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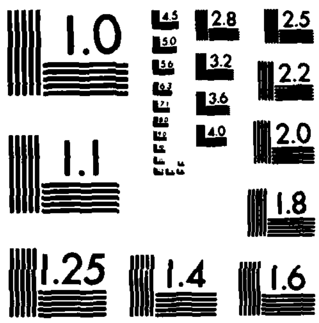
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A Fast Ray Tracing Routine for Laterally Inhomogeneous Media

by

Paul Docherty

Partially supported by the Consortium Project of the Center for Wave Phenomena and by the Selected Research Opportunities Program of the Office of Naval Research

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ABSTRACT

This document ~~We~~ describes ~~here~~ the use of continuation or homotopy procedures in an efficient algorithm for the calculation of raypaths from points within a region at depth to receivers on the surface. Our work follows closely that of H. B. Keller and his students D. J. Perozzi [1983] and J. A. Fawcett [1983]. Polynomials define the interfaces in a two-dimensional piecewise constant velocity medium. Starting with horizontal layering only, the interfaces are gradually deformed until the desired earth model is achieved. At each deformation step the ray equations determined by Fermat's Principle are solved using Newton's method and the ray from the first source position to a receiver vertically above it is found.

Next ~~we~~ the author employs source continuation, moving the source on a grid within the region of interest. At each source position ~~we~~ he finds the rays to all receivers using continuation in receiver location.

Knowing the raypaths, it is straightforward to construct a table of traveltimes. These traveltimes alone serve to position reflectors in the subsurface.

We ~~give~~ two examples of migration and indicate possible applications to forward modeling, i.e. given. Additional keywords: Numerical methods and procedures, Fortran, computer programs, charts.

1. INTRODUCTION

Structural inversion or migration is possible if traveltimes between subsurface points and surface receivers are known. Indeed, traveltimes alone serve to position reflectors, whereas the strength of a reflection contains information concerning the physical properties of the medium. The calculation of traveltime is straightforward if the sound path between source and receiver is known. To perform a migration many such sound paths must be found and in the past this has proved too expensive to make the method practical. We present a high speed ray tracing routine at the heart of a very accurate migration procedure.

Our ray tracing routine is based upon continuation or homotopy procedures. The method was developed by H. B. Keller and two of his students, D. J. Perozzi [1983] and J. A. Fawcett [1983]. It uses a known ray solution to provide a guess at the solution for a nearby raypath. With migration in mind, we have extended their ideas and created an algorithm suitable for finding large numbers of raypaths and traveltimes.

Some prior knowledge of the subsurface is assumed and the user provides a model of known interfaces and interval velocities. A region of interest is chosen and we consider every point within the region as a possible point source. Ray tracing begins between these points at depth and receivers on the surface.

The first ray is found in a stratified medium, where the depth to an interface is some average depth in the true medium. The interfaces are then gradually *deformed* until they become those in the true model. At each deformation step we solve a system of equations determined by Fermat's Principle. Next we use a continuation procedure for the source location to move the source within the region of interest. At each source position we find raypaths to all receivers using continuation in receiver location.

For each raypath a traveltime is calculated and output. Thus, for each source position, there is a list of traveltimes to receivers. A migrated depth section is produced when common depth point data is summed over curves determined by these traveltimes.

We give two examples of migration of synthetic data. They show that the method positions reflectors accurately, though we make no claims regarding the amplitude of the output. Our ray tracing procedure is also a tool for forward modeling. We indicate several possible applications.

2. FORMULATION OF THE NUMERICAL METHOD

The medium is comprised of constant velocity layers separated by arbitrary interfaces. Each interface is characterized by a polynomial, f, and a constant, c, in an equation of the form

$$z = c_k + \lambda f_k(x) \quad . \tag{1}$$

The significance of the parameter λ will become apparent later. We seek the ray joining a source at depth and a receiver on the surface. The ray is a series of straight line segments completely defined by its points of intersection with each interface, given some source and receiver.

Let the number of interfaces between source and receiver be N. Let the coordinates of the point of intersection of the ray with the k_{th} interface be $\underline{X}_k = (x_k, z_k) = (x_k, c_k + \lambda f_k(x_k))$, and the velocity of the medium on the k_{th} segment be V_k. The source position is $\underline{X}_s = (x_0, z_0)$, and the receiver position is $\underline{X}_R = (x_{N+1}, z_{N+1})$. (See Figure 1).

Define

$$\Delta x_k = x_k - x_{k-1}\ ,\ \Delta z_k = z_k - z_{k-1},$$

$$D_k = \sqrt{\Delta x_k^2 + \Delta z_k^2}\ . \tag{2}$$

Here, D_k is the length of the raypath on the k_{th} segment. Using Fermat's Principle (that travel time is extremal with respect to coordinate perturbations), we may write

$$\frac{\partial t}{\partial x_k} = 0\ , \qquad k = 1,\ldots,N, \tag{3}$$

where,

$$t = \sum_{k=1}^{N+1} D_k \,/\, V_k\ , \tag{4}$$

is the total traveltime.

Then

$$\frac{\partial t}{\partial x_k} = \frac{1}{V_k}\frac{\partial}{\partial x_k} D_k + \frac{1}{V_{k+1}}\frac{\partial}{\partial x_k} D_{k+1} = 0 \quad . \tag{5}$$

Using (2)

$$\phi_k = \frac{V_{k+1}}{D_k}(\Delta x_k + \lambda \Delta z_k f_k'(x_k)) - \frac{V_k}{D_{k+1}}(\Delta x_{k+1} + \lambda \Delta z_{k+1} f_k'(x_k)) = 0 \quad . \tag{6}$$

Thus, there are N equations in the N unknowns, x_k, and in system form we may write

$$\underline{\Phi}(\underline{X}(\lambda),\lambda) = 0 \quad , \tag{7}$$

where,

$$\underline{\Phi} = (\phi, \phi_2, \ldots\ldots, \phi_N)^T \ , \ \underline{X} = (x_1, x_2, \ldots\ldots, x_N)^T \quad . \tag{8}$$

Not surprisingly the same system of equations is obtained if we require that Snell's Law be satisfied at each interface.

3. FINDING THE FIRST RAY - CONTINUATION IN INTERFACES

Setting $\lambda = 0$ in (1) all the interfaces become horizontal; the medium is stratified. The raypath between the source and a receiver vertically above it on the surface is found (it is a straight vertical line). The parameter λ is then incremented by some $\Delta\lambda$ and we seek the solution of (7) using Newton's method. Assuming that we know a solution $\underline{X}(\lambda)$, for some value of λ, (for example, we know it for $\lambda = 0$), then as an initial guess in Newton's method for the solution at $\lambda + \Delta\lambda$ we use

$$\underline{X}^{(0)}(\lambda + \Delta \lambda) = \underline{X}(\lambda) + \Delta\lambda \, \dot{\underline{X}}(\lambda) \cdot \tag{9}$$

In this equation $\dot{\underline{X}}(\lambda) = d\underline{X}/d\lambda$. We differentiate (7) to obtain an equation from which $\dot{\underline{X}}(\lambda)$ can be determined:

$$\frac{d\underline{\Phi}}{d\lambda} = \frac{\partial \underline{\Phi}}{\partial \underline{X}} \, \dot{\underline{X}}(\lambda) + \frac{\partial \underline{\Phi}}{\partial \lambda} = 0 \cdot \tag{10}$$

Now using Newton's method, an improved value for the solution at $\lambda + \Delta\lambda$ is

$$\underline{X}^{(1)} = \underline{X}^{(0)} - (J^{-1} \underline{\Phi})^{(0)}. \tag{11}$$

In this equation, $J = \partial\underline{\Phi}/\partial\underline{X}$ is the Jacobian of the system. From (6) we see that ϕ_k depends only on x_{k-1}, x_k and x_{k+1}; the Jacobian is tridiagonal. To find its inverse we use a banded system solver from the I.M.S.L. library. If Newton's method fails to converge to a solution within, say, four iterations we set $\Delta\lambda = \Delta\lambda/2$ and return to (9). We proceed in this manner until $\lambda = 1$, when the first ray in the true model is found. As Fawcett points out, in almost all cases it is possible to begin by setting $\lambda = 0$, and $\Delta\lambda = 1$ and in one step obtain an initial guess for which Newton's method converges. The algorithm is indeed extremely efficient.

4. CONTINUATION IN RECEIVER LOCATION

Given the ray solution for a receiver at $\underline{X}_R^{(i)}$ we employ continuation procedures to find the ray solution for an adjacent receiver at $\underline{X}_R^{(j)}$. For the receiver position we write

$$\underline{X}_R(\lambda) = \lambda\, \underline{X}_R^{(j)} + (1-\lambda)\; \underline{X}_R^{(i)} \; . \tag{12}$$

As λ varies between 0 and 1, $\underline{X}_R^{(i)}$ is continued into $\underline{X}_R^{(j)}$. (See Fig. 2). Our system is now

$$\underline{\Phi}(\underline{X}(\lambda), \underline{X}_R(\lambda)) = 0 \quad . \tag{13}$$

Again we use (9) along with Newton's method. This time $\dot{\underline{X}}(\lambda)$ is determined as a solution of the derivative of (13):

$$\frac{d\underline{\Phi}}{d\lambda} = \frac{\partial\underline{\Phi}}{\partial\underline{X}}\dot{\underline{X}} + \frac{\partial\underline{\Phi}}{\partial\underline{X}_R}\dot{\underline{X}}_R = 0 \quad , \tag{14}$$

and

$$\dot{\underline{X}}_R = \underline{X}_R^{(j)} - \underline{X}_R^{(i)} \quad . \tag{15}$$

Since only ϕ_N depends on $\underline{X}_R$, the Nx2 matrix $\partial\underline{\Phi}/\partial\underline{X}_R$ has all elements equal to zero except for $\partial\phi_N/\partial x_{N+1}$ and $\partial\phi_N/\partial z_{N+1}$ in the last row. Here we consider the surface of the earth to be flat ($z = 0$). Thus, $\partial\phi_N/\partial z_{N+1} = 0$ also. The non-zero element in (15) is the receiver spacing. We could have included surface topography by writing

$$z = c_{N+1} + \lambda f_{N+1}(x) \quad .$$

In finding the first ray we always place the receiver vertically above the source. This allows us to write down the solution in a stratified medium. In general the receiver will be at some other position and we simply use continuation in receiver location to proceed to that point.

Continuation in receiver location was the work of Perozzi and continuation in interfaces was Fawcett's idea. Our aim here is to create a table of traveltimes between points at depth and receivers on the surface. We could begin at each source position with a stratified earth and proceed as above. More efficiently, we could apply the same ideas of continuation to the source.

5. CONTINUATION IN SOURCE LOCATION

If the ray solution for a source at $\underline{X}_S{}^{(i)}$ is known, we can employ continuation procedures to find the solution for a nearby source location $\underline{X}_S{}^{(j)}$. Writing

$$\underline{X}_S(\lambda) = \lambda\, \underline{X}_S^{(j)} + (1 - \lambda)\, \underline{X}_S^{(i)} \quad , \tag{16}$$

then as λ varies between 0 and 1, $\underline{X}_S{}^{(i)}$ is continued into $\underline{X}_S{}^{(j)}$. Our system is now

$$\underline{\Phi}\, (\underline{X}(\lambda),\, \underline{X}_S(\lambda)) = 0 \quad . \tag{17}$$

As before we use (9) and Newton's method, with $\dot{\underline{X}}(\lambda)$ from

$$\frac{d\underline{\Phi}}{d\lambda} = \frac{\partial \underline{\Phi}}{\partial \underline{X}}\, \dot{\underline{X}} + \frac{\partial \underline{\Phi}}{\partial \underline{X}_S}\, \dot{\underline{X}}_S \quad , \tag{18}$$

and from (16)

$$\dot{\underline{X}}_S = \underline{X}_S^{(j)} - \underline{X}_S^{(i)} . \qquad (19)$$

From (6) we see that only ϕ_1 depends on the source position. Thus, the Nx2 matrix $\partial\underline{\xi}/\partial\underline{X}_S$ has all elements equal to zero except for $\partial\phi_1/\partial x_s$ and $\partial\phi_1/\partial z_s$ in the first row. Since our program uses separate subroutines to move the source either vertically, (Fig. 3), or horizontally, (Fig. 4), one of these two elements is zero in each case. The subroutines may be combined to move the source to any point, though this method is particularly efficient if moving on a grid. Equation (19) reduces to the vertical or lateral spacing of the grid points, which may differ, another advantage of this method.

6. PROGRAM DESCRIPTION

The program was written in FORTRAN 77 and implemented on a DEC 10 at the Colorado School of Mines. There are three entry points to the program. The first is for creating ray diagrams like those in this paper. This part of the program is interactive and outputs directly to the terminal or to the plotter. The user supplies a subroutine defining the interfaces, their first and second derivatives and the interval velocities. Source and receiver positions and type of continuation can then be chosen.

There is no interaction at the second entry point. This is the mass ray tracing procedure. The user supplies a subroutine, as above, and also a data file defining the region of interest; the location of this region, the number and density of source points and the number of receivers.

We start by determining the vertical ray in a horizontally stratified medium. We then use continuation in interfaces to obtain the correct ray for the true model. The interfaces remain fixed once this ray has been found. The region of interest is divided into a grid of source points and we move down or across this grid using source continuation. At each source position we use continuation in receiver location to find the rays to all receivers. Each raypath is stored only temporarily, in order to provide a starting point in the solution for an adjacent ray. After each ray is found, a traveltime is calculated and output, so that for each source position there is a list of traveltimes to receivers. Typically, several hundred thousand traveltimes will be found. These may be used later in this program or in a separate inversion scheme.

In each of our continuation schemes we start by setting $\lambda = 0$, $\Delta\lambda = 1$, and in almost all cases find the ray solution in one step. Typically, two or three Newton iterations are required (and never more than four). If a solution is not found within four iterations we return to the known raypath, set $\Delta\lambda = \Delta\lambda/2$, and try Newton's method again. If $\Delta\lambda$ falls below a user defined value then the procedure is stopped. Either no raypath exists or our continuation procedure has failed to converge. It is sometimes possible to converge on a solution along a different path by using one of the other continuation procedures, for example, by returning to a stratified medium. Fawcett discusses such cases and also the problem of bifurcation, that is, the existence of many solutions. A point of bifurcation might occur, for example, where source and receiver are above a syncline. Since in our scheme the source is at depth, we avoid many such situations.

On completion, the final entry point will output a migrated depth section. Each point at depth is considered a possible point source. The input geophone data is then summed over curves determined by the traveltimes from the ray tracing procedure. Only at points along an interface is there significant output from this summation. This is not a new idea, indeed some of the earliest computer migration schemes used existing common depth point stacking programs that summed the data over hyperbolae. Our medium is not of constant velocity and so the curves are no longer hyperbolae, but still the traveltimes serve to position the reflectors.

It is a "fast and dirty" migration and we make no claims regarding the amplitude of the output.

7. EXAMPLES AND APPLICATIONS

(I) Forward Modeling.

Here we display the versatility of our technique. Though the code was designed primarily for use in inversion it can be applied to a range of forward modeling problems. The user supplied subroutine defines the order of interfaces between source and receiver and can thus specify both reflections and refractions. In Figure 5 we have also included multiples.

By placing the source and receiver together on the surface we force a specular reflection from the deepest interface. To step across the surface we first move the receiver and then the source. At each coincident source/receiver position the ray is plotted or the traveltime is calculated. (See Fig. 6).

In a similar fashion, and motivated by our interest in offset problems, we first separate source and receiver before stepping across the surface to create a constant offset plot, (Fig. 7).

Vertical seismic profiling is a natural application for vertical source continuation, (Fig. 8), and in conjunction with receiver continuation, (Fig. 9), we can find all of the raypaths. It is also straightforward to find rays reflected from interfaces below the receivers in the well.

Knowing the raypath we can find the angle at which it strikes an interface. Also, as part of the calculation of traveltime, we found the distance travelled within each layer. Thus, we already have some factors in the calculation of amplitude of synthetic data.

(II) Migration.

The only synthetic data generator available was for beds of constant dip, though variations in thickness and interval velocity were allowed. We give two examples.

Model 1.

Figure 10 shows three interfaces dipping at 30 degrees. The interval velocities are 5000, 6500, 10000 and 12000 feet/sec. The synthetic data (Fig. 11) is for fifty receivers spaced at 100 feet. The frequency of the data is 4 - 30 Hz. As we might expect from the "migrator's equation", (tan (apparent dip) = sin(true dip)), the slope of the events on the zero offset time section is 26.5 degrees. Our migration is for some region of interest above which the positions of interfaces and the interval velocities are known. We chose the region bounded by dashed lines in Figure 10. The region extends from 4100 feet to 7050 feet in depth and is 4900 feet across. The horizontal spacing of source points within the region was chosen to be 100 feet and the vertical spacing 50 feet, for a total of 3000 source positions. 150,000 raypaths were found in tracing to all fifty receivers. C.P.U. time to determine these raypaths and the associated traveltimes was 15 minutes under timesharing conditions. The summation that provided the migrated output shown in Figure 12 took 2 C.P.U. minutes. The interface has been positioned accurately and its dip is 30 degrees. More receivers are necessary to map the interface completely within the region chosen.

Model 2.

Here we have increased the dip to 75 degrees, (Fig. 13). The synthetic data, (Fig. 14), was generated using interval velocities 10000, 12000, 12500 and 14000 feet/sec, with 100 receivers spaced at 150 feet. The frequency of the data is again 4 - 30 Hz. The region of interest, shown dashed in Figure 13, measures 900 feet across and extends from 700 feet to 4650 feet in depth. The horizontal source spacing was chosen to be 100 feet and the vertical spacing 50 feet. The output of the migration is shown in Figure 15. The dip is 74.5 degrees.

8. CONCLUSIONS

We have extended the ideas of Perozzi and Fawcett to create an algorithm suitable for finding large numbers of raypaths and traveltimes. We have shown that the technique is fast, even on a computer of limited capacity. Using only the traveltimes from the ray tracing procedure we have produced highly accurate structural inversions for two specific examples. Possible applications to forward modeling problems have been indicated.

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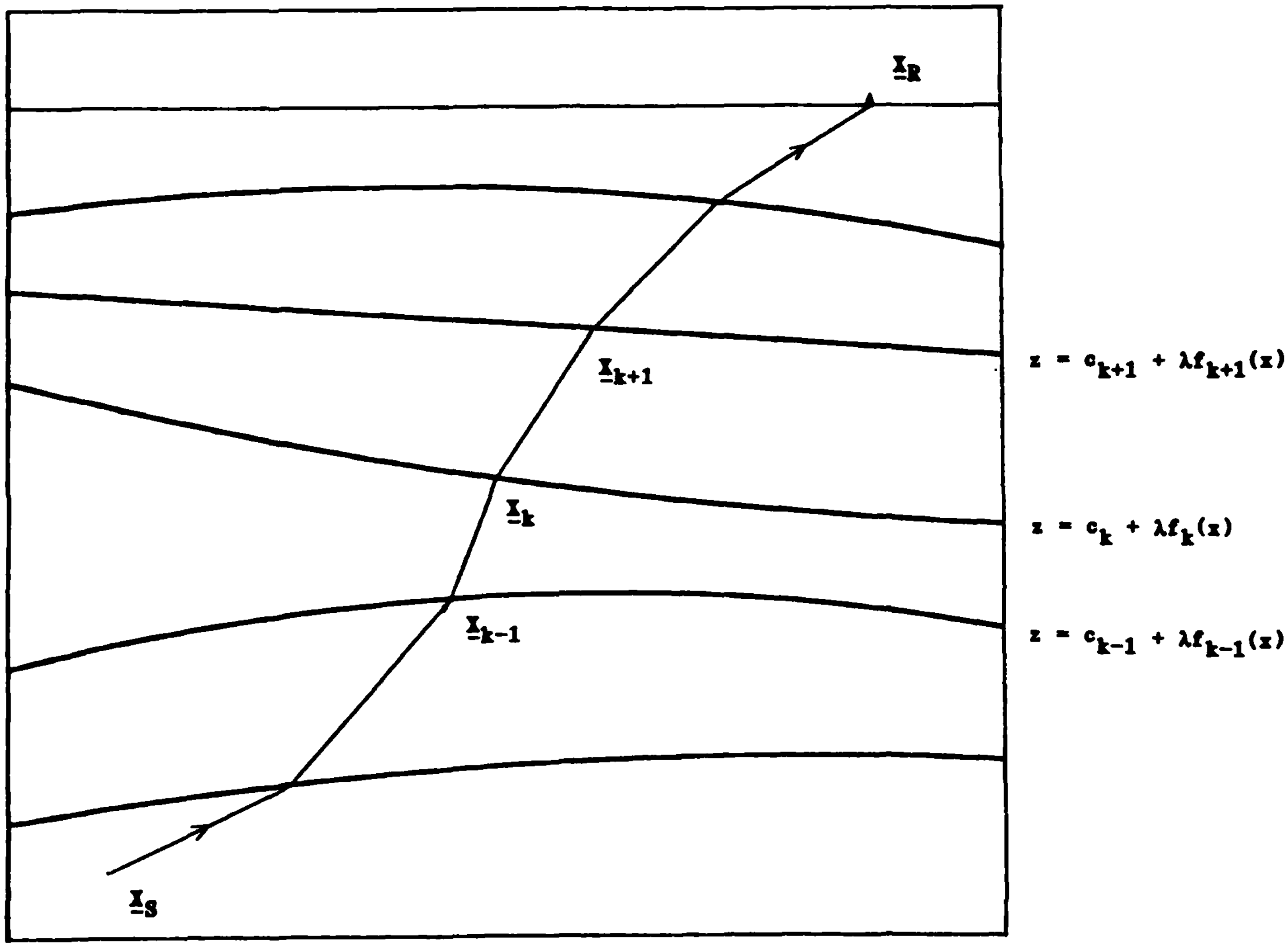

$\underline{X}_R$
$\underline{X}_{k+1}$
$\underline{X}_k$
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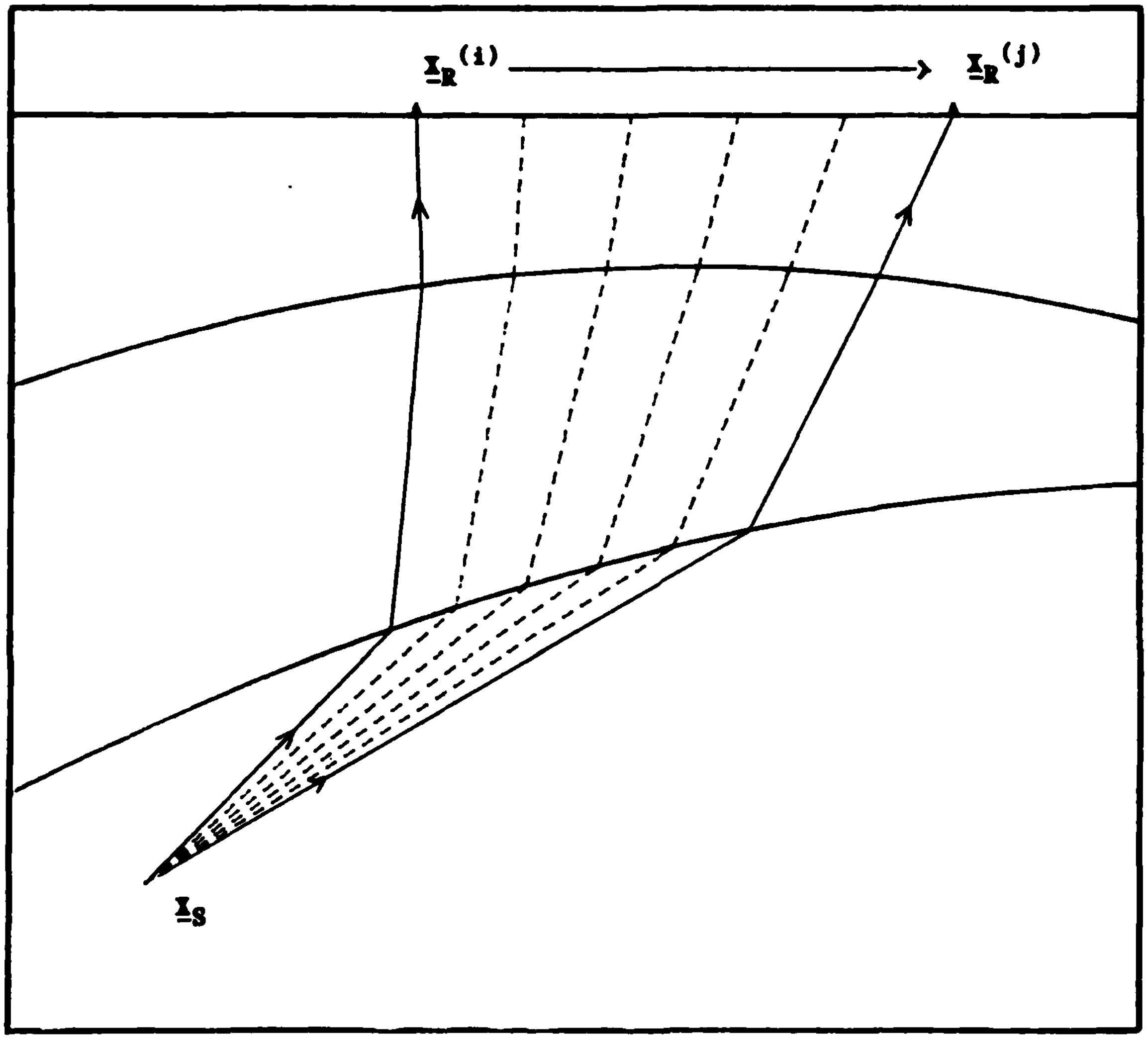
$\underline{X}_S$
$z = c_{k+1} + \lambda f_{k+1}(x)$
$z = c_k + \lambda f_k(x)$
$z = c_{k-1} + \lambda f_{k-1}(x)$

FIGURE 1



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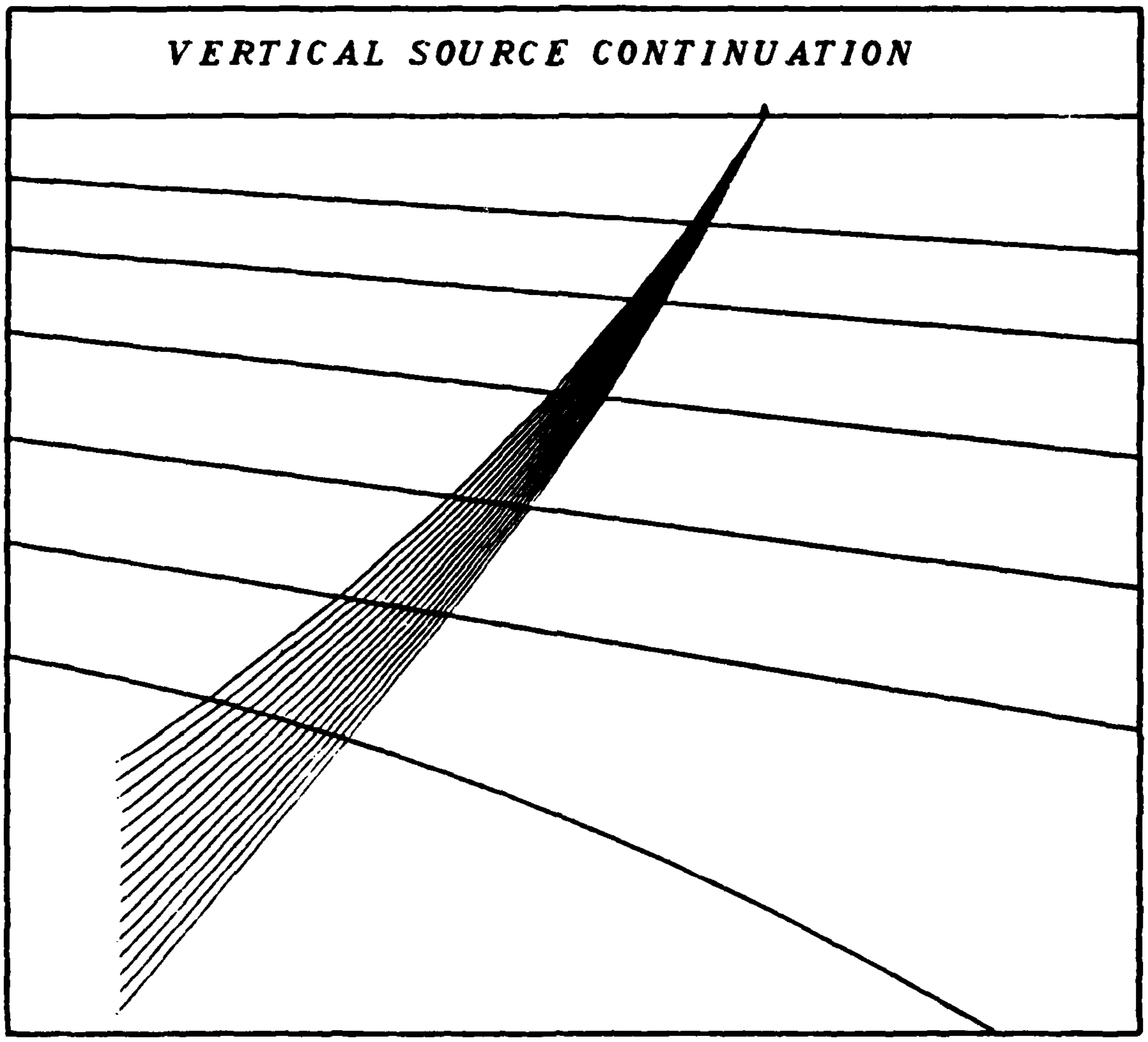

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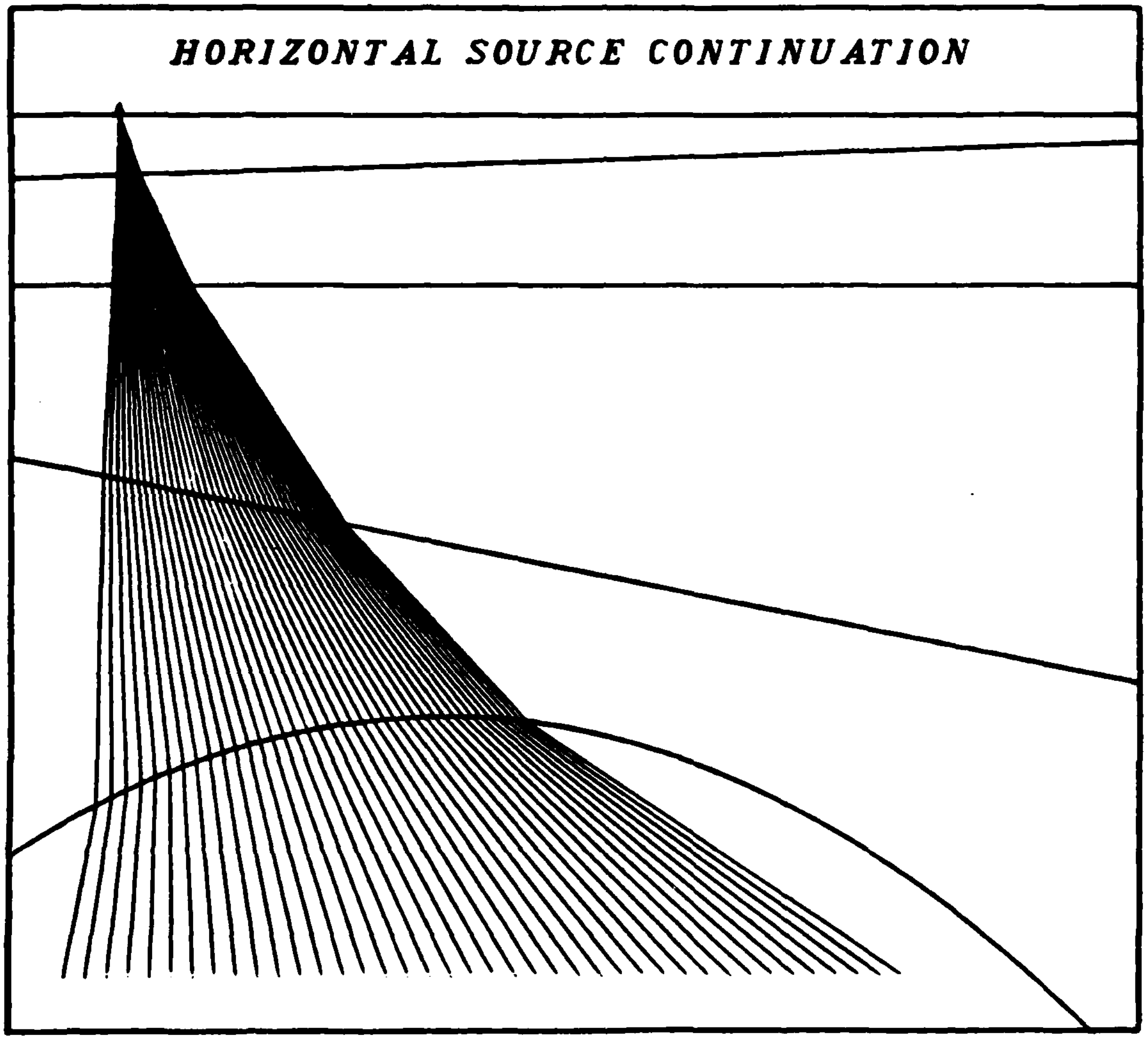

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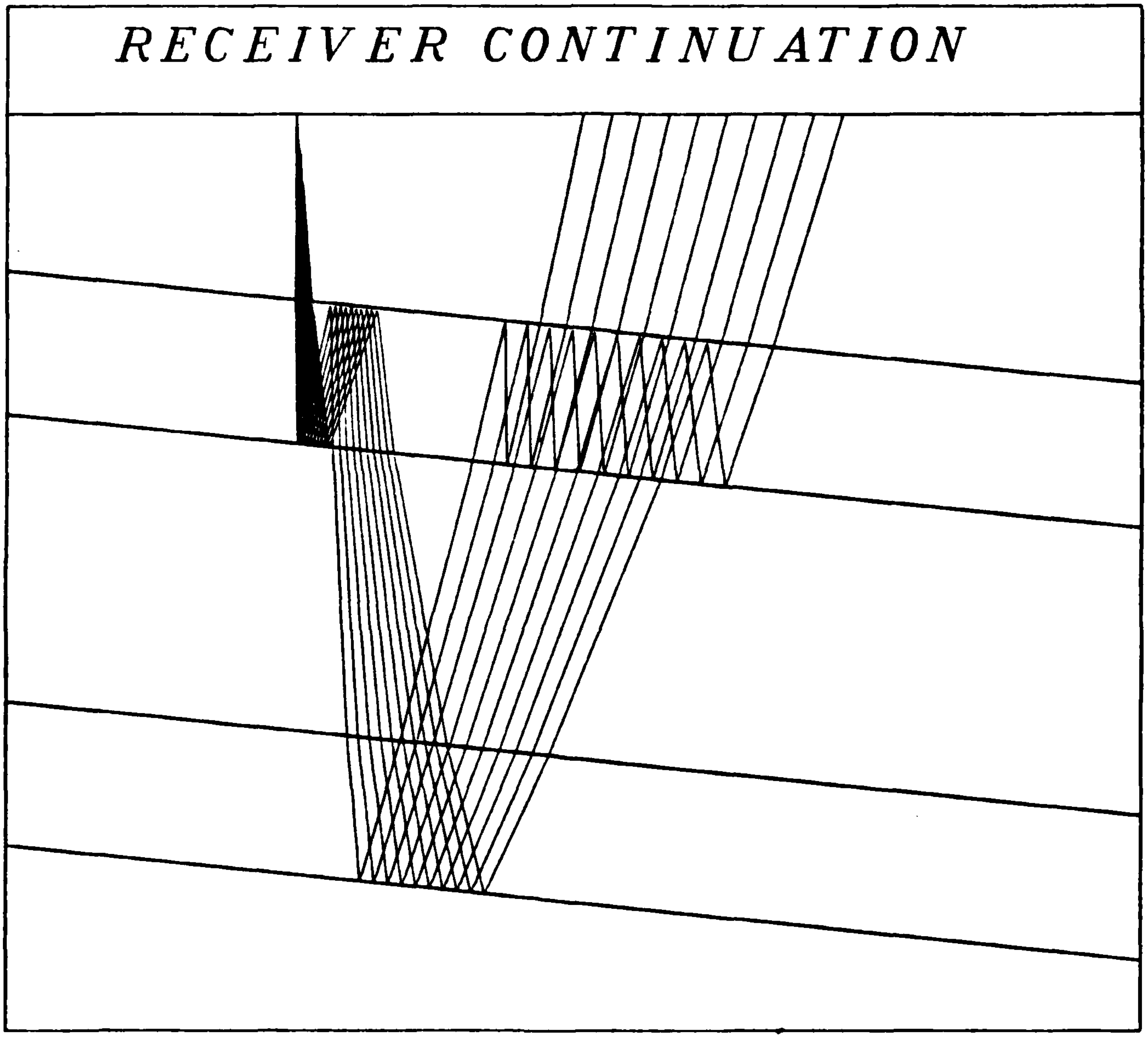

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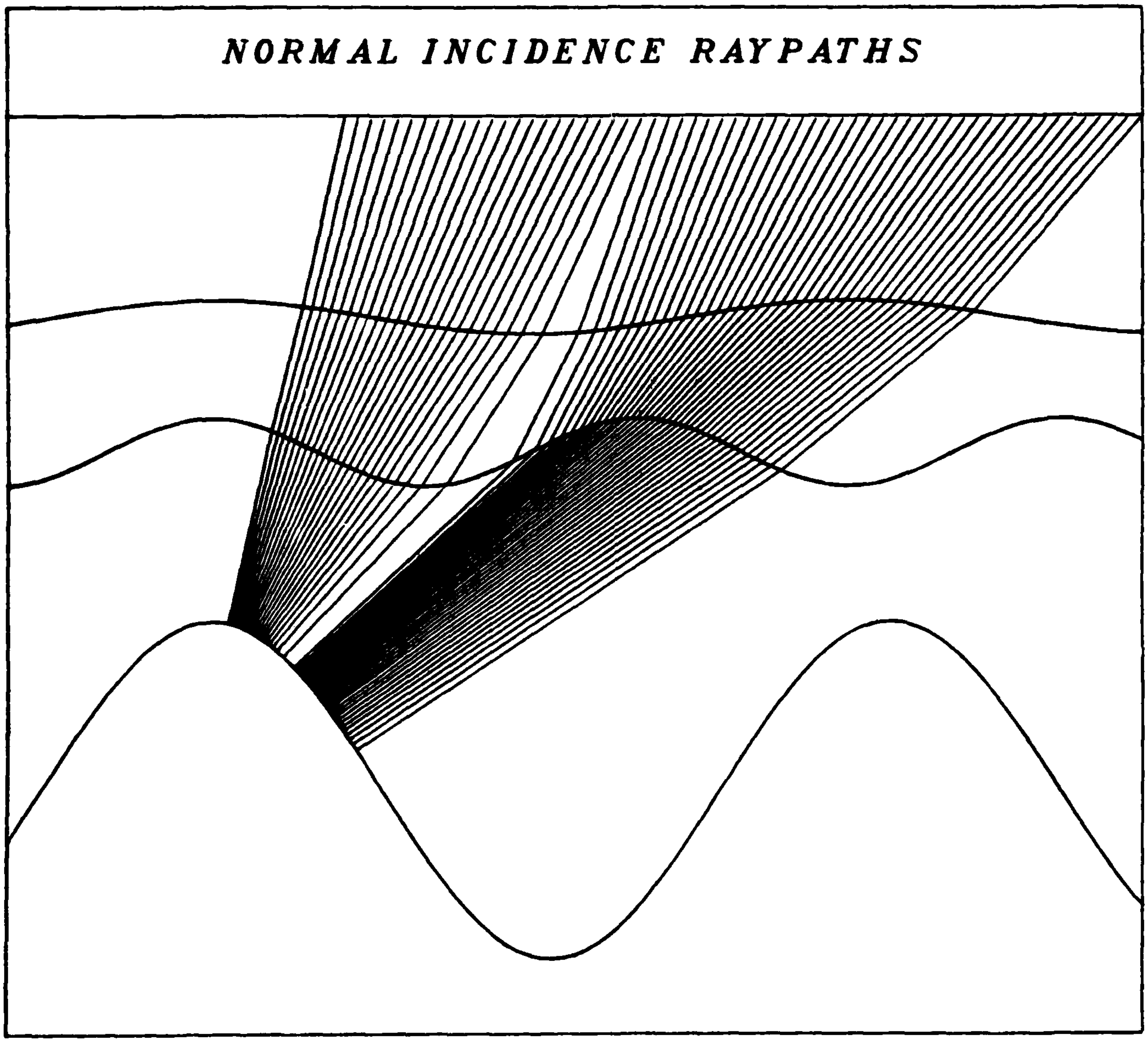

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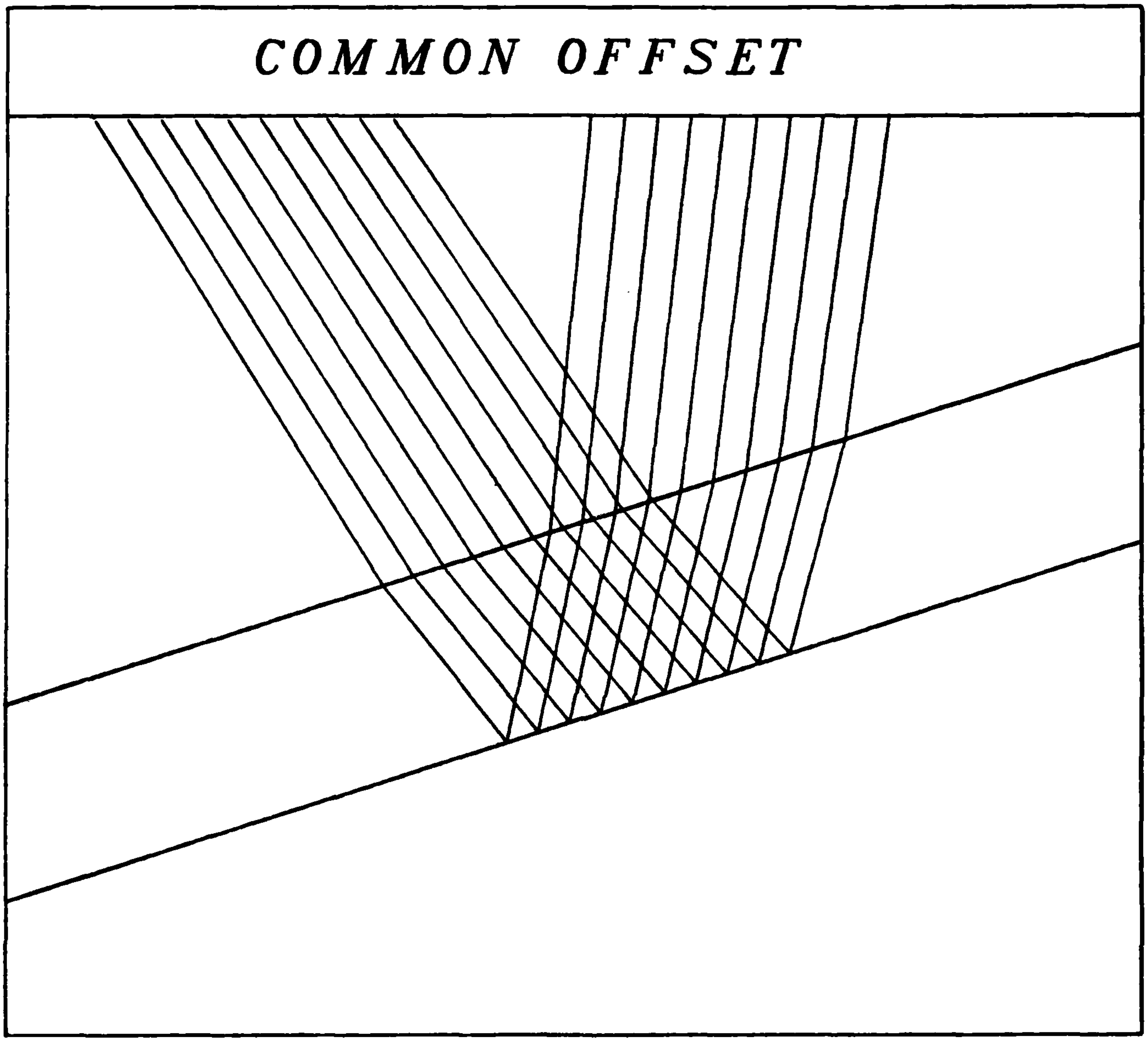

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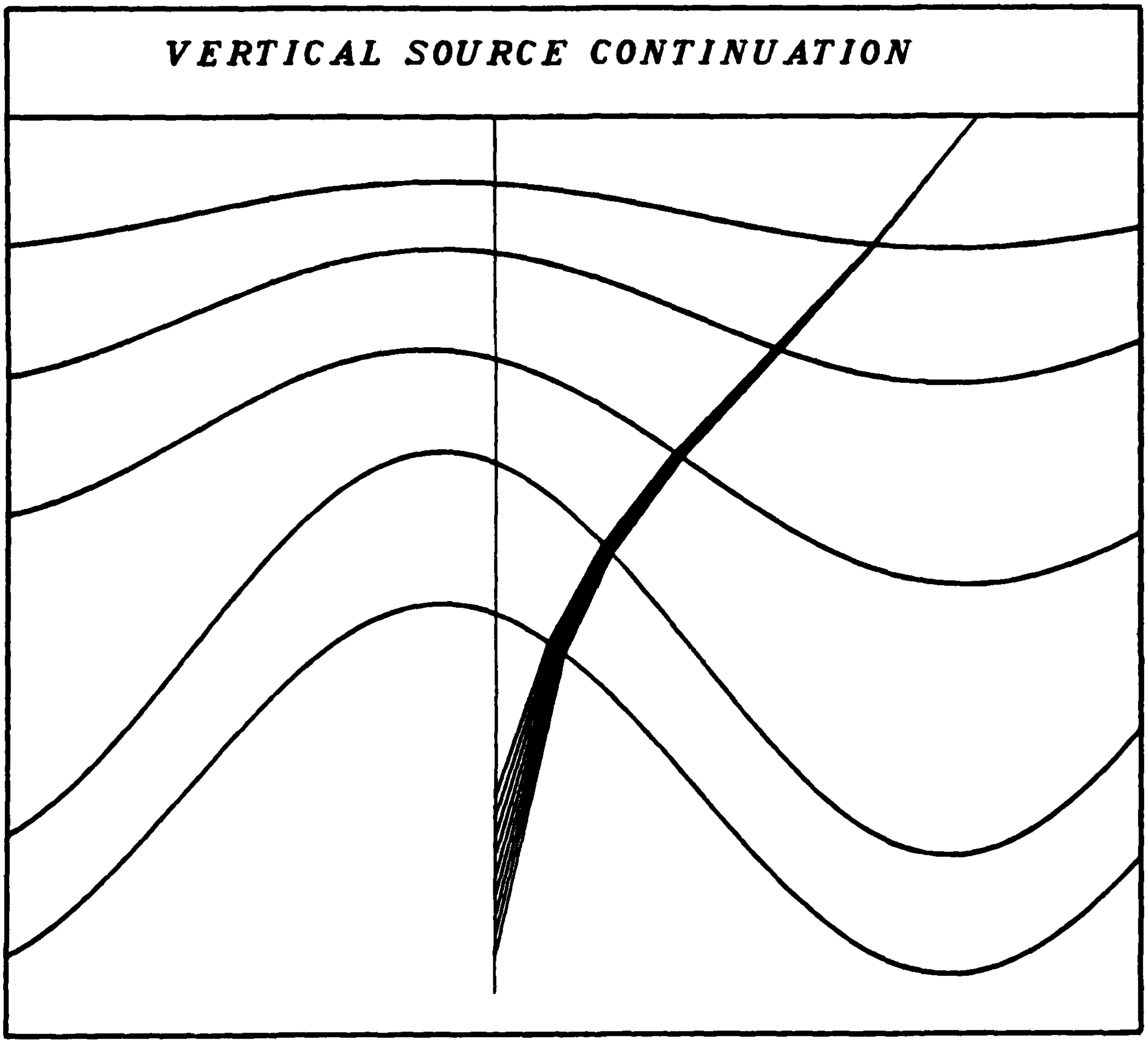

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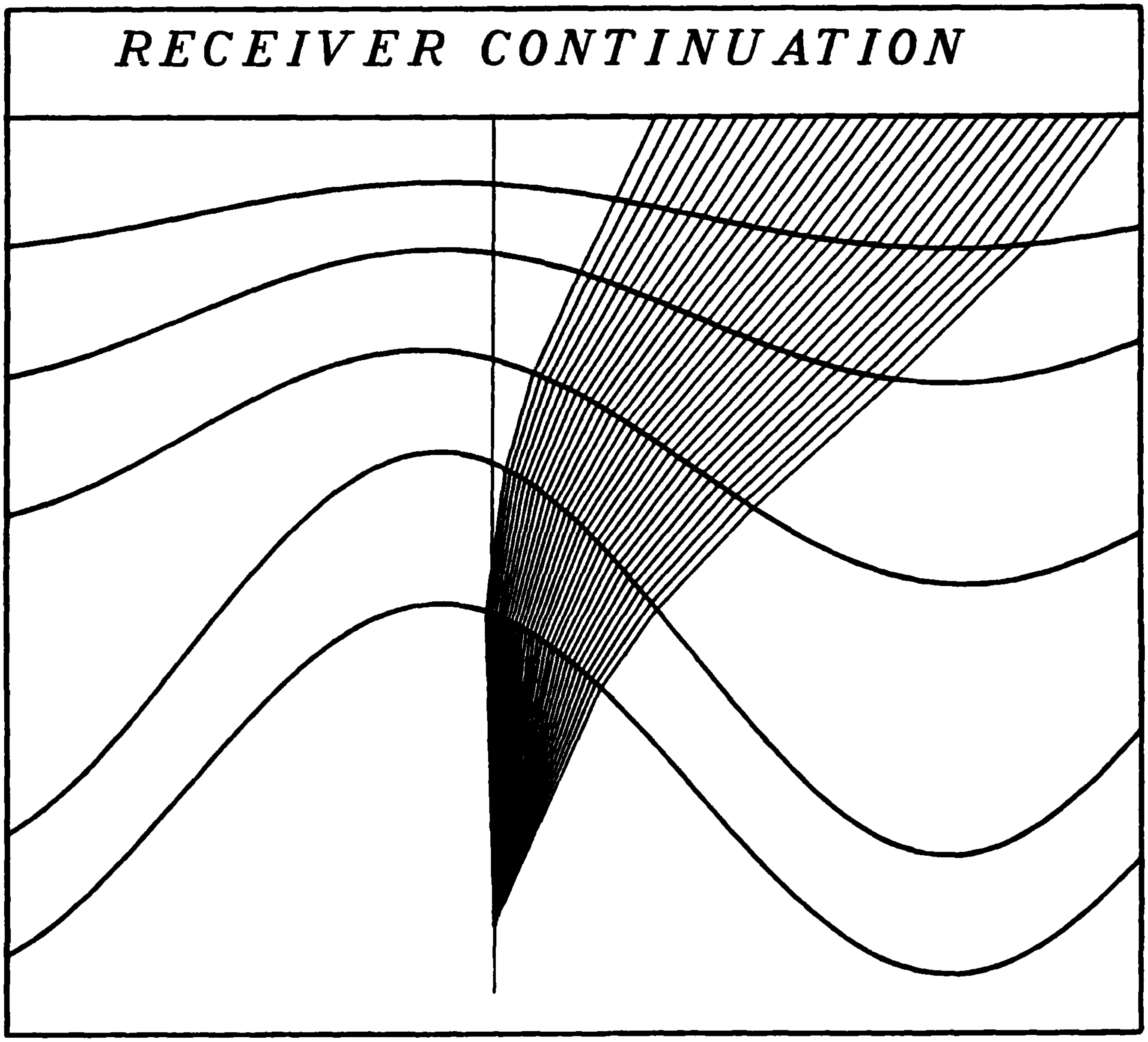

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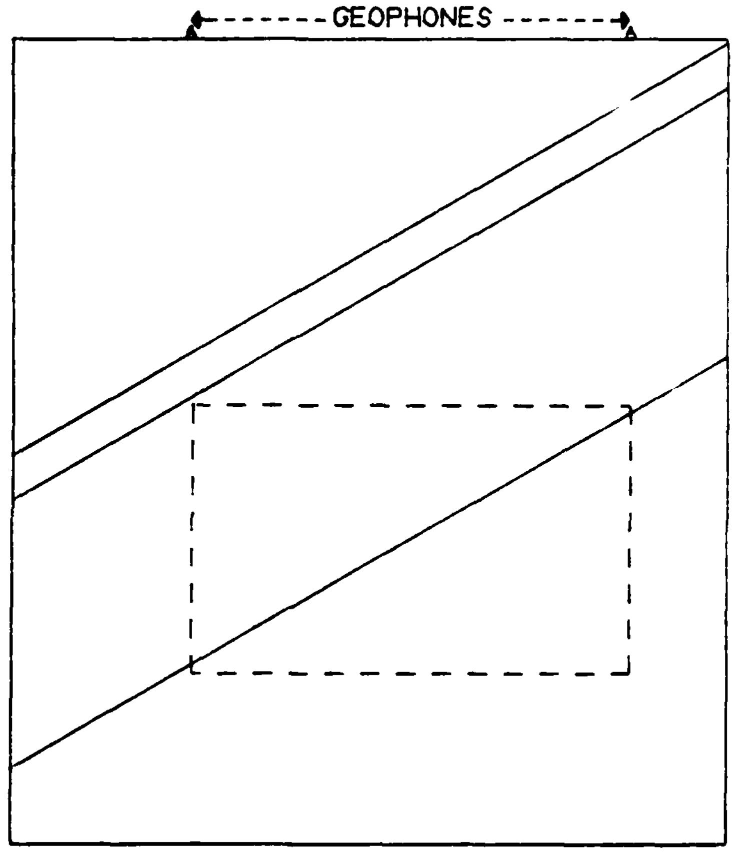

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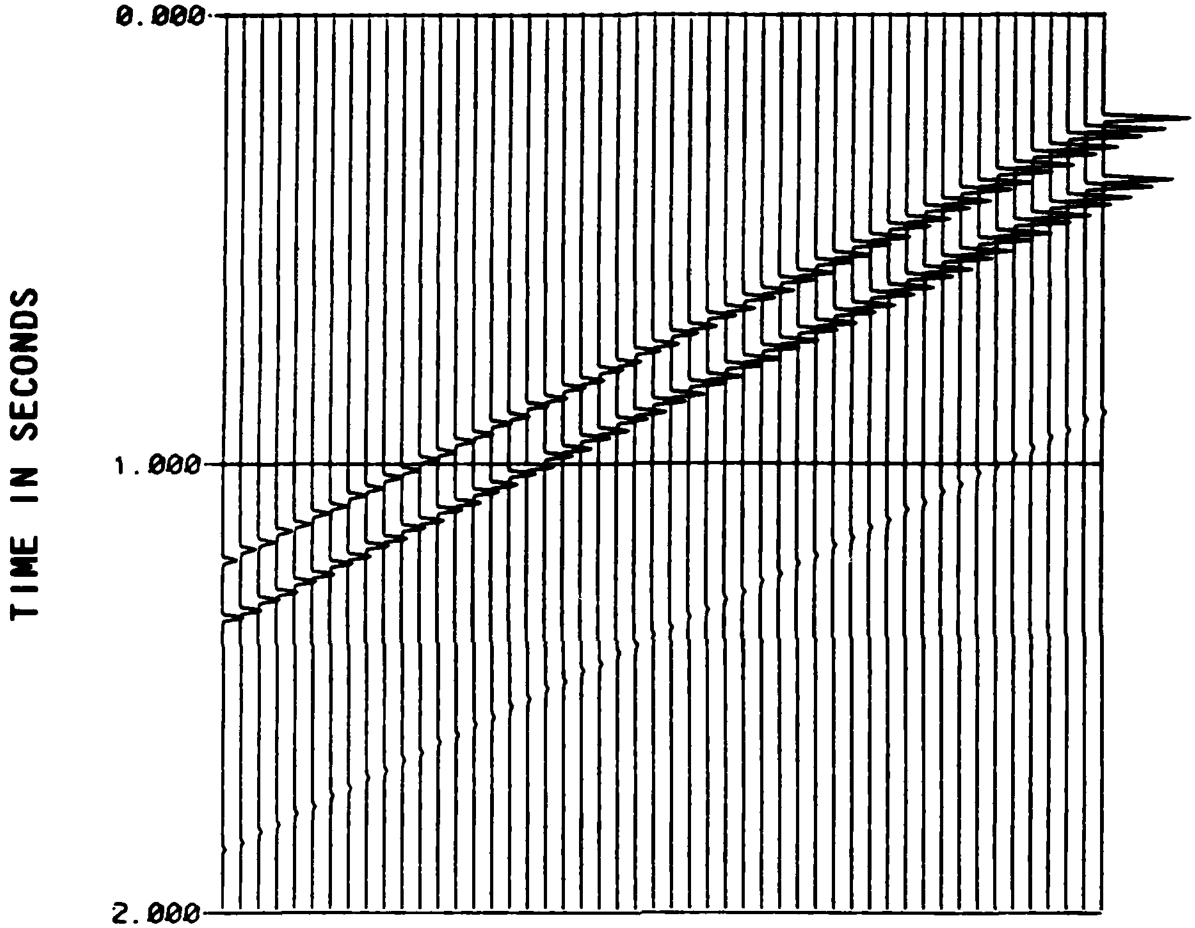

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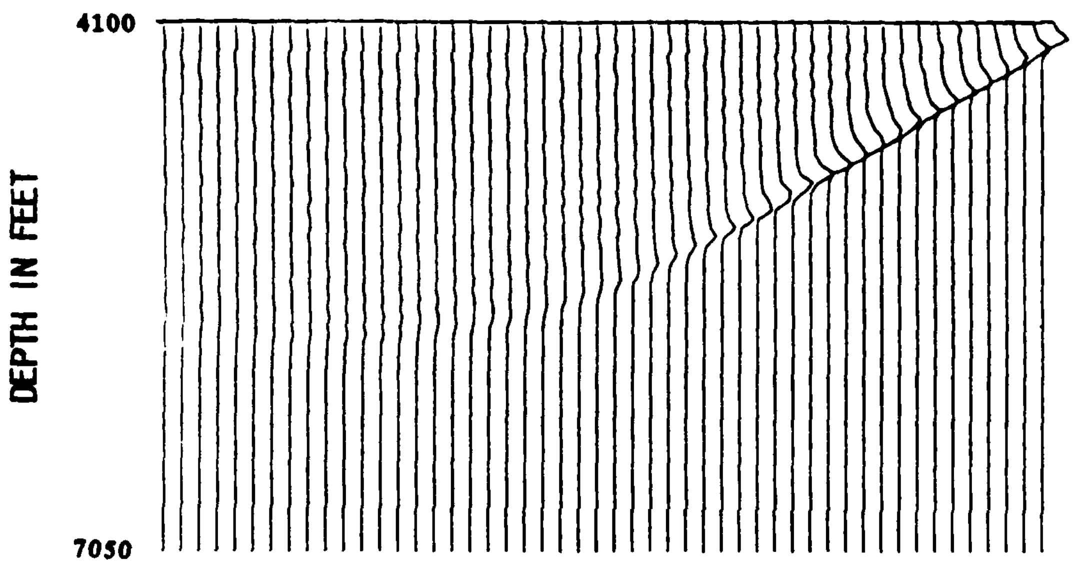

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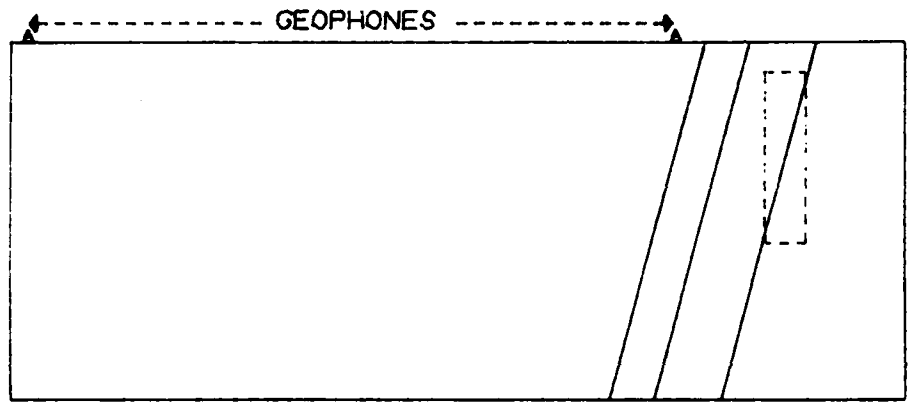

FIGURE 13

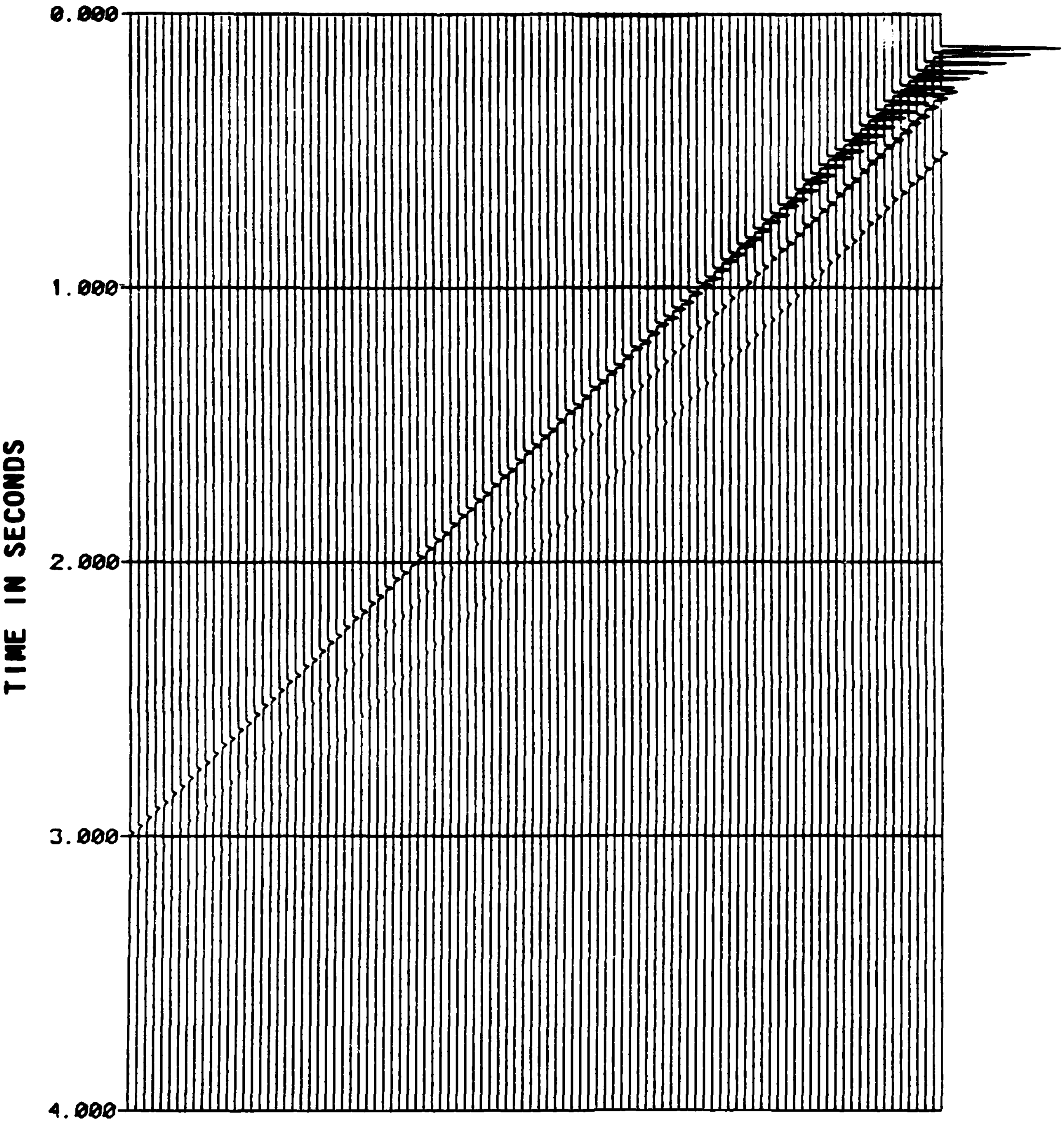

FIGURE 14

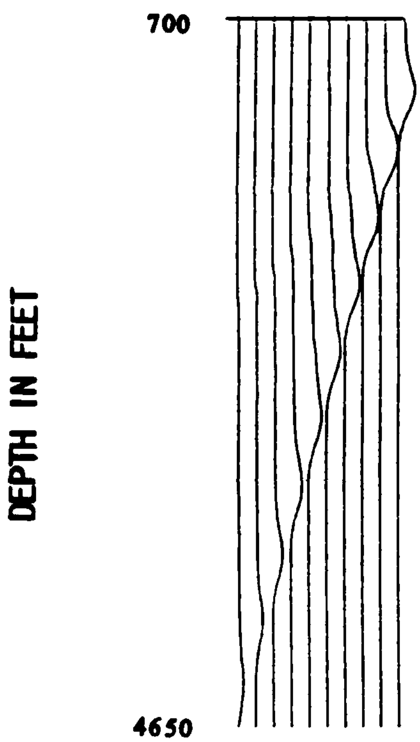

FIGURE 15

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ray tracing, laterally varyng velocity, continuation procedures, homotopy procedures, migration, forward modeling

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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ABSTRACT

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