MICROCOPY RESOLUTION TEST CHART
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This report documents the results of three years of operation of the Reliability Center at Florida State University. During this period 45 technical reports and 43 papers in journals were published on statistical aspects of reliability. In addition, 15 visiting researchers were supported at the Center.
Highlights of Research
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Statistical Aspects of Reliability, Maintainability, and Availability.

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1. **Piecewise Exponential Estimator (PEXE)**

We wish to estimate an unknown life distribution $F$ from censored data. More specifically, let censored observation $X_i = \min(Y_i, Z_i)$; $\delta_i = 1$ if $Y_i \leq Z_i$, 0 if $Y_i > Z_i$. (We assume throughout $i = 1, \ldots, n$.) $\delta_i$ indicates whether the observation is a lifelength (observation from $F$) or a censored value (observation from unknown censoring distribution $G$).

The standard method of estimating from censored data is the Kaplan-Meier estimator (KME) obtained by maximizing likelihood. We have shown that our PEXE estimator is superior to the KME.

1. The PEXE provides a continuous estimator of $F$ and a step function estimator of $r$, the failure rate function. The KME provides a step function estimator of $F$ and no estimator of $r$. Most applied people would feel more comfortable with a continuous $F$ estimator.

2. The PEXE provides a simple rule for estimating $F(t)$ for $t > \max(X_1, \ldots, X_n)$; the KME does not provide any rule.

The two estimators are asymptotically alike in their behavior. For finite sample size, they interlace, showing that they never deviate very much from each other.

In the near future, we plan to investigate the piecewise geometric estimator (PEGE) for estimating discrete life distributions. The PEGE results should correspond closely to those obtained for the PEXE.

2. **Negative Association.**

One of the extremely useful concepts developed for measuring "relatedness" is that of (positive) association. Developed originally for use in reliability, it soon found application in many other areas, as described in a very large number of papers on association and its extensions.
The definition of negative association differs from that of positive association in two respects.

Definition. \( X = (X_1, X_2, \ldots, X_n) \) is negatively associated if \( \text{Cov}[f(X^{(1)}), g(X^{(2)})] \leq 0 \) for all nondecreasing functions \( f, g \), and mutually exclusive subsets \( X^{(1)}, X^{(2)} \subseteq X \).

1. The pairs of subsets \( X^{(1)}, X^{(2)} \) are mutually exclusive.
2. The covariance is \( \leq 0 \) rather than \( \geq 0 \).

We expect that the concept of negative association will have almost as many applications as that of positive association and will elicit a good deal of further research. Our initial paper [58] on the subject gives a good start in this direction.

3. Influence of Each Factor of the Bayesian Model on Other Factors.

In [51] we show that in the Bayesian model the larger the sample observations are stochastically, then the larger stochastically is the posterior distribution.

This type of result can be extended under appropriate conditions to other pairs of factors in the Bayesian model: the choice of prior and the resulting posterior; the sample and the likelihood function; etc.

The resulting relationships should be of considerable value in robustness studies, design of experiments, predicting the cost and accuracy of competing experiments, etc.

4. Percentile Residual Life Functions.

The mean residual life function has been studied theoretically and applied in practice. A closely related concept, the percentile residual life function (PRLF), received very little attention until the recent papers of Joe and Proschan [69], [70], [78] and [80].
Definition. The 100α percentile residual life function αₜ of a continuous life distribution F satisfies:

$$F(t + 100\alpha t) = 100\alpha t \quad \text{for} \quad -\infty < t < \infty \quad \text{and} \quad 0 < \alpha < 1.$$ 

Joe and Proschan first study the basic probabilistic properties of the PRLF. Then they develop tests for the properties of PRLF's. They also produce estimators of the PRLF's. Finally they show how to compare two life distributions on the basis of their PRLF's.

We expect that the approach based on PRLF's will prove popular and useful in the analysis of data from life distributions now that some of the basic technology has been developed. We also expect further theoretical developments in this area.


In the various selection problems treated and solved in the past, it has been standard practice to demonstrate by individual and generally complicated methods that the probability of correct selection (PCS) is a monotone function of the various factors affecting PCS. By the use of the recently developed decreasing in transposition (DT) inequality theory, Berger and Proschan developed a unified method for showing the monotonicity of PCS; moreover, the proofs involved were very much more simple, requiring only 1 or 2 lines to obtain results of much greater power and generality than the previous proofs in the literature. See [87].

6. Imperfect Maintenance.

In maintenance models, it is generally assumed that the maintenance actions required are performed as scheduled. Unfortunately, in actual practice, maintenance performance is imperfect due to human error.
Probabilistic models are now being constructed specifically incorporating the effects of human error. The solutions of these models are more realistic and approximate the actual outcome in practice more closely than previous models which naively assumed no human error.

See [75] for a detailed mathematical analysis in the imperfect repair model. See [59] for a description (without solution) of other imperfect maintenance models.

7. Exact Results for Survival Function Estimators.

Consider the randomly censored model where we observe \( \xi_i = \min(X_i, Y_i) \), where \( X_i \) is the time to failure of the \( i \)th item, \( Y_i \) is the time to censorship, and \( \xi_i \) is 1 if we observe a failure time, 0 if we observe a censored time. Assuming \( X \) has distribution \( F \), \( Y \) has distribution \( G \), and \( X, Y \) independent, the problem is to estimate \( \hat{F}(t) = P(X > t) \) when \( F, G \) are completely unknown. Related problems are to estimate the mean time or median time to failure, in the presence of censoring. A central estimator in these problems is the Kaplan-Meier estimator (KME) which can be expressed as

\[
\hat{F}_n(t) = \prod_{i=1}^{nK_n(t)} C_{in} \delta(i) I\{Z(n) > t\}
\]

where \( K_n(t) \) is the empirical distribution of the \( Z \)'s, \( C_{in} = (n - i)(n - i + 1)^{-1} \), \( Z(1) < \ldots < Z(n) \) are the ordered \( Z \)'s and \( \delta(i) \) is the \( \delta \) corresponding to \( Z(i) \). Virtually all research in the past has used the asymptotic distribution of \( n^{1/2} \hat{F}_n(t) \) to study properties of the estimator. Chen, Hollander, and Langberg [48] derived exact expressions for the moments of \( \hat{F}_n(t) \) in the proportional hazards model where \( \delta = F^B \) for \( B > 0 \). This enables us to study, under proportional hazards, the bias of the KME and to compare the exact variance of the KME to its asymptotic variance. Current research uses the methods of [48] to study exact (i.e., for finite \( n \)) properties of competing estimators of \( F \) and also estimators
of functions of $F$, such as $\mu(F) = \int F(x) \, dx$. These results quantify the deleterious effects of censoring on the estimators. The techniques of [48] are also being used by other authors to study exact properties of procedures for censored data.

8. **Nonparametric Bayesian Estimation.**

Ferguson's Dirichlet process prior enables the incorporation of prior information into the estimation of an unknown distribution $F$. If one knows a priori that the distribution being estimated possesses specific structure (e.g., symmetric about a point, or exchangeable in its coordinates), then Ferguson's Bayes estimator of $F$ is no longer Bayes in this "structured" model, and one can turn to Dalal's $G$-invariant Dirichlet to develop a Bayes estimator in the structured model. Hannum and Hollander [66] derived an explicit expression for the Bayes risk (using weighted squared error loss) of Dalal's Bayes estimator of a symmetric distribution under a $G$-invariant Dirichlet process prior. They assess the savings in risk attained by incorporating known symmetry structure in the model and provide information about the robustness of Ferguson's estimator against a prior for which it is not Bayes.

9. **Tests for Nonparametric Classes of Life Distributions.**

Most tests developed for the IFR, IFRA, NBU, NBUE classes have been restricted to the uncensored case. Chen, Hollander, and Langberg [57] developed a test for the NBU class when the data are censored. The NBU class arises naturally in studies of replacement policies, multistate systems, and shock models and it is important to have a means of assessing whether a distribution is NBU or not. Chen, Hollander, and Langberg [61] also devised a test for decreasing mean residual life alternatives (DMRL) for the censored-data case.
We have also introduced new classes of life distributions and corresponding tests for both uncensored and censored data models. Guess, Hollander, and Proschan [33] introduced the "increasing initially, then decreasing mean residual life" (IDMRL) class of distributions and its dual, the "decreasing initially, then increasing mean residual life" (DIMRL) class of distributions, and showed how these classes can be used for model building. Hollander, Park, and Proschan [26], [89] introduce the "new better than used at age t_0" (NBU-t_0) class and its dual the "new worse than used at age t_0" (NWU-t_0) class. Here t_0 is specified. These classes and their corresponding tests can be used, for example, to determine from operational data whether an airplane engine after t_0 hours of flight is stochastically as good as a new engine.
Basic Research - Technical Reports and Published Papers.

REPORTS.


44. AFOSR 84-171  Wai Chan, Frank Proschan and J. Sethuraman. "Schur-Ostrowski Theorems for Functionals on L₁(0,1).", August 1984.


PAPERS.


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