underlying statistical distributions, but rather on sample moments. The basic BTC technique employs a two level quantizer whose output levels are obtained by matching sample moments. The quantizer's threshold is either taken a priori to equal the sample mean (and thus the output levels are chosen so that the first two sample moments are preserved), or the threshold is chosen so as to preserve a third sample moment. The latter approach, while somewhat more complicated to implement, often yields improved performance since three moments are preserved instead of two.

The developers of the technique presented formulas specifying the quantizer threshold and output levels so that the first three sample moments were supposedly preserved. Unfortunately, however, the formula specifying the quantizer threshold contained an error, and one result we therefore obtained was a correction of this quantity. Moreover, the developers of the technique noted that rounding a certain quantity to an integer was at times necessary in practice; however, there existed ambiguity in precisely at what point in the algorithm the rounding should occur. We therefore investigated the perturbation of the relevant quantity, and found that it was fortunately preferable from the standpoint of all three moments for the rounding to be done consistently in one way. We furthermore investigated the effects of rounding the quantity on the performance and noted that for very many cases it should not be serious. These results were delineated in #10 of the publication list.

Noting that the BTC approach preserves sample moments (two moments with one method and three with the other), we also investigated whether or not improved performance could be obtained by employing moments other than the first two or three. We found that the original BTC scheme could be generalized to yield a family of moment preserving quantizers using higher sample moments. In particular, we found closed form expressions for the quantizer output levels which preserved the n-th and 2n-th moments when a threshold equal to the $L_n$ norm of the samples was employed; we also found closed form expressions specifying the quantizer output levels and threshold which preserved the n-th, 2n-th, and 3n-th moments. We then applied these results to various example images and found that improvement in performance was obtainable by the use of higher moments, both from the standpoint of mean absolute and mean squared error. Finally, we noted that there was a subclass of this family of moment preserving quantizers for which practical difficulties in implementation exist; we then showed that frequently this subclass could be avoided to still obtain good performance. These results were delineated in #1 and #13 of the publication list.
Noting the general applicability of sample moments, we also considered an alternative to the BTC scheme which used sample moments as a measure of the rapidity of image variation over pixel blocks. This approach had as a particular goal the high quality restoration of those portions of the image with most rapid variation in gray level (such as edges). We designed an algorithm which not only was successful in accomplishing this goal but also was accompanied by the somewhat surprising consequence that certain nontrivial images were restored with no degradation whatsoever. The latter consequence was in force whenever the number of edges present in the image was no larger than a specified number which depended on the desired image compression ratio. These results were delineated in #8 of the publication list.

A number of results pertaining to the detection of signals in non-Gaussian noise were obtained. For example, a classical detector of time varying deterministic signals in dependent noise is the matched filter, which maximizes the output signal to noise power ratio; it is also Neyman-Pearson optimal in the particular case when the noise is Gaussian. In many situations it is reasonable to expect that the signal will be known at discrete instants, thus admitting the design of the discrete time matched filter. However, it is often a very different matter to assume that the signal will be known exactly as a closed form analytical expression over, for example, an interval of time, which would be necessary for the design of a matched filter in continuous time. While the signal may thus be incompletely known, it is reasonable to expect that in many cases it could be modeled as bandlimited. If we furthermore assume the signal is known at a fixed number of instants, we might hope that a continuous time filter could be designed which would be insensitive to the remaining inexactness in our knowledge of the signal. Such a filter might also have the potential for improved performance over the discrete time filter. Using a saddlepoint criterion, we obtained a result which specified the form of such a filter. We then showed the saddlepoint criterion did indeed impart robustness into the filter; in fact, the output signal to noise power ratio was shown to be invariant over all appropriate continuous time signals considered. Moreover, we also showed that the performance (as measured by output signal to noise power ratio) of this robust continuous time filter upper bounded that of the discrete time filter. Finally, we showed by way of example that strict improvement in performance over the discrete time filter was possible. Employment of continuous time would thus seem to be useful even in some cases which appear appropriate to discrete time. These results were delineated in #11 of the publication list.

We also investigated a situation where inexact knowledge of the statistics of the noise was present for the discrete time detection of time varying deterministic signals in i.i.d. non-Gaussian noise. In earlier work employing the canonical form of the locally optimal detector, some other authors showed how the design of the asymptotically robust detector for the Huber-Tukey mixture class of noises could be obtained. As might have been expected, the results led to censoring the factors of the test statistic to impart robustness, which led to the employment of a detector nonlinearity which limited observations of large magnitude. However, this work did not take into account the common situation where more is known about a noise density near the origin than on the tails. In fact, the noise often arises in practice from the sum of a number of "nearly" independent sources, and
thus frequently the noise density resembles the Gaussian near the origin but may differ markedly on the tails. Such knowledge had the potential to be exploited for improved performance, where the robustness is imparted to account for our lack of knowledge of the tails of the noise density. We investigated this situation and obtained results which specified an asymptotically robust detector for the case where the noise density was known on an interval about the origin. We then showed by way of example that the approach led to improved performance over the previous one; in fact, the improvement was quite dramatic in certain cases. These results were delineated in \#2 and \#16 of the publication list.

In addition, we considered the discrete time robust detection of a known constant signal in additive independent and identically distributed nominally Laplace noise. The heavy tailed nature of the Laplace density function makes it a popular model for such non-Gaussian noise as that encountered in atmospheric or undersea environments. We observed that applying some existing results of Huber allowed deducing the form of the test statistic as a censored version of the test statistic which is Neyman-Pearson optimal. Since the performance of robust detectors is an important subject which has been largely unexplored, we pursued such an investigation with the hope of exploiting the often tractable Laplace model for the nominal distribution. We were able to obtain closed form solutions for the distribution of the censored test statistic under both hypotheses, with the result that the performance could be measured via receiver operating characteristic curves which plotted detection probability versus false alarm rate. Such curves were then determined for various cases of interest. A comparison of these curves with those for the Neyman-Pearson optimal case was then performed, thus showing the effect of various degrees of contamination of the nominal distribution on the performance of the robust detector when compared to the Neyman-Pearson optimal one. These results were delineated in \#4 of the publication list.

In addition to the aforementioned robustness applications, we also considered a non-parametric detector, the modified sign detector, for the discrete time detection of a constant signal in dependent noise. Employing the very general strong mixing class of dependent noises, we specified the design of the modified sign detector both under asymptotic and finite sample criteria. We found that this design varied greatly from that for a simpler model of dependency which used m-dependent processes. Our results showed how the detector's design depended on a quantitative measure of the rate of "decrease" in dependency between samples as the samples were more widely separated in time. An example was presented which illustrated the complete design of the detector for the case where the dependency rolloff was consistent with that of an autoregressive strong mixing process. These results were delineated in \#15 of the publication list.

The area of memoryless discrete time detection of signals in $\phi$-mixing noise has been studied by us in the past for both the case when the signal was constant and when it was also a random process. The detector structure employed consisted of a memoryless nonlinearity followed by an accumulator and threshold comparator, which was Neyman-Pearson optimal in the independent noise case. Using the criterion of asymptotic relative efficiency, these
earlier results accounted for the dependency by specifying the detector nonlinearity which optimized performance. This nonlinearity appeared as the solution of an integral equation which unfortunately involved second order statistics of the random processes present. Since such statistical knowledge may often be incomplete, we investigated alternative approaches which required less statistical knowledge and which were easy to implement.

For example, we considered the detection of a constant signal in weakly dependent $\phi$-mixing noise using two approaches. For the first, the locally optimal detector was employed, resulting in the need to know only the univariate noise density. Since this approach ignores the dependency, it is of course important to inquire into its performance. We therefore obtained a bound on the degradation in performance which was expressed in terms of the $\phi$-representation of the noise. This bound showed quantitatively how close the performance of the locally optimal detector was to optimal for various $\phi$-representations. The second approach considered a more general form, consisting of the locally optimal nonlinearity plus a linear correction term. Such a structure has been shown to yield improved performance under a moving average weak dependence model. We then showed that in the much more general $\phi$-mixing case, such improved performance is unfortunately not possible uniformly over the class of noise processes possessing a given $\phi$-representation. In fact, for any appropriate $\phi$-representation, we showed that there must exist such a process for which the locally optimal detector outperforms the one with the linear correction term. Because of this result, we might in practice simply wish to employ the locally optimal detector in the absence of the required statistical knowledge. These results were delineated in #9 of the publication list.

Motivated by the work immediately above, we then investigated the effect on the detector's performance induced by more general alterations of the form of the detector nonlinearity from that of the asymptotically optimal nonlinearity. These results, which applied to the detection of a constant signal in weakly dependent $\phi$-mixing noise, showed that the resultant degradation in performance can be bounded in terms of the $L_2$ distance between the optimal nonlinearity and the nonlinearity of interest. As might be expected, this bound also involved the $\phi$-representation of the noise. In particular, our results showed that asymptotic relative efficiency can be viewed as a mapping between metric spaces which is continuous at the point of interest. These results were delineated in #6 and #17 of the publication list.

We also considered an alternative approach which was applied to memoryless discrete time detection of random signals in noise, where in this case both the signal and noise were random processes (not necessarily independent of each other) whose maximal correlation coefficient sequences were summable. Such processes are strong mixing, moreover this class includes as a proper subset the class of $\phi$-mixing processes. In some earlier work we showed that the asymptotically optimal nonlinearity again satisfied an integral equation, which in this case involved second order statistics of both the signal and the noise. Noting that finding an exact solution was often limited in practice by the numerical techniques employed as well as the expected inexact statistical knowledge, it was concluded that the actual nonlinearity obtained was only approximately optimal. We therefore won-
dered whether other approximations might offer certain advantages, and thus pursued an
investigation into the general problem of approximating a detector nonlinearity. This led
to a result which established sufficiency conditions for the performance of a sequence of
approximations to converge to the performance of a given detector. We then applied these
conditions to the case where the optimal detector nonlinearity was approximated by a
quantizer or by a polynomial. Our results specified the form of the optimal \( M \)-level quan-
tizer; moreover, it was shown that the performance of the resultant detector converged to
optimal as the number of quantization levels approached infinity. Since one may be inter-
ested in quantizing to implement the detector digitally, the use of such quantizers might
be attractive. We also specified the form of the optimal polynomial of a given degree, and
found that it could be obtained simply by solving a set of linear equations which involved
joint moments rather than densities. We then showed that for a large class of noises the
resultant detector’s performance converged to optimal as the degree of the polynomial
approached infinity. Because of these convergence results, we therefore concluded that the
employment of a quantizer or polynomial detector allowed performance arbitrarily close
to optimal. These results were delineated in \#12 of the publication list.

As with much work which deals with memoryless detection, the results described
above were obtained in conjunction with stationarity assumptions which were imposed on
the signal. Because there are many cases where a stationarity assumption regarding the
signal may not be appropriate due to, for example, manmade factors affecting the signal,
we then sought to investigate applications of memoryless discrete time detection to the
case of a time varying deterministic signal in dependent non-Gaussian noise. Using the
criterion of asymptotic relative efficiency, we first specified the form of the optimal detector
for the case when the detector class considered consisted of a time invariant memoryless
nonlinearity followed by an accumulator and threshold comparator, the same detector class
considered by us in other contexts for various stationary signal cases. Having noted that
the nonstationarity of the signal might make appropriate a different detector model, we
then employed a model which included a time varying memoryless nonlinearity. We again
specified the form of the optimal detector for this detector class. Several specific examples
were considered, and graphs of the appropriate optimal nonlinearities were provided. We
then evaluated the asymptotic performance of the optimal detectors of the two classes,
and it was seen that for the examples considered the time varying nonlinearity led to
superior performance. Finally, in order to reduce the amount of statistical knowledge of
the noise required for the design of the detector, an approach was presented which involved
employing polynomial approximations of the nonlinearities of interest. Graphs were then
obtained of the optimal polynomials of a given degree for various examples. These results
were delineated in \#3 and \#18 of the publication list.

We have also observed that in order to obtain improved performance it might be
desirable to allow the detector to possess memory. The presence of a nonstationary signal
in conjunction with a dependent noise process often complicates the design of the detector;
moreover, in some such cases applications of known approaches which account for the noise
dependency are inhibited by lack of knowledge of the higher order noise statistics. If the
noise dependency were weak the temptation to ignore it might therefore arise. We were
thus led to consider the discrete time detection with memory of time varying deterministic signals in weakly dependent noise. We investigated quantitative conditions which allowed determining when the dependency could be ignored, and presented a result which allowed bounding the variation in false alarm rate and detection probability induced by ignoring the dependency. The case of stationary Gaussian noise was then considered, and we showed that our result related the bound on the variation in false alarm rate and detection probability to the rate of descent of the noise autocorrelation. These results were delineated in #5 and #14 of the publication list.

Finally, we investigated more generally the effect induced by the aforementioned inexact knowledge of the noise dependency on the detector data processor. Noting that, under a variety of fidelity criteria, this data processor could be expressed in terms of the relevant likelihood ratio, we considered the variation in the likelihood ratio as the noise distribution functions were varied about their nominal values. Our results, which first were applied to the detection of a time varying deterministic signal in additive noise and then were extended to a more general situation in which the signal possessed a random amplitude, characterized a class of noise contaminants over which the likelihood ratio was invariant. It was then shown by example that such a class contained distribution functions that differed greatly from the nominal distribution function, yet still yielded the same likelihood ratio. A key element common to these results was that a contaminant which was a periodic multiple (with period equal to the signal) of the nominal density led to an unperturbed likelihood ratio. These results were delineated in #7 of the publication list.
DIRECTIONS OF CURRENT RESEARCH

Current research is directed toward the areas of signal detection and data compression for image processing. Most of this work has as a goal the admission of non-Gaussian processes with dependency. In addition, we are heavily interested in approaches which require only moderate amounts of statistical knowledge, thus yielding greater utility in practice. This is resulting in the attractiveness of robust or nonparametric schemes. Because in many cases we might expect the presence of dependency in the underlying random processes, we are particularly interested in studying robust or nonparametric schemes with application to dependent processes. These are areas for which few results exist in the dependent non-Gaussian case, and thus the employment of unconventional techniques might be necessary.