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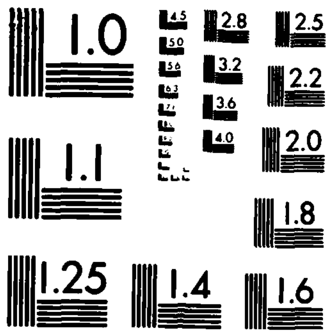

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MATHEMATICAL MODELING AND CONTROL OF A LARGE SPACE STRUCTURE AS APPLIED TO A SHUTTLE-ANTENNA CONFIGURATION

THESIS

John O. Dunstan
Captain, USAF

AFIT/GA/AA/84D-3

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
John O. Dunstan
Capt USAF
Graduate Astronautical Engineering

December 1984

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PREFACE

I would like to express my sincerest thanks to my thesis advisor, Dr. R. A. Calico, for his unending patience and assistance in developing and completing this project.

To my fellow students, I wish to express my appreciation for their support and companionship throughout this endeavor. I truly doubt that I could have made it without them. I have learned a lot from each and every one, and I will deeply miss their insight, humor, and comradeship in my next assignment and for years to come.

Finally, to my darling wife, Kathy. Although she could only lend her support from afar, just knowing that her love and affection were waiting for me when this was done and we could be stationed together again gave me the motivation and drive I needed to see this through.

John O. Dunstan

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ABSTRACT

The equations of motion of a flexible shuttle-beam-antenna system are developed and discretized using an assumed modes approximation. The system was modeled as a cantilever beam rigidly attached to the shuttle with a rigid antenna attached to the free end of the beam. The mass and dimension data for the model was taken from a NASA/IEEE Design Challenge Paper [2] dated June 1984. The equations of motion for both the shuttle-beam-antenna rigid body movement and the vibration of the beam with respect to the shuttle were developed making some simplifyng modifications to fit the modeling assumptions. Two proof-mass actuators, capable of producing a force in the x and y directions only with no torsional control about the z axis, were modeled at positions along the beam. The moments resulting from any torque on the shuttle (due to reaction jets firing, for example), and moments applied to the antenna at the attach point were also modeled. The equations of motion, with the forces and moments evaluated, were put in matrix form. The matrices were diagonalized, resulting in an identity mass matrix and diagonal damping and stiffness matrices.

A controller was developed for a cursory investigation into the controllability of the system. The development made use of linear optimal regulator techniques which produce feedback gains proportional to the state. The state was

truncated to the amplitudes and velocities of the twelve modes having the lowest frequencies. Since these amplitudes and velocities could not be measured directly, state estimation was used. The feedback gains were developed using steady state optimal regulator theory. The closed loop damping coefficient was used as a measure of control improvement. The system was shown to be stable on the very first control attempt, with a closed loop damping coefficient better than the targetted value. Elimination of observation spillover improved the controllability slightly for this first run. More runs were made with different weightings of the controlled modes with similar results.

MATHEMATICAL MODELING AND CONTROL OF A LARGE SPACE STRUCTURE AS APPLIED TO A SHUTTLE-ANTENNA CONFIGURATION

I. Introduction

With the advent of the Space Shuttle, the opportunities to place large, flexible structures into space are becoming more and more commonplace. These large structures, along with all their advantages, bring with them control problems on a scale never before encountered. Extensive research is being conducted to solve these control problems--research which is constantly being updated and improved. The problem of controlling a large structure containing a virtually infinite number of vibrational modes with limited onboard computer resources, sensors, and actuators has become the focus for intense study. Of the control techniques attempted to date, modern state space control methods seem to be the most promising.

In January 1984 an NASA Design Challenge [1] was offered. It is called the Spacecraft Control Laboratory Experiment (SCOLE) and serves as the focus of a design challenge for the purpose of comparing different approaches to control synthesis, modeling, order reduction, state estimation, and system identification. The SCOLE itself is presented as a large antenna attached to the shuttle by a flexible beam. The Challenge consisted of two parts-- a mathematical analysis and a laboratory experiment. Only the mathematical analysis is addressed in this paper.

The first portion of this paper concerns the mathematical modeling of the shuttle-beam-antenna system. Unfortunately, there were many errors in the reference [1]. The paper was re-released in June 1984 [2] with some of the errors corrected. There were, however, still numerous errors in [2] requiring that almost all of the mathematical modeling be done from scratch, precluding an extensive investigation into the controlling of the system. This thesis therefore provides a detailed mathematical model of the system. The system equations of motion were developed assuming the shuttle and antenna to be rigid and the beam to be flexible. The beam was assumed to be capable of transverse bending in each of two orthogonal directions and to undergo torsional motion about it's long axis. The assumed modes method was used to discretize the beam motion and a set of linear first-order equations were developed for the system.

Active control of the system was achieved using a truncated dynamical model, using linear optimal regulator theory and modal suppression techniques as outlined in [6] and [7].

II. Model Configuration

The physical model of the SCOLE is shown in Fig 1. It consists of the shuttle, a 130 foot flexible beam attached to the shuttle's center of mass (an assumption made for modeling purposes), and a rigid antenna attached at one corner to the beam. The axis system is as shown: The x (or roll) axis points out of the nose of the shuttle, the y (or pitch) axis points out the shuttle's right wing, and the z (or yaw) axis points out the bottom of the shuttle, which is nominally toward the Earth. The xyz reference frame is considered attached to the shuttle with its origin at the center of mass (beam attachment point). This frame is free to rotate about an XYZ frame, in which the X axis points along the velocity vector of the shuttle in orbit, the Z axis points to the Earth's center, and the Y axis completes the right-handed system. The XYZ frame is considered inertial for the purposes of this paper.

Another axis system, fixed to the antenna at the beam-antenna attachment point, is shown in Fig 2. When the beam is not deformed, this $x_4y_4z_4$ axis system aligns itself with the xyz axis system. When the beam is deformed, the $x_4y_4z_4$ axes will be displaced from xyz by what are assumed to be very small angles. This axis system will be used in the mathematical development of the equations of motion in the

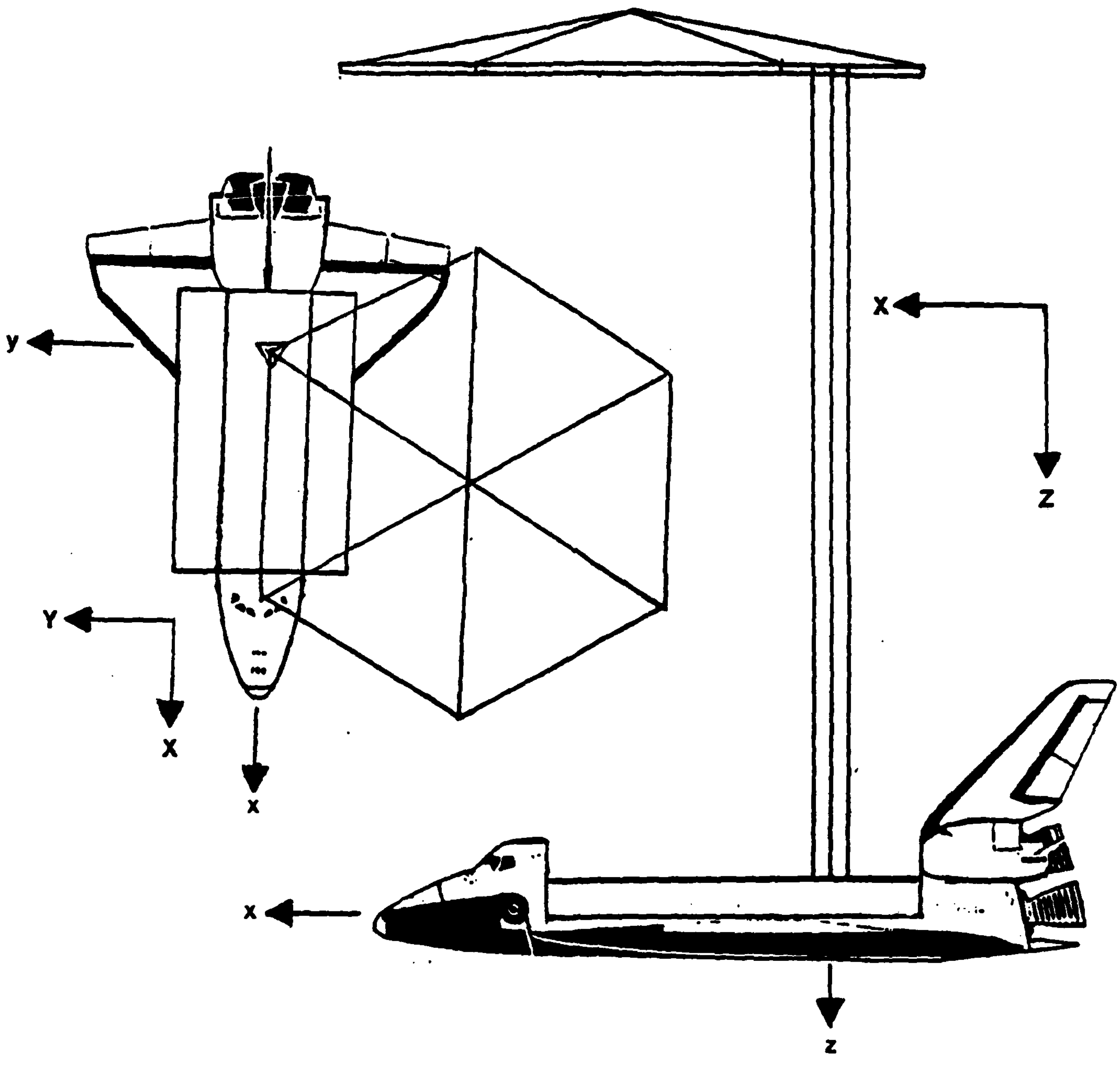


Fig. 1. SCOLE Model

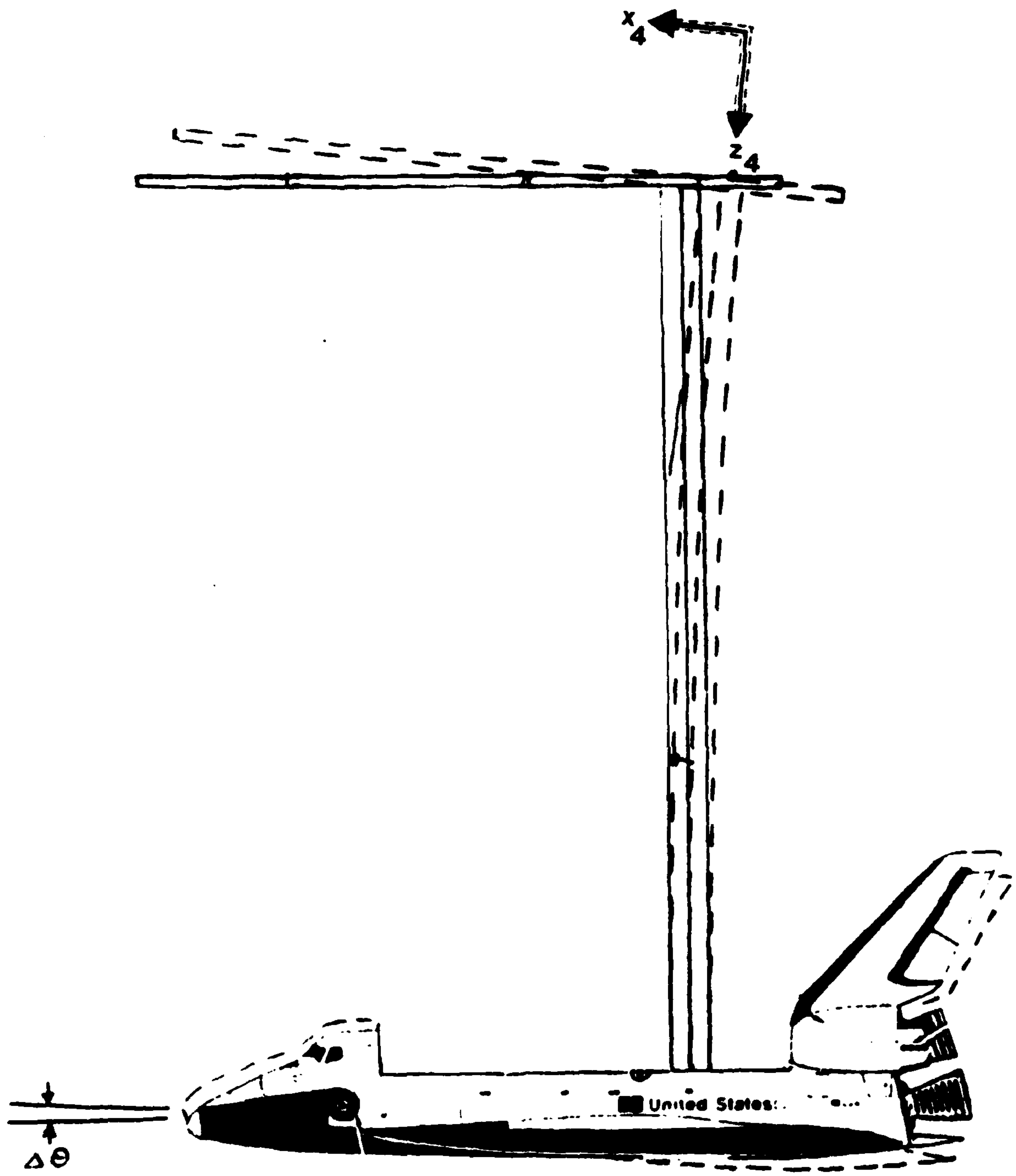


Fig. 2. Flexing Beam Illustration

next chapter.

Reference [2] furnished the information on the values for the masses and moments of inertia for the shuttle, reflector, and entire shuttle-beam-antenna system. This information is presented in Table I. Also included are the modal damping and stiffness coefficients for the vibration of the beam, along with the masses of the proof-mass actuators.

Before developing the mathematical model, the main types of motion will be described, along with the assumptions made for both simplification and clarification.

Referring to Fig 1, the first type of motion considered is the tumbling of the entire shuttle-beam-antenna system, expressed as some arbitrary rotation of the xyz frame about the inertial XYZ frame. The angular velocity is given as $\bar{\omega}$ (no subscript). A fundamental assumption for the purposes of this paper is that the shuttle wishes to stay aligned with the XYZ frame, so any rotation out of that alignment will be met with the shuttle firing its attitude control jets. This means that any rotation will be small and of short duration. The firing of the reaction jets will cause the beam to flex, resulting in the second type of motion to be discussed.

The flexing of the beam adds a tremendous complexity to the problem. Referring to Fig 2, it can be seen that the bending of the beam will change the overall shape of the shuttle-beam-antenna system very slightly. An assumption made here is that the deformations of the beam is small along

<u>Table I</u>

Physical Characteristics of SCOLE

Shuttle	mass = m_1 = 6366.46 slugs
I_1 (slug-ft^2) =	$\begin{bmatrix} 905,443 & 0 & -145,393 \\ 0 & 6,789,100 & 0 \\ -145,393 & 0 & 7,086,601 \end{bmatrix}$
Antenna	mass = m_4 = 12.42 slugs
I_4 (slug-ft^2) =	$\begin{bmatrix} 4,969 & 0 & 0 \\ 0 & 4,969 & 0 \\ 0 & 0 & 9,938 \end{bmatrix}$
Beam	mass = m_B = 12.42 slugs
Roll Bending:	ρA = 0.09556 slugs/ft
	EI = 4.0×10^7 lb-ft^2
	ξ = 0.003
Pitch Bending:	ρA = 0.09556 slugs/ft
	EI = 4.0×10^7 lb-ft^2
	ξ = 0.003
Yaw Torsion	ρJ = 0.9089 slug-ft
	GJ = 4.0×10^7 lb-ft^2
	ξ = 0.003
Proof-Mass Actuators:	mass = m_2 = m_3 = 0.3108 slugs

its entire length, and thus also at the beam-antenna attachment point.

The approach to setting up the mathematical model in this paper will be to assume the beam and shuttle form a dynamical system and that the antenna effect is to subject the beam's free end to forces and moments. Since the antenna is assumed to be rigidly attached to the beam, the motion of the beam is totally defined by the rigid-body motion of the shuttle-beam system plus the elastic motion of the beam tip. The center of mass of the shuttle-beam-antenna system is assumed to be unaffected by the elastic motion. This is a good assumption, considering that the mass of the shuttle is over 500 times that of the beam-antenna combination (see Table I).

The overall motion of the beam-antenna with respect to the shuttle is to be controlled by force and moment actuators located on the antenna, along with two proof-mass actuators (Fig 1), located at positions sn2 and sn3 along the beam to be chosen by the analyst. The optimum location for these actuators could be the subject for an entire study. For this paper, however, the actuators will be assumed to be located at the 40- and 80-foot positions along the beam (see Appendix A). The actuators operate by moving a mass, which causes forces in the x and y directions only. There is no torsional input from these actuators. There is, however, a moment created about the system's mass center which will tend to rotate the shuttle-beam-antenna system out of its desired attitude. These moments, although small, will be included in this study.

Other assumptions made are:

1) The beam does not appreciably stretch, meaning that it does not deform in the z direction.

2) Any forces in the z direction from the motion of the antenna are insignificant.

3) There are no specific forces or moments, such as those due to meteor collisions, solar radiation pressure, gravity torques, or magnetic or atmospheric effects modeled in this study. These forces, if they exist, will be small in comparison to the control torques available.

The next chapter will develop the mathematical model of the system, which in its general form would be nonlinear and contain both partial and ordinary differentials. For the study at hand a linear discrete model was produced using an assumed modes approach and by considering only small motions from an undeformed equilibrium position.

III. Mathematical Development

This section will attack the mathematical development of the system model in four parts. The first will be the choosing of functions to represent the flexing of the beam. The second will be the derivation of the equations of motion of the entire shuttle-beam-antenna system, incorporating the flexing of the beam, and accounting for the rigid antenna through forces and moments acting on the antenna end of the beam. The third will be the development of the equations of motion of the antenna in terms of the displacements and rotations at the end of the beam, along with the development of the forces due to the proof-mass actuators. The fourth section will put the equations of motion together with the generalized forces and express the system in matrix equation form for the purposes of applying a control law.

Choosing Proper Functions

A discretization approach will be employed in this paper in choosing functions to represent the motion of the vibrating beam with respect to the shuttle. This is known as an assumed modes approximation. The more modes modeled, the more accurate the system will be, so this paper will use a fourteen-mode approximation to ensure that the first several system modes have converged.

The differential equation of motion for a beam in bending vibration is given by Meirovitch [1:208-209] as

$$\frac{-\partial^2}{\partial x^2}\left[EI(x)\,\frac{\partial^2 y(x,t)}{\partial x^2}\right] + f(x,t) = M(x)\,\frac{\partial^2 y(x,y)}{\partial t^2} \qquad (1)$$

If the beam is uniform, this is reduced to

$$M\,\frac{\partial^2 y(x,t)}{\partial t^2} + EI\,\frac{\partial^4 y(x,t)}{\partial x^4} = f(x,t) \qquad (2)$$

where $y(x,t)$ is the transverse displacement, x is a coordinate along the beam length, M is the mass per unit length, and EI is the bending stiffness. For free vibration the distributed force $f(x,t) = 0$. For the case of a cantilever beam the associated boundary conditions are

$$y(0,t) = 0$$

$$\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = 0$$

$$\left.\frac{EI\partial^2 y(x,t)}{\partial x^2}\right|_{x=L} = 0 \qquad (3)$$

$$\left.\frac{EI\partial^3 y(x,t)}{\partial x^3}\right|_{x=L} = 0$$

Denoting the running length of the beam by s, the mass per unit length by ρA, and defining the two orthogonal components of bending by u_r and u_p (see Fig. 3) and assuming the beam has equal moments of inertia in these two directions, two equations of motion can be written:

$$\rho A \frac{\partial^2 u_r(s,t)}{\partial t^2} + EI \frac{\partial^4 u_r(s,t)}{\partial s^4} = 0$$

$$\rho A \frac{\partial^2 u_p(s,t)}{\partial t^2} + EI \frac{\partial^4 u_p(s,t)}{\partial s^4} = 0 \tag{4}$$

subject to the conditions

$$u_r(0,t) = u_p(0,t) = 0$$

$$\frac{\partial u_r(0,t)}{\partial s} = \frac{\partial u_p(0,t)}{\partial s} = 0 \tag{5}$$

$$EI \left.\frac{\partial^2 u_r(s,t)}{\partial s^2}\right|_{s=L} = EI \left.\frac{\partial^2 u_p(s,t)}{\partial s^2}\right|_{s=L} = 0$$

$$EI \left.\frac{\partial^3 u_r(s,t)}{\partial s^2}\right|_{s=L} = EI \left.\frac{\partial^3 u_p(s,t)}{\partial s^3}\right|_{s=L} = 0$$

Since the two deformations u_r and u_p satisfy identical relationships, the development will be restricted to u_r and the results applied to both u_r and u_p. Assume that u_r can be written as

$$u_r(s,t) = R(s)U_r(t) \tag{6}$$

Substituting eq (6) into (4a) and dividing by the product $R(s)U_r(t)$ yields

$$\frac{\ddot{U}_r}{U_r} + \frac{EI}{\rho A}\frac{R^{iv}}{R} = 0 \tag{7}$$

which implies that

$$R^{iv} - \alpha_r{}^4 R = 0 \tag{8}$$

where

$$\alpha_r{}^4 = \frac{-\rho A\ \ddot{U}_r}{EI\ U_r} = \frac{\rho A\ \omega_r{}^2}{EI} \tag{9}$$

The above conditions have the general solution

$$R = A_r \sin\alpha_r s + B_r \cos\alpha_r s + C_r \sinh\alpha_r s + D_r \cosh\alpha_r s \tag{10}$$

Substituting eq (6) into the boundary conditions yields

$$R(0) = R'(0) = 0 \tag{11}$$

$$R''(L) = R'''(L) = 0$$

Solving these four equations simultaneously, the results are three important constraints which drive the rest of the development:

$$A_r = -C_r$$

$$B_r = -D_r = \frac{-A_r(\sin\alpha_r L + \sinh\alpha_r L)}{(\cos\alpha_r L + \cosh\alpha_r L)} \tag{12}$$

$$\cos\alpha_r L \cosh\alpha_r L = -1$$

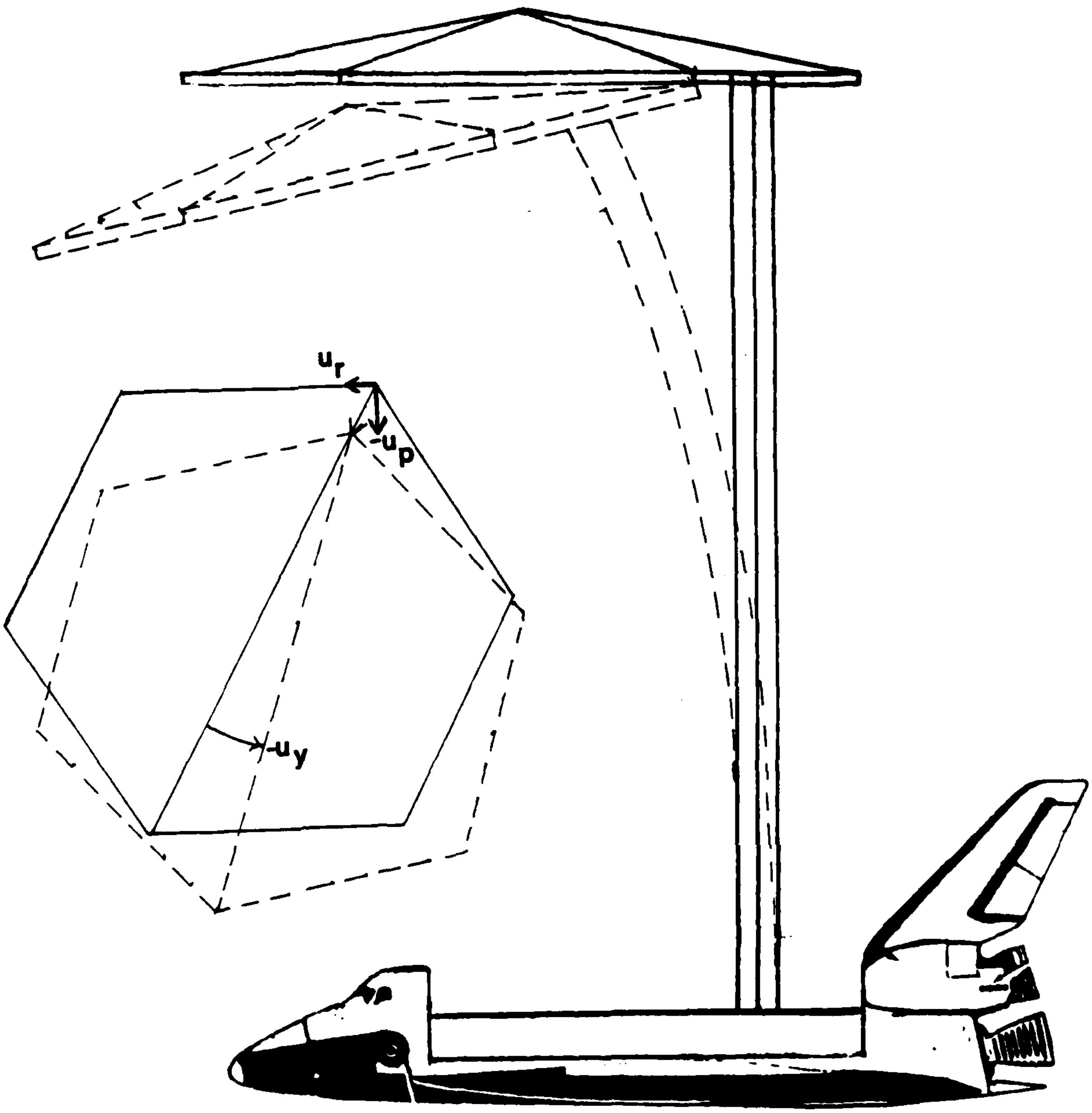


Fig. 3. Deformation of the Antenna

The last of eqs (12) has an infinite number of solutions. These solutions will be denoted by α_{ri} and the associated amplitudes by A_{ri}, B_{ri}, C_{ri}, and D_{ri}. The mode shapes are then given by

$$R_{ri} = A_{ri}\left[(\sin\alpha_{ri}s - \sinh\alpha_{ri}s) + \frac{(\sin\alpha_{ri}L + \sinh\alpha_{ri}L)}{(\cos\alpha_{ri}L + \cosh\alpha_{ri}L)}(\cosh\alpha_{ri}s - \cos\alpha_{ri}s)\right] \tag{13}$$

The amplitude A_{ri} is arbitrary. For convenience, the magnitude A_{ri} will be chosen such that

$$\int_0^L \rho A\, R_i R_i\, ds = 1 \tag{14}$$

Expanding (14) and integrating each individual term directly takes a great deal of bookkeeping and substitution (see Appendix B), but the solution reduces to:

$$\begin{aligned} A_{ri} &= \left(\frac{1}{(\rho AL)}\right)^{1/2} \frac{(\cos\alpha_{ri}L + \cosh\alpha_{ri}L)}{(\sin\alpha_{ri}L + \sinh\alpha_{ri}L)} \\ B_{ri} &= -\left(\frac{1}{(\rho AL)}\right)^{1/2} \end{aligned} \tag{15}$$

Since the mode shapes have been normalized, they possess the convenient property

$$\int_0^L \rho A\, R_i R_j\, ds = \delta_{ij} \tag{16}$$

where δ_{ij} is the Kronecker delta.

The preceeding derivation applies to a cantilever beam in bending. The beam element in the model of Fig (3) is assumed to behave as a cantilever in bending in each of two orthogonal directions (roll and pitch), and in addition to undergo torsional deformation. The normal modes given by eq (13) may also be used for pitch bending if the subscripts r are replaced with p throughout. The roll motion is therefore given by:

$$u_r = \sum_{i=1}^{n_r} R_i(s)U_{ri}(t) \tag{17}$$

and the pitch motion as

$$u_p = \sum_{i=1}^{n_p} P_i(s)U_{pi}(t) \tag{18}$$

where n_r is the number of roll modes and n_p is the number of pitch modes.

The torsional motion about the z (yaw) axis, however, must be treated separately. The differential equation for the torsional motion on a beam is given by

$$\frac{\rho J\partial^2 u_y}{\partial t^2} - \frac{GJ\partial^2 u_y}{\partial s^2} = 0 \tag{19}$$

where J is the polar area moment of inertia. The associated boundary conditions are given by

$$\begin{aligned} u_y(0,t) &= 0 \\ u'_y(L,t) &= 0 \end{aligned} \tag{20}$$

Proceeding as in the case of bending vibration, the torsional mode shapes are given by

$$Y_i = C_{yi} \sin\beta_{yi} s \tag{21}$$

where

$$\beta_{yi} L = (2n-1)\pi/2 \qquad n = 1,2,3,\ldots \tag{22}$$

As before, it is convenient that the mode shapes be orthogonal, so the condition that must be satisfied is

$$\int_0^L \rho J \; Y_i Y_i ds = 1 \tag{23}$$

Substituting (21) into (23) results in

$$\int_0^L \rho J \; C_{yi}{}^2 \sin^2\beta_{yi} s \; ds = 1 \tag{24}$$

which, after integrating and applying the limits, gives

$$C_{yi} = \left(\frac{4\beta_{yi}}{\rho J(2\beta_{yi}L - \sin 2\beta_{yi}L)} \right)^{1/2} = \left(\frac{2}{\rho JL} \right)^{1/2} \tag{25}$$

The sine term goes to zero because 2*beta*L will result in an integer multiple of pi for all betas.

As before, these functions demonstrate the property

$$\int_0^L \rho J \; Y_i Y_j \; ds = \delta_{ij} \tag{26}$$

The torsional motion may now be described by the relationship

$$u_y(s,t) = \sum_{i=1}^{n_y} Y_i(s) U_{yi}(t) \tag{27}$$

Table II

Angles, Coefficients, and Squared Frequencies for the Roll and Pitch Functions

$\alpha_i L$ (radians)	A_i	B_i	ω_i^2 $(\mathrm{rad/sec})^2$
1.875102	.2082777241	−.2837201974	18.11807
4.694091	.2889597487	−.2837201974	711.56816
7.854757	.2835001714	−.2837201974	5578.80978
10.995541	.2837297171	−.2837201974	21422.82836
14.137168	.2837197860	−.2837201974	58541.00381
17.278759	.2837202151	−.2837201974	130635.34107
20.420352	.2837201966	−.2837201974	254837.51330
23.561945	.2837201974	−.2837201974	451705.09099
26.703538	.2837201974	−.2837201974	745221.94382
29.845130	.2837201974	−.2837201974	1162798.20573
32.986723	.2837201974	−.2837201974	1735270.27761
36.128315	.2837201974	−.2837201974	2496900.82710
39.269908	.2837201974	−.2837201974	3485378.78863
42.411501	.2837201974	−.2837201974	4741819.36334

where n_y is the number of yaw modes, $U_{yi}(t)$ are time dependent modal amplitudes and the $Y_i(s)$ are given by eq (21).

A computer program was written to calculate the roots of eq (12c). Using the first fourteen roots, the first fourteen roll and pitch functions were evaluated, solving for each A_i and the (constant) B_i's. The ω_i^2's were also computed, and the data is tabulated in Table II. The betas, being multiples of pi, were found directly and the C_{yi}'s and ω_{yi}^2's were calculated for the yaw torsion functions. This data is presented in Table III.

In the next section, certain integral relationships in-

Table III

Angles, Coefficients, and Squared Frequencies for Yaw Torsion Equation

$\beta_i L$ (radians)	C_i	ω_i^2 $(\text{rad/sec})^2$
1.570796327	.1301023886	6425.3522
4.712388980	.1301023886	57828.1697
7.853981634	.1301023886	160633.8047
10.99557429	.1301023886	314842.2572
14.13716694	.1301023866	520453.5273
17.27875959	.1301023866	777467.6148
20.42035225	.1301023866	1085884.520
23.56194490	.1301023866	1445704.242
26.70353756	.1301023866	1856926.782
29.84513021	.1301023866	2319552.140
32.98672286	.1301023866	2833580.315
36.12831552	.1301023866	3399011.308
39.26990817	.1301023866	4015845.118
42.41150082	.1301023866	4684081.745

volving the continuous coordinates u_r, u_p, and u_y will be needed. Specifically, the kinetic and potential energies associated with these variables are of interest. These energies involve the following six integrals:

$$In_1 = 1/2\int_0^L \rho A \dot{u}_r^2 ds$$

$$In_2 = 1/2\int_0^L \rho A \dot{u}_p^2 ds$$

$$In_3 = 1/2\int_0^L \rho J \dot{u}_y^2 ds \qquad (28)$$

$$In_4 = 1/2\int_0^L EI\left(\frac{\partial^2 u_r}{\partial s^2}\right)^2 ds$$

$$In_5 = 1/2\int_0^L EI\left(\frac{\partial^2 u_p}{\partial s^2}\right)^2 ds$$

$$In_6 = 1/2\int_0^L GJ\left(\frac{\partial u_y}{\partial s}\right)^2 ds$$

Direct substitution of the modal approximations into these integrals yields

$$In_1 = 1/2\sum_{i=1}^{n_r} \dot{U}_{ri}^{\,2}(t)$$

$$In_2 = 1/2\sum_{i=1}^{n_y} \dot{U}_{pi}^{\,2}(t)$$

$$In_3 = 1/2\sum_{i=1}^{n_y} \dot{U}_{yi}^{\,2}(t) \tag{29}$$

$$In_4 = 1/2\sum_{i=1}^{n_r} \omega_{ri}^{\,2} U_{ri}^{\,2}$$

$$In_5 = 1/2\sum_{i=1}^{n_p} \omega_{pi}^{\,2} U_{pi}^{\,2}$$

$$In_6 = 1/2 \sum_{i=1}^{n_y} \omega_{yi}{}^2 U_{yi}{}^2$$

where

$$\omega_{ri}{}^2 = \alpha_{ri}{}^4 \frac{EI}{\rho A}$$

$$\omega_{pi}{}^2 = \alpha_{pi}{}^4 \frac{EI}{\rho A} \qquad (30)$$

$$\omega_{yi}{}^2 = \beta_{yi}{}^2 \frac{GJ}{\rho J}$$

This section has determined the orthonormal eigenfunctions for a fixed-free uniform rod in both bending and torsion. These functions will be used as assumed modes for the beam part of the shuttle-beam-antenna system in the next section. The properties of these functions given in eqs (29) and (30) will prove useful in this development.

System Equations

The system's equations of motion will be developed using a combination of Lagrange's equations for the elastic motion of the beam and Euler's moment equations for the overall motion of the shuttle-beam combination. To that end the kinetic and potential energies for the shuttle-beam combination will be developed. The effect of the antenna will be taken into account through the generalized forces due to the forces and moments present at the beam-antenna attachment point. The beam is assumed to be rigidly attached to the shuttle at the center of mass of the system. It is further assumed to bend in two orthogonal directions normal to its long axis and to undergo torsional motion around its long axis. The location of a general point in the shuttle-beam system relative to the system's mass center is given by

$$\bar{R} = \bar{r} + \bar{u} \tag{31}$$

where $\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$ locates a generic point and $\bar{u}$ denotes the elastic displacement of the mass particle at $\bar{r}$. Defining $\bar{R}_s$ and $\bar{R}_b$ as position vectors of points in the shuttle and beam respectively, the equation becomes

$$\begin{aligned} \bar{R}_s &= x\hat{x} + y\hat{y} + z\hat{z} \\ \bar{R}_b &= (u_r(z,t) + x)\hat{x} + (u_p(z,t) + y)\hat{y} + z\hat{z} \end{aligned} \tag{32}$$

The system kinetic energy is thus given by

$$\begin{aligned} T = 1/2\ m_t V_c{}^2 &+ 1/2 \int_{m_s} \dot{\bar{R}}_s{}^I \cdot \dot{\bar{R}}_s{}^I \, dm_s \\ &+ 1/2 \int_{m_b} \dot{\bar{R}}_b{}^I \cdot \dot{\bar{R}}_b{}^I \, dm_b \end{aligned} \tag{33}$$

where m_t is the total mass, V_c is the velocity of the mass center, m_s and m_b are the masses of the shuttle and beam respectively, and the superscript I denotes inertial derivatives. Denoting the angular velocity of the body-fixed shuttle axes by $\bar{\omega}$ and that of an element of the beam by $\bar{\omega}_b$, the kinetic energy can be rewritten as

$$T = 1/2\ m_t V_c^2 + 1/2 \int_{m_s} (\bar{\omega} \times \bar{R}_s)\cdot(\bar{\omega} \times \bar{R}_s)\,dm_s \tag{34}$$

$$+ 1/2 \int_{m_b} (\dot{\bar{R}}_b^{\,B} + \bar{\omega}_b \times \bar{R}_b)\cdot(\dot{\bar{R}}_b^{\,B} + \bar{\omega}_b \times \bar{R}_b)\,dm_b$$

where the superscript B denotes derivatives seen by an observer fixed in the shuttle body axis, so $\dot{\bar{R}}_s^{\,B} = 0$ since the shuttle is rigid. Furthermore it should be noted that

$$\dot{\bar{R}}_b^{\,B} = \dot{u}_r(z,t)\hat{x} + \dot{u}_p(z,t)\hat{y} \tag{35}$$

and that $\bar{\omega}$ and $\bar{\omega}_b$ are related by

$$\bar{\omega}_b = \bar{\omega} + \bar{\omega}_{b/s} = \bar{\omega} + \dot{u}_y(z,t)\hat{z} \tag{36}$$

where $u_y(z,t)$ is the torsional angular displacement of the beam element at position z. The kinetic energy is expanded and expressed in matrix notation as

$$T = 1/2\{\omega\}^T\,[I_s]\,\{\omega\} + 1/2\int_{mb}\left[\begin{Bmatrix}\dot{u}_r\\ \dot{u}_p\\ 0\end{Bmatrix}^T \begin{Bmatrix}\dot{u}_r\\ \dot{u}_p\\ 0\end{Bmatrix} + 2\begin{Bmatrix}\dot{u}_r\\ \dot{u}_p\\ 0\end{Bmatrix}^T [\tilde{\omega}]\left(\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} + \begin{Bmatrix}u_r\\ u_p\\ 0\end{Bmatrix}\right)\right.$$

$$+ 2\begin{Bmatrix}\dot{u}_r\\ \dot{u}_p\\ 0\end{Bmatrix}^T \left[\tilde{\omega}_{b/s}\right] \left(\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} + \begin{Bmatrix}u_r\\ u_p\\ 0\end{Bmatrix}\right)$$

$$+ \left(\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} + \begin{Bmatrix}u_r\\ u_p\\ 0\end{Bmatrix}\right)^T \left[\tilde{\omega}\right]^T \left[\tilde{\omega}\right]\left(\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} + \begin{Bmatrix}u_r\\ u_p\\ 0\end{Bmatrix}\right)$$

$$+\ 2\left(\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} u_r \\ u_p \\ 0^p \end{Bmatrix}\right)^T \left[\tilde{\omega}\right]^T \left[\tilde{\omega}_{b/s}\right] \left(\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} u_r \\ u_p \\ 0^p \end{Bmatrix}\right)$$

$$+ \left(\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} u_r \\ u_p \\ 0^p \end{Bmatrix}\right)^T \left[\tilde{\omega}_{b/s}\right]^T \left[\tilde{\omega}_{b/s}\right] \left(\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{Bmatrix} u_p \\ u_p \\ 0^p \end{Bmatrix}\right)\Bigg]\, dm_b \qquad (37)$$

where I_s is the moment of inertia matrix for the shuttle. The matrices $\tilde{\omega}$ and $\tilde{\omega}_{b/s}$ in eq (37) are formed from the components of the $\bar{\omega}$ and $\bar{\omega}_{b/s}$ vectors in the form

$$[\tilde{\omega}] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \qquad (38)$$

The kinetic energy in eq (37) contains terms through order four in the variables u_r, u_p, ω_x, ω_y, and u_y and their rates. Since it is desirable to provide a linear set of equations for the motion of the system in the neighborhood of an equilibrium where all of these variables and their rates are small, T will be reduced to include only those terms of order two or less in these variables. This yields the kinetic energy as

$$T = 1/2\ \{\omega\}^T \left[I_{s+b}\right] \{\omega\}$$

$$+\ 1/2\int_{mb}\left[\begin{Bmatrix} \dot{u}_r \\ \dot{u}_p \\ 0^p \end{Bmatrix}^T \begin{Bmatrix} \dot{u}_r \\ \dot{u}_p \\ 0^p \end{Bmatrix} + 2\begin{Bmatrix} \dot{u}_r \\ \dot{u}_p \\ 0^p \end{Bmatrix}^T \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \{\omega\}\right.$$

$$+\ 2\begin{Bmatrix} \dot{u}_r \\ \dot{u}_p \\ 0^p \end{Bmatrix}^T \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{u}_y \end{Bmatrix} + 2\ \{\omega\}^T \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}^T \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \dot{u}_y \end{Bmatrix}$$

$$+ \left\{\begin{matrix} 0 \\ 0 \\ \dot{u}_y \end{matrix}\right\}^T \left[\begin{matrix} z^2+y^2 & -xy & -xz \\ -xy & z^2+x^2 & -yz \\ -xz & -yz & x^2+y^2 \end{matrix}\right] \left\{\begin{matrix} 0 \\ 0 \\ \dot{u}_y \end{matrix}\right\} \Bigg] dm_b \tag{39}$$

Expansion of eq (39) yields numerous terms including integrals involving the variables x and y linearly and the combinations xy, xz, and yz. Since the elastic deformations u_r, u_p, and u_y spatially depend only on z, these integrals are all zero from symmetry considerations. Setting these terms equal to zero yields the kinetic energy in the form

$$T = 1/2\{\omega\}^T \left[I_{s+b} \right]\{\omega\} + 1/2 \int_0^L \rho \left\{\begin{matrix} \dot{u}_r \\ \dot{u}_p \\ \dot{u}_y \end{matrix}\right\}^T \left[\begin{matrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & J \end{matrix}\right] \left\{\begin{matrix} \dot{u}_r \\ \dot{u}_p \\ \dot{u}_y \end{matrix}\right\} dz$$

$$+ \int_0^L \rho A \left\{\begin{matrix} \dot{u}_r \\ \dot{u}_p \\ \dot{u}_y \end{matrix}\right\}^T \left[\begin{matrix} 0 & z & 0 \\ -z & 0 & 0 \\ 0 & 0 & x^2+y^2 \end{matrix}\right] \{\omega\} \; dz \tag{40}$$

Substituting the assumed modes from the previous section results in

$$T = 1/2 \; \{\omega\}^T \left[I_{s+b} \right] \{\omega\} + 1/2 \; \{\dot{U}\}^T \{\dot{U}\} + \{\dot{U}\}^T \left[Sz \right]^T \{\omega\} \tag{41}$$

where

$$\{U\}^T = (U_{r,1} \quad U_{r,2} \quad \cdots \quad U_{r,n^r} \quad U_{p,1} \quad U_{p,2} \quad \cdots$$
$$U_{p,n^p} \quad U_{y,1} \quad U_{y,2} \cdots \quad U_{y,n^y}) \tag{42}$$

and the Sz matrix in eq (41) has dimension 3 by $(n_r+n_p+n_y)$.

Three blocks of terms in the Sz matrix warrant closer attention. The lower right corner block is $x^2 + y^2$ and, after matrix multiplication and integration, will result in

$$Sz(3,k) = \int_0^L 2\omega_3 J \; u_{yi} dz \qquad (43)$$

where

$$i = 1, 2, 3, \ldots, n_y \qquad k = n_r + n_p + i$$

These terms are considered insignificant for this analysis, since a typical value of J will make the terms small compared to the others in the matrix. Therefore these terms will be assumed to be zero.

The other non-zero terms in the matrix are

$$Sz(1,j) = -\int_0^L \rho A z P_j(z) dz \qquad (44)$$

and

$$Sz(2,i) = -Sz(1,j) \qquad (45)$$

where

$$i = 1, 2, 3, \ldots, n_r, \qquad j = n_r + 1, n_r + 2, \ldots, n_r + n_p$$

Substituting the eigenfunction expressions into (44) yields

$$Sz(1,j) = -\rho A \int_0^L \left[zA_{pj} \sin\alpha_{pj} z dz + zB_{pi} \cos\alpha_{pj} z dz \right. \qquad (46)$$

$$\left. - zA_{pj} \sinh\alpha_{pj} z dz - zB_{pj} \cosh\alpha_{pj} z \right] dz$$

Integrating each of the four terms separately, and taking advantage of the fact that

$$B_{pj}(\cos\alpha_{pj}L + \cosh\alpha_{pj}L) = -A_{pj}(\sin\alpha_{pj}L + \sinh\alpha_{pj}L) \quad (47)$$

and

$$B_{pj} = -1/(\rho AL)^{1/2} \quad (48)$$

eq (46) becomes

$$Sz(1,j) = \frac{2(\rho AL^3)^{1/2}}{(\alpha_{pj}L)^2} \quad (49)$$

The Sz(2,i) term will differ by an algebriac sign, and the subscripts will become ri instead of pj. The potential energy can be expressed as

$$V = 1/2\int_0^L EI\left(\frac{\partial^2 u_r}{\partial z^2}\right)^2 dz + 1/2\int_0^L EI\left(\frac{\partial^2 u_p}{\partial z^2}\right)^2 dz$$

$$+ 1/2\int_0^L GJ\left(\frac{\partial u_y}{\partial z}\right)^2 dz \quad (50)$$

Using the assumed mode expressions, this becomes

$$V = 1/2 \ \{U\}^T \left[\diagdown \omega^2 \diagdown\right] \{U\} \quad (51)$$

So, forming the Lagrangian and taking derivatives,

$$L = T - V$$

$$\frac{\partial L}{\partial \{\dot{U}\}} = \left\{\dot{U}_i\right\} + \left[Sz\right]^T \left\{\omega\right\} \quad (52)$$

$$\frac{\partial L}{\partial \{U\}} = -\begin{bmatrix} \diagdown \omega^2 \diagdown \end{bmatrix} \{U\}$$

Forming Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{U}_i}\right) - \frac{\partial L}{\partial U_i} = Q_i \tag{53}$$

where Q_i are the generalized forces to be developed in the next section. Substituting in the terms just developed, Lagrange's equations for this problem become

$$\{\ddot{U}\} + \begin{bmatrix} Sz \end{bmatrix}^T \{\dot{\omega}\} + \begin{bmatrix} \diagdown \omega^2 \diagdown \end{bmatrix} \{U\} = \{Q\} \tag{54}$$

This equation assumes no damping in the beam. The damping will be figured in as one of the last steps before applying the control law.

Eq (54) is but one of two equations which can be obtained from the Lagrangian method. If derivatives are taken with respect to ω, the result is

$$\left\{\frac{\partial T}{\partial \omega}\right\} = \begin{bmatrix} I_T \end{bmatrix} \{\omega\} + \begin{bmatrix} Sz \end{bmatrix} \{\dot{U}\} \tag{55}$$

Applying Lagrange's method, the result is

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \omega}\right\} + \begin{bmatrix} \tilde{\omega} \end{bmatrix} \left\{\frac{\partial T}{\partial \omega}\right\} = \{M\} \tag{56}$$

where M is the sum of all moments on the system. The second term on the left side of the equation can be ignored since it is of higher order. The remaining terms give the equation

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \omega}\right\} = \left\{M\right\} \tag{57}$$

which are generally referred to as Lagrange's equations in quasi-coordinates. Again, substituting in the previously derived terms, the equation becomes

$$\left[I_T\right]\left\{\dot{\omega}\right\} + \left[Sz\right]\left\{\ddot{U}\right\} = \left\{M\right\} \tag{58}$$

This equation along with eq (54) will serve as the two equations of motion for the entire shuttle-beam-antenna system. Eq (54) is the rigid body equation of motion with a modification taking into account the flexing of the beam. Eq (58) is Euler's moment equation with a coupling term, again to account for the beam's flexing.

Now that the functions have been chosen and the two equations of motion derived, it is time to turn to the development of the forces and moments on the cantilever beam. This next section will focus on the term by term derivation of these forces and moments.

Force and Moment Development

Now that the unforced, undamped equations for the roll, pitch, and yaw bending have been obtained, the next step is to develop the generalized forces Q_i and the moments M of eqs (54) and (58).

The generalized forces are determined from [4] as

$$Q_i = \sum_{j=2}^{N} \bar{f}_j \cdot \frac{\partial \dot{\bar{r}}_j}{\partial \dot{U}_i} + \sum_{k=2}^{M} \bar{g}_k \cdot \frac{\partial \bar{\omega}_k}{\partial \dot{U}_i} \tag{59}$$

where the $\bar{f}_j$ are the applied forces, the $\dot{\bar{r}}_j$ is the velocity of the point of application of the force, and the $\dot{U}_i$ are the modal velocities from eq (42), where i runs from 1 to $n_r + n_p + n_y$. The $\bar{g}_k$ are the applied torques and $\bar{\omega}_k$ is the angular velocity of the element at which the torque is applied. For the problem at hand the only applied forces of interest are those shown in Fig 4 due to the actuators and the forces and moments due to the antenna. From Fig 4 it can be seen that the forces and moments are applied at three specific locations. Specifically, the forces are applied at points sn2 (which is 40), sn3 (which is 80), and at the beam-antenna attachment point sn4 (which is 130). Notice that the index j ranges from 2 to 4. This is because all forces at sn1 (the shuttle-beam attachment point) are zero due to the cantilever model.

Moments are applied only at sn4. As before, all moments at sn1 are zero, so the index k ranges from 2 to 4. Each of the forces and moments from eq (59) must now be individually identified.

The proof-mass actuators are designed to apply a force in the X and Y directions only. The X-direction forces are:

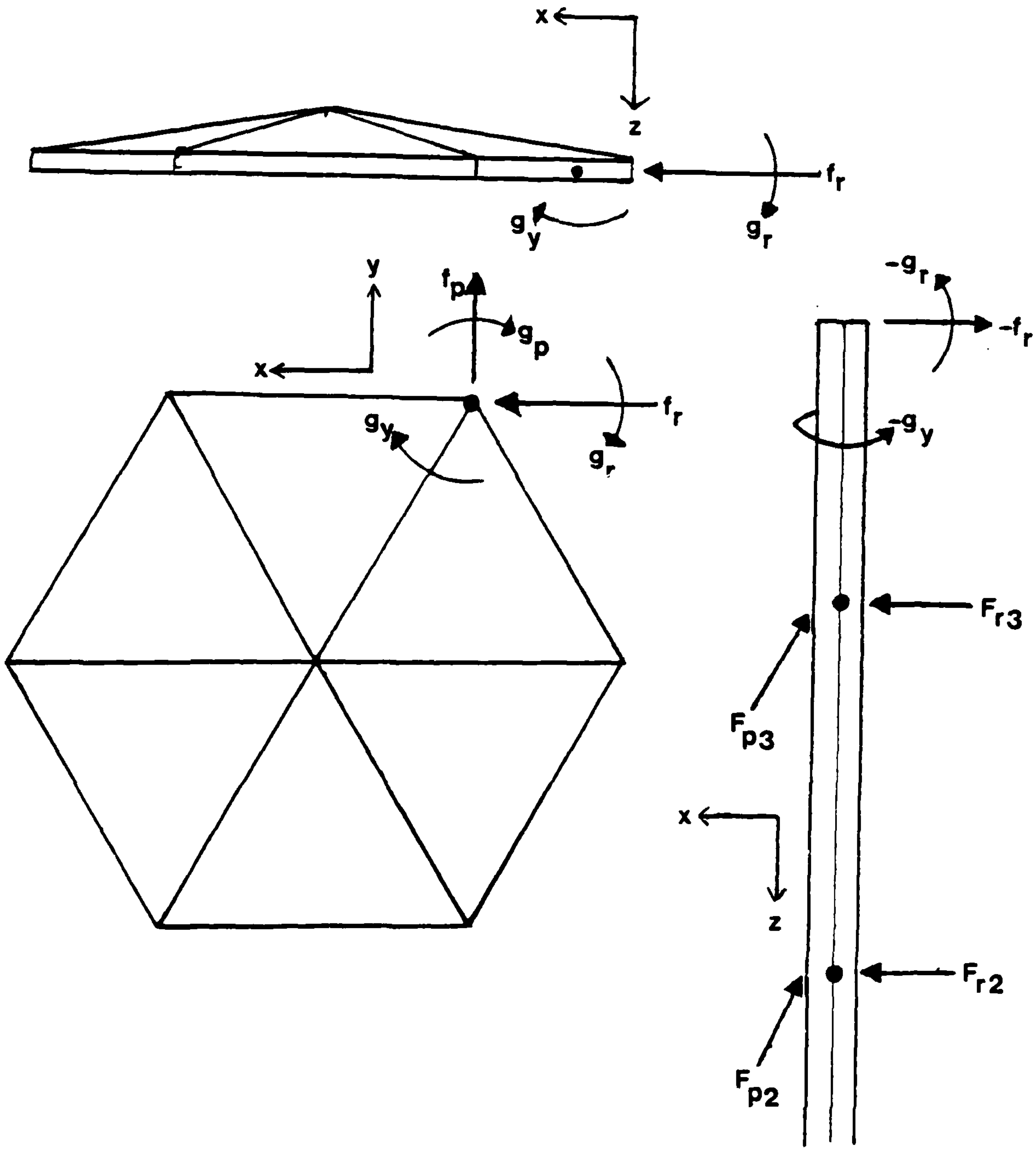


Fig. 4. Beam and Antenna Forces and Moments

$$f_{r,2} = m_2 \left.\frac{\partial^2 u_r}{\partial t^2}\right|_{s=40} + F_{r,2}$$

$$f_{r,3} = m_3 \left.\frac{\partial^2 u_r}{\partial t^2}\right|_{s=80} + F_{r,3} \tag{60}$$

where the first term in each equation is the mass of the actuator multiplied by the acceleration of the beam-actuator attachment point, and the second term is the force metered out by the actuator itself. The subscript 2 or 3 denotes the actuator at position sn2 or position sn3 respectively. The Y-direction forces have an identical form:

$$f_{p,2} = m_2 \left.\frac{\partial^2 u_p}{\partial t^2}\right|_{s=40} + F_{p,2}$$

$$f_{p,3} = m_3 \left.\frac{\partial^2 u_p}{\partial t^2}\right|_{s=80} + F_{p,3} \tag{61}$$

The last forces and moments to contend with are the forces and moments on the beam-antenna attachment point. The NASA paper [2] gives expressions for these forces which appear to be in error. Therefore the expressions for the forces and moments are derived as follows.

Looking at the free-body diagram of the antenna (Fig 4), the forces will be developed from the force equation. The position, velocity, and acceleration of the center of mass of the antenna with respect to the attachment point p are:

$$\bar{r}_c = 18.75\hat{x}_4 - 32.5\hat{y}_4$$

$$\bar{v}_c = \frac{d\bar{r}_c}{dt} + \bar{\omega}_4 \times \bar{r}_c = \bar{\omega}_4 \times \bar{r}_c$$

$$\bar{a}_c = \frac{d(\bar{\omega}_4 \times \bar{r}_c)}{dt} + \bar{\omega}_4 \times (\bar{\omega}_4 \times \bar{r}_c) = \dot{\bar{\omega}}_4 \times \bar{r}_c \tag{62}$$

These equations make use of the fact that the antenna is modeled as a rigid body, so the time rate of change of the position vector in this frame is zero. All higher order terms have been neglected. Now, using $\bar{F}=M\bar{A}$, and denoting the acceleration of the center of mass of the antenna in this frame as $\bar{a}_c$:

$$\bar{f}_4 = m_4\bar{a}_c = m_4(\bar{a}_p + \bar{a}_{c/p}) \tag{63}$$

where $\bar{a}_p$ is the acceleration of the attachment point and $\bar{a}_{c/p}$ is the acceleration of the mass center with respect to point p. The acceleration of point p is, in vector form:

$$\bar{a}_p = \frac{\partial^2 u_r\hat{x}}{\partial t^2} + \frac{\partial^2 u_p\hat{y}}{\partial t^2} \tag{64}$$

The acceleration of the mass center wrt point p takes a little more development. Since the antenna is rigidly attached to the beam, $\bar{\omega}_4$ is given by

$$\bar{\omega}_4 = \begin{bmatrix} \left.\dfrac{\partial^2 u_r}{\partial s \partial t}\right|_{s=130} \\ \left.\dfrac{\partial^2 u_p}{\partial s \partial t}\right|_{s=130} \\ \left.\dfrac{\partial u_y}{\partial t}\right|_{s=130} \end{bmatrix} + \bar{\omega} = \begin{bmatrix} \omega_{4a} \\ \omega_{4b} \\ \omega_{4c} \end{bmatrix} + \bar{\omega} \tag{65}$$

where ω is the rotation of the entire shuttle-beam-antenna system. This effect was addressed in the rigid body equation derivation and will not appear in the antenna-fixed reference frame. Therefore, the time-derivative of ω_4 is found to be:

$$\dot{\bar{\omega}}_4 = \begin{bmatrix} \left.\dfrac{\partial^3 u_r}{\partial s \partial t^2}\right|_{s=130} \\ \left.\dfrac{\partial^3 u_p}{\partial s \partial t^2}\right|_{s=130} \\ \left.\dfrac{\partial^2 u_y}{\partial t^2}\right|_{s=130} \end{bmatrix} \tag{66}$$

which can be written in vector form as:

$$\dot{\bar{\omega}}_4 = \left\{ \frac{\partial^3 u_r \ \hat{x}}{\partial s \partial t^2} + \frac{\partial^3 u_p \ \hat{y}}{\partial s \partial t^2} + \frac{\partial^2 u_y \ \hat{z}}{\partial t^2} \right\}_{s=130} \tag{67}$$

Before taking the cross product of the $\bar{\omega}_4$ and the position vector, the vectors must be in compatable frames. It can be shown that for very small angles, the $x_4 y_4 z_4$ frame and the xyz frame are essentially equal. This derivation will obtain the same result, but will go through the cross-product step by step, showing what assumptions are necessary.

Referring to Fig. (5), three possible rotations out of the $x_4y_4z_4$ are shown, along with their respective rotation matrices. Using the small angle assumption that the cosine of an angle is approximately equal to 1 and the sine is approximately equal to the angle itself, these three matrices become

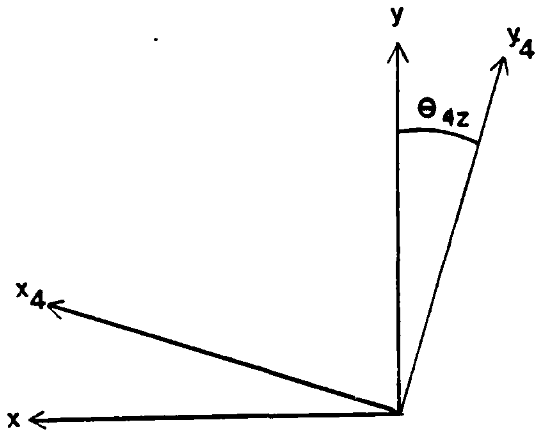
$$R_1 = \begin{bmatrix} 1 & -\theta_{4z} & 0 \\ \theta_{4z} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\theta_{4x} \\ 0 & \theta_{4x} & 1 \end{bmatrix}$$

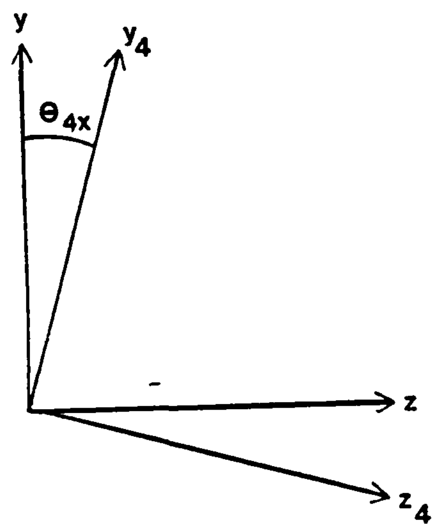
$$R_3 = \begin{bmatrix} 1 & 0 & \theta_{4y} \\ 0 & 1 & 0 \\ -\theta_{4y} & 0 & 1 \end{bmatrix}$$

Multiplying these together and ignoring any non-linear terms, the rotation matrix from the $x_4y_4z_4$ frame into the xyz frame is

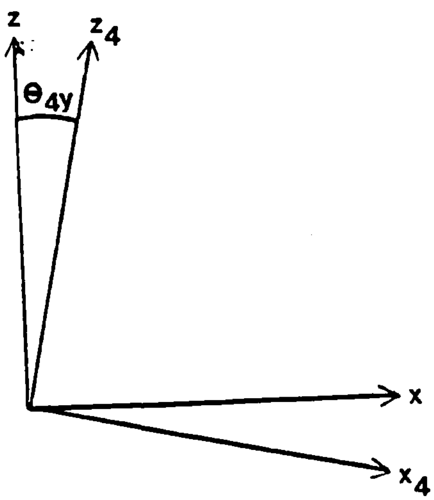
$$R_{x/4} = \begin{bmatrix} 1 & -\theta_{4z} & \theta_{4y} \\ \theta_{4z} & 1 & -\theta_{4x} \\ -\theta_{4y} & \theta_{4x} & 1 \end{bmatrix} \tag{68}$$



$$R_1 = \begin{bmatrix} \cos\theta_{4z} & -\sin\theta_{4z} & 0 \\ \sin\theta_{4z} & \cos\theta_{4z} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{4x} & -\sin\theta_{4x} \\ 0 & \sin\theta_{4x} & \cos\theta_{4x} \end{bmatrix}$$



$$R_3 = \begin{bmatrix} \cos\theta_{4y} & 0 & \sin\theta_{4y} \\ 0 & 1 & 0 \\ -\sin\theta_{4y} & 0 & \cos\theta_{4y} \end{bmatrix}$$

Fig. 5. Rotations

where the $R_{x/4}$ denotes rotation of the $x_4y_4z_4$ frame with respect to the xyz frame. Therefore, the position vector becomes, in column vector form

$$\bar{r}_c = \begin{Bmatrix} 18.75 + 32.5\theta_{4z} \\ 18.75\theta_{4z} - 32.5 \\ -18.75\theta_{4y} - 32.5\theta_{4x} \end{Bmatrix} \tag{69}$$

Taking the cross product using matrices, the $\dot{\omega}_4$ vector is written in 'tilda' form and the cross product becomes

$$\bar{a}_{c/p} = \begin{bmatrix} 0 & -\dot{\omega}_{4c} & \dot{\omega}_{4b} \\ \dot{\omega}_{4c} & 0 & -\dot{\omega}_{4a} \\ -\dot{\omega}_{4b} & \dot{\omega}_{4a} & 0 \end{bmatrix} \begin{Bmatrix} 18.75 + 32.5\theta_{4z} \\ 18.75\theta_{4z} - 32.5 \\ -18.75\theta_{4y} - 32.5\theta_{4x} \end{Bmatrix} \tag{70}$$

Therefore, the acceleration of the mass center wrt point p is given by (ignoring non-linear terms):

$$\bar{a}_{c/p} = 32.5\dot{\omega}_{4c}\hat{x} + 18.75\dot{\omega}_{4c}\hat{y} + (-18.75\dot{\omega}_{4b} - 32.5\dot{\omega}_{4a})\hat{z}$$

$$= \frac{32.5\partial^2 u_y \hat{x}}{\partial t^2} + \frac{18.75\partial^2 u_y \hat{y}}{\partial t^2} - \left(\frac{32.5\partial^3 u_r}{\partial s \partial t^2} + \frac{18.75\partial^3 u_p}{\partial s \partial t^2}\right)\hat{z} \tag{71}$$

The z component will be ignored, since the assumption is that the beam is not stretched in the z-direction. So, putting all of the terms together in the force equation,

$$\bar{f}_4 = m_4\left[\left(\frac{\partial^2 u_r}{\partial t^2} + \frac{32.5\partial^2 u_y}{\partial t^2}\right)\hat{x} + \left(\frac{\partial^2 u_p}{\partial t^2} + \frac{18.75\partial^2 u_y}{\partial t^2}\right)\hat{y}\right] \tag{72}$$

Breaking these forces into their respective components,

$$f_{r4} = m_4 \left.\frac{\partial^2 u_r}{\partial t^2}\right|_{s=130} + 32.5 m_4 \frac{\partial^2 u_y}{\partial t^2}$$

$$f_{p4} = m_4 \left.\frac{\partial^2 u_p}{\partial t^2}\right|_{s=130} + 18.75 m_4 \frac{\partial^2 u_y}{\partial t^2}$$

These are all of the forces to be modeled in this study. The expressions for the moments will now be developed.

The only moments to derive are those about the beam-antenna attachment point (see Fig. 4). These are obtained from Euler's moment equations and are

$$\begin{Bmatrix} g_{r,4} \\ g_{p,4} \\ g_{y,4} \end{Bmatrix} = \Big[I_4\Big]\Big\{\dot{\omega}_4\Big\} + \Big[\tilde{\omega}_4\Big]\Big[I_4\Big]\Big\{\omega_4\Big\} + \Big\{M_4\Big\} \tag{73}$$

The middle term will be neglected because it is of second order. The $\bar{\omega}_4$ was given by eq (65) as

$$\bar{\omega}_4 = \begin{bmatrix} \left.\dfrac{\partial^2 u_r}{\partial s \partial t}\right|_{s=130} \\ \left.\dfrac{\partial^2 u_p}{\partial s \partial t}\right|_{s=130} \\ \left.\dfrac{\partial u_y}{\partial t}\right|_{s=130} \end{bmatrix} + \bar{\omega} \tag{65}$$

Taking the time derivative of eq (65) and substituting in the mode approximations, the result is

$$\dot{\omega}_4 = \begin{Bmatrix} \ddot{U}_r R'(130) \\ \ddot{U}_p P'(130) \\ \ddot{U}_y Y(130) \end{Bmatrix} \tag{74}$$

From Table I, I-4 is given as

$$I_4 = \begin{bmatrix} 4969 & 0 & 0 \\ 0 & 4969 & 0 \\ 0 & 0 & 9938 \end{bmatrix}$$

So the moments become

$$\begin{Bmatrix} g_{r,4} \\ g_{p,4} \\ g_{y,4} \end{Bmatrix} = \begin{bmatrix} 4969 & 0 & 0 \\ 0 & 4969 & 0 \\ 0 & 0 & 9938 \end{bmatrix} \begin{Bmatrix} \ddot{U}_r R'(130) \\ \ddot{U}_p P'(130) \\ \ddot{U}_y Y(130) \end{Bmatrix} + \{M_4\} \tag{75}$$

which reduces to

$$\begin{Bmatrix} g_{r,4} \\ g_{p,4} \\ g_{y,4} \end{Bmatrix} = \begin{Bmatrix} 4969\ddot{U}_r R'(130) + M_{4x} \\ 4969\ddot{U}_p P'(130) + M_{4y} \\ 9938\ddot{U}_y Y(130) + M_{4z} \end{Bmatrix} \tag{76}$$

Now that each force and moment has been identified, each Q_i must be developed. A sample derivation of Q_1 follows. Every other Q_i ranging from Q_2 through $Q_{(n_r+n_p+n_y)}$ will have a similar development.

From eq (59), the Q_1 will be given by

$$Q_1 = \sum_{j=2}^{4} \bar{f}_j \cdot \frac{\partial \dot{\bar{r}}_j}{\partial \dot{U}_1} + \sum_{k=2}^{4} \bar{g}_k \cdot \frac{\partial \bar{\omega}_k}{\partial \dot{U}_1} \tag{77}$$

The r_2 term is derived from

$$\bar{r}_2 = (x + u_r)\hat{x} + (y + u_p)\hat{y} - sn2\hat{z}$$

$$\dot{\bar{r}}_2 = \bar{\omega} \times \bar{r}_2 + \sum_{i=1}^{n_r} \dot{U}_i R_i(40)\hat{x} + \sum_{j=1}^{n_p} \dot{U}_j P_j(40)\hat{y} \tag{78}$$

So the partial derivative with respect to $\dot{U}_1$ is

$$\frac{\partial \dot{\bar{r}}_2}{\partial \dot{U}_1} = R_1(40)\hat{x} \tag{79}$$

Similarly,

$$\frac{\partial \dot{\bar{r}}_3}{\partial \dot{U}_1} = R_1(80)\hat{x} \tag{80}$$

and

$$\frac{\partial \dot{\bar{r}}_4}{\partial \dot{U}_1} = R_1(130)\hat{x} \tag{81}$$

The angular velocity $\bar{\omega}_4$ was given by eq (65). Substituting the assumed modes yields

$$\bar{\omega}_4 = \sum_{i=1}^{n_r} \dot{U}_i R_i'(130)\hat{x} + \sum_{j=1}^{n_p} \dot{U}_j P_j'(130)\hat{y}$$

$$+ \sum_{k=1}^{n_y} \dot{U}_k Y(sn4) z + \bar{\omega} \qquad (82)$$

The derivative with respect to $\dot{U}_1$ is then

$$\frac{\partial \bar{\omega}_4}{\partial \dot{U}_1} = R_1'(130)\hat{x} \qquad (83)$$

The forces are as given in eqs (60), (61), and (72). The moment is given by

$$\bar{g}_4 = \bar{g}_{r,4} = 4969 \sum_{i=1}^{n_r} \ddot{U}_i R_i'(130)\hat{x} + M_{4x}\hat{x} \qquad (84)$$

Substituting these expressions into eq (77) gives the first generalized force as

$$Q_1 = m_2 \sum_{i=1}^{n_r} \ddot{U}_i R_i(40) R_1(40) + F_{r,2} R_1(40)$$

$$+ m_3 \sum_{i=1}^{n_r} \ddot{U}_i R_i(80) R_1(80) + F_{r,3} R_1(80)$$

$$+ m_4 \sum_{i=1}^{n_r} \ddot{U}_i R_i(130) R_1(130) + 32.5 m_4 \sum_{k=1}^{n_y} \ddot{U}_k Y_k(130) R_1(130)$$

$$+ 4969 \sum_{i=1}^{n_r} \ddot{U}_i R_i'(130) R'(130) + M_{4x} R_1'(130) \qquad (85)$$

where

$i = 1, 2, 3, \ldots, n_r \qquad k = n_r+n_p+1,\ n_r+n_p+2,\ \ldots,\ n_r+n_p+n_y$

Each of the $n_r+n_p+n_y$ generalized forces can be derived into similar expressions. Since for this model 14 modes for each of the roll, pitch, and yaw motions are assumed, there are 42 Q_i's in all. The first 14 will have terms associated with the second derivative of the roll amplitudes, terms associated with the second derivative of the yaw amplitudes, and terms associated with the proof-mass actuator forces in the x-direction, as shown by eq (85). Expanding these terms in matrix form results in three matrices. For example, using just the 14 roll terms, the matrix associated with the second derivative of the roll amplitudes is

$$T_{rr} = \begin{bmatrix} trr_{1,1} & trr_{1,2} & trr_{1,3} & \cdots & trr_{1,14} \\ trr_{2,1} & trr_{2,2} & trr_{2,3} & \cdots & trr_{2,14} \\ trr_{3,1} & trr_{3,2} & trr_{3,3} & \cdots & trr_{3,14} \\ \vdots & \vdots & \vdots & & \vdots \\ trr_{14,1} & trr_{14,2} & trr_{14,3} & \cdots & trr_{14,14} \end{bmatrix} \quad (86)$$

where

$$\begin{aligned} trr_{i,j} = & - m_2 R_i(40) R_j(40) - m_3 R_i(80) R_j(80) \\ & - m_4 R_i(130) R_j(130) - 4969 R_i'(130) R_j(130) \end{aligned}$$

The matrix associated with the second time derivative of the yaw amplitudes looks like

$$
T_{ry} = \begin{bmatrix} try_{1,1} & try_{1,2} & try_{1,3} & \cdots & try_{1,14} \\ try_{2,1} & try_{2,2} & try_{2,3} & \cdots & try_{2,14} \\ try_{3,1} & try_{3,2} & try_{3,3} & \cdots & try_{3,14} \\ \vdots & \vdots & \vdots & & \vdots \\ try_{14,1} & try_{14,2} & try_{14,3} & \cdots & try_{14,14} \end{bmatrix} \quad (87)
$$

where

$$
try_{i,j} = -32.5 m_4 R_i(130) Y_j(130)
$$

and the matrix of forces and moments is

$$
T_{rc} = \begin{bmatrix} -F_{r,2}R_1(40) - F_{r,3}R_1(80) - M_{4x}R_1'(130) \\ -F_{r,2}R_2(40) - F_{r,3}R_2(80) - M_{4x}R_2'(130) \\ -F_{r,2}R_3(40) - F_{r,3}R_3(80) - M_{4x}R_3'(130) \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ -F_{r,2}R_{14}(80) - F_{r3}R_{14}(80) - M_{4x}R_{14}'(130) \end{bmatrix} \quad (88)
$$

The pitch equation will have a right side in an identical form with the only change being that any roll functions are replaced by pitch functions. The yaw equation simplifies into a somewhat simpler form, but still has a matrix associated with the second time derivative of the yaw amplitudes and a matrix of force and moment terms. There is no coupling in the yaw equation with either pitch or roll, which is expected for a fixed-free model. The yaw matrices are

$$
T_{yy} = \begin{bmatrix} tyy_{1,1} & tyy_{1,2} & tyy_{1,3} & \cdots & tyy_{1,14} \\ tyy_{2,1} & tyy_{2,2} & tyy_{2,3} & \cdots & tyy_{2,14} \\ tyy_{3,1} & tyy_{3,2} & tyy_{3,3} & \cdots & tyy_{3,14} \\ \vdots & \vdots & \vdots & & \vdots \\ tyy_{14,1} & tyy_{14,2} & tyy_{14,3} & \cdots & tyy_{14,14} \end{bmatrix} \qquad (89)
$$

where

$$
tyy_{i,j} = -7{,}086{,}601 Y_i(0) Y_j(0) - 9938 Y_i(130) Y_j(130)
$$

and

$$
T_{yc} = \begin{bmatrix} -M_{4z} Y_1(130) \\ -M_{4z} Y_2(130) \\ -M_{4z} Y_3(130) \\ \vdots \\ -M_{4z} Y_{14}(130) \end{bmatrix} \qquad (90)
$$

These matrices come from the last 14 Q_i's, that is, Q_{29} through Q_{42}.

Final Model

Before tying together the equations of motion incorporating the generalized forces, eq (54) will be slightly modified to represent the angular rate of the shuttle-beam-antenna system and its time derivative in terms of the corresponding angular displacements and their time derivatives. The middle term on the left side of eq (54) can be written as

$$\left[Sz \right]^T \left\{ \dot{\omega} \right\} = \left[Sz \right]^T \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} = \left[Sz \right]^T \left\{ \ddot{\theta} \right\} \tag{91}$$

where θ_1, θ_2, and θ_3 are the angles through which the shuttle system rotates. Similarly, the first term on the left side of eq (58) can be rewritten as

$$\left[I_T \right] \left\{ \dot{\omega} \right\} = \left[I_T \right] \left\{ \ddot{\theta} \right\} \tag{92}$$

The left side of eq (54) is in a very simple form, thanks to the judicious choice of assumed modes. The right side, however, contains terms in the generalized forces which will combine with terms on the left side and complicate them somewhat. Specifically, each of the generalized forces contains terms which will combine with the identity matrix (the mass matrix) on the left side of the equation. If eq (54) is rewritten to incorporate the generalized forces, it can be expressed as

$$\left[\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & I & 0 \\ \hline 0 & 0 & I \end{array} \right] \begin{Bmatrix} \ddot{U}_r \\ \ddot{U}_p \\ \ddot{U}_y \end{Bmatrix} + Sz^T \left\{ \dot{\omega} \right\} + \omega^2 \left\{ U \right\} = \left[\begin{array}{c|c|c} T_{rr} & 0 & T_{ry} \\ \hline 0 & T_{pp} & T_{py} \\ \hline 0 & 0 & T_{yy} \end{array} \right] \begin{Bmatrix} \ddot{U}_r \\ \ddot{U}_p \\ \ddot{U}_y \end{Bmatrix} + \begin{Bmatrix} T_{rc} \\ T_{pc} \\ T_{yc} \end{Bmatrix} \tag{93}$$

where each single term in the matrices accociated with the second derivative of the amplitudes represents a 14 x 14 block, and each I represents a 14 x 14 identity matrix. The first matrix on the right side can be combined with the first matrix on the left side. The resulting equation can be written as

$$\begin{bmatrix} M_r & 0 & -T_{ry} \\ 0 & M_p & -T_{py} \\ 0 & 0 & M_y \end{bmatrix} \begin{Bmatrix} \ddot{U}_r \\ \ddot{U}_p \\ \ddot{U}_y \end{Bmatrix} + \begin{bmatrix} Sz \end{bmatrix}^T \left\{ \dot{\omega} \right\} \begin{bmatrix} \diagdown \omega^2 \diagdown \end{bmatrix} \left\{ U \right\} = \begin{Bmatrix} T_{rc} \\ T_{pc} \\ T_{yc} \end{Bmatrix} \tag{94}$$

where

$$M_r = I - T_{rr}$$

$$M_p = I - T_{pp}$$

$$M_y = I - T_{yy}$$

A new vector must now be defined to augment the U vector. It is formed by incorporating the θ's and can be written:

$$\bar{X} = \left\{ \theta \quad U_r \quad U_p \quad U_y \right\}^T \tag{95}$$

This is vector of 45 elements with the first 3 elements being the angles through which the shuttle-beam-antenna system rotates. Eqs (94) and (58) can now be added, and their sum expressed as

$$\left[\begin{array}{c|ccc} I_T & & Sz & \\ \hline & M_r & 0 & -T_{ry} \\ Sz^T & 0 & M_p & -T_{py} \\ & 0 & 0 & M_y \end{array}\right] \begin{Bmatrix} \ddot{\theta} \\ \ddot{U}_r \\ \ddot{U}_p \\ \ddot{U}_y \end{Bmatrix} + \left[\begin{array}{c|c|c|c} 0 & 0 & 0 & 0 \\ \hline 0 & \omega_r^2 & 0 & 0 \\ \hline 0 & 0 & \omega_p^2 & 0 \\ \hline 0 & 0 & 0 & \omega_y^2 \end{array}\right] \begin{Bmatrix} \theta \\ U_r \\ U_p \\ U_y \end{Bmatrix} = \begin{Bmatrix} M \\ T_{rc} \\ T_{pc} \\ T_{yc} \end{Bmatrix}$$

(96)

remembering that Sz is not a square matrix (in fact, it is a

3 by 42 for a 14-mode approximation). The new mass matrix in this equation will now be called M (no subscript), and the force and moment matrix on the right side will be called F. The stiffness matrix, which now has zeros as the first three diagonal elements, will continue to be called ω^2.

One final operation will be performed on this equation for convenience. It is desirable for the mass matrix to be the identity matrix as it was prior to the incorporation of the generalized forces so that the state-space form will be easier with which to work. It is possible to accomplish this and still keep the stiffness matrix in diagonal form by using some of the properties of matrix manipulation. If an eigenvalue analysis is done on the unforced (homogeneous) form of eq (96), the resulting eigenvectors can be put, column by column, into a transfer matrix [5:182-186]. The vector x can be expressed as

$$\bar{x} = \emptyset\ \bar{\eta} \tag{97}$$

so eq (96) can be rewritten as

$$\left[M\right]\left[\emptyset\right]\left\{\ddot{\eta}\right\} + \left[\diagdown\omega^2\diagdown\right]\left[\emptyset\right]\left\{\eta\right\} = \left\{F\right\} \tag{98}$$

Now, pre-multiplying eq (98) by

$$\Big[\left[\emptyset\right]\left[M\right]\Big]^{-1} = \left[M\right]^{-1}\left[\emptyset\right]^{-1} \tag{99}$$

the results will be

$$\left[\diagdown 1\diagdown\right]\left\{\ddot{\eta}\right\} + \left[\diagdown K\diagdown\right]\left\{\eta\right\} = \left[E\right]\left\{F\right\} \tag{100}$$

where

$$\left[K \right] = \left[M \right]^{-1} \left[\phi \right]^{-1} \left[\diagdown \omega^2 \diagdown \right] \left[\phi \right] \qquad (101)$$

$$\left[E \right] = \left[M \right]^{-1} \left[\phi \right]^{-1}$$

The mass matrix has thus been reduced to the identity matrix, and the new stiffness matrix, K, keeps its diagonal form. The terms on the diagonal of K also happen to be the eigenvalues of the unforced system. The term on the right side of the equation will now be called EF, and will figure prominently in the control portion in the next chapter.

Finally, the damping of the beam must be taken into account. The damping matrix can be defined as

$$\left[D \right] = \left[2 \xi \omega \right] \qquad (102)$$

where the ξ was defined in the reference [2] as .003 for roll, pitch, and yaw motion. This leaves the final mathematical representation of this system as

$$\left[\diagdown 1 \diagdown \right] \left\{ \ddot{\eta} \right\} + \left[\diagdown D \diagdown \right] \left\{ \dot{\eta} \right\} + \left[\diagdown K \diagdown \right] \left\{ \eta \right\} = \left[E \right] \left\{ F \right\} \qquad (103)$$

Each matrix on the left side is a 45 x 45 diagonal matrix, and the matrix on the right side is a 45 x 1 matrix, which will be modified in the next chapter.

The mathematical model of the shuttle-beam-antenna system has now been derived, resulting in a single matrix equation. The next chapter will apply apply linear control theory to it to investigate how the system might be controlled.

IV. Control Model

The previous chapters have taken a complicated physical system and reduced it into a mathematical model. This model has simplified the system somewhat by ignoring numerous non-linear terms. The final mathematical expression consists of a fairly large matrix equation which will now be used to develop a control law.

The bulk of the control work done in this paper is a direct result of in-depth analyses done by Janiszewski [6] and Aldridge [7]. The computer programs generated by this work were modified for use on this model, and other than observing results, no attempt was made to further the study of the control techniques employed. A brief outline of the theory will be presented in this chapter. However, anyone wishing detailed study of this control method should refer to the works referenced throughout the chapter.

Since it would be impossible to control all of the modes of a vibrating structure, a method must be found to control the most excitable modes while being careful not to drive any uncontrolled modes unstable. Janiszewski [6] shows how this is done by dividing the modes into three categories: controlled, suppressed, and residual. The controlled modes will be actively controlled. The suppressed modes will not be actively controlled, but care will be taken to avoid exciting them while working on the controlled modes. The residual modes will not be controlled or suppressed, with the assumption that their frequencies are either too high to excite significantly or, if excited, will dampen out through the natural damping of the beam.

The first step towards controlling the system is to truncate the mathematical model to a reasonable number of modes with which to work. The ACOSS program developed and modified by Janiszewski and Aldridge could be easily modified for this model and handled twelve modes. Since the modes with the lowest frequencies tend to be the most excitable, the model's twelve lowest modes were used. To get just those modes, eq (103) was put in the proper form by judicious use of the Φ matrix. The eigenvectors of the eigenvalue problem solution, which are the columns of Φ, were ordered so that the eigenvalues which appear on the diagonal of the K matrix were in ascending order. The three lowest frequencies (the square root of the eigenvalues), which are zero, correspond to the rigid body modes and must necessarily be controlled to keep the shuttle in the proper attitude. The next nine modes are the lowest of the roll, pitch, and yaw beam modes, ordered from lowest to highest without regard to which axis they correspond. It is these modes which are the most excitable and must be controlled or suppressed. To properly truncate eq (103), the left side will have the top left 12 x 12 sub-matrix extracted from each 45 x 45 matrix. The right side will have the top 12 x 10 sub-matrix extracted from the 45 x 10 matrix which results after all matrix multiplication has been performed. This is done within the computer program SHUTBM (see Appendix C) and passed to the ACOSS program. The smaller matrix equation can now be put in state-space form.

If the state vector is defined as

$$\bar{x} = \left| \theta \quad U_r \quad U_p \quad U_y \quad \dot{\theta} \quad \dot{U}_r \quad \dot{U}_p \quad \dot{U}_y \right|^T \tag{104}$$

then the state equation can be written as

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \tag{105}$$

where

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -K & -D \end{array}\right] \qquad B = \left[\begin{array}{c} 0 \\ \hline B_f \end{array}\right] \tag{106}$$

The matrix B_f will depend on what is chosen as the control vector u(t). For this analysis, the shuttle can be torqued about all three axes as can the antenna. The two proof-mass actuators can each produce forces in the X and Y directions. Using moments and forces for the control vector, it can be written as

$$u(t) = \left| F_{r2} \quad F_{p2} \quad F_{r3} \quad F_{p3} \quad M_{1x} \quad M_{4x} \quad M_{1y} \quad M_{4y} \quad M_{1z} \quad M_{4z} \right|^T \tag{107}$$

where

F_{r2} and F_{p2} are the forces in the roll and pitch axes directions made by the actuator at S=40.

F_{r3} and F_{p3} are the forces in the roll and pitch axes directions made by the actuator at S=80.

M_{1x}, M_{1y}, and M_{1z} are the moments applied to the shuttle.

M_{4x}, M_{4y}, and M_{4z} are the moments applied to the antenna.

This control vector must be factored out of the term on the right side of eqn (103) before it is truncated. What is left of the F matrix after factoring (and before premultiplication by the E matrix) is denoted as F_b. This matrix must be modified slightly so that the elements of the control vector will be roughly of the same magnitude. Reference [2] infers that the actuator forces will be in units of pounds (since the masses being moved weigh 10 pounds and are driven a distance of only one foot), or perhaps tens of pounds. The moments, however, are limited by the NASA paper [2] to 10,000 ft-lbs. If these moments are expressed in the control vector in units of thousands of ft-lbs, then the forces and moments will be of roughly the same order. To make the conversion, all elements of the F_b matrix corresponding to a moment in the control vector must be multiplied by 1000. This has been done in the computer program MODEL (see Appendix C).

The sensor output can be expressed as

$$\bar{y} = C\,\bar{x} \tag{108}$$

For the problem at hand, the output vector $\bar{y}$ is given by

$$\bar{y}^T = \left| \theta_1 \quad \theta_2 \quad \theta_3 \quad u_r(sn2) \quad u_p(sn2) \quad u_r(sn3) \quad u_p(sn3) \right| \tag{109}$$

The ACOSS program takes the truncated matrix equation and forms the state-space equation. It also must be given the C matrix before it can start the control algorithm. Once it has this information along with input options, it can begin forming the control law, a brief description of which follows.

As both Janiszewski [6] and Aldridge [7] point out, the state x cannot be measured directly. It can only be measured through the output y, so a state estimator has been developed for use with this control technique. This estimator takes the output y and makes a best estimate of the state x which corresponds to y. The result of this estimator is an equation for the estimated error:

$$\dot{\bar{e}}_c(t) = (A_c - K_cC_c)\, \bar{e}_c(t) \tag{110}$$

which includes the estimator gain K_c, which was found through minimizing the quadratic regulator performance index [8:537].

The same process is applied in finding the control gain G to be used for feedback purposes in the equations:

$$\dot{\bar{x}}_c(t) = (A_c + B_cG)\bar{x}_c(t) + B_cG\bar{e}(t) \tag{111}$$

$$\dot{\bar{x}}_s = A_s\bar{x}_s(t) + B_sG\bar{x}_c(t) + B_sG\bar{e}(t)$$

The controller and observer gain matrices K and G are determined such that the performance indices J_c and J_o are minimized where

$$J_c = \int_0^\infty (\bar{x}_c{}^T Q_c \bar{x}_c + \bar{u}^T R_c \bar{u})\, dt \tag{112}$$

$$J_o = \int_0^\infty (\bar{e}_c{}^T Q_o \bar{e}_c + \bar{v}^T R_o \bar{v})\, dt$$

The matrices Q_c, Q_o, R_c, and R_o are chosen by the control designer. If Q_c and Q_o are chosen positive semi-definite and R_c and R_o positive definite, then A_c+B_cG and A_c-KC_c are guaranteed stable.

The closed loop system model can now be formed, as Janiszewski [6] by defining a new state vector:

$$\bar{z}(t) = \left| \bar{x}_c{}^T(t) \;\vdots\; \bar{e}_c{}^T(t) \;\vdots\; \bar{x}_s{}^T(t) \right|^T \tag{113}$$

which incorporates the controlled states, suppressed states, and estimator error. The closed loop system model can then be expressed as:

$$\dot{\bar{z}}(t) = \left[\begin{array}{c|c|c} A_c + B_cG & B_cG & 0 \\ \hline 0 & A_c - KC_c & KC_s \\ \hline B_sG & B_sG & A_s \end{array}\right] \bar{z}(t) \tag{114}$$

The eigenvalues of the above matrix will show the stability (or instability) of the system. All negative eigenvalues (or complex conjugate eigenvalues with a negative real part) will show the system to be stable within the limitations of the model. Any positive real parts of eigenvalues will show the system to be unstable. This would be caused by a coupling effect of the KC_s term (called observation spill-over) and/or the B_sG terms (called control spillover) since the optimal regulator theory insures that A_c+B_cG and A_c-KC_c are stable matrices.

Once the ACOSS program has built eqn (114), it runs the eigenvalue problem. At this point it is an unsuppressed run. The program will then run the suppression algorithm, which effectively stabilizes the system if it was initially unstable.

The suppression algorithm can be implemented in one of two ways. The intent is to drive eqn (114) into an upper or lower diagonal form so that the eigenvalues of the matrix will be the eigenvalues of the terms on the diagonal, which all have a negative real part. This can be done by either driving

$$B_sG = 0$$

or

$$KC_s = 0$$

Care must be taken to keep from allowing B_cG or KC_c to become zero, lest control or observation be completely lost.

The method used by ACOSS is to find a transformation matrix T such that the new control vector U(t) will become

$$\bar{u}(t) = T\bar{v}(t) \tag{115}$$

This is done through a technique known as Singular Value Decomposition [9]. Aldridge [7] gives a straightforward explanation of SVD on pages 42-48.

After having found the T matrix, ACOSS then reruns the eigenvalue problem, with the results being eigenvalues with negative real parts. The system is thus shown to be stable within the bounds of the mathematical model.

The next chapter will show what the program did to the shuttle-beam-antenna system. It is meant as a guide to show that the system can indeed be stabilized, and what effect different control weightings have on its stability.

V. Results

The intent of this investigation was to show whether or not the mathematical model developed in Chapter III could be stabilized, and how the closed loop damping could be improved over the open loop damping.

No time response was calculated for this system. The measure of performance was chosen to be the closed loop modal damping coefficients, ξ_{ci}. After each run, this is compared to each open loop damping coefficient, ξ_{oi}, which was given (see Table I) as 0.003. A target value for improvement was set at .03, which is a factor of 10 above the open loop coefficient.

An initial run of the ACOSS program was made, using zero initial conditions. Initial weighting values of 1 were given to the weighting matrix F of the matrix-Riccati equation [8:541]. The modes selected for control were those with the six lowest frequencies. These were the three rigid body modes and the three lowest roll, pitch, and yaw modes. The suppressed modes were those four with the next lowest frequencies, and the residual modes were the remaining two, which had the highest frequencies of those modeled. ACOSS ran both the unsuppressed and the suppressed algorithms. The results are shown in Table IV and Table V. Table IV shows all of the eigenvalues of the modeled system to have negative real parts. Table V shows that the .03 performance index was easily surpassed with these weightings. There is very little difference between the unsuppressed and suppressed portions of the run, mainly because the system was stable to begin with, and no suppression was needed.

Another run was made with the weightings on the controlled flexible modes set at 50 to determine the effect on system eigenvalues. The results, shown in Table VI and Table VII, as expected, show very heavy damping on the flexible modes and a very slight change on the rigid body mode damping. The system is still stable without suppression, so the suppression portion showed little change as before.

Another run was made to see if the system eigenvalues could be driven unstable. The weighting of the flexible yaw mode was set at 1000. The system did indeed become unstable, as shown by the overall system eigenvalues in Table VIII. There are three complex conjugate pairs of eigenvalues with positive real parts resulting from this weighting. The suppression algorithm stabilized the system, however, as shown in Table VIII and Table IX. The eigenvalues again all have negative real parts, showing the system to be stable. This shows that the ACOSS program works on this model as it was designed.

Table IV

Overall System Eigenvalues

Initial Run

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

$Q_c = Q_o = \lceil 1 \rfloor$

$R_c = R_o = \lceil 1 \rfloor$

Before Suppression	After Suppression
-0.24024311 ± 80.06607536i	-0.24019691 ± 80.06608979i
-0.24055247 ± 80.18813210i	-0.24056484 ± 80.18812917i
-0.10488282 ± 34.81784403i	-0.10445463 ± 34.81805332i
-0.10461379 ± 34.94614859i	-0.10483854 ± 34.94602274i
-0.03907432 ± 13.00359541i	-0.03902331 ± 13.00771146i
-0.03814158 ± 13.13582349i	-0.03942096 ± 13.14026087i
-2.39955811 ± 5.086878294i	-2.40027385 ± 5.076907440i
-0.16797683 ± 5.594573741i	-0.01679353 ± 5.597809809i
-0.76047227 ± 1.785963613i	-0.75564842 ± 1.762731235i
-0.70649699 ± 1.803664617i	-0.70786680 ± 1.776586936i
-0.15720756 ± 1.845696247i	-0.00556498 ± 1.854871652i
-0.15640709 ± 1.836151481i	-0.00553401 ± 1.844585698i
-0.86594036 ± 0.499810091i	-0.86602637 ± 0.499994808i
-0.86622385 ± 0.500038680i	-0.86603015 ± 0.499998044i
-0.02637183 ± 0.026597291i	-0.02661848 ± 0.026601183i
-0.00990540 ± 0.009912217i	-0.00991154 ± 0.009910617i
-0.00994017 ± 0.010025435i	-0.01002882 ± 0.010027879i
-0.86602540 ± 0.500000000i	-0.86602540 ± 0.499999999i

Table V

Eigenvalues and Closed Loop Damping Coefficients

Initial Run

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

$Q_c = Q_o = [\grave{}1\lrcorner]$

$R_c = R_o = [\grave{}1\lrcorner]$

Controlled Modes:

Before Suppression		After Suppression	
$-0.00991 \pm 0.00991i$	$\xi = .707$	$-0.00991 \pm 0.00991i$	$\xi = .707$
$-0.01003 \pm 0.01003i$	$\xi = .707$	$-0.01003 \pm 0.01003i$	$\xi = .707$
$-0.02662 \pm 0.02660i$	$\xi = .707$	$-0.02662 \pm 0.02660i$	$\xi = .707$
$-2.40027 \pm 5.07691i$	$\xi = .427$	$-2.00427 \pm 5.07691i$	$\xi = .427$
$-0.70787 \pm 1.77659i$	$\xi = .370$	$-0.70787 \pm 1.77659i$	$\xi = .370$
$-0.75565 \pm 1.76273i$	$\xi = .394$	$-0.75565 \pm 1.76273i$	$\xi = .394$

Suppressed Modes:

Before Supression		After Suppression	
$-0.03902 \pm 13.0077i$	$\xi = .003$	$-0.03902 \pm 13.0077i$	$\xi = .003$
$-0.03942 \pm 13.1403i$	$\xi = .003$	$-0.03942 \pm 13.1403i$	$\xi = .003$
$-0.10445 \pm 34.8181i$	$\xi = .003$	$-0.10445 \pm 34.8181i$	$\xi = .003$
$-0.10484 \pm 34.9460i$	$\xi = .003$	$-0.10484 \pm 34.9460i$	$\xi = .003$

Residual Modes:

Before Suppression		After Suppression	
$-0.24019 \pm 80.0661i$	$\xi = .003$	$-0.24019 \pm 80.0661i$	$\xi = .003$
$-0.24056 \pm 80.1881i$	$\xi = .003$	$-0.24056 \pm 80.1881i$	$\xi = .003$

Table VI

Overall System Eigenvalues

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

$$Q_c = Q_o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix}$$

$$R_c = R_o = \begin{bmatrix} 1 \end{bmatrix}$$

Before Suppression	After Suppression
$-0.24000054 \pm 80.06585026i$	$-0.24018449 \pm 80.06609121i$
$-0.23968480 \pm 80.18829557i$	$-0.24056131 \pm 80.18812968i$
$-32.9177464 + 0i$	$-32.9345667 + 0i$
$-0.09589029 \pm 34.81961022i$	$-0.10445463 \pm 34.81805331i$
$-0.09286895 \pm 34.95197391i$	$-0.10483854 \pm 34.94602274i$
$-0.07403583 \pm 12.88756352i$	$-0.03902331 \pm 13.00771146i$
$-0.06884054 \pm 13.01455579i$	$-0.03942096 \pm 13.14026087i$
$-8.07978974 + 0i$	$-8.43563283 + 0i$
$-7.47846553 + 0i$	$-7.96961656 + 0i$
$-0.27405494 \pm 5.586351641i$	$-0.01679357 \pm 5.597809809i$
$-0.55446360 \pm 1.858837822i$	$-1.11786810 + 0i$
$-0.53340846 \pm 1.819136002i$	$-0.00556567 \pm 1.854871646i$
$-1.48750669 + 0i$	$-0.00553445 \pm 1.844585695i$
$-0.66519955 + 0i$	$-0.39513958 + 0i$
$-0.50217705 + 0i$	$-0.43372921 + 0i$
$-0.02355995 \pm 0.025848884i$	$-0.02705426 \pm 0.026160978i$
$-0.00994269 \pm 0.009989706i$	$-0.00993458 \pm 0.009887539i$
$-0.00877420 \pm 0.009894585i$	$-0.01005245 \pm 0.010004210i$
$-1.94580388 \pm 1.813202193i$	$-1.94563452 \pm 1.812601329i$
$-1.94524442 \pm 1.812357534i$	$-1.94563911 \pm 1.812608398i$
$-1.94564483 \pm 1.812604131i$	$-1.94564485 \pm 1.812604177i$

Table VII

Eigenvalues and Closed Loop Damping Coefficients

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

Weightings: See Table VI

Controlled Modes:

Before Suppression		After Suppression	
$-0.00993 \pm 0.00989i$	$\xi = .709$	$-0.00993 \pm 0.00989i$	$\xi = .709$
$-0.01005 \pm 0.01000i$	$\xi = .709$	$-0.01005 \pm 0.01000i$	$\xi = .709$
$-0.02705 \pm 0.02616i$	$\xi = .719$	$-0.02705 \pm 0.02616i$	$\xi = .719$
$-0.39514 + 0i$	$\xi = 1.0$	$-0.39514 + 0i$	$\xi = 1.0$
$-0.43373 + 0i$	$\xi = 1.0$	$-0.43373 + 0i$	$\xi = 1.0$
$-1.11787 + 0i$	$\xi = 1.0$	$-1.11787 + 0i$	$\xi = 1.0$
$-7.96596 + 0i$	$\xi = 1.0$	$-7.96596 + 0i$	$\xi = 1.0$
$-8.43561 + 0i$	$\xi = 1.0$	$-8.43561 + 0i$	$\xi = 1.0$
$-32.9346 + 0i$	$\xi = 1.0$	$-32.9346 + 0i$	$\xi = 1.0$

Suppressed Modes:

Before Suppression		After Suppression	
$-0.03902 \pm 13.0077i$	$\xi = .003$	$-0.03902 \pm 13.0077i$	$\xi = .003$
$-0.03942 \pm 13.1403i$	$\xi = .003$	$-0.03942 \pm 13.1403i$	$\xi = .003$
$-0.10445 \pm 34.8181i$	$\xi = .003$	$-0.10445 \pm 34.8181i$	$\xi = .003$
$-0.10483 \pm 34.9460i$	$\xi = .003$	$-0.10483 \pm 34.9460i$	$\xi = .003$

Residual Modes:

Before Suppression		After Suppression	
$-0.24019 \pm 80.0661i$	$\xi = .003$	$-0.24019 \pm 80.0661i$	$\xi = .003$
$-0.24057 \pm 80.1881i$	$\xi = .003$	$-0.24057 \pm 80.1881i$	$\xi = .003$

Table VIII

Overall System Eigenvalues

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

$$Q_c = Q_o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix}$$

$$R_c = R_o = [1]$$

Before Suppression	After Suppression
-150.26995172 + 0i	-150.300112 + 0i
-0.2411541224 ± 80.0589083i	-0.24018206 ± 80.0660909i
-0.2059402063 ± 80.2074037i	-0.24056305 ± 80.1881308i
-38.280270629 + 0i	-39.0747709 + 0i
-0.0865376915 ± 34.8052009i	-0.10445463 ± 34.8180533i
+0.1107479215 ± 35.2931502i	-0.10483854 ± 34.9460227i
+0.0272984940 ± 12.8537225i	-0.03902331 ± 13.0077115i
+0.5538430430 ± 11.0878564i	-0.03942096 ± 13.1402609i
-7.3933793757 + 0i	-8.19451804 + 0i
-0.9721057460 ± 5.50363776i	-0.01679430 ± 5.59780981i
-2.8091057926 ± 1.32575141i	-0.00558402 ± 1.85487153i
-0.5164799305 ± 1.73838578i	-0.00553445 ± 1.84458569i
-1.3225958070 + 0i	-0.45336639 + 0i
-0.4337955132 + 0i	-0.21174024 + 0i
-0.2052694774 + 0i	-0.09184072 + 0i
-0.0224186944 ± 0.02557039i	-0.02703085 ± 0.02618073i
-0.0091808952 ± 0.00986227i	-0.01003086 ± 0.01002382i
-0.0070312672 ± 0.00942698i	-0.00992552 ± 0.00989796i
-1.9451667499 ± 1.81280739i	-1.94563263 ± 1.81260802i
-1.9457737116 ± 1.81253073i	-1.94564374 ± 1.81260481i
-1.9456447513 ± 1.81260413i	-1.94564486 ± 1.81260418i

Table IX

Eigenvalues and Damping Coefficients

Controlled Modes: 1,2,3,4,5,6

Suppressed Modes: 7,8,9,10

Residual Modes: 11,12

Weightings: See Table VII

Before Suppression		After Suppression	
$-0.00992 \pm 0.00998i$	$\xi = .708$	$-0.00992 \pm 0.00998i$	$\xi = .708$
$-0.01003 \pm 0.01002i$	$\xi = .707$	$-0.01003 \pm 0.01002i$	$\xi = .707$
$-0.02731 \pm 0.02618i$	$\xi = .718$	$-0.02731 \pm 0.02618i$	$\xi = .718$
$-0.09184 + 0i$	$\xi = 1.0$	$-0.09184 + 0i$	$\xi = 1.0$
$-0.21174 + 0i$	$\xi = 1.0$	$-0.21174 + 0i$	$\xi = 1.0$
$-0.45336 + 0i$	$\xi = 1.0$	$-0.45336 + 0i$	$\xi = 1.0$
$-8.19445 + 0i$	$\xi = 1.0$	$-8.19445 + 0i$	$\xi = 1.0$
$-39.0747 + 0i$	$\xi = 1.0$	$-39.0747 + 0i$	$\xi = 1.0$
$-150.300 + 0i$	$\xi = 1.0$	$-150.300 + 0i$	$\xi = 1.0$

Suppressed Modes:

Before Suppression		After Suppression	
$-0.03902 \pm 13.0077i$	$\xi = .003$	$-0.03902 \pm 13.0077i$	$\xi = .003$
$-0.03942 \pm 13.1403i$	$\xi = .003$	$-0.03942 \pm 13.1403i$	$\xi = .003$
$-0.10445 \pm 34.8181i$	$\xi = .003$	$-0.10445 \pm 34.8181i$	$\xi = .003$
$-0.10483 \pm 34.9460i$	$\xi = .003$	$-0.10483 \pm 34.9460i$	$\xi = .003$

Residual Modes:

Before Suppression		After Suppression	
$-0.24019 \pm 80.0661i$	$\xi = .003$	$-0.24019 \pm 80.0661i$	$\xi = .003$
$-0.24056 \pm 80.1881i$	$\xi = .003$	$-0.24056 \pm 80.1881i$	$\xi = .003$

V. Conclusions

A mathematical model was developed for a shuttle-beam-antenna system. The system was discretized using an assumed modes approximation. The equations of motion were developed from scratch and linearized by ignoring higher-order terms and assuming small beam deflections. The coupling between the rigid-body and flexible motions was taken into account in the equations of motion. Fourteen modes were assumed in each of the three directions of motion (roll and pitch bending and yaw torsion). The equations of motion were put in matrix form, and the matrices diagonalized.

Linear optimal regulator theory was applied to examine the stability of the mathematical model. A target value of .03 for the closed loop modal damping coefficient was used as a measure of improvement between the open loop and closed loop system. The target value was surpassed on the first try, using equal weightings on all of the modeled modes. The weightings were then modified to examine the eigenvalues of the system for open loop instability. This was found with a very large weighting of the flexible yaw mode. The closed loop suppression, as expected, re-stabilized the system.

VII. Recommendations

The major emphasis of this analysis was on the mathematical modeling of the shuttle-beam-antenna system. Unfortunately, only a cursory investigation of the control of the system could be accomplished. A complete study should be made of the time response of the system, using varying initial conditions to measure how quickly the system can be controlled.

As mentioned in Appendix A, the choice for the locations of the two actuators was based solely upon the mode shapes of a cantilever beam with no mass attached to its free end. An analysis could be made to determine the best locations for the actuators, incorporating the mass of the antenna into the mode shapes of the beam.

The NASA Challenge [2] also gave information (which needs to be carefully corrected) on line-of-sight calculations. This would be a better measure of system performance than the closed loop damping coefficient and should be the subject of further study. The paper also sets up a maneuvering problem, incorporating attitude changes and a slew maneuver of the antenna. A complete study can be made on this subject alone.

The major building blocks of a thorough investigation of the control of the system have now been created. The computer programs have been built with the flexibility to alter the locations of the actuators, the weightings of the controlled modes, the number of modes to be controlled and suppressed, and many other options. An investigation to find a complete control law for this system can now be attempted.

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Appendix A

Cantilever Mode Shapes

The NASA paper [2] gives graphs for the roll, pitch, and yaw mode shapes of the beam without reference to the type model used. An investigation into a fixed-free cantilever beam and a free-free beam showed that the mode shapes of a fixed-free beam closely resemble those in the NASA paper. The associated frequencies do not match exactly, but no clue was given as to the number of modes assumed by the paper, and therefore the accuracy of the frequencies cannot be measured. The next few pages show the first seven calculated mode shapes for a fixed-free cantilever beam using 14 modes for the roll (and pitch) motion, 14 modes for the yaw motion. Since the beam bending is identical for both roll and pitch, the roll graphs also accurately depict the pitch modes. A comparison with the graphs in the NASA paper shows the close resemblance.

It was necessary to choose the positions along the beam for the locations of the two proof-mass actuators. This was done by inspection of the roll (and therefore pitch) mode shapes using the 14-mode approximation graph. The higher the amplitude of any mode shape at a point along the beam, the more that particular mode will be influenced by an actuator located at that point. A best fit was thus made to the seven curves inspected. The 40-foot point showed adequate amplitude for all seven modes and was chosen as the position of actuator number one. The 80-foot point showed adequate amplitude for all but mode number 5. Since the frequency for mode 5 is fairly high and adequate amplitude was shown at the 40-foot point, it was decided that the 80-foot point would be adequate for the position of actuator number two. It should be kept in mind that these are the mode shapes for a cantilever beam without any mass at its free end. Since the shuttle-beam-antenna system does indeed have such a condition, the best positions for the actuator locations could very well be different from those used here. This would represent a completely different study, and the positions chosen here will serve the purposes of this investigation.

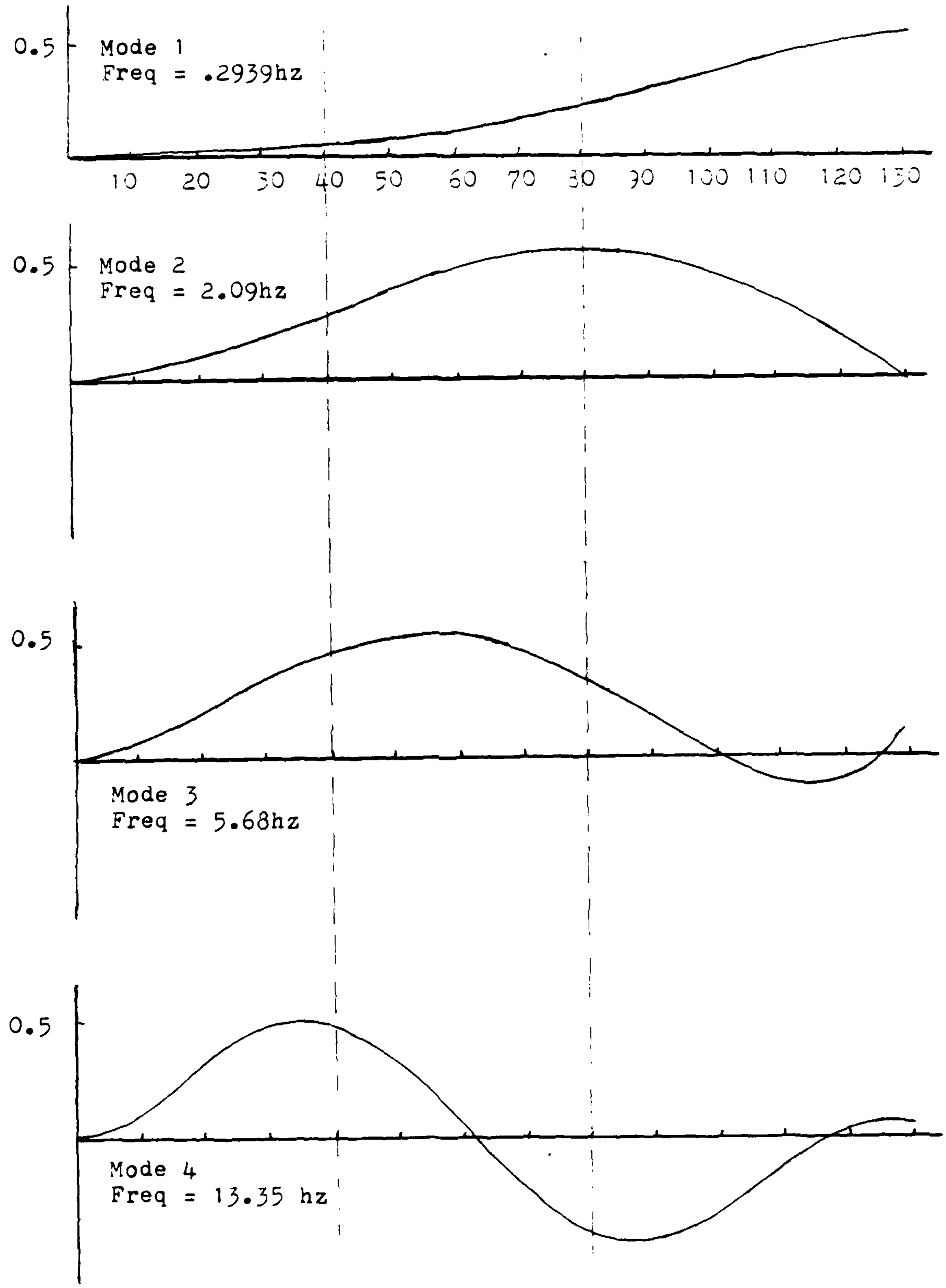

Fig. 6. Roll and Pitch Mode Shapes

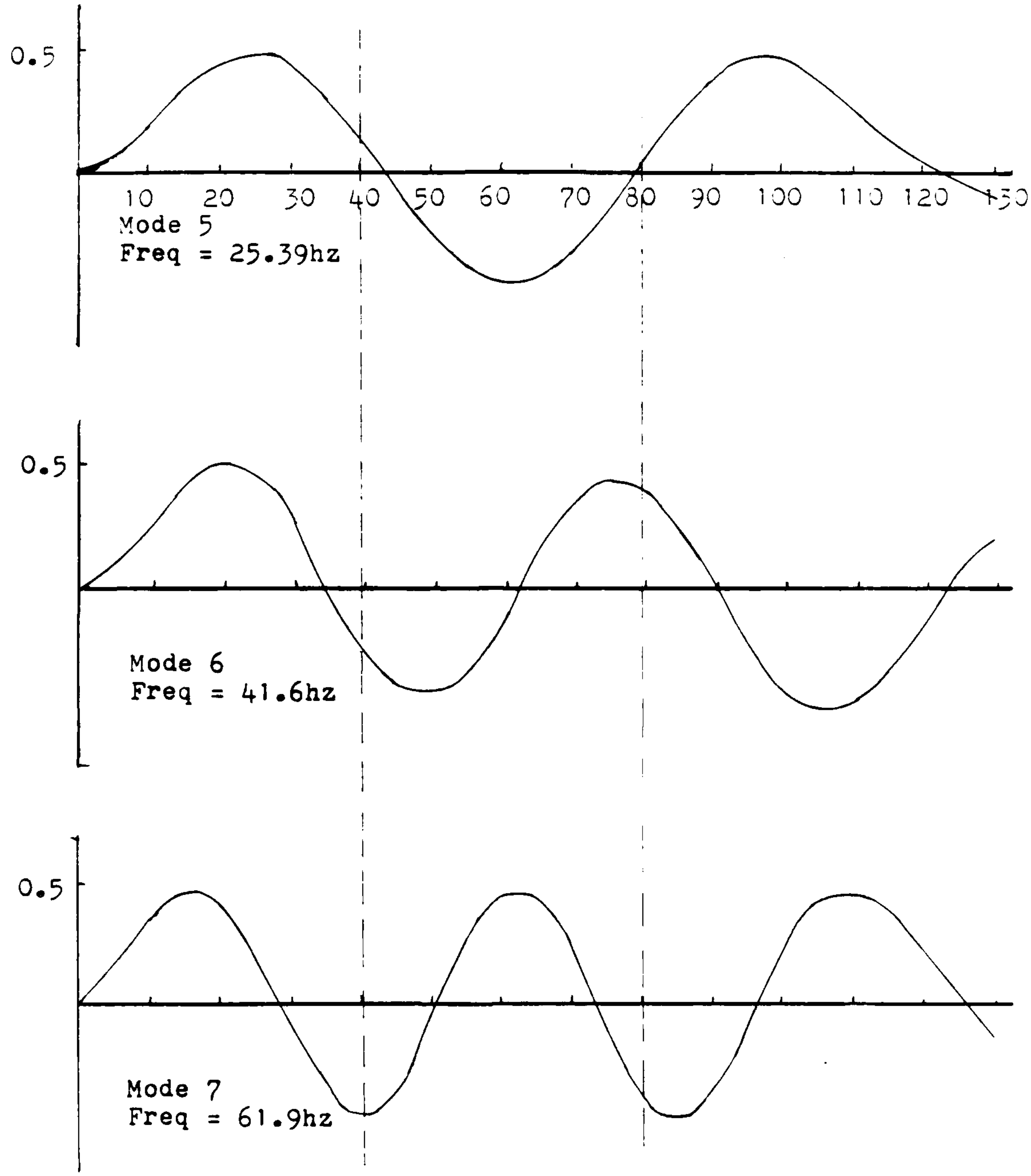

Fig. 7. Roll and Pitch Mode Shapes

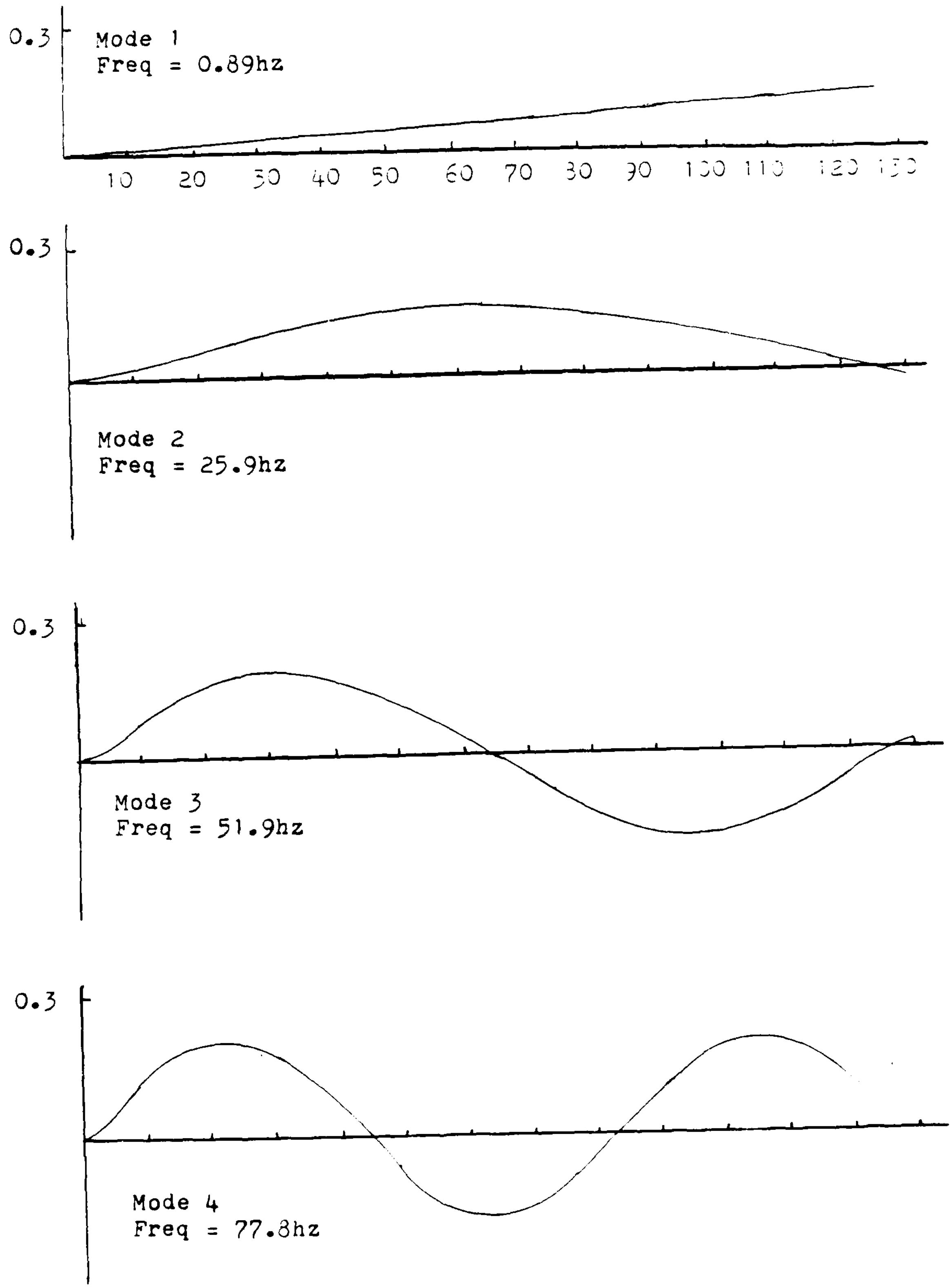

Fig. 8. Yaw Mode Shapes

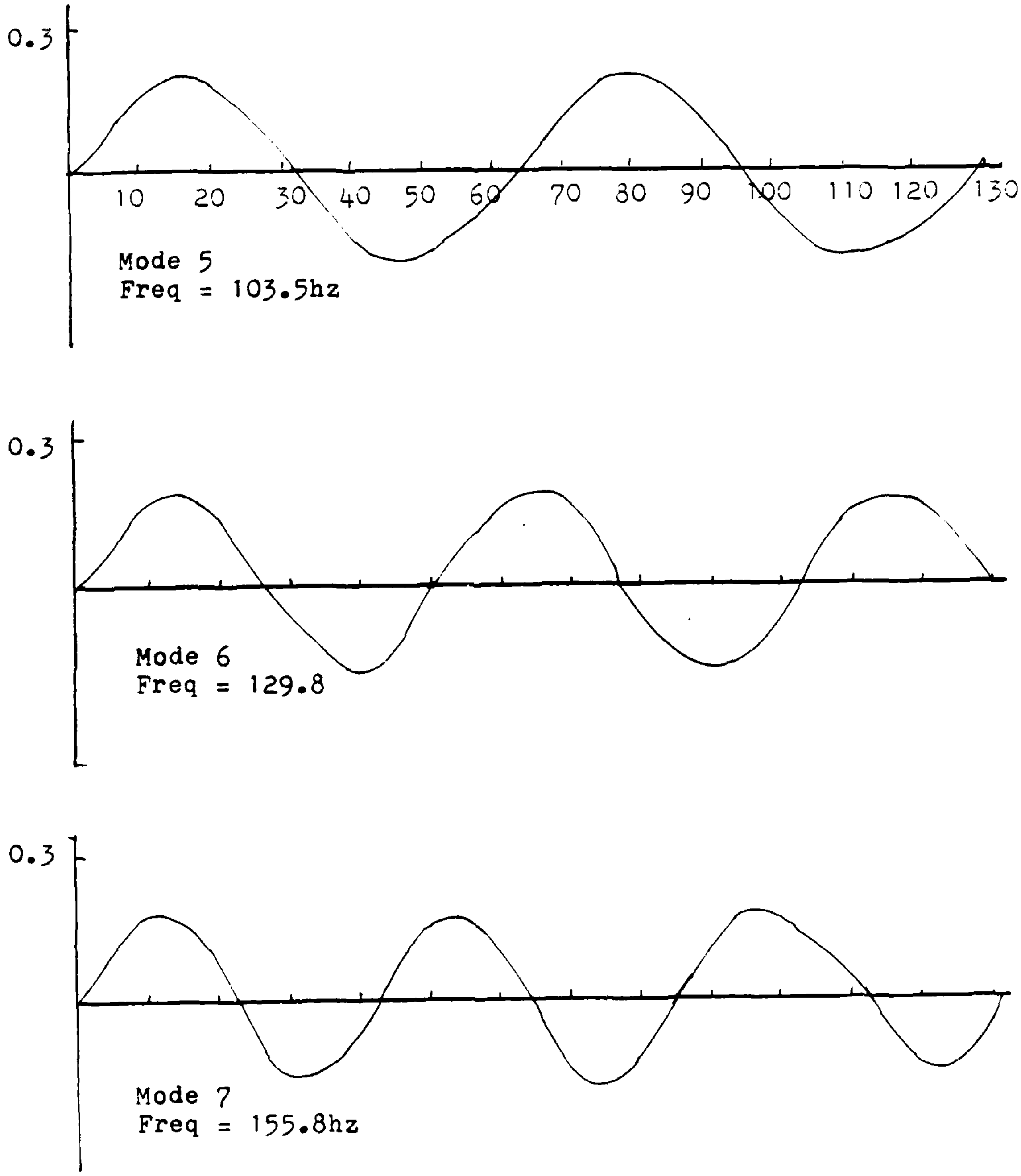

Fig. 9. Yaw Mode Shapes

Appendix B

Derivation of Coefficients for Roll and Pitch Functions

This section shows the development used to solve for the coefficients of the equation

$$R_i = A_i \sin\alpha L + B_i \cos\alpha L + C_i \sinh\alpha L + D_i \cosh\alpha L$$

with the constraint

$$\int_0^L \rho A R_i R_i \, ds = 1$$

The i subscript will be dropped for convenience, and the ρA will be rewritten as ρa to avoid confusion. Several other constraints also apply to this problem:

$$A = -C$$

$$B = -D$$

$$\cos\alpha L * \cosh\alpha L = -1$$

$$B = \frac{-A(\sin SL + \sinh\alpha L)}{(\cos\alpha L + \cosh\alpha L)} = -QA$$

Thus the function becomes

$$R = A\sin\alpha L - QA\cos\alpha L - A\sinh\alpha L + QA\cosh\alpha L$$

Substituting this function into the constraint equation:

$$1 = \rho a \int_0^L \left[A\sin\alpha s - QA\cos\alpha s - A\sinh\alpha s + QA\cosh\alpha s\right]^2 ds$$

Carrying out the multiplication results in

$$1/\rho a = A^2 \int_0^L \left[\sin^2\alpha s - 2Q\sin\alpha s - 2\sin\alpha s\sinh\alpha s\right.$$

$$+ 2Q\sin\alpha s\cosh\alpha s + Q^2\cos^2\alpha s + 2Q\cos\alpha s\sinh\alpha s$$

$$-2Q^2\cos\alpha s\cosh\alpha s + \sinh^2\alpha s - 2Q\sinh\alpha s\cosh\alpha s$$

$$\left. + Q^2\cosh^2\alpha s\right] ds$$

Integrating each term separately yields

$$1/\rho a = A^2 \left\{ \left[\frac{s}{2} - \frac{\sin 2\alpha s}{4\alpha}\right]_0^L - 2Q\left[\frac{\sin^2\alpha s}{2\alpha}\right]_0^L\right.$$

$$- 2\left[\frac{\cosh\alpha s\sin\alpha s - \sinh\alpha s\cos\alpha s}{2\alpha}\right]_0^L$$

$$+ 2Q\left[\frac{\sinh\alpha s\sin\alpha s - \cosh\alpha s\cos\alpha s}{2\alpha}\right]_0^L$$

$$+ Q^2\left[\frac{s}{2} + \frac{\sin 2\alpha s}{4\alpha}\right]_0^L$$

$$+ 2Q\left[\frac{\cosh\alpha s\cos\alpha s + \sinh\alpha s\sin\alpha s}{2\alpha}\right]_0^L$$

$$- 2Q^2\left[\frac{\sinh\alpha s\cos\alpha s + \cosh\alpha s\sin\alpha s}{2\alpha}\right]_0^L$$

$$+ \left[\frac{\sinh\alpha s\cosh\alpha s}{2\alpha} - \frac{s}{2}\right]_0^L - 2Q\left[\frac{\sinh^2\alpha s}{2\alpha}\right]_0^L$$

$$\left. + Q^2\left[\frac{s}{2} + \frac{\sinh\alpha s\cosh\alpha s}{2\alpha}\right]_0^L \right\}$$

Plugging in the limits and rearranging terms gives:

$$1/\rho a = A^2 \Bigg[\left(\frac{Q^2 - 1}{4\alpha}\right) \sin 2\alpha L - \frac{Q\sin^2\alpha L}{\alpha}$$

$$- \left(\frac{Q^2 + 1}{\alpha}\right) \cosh\alpha L \sin\alpha L + \left(\frac{1 - Q^2}{\alpha}\right) \sinh\alpha L \cos\alpha L$$

$$+ \frac{2Q\sinh\alpha L \sin\alpha L}{\alpha} + \left(\frac{Q^2 + 1}{2\alpha}\right) \sinh\alpha L \cosh\alpha L$$

$$+ Q^2 L - \frac{Q\sinh^2\alpha L}{\alpha} \Bigg]$$

To simplify this further, a very close look must be taken at

Q, Q^2, $Q^2 - 1$, and $Q^2 + 1$:

$$Q^2 = \left(\frac{\sin\alpha L + \sinh\alpha L}{\cos\alpha L + \cosh\alpha L}\right)^2$$

$$= \frac{\sin^2\alpha L + 2\sin\alpha L \sinh\alpha L + \sinh^2\alpha L}{\cos^2\alpha L + 2\cos\alpha L \cosh\alpha L + \cosh^2\alpha L}$$

but

$$\cos^2\alpha L + 2\cos\alpha L\cosh\alpha L + \cosh^2\alpha L$$

$$= \cos^2\alpha L - 2 + \cosh^2\alpha L$$

$$= (\cos^2\alpha L - 1) + (\cosh^2\alpha L - 1)$$

$$= -\sin^2\alpha L + \sinh^2\alpha L$$

so, substituting:

$$Q^2-1 = \frac{\sin^2\alpha L + 2\sin\alpha L\sinh\alpha L + \sinh^2\alpha L - \sin^2\alpha L + \sinh^2\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L}$$

$$= \frac{2\sin^2\alpha L + 2\sin\alpha L\sinh\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L}$$

in the same manner:

$$Q^2 + 1 = \frac{2\sin\alpha L\sinh\alpha L + 2\sinh^2\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L}$$

Substituting in these expressions for Q, Q^2, Q^2-1, and Q^2+1:

$$1/\rho a = A^2\left[\frac{\sin\alpha L}{2\alpha}\left(\frac{\sin\alpha L + \sinh sL}{-\sin^2\alpha L + \sinh^2 sL}\right)\sin 2\alpha L\right.$$

$$- \frac{(\sin\alpha L + \sinh\alpha L)(\sin^2\alpha L)}{\alpha(\cos\alpha L + \cosh\alpha L)}$$

$$-2\,\frac{(\sin\alpha L\sinh\alpha L\cosh\alpha L)(\sin\alpha L + \sinh\alpha L)}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}$$

$$- \frac{2\sin^2\alpha L\cos\alpha L(\sin\alpha L + \sinh\alpha L)}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}$$

$$+ \frac{2\sin\alpha L\sinh\alpha L(\sin\alpha L + \sinh\alpha L)}{\alpha(\cos\alpha L + \cosh\alpha L)}$$

$$+ \frac{\sinh^2\alpha L\cosh\alpha L(\sin\alpha L + \sinh\alpha L)}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}$$

$$+ \frac{L(\sin^2\alpha L + 2\sin\alpha L\sinh\alpha L + \sinh^2\alpha L)}{-\sin^2\alpha L + \sinh^2\alpha L}$$

$$\left. - \frac{\sinh^2\alpha L(\sin\alpha L + \sinh\alpha L)}{\alpha(\cos\alpha L + \cosh\alpha L)} \right]$$

now, let

$$\frac{\sin\alpha L + \sinh\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L} = x$$

so:

$$1/\rho a = A^2 \quad \frac{x}{\alpha}\sin^2\alpha L\cos\alpha L - \frac{x}{\alpha}\sin^2\alpha L(\cos\alpha L + \cosh\alpha L)$$

$$- 2\frac{x}{\alpha}\sin\alpha L\sinh\alpha L\cosh\alpha L - 2\frac{x}{\alpha}\sin\alpha L\cos\alpha L\sinh\alpha L$$

$$+ 2\frac{x}{\alpha}(\cos\alpha L + \cosh\alpha L)\sinh\alpha L\sin\alpha L + \frac{x}{\alpha}\sinh^2\alpha L\cosh\alpha L$$

$$+ L\left(\frac{\sin\alpha L + \sinh\alpha L}{\cos\alpha L + \cosh\alpha L}\right)^2 - \frac{x}{\alpha}(\cos\alpha L + \cosh\alpha L)\sinh^2\alpha L$$

Combining and cancelling appropriate terms:

$$1/\rho a = A^2 \quad -\frac{x}{\alpha}\sin^2\alpha L\cosh\alpha L - \frac{x}{\alpha}\sinh^2\alpha L\cos\alpha L$$

$$+ L\left(\frac{\sin\alpha L + \sinh\alpha L}{\cos\alpha L + \cosh\alpha L}\right)^2$$

Combining the x/a terms and re-substituting for x:

$$-x(\sin^2\alpha L\cosh\alpha L + \sinh^2\alpha L\cos\alpha L) = -\left[\frac{\sin^3\alpha L + \sin^3\alpha L\cos\alpha L}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right.$$

$$\left. + \frac{\sin^2\alpha L\cosh\alpha L\sinh\alpha L + \sin\alpha L\cos\alpha L\sinh\alpha L}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right]$$

$$= -\left[\frac{\sin\alpha L\cosh\alpha L(1 - \cos^2\alpha L) + \cosh\alpha L\sinh\alpha L(1 - \cos^2\alpha L)}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right.$$

$$\left. + \frac{\sin\alpha L\cos\alpha L(\cosh^2\alpha L - 1) + \sinh\alpha L\cos\alpha L(\cosh^2\alpha L - 1)}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right]$$

$$= -\left[\frac{\sin\alpha L\cosh\alpha L - \sin\alpha L\cosh\alpha L\cos^2\alpha L + \cosh\alpha L\sinh\alpha L}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right.$$

$$+ \frac{-\cosh\alpha L\sinh\alpha L\cos^2\alpha L + \sin\alpha L\cos\alpha L\cosh^2\alpha L}{\alpha(-\sinh^2\alpha L + \sinh^2\alpha L)}$$

$$\left. + \frac{-\sin\alpha L\cos\alpha L + \cos\alpha L\sinh\alpha L\cosh^2\alpha L - \sinh\alpha L\cos\alpha L}{\alpha(-\sin^2\alpha L + \sinh^2\alpha L)}\right]$$

Every cos*cosh product can be replaced by -1, so:

$$-\frac{1}{\alpha}\left[\frac{\sin\alpha L\cosh\alpha L + \sin\alpha L\cos\alpha L + \cosh\alpha L\sinh\alpha L + \sinh\alpha L\cos\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L}\right.$$

$$\left. + \frac{-\sin\alpha L\cosh\alpha L - \sin\alpha L\cos\alpha L - \sinh\alpha L\cosh\alpha L - \sinh\alpha L\cos\alpha L}{-\sin^2\alpha L + \sinh^2\alpha L}\right]$$

$$= 0$$

So, the remaining term is the only non-zero term, resulting in:

$$1/\rho a = A^2 L \left(\frac{\sin\alpha L + \sinh\alpha L}{\cos\alpha L + \cosh\alpha L}\right)^2$$

Solving for A:

$$A = \left(\frac{1}{\rho a L}\right)^{1/2} \left(\frac{\cos\alpha L + \cosh\alpha L}{\sin\alpha L + \sinh\alpha L}\right)$$

And solving for B:

$$B = -QA = -\left(\frac{1}{\rho a L}\right)^{1/2}$$

Appendix C

Computer Listings

```
      PROGRAM MODEL
C
C *******************************************************************
C
C     THIS PROGRAM TAKES THE ANGLE DATA FROM 'FDATA1' AND COMPUTES
C   THE A-, B-, AND C- COEFFICIENTS FOR THE ROLL, PITCH, AND YAW
C   EQUATIONS.  IT ALSO COMPUTES THE OMEGA-SQUARES FOR THESE EQUA-
C   TIONS.  IT THEN EVALUATES THESE EQUATIONS AT BOTH ENDS OF THE
C   FIXED-FREE BEAM.  IT INCORPORATES SOME CORRECTION FACTORS DUE
C   TO THE MOTION OF THE PROOF-MASS ACTUATORS (LOCATED AT POSITIONS
C   SN1 AND SN2).  THE PROGRAM PUTS ALL OF THIS INFORMATION IN MATRIX
C   FORM, CREATING TWO MATRICES--A MASS MATRIX 'M' (STORED IN DATA
C   FILE 'FMASTM'), AND A STIFFNESS MATRIX 'O' (MADE UP OF THE OMEGA-
C   SQUARES AND STORED IN DATA FILE 'FMASTK').  THE REST OF THE PRO-
C   GRAM SOLVES THE EIGENVALUE PROBLEM FOR THESE TWO MATRICES FOR
C   COMPARISON WITH THE NASA PAPER.
C
C *******************************************************************
C
      DOUBLE PRECISION A(14),B(14),C(14),ALPHAL(14),ALPHA(14)
      DOUBLE PRECISION BETAL(14),BETA(14),OMESQR(14),OMESQP(14)
      DOUBLE PRECISION PHI(14),PHIPR(14),PHIZ(14),PHIPRZ(14),OMESQY(14)
      DOUBLE PRECISION THETA(14),THEPR(14),THETAZ(14),THEPRZ(14)
      DOUBLE PRECISION PSI(14),PSIPR(14),PSIZ(14),PSIPRZ(14)
      DOUBLE PRECISION PHICR1(14),PHICR2(14),THECR1(14),THECR2(14)
      DOUBLE PRECISION PHICOR(14,14),THECOR(14,14),PMAT(7,45)
C
      DOUBLE PRECISION FR(14,14),GR(14,14),FP(14,14),GP(14,14),ONE
      DOUBLE PRECISION FY(14,14),GY(14,14),M(42,42),O(42,42)
      DOUBLE PRECISION A1,A4,B1,B4,C1,C4,D1,M1,M4,Y4,Z4,M2,M3,SN1,SN2
      DOUBLE PRECISION FBMAT(45,10)
C
      REAL AA(42,42),BB(42,42),COMP1,COMP2,VREAL(42,42),HERTZ(42)
      REAL WK(1888),BBETA(42),FBMATX(45,10),PMATX(7,45)
C
      COMPLEX ALFA(42),VEC(42,42),VECT(42,42),EIGENV(42),OMEGA(42)
      COMPLEX TEMP(42)
      INTEGER I,J,K,L,Q,U,V,W,X,Y,Z
      INTEGER IA,IB,N,IJOB,IZ,IER
C
      PARAMETER (A1=905443.00,A4=4969.0,B1=6789100.0,B4=4969.0,
     1     C1=7086601.0,C4=9938.0,D1=-145393.0,M1=6366.46,M4=12.42,
     1     M2=.3108,M3=.3108,SN1=40.0,SN2=80.0)
      PARAMETER (V=14,W=15,X=28,Y=29,Z=42)
C
C *******************************************************************
C  INITIALIZATIONS
C *******************************************************************
      Y4=-18.75*M4
      Z4=-32.5*M4
      ONE=-1.0
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  READ IN THE ANGLE INFORMATION:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      OPEN(UNIT=7,FILE='FDATA1',ACCESS='SEQUENTIAL',STATUS='OLD')
      REWIND(UNIT=7)
10     FORMAT(E31.24)
      DO 20 I=1,14
      READ(7,10)ALPHAL(I)
         B(I)=-1/(SQRT(.09556*130))
      A(I)=(-B(I))*(COS(ALPHAL(I))+COSH(ALPHAL(I)))/(+SIN(ALPHAL(I))
     1 +SINH(ALPHAL(I)))
        OMESQR(I)=(4E7/(.09556*130**4))*(ALPHAL(I)**4)
        OMESQP(I)=(4E7/(.09556*130**4))*(ALPHAL(I)**4)
20    CONTINUE
C
                IF (I.EQ.111) THEN
C
      PRINT*,'YOUR COEFFICIENTS ARE:'
      PRINT*,' '
      DO 40 I=1,14
        WRITE(*,10)(A(I))
40    CONTINUE
C
             ENDIF
C
      DO 50 I=1,14
       BETAL(I)=(2.0*I-1)*(DACOS(ONE))/2.0
      BETA(I)=BETAL(I)/130
50    CONTINUE
      DO 60 I=1,14
        C(I)=SQRT(2/(.9089*130))
        OMESQY(I)=(4E7/(.9089*130**2))*(BETAL(I)**2)
60    CONTINUE
C
C
              IF(I.EQ.111) THEN
C
      PRINT*,'THE BETA-LS ARE:'
      DO 70 I=1,14
        WRITE(*,10)BETAL(I)
70    CONTINUE
      PRINT*,'THE BETAS ARE:'
      DO 80 I=1,14
        WRITE(*,10)BETA(I)
80    CONTINUE
C
      PRINT*,'AND ALL THE CπS ARE:'
      WRITE(*,10)C(1)
C
              ENDIF
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  CALCULATE THE PHI, PHI-PRIME, AND PHI CORRECTION FACTORS:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      DO 90 I=1,14
        ALPHA(I)=ALPHAL(I)/130
90    CONTINUE
      DO 100 I=1,14
       PHI(I)=A(I)*SIN(ALPHAL(I))+B(I)*COS(ALPHAL(I))
     1       -A(I)*SINH(ALPHAL(I))-B(I)*COSH(ALPHAL(I))
C
        PHICR1(I)=A(I)*SIN(ALPHA(I)*SN1) + B(I)*COS(ALPHA(I)*SN1)
     1          - A(I)*SINH(ALPHA(I)*SN1) - B(I)*COSH(ALPHA(I)*SN1)
C
        PHICR2(I)=A(I)*SIN(ALPHA(I)*SN2) + B(I)*COS(ALPHA(I)*SN2)
     1          - A(I)*SINH(ALPHA(I)*SN2) - B(I)*COSH(ALPHA(I)*SN2)
C
100   CONTINUE
      DO 110 I=1,14
        PHIPR(I)=(ALPHA(I))*(A(I)*COS(ALPHAL(I))-B(I)*SIN(ALPHAL(I))
     1           -A(I)*COSH(ALPHAL(I))-B(I)*SINH(ALPHAL(I)))
110   CONTINUE
      DO 120 I=1,14
          PHIZ(I)=0
120   CONTINUE
      DO 130 I=1,14
        PHIPRZ(I)=0
130   CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  CALCULATE THE THETA, THETA-PRIME, PSI, PSI-PRIME, AND THETA-
C  CORRECTION FACTORS:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 140 I=1,14
      THETA(I)=A(I)*SIN(ALPHAL(I))+B(I)*COS(ALPHAL(I))
     1       -A(I)*SINH(ALPHAL(I))-B(I)*COSH(ALPHAL(I))
C
        THECR1(I)=A(I)*SIN(ALPHA(I)*SN1) + B(I)*COS(ALPHA(I)*SN1)
     1           -A(I)*SINH(ALPHA(I)*SN1) - B(I)*COSH(ALPHA(I)*SN1)
C
        THECR2(I)=A(I)*SIN(ALPHA(I)*SN2) + B(I)*COS(ALPHA(I)*SN2)
     1           -A(I)*SINH(ALPHA(I)*SN2) - B(I)*COSH(ALPHA(I)*SN2)
C
140   CONTINUE
      DO 150 I=1,14
        THEPR(I)=(ALPHA(I))*(A(I)*COS(ALPHAL(I))-B(I)*SIN(ALPHAL(I))
     1           -A(I)*COSH(ALPHAL(I))-B(I)*SINH(ALPHAL(I)))
150   CONTINUE
      DO 160 I=1,14
        THETAZ(I)=0
```

```
160    CONTINUE
       DO 170 I=1,14
         THEPRZ(I)=0
170    CONTINUE
       DO 180 I=1,14
         PSI(I)=C(I)*SIN(BETAL(I))
180    CONTINUE
       DO 190 I=1,14
         PSIPR(I)=(BETA(I))*COS(BETAL(I))*C(I)
190    CONTINUE
       DO 200 I=1,14
         PSIZ(I)=0
200    CONTINUE
       DO 210 I=1,14
         PSIPRZ(I)=C(I)*BETA(I)
210    CONTINUE
C
C   CHECK FOR PROPER CALCULATIONS
C
                   IF (I.EQ.111) THEN
C
       PRINT*,'THE PHI   FUNCTIONS EVALUATED AT ZERO ARE:'
       DO 220 I=1,14
         WRITE(*,10)PHIZ(I)
220    CONTINUE
       PRINT*,'                         '
       BRINT0,I=THE1PHI   FUNCTIONS AT 130 ARE:'
         WRITE(*,10)PHI(I)
230    CONTINUE
       PRINT*,' '
       PRINT*,'THE   PHI-PRIME FUNCTIONS AT ZERO ARE:'
       DO 240 I=1,14
         WRITE(*,10)PHIPRZ(I)
240    CONTINUE
       PRINT*,' '
       PRINT*,'THE   PHI-PRIME FUNCTIONS AT 130 ARE:'
       DO 250 I=1,14
         WRITE(*,10)PHIPR(I)
250    CONTINUE
       PRINT*,' '
C
C
       PRINT*,'THE THETA FUNCTIONS EVALUATED AT ZERO ARE:'
       DO 260 I=1,14
         WRITE(*,10)THETAZ(I)
260    CONTINUE
       PRINT*,'                         '
       PRINT*,'THE THETA FUNCTIONS AT 130 ARE:'
       DO 270 I=1,14
         WRITE(*,10)THETA(I)
270    CONTINUE
       PRINT*,' '
```

```
      PRINT*,'THE THETA-PRIME FUNCTIONS AT ZERO ARE:'
      DO 280 I=1,14
        WRITE(*,10)THEPRZ(I)
280   CONTINUE
      PRINT*,' '
      PRINT*,'THE THETA PRIME FUNCTIONS AT 130 ARE:'
      DO 290 I=1,14
        WRITE(*,10)THEPR(I)
290   CONTINUE
      PRINT*,' '
C
      PRINT*,'THE ASSOCIATED PSI FUNCTIONS AT ZERO ARE:'
      DO 300 I=1,14
        WRITE(*,10)PSIZ(I)
300   CONTINUE
      PRINT*,' '
      PRINT*,'THE PSI FUNCTIONS AT 130 ARE:'
      DO 310 I=1,14
        WRITE(*,10)PSI(I)
310   CONTINUE
      PRINT*,' '
      PRINT*,'THE PSI-PRIME FUNCTIONS AT ZERO ARE:'
      PRINT*,' '
      DO 320 I=1,14
        WRITE(*,10)PSIPRZ(I)
320   CONTINUE
      PRINT*,' '
      PRINT*,'THE PSI-PRIME FUNCTIONS AT 130 ARE:'
      PRINT*,' '
      DO 330 I=1,14
        WRITE(*,10)PSIPR(I)
330   CONTINUE
C
              ENDIF
C
C ****************************************************************
C
C      STORE THE PHIPR(I), THEPR(I), PSIPR(I), PHICR1(I), PHICR2(I),
C   THECR1(I), AND THECR2(I) ARRAYS IN A SEQUENTIAL FILE CALLED 'FRHS'
C   FOR USE BY SHUTBM.
C
C ****************************************************************
C
      OPEN(UNIT=13,FILE='FRHS',ACCESS='SEQUENTIAL',STATUS='NEW')
C
      DO 321 I=1,14
        WRITE(13,10)PHIPR(I)
321   CONTINUE
C
      DO 322 I=1,14
        WRITE(13,10)THEPR(I)
322   CONTINUE
```

AD-A153 269 MATHEMATICAL MODELING AND CONTROL OF A LARGE SPACE STRUCTURE AS APPLIED T.. (U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. J O DUNSTAN 2/2

UNCLASSIFIED DEC 84 AFIT/GA/AA/84D-3 F/G 22/2 NL

END

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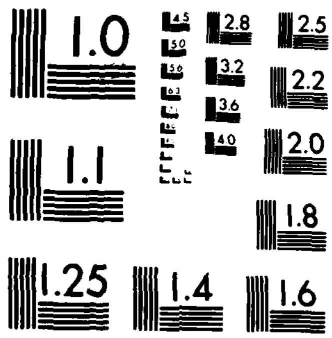

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
C
      DO 323 I=1,14
        WRITE(13,10)PSI(I)
323   CONTINUE
C
      DO 324 I=1,14
        WRITE(13,10)PHICR1(I)
324   CONTINUE
C
      DO 325 I=1,14
        WRITE(13,10)PHICR2(I)
325   CONTINUE
C
      DO 326 I=1,14
        WRITE(13,10)THECR1(I)
326   CONTINUE
C
      DO 327 I=1,14
        WRITE(13,10)THECR2(I)
327   CONTINUE
C
C
C ******************************************************************
C
C     STORE THE ANGLES AND CORRECTION IN THE DATA FILE 'FPHCRS' FOR
C  CLOSE INSPECTION, IF DESIRED:
C
C ******************************************************************
C
      OPEN (UNIT=9,FILE='FPHCRS',ACCESS='SEQUENTIAL',STATUS='NEW')
      WRITE(9,*)'THE ALPHAπS ARE:'
      WRITE(9,*)' '
      DO 311 I=1,14
        WRITE(9,10)ALPHA(I)
311   CONTINUE
      WRITE(9,*)' '
      WRITE(9,*)'THE ALPHA-40S ARE:'
      WRITE(9,*)' '
      DO 312 I=1,14
        WRITE(9,10)ALPHA(I)*SN1
312   CONTINUE
      WRITE(9,*)' '
      WRITE(9,*)'THE PHI FUNCTIONS AT 40 ARE:'
      WRITE(9,*)' '
      DO 331 I=1,14
        WRITE(9,10)PHICR1(I)
331   CONTINUE
      WRITE(9,*)' '
      WRITE(9,*)'THE PHI FUNCTIONS AT 80 ARE:'
      WRITE(9,*)' '
      DO 332 I=1,14
        WRITE(9,10)PHICR2(I)
```

```
332   CONTINUE
      WRITE(9,*)' '
      WRITE(9,*)'THE THETA FUNCTIONS AT 40 ARE:'
      WRITE(9,*)' '
      DO 333 I=1,14
        WRITE(9,10)THECR1(I)
333   CONTINUE
      WRITE(9,*)' '
      WRITE(9,*)'THE THETA FUNCTIONS AT 80 ARE:'
      WRITE(9,*)' '
      DO 334 I=1,14
        WRITE(9,10)THECR2(I)
334   CONTINUE
      ENDFILE(UNIT=9)
C
C ****************************************************************
C
C     THIS NEXT SECTION CREATES THE MATRICES FR, GR, FP, GP, GY,
C  AND FY.  FR IS THE ROLL MASS MATRIX MODIFYING TERM DUE TO THE
C  MOTION OF THE MASSES ON EACH END OF THE BEAM.  FP AND GY ARE
C  THE PITCH AND YAW MODIFYING TERMS DUE TO THE SAME THING.  GR,
C  GP, AND FY (WHICH ENDS UP BEING ZERO FOR THE FIXED-FREE MODEL)
C  ARE MASS MATRIX MODIFYING TERMS DUE TO THE COUPLING BETWEEN
C  ROLL, PITCH, AND YAW WHEN THE BEAM IS DEFORMED.
C
C ****************************************************************
C
      DO 340 I=1,14
        DO 350 J=1,14
       FR(I,J)=-(M1*PHIZ(J)*PHIZ(I) + M4*PHI(J)*PHI(I)
     1        + A1*PHIPRZ(J)*PHIPRZ(I) + A4*PHIPR(J)*PHIPR(I))
350   CONTINUE
340   CONTINUE
C
      DO 360 I=1,14
        DO 370 J=1,14
        GR(I,J)=   Z4*(PSI(J)*PHI(I)) + D1*PSIZ(J)*PHIPRZ(I)
370   CONTINUE
360   CONTINUE
C
      DO 380 I=1,14
        DO 390 J=1,14
       FP(I,J)=-(M1*THETAZ(J)*THETAZ(I) + M4*THETA(J)*THETA(I)
     1         + B1*THEPRZ(J)*THEPRZ(I) + B4*THEPR(J)*THEPR(I))
390   CONTINUE
380   CONTINUE
C
      DO 400 I=1,14
        DO 410 J=1,14
         GP(I,J)=    Y4*(PSI(J)*THETA(I))
410   CONTINUE
400   CONTINUE
```

```
C
C
      DO 420 I=1,14
        DO 430 J=1,14
        GY(I,J)=(-D1)*(PSIZ(I))*(PHIPRZ(J))
430   CONTINUE
420   CONTINUE
C
      DO 440 I=1,14
        DO 450 J=1,14
       FY(I,J)=-(C1*PSIZ(I)*PSIZ(J) + C4*PSI(I)*PSI(J))
450   CONTINUE
440   CONTINUE
C
C *****************************************************************
C
C     THIS SECTION CREATES THE ROLL AND PITCH 'CORRECTION MATRICES'.
C  THESE ARE ACTUALLY ADDITIONAL TERMS ORIGINALLY IGNORED IN THE
C  DEVELOPMENT OF THE EQUATIONS OF MOTION FOR THE SYSTEARE
C  DUE TO THE MOTION OF THE MASSES IN THE PROOF-MASS ACTUATORS.
C
C ********************************************************************
C
      DO 451 I=1,14
        DO 452 J=1,14
          PHICOR(I,J)=-M2*(PHICR1(I)*PHICR1(J)+ PHICR2(I)*PHICR2(J))
          THECOR(I,J)=-M2*(THECR1(I)*THECR1(J)+ THECR2(I)*THECR2(J))
452   CONTINUE
451   CONTINUE
C
C *********************************************************
C
      OPEN(UNIT=19,FILE='FCORS',ACCESS='SEQUENTIAL',STATUS='NEW')
16    FORMAT(5(E19.12))
17    FORMAT(4(E19.12))
      WRITE(19,*)' '
      WRITE(19,*)'THE PHI CORRECTION MATRIX IS:'
      WRITE(19,*)' '
      K=1
      L=5
455   DO 456 I=1,14
        WRITE(19,16)(PHICOR(I,J),J=K,L)
456   CONTINUE
      WRITE(19,*)' '
      K=K+5
      L=L+5
      IF (L.NE.15) GOTO 455
      DO 457 I=1,14
        WRITE(19,17)(PHICOR(I,J),J=11,14)
457   CONTINUE
      ENDFILE(UNIT=19)
C
```

```
C
                IF (I.EQ.111) THEN
C
      PRINT*,' '
      PRINT*,'THE U-PHI ROLL MATRIX IS:'
      PRINT*,' '
      DO 460 I=1,14
        WRITE(*,10)(FR(I,J),J=1,14)
460   CONTINUE
      PRINT*,' '
      PRINT*,'THE U-PSI ROLL MATRIX IS:'
      PRINT*,' '
      DO 470 I=1,14
        WRITE(*,10)(GR(I,J),J=1,14)
470   CONTINUE
C
C
C
      PRINT*,' '
      PRINT*,'THE U-THETA PITCH MATRIX IS:'
      PRINT*,' '
      DO 480 I=1,14

        WRITE(*,10)(FP(I,J),J=1,14)
480   CONTINUE
      PRINT*,' '
      PRINT*,'THE U-PSI PITCH MATRIX IS:'
      PRINT*,' '
      DO 490 I=1,14
        WRITE(*,10)(GP(I,J),J=1,14)
490   CONTINUE
C
      PRINT*,'THE U-PHI YAW MATRIX IS:'
      PRINT*,' '
      DO 500 I=1,14
        WRITE(*,10)(GY(I,J),J=1,14)
500   CONTINUE
      PRINT*,' '
C
C
      PRINT*,'THE U-PSI YAW MATRIX IS:'
      PRINT*,' '
      DO 510 I=1,14
        WRITE(*,10)(FY(I,J),J=1,14)
510   CONTINUE
C
                ENDIF
C
C
      Q=42
C
C ******************************************************************
```

```
C
C      THIS SECTION CREATES THE MASS MATRIX 'M' AND OMEGA-SQUARED
C   MATRIX 'O', WHICH WILL HAVE AN EIGENVALUE PROBLEM DONE ON THEM.
C
C ********************************************************************
C
      DO 520 I=1,V
        DO 530 J=1,V
          IF (I.EQ.J) THEN
          M(I,J)=1-FR(I,J)-PHICOR(I,J)
          ELSE
            M(I,J)=-FR(I,J)-PHICOR(I,J)
          ENDIF
530   CONTINUE
520   CONTINUE
C
      DO 540 I=1,V
        DO 550 J=W,X
          M(I,J)=0
550   CONTINUE
540   CONTINUE
C
      DO 560 I=1,V
        DO 570 J=Y,Z
          L=J-X
          M(I,J)=-GR(I,L)
570   CONTINUE
560   CONTINUE
C
C
      DO 580 I=W,X
        K=I-V
        DO 590 J=1,V
          M(I,J)=0
590   CONTINUE
580   CONTINUE
C
      DO 600 I=W,X
        K=I-V
        DO 610 J=W,X
          L=J-V
            IF (I.EQ.J) THEN
          M(I,J)=1-FP(K,L)-THECOR(K,L)
          ELSE
            M(I,J)=-FP(K,L)-THECOR(K,L)
          ENDIF
610   CONTINUE
600   CONTINUE
C
      DO 620 I=W,X
        K=I-V
        DO 630 J=Y,Z
```

```
            L=J-X
            M(I,J)=-GP(K,L)
630    CONTINUE
620    CONTINUE
C
C
       DO 640 I=Y,Z
         K=I-X
         DO 650 J=1,V
           M(I,J)=-GY(K,J)
650    CONTINUE
640    CONTINUE
C
       DO 660 I=Y,Z
         DO 670 J=W,X
           M(I,J)=0
670    CONTINUE
660    CONTINUE
C
       DO 680 I=Y,Z
         K=I-X
         DO 690 J=Y,Z
           L=J-X
           IF (I.EQ.J) THEN
           M(I,J)=1-FY(K,L)
           ELSE
             M(I,J)=-FY(K,L)
           ENDIF
690    CONTINUE
680    CONTINUE
C
C WRITE THE M MATRIX
C
       IF (I.EQ.111) THEN
       PRINT*,'THE M MATRIX IS:'
       PRINT*,'     '
       DO 700 I=1,Z
         DO 710 J=1,Z
           WRITE(*,10)M(I,J)
710     CONTINUE
700     CONTINUE
       ENDIF
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  PUT THE M MATRIX IN A FILE CALLED FMASTM
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
        OPEN(UNIT=8,FILE='FMASTM',ACCESS='SEQUENTIAL',STATUS='NEW')
       DO 711 I=1,Z
         DO 712 J=1,Z
            WRITE(8,10)M(I,J)
712    CONTINUE
711    CONTINUE
       ENDFILE(UNIT=8)
```

```
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C CREATE THE OMEGA-SQUARED MATRIX
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 720 I=1,V
        DO 730 J=1,Z
          IF (I.EQ.J) THEN
            O(I,J)=OMESQR(I)
          ELSE
            O(I,J)=0
          ENDIF
730   CONTINUE
720   CONTINUE
C
      DO 740 I=W,X
        K=I-V
        DO 750 J=1,Z
          IF (I.EQ.J) THEN
            O(I,J)=OMESQP(K)
          ELSE
            O(I,J)=0
         ENDIF
750   CONTINUE
740   CONTINUE
C
      DO 760 I=Y,Z
        K=I-X
        DO 770 J=1,Z
          IF (I.EQ.J) THEN
            O(I,J)=OMESQY(K)
          ELSE
            O(I,J)=0
        ENDIF
770   CONTINUE
760   CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  WRITE THE O MATRIX INTO A FILE CALLED FMASTK
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      OPEN(UNIT=11,FILE='FMASTK',ACCESS='SEQUENTIAL',STATUS='NEW')
C
      DO 761 I=1,Z
        DO 762 J=1,Z
          WRITE(11,10)O(I,J)
762   CONTINUE
761   CONTINUE
      ENDFILE(UNIT=11)
C
      DO 780 I=1,Z
        DO 790 J=1,Z
```

```
            AA(I,J)=REAL(O(I,J))
          BB(I,J)=REAL(M(I,J))
790   CONTINUE
780   CONTINUE
C
C
C
C *****************************************************************
C
C
C     THIS SECTION BUILDS THE [FB] MATRIX 'FBMAT', WHICH IS WHAT IS
C  LEFT OVER ON THE RIGHT HAND SIDE (OTHER THAN THE INVERSE TRANS-
C  FORMATION MATRIX) AFTER THE CONTROL VECTOR [FC] IS BROKEN OUT.
C  THE RIGHT HAND SIDE LOOKS LIKE:  RHS = [E][FB]FC.
C
C *****************************************************************
C
C
      DO 1000 I=1,45
        DO 1010 J=1,10
          FBMAT(I,J)=0
1010  CONTINUE
1000  CONTINUE
C
      FBMAT(1,2)=SN1
      FBMAT(1,4)=SN2
      FBMAT(1,5)=1000.0
      FBMAT(1,6)=1000.0
      FBMAT(2,1)=-SN1
      FBMAT(2,3)=-SN2
      FBMAT(2,7)=1000.0
      FBMAT(2,8)=1000.0
      FBMAT(3,9)=1000.0
      FBMAT(3,10)=1000.0
C
      DO 1020 I=4,17
        K=I-3
        FBMAT(I,1)=-PHICR1(K)
        FBMAT(I,3)=-PHICR2(K)
        FBMAT(I,6)=-PHIPR(K)*1000.0
1020  CONTINUE
C
      DO 1030 I=18,31
        K=I-17
        FBMAT(I,2)=-THECR1(K)
        FBMAT(I,4)=-THECR2(K)
        FBMAT(I,8)=-THEPR(K)*1000.0
1030  CONTINUE
C
      DO 1040 I=32,35
        K=I-31
        FBMAT(I,10)=PSI(K)*1000.0
```

```
1040   CONTINUE
C
       DO 1050 I=1,45
         DO 1060 J=1,10
           FBMATX(I,J)=REAL(FBMAT(I,J))
1060   CONTINUE
1050   CONTINUE
C
C ****************************************************************
C
C      THIS SECTION CREATES THE P-MATRIX, WHICH IS FROM THE EQUATION
C   Y=P*X.  THE P-MATRIX WILL BE MULTIPLIED BY THE MATRIX OF EIGEN-
C   VALUES IN 'SHUTBM' TO PUT THE STATE VECTOR IN MODAL COORDINATES.
C
C ****************************************************************
C
C
       DO 1070 I=1,7
         DO 1080 J=1,45
           PMAT(I,J)=0
1080   CONTINUE
1070   CONTINUE
C
       PMAT(1,1)=1.0
       PMAT(2,2)=1.0
       PMAT(3,3)=1.0
C
       DO 1090 J=4,17
         K=J-3
         PMAT(4,J)=PHICR1(K)
         PMAT(6,J)=PHICR2(K)
1090   CONTINUE
C
       DO 1100 J=18,31
         K=J-17
         PMAT(5,J)=THECR1(K)
         PMAT(7,J)=THECR2(K)
1100   CONTINUE
C
       DO 1110 I=1,7
         DO 1111 J=1,45
           PMATX(I,J)=REAL(PMAT(I,J))
1111   CONTINUE
1110   CONTINUE
C
C ****************************************************************
C      STORE THE FBMATX AND THE PMATX IN A FILE CALLED 'FBMATX':
C ****************************************************************
C
C
       OPEN(UNIT=12,FILE='FBMATX',ACCESS='SEQUENTIAL',STATUS='NEW')
C
```

```
12     FORMAT(E14.7)
C
       DO 1120 I=1,45
         WRITE(12,12)(FBMATX(I,J),J=1,10)
1120   CONTINUE
C
       DO 1130 I=1,7
         WRITE(12,12)(PMATX(I,J),J=1,45)
1130   CONTINUE
C
       ENDFILE(UNIT=12)
C
C
C
C *******************************************************************
C
C      THIS SECTION SOLVES THE EIGENVALUE PROBLEM A*X = LAMBDA*B*X,
C   WHERE A IS THE 'O' MATRIX AND B IS THE 'M' MATRIX.  IT USES THE
C   IMSL ROUTINE EIGZF, WHICH RETURNS THE EIGENVALUES (EIGENV) AND
C   EIGENVECTORS (VEC).
C
C *******************************************************************
C
C
       IA=Z
       IB=Z
       IZ=Z
       N=Q
       IJOB=2
C
       CALL EIGZF (AA,IA,BB,IB,N,IJOB,ALFA,BBETA,VEC,IZ,WK,IER)
C
C
       DO 800 I=1,Q
           IF (BBETA(I).NE.0) THEN
             EIGENV(I)=ALFA(I)/BBETA(I)
           ELSE
             EIGENV(I)=999999.9
           ENDIF
800    CONTINUE
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C SORT THE EIGENVALUES AND EIGENVECTORS
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
            IF (I.NE.111) GOTO 861
       U=Q-1
810    IF (U.NE.0) THEN
         DO 820 I=1,U
           K=I+1
         COMP1=REAL(EIGENV(K))
```

```
          COMP2=REAL(EIGENV(I))
            IF (COMP1.LT.COMP2) THEN
              TEMP(I)=EIGENV(I)
              EIGENV(I)=EIGENV(K)
              EIGENV(K)=TEMP(I)
              DO 830 J=1,Q
                VECT(J,I)=VEC(J,I)
                VEC(J,I)=VEC(J,K)
                VEC(J,K)=VECT(J,I)
830           CONTINUE
              ENDIF
820       CONTINUE
        U=U-1
        GOTO 810
        ENDIF
C
C
861     CONTINUE
C
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MAKE THE DOUBLE PRECISION NUMBERS INTO REAL
C  CALCULATE THE FREQUENCIES IN RADIANS PER SECOND (OMEGA)
C  CALCULATE THE FREQUENCIES IN CYCLES PER SECOND (HERTZ)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
        DO 840 I=1,Q
          DO 850 J=1,Q
            VREAL(I,J)=REAL(VEC(I,J))
850     CONTINUE
840     CONTINUE
C
        DO 860 I=1,Q
          OMEGA(I)=SQRT(EIGENV(I))
          HERTZ(I)=OMEGA(I)/(2*ACOS(-1.0))
860     CONTINUE
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  STORE THE INFORMATION IN FILES 'FFREQQ' AND 'FIGVCT'.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
        OPEN(UNIT=15,FILE='FFREQQ',ACCESS='SEQUENTIAL',STATUS='NEW')
7       FORMAT(A24)
           WRITE(15,7)'                        '
          WRITE(15,7)'THE EIGENVALUES ARE:    '
          WRITE(15,7)'                        '
        DO 870 I=1,42
          WRITE(15,12)EIGENV(I)
870     CONTINUE
```

```
      ENDFILE(UNIT=15)
C
C
C
C
      OPEN(UNIT=10,FILE='FIGVCT',ACCESS='SEQUENTIAL',STATUS='NEW')
C
11    FORMAT(6(E14.7))
C
      WRITE(10,*)'THE UNSORTED EIGENVECTOR MATRIX IS:'
      WRITE(10,*)' '
      K=1
      L=6
950   DO 940 I=1,42
        WRITE(10,11)(VREAL(I,J),J=K,L)
940   CONTINUE
      WRITE(10,*)' '
      K=K+6
      L=L+6
      IF (L.NE.48) GOTO 950
      ENDFILE(UNIT=10)
C
C
1     END
```

```
      PROGRAM SHUTBM
C
C     THIS PROGRAM SOLVES AN EIGENVALUE PROBLEM OF THE FORM
C  A*X = LAMBDA*B*X.  THE A MATRIX IS A 45X45 MATRIX MADE UP
C  OF THE 42X42 MATRIX 'O' FROM FMASTK.  THE UPPER LEFT CORNER
C  HAS THREE ZEROS ON THE DIAGONAL.  THE B MATRIX IS A 45X45
C  MADE OF FOUR PARTS.  THE UPPER LEFT 3X3 IS THE MOMENT OF INERTIA
C  MATRIX FROM THE NASA PAPER.  THE LOWER RIGHT 42X42 IS THE M
C  MATRIX FROM FMASTM.  THE UPPER RIGHT 3X42 (THE LOWER LEFT 42X3
C  IS THE TRANSPOSE) IS A CORRECTION MATRIX WHICH TAKES INTO AC-
C  COUNT THE ROTATION OF THE SHUTTLE-BEAM-REFLECTOR SYSTEM.  IT
C  IS FILLED WITH ZEROS EXCEPT FOR THE LOWER LEFT CORNER, WHICH
C  CONTAINS THE SZ'S.
C
      DOUBLE PRECISION DALFAL(14),DPM(42,42),DPO(42,42)
      DOUBLE PRECISION DPHIPR(14),DTHEPR(14),DPSI(14),DHICR1(14)
      DOUBLE PRECISION DHICR2(14),DHECR1(14),DHECR2(14)
      REAL    BETA(45),ALPHAL(14),M(42,42),O(42,42),A(45,45),B(45,45)
      REAL    WK(2888),SZ(14),SZMAT(3,42),INRTIA(3,3),EIGENV(45)
      REAL EIGVEC(45,45),MTILDA(45,45),FBMATX(45,10),PROD1(45,45)
      REAL INVRSE(45,45),PROD2(45,45),PROD3(45,45),KTILDA(45,45)
      REAL DTILDA(45,45),PHIPR(14),THEPR(14),PSI(14),PHICR1(14)
      REAL PHICR2(14),THECR1(14),THECR2(14),DMAT(45,10),A4,B4,C4
      REAL CMAT(7,45),PMAT(7,45)
C
      COMPLEX ALFA(45),DIGENV(45),DIGVEC(45,45),OMEGA(45)
      INTEGER I,J,K,L,IDGT ,LL,MM,NN,IC
      INTEGER IA,IB,N,IJOB,IZ,IER
C
      PARAMETER (A4=4969.0,B4=4969.0,C4=9938.0)
C
C ******************************************************************
C
C     READ IN THE INFORMATION FROM 'FDATA2' (THE ANGLES),
C  'FMASTM' (THE MASS MATRIX), AND 'FMASTK' (THE STIFFNESS MATRIX)
C
C ******************************************************************
C
      OPEN(UNIT=7,FILE='FDATA2',ACCESS='SEQUENTIAL',STATUS='OLD')
      OPEN(UNIT=8,FILE='FMASTM',ACCESS='SEQUENTIAL',STATUS='OLD')
      OPEN(UNIT=9,FILE='FMASTK',ACCESS='SEQUENTIAL',STATUS='OLD')
      OPEN(UNIT=12,FILE='FRHS',ACCESS='SEQUENTIAL',STATUS='OLD')
      OPEN(UNIT=11,FILE='FBMATX',ACCESS='SEQUENTIAL',STATUS='OLD')
      REWIND(UNIT=7)
      REWIND(UNIT=8)
      REWIND(UNIT=9)
      REWIND(UNIT=12)
      REWIND(UNIT=11)
C
10    FORMAT(E31.24)
11    FORMAT(E14.7)
13    FORMAT(10(E11.4))
```

```
14    FORMAT(4(3(4X,E14.7),/),/)
16    FORMAT(5(E14.7))
C
      DO 20 I=1,14
        READ(7,10)DALFAL(I)
        ALPHAL(I)=REAL(DALFAL(I))
20    CONTINUE
C
      DO 30 I=1,42
        DO 40 J=1,42
          READ(8,10)DPM(I,J)
            M(I,J)=REAL(DPM(I,J))
          READ(9,10)DPO(I,J)
            O(I,J)=REAL(DPO(I,J))
40    CONTINUE
30    CONTINUE
C
      DO 41 I=1,14
        READ(12,10)DPHIPR(I)
        PHIPR(I)=REAL(DPHIPR(I))
41    CONTINUE
C
      DO 42 I=1,14
        READ(12,10)DTHEPR(I)
        THEPR(I)=REAL(DTHEPR(I))
42    CONTINUE
C
      DO 43 I=1,14
        READ(12,10)DPSI(I)
        PSI(I)=REAL(DPSI(I))
43    CONTINUE
C
      DO 44 I=1,14
        READ(12,10)DHICR1(I)
        PHICR1(I)=REAL(DHICR1(I))
44    CONTINUE
C
      DO 45 I=1,14
        READ(12,10)DHICR2(I)
        PHICR2(I)=REAL(DHICR2(I))
45    CONTINUE
C
      DO 46 I=1,14
        READ(12,10)DHECR1(I)
        THECR1(I)=REAL(DHECR1(I))
46    CONTINUE
C
      DO 47 I=1,14
        READ(12,10)DHECR2(I)
        THECR2(I)=REAL(DHECR2(I))
47    CONTINUE
C
```

```
      DO 48 I=1,45
        READ(11,11)(FBMATX(I,J),J=1,10)
48    CONTINUE
C
      DO 49 I=1,7
        READ(11,11)(PMAT(I,J),J=1,45)
49    CONTINUE
C
      ENDFILE(UNIT=7)
      ENDFILE(UNIT=8)
      ENDFILE(UNIT=9)
      ENDFILE(UNIT=12)
C
CCCCCCCCCCCCCCCCCCCCCCCCC
C  FORM THE INRTIA MATRIX:
CCCCCCCCCCCCCCCCCCCCCCCCC
C
      INRTIA(1,1)=975423.0
      INRTIA(1,2)=0
      INRTIA(1,3)=-145393.0
      INRTIA(2,1)=0
      INRTIA(2,2)=6859080.0
      INRTIA(2,3)=0
      INRTIA(3,1)=-145393.0
      INRTIA(3,2)=0
      INRTIA(3,3)=7086601.0
C
CCCCCCCCCCCCCCCCCCCCCC
C  CALCULATE THE SZ'S:
CCCCCCCCCCCCCCCCCCCCCC
C
      DO 50 I=1,14
        SZ(I)=(2*130*SQRT(.09556*130))/(ALPHAL(I)**2)
50    CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCC
C  BUILD THE SZMAT MATRIX:
CCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DO 60 I=1,3
        DO 70 J=1,42
          SZMAT(I,J)=0
70    CONTINUE
60    CONTINUE
C
C
C
C
      DO 80 J=1,14
        SZMAT(1,J)=(A4*PHIPR(J))
        SZMAT(2,J)=+SZ(J)
80    CONTINUE
```

```
C
C
      DO 81 J=15,28
        K=J-14
        SZMAT(2,J)=B4*THEPR(K)
        SZMAT(1,J)=-SZ(K)
81    CONTINUE
C
      DO 82 J=29,42
        K=J-28
        SZMAT(3,J)=C4*PSI(K)
82    CONTINUE
C
C
C *******************************************************************
C
C      THIS SECTION FORMS THE LARGE 'A' MATRIX, WHICH CONSISTS OF
C   THE OMEGA-SQUARES ON THE DIAGONAL WITH ZEROS AS THE FIRST THREE
C   OMEGAS.  IT THEN BUILDS THE 'B' MATRIX, WHICH IS MADE UP OF
C   THE INERTIA MATRIX, THE MASS MATRIX, AND THE SZ-MATRIX (SZMAT)
C   IT WILL LOOK LIKE:
C
C         L                 L                        L
C         L IIIIIIIII       L   SSSSSSSS             L
C         L     I           L   S                    L
C         L     I           L   S                    L
C         L     I           L   SSSSSSSS  ZZZZZ      L
C         L     I           L          S     Z       L
C         L     I           L          S    Z        L
C         L IIIIIIIII       L   SSSSSSSS  ZZZZZ      L
C         L                 L                        L
C         -------------------------------------------
C         L             TTT L                        L
C         L SSSSSSS      T  L      MM       MM       L
C         L S            T  L      M M     M M       L
C         L S               L      M  M   M  M       L
C         L SSSSSSS  ZZZZZ  L      M   MM    M       L
C         L       S     Z   L      M         M       L
C         L       S    Z    L      M         M       L
C         L SSSSSSS  ZZZZZ  L      M         M       L
C         L                 L                        L
C
C ***********************************************************
C
995   DO 100 I=1,45
        DO 110 J=1,45
          A(I,J)=0
110   CONTINUE
100   CONTINUE
C
      DO 120 I=4,45
        K=I-3
```

```
          DO 130 J=4,45
            L=J-3
            A(I,J)=O(K,L)
130   CONTINUE
120   CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  CREATE THE LARGE B MATRIX:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      DO 140 I=1,3
        DO 150 J=1,3
          B(I,J)=INRTIA(I,J)
150   CONTINUE
140   CONTINUE
C
      DO 160 I=1,3
        DO 170 J=4,45
          L=J-3
          B(I,J)=SZMAT(I,L)
170   CONTINUE
160   CONTINUE
C
      DO 180 I=4,45
        K=I-3
        DO 190 J=1,3
          B(I,J)=SZMAT(J,K)
190   CONTINUE
180   CONTINUE
C
      DO 200 I=4,45
        K=I-3
        DO 210 J=4,45
          L=J-3
          B(I,J)=M(K,L)
210   CONTINUE
200   CONTINUE
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C CHECK THE INRTIA AND SZMAT MATRICES:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
            IF (I.EQ.111) THEN
      PRINT*,'THE INERTIA MATRIX IS:'
      PRINT*,' '
      DO 220 I=1,3
        WRITE(*,11)(INRTIA(I,J),J=1,3)
220   CONTINUE
C
C
C
```

```
C
19      FORMAT(3(E14.7))
C
       PRINT*,' '
       PRINT*,'THE SZ TRANSPOSE MATRIX IS:'
       PRINT*,' '
       DO 221 I=1,42
         WRITE(*,19)(SZMAT(J,I),J=1,3)
221    CONTINUE
C
              ENDIF
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C ******************************************************************
C
C      THIS SECTION SOLVES THE EIGENVALUE PROBLEM A*X = LAMBDA*B*X,
C   GIVING BACK THE EIGENVALUES AND EIGENVECTORS OF THE SYSTEM.
C
C ********************************************************************
C
       IA=45
       IB=45
       N=45
       IJOB=2
       IZ=45
C
       CALL EIGZF(A,IA,B,IB,N,IJOB,ALFA,BETA,DIGVEC,IZ,WK,IER)
C
       PRINT*,'THE IER FOR EIGZF IS: ',IER
       PRINT*,'THE PERFORMANCE INDEX FOR EIGZF IS: ',WK(1)
C
       DO 250 I=1,45
         IF (BETA(I).NE.0) THEN
           DIGENV(I)=ALFA(I)/BETA(I)
         ELSE
           DIGENV(I)=99999999.9
         ENDIF
250    CONTINUE
C
       DO 251 I=1,45
         EIGENV(I)=REAL(DIGENV(I))
         DO 252 J=1,45
           EIGVEC(I,J)=REAL(DIGVEC(I,J))
252    CONTINUE
251    CONTINUE
C
C
C ************************************************************
C
C      SORT THE EIGENVALUES FROM LOWEST TO HIGHEST, SORTING
C   THIER RESPECTIVE EIGENVALUE ALSO.
C
```

```
C ********************************************************************
C
      CALL SORT(EIGENV,EIGVEC,45)
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  CHECK THE EIGENVALUE AND EIGENVECTORS:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      OPEN(UNIT=10,FILE='FIGENV',ACCESS='SEQUENTIAL',STATUS='NEW')
C
17    FORMAT(5(E14.7))
      DO 260 I=1,45
        WRITE(10,17)EIGENV(I)
260   CONTINUE
      WRITE(10,*)' '
C
      K=1
      L=5
261   DO 262 I=1,45
        WRITE(10,17)(EIGVEC(I,J),J=K,L)
262   CONTINUE
      WRITE(10,*)' '
      K=K+5
      L=L+5
      IF (L.NE.50) GOTO 261
      ENDFILE(UNIT=10)
C
C
C
998   LL=45
      MM=45
      NN=45
      IC=45
      IDGT=5
C
C*****************************************************
C
C
C
C
C
C
C*****************************************************
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY B*TRANSFORMATION MATRIX:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CALL VMULFF(B,EIGVEC,LL,MM,NN,IA,IB,PROD1,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (B,EIGVEC) IS: ',IER
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  INVERT THIS PRODUCT
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL LINV2F (PROD1,N,IA,INVRSE,IDGT,WK,IER)
      PRINT*,'THE IER FOR LINV2F (PROD1) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY THE INVERSE AND B:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL VMULFF (INVRSE,B,LL,MM,NN,IA,IB,PROD2,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (INVERSE,B) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY THIS RESULT BY THE TRANSFORMATION MATRIX:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL VMULFF(PROD2,EIGVEC,LL,MM,NN,IA,IB,MTILDA,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (PROD2,EIGVEC) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  PREMULTIPLY THE INVERSE TRANSFORMATION MATRIX BY A:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL VMULFF(INVRSE,A,LL,MM,NN,IA,IB,PROD3,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (INVERSE,A) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY THIS RESULT BY THE TRANSFORMATION MATRIX
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL VMULFF(PROD3,EIGVEC,LL,MM,NN,IA,IB,KTILDA,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (PROD3,EIGVEC) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY INVRSE AND FBMATX TO GET DMAT,  WHICH WILL BE THE RHS
C  MATRIX IN 'FCNTRL'.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      CALL VMULFF(INVRSE,FBMATX,LL,MM,10,IA,IB,DMAT,IC,IER)
C
      PRINT*,'THE IER FOR VMULFF (INVRSE,FBMATX) IS: ',IER
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  MULTIPLY PMAT AND EIGVEC TO GET CMAT, WHICH WILL BE READ INTO
C  ACOSS DIRECTLY:
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
2222  CALL VMULFF  (PMAT,EIGVEC,7,45,45, 7,45,CMAT, 7,IER)
```

```
C
      PRINT*,'THE IER FOR VMULFF (PMAT,EIGVEC) IS: ',IER
C
C
C ***************************************************************
C
C  PUT MTILDA AND KTILDA THROUGH A FILTER TO TAKE OUT THE VERY
C  SMALL NUMBERS (SET THEM TO ZERO)
C
C ***************************************************************
C
      DO 510 I=1,45
        DO 520 J=1,45
          IF (MTILDA(I,J).LT.1E-4) THEN
            MTILDA(I,J)=0
          ENDIF
          IF (KTILDA(I,J).LT.1E-4) THEN
            KTILDA(I,J)=0
          ENDIF
520   CONTINUE
510   CONTINUE
C
C
C
C
C ********************************************************************
C
C     THIS SECTION CREATES THE DAMPING MATRIX 'DTILDA'.  IT TAKES
C  THE SQUARES OF THE EIGENVALUES, WHICH ARE ON THE DIAGONAL OF
C  'KTILDA', AND FORMS 2*ZETA*OMEGA, WHERE ZETA IS A CONSTANT .003,
C  AND OMEGA IS THE SQUARE ROOT OF EACH EIGENVALUE.  THE RESULT
C  IS A DIAGONAL MATRIX CORRESPONDING TO THE DIAGONAL STIFFNESS
C  MATRIX (KTILDA).
C
C ********************************************************************
C
C
      DO 530 I=1,45
        DO 540 J=1,45
          IF (I.EQ.J) THEN
            DTILDA(I,J)=2.0*.003*SQRT(KTILDA(I,J))
          ELSE
            DTILDA(I,J)=0
          ENDIF
540   CONTINUE
530   CONTINUE
C
      OPEN(UNIT=15,FILE='FMTILD',ACCESS='SEQUENTIAL',STATUS='NEW')
      OPEN(UNIT=17,FILE='FKTILD',ACCESS='SEQUENTIAL',STATUS='NEW')
      OPEN(UNIT=18,FILE='FDTILD',ACCESS='SEQUENTIAL',STATUS='NEW')
      OPEN(UNIT=16,FILE='FTRANS',ACCESS='SEQUENTIAL',STATUS='NEW')
      OPEN(UNIT=13,FILE='FDMATX',ACCESS='SEQUENTIAL',STATUS='NEW')
```

```
C
      K=1
      L=5
      WRITE(15,*)'THE DIAGONAL MASS MATRIX-UNSORTED-IS:'
      WRITE(17,*)'THE DIAGONAL STIFFNESS MATRIX-UNSORTED-IS:'
      WRITE(18,*)'THE DIAGONAL DAMPING MATRIX-UNSORTED-IS:'
      WRITE(15,*)' '
      WRITE(17,*)' '
      WRITE(18,*)' '
499   DO 500 I=1,45
          WRITE(15,16)(MTILDA(I,J),J=K,L)
          WRITE(17,16)(KTILDA(I,J),J=K,L)
          WRITE(18,16)(DTILDA(I,J),J=K,L)
500   CONTINUE
      WRITE(15,*)' '
      WRITE(17,*)' '
      WRITE(18,*)' '
      K=K+5
      L=L+5
      IF (L.NE.50) GOTO 499
C
      DO 600 I=1,45
        WRITE(16,16)(INVRSE(I,J),J=1,45)
600   CONTINUE
C
      WRITE(13,*) 1, 1, 1
      WRITE(13,*) 1.0
      WRITE(13,*) 6, 4, 2 ,10, 7 ,0.003
      DO 601 I=1,12

          WRITE(13,14)(EIGVEC(I,J),J=1,12)
601   CONTINUE
      WRITE(13,'(//)')
C
C
      DO 610 I=1,12
        WRITE(13,13)(DMAT(I,J),J=1,10)
610   CONTINUE
      WRITE(13,'(///)')
C
C  WRITE IN THE FIRST 12 ROWS OF CMAT TRANSPOSED, WHICH IS THE
C  PROPER FORM FOR ACOSS TO HANDLE
C
      DO 611 I=1,12
        WRITE(13,13)(CMAT(J,I),J=1,7)
611   CONTINUE
      WRITE(13,'(///)')
C
C
2218  FORMAT(2F3.1)
2219  FORMAT(12F3.1)
      DO 620 I=1,12
```

```
          WRITE(13,11)SQRT(KTILDA(I,I))
620     CONTINUE
C
        WRITE(13,'(//)')
        DO 2200 I=1,4
          WRITE(13,2219)(0.0,J=1,12)
2200    CONTINUE
        WRITE(13,'(//)')
        DO 2201 I=1,12
          WRITE(13,2218)(0.0,J=1,2)
2201    CONTINUE
C
        ENDFILE(UNIT=15)
        ENDFILE(UNIT=17)
        ENDFILE(UNIT=18)
        ENDFILE(UNIT=16)
        ENDFILE(UNIT=13)
C
999     END
C
        SUBROUTINE SORT(A,B,D)
C
        REAL A(45),B(45,45),TEMP(45),VECT(45,45)
        INTEGER I,J,K,D,U
C
        U=D-1
20      IF (U.NE.0) THEN
          DO 30 I=1,U
            K=I+1
            IF (A(K).LT.A(I)) THEN
              TEMP(I)=A(I)
              A(I)=A(K)
              A(K)=TEMP(I)
              DO 40 J=1,45
                VECT(J,I)=B(J,I)
                B(J,I)=B(J,K)
                B(J,K)=VECT(J,I)
40            CONTINUE
            ENDIF
30        CONTINUE
        U=U-1
        GO TO 20
        ENDIF
        END
```

VITA

Captain John O. Dunstan was born on 21 October 1954 in Berkeley, California. He graduated from McClintock High School in Tempe, Arizona, in 1972. In July 1972 he entered the United States Air Force Academy as a cadet, graduating in June 1976 with a degree of Bachelor of Science in Engineering Sciences. He then entered Ungergraduate Pilot Training at Williams AFB, Arizona, receiving his wings in September 1977. He was then assigned to the 96th Bomb Wing, Dyess AFB, Texas, where he served as a B-52 copilot and aircraft commander from April 1978 until his entry into the School of Engineering, Air Force Institute of Technology, in May 1983.

Permanent address: 5426 Teresa Lane
Abilene, Texas 79606

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The equations of motion of a flexible shuttle-beam-antenna system are developed and discretized using an assumed modes approximation. The system was modeled as a cantilever beam rigidly attached to the shuttle with the motion of the antenna accounted for as forces and moments on the beam's free end. The equations of motion for the system were formed from the linearized kinetic and potential energies using Lagrange's method. The equations were put into matrix form, and the matrices were diagonalized.

Linear optimal regulator techniques were employed to control the system. Two proof-mass actuators were modeled at the 40- and 80-foot positions along the beam. The shuttle was assumed to be able to be torqued about it's axes, as was the antenna. Initial runs of the ACOSS computer program showed the system to be stable with unit weighting on the weighting matrix. The system was driven unstable by changing the weightings on the flexible modes. The suppression algorithm in the program re-stabilized the system, as expected.

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