AD-A151 866

AFOSR-TR- 85-0269

(2)

THE UNIVERSITY OF NEW MEXICO
COLLEGE OF ENGINEERING

BUREAU OF ENGINEERING RESEARCH

ELASTIC-WORKHARDENING SDF SYSTEM SYBJECTED

TO RANDOM BLAST EXCITATIONS

DTIC
ELECTE
MAR 29 1985
S D

by

Frederick D. Ju
Thomas L. Paez
Fashin Chang

DTIC FILE COPY

Work supported by Air Force Office of Scientific Research
Contract No. 81-0086

Interim Report
ME-130(84)AFOSR-993-1

Reproduced From
Best Available Copy

November 1984

20000807030

Approved for public release;
distribution unlimited.

85 03 12 112

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release Distribution Unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) ME-130(84)AFOSR-993-1	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 85-0269
6a. NAME OF PERFORMING ORGANIZATION UNIVERSITY OF NEW MEXICO 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Directorate of Aerospace Sciences Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) ALBUQUERQUE, NM 87131	7b. ADDRESS (City, State and ZIP Code) BOLLING AFB, DC 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AIR FORCE OFFICE OF SCIENTIFIC RESEARCH 8b. OFFICE SYMBOL (If applicable) AFOSR/NA	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-81-0086
8c. ADDRESS (City, State and ZIP Code) BOLLING AFB, DC 20332	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2307; TASK NO. C2; WORK UNIT NO.
11. TITLE (Include Security Classification) ELASTIC-WORKHARDENING SDF SYSTEM SUBJECTED TO RANDOM BLAST EXCITATIONS (UNCLASSIFIED)	
12. PERSONAL AUTHOR(S) JU, FREDERICK D., PAEZ, THOMAS L., CHANG, FASHIN	
13a. TYPE OF REPORT Interim 13b. TIME COVERED FROM 7/83 TO 7/84	14. DATE OF REPORT (Yr., Mo., Day) 84, 11, 26 15. PAGE COUNT 51
16. SUPPLEMENTARY NOTATION from p. 1	
17. COSATI CODES FIELD / GROUP / SUB. GR.	18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Bilinear work-hardening, Permanent set, Maximum displacement response, Energy of Dissipation, Computer Program, Fortran,

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The present phase of investigation developed a model to characterize the permanent set of a SDF, bilinear hysteretic system subject to blast type loading. From that model, the inelastic structural response was characterized. A computer program VARE.F was developed compute the moments of critical measures of inelastic response. The means and variances the maximum displacement response of the permanent set and of the energy of dissipation a all useful in probabilistic analyses in structural design or in damage assessment of structures subjected to blast-type loadings. Operator supplied key words included.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED X SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Lt. Col. Lawrence D. Hokanson	22b. TELEPHONE NUMBER (Include Area Code) 202/767-4935 22c. OFFICE SYMBOL AFOSR/NA

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DTIC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12.
Distribution is unlimited.
MATTHEW J. KERPER
Chief, Technical Information Division

ELASTIC-WORKHARDENING SDF SYSTEM SUBJECTED TO RANDOM BLAST EXCITATIONS

by

Frederick D. Ju

Thomas L. Paez

Fashin Chang

The University of New Mexico
Department of Mechanical Engineering

DTIC
COPY
INSPECTED
1

Accession For
NTIS GRA&I ☒
DTIC TAB ☐
Unannounced ☐
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or Special
A-1

Interim Report

ME-130(84)AFOSR-993-1

Air Force Office of Scientific Research

November 1984

ACKNOWLEDGEMENT

The authors wish to acknowledge the support of the Air Force Office of Scientific Research in this study. This study was performed under AFOSR contract number 81-0086.

Preceding Page's Blank

TABLE OF CONTENTS

Section		Page
1.0	Introduction	1
2.0	The Analysis of Maximum Response	5
3.0	Probabilistic Analysis of Response	11
	3.1 The Maximum Response Random Process	14
	3.2 The Permanent Offset Random Process	15
	3.3 The Energy Dissipated Random Process	15
4.0	Numerical Example	17
	4.1 The Maximum Displacement	17
	4.2 The Permanent Offset	20
	4.3 The Energy Dissipated	22
5.0	Summary	24
REFERENCES		25
APPENDIX - COMPUTER PROGRAMS		26

NOMENCLATURE

Symbol	
A	excitation amplitude
A_7	post-yield stiffness
E_d	energy of dissipation
g	a function of random variables
h	permanent set
k	stiffness in the elastic range
m	system mass
R	maximum displacement, z_{max}
$R(z)$	restoring force
t	time
t_{max}	the time the system reaches its maximum displacement, z_{max}
t_7	the time the system first reaches its yield displacement, u_7
u_7	yield displacement
z	response displacement
z_{max}	maximum displacement response, R
α	excitation decay rate
ζ	damping ratio
μ	mean
γ	random variable
ρ	correlation coefficient
σ	standard deviation
β	random variable
ω_n	natural frequency (small displacement)
ω_d	damped frequency (small displacement)
ω_y	post-yield frequency ($= \sqrt{A_7/m}$)

1.0 INTRODUCTION

An above ground explosion generates a shock wave in air, or airblast, and is accompanied by some duration of strong wind. Especially, the airblast from a nuclear detonation can cause extremely high air pressures and has a relatively long period of duration. When structures under design may be subjected to this sort of loading condition, it is necessary to analyze the behaviors of the stuctures when subjected to this kind of input to determine whether the structures can survive or not. → p. 1423

When an airblast produces a high air pressure and has a long period of duration, it will cause many structures to have an extreme response. If the structure can execute a response that has plastic deformation, the designer must have some technique to analyze inelastic response in the design process. The designer needs the capability to predict such measures of structural response as the plastic deformation and maximum displacement.

When a structure executes an extreme response, it may behave inelastically, and during the response, offsets from the initial elastic configuration may occur. When these offsets are large, residual deformation in the structure may exist at the completion of structural motion, and the magnitudes of these residual deformations may be important. When they are important, it is desirable to include the potential for these residual deformations in the mathematical model of the structure so that they may be predicted in the structural analysis. The models used in References [1], [2], [3] do account for some features of inelastic response, but do not permit permanent offset (residual deformation). The model used in the current investigation does account for permanent offset.

The properties of nuclear and high explosive airblasts are random. The reason is that the material and geometric properties of blast sources are random. In view of this, an airblast measurement is never repeated even when nominally identical blast sources are used. An airblast signal usually consists of a rapidly increasing pressure followed by a gradual decay. Two properties of the pressure wave are of great concern. The first is the amplitude of the airblast; the second is the decay rate. The first property describes the highest air pressure the airblast produces

throughout the time history. In general, the rise time is so brief that the peak amplitude is assumed to occur at the beginning of explosion. The second property deals with the rate at which the airblast dies out. In this report, these two properties are considered random.

In most applications the structure itself should also be considered random. The material properties of the structure determine the elastic stiffness, yield stiffness, yield displacement, etc. Consequently, the natural frequency is affected strongly by the material properties. When the material of the structure is considered to be random, then the properties of the structure must also be considered as random. In this report, the material properties considered as random are natural frequency, yield stiffness, yield displacement and damping ratio. These four properties are considered random because they affect the response significantly.

The previous discussion points out that both the excitation and structural system can be regarded as random; in this study both sources of randomness are considered. The response of the system, therefore, can be regarded as a response random process, and there is a relationship that characterizes the response in terms of the excitaion and system random variables.

These random variables include input random variables which are peak amplitude of air pressure and decay rate, and system random variables which are natural frequency, yield stiffness, yield displacement, and damping ratio. It is desirable to develop a method to establish the relationship among the input characteristics, the system characteristics, and the response characteristics. The response random process is then characterized in terms of the relationship.

Many measures of structural response concern the designer. Some measures of special importance are plastic deformation, maximum displacement response, and energy dissipated by the system spring, or by the damper. The plastic deformation, which is the offset from the static configuration, must be controlled in structures. The maximum displacement response must be less than the design value. The energy dissipated must be less than an allowable value since it is related to the fatigue and failure

of a structure. When the probabilistic character of the input excitaion and system random variables are known, and when the relationship between measures of response and the input and system variables has been established, then it is possible to specify the response random process. Certain probabilistic measures of structural response are obtained. Specifically, the mean and variance of permanent offset, peak response, and energy dissipted in an inelastic structure are obtined. The results can be used to determine whether design criteria are satisfied.

In past investigations, several mathematical models have been used with structural identification procedures to study structural behavior. Lutes and Hseih [1] used linear, third and fourth order equations to approximately simulate the response of an SDF hysteretic system. They found that the spectral density, or the autocorrelation functon of the linear and hysteretic systems can be matched. Toussi and Yao [2] used the response of a ten-story reinforced concrete test structure as a means to study the feasibility and practicallity of a hysteresis identification technique. They considered the hysteresis to be a measure of damage. Yao, Toussi and Sozen presented the state-of-the art on damage assessment of existing structures in Reference [3]. Morrison and Paez [4] proposed a procedure for predicting the probability of survival of inelastic structures excited by blast. In this study the failure criterion was related to peak strain in the structural repsonse. A method for establishing the probabilistic character of nonlinear structural repsonse was developed. Rohani [5] presented a probabilistic solution of one-dimensional wave propagation phenomenon in earth media by using a deterministic model. Bennett and Paez [6] provided a means for relating uncertainties of input parameters to the uncertainty in response in a reinforced concrete slab. The method used in the last three references is adopted in this investigation to characterize nonlinear measures of inelastic structure response.

The objectives of this investigation are, first, to develop a model to define the permanent offset in an inelastic structure. Next, we develop the relationship among input characteristic, system characteristic and response characteristic through which the response random process is established. Based on this response random process, and given the moments

of input and system variables, the response characteristics are computed. This provides a means to analyze the structural response when the structure is subjected to blast type loading and both the input and the system are considered random.

The structure considered here is a single-degree-of-freedom (SDF) bilinear hysteretic system which possesses an elastic stiffness and yield stiffness. The natural frequency and yield natural frequency are therefore defined. The hysteresis loop of a spring, strongly influences the energy dissipated in a structure; this will be discussed in detail. The airblast input is modeled by a decaying exponential function. The value of peak air pressure and decay rate of the input are obtained from the Air Force Design Manual [7].

2.0 THE ANALYSIS OF MAXIMUM RESPONSE

The maximum response of a structure when subjected to blast type load can be predicted through basic structure response analysis. Intuitively, the maximum response must relate to the input parameters, which are peak air pressures and decay rate, and system parameters, which are natural frequency, damping ratio, etc. All these properties affect the response. Specifically, we can establish a mathematical function which governs the maximum reponse. The response random process can be simply expressed. Given the statistical moments of the input and system parameters, the statistical moments of the responses can be computed. The response considered here can be inelastic.

It is assumed that the system under consideration has significant elastic and yield stiffness so that the bilinear hysteretic system can be used as the structural model. The offset function for blast type input will behave as in Figure 2-1.

Again consider an SDF, bilinear hysteretic system, with small damping ratio, ζ, system mass, m, elastic stiffness, k. The equation of motion is given by

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \frac{1}{m}R(z) = Ae^{-\alpha t} \qquad t \geq 0 \tag{2-1}$$

Assume that the system starts from rest, $z(0) = 0$ and $z'(0) = 0$. Let t_7 denote the time the system first reaches its yield displacement, u_7.

Within time $t < t_7$, the system remains linear, so that Equation (2-1) can be written as

$$\ddot{z} + 2\zeta\,\omega_n\dot{z} + \omega_n^2 z = Ae^{-\alpha t} \qquad \text{for } t < t_7 \tag{2-2}$$

Because the system remains linear during the initial portion of response, the above equation can be solved immediately by using convolution integral

$$z(t) = \int_0^t h(\tau)\, x\,(t - \tau)\, d\tau \tag{2-3}$$

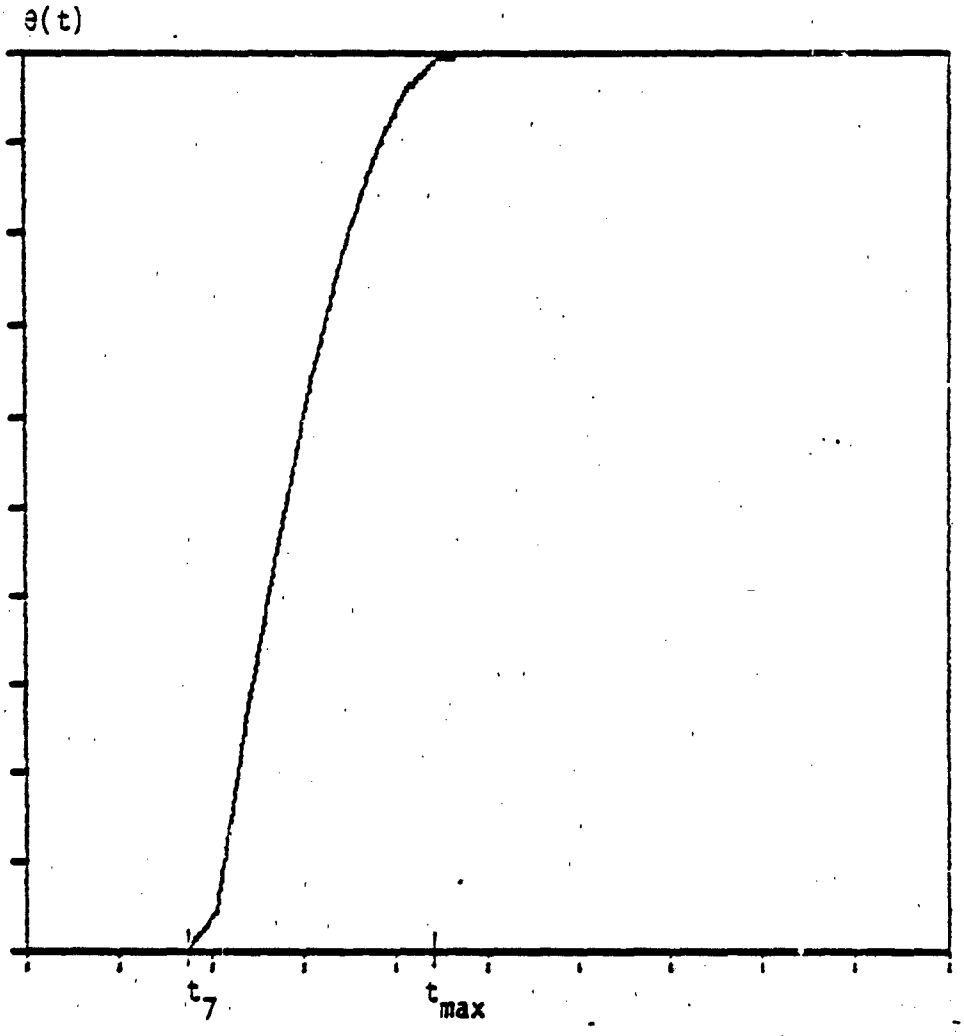

Figure 2-1. The Offset Function in Time Domain.

where h(t) is the linear system impulse response function and x(t) is the input forcing function. The impulse response function is

$$h(t) = \frac{e^{-\zeta\omega_n t}}{\omega_d} \cdot \sin(\omega_d t) \tag{2-4}$$

where $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ and $\ddot{x}(t) = Ae^{-\alpha t}$. Carry out the integral; then

$$z(t) = \frac{A}{\left[\omega_d (\alpha - \zeta\omega_n)^2 + \omega_d^2\right]} \cdot \left\{e^{-\zeta\omega_n t}[(\alpha - \zeta\omega_n) \sin(\omega_d t) - \omega_d \cos(\omega_d t)] + \omega_d e^{-\alpha t}\right\}$$

$$\text{for } 0 \leq t \leq t_7 \tag{2-5}$$

At time $t = t_7$, the displacement response should be equal to u_7.

$$z(t_7) = \frac{A}{\omega_d\left[(\alpha - \zeta\omega_n)^2 + \omega_d^2\right]}\left\{e^{-\zeta\omega_n t_n}[(\alpha - \zeta\omega_n)\sin(\omega_d t_7) - \omega_d \cos(\omega_d t_7) + \omega_d e^{-\alpha t_7}\right\} \tag{2-6}$$

The time t_7 at which yielding first occurs must be determined by solving the above equation for t_7. Since ζ and t_7 are both small; $e^{-\zeta\omega_n t_7} \doteq 1$. Simplify the notation by letting $\alpha - \zeta\omega_n = p$; $\omega_d[p^2 + \omega_d^2] = x_1$; $A/x_1 = x_2$. Rewrite Equation (2-6) using the above notation.

$$x_2 \cdot \left[p \sin(\omega_d t_7) - \omega_d \cos(\omega_d t_7) + \omega_d e^{-\alpha t_7}\right] = u_7 \tag{2-7}$$

The value of α is about from 0.5 - 6.0. This is usually much smaller than the value of ω_n; hence,

$$p \cdot \sin(\omega_d t_7) \ll \omega_d \cos(\omega_d t_7)$$

and the quantity on the left is negligible.

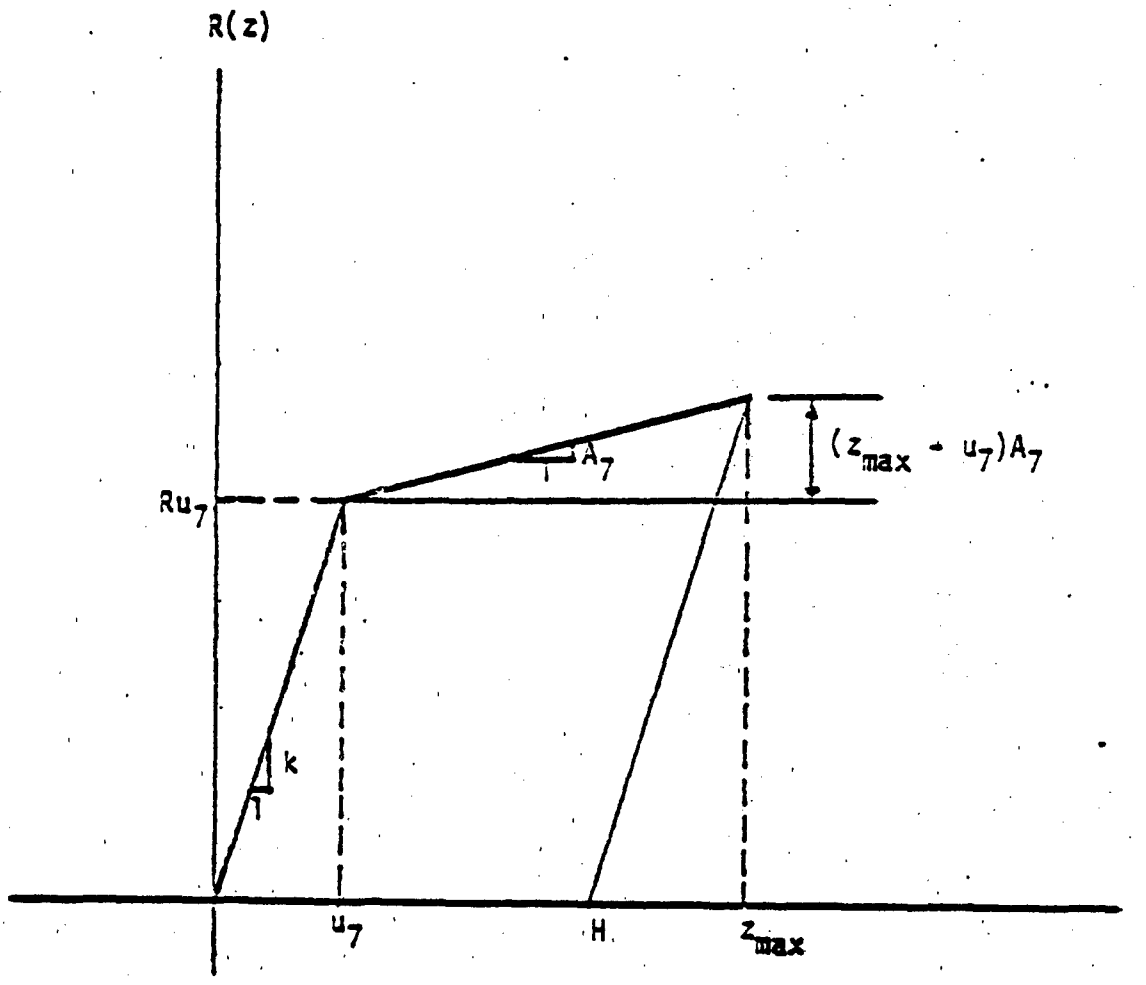

Figure 2-2. The Behavior of Bilinear Hysteretic Restoring Force.

Using Taylor's expansion to express $e^{-\alpha t_7}$ and $\cos(\omega_d t_7)$ and neglecting the high order term, t_7 can finally be evaluated

$$t_7 \doteq \frac{1}{2}\left(B + \sqrt{B + 4C}\right) \text{ where } B = 2\alpha/(\omega_d^2 + \alpha^2),\ C = 2(u_7)/A. \qquad (2\text{-}8)$$

After solving for t_7, the velocity at $t = t_7$ can be computed immediately. Let $\dot{z}(t_7) = \dot{z}_7$.

To this point, the only assumption used in determining the time of first yield and the velocity at that time is that the system viscous damping factor is relatively small. No assumptions regarding the input level or yield level have been used.

Next, we consider the phase of the response when yielding starts to occur. Let t_{max} be the time the response reaches its maximum displacement value. During the time interval (t_7, t_{max}) the system has entered the plastic region and permanent offset starts to occur. If we let A_7 denote the yield stiffness, then for time $t_{max} < t_7$, the restoring force function can be written as (refer to Fig. 2-2):

$$z + 2\zeta\omega_n \dot{z} + \frac{1}{m}[A_7 z + (k - A_7)\, u_7] = Ae^{-\alpha t}, \quad t_{max} \geq t \geq t \qquad (2\text{-}9)$$

or

$$z + 2\zeta\omega_n \dot{z} + \omega_y^2 z = Ae^{-\alpha t} - (\omega_n^2 - \omega_y^2)u_7, \quad t_{max} > t > t \qquad (2\text{-}10)$$

where $\omega_y^2 = A_7/m$

The solution for $z(t)$ can be expressed

$$z(t) = e^{-\zeta\omega_n t}(c_1 \cos(\bar{\omega}_d t) + c_2 \sin \bar{\omega}_d t) + z_p, \quad t_y \leq t \leq t_{max} \qquad (2\text{-}11)$$

where $\bar{\omega}_d = \sqrt{(k_y/m) - (\zeta\omega_n)^2}$

$$z_p = q_1 + q_2 e^{-\alpha t}$$

$$q_1 = [(k_y/m - \omega_n^2] z_y m/k_y$$

$$q_2 = A/[\alpha^2 - 2\zeta\omega_n\alpha + (k_y/m)]$$

By using q_1 and q_2 in Equation 2-11, the constants C_1 and C_2 can be solved by using as initial condition, $z(t_7) = u_7$ and $z(t_7) = \dot{z}_7$ which were obtained in the previous discussion.

Since the displacement response has been determined in Equation (2-11), the maximum value can be obtained by maximizing Equation (2-11). The maximum occurs when

$$\dot{z}\ (t_{max}) = 0 \tag{2-12}$$

The t_{max} can be obtained from Equation (2-12). The corresponding z_{max} then can be evaluated by using t_{max} in Equation (2-11).

3.0 PROBABILISTIC ANALYSIS OF RESPONSE

In the previous section, the relationship among the maximum response and input parameters and system parameters was established. The response random process is then characterized in terms of the established relationship. In many cases, the established function may be complicated. In order to analyze some measures of the response, a method using Taylor's expansion is available and has been widely used. A brief discussion of that method is presented here.

Let R denote a random variable that is a measure of the structural response random process. It is assumed that R is a function of n random variables. These n random variables are the parameters of the input and the structural system. Let the functional relation be denoted

$$R = g(\gamma_1, \gamma_2, \ldots\ldots\ldots \gamma_n) \tag{3.1}$$

The γ_i, i=1...n are the random variable with mean values μ_{γ_i}, i=1,...n. The function g can be expanded in a Taylor series about the means of the parameters:

$$R = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \ldots\ldots \mu_{\gamma_n}) + \sum_{i=1}^{n} (\gamma_i - \mu_i) \left.\frac{\partial g}{\partial \gamma_i}\right|_{\{\mu\}} + \ldots. \tag{3-2}$$

where $\{\mu_{\gamma_i}\}$ is the vector of mean values of the γ_i.

The moments of R can be evaluated by using some portion of the series in Equation (3-2). For example, the series can be truncated following the linear terms. When this is done, the mean of R is

$$E[R] = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \ldots. \mu_{\gamma_n}) \tag{3-3}$$

This mathematical expression shows that the mean value of the dependent random variable, R, is closely related to the mean values of the underlying random variables, μ_{γ_i}, and the function expression $g(\mu_{\gamma_1}, \mu_{\gamma_2} \cdots \mu_{\gamma_n})$.

The variance of R can be computed as:

$$
\begin{aligned}
Var[R] &= E[R^2] - E[R]^2 \\
&= E\left[g^2\left(\mu_{\gamma 1}, \mu_{\gamma 2}, \cdots \mu_{\gamma n}\right)\right] + E\left[2g\left(\mu_{\gamma 1}, \mu_{\gamma 2}, \cdots \mu_{\gamma n}\right) \cdot \right. \\
&\qquad \left. \sum_{i=1}^{n} \left(\gamma_i - \mu_{\gamma_i}\right) \frac{\partial g}{\partial \gamma}\bigg|_{\{\mu_{\gamma_i}\}}\right] \\
&\quad + E\left[\sum_{j=1}^{n} \sum_{i=1}^{n} \left(\gamma_i - \mu_{\gamma_i}\right) \frac{\partial g}{\partial \gamma_i}\bigg|_{\{\mu_{\gamma_i}\}} \cdot \left(\gamma_j - \mu_{\gamma_j} \frac{\partial g}{\partial \gamma_j}\right)_{\{\mu_{\gamma_j}\}}\right] - E[R]^2 \\
&= \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}\, \sigma_i \sigma_i \frac{\partial g}{\partial \gamma_i}\bigg|_{\{\mu_{\gamma_i}\}} \frac{\partial g}{\partial \gamma_j}\bigg|_{\{\mu_{\gamma_j}\}} \qquad (3\text{-}4)
\end{aligned}
$$

The σ_i and σ_j represent the standard deviations of γ_i and γ_j and ρ_{ij} here represents the coefficient of correlation between γ_i and γ_j.

The analysis technique outlined above can be used in a wide variety of cases; especially, it is useful in the characteristics of the response of a nonlinear system. When the moments of the underlying random variables are known (means, variances, and covariances), then it is easy to apply. But there are some cases where the random variable R depends on some underlying random variables whose characteristics are unknown, a priori, as well as random variables whose characteristics are known. For example, we may be interested in establishing the moments of energy dissipated during inelastical response. This quantity can be expressed as a function of

input parameters, system parameters and the peak response. The moments of the input and system parameters are probably known, a priori, but the moments of peak response certainly are not. To account for this, the first phase of an investigation can be aimed at analyzing peak response and its statistical relation to the input and system parameters. After this is done, the energy dissipated can be analyzed using the general approach outlined above. In general, any sequence of analyses like this one can be executed. At each step, a new random variable characterizing the response can be introduced.

For example, the analysis referred to above can be accomplished in the following way. Let R be a function of n random variables, β_i $i=1,2,\ldots m, m+1, m+2, \ldots n$. The moments and cross moments of the β_i, $i=1,2,\ldots m$, are known, a priori, and the moments and cross moments of the $\beta_i, i=m+1,\ldots n$, are not known. However, the moments of the $\beta_i, i=m+1,\ldots n$, can be determined from the $\beta_i, i=1,\ldots m$. Let $\beta_i, i=m+1,\ldots n$ be expressed as follows:

$$\beta_i = h_i(\beta_1, \beta_2 \ldots \beta_m), \qquad i=m+1,\ldots n \tag{3-5}$$

The Taylor expansion for the above expression can be written as in Equation (3-2). The series can be truncated at some point and the result can be used to approximate the moments and cross moments for all the $\beta_i, i=m+1,\ldots n$. For example, when the series are truncated following the linear term, the covariance between two of the β_i's, one from the group $i=1,\ldots m$, and the other from the group, $i=m+1,\ldots n$, can be expressed as

$$\mathrm{Cov}(\beta_p \beta_q)$$

$$= E\left[\beta_p \left\{h_q\left(\mu_{\beta_1}, \mu_{\beta_2}, \ldots \mu_{\beta_r}\right) + \sum_{j=1}^{r} \left(\beta_j - \mu_{\beta_j}\right) \frac{\partial h_q}{\partial \beta_j}\bigg|_{\{\mu_{\beta_j}\}} + \ldots\right\}\right]$$

$$- E[\beta_p]\, E[h_q] \doteq \sum_{j=1}^{r} \left\{E[\beta_p \beta_j] - E[\beta_p]\, \mu_{\beta_j}\right\} \frac{\partial h_q}{\partial \beta_j}\bigg|_{\{\mu_{\beta_j}\}}$$

$$= \sum_{j=1}^{r} \rho_{pj}\, \sigma_{\beta_p}\, \sigma_{\beta_j} \left.\frac{\partial h_q}{\partial \beta_j}\right|_{\{\mu_{\beta_j}\}}, \qquad \text{for } 1 \le p \le m, \quad m+1 \le q \le n, \quad r \le m \tag{3-6}$$

The covariance between two of the β_i's, for which $i=m+1,\ldots.n$, can be expressed as

$$\mathrm{Cov}\,[\beta_p \beta_q]$$

$$= E\left[\left\{h_p\left(\mu_{\beta_1},\ldots\mu_{\beta_r}\right) + \sum_{j=1}^{r}\left(\beta_j - \mu_{\beta_j}\right) \left.\frac{\partial h_p}{\partial \beta_j}\right|_{\mu_{\beta_j}} + \ldots\right\} \cdot\right.$$

$$\left.\left\{h_q\left(\mu_{\beta_1},\ldots\mu_{\beta_s}\right) + \sum_{i=1}^{s}\left(\beta_i - \mu_{\beta_i}\right) \left.\frac{\partial h_s}{\partial \beta_i}\right|_{\{\mu_{\beta_i}\}} + \ldots\right\}\right] - E[h_p]\, E[h_q]$$

$$= \sum_{j=1}^{r} \sum_{i=1}^{s} \rho_{ij}\, \sigma_{\beta_j}\, \sigma_{\beta_j} \left.\frac{\partial h_p}{\partial \beta_j}\right|_{\{\mu_{\beta_j}\}} \left.\frac{\partial h_s}{\partial \beta_i}\right|_{\{\mu_{\beta_i}\}}, \quad \text{for } m+1 \le p, \quad q \le n, \; r,s \le m \tag{3-7}$$

The means and variances of the β_i, $i=m+1,\ldots.n$ are obtained in the same way as Equations (3-3) and (3-4). Given these moments and cross moments, the mean and variance of R can be obtained using Equation (3-3) and (3-4), as they are written.

The response random process can be accurately characterized using three measures: the permanent offset, the maximum response, and the energy dissipated. Using the techniques described above, the means and variances of each random variable can be computed. The following section will present more detail about those random variables.

3.1 The Maximum Response Random Process

From Section 2.0, the maximum displacement response is governed by 6 random parameters and can be represented as:

$$R = g(\gamma_1, \gamma_2 \cdots \gamma_6)$$

where R = maximum displacement response

γ_1 = input peak air pressures (A), γ_2 = decay rate (α)

γ_3 = natural frequency (ω_n), $\gamma_4 = \sqrt{A_7/m}$

γ_5 = yield displacement u_7, γ_6 = damping ratio (ζ)

Using Equations (3-3), (3-4), the mean and variance of R can then be computed.

3.2 The Permanent Offset Random Process

The relation between permanent offset, H, and maximum displacement, R, is:

$$H = (R - u_7)(1 - \frac{A_7}{k}) \tag{3-8}$$

So we can let

$$H = h(\beta_1, \beta_2, \beta_3, \beta_4)$$

where $\beta_1 = \omega_n$, $\beta_2 = \omega_y$, $\beta_3 = u_7$, $\beta_4 = R$. Then the mean and variance of H can be computed using the technique as stated above.

3.3 The Energy Dissipated Random Process

The energy dissipated by a bilinear hysteretic system includes two parts. The first part is the energy dissipated by the spring. The second part is the energy dissipated by the damper. The system will reach its maximum offset within the first cycle. Then the energy is dissipated by the spring only during this time. After the system reaches its maximum offset, then the system remains linear in a new equilibrium configuration. The energy is then dissipated by the damper only. The energy dissipated by a linear system is related to the damper only and has already been studied (Reference 8). In this report, only energy dissipated by the spring is considered.

Refer to Figure (2-2). The energy dissipated by the spring can be computed as:

$$E_d = .5 \cdot k \cdot u_7^2 + .5\ (z_{max} - u_7)A_7 + k \cdot u_7(z_{max} - u_7) - \frac{1}{2}\, k(z_{max} - H)^2 \tag{3-9}$$

therefore $E_d = m(\frac{1}{2}\,\omega_n^2 u_7^2 + \frac{1}{2}\,\omega_y^2(z_{max} - u_7)^2 + \omega_n^2 u_7(z_{max} - u_7)$

$$- \frac{1}{2}[(z_{max} - u_7)\omega_y^2/\omega_n + u_7\omega_n]^2$$

In other words, the energy dissipated can be expressed as:

$$E_d = s(\eta_1, \eta_2, \eta_3, \eta_4), \tag{3-10}$$

where we denote $\eta_1 = \omega_n$, $\eta_2 = \omega_y$, $\eta_3 = u_7$, $\eta_4 = z_{max}$. Again using Equation (3-3), (3-4), and (3-6), the mean and variance of energy dissipated can be computed.

4.0 NUMERICAL EXAMPLE

In Section 2.0 an approach for computing the maximum displacement for a bilinear hysteretic system subjected to blast type load was developed. In Section 3.0 an approach for the probabilistic analysis of the response was developed. A computer program, called VARE.F is used in this section to calculate the means and variances of the response, which includes maximum displacement, permanent offset and energy dissipated by the sprin.

In Section 3.0 the technique used to evaluate the variance of a response random variable, which depends on n underlying random variables and whose statistical moments are known, refers to the partial derivatives with respect to the underlying random variables. In order to calculate the partial derivatives with respect to the underlying variables, an approximated method is used here. let R be a random variable which depends on n underlying variables γ_i, i=1,....n (for example, maximum displacement which is expressed in Equation 3-1.). Then

$$R = g(\gamma_1, \gamma_2, \ldots \gamma_n) \tag{4-1}$$

and the partial derivative of R with respoect to γ_i can be approximated

$$\frac{\partial g}{\partial \gamma_i} = \frac{g(\gamma_1, \gamma_2 \ldots \gamma_i + \Delta\gamma_i \ldots \gamma_n) - g(\gamma_1, \gamma_2 \ldots \gamma_i - \Delta\gamma_i \ldots \gamma_n)}{2\Delta\gamma_i} \tag{4-2}$$

This approximation can be computed even though $g(\gamma_1, \gamma_2, \ldots \gamma_n)$ may not be a function that can be written explicitly. Writing $g(\gamma_1, \gamma_2 \ldots \gamma_n)$ here implies that a computer program can be run using the γ_i, i=1,...n, as inputs to obtain a result R. The program can be run twice to obtain the above approximation. using above approximation in the program VARE.F the first order partial derivative with respect to any given underlying random variable can be calculated. The maximum displacement, permanent offset and energy dissipated are considered in the following section.

4.1 The Maximum Displacement

It is mentioned in Section 3.1 that the maximum displacement is related to six random parameters. The functional expression is

$$R = g(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_4, \gamma_5, \gamma_6)$$

where R is maximum displacement, γ_1 = input peak air pressure

γ_2 = decay rate of input force, γ_3 = natural frequency, ω_n

$\gamma_4 = \sqrt{A_7/m}$ γ_5 = damping ratio,

γ_6 = yield displacement, u_7

The means and standard deviations of the above six random variables are given in Table 4.1.

Table 4.1

The Means and Standard Deviation of γ_i, i=1,6

γ_i	Mean Value, μ_{γ_i}	Standard Deviation, σ_{γ_i}
i=1	0.121428e+04	.1 μ_{γ_1}
i=2	0.120000e+01	.1 μ_{γ_2}
i=3	0.358568e+02	.1 μ_{γ_3}
i=4	0.113389e+02	.1 μ_{γ_4}
i=5	0.119523e-01	.2 μ_{γ_5}
i=6	0.100000e+01	.2 μ

The covariances between random variable pairs in γ_i, i=1,6 are given in Table 4.2.

Table 4.2

The Covariance Matrix

$.01\ \mu_1^2$	$.001\ \mu_1\ \mu_2$	0.	0.	0.	0.
	$.01\ \mu_2^2$	0.	0.	0.	0.
		$.01\ \mu_3^2$	$.001\ \mu_3\ \mu_4$	$.001\ \mu_3\ \mu_5$	$.0001\ \mu_3\ \mu_6$
(symmetric)			$.01\ \mu_4^2$	0.	0.
				$.04\ \mu_5^2$	0
					$.04\ \mu_6^2$

The $\partial R/\partial \gamma_i$, i=1,6 are computed using Equation and the computer program VARE.F. The results are shown in Table 4.3.

TABLE 4.3

Partial Derivative of Peak Response

i	$\partial R/\partial \gamma_i$
i=1	0.57023e-02
i=2	-0.36344e+00
i=3	-0.17445e+00
i=4	-0.34095e-01
i=5	-0.13495e+02
i=6	-0.42679e+01

Using Equation 3-3 and the computer program VARE.F, the mean of maximum displacement is computed. The results is:

$E[R] = 2.65$ (in)

Using Equation 3-4 and the computer program VARE.F, the variance of maximum displacement is computed. Each term in Equation 3-4 is listed in Table 4.4.

Table 4.4.

The Terms in Variance R

.47943+00	-.302e-02	0.	0.	0.	0.
	.1902e-02	0.	0.	0.	0.
		.3913e+00	-.2418e-02	.2017e-02	.5339e-02
			.1494e-02	0.	0.
(symmetric)				.1041e-02	0.
					.7268e+00

The variance of maximum displacement is the sum of every term as shown in Table 4.4. The result is

$$Var[R] = 1.617 \ (in^2)$$

Table 4.4 shows that only the variance of input peak air pressure, natural frequency and yield displacement have significant effect on the variance of maximum displacement. The rest of the terms are almost negligible.

4.2 The Permanent Offset

In Section 3.2, the permanent offset is expressed using Equation 3-8. Let H represent the permanent offset; then

$$H = h(g_1, g_2, g_3, g_4)$$

where H = permanent offset,

g_1 = natural frequency, ω_n

$g_2 = \sqrt{A_7/m}$

g_3 = yield displacement, u_7

g_4 = maximum displacement, R

The means and standard deviations of g_i, i=1,4 are given in Table 4.5.

Table 4.5

The Means and Standard Deviations of β_i, i=1,4

z	Mean μ_{β_i}	Standard Deviation σ_{β_i}
		i
i=1	0.358569e+02	.1 μ_{β_1}
i=2	0.113389e+02	.1 β_{β_2}
i=3	0.100000e+01	.2 u_{β_3}
i=4	0.265600e+01	.12717

Again using Equation 3-3 and the computer program VARE.F, the mean value of H is evaluated and the result is

E[H] = 1.4907 (in)

The first order derivatives of H with respect to β_i,i=1,4 are computed directly from Equation (3-8) and are shown in Table 4.6.

Table 4.6

Partial Derivative of H

i	i=1	i=2	i=3	i=4
$\partial H/\partial \beta_i$	0.92387e-02	-0.29215e-01	-0.90000e+00	0.90000e+00

The covariance between β_4 and β_i,i=1,3 is computed using Equation (3-6). The covariance between two β_i,i=1,3 is a known quantity. Therefore, when the variance of permanent offset is expressed by the form of Equation (3-4), in which g represents permanent offset at this stage, each term in Equation (3-4) is computed and is listed in Table 4.7.

Table 4.7

The Terms in Variance H

.0011	- .00011	-.00006	0.0191
	.0011	0	.00302
		.0324	.1393
(symmetric)			1.13100

The variance of H is the sum of every term of Table 4.7; the result is

$Var\ (H) = 1.5906\ (in^2)$

From Table 4.7, it is observed that the maximum displacement has great influence on the variance of permanent offset. The covariance between maximum displacement and yield displacement has less influence, and the rest of the terms are almost negligible.

4.3 The Energy Dissipated

In Section 3.3, the energy dissipated in a bilinear hysteretic system is separated into two parts. The first part is energy dissipated in the spring and the second part is energy dissipated in the damper. Here, only the energy dissipated by the spring is considered. From Equation 3-9, the energy dissipated by the spring can be expressed as

$$E_d = S\ (\eta_1,\ \eta_2,\ \eta_3,\ \eta_4,)$$

where $\eta_1 = \omega_n$, $\eta_2 = \sqrt{A_7/m}$, $\eta_3 = u_7$, $\eta_4 = R$.

The means and variances of the η_i, i=1,4 are the same as given in Table 4.5. The first order derivatives of E_d with respect to η_i, i=1,4 are obtained directly from the expression of E_d which is given in Equation (3-10). The results are

Table 4.8

The Derivatives of E_d

i	i=1	i=2	i=3	i=4
$\partial Ed/\eta_i$	.838372e+03	- .887303e+02	.56783e+03	.944165e+04

The mean value of E_d is computed by using Equation (3-3) and computer program VARE.F and the result is $E(E_d)$ = .161578e+05 (lb-in). When using Equation (3-4) to compute the variance of E_d, the variable g in Equation (3-4) represents E_d at this stage. Each element in Equation 3-4 can be obtained using the approach of Section 4-2. Table 4.9 lists each term of Equation (3-4).

Table 4.9

The Terms in Variance E_d

.903687e+07	-.302450e+05	.341400e+04	-.181982e+08
	.101225e+05	0	.961467e+05
		.128976e+05	-.921978e+06
			.144175e+09

The variance of E_d is the sum of every element of Table 4.9. The result is

$$\text{Var}\ (E_d) = 0.115133e+09\ (lb^2 - in^2)$$

From Table 4.9, it is observed that the maximum displacement has great influence on the variance of E_d, and the rest of the terms are almost negligible.

5.0 SUMMARY

The objectives of this study were, first, to develop a model that characterizes the permanent offset of an SDF, bilinear hysteretic system subjected to blast type load; and second, to probabilistically characterize the features of the inelastic structural response. In this report, the response characteristics considered were: (1) maximum displacement response (2) permanent offset (3) energy dissipated by the inelastic spring. The statistical properties characterizing these measures of response are the mean and variance.

In Section 2, the maximum displacement response was computed. The response random process was then established. Section 3 discussed the techniques of probabilistic analysis of a complicated function. Since the response random process which was established in Section 2 could not be expressed in a closed form, the computer program VARE.F was developed to compute the moments of critical measures of inelastic response.

Several numerical results were shown in Section 4. One of the important results showed that the maximum displacement response has great influence on the variance of permanent offset and energy dissipated. The results developed in this study are restricted to blast type inputs, and bilinear hysteretic, SDF systems.

The results developed in this report can be used in the probabilistic design process. For example, the computer program VARE.F may be used to determine the mean and variance of some critical response measures and the designer may determine if the response satisfies certain design criteria. Or, the response may be restricted to some extreme level, then the system parameters may be determined by using computer program VARE.F with a trial and error method. Also we can predict the mean and variance of the response in an assessment of an existing building.

Future study might apply the present techniques to multi-degree-of-freedom systems. Or response models that characterize response to other than blast inputs might be sought.

REFERENCES

1. Lutes, L. D., Hseih, J., "Higher Order Equivalent Linearization in Random Vibration," Proceedings, Third Engineering Mechanics Division Specialty Conference, ASCE, Austin, TX, September 1979.

2. Toussi, S., Yao, J. T. P., "Hysteresis Identification of Multi-Story Buildings," Technical Report No. CE-STR-81-5, School of Civil Engineering, Purdue University, West Lafayette, IN, May 1981.

3. Yao. J. T. P., Toussi, S., and Sozen, M. A., "Damage Assessment from Dynamic Response Measurements." Presented at the Symposium on Structural Reliability and Damage Assessment, Ninth U.S. National Congress of Applied Mechanics, Cornell University, Ithaca, NY, June 1982.

4. Morrison, D., Paez, T. L., "Probabilistic Multiple Point Failure Analysis," Proceedings of the Fourth ASCE Conference on Probabilistic Mechanics and System Reliability, Berkeley, CA, 1984.

5. Rohani, B., "Probabilistic Solution for One-Dimensional Plane Wave Propagation in Homogeneous Bilinear Hysteretic Materials," SL-82-7, U.S. Army Engineer Waterways Experiment Station Structures Laboratory, June 1982.

6. Bennett, R., Paez, T. L., "Fragility of Concrete Slabs," Proceedings of the Fourth ASCE Conference on Probabilistic Mechanics and System Reliability, Berkeley, CA, 1984.

7. Crawford, R.E., Higgins, C. J., Bultmann, E. H., "The Air Force Manual for Design and Analysis of Hardened Structures," Fifth Printing, Air Force Weapons Laboratory, Kirtland Air Force Base, 1978.

8 Chang, F. C., Paez, T. L., Ju, F. D., "Least Favorable Response of Inelastic Structures," Tech. Report No. CE-63(83) AFOSR-993-1, Bureau of Engineering Research, The University of New Mexico, Albuquerque, NM, March 1983.

APPENDIX

COMPUTER PROGRAM VARE.F

```
C*************************************************************
c
c
c
c this program compute the var [ED] and E [ED]
c
c  Equation of motion :
c  m7 y" + c7 y' + k7 ( y-O ) = fm exp(-alpha t )
c  or
c       y" + 2 zita wn y' + wn**2 ( y-O )=A exp (-alpha t)
c  where :
c      g is z-max =G (bata1,bata2,.....,bata6)
c      bata1=A,          and xmu(1)=E [bata1]
c      bata2=alpha,      and xmu(2)=E [bata2]
c      bata3=wn,         and xmu(3)=E [bata3]
c      bata4=wn1,        and xmu(4)=E [bata4]
c      bata5=zita,       and xmu(5)=E [bata5]
c      bata6=v7,         and xmu(6)=E [bata6]
c
c  and :
c       xrho(i,j) are coefficient of correlation between bata(i)
c       xsigma(i) are corresponding STANDARD DEVIATION
c       xvar (i,j) are variance matrix
c       pgwb(i) =d G /d bata(i)
c
c    offset is the max offset  of the response,ie:
c      offset=H(gamma1,gamma2,gamma3,gamma4)
c      gamma1=G          and zmu1=E [gamma1]
c      gamma2=wy1         and zmu2=E [gamma2]
c      gamma3=wn         and zmu3=E [gamma3]
c      gamma4=v7         and zmu4=E [gamma4]
c   and zrho(i,j) zsigma(i),zvar(i,j),ph(i) represent the same
c    characteristic as in xmu
c
c   In energy  dissipation part ,
c      smu(i) are related to the spring
c   therefore :
c      smu(1)=E[wn]
c      smu(2)=E[wy1]
c      smu(3)=E[v7]
c      smu(4)=E[zmax]
c   The svar(i,j),srho(i,j),pes(i) are corresponding
c   variance matrix, coeff. of correlation, and first order
c   derivitive w.r.p to those parameter.
c
c      dmu(i) are related to the damping
c   Therefore :
c      dmu(1)=a
c      dmu(2)=alpha
c      dmu(3)=wn
c      dmu(4)=zita
c      dmu(5)=thita
c
```

```
c
C ***************************************************************
c
      dimension xmu(6),xsigma(6),xrho(6,6),xvar(6,6),pgwb(6)
      dimension zmu(4),zsigma(4),zrho(4,4),zvar(4,4),ph(4)
      dimension smu(4),ssigma(4),srho(4,4),svar(4,4),pes(4)
      dimension dmu(5),dsigma(5)
      dimension ff(1024),offset(1024),tryl(1000),qql(1000),v(1024)
      real c7, m7
c
c  define input parameter
      fm=8500.
      c7=6.
      m7=7.
      k7=9000.
      a7=.1*k7
      v7=1.
      dt=0.01
      nb=1024
      a=fm/m7
      alpha=1.2
      wn=(k7/m7)**.5
      wyl=(a7/m7)**.5
      zita=c7/(2.*m7*wn)
c
c
      do 80 i=1,nb
      tx=(i-1)*dt
      ff(I)=fm*exp(-alpha*tx)
 80   continue
c
c  define the moment value of random parameter
      xmu(1)=a
      xmu(2)=alpha
      xmu(3)=wn
      xmu(4)=wyl
      xmu(5)=zita
      xmu(6)=v7
c
      xsigma(1)=.1*a
      xsigma(2)=0.1*xmu(2)
      xsigma(3)=0.1*xmu(3)
      xsigma(4)=0.1*xmu(4)
      xsigma(5)=.2*zita
      xsigma(6)=.2*v7
c
      xrho(1,2)=.1
      xrho(3,4)=.1
      xrho(3,5)=.1
      xrho(3,6)=.01
      do 90 i=1,6
      do 90 j=1,6
      if (i.eq.j) go to 91
```

```
      xrho(j,i)=xrho(i,j)
      go to 90
   91 xrho(i,j)=1.
   90 continue
c
c
c
c  find the value of q which is the parameter to be identify
c  in offset
      call sbilin (FF,c7,m7,k7,a7,v7,dt,nb,v,offset,ened)
      thcon=offset(nb)
      write (6,10) ened, thcon
 10   format ('Energy dissipated and offset from sbilin is ',
     c         e13.6,2x,f9.5)
C
C
C   In order to prevent offset oscillating, check if the max offset
c   equal to perment offset , if not, it means that the forcinG
c   function , or the system itself , does not practical.
c
      offmax=0.
      nb1=nb-1
      do 75 i=1,nb1
      offmax=amax1 (offmax,offset(i))
 75   continuE
      if (offmax.eq.0.) go to 3000
      if (offmax.ne.thcon) go to 3001
c
c
c  identify the parameter q1 and q2
      q1=1000.
      qx=q1
      p=thcon*2./3.1415926
      diff=1.
      cycle=1.
c
1001  call search (offset,p,q1,nb,dt,epsiln)
      try1(1)=epsiln
      ku=1
      qq1(ku)=q1
C
      dq1=qx/(diff*10.7)
C
1021  q1=q1+dq1
      call search (offset,p,q1,nb,dt,epsiln)
      ku=ku+1
      qq1(ku)=q1
      try1(ku)=epsiln
      if (try1(ku).lt.try1(ku-1)) go to 1021
      if (ku.ge.3) go to 2000
C
      q1=q1-dq1
1010  q1=q1-dq1
```

```
t,p,ql,nb,dt,epsiln)

y1(ku-1)) go to 2000


O TO 1001

.6)

*****************************************
ar[G]

*****************************************
2),xmu(3),xmu(4),xmu(5),xmu(6),
,thir)

i),xsigma(i),i=1,6)

= zmax is ',f16.9)
s :',6x,'xsigma[i] is ')


f,k7,a7,dt,nb,pmox,pg)


(i),i=1,6)
ata is :')


xsigma(iqq)
xrho(ipp,iqq).eq.0.) go to 99
to 98
to 97
```

```
      pga=pgwb(icn)
      icn=iqq
      pgb=pgwb(icn)
      xvar(ipp,iqq)=pga*pgb*sigx*xrho(ipp,iqq)
      go to 100
   97 icn=ipp
      pga=pgwb(icn)
      xvar(ipp,iqq)=(pga)**2*sigx*xrho(ipp,iqq)
      go to 100
c
   98 xvar(ipp,iqq)=xvar(iqq,ipp)
      go to 100
c
   99 xvar(ipp,iqq)=0.
  100 continue
c
c
      do 200 i=1,6
      do 200 j=1,6
      varx=varx+xvar(i,j)
  200 continue
      write (6,24)
      write (6,25) ((xvar(i,j),j=1,6),i=1,6)
      write (6,26) varx
   24 format (/'X-variance matrix : ')
   25 format (6e16.6)
   26 format (/' var [G] is ',2x,e13.6)
c
c
c
c
c
c
c  compute E [offset]
      off=(pmr-v7)*(1.-a7/k7)
c
c
c ***************************************************************
c
c
c
c   COMPUTE VAR [offset]
c
c   ph(1)=dh/d(gamma1)
c   ph(2)=dh/d(gamma2)
c   ph(3)=dh/d(gamma3)
c   ph(4)=dh/d(gamma4)
c
c ***************************************************************
c
c
      ph(1)=1.-(wy1/wn)**2
      ph(2)=(pmr-v7)*(-2.)*wy1/wn**2
```

```
      ph(3)=(pmr-v7)*2.*wyl**2/wn**3
      ph(4)=-1.*(1.-(wyl/wn)**2)
c
      zmu(1)=pmr
      zmu(2)=wyl
      zmu(3)=wn
      zmu(4)=v7
c
      zsigma(1)=varx**.5
      zsigma(2)=xsigma(4)
      zsigma(3)=xsigma(3)
      zsigma(4)=xsigma(6)
      write (6,55)
      write (6,56)(zmu(i),zsigma(i),i=1,4)
      write (6,27) off
   27 format (/'E [offset] = ',f15.8)
   55 format (' zmu(i) is :',6x,'zsigma is :')
   56 format (e14.6,3x,e14.6)
c
c
c
c   define zrho(i,j)
      zrho(2,3)=xrho(3,4)
      zrho(3,4)=xrho(3,6)
      do 320 i=1,4
      do 320 j=1,4
      if (i.eq.j) go to 321
      zrho(j,i)=zrho(i,j)
      go to 320
  321 zrho(i,j)=1.
  320 continue
c
c
c compute cov[G wyl], cov[G wn], cov[G v7], E[G**2]
      do 350 i=1,4
      if (i.eq.1) go to 351
      call egr (xmu,xsigma,xrho,pgwb,zmu(i),epara)
      zvar(1,i)=epara*ph(1)*ph(i)
      go to 350
  351 zvar(1,i)=(zsigma(1)*ph(1))**2
  350 continue
c
c
      do 400 i=2,4
      do 400 j=2,4
      if (i.gt.j) go to 400
      zvar(i,j)=zrho(i,j)*zsigma(i)*zsigma(j)*ph(i)*ph(j)
  400 continue
      do 410 i=1,4
      do 410 j=1,4
      zvar(j,i)=zvar(i,j)
  410 continue
      write (6,28)
```

```
      write (6,29) (ph(i),i=1,4)
      write (6,30)
      write (6,31) ((zvar(i,j),j=1,4),i=1,4)
   28 format (/'ph(i) is :')
   29 format (e16.7)
   30 format (/'Z-variance matrix is :')
   31 format (4f15.6)
c
      varof=0.
      do 500 i=1,4
      do 500 j=1,4
      varof=varof+zvar(i,j)
  500 continue
c
      write (6,32) varof
   32 format (/'Var [offset] = ',f16.8)
c
c
c*****************************************************************
c
c
c
c  compute E [ED]
c    ED : ENERGY DIssipated
c    ED1= dissipate by spring
c    ED2= dissipate by damping
c
c*****************************************************************
c
c
c define meam value and standard deviation
      smu(1)=wn
      smu(2)=wy1
      smu(3)=v7
      smu(4)=pmr
c
      ssigma(1)=xsigma(3)
      ssigma(2)=xsigma(4)
      ssigma(3)=xsigma(6)
      ssigma(4)=varx**.5
c
      dmu(1)=a
      dmu(2)=alpha
      dmu(3)=wn
      dmu(4)=zita
      dmu(5)=off
c
      dsigma(1)=xsigma(1)
      dsigma(2)=xsigma(2)
      dsigma(3)=xsigma(3)
      dsigma(4)=xsigma(5)
      dsigma(5)=varof**.5
c
```

```
      call engl(smu,m7,edl)
      call eng2(dmu,nb,q,m7,ed2)
      write (6,60)
      write (6,61) (smu(i),dmu(i),i=1,4)
      write (6,62) dmu(5)
      write (6,33) edl,ed2
   60 format (' smu(i) is ',8x,'dmu(i) is ')
   61 format (2el5.6)
   62 format (15x,el5.6)
   33 format (/'E[Edl] is ',el5.6,'  E[Ed2] IS ',el3.6)
c
c
c
c
c
c  compute var [Ed]
      do 600 cn=1,4
      call pengl(cn,smu,m7,pedl)
      icn=cn
      pes(icn)=pedl
  600 continue
      write (6,34)
      write (6,35) (pes(i),i=1,4)
   34 format (/'pes(i) is :')
   35 format (el3.6)
c
c
c  define srho(i,j),j=1,3 i=1,3
c  where srho(i,j) is the coefficient of correlation
c  in spring energy dissipate case
c
      srho(1,2)=xrho(3,4)
      srho(1,3)=xrho(3,6)
      do 650 i=1,4
      do 650 j=1,4
      if (i.eq.j) go to 651
      srho(j,i)=srho(i,j)
      go to 650
  651 srho(i,j)=1.
  650 continue
c
c
      do 700 i=1,3
      do 700 j=1,3
      if (i.gt.j) go to 700
      svar(i,j)=srho(i,j)*ssigma(i)*ssigma(j)*pes(i)*pes(j)
  700 continue
c
      svar(4,4)=varx*pes(4)**2
c
      do 800 i=1,3
      call egr(xmu,xsigma,xrho,pgwb,smu(i),covhs)
      svar(i,4)=covhs*pes(i)*pes(4)
```

```
 800  continue
c
c
      do 900 i=1,5
      do 900 j=1,5
      svar(j,i)=svar(i,j)
 900  continue
      write (6,36)
      write (6,37)((svar(i,j),j=1,4),i=1,4)
  36  format (/'S-variance matrix is :')
  37  format (4e15.6)
c
      do 920 i=1,4
      do 920 j=1,4
      svar1=svar1+svar(i,j)
 920  continue
c
      write (6,38) svar1
  38  format (/'var [ED1] IS ',e13.6)
      go to 6000
 3000 WRITE (6,18)
 18   FORMAT (' NO YIELDING OCCUR')
      GO TO 6000
 3001 WRITE (6,19)
  19  FORMAT ( ' THIS CASE DOEN NOT PRACTICAL')
C
 6000 STOP
      END
c
c
c
      subroutine pg1(icn,xmu,ff,k7,a7,dt,nb,pmo,pg)
c  This subroutine compute dG/db, denote it as pg ;
c     Where b is random parameter.
c     cn = control number ;
c       if :
c     cn=1. : compute dG/dA
c       =2. : compute dG/dalpha
c       =3. : compute dG/dwn
c       =4. : compute dG/dwy1
c       =5. : compute dG/dzita
c       =6. : compute dG/dv7
c
      dimension ff(1024),xmu(6)
      real K7,M7,k71,k72
c
      a=xmu(1)
      alpha=xmu(2)
      wn=xmu(3)
      wy1=xmu(4)
      zita=xmu(5)
```

```
      v7=xmu(6)
c
c      read (5,*) error
      error=500.
c
      if (icn.eq.6) go to 6
      if (icn.eq.5) go to 5
      if (icn.eq.4) go to 4
      if (icn.eq.3) go to 3
      if (icn.eq.2) go to 2
      if (icn.eq.1) go to 1
      go to 4999
c
    1 m7=k7/(wn**2)
      fm=a*m7
      dfm=fm/error
      da=dfm/m7
      a1=a+da
      a2=a-da
      call g(a1,alpha,wn,wy1,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
c      write (6,22) thir
c
      g1=spmo
      call g(a2,alpha,wn,wy1,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
      g2=spmo
      pg=(g1-g2)/(2.*da)
      pmo=g1
      go to 5000
c
    2 dalpha=alpha/error
      alpha1=alpha+dalpha
      alpha2=alpha-dalpha
      call g(a,alpha1,wn,wy1,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
c      write (6,22) thir
c
      g1=spmo
      call g(a,alpha2,wn,wy1,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
      g2=spmo
      pg=(g1-g2)/(2.*dalpha)
      pmo=g1
      go to 5000
c
    3 dk7=k7/error
      k71=k7+dk7
      k72=k7-dk7
      m7=a7/(wy1**2)
      wn1=(k71/m7)**.5
      wn2=(k72/m7)**.5
      dwn=wn1-wn2
      call g(a,alpha,wn1,wy1,zita,v7,ff,k71,a7,dt,nb,spmo,thir)
c      write (6,22) thir
c
      g1=spmo
```

```
      call g(a,alpha,wn2,wyl,zita,v7,ff,k72,a7,dt,nb,spmo,thir)
      g2=spmo
      pg=(gl-g2)/(2.*dwn)
      pmo=gl
      go to 5000
c
4     da7=a7/error
      a71=a7+da7
      a72=a7-da7
      m7=k7/(wn**2)
      wyll=(a71/m7)**.5
      wyl2=(a72/m7)**.5
      dwyl=wyll-wyl2
      call g(a,alpha,wn,wyll,zita,v7,ff,k7,a71,dt,nb,spmo,thir)
c      write (6,22) thir
c
      gl=spmo
      call g(a,alpha,wn,wyl2,zita,v7,ff,k7,a72,dt,nb,spmo,thir)
      g2=spmo
      pg=(gl-g2)/(2.*dwyl)
      pmo=gl
      go to 5000
c
  5   dzita=zita/error
      zital=zita+dzita
      zita2=zita-dzita
      call g(a,alpha,wn,wyl,zital,v7,ff,k7,a7,dt,nb,spmo,thir)
c      write (6,22) thir
c
      gl=spmo
      call g(a,alpha,wn,wyl,zita2,v7,ff,k7,a7,dt,nb,spmo,thir)
      g2=spmo
      pg=(gl-g2)/(2.*dzita)
      pmo=gl
      go to 5000
c
  6   dv7=v7/error
      v71=v7+dv7
      v72=v7-dv7
      call g(a,alpha,wn,wyl,zita,v71,ff,k7,a7,dt,nb,spmo,thir)
c      write (6,22) thir
c
      gl=spmo
      call g(a,alpha,wn,wyl,zita,v72,ff,k7,a7,dt,nb,spmo,thir)
      g2=spmo
      pg=(gl-g2)/(2.*dv7)
      pmo=gl
      go to 5000
c
   22 format ('thita ratio is ',f13.7)
 4999 write (6,19)
   19 format ('Control Number Wrong')
c
```

```
 5000 return
      end
c
c
      subroutine g(a,alpha,wn,wyl,zita,v7,ff,k7,a7,dt,nb,pmr,thir)
C  This subroutine is to  predict the max offset for blast load
C  pmo=predicted max response
c   pmr=100:  represent jmm expired (in do loop)
c   pmr=101:  represent something trouble in finding tmax
c
      dimension ff(1024),VV(1024),OFFSET(1024),SLOPE(30)
      REAL K7,M7
      WD=((1.-ZITA**2)**.5)*WN
      m7=k7/(wn**2)
      c7=2.*zita*wn*m7
      V=-ZITA*WN
      F=ALPHA+V
C
C
      wy=(wyl**2-(zita*wn)**2)**.5
      D=(WN**2-WY1**2)*V7
C
c
      call sbilin (ff,c7,m7,k7,a7,v7,dt,nb,vv,offset,ened)
c
C
C  CALCULATE T7
      B=2.*ALPHA/(WD**2+ALPHA**2)
      C=2.*V7/A
      T7=(B+(B**2+4.*C)**.5)/2.
      T71=T7
c      WRITE (6,1) T7
c  1   FORMAT ('THE TEMPERATORY   T7 = ',F11.7)
c
C  CALCULATE EXACT  T7
      DO 700 JII=1,2000
      XX=JII*.00001
      T7=T71+XX
      X1=((ALPHA-ZITA*WN)**2+WD**2)*WD
      X2=A/X1
      X3=EXP (-ZITA*WN*T7)
      X4=(ALPHA-ZITA*WN)*SIN(WD*T7)
      X5=WD*COS(WD*T7)
      X6=WD*EXP(-ALPHA*T7)
C
C  CHECK Z AT T7 IF IT IS EQUAL TO V7
      ZT7=X2*(X3*(X4-X5)+X6)
      ERR1=ABS(1.-ZT7/V7)
      IF (ERR1.LE.0.0005) GO TO 701
  700 CONTINUE
c
c compute the z'(t7)
  701 X7=WD*(ALPHA-ZITA*WN)*COS(WD*T7)
```

```
      X8=WD**2*SIN(WD*T7)
      X9=-ZITA*WN*X3*(X4-X5)
      ZDOT7=X2*(X9+X3*(X7+X8)-ALPHA*X6)
C
c
c calculate tmax (using approached method)
      rx=2.*zita*wn
      g1=a*(1.-exp(-rx*t7))
      g2=(wn**2)*v7*(exp(-rx*t7)-1.)
      gx=-(g1+g2)+zdot7*rx
c
      h1=a*exp(alpha*t7)
      h2=a*rx/(alpha-rx)+(wn**2)*v7
      h3=a*alpha*exp(t7*(alpha-rx))/(alpha-rx)
      h4=gx*exp(rx*t7)
      h5=h1-h3
      h6=h2+h4
c
      p1=h5*alpha**2+h6*(rx**2)
      p2=h5*alpha+h6*rx
      p3=h5+h6-(wn**2)*v7
c
      ux=p2**2-4.*p1*p3
      if(ux.le.0.) go to 2700
      uu=ux**.5
      tmax1=(p2+uu)/(2.*p1)
      tmax2=(p2-uu)/(2.*p1)
      if (tmax1.le.0.17.and.tmax1.ge.0.) go to 70
      tmax=tmax2
      go to 71
  70  tmax=tmax1
  71  tmaxt=tmax
c
c
c  compute c1 and c2
      q1=-d/(wy1**2)
c
      qx1=alpha**2
      qx2=-2.*zita*wn*alpha
      qx3=wy1**2
      q2=a/(qx1+qx2+qx3)
c
      xa1=exp(-zita*wn*t7)
      xa2=-zita*wn*cos(wy*t7)
      xa3=-wy*sin(wy*t7)
      xat7=xa1*(xa2+xa3)
c
      xb1=-zita*wn*sin(wy*t7)
      xb2=wy*cos(wy*t7)
      xbt7=xa1*(xb1+xb2)
c
      xct7=-alpha*q2*exp(-alpha*t7)
c
```

```
      xdt7=xa1*cos(wy*t7)
c
      xet7=xa1*sin(wy*t7)
c
      xft7=q1+q2*exp(-alpha*t7)
c
      xgt7=v7-xft7
c
      xht7=zdot7-xct7
c
      deno=xat7*xet7-xdt7*xbt7
      c1=(xht7*xet7-xbt7*xgt7)/deno
      c2=(xat7*xgt7-xdt7*xht7)/deno
c
c compute exact tmax, since z'=0 when t=tmax
      tmx=tmaxt
      dtx=tmaxt
  880 tx=dtx/30.
      do 900 jmm=1,30
      tmx=tmx-tx
      xa1=exp(-zita*wn*tmx)
      xa2=-zita*wn*cos(wy*tmx)
      xa3=-wy*sin(wy*tmx)
      xatmx=xa1*(xa2+xa3)
c
c
      xb1=-zita*wn*sin(wy*tmx)
      xb2=wy*cos(wy*tmx)
      xbtmx=xa1*(xb1+xb2)
c
      xctmx=-alpha*q2*exp(-alpha*tmx)
c
      zdtmx=xatmx*c1+xbtmx*c2+xctmx
      slope(jmm)=zdtmx
      if (jmm.eq.1.and.slope(jmm).gt.0.) go to 899
      if (jmm.eq.1) go to 900
      prod=slope(jmm)*slope(jmm-1)
      if (prod.le.0.) go to 902
  900 continue
      write (6,10)
  10  format ('something WRONG in t-max[subG]')
  902 if (slope(jmm-1).gt.-.0001) go to 901
      dtx=dtx/30.
      tmx=tmx+tx
      go to 880
c
  899 do 920 jmm=1,1000
      tx=(jmm-1)*0.0001
      tmx=tmaxt+tx
      xa1=exp(-zita*wn*tmx)
      xa2=-zita*wn*cos(wy*tmx)
      xa3=-wy*sin(wy*tmx)
```

```
      xatmx=xa1*(xa2+xa3)
c
c
      xb1=-zita*wn*sin(wy*tmx)
      xb2=wy*cos(wy*tmx)
      xbtmx=xa1*(xb1+xb2)
c
      xctmx=-alpha*q2*exp(-alpha*tmx)
c
      zdtmx=xatmx*c1+xbtmx*c2+xctmx
      if (zdtmx.le.0.01) go to 901
 920  continue
      go to 2100
c
c  compute exact z(at exact tmax)
 901  xdtmx=xa1*cos(wy*tmx)
      xetmx=xa1*sin(wy*tmx)
      xftmx=q1+q2*exp(-alpha*tmx)
      ztmx=xdtmx*c1+xetmx*c2+xftmx
c
c calculate predicted max-offset
      pmo=(ztmx-v7)*(1.-a7/k7)
      thir=pmo/offset(nb)
      pmr=ztmx
c
      go to 3000
c
 2100 write (6,98)
   98 format ('fmm expire')
      pmr=100.
      go to 3000
 2700 write (6,99)
   99 format ('U less than zero')
      pmr=101.
3000  return
      END
C
      SUBROUTINE SBILIN(F,C7,M7,K7,A7,V7,D9,N,V,V0,ened)
      DIMENSION F(1024),V0(1024),V(1024)
      DIMENSION V1(1024),V2(1024)
      REAL M7,K7,K8,K9
      U7=K7*V7
      K9=1.-A7/K7
C
C INITIALIZE VARIABLES
      V(1)=0.
      V0(1)=0.
      V1(1)=0.
      V2(1)=F(1)/M7
C  START THE RESPONSE CYCLE
      Q1=6.*M7/D9**2
      Q2=3.*C7/D9
      Q3=6.*M7/D9
```

```
      Q4=3.*M7
      Q5=3.*C7
      Q6=0.5*D9*C7
      Q7=3./D9
      Q8=3.
      Q9=.5*D9
      K8=K7
      NM=N-1
      ENED=0.
      DO 1199 I=1,NM
      I1=I+1
      U1=Q1+Q2+K8
      U2=Q3*V1(I)+Q4*V2(I)
      U3=Q5*V1(I)+Q6*V2(I)
      V5=(F(I1)-F(I)+U2+U3)/U1
      V6=Q7*V5-Q8*V1(I)-Q9*V2(I)
      V(I1)=V(I)+V5
      V1(I1)=V1(I)+V6
C COMPUTE THE STIFFNESS AT T+DT
      X0=K7*(V(I1)-V0(I))
      X1=A7*(V(I1)-V7)+U7
      X2=A7*(V(I1)+V7)-U7
      IF (X0.GT.X1) GO TO 1150
      IF (X0.LT.X2) GO TO 1160
      K8=K7
      V0(I1)=V0(I)
      GO TO 1170
 1150 IF (V1(I1).GT.0.) K8=A7
      IF (V1(I1).LE.0.) K8=K7
      V0(I1)=(V(I1)-V7)*K9
      GO TO 1170
 1160 IF (V1(I1).LT.0.) K8=A7
      IF (V1(I1).GE.0.) K8=K7
      V0(I1)=(V(I1)+V7)*K9
 1170 V2(I1)=(F(I1)-C7*V1(I1)-K7*(V(I1)-V0(I1)))/M7
      ened=ened+d9*.5*(v1(i)*(f(i)-m7*v2(i))+
     c                v1(i1)*(f(i1)-m7*v2(i1)))
 1199 CONTINUE
      RETURN
      END
c
c
      subroutine fft (a,n,nb,isgn,dt)
      complex a(nb),u,w,t
c
c     dividing all element by nb
      do 1 j=1,nb
    1 a(j)=a(j)/nb
c
c     reorder sequencr
      nbd2=nb/2
      nbm1=nb-1
      j=1
```

```
      do 4 l=1,nbml
      if (l.ge.j) go to 2
      t=a(j)
      a(j)=a(l)
      a(l)=t
    2 k=nbd2
    3 if (k.ge.j) go to 4
      j=j-k
      k=k/2
      go to 3
    4 j=j+k
c     calculate fft
      pi=3.1415926
      do 6 m=1,n
      u=(1.0,0.0)
      ME=2**M
      k=me/2
      w=cmplx(cos(pi/k),isgn*sin(pi/k))
      do 6 j=1,k
      do 5 l=j,nb,me
      lpk=l+k
      t=a(lpk)*u
      a(lpk)=a(l)-t
    5 a(L)=a(l)+t
    6 u=u*w
c
      tt=(nb-1)*dt
c
      if (isgn.eq.1) go to 99
      do 110 i=1,nb
      a(i)=a(i)*tt
  110 continue
      go to 100
c
   99 do 120 i=1,nb
      a(i)=a(i)/dt
  120 continue
  100 return
      end
c
c
c
      subroutine engl (smu,m7,endl)
c    this subroutine compute the E [ED1]
      dimension smu(4)
      real k7,m7
      k7=(smu(1)**2)*m7
      a7=(smu(2)**2)*m7
      v7=smu(3)
      zmax=smu(4)
      thita=(1.-a7/k7)*(zmax-v7)
c
      xl=.5*k7*v7**2
```

```
      x2=a7*(zmax-v7)**2/2.
      x3=k7*v7*(zmax-v7)
      x4=k7*(zmax-thita)**2/2.
c
      end1=x1+x2+x3+x4
      return
      end
c
c
c
      subroutine eng2 (dmu,nb,q,m7,ed2)
c  this subroutine compute E [ED2]
      dimension dmu(5),f1(1024),f2(1024),w(1024),deno(512)
      complex ff1(1024),ff2(1024),ff(512),hh(512)
      real m7
c
      a=dmu(1)
      alpha=dmu(2)
      wn=dmu(3)
      zita=dmu(4)
      thita=dmu(5)
      p=thita*2./3.1415926
c
      dt=0.01
      tt=(nb-1)*dt
c
      do 100 i=1,1024
      tx=(i-1)*dt
      f1(i)=a*exp(-alpha*tx)
      f2(i)=wn**2*p*atan(q*tx**4)
      ff1(i)=cmplx (f1(i),0.)
      ff2(i)=cmplx (f2(i),0.)
      w(i)=(i-1)*2.*3.1415926/tt
  100 continue
c
      call fft(ff1,10,nb,-1,dt)
      call fft(ff2,10,nb,-1,dt)
c
      nb2=nb/2
      do 200 i=1,nb2
      ff(i)=ff1(i)+ff2(i)
      re=wn**2-w(i)**2
      xim=-2.*zita*wn*w(i)
      deno(i)=re**2+xim**2
      hh(i)=cmplx (re,xim)/deno(i)
  200 continue
c
      end2=0.
      do 400 i=1,nb2
      xe1=cabs(hh(i))
      xe2=cabs(ff(i))
      ed2=ed2+(xe1*xe2*w(i))**2
  400 continue
```

```
      ed2=2.*ed2
      return
      end
c
c
c
      subroutine pengl(cn,smu,m7,pedl)
c   this subroutine compute d ED1/d bata
c     cn =1 d ED1/d wn
c     cn =2 d ED1/d wyl
c     cn =3 d ED1/dv7
c     cn =4 d ED1/d zmax
c
      dimension smu(4)
      real m7
      wn=smu(1)
      wyl=smu(2)
      v7=smu(3)
      zmax=smu(4)
      const=(zmax-v7)*wyl**2/wn+v7*wn
c
      if (cn.eq.1.) go to 100
      if (cn.eq.2.) go to 200
      if (cn.eq.3.) go to 300
      if (cn.eq.4.) go to 400
      go to 600
c
 100  const1=const*(v7-(zmax-v7)*(wyl/wn)**2)
      pedl=(wn*v7**2+2.*wn*v7*(zmax-v7)-const1)*m7
      go to 700
 200  pedl=(wyl*(zmax-v7)**2-2.*(zmax-v7)*wyl*const/wn)*m7
      go to 700
 300  const2=const*(wn**2-wyl**2)/wn
      pedl=(zmax-v7)*(wn**2-wyl**2)-const2
      go to 700
 400  pedl=((zmax-v7)*wyl**2+v7*wn**2-const*wyl**2/wn)*m7
      go to 700
 600  write (6,1)
  1   format (' control number wrong')
 700  return
      end
c
c
c
c
c
c
      subroutine egr (xmu,xsigma,xrho,pgwb,gamma,epara)
c  this subroutine compute E [R,gamma]
c  where R=zmax
c  xmu: mean value of bata
c                        where bata see main program
```

```
c  pgwb: d g/d bata(i)
c
      dimension xmu(6),xsigma(6),xrho(6,6),pgwb(6)
c
      do 100 i=1,6
      if (xmu(i).eq.gamma) go to 101
 100  continue
      write (6,1)
 1    format ('gamma is not equal to xmu(i) in subroutine egr,WRONG!!')
 101  nj=i
c
      epara=0.
      do 200 j=1,6
      ex1=xrho(nj,j)*xsigma(nj)*xsigma(j)
      ex2=(ex1)*pgwb(j)
      epara=epara+ex2
 200  continue
      return
      end
c
c
c
      subroutine search (thita,p,a,nb,dt,epsiln)
      dimension thita(nb)
      sum1=0.
      do 100 i=1,nb
      tx=(i-1)*dt
      ty=p*atan(a*(tx**4))
      sum1=sum1+(thita(i)-ty)**2
 100  continue
      epsiln=sum1
      return
      end
```

END

FILMED

5-85

DTIC