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RAYLEIGH CRITERIA RESOLUTION OF OPTICAL CORRELATIONS

Don A. Gregory
Research Directorate
US Army Missile Laboratory

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Redstone Arsenal, Alabama 35898

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This report describes an investigation into determining the resolvability between correlations produced by Fourier transform matched filters made of very similar scenes. Knowledge of this type is needed to accurately predict the total number of matched filters needed to completely identify a scene or object regardless of its rotational position. Similar experiments may be done for scale and tilt as the variable. This report introduces the Rayleigh criteria as the standard by which distinguishability may be measured in a Vander Lugt type optical recognition system.
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I. INTRODUCTION

Considerable interest has been generated in using Fourier transform matched filters for object discrimination and tracking. In order to recognize several objects or several orientations of the same object many matched filters may be needed. Interest has been shown in storing several of these filters in a small area [1,2]. This has been done by multiply exposing the photographic plate or by using arrays produced by holographic elements or other means [3,4]. Arguments are presented here which investigate how closely packed these filters can be and still have good resolution between them. In this instance close packing refers to double-exposing a photographic plate to an input scene at two slightly different rotation positions. Classic line-shapes are used to model examples of experimental results and some conclusions are drawn using the Rayleigh criteria as a guideline.
The Gaussian intensity distribution has the form

\[ I_1 = I_0 \, e^{-2\frac{r^2}{w^2}} \]  

(1)

where \( r \) is the radius from the center of the distribution and \( 2w \) is the full line width at \( I = (1/e^2)I_0 \). If another distribution of the same form is located a small distance \( \delta_1 \) away

\[ I_2 = I_0 \, e^{-2\frac{(r-\delta_1)^2}{w^2}} \]  

(2)

then the sum of these distributions is

\[ I = I_1 + I_2 = I_0 \left[ e^{-2\frac{r^2}{w^2}} + e^{-2\frac{(r-\delta_1)^2}{w^2}} \right] \]  

(3)

Let

\[ \rho_1 = \frac{I(r=0)}{I(r=\frac{\delta_1}{2})} \]  

(4)

Substituting from the above gives

\[ \rho_1 = \frac{1}{2} \left( e^{\frac{\delta_1^2}{2w^2}} + e^{-3\frac{\delta_1^2}{2w^2}} \right) \]  

(5)

\( \rho_1 \) is thus the ratio of the peak at \( r = 0 \) to the height of the valley between the two distributions. Solving for \( \delta_1 \) using the fact that the second term is much smaller than the first gives

\[ \delta_1 = \frac{w}{2 \ln (2\rho)} \]  

(6)
Using the Rayleigh criteria as a rough guideline, \( \rho_1 = 1.23 \) and substituting this into Eq. (6) yields

\[
\delta_1 = 1.34 \, w .
\]

(7)

The same sort of analysis may be applied to a Lorentzian distribution having the form

\[
I = \frac{A}{r^2 + (\sigma/2)^2} + \frac{A}{(r-\delta_2)^2 + (\sigma/2)^2}
\]

(8)

where \( A \) is a constant equal to \( I_o(\sigma/2)^2 \), \( r \) is the radius and \( \sigma \) the full linewidth at half the maximum intensity. This produces a \( \delta_2 \) of

\[
\delta_2 = \frac{\sigma}{2} \left[ 4 \rho_2 - 3 + \sqrt{16 \rho_2 (\rho_2 - 1) + 1} \right]^{1/2}
\]

(9)

Then for \( \rho_2 = 1.23 \) as a value of the intensity ratio

\[
\delta_2 = 1.03 \, \sigma.
\]

(10)

For a \( \text{sinc}^2 \) distribution

\[
I = I_o \left( \frac{\sin \alpha r}{\alpha r} \right)^2 + I_o \left[ \frac{\sin \alpha (r-\delta_3)}{\alpha (r-\delta_3)} \right]^2
\]

(11)

where again \( r \) is the radius from the center of the distribution and

\[
\alpha = \pi/r(o)
\]

(12)

where \( r(o) \) is the value of \( r \) at the first zero of intensity. This distribution yields

\[
\rho_3 = \frac{\alpha^2 \delta_3^2 + \sin^2 \delta_3}{8 \sin^2 (\alpha \delta_3/2)}
\]

(13)

which must be solved graphically or by iteration using \( \rho_3 = 1.23 \). For small values of \( \delta_3 \), note that \( \alpha^2 \delta_3^2 > > \sin^2 \delta_3 \).
III. EXPERIMENTAL RESULTS

Figure 1 shows three distributions compared with experimental data taken using visible (Helium-Neon) real-time optical correlation methods. The input scene is fed via an RCA television camera (model number TC 1005) into a Videotek Monitor (model VM-12PR) which is used as the input to a Hughes liquid crystal light valve (the same one used in Ref. 4) which produces the coherent image that is fed into a real-time Vander Lugt type correlator. Experiments of this type are described in detail elsewhere [5]. All three distributions have been arbitrarily fit to experimental data at \( r = 0 \) and \( r = 2.94 \) using \( w = 2.80^\circ \), \( \sigma = 2.07^\circ \), and \( \alpha = 0.775 \) deg\(^{-1}\). This gives \( \delta_1 = 3.8^\circ \), \( \delta_2 = 2.13^\circ \), and \( \delta_3 = 4.0^\circ \). Note that these widths should depend upon the spatial frequency of the input scene as well as other factors and thus serve only as examples here. In this particular experiment the input scene was a low resolution black and white aerial photograph of Huntsville, Alabama. The photograph was placed on a rotatable stage and matched filters made at the desired rotation angles. Experimental data in Figure 1 is the correlation intensity for a filter made at \( 0^\circ \) rotation of the input scene. After development of the film plate (Kodak 649F) it was replaced in the correlator and the input scene rotated slowly as the corresponding correlation intensity was measured using an RCA CCD camera fed into a television monitor then digitized by a Colorado Video Analyzer (model 321). This signal was then recorded by a Hewlett-Packard model 680 strip chart recorder. Figure 2 is representative of actual data taken as the input scene was rotated through the position \( (0^\circ \text{ rotation}) \) where the matched filter was made.

In order to investigate multiply stored filters, a single plate was exposed twice with different rotation angles of the input scene. After development using standard techniques for Kodak 649F plates, the filter was reinserted into the correlator and data taken as before. Figure 3 shows the actual data and a plot of Equation 8 using the Rayleigh criteria. The exposures were made using the same criteria for \( \delta_2 \) of \( 2.13^\circ \) of scene rotation. The agreement is reasonably good. Some variation is expected due to the difficulty in obtaining equally intense correlations for the two filters. This depends upon the exposure times used, film response, and several other variables. In the theoretical analysis the calculations could easily have been done using distributions having different peak heights but then the value of \( \delta \) would depend upon these heights which can't be known with any certainty in advance of doing the experiment. In the calculations presented, it was also assumed that individual correlation intensities should be added directly rather than adding the corresponding fields then squaring for the intensity. This has been justified experimentally by storing one filter made of \( 0^\circ \) scene rotation and another filter at \( 90^\circ \) scene rotation then adding the resulting correlation intensities algebraically. This distribution was then compared with data similar to Figure 3. The results were found to be essentially equal. This indicates that there is little if any phase addition contribution to the sum of the two correlation intensities.
IV. CONCLUSIONS

In this brief report a method of analyzing closely spaced optical matched filters has been presented along with experimental data which tends to support the findings. The use of the Rayleigh criteria may not be strictly correct in that this criteria is for a Bessel function intensity distribution only but it serves as an initial guideline [6]. Experimental data seems to suggest that the separation criteria will become more stringent as laboratory methods improve.
Figure 1. Correlation intensity vs angle of rotation of input scene

Gaussian
Lorentzian
Sinc²
Experimental
Figure 2. Correlation intensity as input scene was rotated through 0° where matched filter was made.
Figure 3. Plot of Eq. (8) compared with actual data for two filters stored 2.11° of rotation apart.

INTENSITY (ARBITRARY UNITS)

SCENE ROTATION (DEGREES)
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Professor Anil K. Jain
Department of Electrical Engineering
University of California, Davis
Davis, CA 95616

Terry Turpin
Department of Defense
9800 Savage Road
Fort George G. Meade, MD 20755

Dr. Stuart A. Collins
Electrical Engineering Department
Ohio State University
1320 Kenneair Road
Columbus, OH 43212

US Army Materiel Systems Analysis Activity
ATTN: AMXS−MP
Aberdeen Proving Ground, MD 21005

US Army Night Vision Laboratory
ATTN: DELNV−L, Dr. R. Buser
Ft. Belvoir, VA 22060

Dr. F. T. S. Yu
Penn State University
Department of Electrical Engineering
University Park, PA 16802

Dr. William P. Bleha
Liquid Crystal Light Valve Devices
Hughes Aircraft Company
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Dr. J. G. Castle
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ATTN: Document Control
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Arlington, VA 22209

Dr. J. W. Goodman
Information Systms Laboratory
Department of Electrical Engineering
Stanford University
Stanford, CA 04305

Eric G. Johnson, Jr.
National Bureau of Standards
325 S. Broadway
Boulder, CO 80302

Dr. nicholas George
The Institute of Optics
University of Rochester
Rochester, NY 14627

Naval Avionics Facility
Indianapolis, IN 46218

Dr. David Cassasent
Carnegie Mellon University
Hamerschage Hall, Room 106
Pittsburg, PA 15213