ADVANCED LIFE ANALYSIS

METHODS - Crack Growth

Analysis Methods for
Attachment Lugs

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This technical report has been reviewed and is approved for publication.

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Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.
Analysis procedures for the assessment of the damage tolerance of aircraft attachment lugs are developed and presented. Stress and fracture analyses are conducted for simple male straight shank lugs subjected to symmetric loading and tapered lugs subjected to symmetric and off-axis loadings. Solutions of stress distributions and stress intensity factors are obtained for various parameters. The parametric variations include lug outer-to-inner radius ratios (R/R_i), corner and through-the-thickness cracks, crack lengths and aspect ratios, with and without interference-fit bushing and loads above and below yield of the lugs. Methods of varying degree of sophistication are considered in the development of stress intensity factor solutions for cracks in attachment lugs. For through-the-thickness crack problems, simple compounding method, two-dimensional cracked finite element method, and the weight function method are used. In the case of corner cracks, solutions are obtained using simple compounding method.
ADVANCED LIFE ANALYSIS METHODS - Crack Growth Analysis Methods for Attachment Lugs (Unclassified)

18. (Continued)

Corner, Interference-Fit Bushing, Elasto-Plastic Analysis, Crack Growth Analysis,

19. (Continued)

Weight function method and a rigorous three-dimensional hybrid displacement cracked finite element method. The stress distributions and the stress intensity factors are consolidated into a crack growth analysis procedure including several crack growth models and spectrum retardation models to predict the fracture and crack growth behaviors in attachment lugs. Original Furnished Keywords include:
FOREWORD

This is Volume II of six final report volumes on Contract F33615-80-C-3211, "Advanced Life Analysis Methods." The work reported herein was conducted jointly by Lockheed-Georgia Company and Lockheed-California Company under contract with Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB. J. L. Rudd is the Air Force project leader.
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SECTION I

PROGRAM SUMMARY

The objective of the program is to develop the design criteria and analytical methods necessary to ensure the damage tolerance of aircraft attachment lugs. As planned, the program proceeds logically from an extensive cracking data survey and nondestructive inspection (NDI) assessment, through method development and evaluation, to the preparation of damage tolerance design criteria for aircraft attachment lugs.

The program consists of three phases involving seven tasks. Phase I consists of Tasks I, II and III; Phase II consists of Tasks IV, V and VI; and Phase III consists of Task VII. A roadmap shown in Figure 1-1 summarizes the major activities by task, decision points and their interrelationships.

Task I involves a survey of structural cracking data such as the initial flaw size, shape and location which occur in aircraft attachment lugs. Sources for these data include open literature, available Lockheed data, and visits to the five Air Force Air Logistics Centers (ALCs). The types of aircraft structure used to obtain these data include service aircraft, full-scale test articles, component test articles, and coupon specimens.

Task II assesses the current NDI capability to find these flaws or cracks. This assessment is to be based upon information obtained from the open literature, available Lockheed NDI data and experience, and Air Force ALC data. The NDI techniques capable of finding flaws in attachment lugs and the flaw sizes these techniques are capable of finding are identified. Where possible, the probability of detecting a flaw of a particular size for the NDI technique involved is specified as well as the confidence level assigned to that probability. The results obtained from Tasks I and II will be used in the formulation of the initial flaw assumptions to be developed in Task VII as part of the damage tolerant design criteria for attachment lugs.

Task III involves three different levels of complexity and degrees of sophistication for determining stress intensity factors for single corner cracks and single through-the-thickness cracks in aircraft attachment lugs, and the development of crack growth analysis capable of
Figure 1. Roadmap of the Program
predicting the growth behavior of these cracks and residual strength of these lugs. These stress intensity factors and crack growth analyses are used in Task IV to predict the residual strength and the crack growth behavior for a number of different geometries and test conditions defined in the experimental program. These predictions are made prior to testing. Two groups of attachment lug geometries are tested and experimental test data are generated in Task V. The analytical methods developed in Task III are evaluated by correlating the analytical predictions made in Task IV with the Group I experimental test data (based upon accuracy and cost) for use in prediction of Group II tests. Further evaluation of the selected method is made by correlating the analytical predictions for the Group II tests (Task IV) with the experimental test results (Task V). These correlations indicate what improvements are necessary for the selected analytical method. The results are presented in parametric format useful to designers and analysts. Damage tolerant design criteria for aircraft attachment lugs are developed in Task VII. These criteria are similar in nature to those of Military Specification MIL-A-83444, and require crack growth analyses by the types of methods developed and verified in Tasks III through VI. The criteria include initial flaw assumptions (e.g., initial flaw type, shape, size, etc.) based upon the cracking data survey of Task I, NDI assessment of Task II, and crack initiation tests of Task V.

As Figure 1-1 shows, the following sequence of final report volumes is generated under this project:

Volume I. Cracking Data Survey and NDI Assessment for Attachment Lugs
Volume II. Crack Growth Analysis Methods for Attachment Lugs
Volume III. Experimental Evaluation of Crack Growth Analysis Methods for Attachment Lugs
Volume IV. Tabulated Test Data for Attachment Lugs
Volume V. Executive Summary and Damage Tolerance Criteria Recommendations for Attachment Lugs
Volume VI. User's Manual for "LUGRO" Computer Program to Predict Crack Growth in Attachment Lugs

This is Volume II on Crack Growth Analysis Methods for Attachment Lugs, which is the result of Task III efforts. It contains the developed analytical methodologies for determining the stress intensity factors for single corner cracks and single through-the-thickness cracks in aircraft attachment lugs, and for predicting the growth behavior of these cracks.
SECTION II

INTRODUCTION

In aircraft structures, lug-type joints are frequently used to connect major structural components or in linkage structure. The lug joint is normally connected by a single bolt or pin, creating a simple joint that is easy to assemble and disassemble. Since clamping of the joint is not normally allowed, the lug can act as a pivot. But the elastic gross section stress concentration for normal lugs is very high, resulting in a relatively short crack initiation and crack growth life. To improve the crack initiation life, the stress concentration factor can be effectively reduced by cold working the hole or by installing an interference-fit bushing prior to pin fitting. To minimize the wear in either the lug or the pin, most aircraft lugs have an oil fitting and/or lubrication provisions.

During the past decade, the influence of fracture mechanics on the design, manufacture, and maintenance of aircraft has steadily increased. Also, nondestructive inspection techniques have been improved significantly. However, some cracks still cannot be detected during routine maintenance inspection. Under service loading, such cracks will grow and fracture can occur if the crack length reaches a critical dimension before it can be detected and the part repaired or replaced. To assure aircraft safety, the U. S. Air Force has imposed damage-tolerance requirements (MIL-A-83444) [1] which include the prediction of fatigue crack growth life and residual strength of the structure by assuming that small initial flaws exist at critical locations of new structure due to various material and manufacturing and process operations. Assumptions regarding the initial size, shape, location, multiplicity, etc. for these flaws are specified in MIL-A-83444. However, these assumptions were established primarily for skin-stringer structure and may not be applicable for attachment lugs.

Attachment lugs are some of the most fracture critical components in aircraft structure, and the consequences of a structural lug failure can be very severe. Therefore, it is necessary to develop damage tolerance design requirements, similar to MIL-A-83444, for attachment lugs to ensure the safety of aircraft. The development of these damage tolerance requirements will be based upon actual cracking data for attachment lugs and current
nondestructive inspection capability. Once the damage tolerance design requirements for aircraft attachment lugs are established, the analytical methods necessary to satisfy the crack growth and residual strength requirements are needed. In particular, stress intensity factors for cracks in attachment lugs are needed. Such stress intensity factors will depend upon the complexities of structural configuration, crack geometry, applied loads, and the fit between the pin and the lug.

There are a number of different methods for determining the stress intensity factors, $K$, for cracks in aircraft attachment lugs. Schijve and Hoeymakers [2] and Wanhill [3] derived empirical $K$-solutions from the growth rate data for through cracks under constant amplitude loading using a back-tracking method such as that proposed by James and Anderson [4]. Analytically, Liu and Kan [5] and Kirkby and Rooke [6] used the simple compounded solution method which involves superimposing known solutions, such as in Reference [7] to estimate the stress intensity factors. Aberson and Anderson [8] used a special crack-tip singularity element to compute the stress intensity factors for a crack in a nonsymmetrical aft lug of an engine pylon. Pian et al [9] used the hybrid finite element method to compute the $K$-values for cracks oriented in various angles from the axial direction of straight lugs. Impellizzeri and Rich [10] modified the exact weight function derived by Bueckner [11], for an edge crack in a semi-infinite plate, to include a series of geometry correction factors. Then they computed the $K$-values using the weight function method. Except for Reference [8], all of these works made the assumption that the assumed or computed pin-bearing pressure distribution for an uncracked case remains unchanged even after the crack has initiated and propagated. Based on the parametric study conducted in Reference [9], it was found that, for any given crack length, the difference in the stress intensity factor computed using the uniform and cosine pin-bearing pressure distributions was as much as 30 percent. Therefore, it is salient that the correct representation of the pin-bearing pressure distribution during the crack growth process is essential to the calculation of accurate stress intensity factors. An analysis procedure using a finite element method with inclusion of a crack-tip singularity element for analyzing cracks in both straight and tapered lugs having a neat fit between the pin and the hole [12] has shown that it can accurately account for the change of pin-bearing pressure distribution with the change in crack length, and provide stress intensity factors which are in excellent agreement with the available data.
This approach was extended for the development of analytical methodology for analyzing cracks in attachment lugs with and without the presence of residual stress resulting from the installation of the interference-fit bushing [13]. The analytical procedure developed for computing the stress intensity factors for cracks in attachment lugs having residual stresses around the hole prior to the application of pin-bearing loads consists of two major steps. First, the effective unflawed stress distribution on the prospective crack surface was obtained by superimposing the residual hoop stresses due to the installation of an interference-fit bushing on the applied tangential stresses obtained due to the application of a pin loading. Second, a through crack was introduced in this stress field by removing the tractions on the crack faces and computing the corresponding stress intensity factor using the weight function approach.

This report describes the analytical methods and procedures for obtaining the stress intensity factors and for predicting the fatigue crack growth life for cracks at attachment lugs with and without the presence of residual stresses. Two types of attachment lugs are considered in the analysis. They are: (1) straight-shank male attachment lugs and (2) tapered male attachment lugs as shown in Figure 2-1. Three different outer-to-inner radius ratios, $R_o/R_i = 1.50, 2.25$ and $3.0$, are considered in the analysis. The straight attachment lugs are subjected to axial pin loading only, while the tapered attachment lugs are subjected to axial, off-axis and transverse pin loadings. Types of cracks considered include single through-the-thickness crack and single corner crack as depicted in Figure 2-2.

For through-the-thickness cracks, the methods developed and described in this report include: (1) the compounding solution method which involves the superposition of known solutions for idealized cracked geometries; (2) the two-dimensional cracked finite element method which is capable of characterizing the crack-tip stress singularity internally; and (3) the weight function (also known as Green's function) method which computes the stress intensity factors from the knowledge of the unflawed stress distribution and the superposition technique. For corner cracks, the methods developed and described include: (1) the one parameter compounding method which assumes a constant crack shape and estimates the stress intensity factor at only one location on the crack front (at lug surface); (2) a two-dimensional approach which estimates the stress intensity factors along the crack front.
Figure 2-1. Types of Attachment Lugs
Considered in the Analysis
Figure 2-2. Types of Cracks Considered in the Attachment Lug Analysis
of a corner crack by modifying the Green's function for through-the-thickness crack solutions with appropriate hole curvature, flaw shape, front free surface and back surface correction factors; (3) the rigorous three-dimensional cracked finite element method, which was developed using a hybrid displacement finite element procedure.

After the stress intensity factor is determined as a function of crack length for a particular lug and loading condition, its range (ΔK) can be computed for the Nth cycle in a given load spectrum. Then a numerical integration of da/dN versus ΔK relation can be carried out to predict the crack growth characteristics. An automated computer program was developed utilizing the state-of-the-art fracture mechanics methodologies for the prediction of fatigue crack growth history under fatigue loading. The program is capable of predicting the crack growth behavior of single corner cracks and single through-the-thickness cracks in attachment lugs using block-by-block integration technique. In the case of corner crack problems, a transitional crack growth criterion from part-through crack to through-the-thickness crack was also developed and incorporated. Several load-interaction models were also included in the crack growth analysis program for spectrum loadings.

The prediction methods presented in this volume of the report are to be evaluated against fatigue crack growth test results. The test program consists of two parts. Group I tests consist of straight-shank male attachment lugs. Both constant amplitude and block spectrum loadings are considered in this group. Variational tests, such as different thickness and with residual stresses due to interference-fit bushings, are also included in Group I testing. Group II tests, performed later, encompass several additional lug configurations (tapered, dogbone, clevis and a simulated wing-pylon attach lug) and include both axial and off-axis loadings.

The stress intensity factor formulations prepared for Group I straight-shank male attachment lugs are described in Section III. A draft of Section III was completed prior to Group I testing. After Group I testing, these formulations are evaluated. Based on this evaluation, a stress intensity factor estimation methodology is selected for Group II testing. Section IV describes such a selected methodology for tapered attachment lugs. The crack growth prediction method which employs these stress intensity factor solutions is presented in Section V.
SECTION III
STRESS INTENSITY FACTORS FOR STRAIGHT ATTACHMENT LUGS

In this section, the analytical methodologies required for the development of stress intensity factors for cracks in simple straight-shank male attachment lugs are described. The stress intensity factor solutions are obtained for single through-the-thickness cracks and single quarter-elliptical corner cracks in attachment lugs. As mentioned previously, the following methods are used for through-the-thickness and corner cracks.

For through-the-thickness crack problems:

1. Simple compounding method
2. Two-dimensional cracked finite element procedure
3. Weight function or Green's function method

For corner crack problems:

1. Simple one parameter compounding method
2. Two-dimensional Green's function solution modification
3. Rigorous three-dimensional cracked finite element procedure

This section also describes the method for developing the stress intensity factor for problems with residual stresses (due to the installation of interference-fit bushings). A stress analysis of unflawed straight attachment lugs was also performed and is presented here. Note that all the specimens tested in the Group I test program are straight attachment lugs and the methodologies described in this section are evaluated using Group I test results.

1. STRESSES IN AN UNFLAWED STRAIGHT ATTACHMENT LUG

A primary item of interest in the analysis of cracked attachment lugs is the stress distribution in the uncracked state. This is important for at least three reasons. The location of the peak tangential stress determines the location of the most critical crack. The stress intensity factor for a very small crack is proportional to the stress concentration factor value. Finally, the weight function method of estimating stress intensity factors requires the stresses on the respective crack surface in the uncracked lug.
The conventional displacement finite element method was used to obtain, for an uncracked straight lug, (1) the tangential stresses along the edge of the hole, (2) the stress distribution on the prospective crack surface, and (3) the pin-bearing pressure distribution along the contact surface between the pin and the lug. The rigidity ratio, $E_{\text{pin}}/E_{\text{lug}}$, is assumed to be 3.0 in the present analysis.

Figure 3-1 shows a typical model used in the analysis. Due to symmetry, only the upper half of the lug was modeled. The lug and the pin were represented by a set of constant-strain triangular and quadrilateral elements. To load the model, a concentrated force was applied at the center of the pin and reacted at the other end of the lug. Spring elements (S) were used to connect the pin and lug at each pair of nodes having identical nodal coordinates all around the periphery. The area of contact was determined iteratively by assigning a very high stiffness to spring elements which were in compression and a very low stiffness (essentially zero) to spring elements which were in tension. A neat-fit and no friction assumptions were made for the pin-to-lug assembly.

The calculated tangential stresses (normalized by the average bearing stress) along the edge of the hole for a pin loading applied in the $0^\circ$ direction are shown in Figure 3-2 for $R_o/R_i$ ratios of 1.50, 2.25 and 3.00. As anticipated, the maximum tangential stresses are located at about $90^\circ$ away from the axis of the lug, and the minimum stresses (small amount in compression) are located at the $180^\circ$ location.

The tangential stresses for pin loading applied in axial compression ($\theta = 180^\circ$) are shown in Figure 3-3. As can be seen from this figure, the resulting stresses at $\theta = 90^\circ$ are small tensile stresses. Thus, reversing the loading direction does not cause a stress reversal at the critical $90^\circ$ location. Furthermore, the tensile stresses produced are relatively small.

The unflawed elastic stress distributions on the prospective crack plane for axial tension applied in the $0^\circ$ direction are presented in Figure 3-4. These values are also tabulated in Table 3-1. The tangential stresses along the x-axis shown in the figure are normalized by the average bearing stress. It is clear that the gradient of the stress distribution close to the edge of the hole is very steep, especially for a lug having a smaller $R_o/R_i$ ratio. The corresponding elastic stress concentration factors at the edge of the hole, $\sigma_{\text{max}}/\sigma_{\text{br}}$, are plotted in Figure 3-5. The equation of the
Figure 3.1: Typical Finite Element Model for Unflawed Stress Analysis
Figure 3-2. Tangential Stress Along the Edge of the Hole for a Pin Loading Applied in 0° Direction
Figure 3-3. Tangential Stress Along the Edge of the Hole for a Pin Loading Applied in 180° Direction

\[ \sigma_{br} = \frac{P}{2 R_i B} \]
\[ \sigma_{br} = \frac{P}{2R_i B} \]

<table>
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<tr>
<th>SYMBOL</th>
<th>( R_o/R_i )</th>
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<tr>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
</tr>
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<td>2.5</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td></td>
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**Figure 3.4.** Unflawed Stress Distributions Along x-Axis for Straight Attachment Lugs Having Various \( R_o/R_i \) Ratios

Normalized Distance from Edge of Hole, \((x-R_i)/R_i\)
<table>
<thead>
<tr>
<th>$K_0/K_1$</th>
<th>$\frac{\sigma_y}{\sigma_0}$</th>
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<td>0.000*</td>
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<tr>
<td>0.95</td>
<td>1.218</td>
<td>0.651</td>
<td>0.529</td>
<td>0.432</td>
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<td>Extrapolated Values</td>
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</table>

* Extrapolated Values
Figure 3-5. Elastic Stress Concentration Factors for Straight Attachment Lugs

\[
K_{lb} = 2.75 \left( \frac{R_o - R_i}{R_i} \right)^{-0.675}
\]
logarithmic straight line shown in this figure fits the computed values within 0.5 percent, as shown in the accompanying table.

The computed pin-bearing pressure distributions along the contact surface are shown in Figure 3-6. The figure shows that the shape of the distribution is close to uniform at the central portion of the contact surface. As the $R_o/R_i$ ratio increases, the pressure decreases near $\theta = \pm 90^\circ$ but increases near $\theta = 0^\circ$.

2. STRESS INTENSITY FACTORS FOR THROUGH-THE-THICKNESS CRACKS

The development of stress intensity factor solutions for single through-the-thickness cracks in attachment lugs by various methods are described in this subsection. Also included in this subsection are the methods for analyzing attachment lugs with interference-fit bushings and elasto-plastic analysis for analyzing lugs which are loaded above yield.

2.1 COMPOUNDING METHOD APPROXIMATIONS

Stress intensity factors for an axially-loaded straight lug with a single through-the-thickness crack can be estimated by the compounding of known $K$ solutions. The compounding method has been used for crack growth analysis of complex structure [14].

As Figure 3-7 shows, the actual lug configuration is approximated as an infinite strip of width $2R_o$ containing a central hole of radius $R_i$. A single through-the-thickness crack is assumed to be present at $\theta = 90^\circ$ to the axis of the lug. The length of the crack is $a_1$, measured from the edge of the hole. The total pin load $P$ is assumed to result from a uniform distribution of radial pressure from $\theta = -67.5^\circ$ to $67.5^\circ$, and to be reacted by a remote uniform stress $\sigma_o = P/(2K_oB)$, where $B$ is the lug thickness.

The stress intensity factor for the configuration shown in Figure 3-7 can be approximated by combining a number of known $K$ solutions. The basic solutions are for cracks emanating from holes in infinite plates. Multiplicative correction factors are used to modify these infinite plate solutions to account for the effects of finite width.
Figure 3-6. Pin-Bearing Pressure Distributions Along the Contact Surface of Straight Lugs Subjected to a Pin Loading Applied in $0^\circ$ Direction
Figure 3-7. Approximation of Axially Loaded Straight Lug
2.1.1 Cracks at Holes in Infinite Plates - The loading shown in Figure 3-7 consists of pin loading in one direction and remote loading in the other. Tweed and Rooke [15] have obtained the solution for a crack emanating from a circular hole in an infinite plate subjected to remote loading. This is Case 31 in the compendium given in Appendix A. The formula for $K_{31}$ fits Tweed and Rooke's solution within 0.5 percent.

The general solution for the same geometry subjected to arbitrary radial loading at the hole is given in Reference [16]. Case 35 in Appendix A gives an equation which closely fits the special case of uniform radial pressure applied from $\theta = -67.5^\circ$ to $\theta = 67.5^\circ$, and from $(180^\circ - \theta) = -67.5^\circ$ to $(180^\circ - \theta) = 67.5^\circ$.

2.1.2 Symmetrical Line Cracks in Strips - The solution for a symmetrical line crack of length $2a_0$ in an infinite strip of width $2b$ under uniform tension stress $\sigma_0$ is well known. It is given in Appendix A as Case 21. The function $\varphi_{21}(\lambda_0)$ is called the width correction factor, because it is applied multiplicatively to the infinite-plate $K$ solution to give the correct solution for the strip.

Case 22 in Appendix A gives an approximate solution for the symmetrically-cracked strip subjected to concentrated splitting forces $P$ applied to both surfaces of the crack at the centerline. This approximation is derived by the similarity method from three known $K$ solutions, as follows:

Figure 3-8 shows the four similar configurations. In all four cases the crack lengths are the same and the shear stresses vanish along the lines $x = \pm b$. The unknown solution is $K_{22}$; the solutions for the other three cases shown are known in closed form. $K_{19}$ is the well-known solution for periodic collinear cracks subjected to uniform loading [17]:

$$K_{19} = S \sqrt{\pi a_0} \sqrt{\frac{2b}{\pi a_0}} \tan \frac{a_0}{2b}$$

The solution $K_{20}$ for periodic collinear cracks subjected to central splitting forces on the crack surfaces is given by Tada, et al [18].
Figure 3.14. Similarity Method for Approximating $K_{22}$
Notice in Figure 3-8 that Cases 21 and 22 are identical geometries, as are Cases 19 and 20. Furthermore, Cases 20 and 22 are identical in the applied loadings, as are Cases 19 and 21.

Thus, the correction factor approach can be applied in two alternative ways to estimate $K_{22}$. The ratio $(K_{21}/K_{19})$, a correction factor to account for the difference between a finite-width strip and a periodic crack array, can be multiplied times $K_{20}$, the solution for the periodic crack array with the Case 22 loading condition. Alternatively, the ratio $(K_{20}/K_{19})$, a correction factor to account for the difference between remote loading and point loading on the crack line, can be multiplied times $K_{21}$, the solution for remote loading of the Case 22 geometry. Either way, the resulting equation for $K_{22}$ is the same:

$$K_{22} = \frac{K_{21}K_{20}}{K_{19}}$$

2.1.3 Effect of Eccentricity — The crack in the lug is eccentrically located, so eccentricity must be considered. Isida [19] has obtained the stress intensity solution for an unsymmetrical crack in a strip under uniform tension. Equations are given in Appendix A, Case 26, which fit Isida's numerical results within about 3 percent (within 1 percent for $a/b_n \leq 0.6$). There are two stress intensity formulas, $K_{26}^{(N)}$ for the crack tip nearest the edge and $K_{26}^{(F)}$ for the crack tip farthest from the edge. For the symmetrical crack both formulas reduce to the equation for $K_{21}$.

Case 27 in Appendix A gives an approximate solution for the unsymmetrical crack subjected to splitting forces on the crack surfaces at the strip centerline. This solution is obtained by the Similarity Method from the five known $K$ solutions shown in Figure 3-9. (The solutions for Cases 12 and 13 are well known and are given in Reference [18].) There are two eccentricities in Case 27: the crack is not centered on the strip centerline, and the load is not centered on the crack centerline. Remove both eccentricities and Case 22 is obtained.
Figure 1-9. Similarity Method for Approximating $K_{27}$
The crack is not centered on the strip in Case 26. Remove this eccentricity and Case 21 is obtained. Therefore, the correction factor to account for eccentricity between the crack and the strip is the ratio \( K_{26}/K_{21} \).

The load is not centered on the crack in Case 13. Remove this eccentricity and Case 12 is obtained. Therefore, the correction factor to account for eccentricity between the loading point and the crack is the ratio \( K_{13}/K_{12} \).

When both correction factors are applied to \( K_{22} \) the approximate formula for \( K_{27} \), given in Appendix A, is obtained:

\[
K_{27}^{(N)} = K_{22} \frac{K_{26}^{(N)}}{K_{21}} \cdot \frac{K_{13}}{K_{12}}
\]  

(4)

2.1.4 Cracks at Holes in Strips — The stress intensity solutions for cracks at holes in infinite plates and for line cracks in strips can be combined to obtain approximate \( K \) formulas for cracks at holes in strips. The resulting formulas can be superimposed to estimate \( K \) for a pin-loaded lug with a crack.

Case 41 in Appendix A gives an approximate solution for a uniformly loaded strip with a crack emanating from a central hole. Referring to Figure 3-10, one would expect the solution for \( K_{41} \) to be approximated by the product \( K_{31} K_{26}/K_{11} \).

This product, however, does not account for the interaction between the hole and the edges of the strip. The gross area stress concentration factor given by Peterson [20] for a center hole in a strip can be approximated by the following formula:

\[
k_c = 3 \sec \left( \frac{\pi r}{2b} \right) - \sqrt{3} \Phi_{21}^2 \left( \frac{r}{b} \right)
\]  

(5)

As the crack length \( a_1 \) approaches zero, \( K_{41} \) has a known asymptotic solution in terms of the stress concentration factor, i.e.

\[
\lim_{a_1/r \to 0} (K_{41}) = k_t \left( \frac{r}{b} \right) \frac{K_{31}}{3} - \Phi_{21}^2 \left( \frac{r}{b} \right) K_{31}
\]  

(6)
Figure 1-10. "Attempted" Similarity Method for Approximating \( K_{41} \)
To satisfy Equation (6), a hole-strip width correction factor $\Phi_{21}(r/b)$ must be included, so that the approximate solution for Case 41 is

$$K_{41} = \frac{K_{31} K_{26}}{K_{11}} \Phi_{21}(r/b)$$ \hspace{1cm} (8)

Case 44 in Appendix A is an approximate solution for a strip with a crack at a central hole, loaded symmetrically at the hole. This solution is derived by the same approach used to derive $K_{44}$. In this case, however, the gross area stress concentration factor for the uncracked case is not known. Figure 3-5, obtained by finite element analysis, could have been used here to estimate the required $k_e$, but to do so would have compromised the independence of the solutions generated by the compounding and finite element methods. Based on engineering judgment, therefore, $\Phi_{21}(r/b)$ is appended to the product solution obtained from Figure 3-11 to approximate $K_{44}$, just as was done for Case 41. Thus, $K_{44}$ is approximated by the equation

$$K_{44} = \frac{K_{35} K_{27}}{K_{13}} \Phi_{21}(r/b)$$ \hspace{1cm} (9)

2.1.5 Cracked Lug - The final step in the derivation of a compounding method stress intensity factor for the pin-loaded straight lug is a simple superposition. As demonstrated in Figure 3-12, the result of superimposing Cases 41 and 44 is identical to the result of superimposing the lug approximation in Figure 3-7 with itself (oriented upside-down and backwards). Thus

$$K_{ Lug} = \frac{1}{2} (K_{41} + K_{44})$$ \hspace{1cm} (10)

Equation (10) was used to compute the stress intensity factors for $K_0/K_1$ ratios of 1.50, 2.25 and 3.0, and the results are tabulated in Table 3-2.

2.2 TWO-DIMENSIONAL CRACKED FINITE ELEMENT METHOD

Because of the ease with which the finite-element method handles complex geometries and boundary conditions, this method has been used extensively to study fracture in complex structures. Two special crack-tip
Figure 3-11. "Attempted" Similarity Method for Approximating $K_{44}$
Figure 3-12. Superposition of Cases 41 and 44
TABLE 3-2. NORMALIZED STRESS INTENSITY FACTORS FOR SINGLE THROUGH-THE-
THICKNESS CRACKS IN STRAIGHT ATTACHMENT LUGS OBTAINED USING
THE COMPOUNDING METHOD

<table>
<thead>
<tr>
<th>( \frac{a_1}{(R_o - R_i)} )</th>
<th>( \frac{K}{(\sigma_{br} \sqrt{\pi a_1})} ), for ( R_o / R_i )</th>
<th>( \frac{K}{(\sigma_{o} \sqrt{\pi a_1})} ), for ( R_o / R_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50</td>
<td>2.25</td>
</tr>
<tr>
<td>0.0</td>
<td>4.472</td>
<td>2.283</td>
</tr>
<tr>
<td>0.1</td>
<td>4.134</td>
<td>1.752</td>
</tr>
<tr>
<td>0.2</td>
<td>3.899</td>
<td>1.449</td>
</tr>
<tr>
<td>0.3</td>
<td>3.751</td>
<td>1.267</td>
</tr>
<tr>
<td>0.4</td>
<td>3.683</td>
<td>1.159</td>
</tr>
<tr>
<td>0.5</td>
<td>3.702</td>
<td>1.103</td>
</tr>
<tr>
<td>0.6</td>
<td>3.827</td>
<td>1.094</td>
</tr>
<tr>
<td>0.7</td>
<td>4.115</td>
<td>1.138</td>
</tr>
<tr>
<td>0.8</td>
<td>4.721</td>
<td>1.273</td>
</tr>
<tr>
<td>0.9</td>
<td>6.287</td>
<td>1.659</td>
</tr>
</tbody>
</table>
singularity elements, usually referred to as cracked elements, developed at Lockheed-Georgia [8] have brought the power and flexibility of the finite-element method to bear much more effectively on fracture mechanics problems. This method has been used to analyze a crack in an unsymmetric lug of a C-5 engine pylon [8]. It has been modified slightly to accurately account for the change in pin-bearing pressure distribution for the change in crack length. The following discussion describes this method as it was used in the calculation of stress intensity factors for cracks at attachment lugs.

Figure 3-13 shows a typical finite element model used for a single crack emanating from a straight attachment lug. A 10-node high-order singularity element [8] was used at the crack tip region for computing the stress intensity factors. The remainder of the finite element modeling details was identical to those described in the stress analysis section for the uncracked lug, including the iterative determination of the contact area. The solution was obtained for several outer-to-inner radius ratios, \( R_o/R_i \), and for a succession of cracks having normalized lengths, \( c/(R_o - R_i) \) ranging from 0.1 to 0.9. As in the case of stress analysis, the rigidity ratio, \( E_{pin}/E_{lug} \), of 3.0 is assumed in the fracture analysis.

The pin-bearing pressure distributions obtained for single cracks emanating from attachment lugs loaded by neat-fit pins are presented in Figures 3-14 through 3-16. These figures show the effect of crack length on the pin-bearing pressure distributions for a straight lug having \( R_o/R_i \) ratios of 1.5, 2.25 and 3.00, respectively. When there is no crack, the distribution is close to uniform at the central portion of the contact surface, unlike the cosine distribution commonly assumed in literature. There is no contact at the expected crack location, \( \theta = -90^\circ \). However, as soon as a crack appears, the contact spreads to one side ("upper lip") of the crack mouth. As the crack opens, the other side ("lower lip") of the crack mouth moves away from the pin resulting in no pin-bearing pressure on the lower lip. Figures 3-14 through 3-16 show that the longer the crack, the higher the pin-bearing pressure on the upper lip of the crack mouth, especially for a small \( R_o/R_i \) ratio. For example, for a \( R_o/R_i \) ratio of 1.5, the normalized pressure at the upper lip of the crack mouth increases from zero for \( c/R_i = 0 \) to more than twice the average pressure for \( c/R_i \geq 0.3 \). The pressure distribution elsewhere on the contact surface also changes with the crack length. For larger crack sizes, the pressure decreases from an initial
Figure 3-13. Typical Finite Element Model for Stress Intensity Factor Analysis
Figure 3-14. Pin-Bearing Pressure Distributions for Single Cracks at a Straight Attachment Lug Having a $R_o/R_i$ Ratio of 1.50
Figure 3-15. Pin-Bearing Pressure Distributions for Single Cracks at a Straight Attachment Lug Having a $R_o/R_i$ Ratio of 2.25
Figure 3-16. Pin-Bearing Pressure Distributions for Single Cracks at a Straight Attachment Lug Having a $r_0/r_1$ Ratio of 1.00
maximum value at the crack mouth to a local minimum and then gradually increases again and approaches another maximum before decreasing to zero at the end of the contact surface.

The effect of pin-bearing pressure distributions on stress intensity factors is shown in Figure 3-17 for single cracks in a straight lug having a $R_o/K_i$ ratio of 1.5. In Figure 3-17, the circle and triangle symbols are results obtained from Reference [9], where it is assumed that the pin-bearing pressure distributions are cosine and uniform distributions along the 180-degree contact surface, respectively, and that such distributions remain unchanged with the crack length. The square symbol represents the results obtained using the present analysis, which has properly accounted for the change in pressure distribution as the result of crack extension. As can be seen from this figure, when the crack is small ($c/R_i < 0.05$), the current computed $K$-value is practically the same as that obtained using a uniform pin-bearing pressure distribution. However, as the crack length increases, the current analysis gives a lower $K$-value than the others. This is because when the crack length increases, the pressure near $\theta = -90^\circ$ increases markedly and exceeds the average pin-bearing pressure (see Figure 3-14). This high pin-bearing pressure, when applied near $\theta = -90^\circ$ in the direction almost parallel to the crack orientation, tend to close the crack surfaces, hence reducing the stress intensity factor as discussed by Brussat [16].

The computed normalized opening mode stress intensity factors using a steel pin and an aluminum lug model are presented in Figure 3-18 as a function of normalized crack length ($c/R_i$) for single cracks emanating from the hole wall of straight attachment lugs with $R_o/R_i$ ratios ranging from 1.5 to 3.0. In all cases, the computed sliding-mode stress intensity factors, $K_{II}$, are much smaller than those of the opening mode, $K_I$, so they are not presented in the figures. It should be noted that the $K$-values were normalized in terms of the average bearing stress, $\sigma_{br}$, instead of far-field gross section stress, $\sigma_o$. To convert these normalized factors in terms of $\sigma_o$, one can simply multiply these normalized factors by the corresponding ratio of $R_o/K_i$. For convenience, the computed $K$-values normalized in terms of the average far-field stress are tabulated in Table 3-3 as a function of

---

*The results are mislabeled in Reference [9]; uniform distribution results are labeled cosine distribution and vice versa.*
Figure 3-17. Sensitivity of Stress Intensity Factors to Pressure Distribution ($R_o/R_1 = 1.50$)
Figure 3-14. Normalized Stress Intensity Factor Solutions for Single Cracks at Attachment Lugs by Finite Element Method
### Table 3-3. Normalized Stress Intensity Factors for Single Through-the-Thickness Cracks in Straight Attachment Lugs Using Cracked Finite Element Method

<table>
<thead>
<tr>
<th>$c/(R_o - R_i)$</th>
<th>$K/(\sigma_{br} \sqrt{\pi c})$</th>
<th>$K/(\sigma_o \sqrt{\pi c})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50</td>
<td>2.00</td>
</tr>
<tr>
<td>0.0</td>
<td>4.916</td>
<td>3.066</td>
</tr>
<tr>
<td>0.1</td>
<td>3.580</td>
<td>2.237</td>
</tr>
<tr>
<td>0.2</td>
<td>3.110</td>
<td>1.789</td>
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<tr>
<td>0.3</td>
<td>2.945</td>
<td>1.639</td>
</tr>
<tr>
<td>0.4</td>
<td>2.816</td>
<td>1.506</td>
</tr>
<tr>
<td>0.5</td>
<td>2.820</td>
<td>1.472</td>
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<tr>
<td>0.6</td>
<td>2.824</td>
<td>1.442</td>
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<tr>
<td>0.7</td>
<td>2.981</td>
<td>1.509</td>
</tr>
<tr>
<td>0.8</td>
<td>3.219</td>
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</tr>
<tr>
<td>0.9</td>
<td>4.080</td>
<td>2.044</td>
</tr>
</tbody>
</table>
normalized crack length (normalized by the net section). Note that at the edge of the hole (c/R_i = 0), the normalized stress intensity factor was obtained by multiplying the stress concentration factor determined from unflawed stress analysis by 1.12, which was derived by Gross et al. [21] for a straight edge crack in a finite-plate specimen loaded in tension. Unflawed elastic distributions on the prospective crack plane and the corresponding stress concentration factors at the edge of the hole have been given in Figures 3-4 and 3-5, respectively.

Studies were also made to investigate the effects of a single crack versus a symmetrical double crack and relative rigidity of the pin and the lug (E_{pin}/E_{lug}). A typical comparison of K-values obtained for a single crack and a symmetrical double crack in a straight aluminum lug loaded by a steel pin (E_{pin}/E_{lug} = 3.0) is shown in Figure 3-19. As anticipated, the computed K-values for a double crack are higher than those of a single crack, and the difference increases with the crack length. It increases from less than 1% for c/R_i ≤ 0.2 to about 37% for c/R_i = 0.9. To study the effect of the relative rigidity of the pin and the lug on the stress intensity factors, the computed K-values for a single crack emanating from the hole wall of a steel lug loaded by a steel pin (E_{pin}/E_{lug} = 1.0) is also included in Figure 3-19. As presented in this figure, the K-values computed for a combination of a steel pin and steel lug is slightly higher than that of the steel pin and the aluminum lug. A comparison of stress distributions along the θ = -90° radial line for steel lug-steel pin and aluminum lug-steel pin combinations is presented in Figure 3-20. These stresses can be used in conjunction with Green's function to calculate the stress intensity factors appropriately.

2.3 THE WEIGHT FUNCTION METHOD

The linear superposition method has been used frequently to obtain the stress intensity factors for various types of crack problems. The principle of superposition of linear elasticity implies that, for the purpose of calculating stress intensity factors, loading the crack faces with G(x) is equivalent to loading the cracked body with loads which produce G(x) on the prospective crack faces in the absence of a crack.

Figure 3-21 shows the scheme of the linear superposition method. The stress intensity factor of problem 3-21a is equivalent to the sum of that of
Figure 7-19. Normalized Stress Intensity Factors for Cracks Emanating From Straight Lugs for Single and Symmetric Cracks and for Various $E_{\text{pin}}/E_{\text{lug}}$ Ratios ($R_0/R_1 = 2.25$)
Figure 3-20. Unflawed Stress Distribution in Straight Attachment Lugs for Various $E_{\text{pin}}/E_{\text{lug}}$ Ratios ($R_o/R_1 = 2.25$)
Figure 3-21. Schematic of Linear Superposition Method

\[ \sigma_0 = \frac{P}{2R_o B} \]
problems 3-21b and 3-21c. Since problem 3-21b is crack free, the stress-intensity factor of problem 3-21a is equivalent to that of problem 3-21c.

By idealizing the stress in problem 3-21c as $N$ discrete loads, $P_1, \ldots, P_N$, then the stress-intensity factor, for a given crack length $c$, can be computed from the following equation

$$K(c) = \sum_{i=1}^{N} K_i = \sum_{i=1}^{N} k_i(x_i, c) P_i(x_i)$$

where $k_i(x_i, c)$ is the normalized stress-intensity factor due to the $i$th load, $P_i$, applied at location $x_i$. For arbitrary distributed stress, $\sigma(x)$, instead of discrete forces, $P_i$, Equation (11) becomes

$$K(c) = \int_{0}^{c} k(x, c) \cdot \sigma(x) \, dx$$

In Equation (12), $k(x, c)$ is the weight function (or Green's function). Bueckner [22] and Rice [23] defined the weight function as

$$k(x, c) = \frac{H}{2K(c)} \cdot \frac{\partial u(x, c)}{\partial c}$$

for a symmetrical load system on a linearly elastic body containing a crack of length $c$. In the above equation, $H$ is an appropriate elastic modulus: it is $E/(1-\nu^2)$ for plane strain and $E$ for generalized plane stress. $K(c)$ is the known stress intensity factor and $u(x, c)$ is the $y$-component of the crack surface displacement at $x$. The weight function was shown [22, 23] to be unique for a given structural geometry and crack size regardless of the loading condition. Therefore, it can be developed for one load condition and then utilized to determine the stress-intensity factor for any other load condition.

The closed form expressions for the weight function for edge cracks [11, 24], center cracks [25] and collinear cracks [26] in a wide panel are available. However, the closed form weight function for cracks emanating from a hole is not available. Therefore, the weight function for a straight center crack has sometimes been used to estimate the stress-intensity factor for radial cracks emanating from a circular hole [27-29], located in the
geometric center of a long plate. Impellizzeri and Rich [10] modified the weight function derived by Bueckner [11] for an edge crack in a semi-infinite plate to include the geometric correction factors for estimating the stress-intensity factor of a crack in an attachment lug. In order to establish the appropriate values of the derivative $\partial u/\partial c$, to be used in Equation (13), Grandt [30] supplemented Bowie's solutions [31] for a radially-cracked hole with a finite element analysis to obtain the corresponding crack-mouth opening displacements. Hsu and Rudd [32] developed the Green's function from the stress-intensity factors computed using the high-order singularity element for a double-radial crack emanating from an open hole and subjected to a pair of concentrated loads on and perpendicular to the crack surface. This approach is capable of accounting for the effect of hole curvature, finite width and the profile of the lug head. Therefore, it was used to develop the weight function for through-the-thickness cracks emanating from the hole of attachment lugs.

By defining the weight function $G = k(c/\pi)^{1/2}$ and $\xi = (x-R_i)/c$ and substituting them into Equation (12), one obtains

$$K(c) = \sigma_o \sqrt{\pi c} \int_0^1 G(c, \xi) \tilde{\sigma}(\xi) \, d\xi$$

(14)

where $\sigma_o$ is the uniform far-field stress and $\tilde{\sigma} = \sigma/\sigma_o$ is the normalized unflawed stress distribution on the prospective crack surface.

The weight function $G$, for a single radial crack emanating from a circular hole of the lug and subjected to a pair of concentrated forces on the crack surfaces, as shown in Figure 3-22a, is obtained from the computed stress-intensity factor using 2-D cracked finite element analysis for various $R_o/R_i$ ratios, crack lengths $c/(R_o-R_i)$, and $(x-R_i)/c$ ratios as follows:

$$G \left( \frac{c}{(R_o-R_i)}, \frac{x}{c}, \frac{R_o}{R_i} \right) = k(x,c) \sqrt{a/\pi}$$

(15)

Finite element methodology becomes difficult to employ when the concentrated forces are applied close to the crack tip, say $(x-R_i)/c > 0.9$. In this range the corresponding weight functions were obtained using the edge crack model as shown in Figure 3-22b. The weight function, $G_1$, for such edge cracks is available in Reference [18], and can be written as
Figure 3-22. Model for Generating the Weight Functions
\[ G_1 = \frac{2}{\pi} \left\{ \frac{3.52(1-(x-R_i)/c)}{\left(1 - \frac{c}{2b}\right)^{3/2}} - \frac{4.35 - 5.28 (x-R_i)/c}{(1 - \frac{c}{2b})^{1/2}} \right\} + \]

\[ \left[ \frac{1.30-0.30 \left((x-R_i)/c\right)^{3/2}}{\sqrt{1 - \left((x-R_i)/c\right)^2}} + 0.83-1.76 \left(x-R_i\right)/c \right] \left[1-(1-(x-R_i)/c) \frac{c}{b}\right] \]  

In the conventional finite element method, the external force can only be applied at nodal points. When the crack length is small, it becomes cumbersome to refine the model such that there will be enough nodes along the crack faces for the purpose of computing the \( K \) and \( G \) values. Therefore, an alternate approach is used. For each crack length \( c \), the \( K \) and \( G \) values were calculated at each available nodal point on the crack face, say \( (x_i-R_i)/c \), using a finite element model for the configuration shown in Figure 3-22a and Equation (15). The weight function \( G_1 \) for an edge crack in a finite width strip (Figure 3-22b) was then calculated at the same locations, \( (x_i-R_i)/c \), using Equation (16), and the ratio was obtained.

\[ r \left((x_j-R_i)/c\right) = G \left((x_j-R_i)/c\right)/G_1 \left((x_j-R_i)/c\right) \]  

Assuming that \( r \) approaches 1 as \( (x_j-R_i)/c \) approaches 1, a least squares polynomial fit is obtained.

\[ r \left((x-R_i)/c\right) = \sum_{j=1}^{M} C_j \left((x-R_i)/c\right)^{j-1} \]  

The weight function at any location on the entire crack surface can then be computed using Equations (16) and (18) as

\[ G \left((x-R_i)/c\right) = r \left((x-R_i)/c\right) G_1 \left((x-R_i)/c\right) \]  

The results obtained using the aforementioned procedure are shown in Figures 3-23 through 3-25 for \( R_o/R_i \) ratios of 1.50, 2.25, and 3.00, respectively. The symbols shown in these three figures are the discrete values obtained using the finite element method and the curves are obtained
Figure 1-23. Original Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_i$ Ratio of 1.50
Figure 3-24. Original Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_i$ Ratio of 2.25
Figure 3-25. Original Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_i$ Ratio of 3.00
using Equation (19). As seen from these figures, this alternate approach gives an excellent estimation of G-values at any desired location.

With a knowledge of the weight functions, G, and the unflawed stress, \( \sigma \), on the prospective crack surface with the crack absent, one can numerically integrate in Equation (14) to obtain the corresponding stress-intensity factor for a radial crack in an attachment lug.

Integration of Equation (14) can be carried out accurately and efficiently by the Gaussian quadrature formula. In order to perform the Gaussian integration, the integral in Equation (14) needs to be transferred into the well-known Newton-Cotes quadrature formula for the range of integration between \(-1\) and \(+1\). This can be easily done by assuming that

\[
\zeta = 2\xi - 1, \quad \text{or} \quad \xi = \frac{1}{2} (\zeta + 1), \quad (20)
\]

and the integral of Equation (14) becomes

\[
\int_{0}^{1} \bar{\sigma}(\xi) G(c, \xi) d\xi = \int_{-1}^{1} F(\zeta) d\zeta = \sum_{i=1}^{n} H_i F(\zeta_i) \quad (21)
\]

where

\[
F(\zeta) = \frac{1}{2} \bar{\sigma} \left( \frac{\zeta + 1}{2} \right) G \left( c, \frac{\zeta + 1}{2} \right) \quad (22)
\]

The abscissae (\( \zeta_i \)) and weight coefficients (\( H_i \)) of the Gaussian quadrature formula, Equation (21), for \( n \) up to 10 are given in Reference [33]. With this transformation, the stress-intensity factor can then be calculated from the following equation:

\[
K = \sigma_0 \sqrt{\pi c} \sum_{i=1}^{n} H_i F(\zeta_i) \quad (23)
\]

An evaluation was made on the calculation of the stress-intensity factor using Equation (23). It was found that, for \( n = 9 \), the computed \( K \) values using Gaussian integration are essentially the same as the ones obtained using trapezoidal rule numerical integration with 400 equal integration intervals.
Therefore, the more economical Gaussian integration was exclusively used in the \( K \) calculation using the weight function approach.

An important point on the development of the Green's function for attachment lugs is to be noted at this point. In the case of an attachment lug loaded by a pin, which is a contact problem, the pin bearing pressure distribution between the lug and the pin varies with the crack length (Figures 3-14 through 3-16). The pin bearing pressure distribution has a significant effect on the unflawed stress distribution (on the prospective crack surface) and on the stress-intensity factors (Figure 3-17). In the case of finite element analysis of cracked lugs, the pin bearing pressure distribution was taken into account in the analysis itself. In the case of the Green's function method, the pin bearing pressure distribution variation can be accounted for by two methods.

The first method is to use the Green's function developed above which should strictly be used only in conjunction with the correct unflawed stress distribution on the prospective crack surface for the varying pin bearing stress distribution as the crack length changes. The problem of calculating the pin bearing pressure distribution as the crack length changes is statically indeterminate, unless a fracture analysis, such as the finite element solution described before, is carried out. But, once a fracture analysis is made, then there is no need for the varying pin bearing pressure distribution, because the needed stress-intensity factors can be obtained from the fracture analysis. Thus, this method becomes redundant and is discarded.

The second method is to use available data of stress-intensity factors (from the finite element procedure, Figure 3-18) and the unflawed stress distribution on the prospective crack surface (Figure 3-4) to modify the Green's functions to account for the varying pin bearing pressure distribution. Such modifications have been made for the Green's function and are presented in Figures 3-26 through 3-28 for outer-to-inner radius ratios of 1.50, 2.25 and 3.00, respectively, whereas Figures 3-23 through 3-25 correspond to the original or unmodified Green's function. For calculating the stress-intensity factors in simple attachment lugs loaded by pins, the Green's functions in Figures 3-26 through 3-28 should be used. The Green's functions given in Figures 3-23 through 3-25 are still useful and needed in the analysis of attachment lugs where the unflawed stress distribution on the
Figure 3-26. Modified Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_t$ Ratio of 1.50
Figure 3-27. Modified Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_i$ Ratio of 2.25
Figure 3-28. Modified Weight Functions for a Single Through-the-Thickness Crack at Straight Attachment Lug Having a $R_o/R_i$ Ratio of 1.00
prospective crack surface does not depend on crack length, for example, residual stresses due to the installation of an interference-fit bushing. Tables 3-4 through 3-9 provide the original and modified Green's function values in a tabular form at several \((x-R_i)/c\) locations and different crack lengths for outer-to-inner radius ratios of 1.50, 2.25 and 3.00.

In order to put the results in perspective, Figure 3-29 compares the stress-intensity factors obtained by the compounding method, finite element method and original and modified Green's function methods. As can be seen from the figure, the results obtained by the modified Green's function method match the finite element solutions. The results of the original or unmodified Green's function method tend to be higher, especially for the \(R_o/R_i\) ratio of 1.50. The difference between the two Green's function methods essentially reflects the effect of pin bearing pressure distribution as the crack length changes.

Figure 3-29 also compares the compounding method solution to the other solutions. For \(R_o/R_i\) ratios of 2.25 and 3.00, the compounding solution gives excellent estimates of stress-intensity factors, except when the crack length is very small. The error at small crack lengths could be corrected by making proper use of the stress concentration factor values for the lugs, \(K_{tb}\), given in Figure 3-5. However, for \(R_o/R_i = 1.50\) the compounding solution tends to overestimate the stress-intensity factors, like the original Green's function method. This error may be partially due to the major differences between the true pin bearing pressure distribution and the idealized distribution used in the compounding method.

2.4 ANALYSIS OF LUGS WITH INTERFERENCE-FIT BUSHINGS

In order to improve the crack initiation and crack growth life, the concept of installing an interference-fit bushing to introduce beneficial residual stresses around the hole of the lug prior to pin fitting has been used in aircraft attachment lug design. For a given fatigue load cycle, the installation of an interference bushing can reduce the effective tangential stress range at the likely location of crack initiation, resulting in improvement in fatigue and crack growth life. It also reduces fretting damage of the hole wall of the lug.
### Table 3-4. Original Weight Function for $R_o/R_1 = 1.50$

<table>
<thead>
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<th>$(x-k_1)/c$</th>
<th>$c/(k_o-k_1)$</th>
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<td>1.4025</td>
<td>1.6096</td>
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**TABLE 3-9. MODIFIED WEIGHT FUNCTION FOR R_o/R_1 = 3.00**
Figure 3.29. Comparison of Stress Intensity Factors Computed Using the Compounding, Weight Function and Cracked Finite Element Methods
The stress-intensity factors for a crack emanating from an attachment lug having an interference-fit bushing can be estimated using the weight function approach similar to the one reported in Reference [34]. The procedure consists of two major steps. First, the effective unflawed stress distribution on the prospective crack surface was obtained by superimposing the residual hoop stresses, $\sigma_{re}$, due to the installation of an interference-fit bushing on the applied tangential stresses, $\sigma_{ap}$, obtained due to the application of a pin loading. Next, a crack was introduced in this stress field by removing the tractions on the crack faces and computing the corresponding effective stress-intensity factor using the developed weight function, $G$, and the following equation:

$$K = \sqrt{\pi c} \int_{0}^{1} (\sigma_{ap} + \sigma_{re}) \cdot G(c, \xi) d\xi$$

(24)

**Unflawed Stress Analysis**

The installation of an interference-fit bushing creates a compressive radial stress and a tensile hoop stress in the lug in a manner similar to a thick-walled cylinder under internal pressure. On this basis, an approach similar to that of Seely and Smith [35] for a thick-walled cylinder under internal pressure is used to compute the residual stresses in the lug due to the installation of an interference-fit bushing. Before the residual stresses in the lug can be computed, the pressure, $P_s$, on the surface of contact between the bushing and the lug must be determined.

Let $R_o$ and $R_i$ denote outer and inner radii of the lug and $r_o$ and $r_i$ denote outer and inner radii of the bushing before the interference-fit installation, respectively. Let $R_i$ be the inner radius of the lug (and also the outer radius of the bushing) after installation, and let $\Delta_R$ be the difference in these radii before installation.

$$\Delta_R = R_o - R_i$$

(75)

After the installation of the bushing, the inner radius of the lug will be larger than its initial value by an amount, $\delta_i$, which is related to the unknown contact pressure as
\[
\delta_1 = \frac{P_s R_1}{E_L} \left( \frac{R_o^2 + R_1^2}{R_o^2 - R_1^2} + \mu_L \right) \tag{26}
\]

where \(E_L\) and \(\mu_L\) are the Young's modulus and Poisson's ratio of the lug.

At the same time, the outer radius of the bushing will be changed (decreased) by an amount, \(\delta_2\), which is given by the following equation:

\[
\delta_2 = \frac{P_s R_1}{E_B} \left( \frac{R_1^2 + r_1^2}{R_1^2 - r_1^2} - \mu_B \right) \tag{27}
\]

where \(E_B\) and \(\mu_B\) are the Young's modulus and Poisson's ratio of the bushing.

At the completion of the installation process, the difference in these original radii, \(\delta_R\), disappears as a result of the changes in the length \(\delta_1\) and \(\delta_2\). Therefore, the sum of the magnitudes of \(\delta_1\) and \(\delta_2\) is equal to \(\delta_R\) i.e.,

\[
\delta_R = \frac{P_s R_1}{E_L} \left[ \left( \frac{R_o^2 + R_1^2}{R_o^2 - R_1^2} + \mu_L \right) + \frac{E_L}{E_B} \left( \frac{R_1^2 + r_1^2}{R_1^2 - r_1^2} - \mu_B \right) \right] \tag{28}
\]

The contact pressure is obtained from the above equation as

\[
P_s = \delta_R E_L R_1 \left[ \left( \frac{R_o^2 + R_1^2}{R_o^2 - R_1^2} + \mu_L \right) + \frac{E_L}{E_B} \left( \frac{R_1^2 + r_1^2}{R_1^2 - r_1^2} - \mu_B \right) \right] \tag{29}
\]

With contact pressure determined, the residual stresses in the lug due to the installation of an interference-fit bushing can be calculated using the following equations:
\[ \sigma_\theta = \frac{P_s R_1^2}{R_2^2 - R_1^2} \left[ \frac{R_1^2}{x^2 + 1} \right] \]

and

\[ \sigma_r = \frac{P_s R_1^2}{R_2^2 - R_1^2} \left[ \frac{R_1^2}{x^2 - 1} \right] \]

\[ (30) \]

where \( x \) is the radial distance as shown in Figure 3-30.

Equation (30) can be used to compute the residual tangential stresses along the radial direction (\( x \)-axis) in attachment lugs due to the installation of interference-fit bushings. In the following discussion, results are presented for attachment lugs with lug outer radius to bushing inner radius ratio \((R_o/r_i)\) of 2.25. The nondimensional parameters considered in the analysis are interference levels \((\delta R/r_i)\), bushing-to-lug rigidity \((E_B/E_l)\) and bushing thickness \((t_B/r_i)\). The effects of these parameters on the stress distribution along the \( x \)-axis in attachment lugs are presented in Figures 3-30 through 3-32. Figure 3-30 shows the increase in residual stresses with increasing diametral interference for a bushing-to-lug rigidity ratio \((E_B/E_l)\) of 3.0 and a bushing thickness \((t_B/r_i)\) of 0.12. Figure 3-31 shows the increase in residual stresses with increasing bushing-to-lug rigidity ratio \((E_B/E_l)\) for a bushing thickness \((t_B/r_i)\) of 0.12 and an interference level \((\delta R/r_i)\) of 0.00533. Figure 3-32 shows the increase in residual stresses with increasing bushing thickness \((t_B/r_i)\) for a bushing-to-lug rigidity ratio \((E_B/E_l)\) of 3.0 and an interference level \((\delta R/r_i)\) of 0.00533.

The tangential stresses along the \( x \)-axis due to the application of pin loading are then computed using the finite element method. In the analysis, it is assumed that the bushing and the lug remain in contact and that no slippage occurs along the hole wall surface during the application of the load. The computed tangential stress normalized by the far-field applied stress for a lug with a \( R_o/r_i \) of 2.25 are shown in Figures 3-33 and 3-34. Figure 3-33 depicts the decrease in stresses due to pin loading with increasing bushing-to-lug rigidity ratio \((E_B/E_l)\) for a constant bushing thickness \((t_B/r_i)\) of 0.12. Figure 3-34 presents the decrease in stresses due to pin loading with increasing bushing thickness \((t_B/r_i)\) for a constant bushing-to-lug rigidity ratio \((E_B/E_l)\) of 3.0.
Figure 3-10. Stress Distributions Along x-Axis in an Attachment Lug
With Bushing for Various Levels of Interference
\[ \frac{R_0}{r_i} = 2.25 \quad \frac{t_B}{r_i} = 0.12 \]
\[ \frac{\delta_R}{r_i} = 0.00533 \]
\[ \mu_B = \mu_L = 0.3 \]

Figure 3-11. Stress Distributions Along x-Axis in an Attachment Lug with Bushing for Various Bushing-to-Lug Rigidity Ratios
\[ \frac{R_o}{r_i} = 2.25, \frac{E_B}{E_L} = 3.0 \]
\[ \frac{\delta R}{r_i} = 0.00533 \]
\[ \mu_B = \mu_L = 0.3 \]

**Figure 3-12. Stress Distributions Along x-Axis in an Attachment Lug With Bushing for Various Bushing Thicknesses**
Figure 3-33. Stress Distributions Along x-Axis in an Attachment Lug With Bushing Due to Pin Loading for Various Bushing-to-Lug Rigidity Ratios
Figure 3-34. Stress Distributions Along x-Axis in an Attachment Lug With Bushing Due to Pin Loading for Various Bushing Thicknesses

\[
\frac{R_o}{r_i} = 2.25 \quad \frac{E_B}{E_L} = 3.0 \\
\mu_c = \mu_L = 0.3
\]
The results presented in Figures 3-30 through 3-34 pertaining to the stress analysis of lugs with bushings should be used with caution, in the sense that they can be used as long as the assumptions are not violated. In other words, these results can be used as long as there is sufficient interference to prevent separation between the bushing and the lug. When the interference level is small and/or when the crack length is large there will be separation, which again depends on the geometry and the magnitude of the applied loading. Also, the above results cannot be used when the total effective stress (residual stress plus applied stress) exceeds the material yield strength. If the total effective stress exceeds the material yield strength, a nonlinear elasto-plastic finite element analysis such as used in Reference [34] should be conducted to obtain the total effective stress. In the current study, only the linear analysis is performed. Once the total tangential stresses are obtained by superposing residual stresses and the applied stresses, then they can be used to calculate the stress intensity factors for a crack emanating from an attachment lug having an interference-fit bushing by using Equation (24) where $G(c, \xi)$ is the original or unmodified Green's function.

Stress-Intensity Factor Analysis

To study the effects of various parameters on actual (dimensional) stress-intensity factors rather than nondimensional, a physical lug having an outer radius of 1.6875 inches with a bushing inner radius of 0.75 inch ($R_o/r_i = 2.25$) subjected to a far field stress ($\sigma_o$) of 6 ksi and stress ratio ($R_{far}$) of 0.1 is considered in this discussion. For this lug configuration and loading, an effective stress ratio ($R_{eff}$) at the lug hole wall $(x = R)$ as a function of interference levels can be computed and is presented in Figure 3-35a. Figure 3-35b shows the variation of $R_{eff}$ along the x-axis of the lug for an interference level ($\delta_R/r_i$) of 0.00533. It is seen from the figure that $R_{eff}$ is constant almost throughout the net section (x-axis) except near the outer surface of the lug.

The stress-intensity factors were computed using Equation (24) and are presented in Figures 3-36 and 3-37. Figure 3-36 shows the effective stress-intensity factor range, $\Delta K$, for various bushing-to-lug rigidities ($E_B/E_L$) for a given bushing thickness $(t_B/r_i)$ of 0.12. The data corresponding to $E_B/E_L = 0$ represent a simple lug with no bushing. It is clear from
Figure 3-35. Effective Stress Ratio in an Attachment Lug with Bushing for Various Interference Levels and Along x-Axis
Figure 3-36. Stress Intensity Factor Ranges for Cracks at an Attachment Lug for Various Bushing-to-Lug Rigidity Ratios
\[ \frac{R_0}{r_i} = 2.25 \; ; \; r_i = 0.75 \text{ in} \]
\[ \frac{E_B}{E_L} = 3.0 \; ; \; E_L = 10^7 \text{ psi} \]
\[ \sigma_0 = 6 \text{ ksi} \; ; \; R = 0.1 \]

Figure 3-37. Stress Intensity Factor Ranges for Cracks at an Attachment Lug With Bushing for Various Bushing Thicknesses
Figures 3-35a and 3-36 that the installation of the interference-fit bushing causes an increase in the effective stress-intensity factor ratio, $R_{\text{eff}} = \sigma_{\text{min}}/\sigma_{\text{max}} = K_{\text{min}}/K_{\text{max}}$, but a significant decrease in the stress-intensity factor range, $\Delta K$. This will result in the reduction of fatigue crack growth rate. The effects of bushing thickness ($t_B/r_i$) on the effective stress-intensity factor range, $\Delta K$, are presented in Figure 3-37. Based on the results shown in Figures 3-36 and 3-37, it may be concluded that: (1) for constant bushing thickness, an increase in the bushing rigidity decreases the effective stress-intensity factor range; and (2) for constant bushing rigidity, an increase in bushing thickness decreases the effective stress-intensity factor range.

3. STRESS-INTENSITY FACTORS FOR CORNER CRACKS

One of the most common types of flaws for which there exists no closed form analytical solution is the corner crack at a circular hole. To date, several approximate methods have been proposed for computing the stress-intensity factors of a quarter-elliptical crack emanating from the corner of an open hole located in the geometric center of a plate. The methods range from an empirical equation which was developed using the fatigue crack growth method of calibrating the measured crack growth rate, $da/dN$, and the stress-intensity factor range, $\Delta K$, to one- and two-dimensional compounded solutions, to the sophisticated three-dimensional finite element analysis. In this section, three methods in an increasing level of complexity and sophistication are presented for the determination of the stress-intensity factors for single corner cracks in aircraft attachment lugs.

3.1 ONE-PARAMETER COMPOUNDING APPROXIMATION

The corner crack is more difficult to analyze than the through-the-thickness crack because it has both surface length and bore depth dimensions and both must be considered in the crack growth analysis. One possible simplification is to assume a fixed relationship between crack depth "a" and surface length "c", so that only one crack length parameter is independent. This was the approach used in Reference [14] to analyze corner cracks at fastener holes. The following stress-intensity correction factor, governing the growth of the corner crack in the length direction, was verified empirically on open hole fatigue coupons in Reference [14]:
\[
\phi(c) = 1 - \frac{0.2886}{1 + 2\left(\frac{a}{c}\right)\left(\frac{c}{b}\right)}
\]

This factor can be multiplied by the through-the-thickness crack solution, \(K_{\text{LUG}}\), obtained by the compounding method and given in Equation (10).

\[
K(c) = K_{\text{LUG}} \phi(c)
\]  

Equation (32) provides a simple one-parameter \(K\) estimate for the point near the lug face. This factor can be used even after \(a > B\), to smoothly change \(K\) as the corner crack becomes a through-the-thickness crack.

Equation (32) was used to estimate the stress-intensity factors for corner cracks at attachment lugs having \(R_o/R_I\) ratios of 1.50, 2.25, and 3.00. These results are tabulated in Tables 3-10 and 3-11 for \(B/R_I = 2/3\) and \(1/3\), respectively. Note that, in the calculation, the depth-to-length ratio \((a/c)\) was assumed to be constant and equal to 1.33, which was found experimentally by Schijve and Hoeymakers [2].

3.2 TWO-PARAMETER WEIGHT FUNCTION APPROXIMATION

In an earlier contractual program with the Flight Dynamics Laboratory, Hsu et al [36] developed a simple procedure for estimating the stress-intensity factors along the boundary of quarter-elliptical corner cracks emanating from fastener holes from corresponding through crack solutions. Correlation between the calculated stress-intensity factors and those deduced from the tests were good. A similar procedure can be used to estimate the stress-intensity factors along the periphery of a quarter-elliptical corner crack in an attachment lug.

The stress-intensity factor for a single through-the-thickness crack of length \(c\) in an attachment lug is given by

\[
K\left(\frac{c}{R_I}\right) = \sigma_o \sqrt{\pi c} \beta_T \left(\frac{c}{R_I}\right)
\]

where \(\beta_T \left(\frac{c}{R_I}\right)\) is the through-the-thickness correction factor.

Then the corresponding stress-intensity factor for a single quarter-elliptical corner crack can be written as
TABLE 3-10. NORMALIZED STRESS INTENSITY FACTORS FOR SINGLE CORNER CRACKS IN STRAIGHT ATTACHMENT LUGS USING COMPOUNDING METHOD - \( b/K_1 = 2/3 \), \( a/c = 1.33 \)

<table>
<thead>
<tr>
<th>( \frac{c}{(K_0 - K_1)} )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 1.50 )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 2.25 )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 3.00 )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 1.50 )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 2.25 )</th>
<th>( K_{\sigma_{cr}} \sqrt{\pi c} ) for ( K_0/K_1 = 3.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>4.772</td>
<td>3.654</td>
<td>3.761</td>
<td>3.181</td>
<td>1.624</td>
<td>1.254</td>
</tr>
<tr>
<td>0.1</td>
<td>4.446</td>
<td>2.930</td>
<td>2.685</td>
<td>2.964</td>
<td>1.302</td>
<td>0.895</td>
</tr>
<tr>
<td>0.2</td>
<td>4.285</td>
<td>2.632</td>
<td>2.269</td>
<td>2.857</td>
<td>1.170</td>
<td>0.756</td>
</tr>
<tr>
<td>0.3</td>
<td>4.249</td>
<td>2.463</td>
<td>1.995</td>
<td>2.833</td>
<td>1.095</td>
<td>0.665</td>
</tr>
<tr>
<td>0.4</td>
<td>4.316</td>
<td>2.356</td>
<td>1.816</td>
<td>2.877</td>
<td>1.047</td>
<td>0.605</td>
</tr>
<tr>
<td>0.5</td>
<td>4.483</td>
<td>2.308</td>
<td>1.718</td>
<td>2.989</td>
<td>1.026</td>
<td>0.573</td>
</tr>
<tr>
<td>0.6</td>
<td>4.776</td>
<td>2.332</td>
<td>1.692</td>
<td>3.184</td>
<td>1.036</td>
<td>0.564</td>
</tr>
<tr>
<td>0.7</td>
<td>5.271</td>
<td>2.457</td>
<td>1.751</td>
<td>3.514</td>
<td>1.092</td>
<td>0.584</td>
</tr>
<tr>
<td>0.8</td>
<td>6.183</td>
<td>2.771</td>
<td>1.948</td>
<td>4.122</td>
<td>1.232</td>
<td>0.649</td>
</tr>
<tr>
<td>0.9</td>
<td>8.389</td>
<td>3.636</td>
<td>2.526</td>
<td>5.593</td>
<td>1.616</td>
<td>0.842</td>
</tr>
<tr>
<td>( \frac{c}{(K_o - K_i)} )</td>
<td>( \frac{K}{\sigma_o \sqrt{\pi c}} ) for ( K_o/K_i = )</td>
<td>( \frac{K}{\sigma_{br} \sqrt{\pi c}} ) for ( R_o/R_i = )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>2.25</td>
<td>3.00</td>
<td>1.50</td>
<td>2.25</td>
<td>3.00</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>4.772</td>
<td>3.654</td>
<td>3.761</td>
<td>3.181</td>
<td>1.624</td>
<td>1.254</td>
</tr>
<tr>
<td>0.1</td>
<td>4.543</td>
<td>3.182</td>
<td>3.002</td>
<td>3.029</td>
<td>1.414</td>
<td>1.001</td>
</tr>
<tr>
<td>0.2</td>
<td>4.568</td>
<td>2.946</td>
<td>2.476</td>
<td>3.045</td>
<td>1.309</td>
<td>0.825</td>
</tr>
<tr>
<td>0.3</td>
<td>4.680</td>
<td>2.701</td>
<td>2.106</td>
<td>3.120</td>
<td>1.200</td>
<td>0.702</td>
</tr>
<tr>
<td>0.4</td>
<td>4.824</td>
<td>2.524</td>
<td>1.881</td>
<td>3.216</td>
<td>1.122</td>
<td>0.627</td>
</tr>
<tr>
<td>0.5</td>
<td>5.017</td>
<td>2.429</td>
<td>1.759</td>
<td>3.345</td>
<td>1.080</td>
<td>0.586</td>
</tr>
<tr>
<td>0.6</td>
<td>5.312</td>
<td>2.424</td>
<td>1.722</td>
<td>3.541</td>
<td>1.077</td>
<td>0.574</td>
</tr>
<tr>
<td>0.7</td>
<td>5.809</td>
<td>2.532</td>
<td>1.774</td>
<td>3.873</td>
<td>1.125</td>
<td>0.591</td>
</tr>
<tr>
<td>0.8</td>
<td>6.747</td>
<td>2.839</td>
<td>1.967</td>
<td>4.498</td>
<td>1.262</td>
<td>0.656</td>
</tr>
<tr>
<td>0.9</td>
<td>9.065</td>
<td>3.707</td>
<td>2.546</td>
<td>6.043</td>
<td>1.648</td>
<td>0.849</td>
</tr>
</tbody>
</table>
\[ K \left( \frac{a}{c}, \phi \right) = \sigma_0 \sqrt{\pi a} \beta \left( \frac{x-R_1}{R_1} \right) \frac{M'_1}{\phi} \left( \frac{a}{c}, \phi \right) M_c \left( \frac{c}{R_1}, \frac{x-R_1}{c} \right) M_B \left( \frac{a}{c}, \frac{a}{B} \right) \]  

(34)

for \( \frac{a}{c} \leq 1 \)

and

\[ K \left( \frac{c}{a}, \phi \right) = \sigma_0 \sqrt{\pi c} \beta \left( \frac{x-R_1}{R_1} \right) \frac{M'_1}{\phi} \left( \frac{c}{a}, \phi \right) M_c \left( \frac{c}{R_1}, \frac{x-R_1}{c} \right) M_B \left( \frac{a}{c}, \frac{a}{B} \right) \]  

(35)

for \( \frac{a}{c} > 1 \)

where

\[ M_c = F + \frac{x-R_1}{c} (1-F) \]  

(36)

in which \( \alpha \) is the elliptical angle measured from the hole wall, \( x \) is the distance from the center of the hole to the particular point of interest on the crack periphery, \( M'_1/\phi \) is the combined front free surface and flaw shape factor, \( M_c \) is the wall curvature correction factor, and \( M_B \) is the back surface correction factor. The values of \( M'_1/\phi \) and \( F \), and \( M_B \) are presented in References [36] and [58], respectively. As in Reference [36], it is assumed that for a given number of applied load cycles, the extension of the quarter-elliptical crack border is controlled by the stress intensity factors at two points; namely, the intersections of the crack periphery with both the hole wall and plate surface (i.e., \( K_A \) and \( K_C \)). In general, the stress intensity factors at these two locations are different, resulting in different crack growth rates. Therefore, the new flaw shape aspect ratio after each crack growth increment differs from the preceding one. The new flaw shape aspect ratio is computed using the new crack lengths at both the hole wall and plate surface. All the correction factors have been reduced to simple equations for locations A and C, and the two-parameter corner crack stress intensity factor formulas are summarized in Figure 3-38.

In Reference [36], the above process was repeated until the crack length along the hole wall was equal to the plate thickness. At that time, the crack was assumed to be a through-the-thickness crack with length \( c \). This assumption was based upon the experimental observation that after the crack penetrates the back surface and the cyclic load application continues, the back surface crack length increases much faster than that of the front surface until the front of the through-the-thickness crack becomes stable.
\[ K_1^{(A)} = \sigma_0 \sqrt{\pi c} \beta_T (0) \sqrt{A_n} \frac{M_{1}^{(A)}}{\sqrt{Q}} \]

\[ K_1^{(C)} = \sigma_0 \sqrt{\pi c} \beta_T \left( \frac{c}{R_i} \right) \sqrt{A_n} \frac{M_{1}^{(C)}}{\sqrt{Q}} \]

... WHERE, IF

\[ A_n = \text{MIN (calc., 1.0)} \]

\[ C_n = \text{MIN (calc., 1.0)} \]

... THEN

\[ \sigma_0 = PM(2R_0) \]

\[ \beta_T = \frac{K_{THRU}}{\sigma_0 \sqrt{\pi c}} \]

\[ K_{THRU} = K_1 \text{ FOR THROUGH-THICKNESS CRACK OF LENGTH } c \]

\[ Q = 1. + 1.464 (A_n C_n)^{1.65} \]

\[ M_{1}^{(A)} = (1. + 0.025 C_n - 0.0985 (1. - A_n) \sqrt{C_n}) \]

\[ M_{1}^{(C)} = (1. + 0.214 C_n - 0.0925 (1. - A_n) \sqrt{A_n}) \]

\[ F = 1. - 2.03 S - 9.635 S^2 - 23.37 S^3 - 25.485 S^4 - 10.403 S^5 \]

\[ S = c \tan \alpha \]

\[ M_B = \begin{cases} 1.0, & a > c \\ 1.0 - \left( 0.32 - 0.82 A_n \right) (1.3 - A_n), & a \leq c \end{cases} \]

**Figure 3-38. Two-Parameter Corner Crack Stress Intensity Factor Formulas**
This transitional crack growth assumption is very reasonable for a crack at a fastener hole, since, at the time the crack grows through the thickness, the remaining net section is usually much larger than the surface crack length. However, for an attachment lug, when a part-through crack grows through the thickness, the remaining net section is usually small in comparison to the surface crack size. Therefore, a proper transitional crack growth criterion is needed for the transition period from the time the crack penetrates the back surface to the time the crack lengths are essentially equal on the front and back surfaces.

Figure 3-39 shows the transitional crack geometry, in which \( c_F \) and \( c_B \) are crack lengths on the front and back surfaces, respectively. Collipriest and Ehret [37] proposed a stress-intensity magnification factor for the crack tip at the back surface of a surface crack as

\[
Q_L = \left[ \frac{1}{1 - \sqrt{1 - \frac{c_B}{c_F}}^2} \right]^{1/2} \quad \text{for } c_B > 0 \tag{37}
\]

When the back-side crack length equals the front-side crack length, the magnification is unity and the through crack has achieved a uniform front. This factor will be used to estimate the stress-intensity factor at the back surface of the transitional crack, i.e.

\[
K_B = \sigma_0 \sqrt{\pi c_B} \beta_T \left( \frac{c_B}{c_F} \right) Q_L \tag{38}
\]

The stress-intensity factor at the front surface will be calculated using Equation (15) and an imaginary crack length along the hole wall direction, \( a' \), i.e.

\[
K_C = \sigma_0 \sqrt{\pi c_F} \beta_T \left( \frac{c_F}{c_F} \right) \frac{1}{\phi} \left( \frac{c_F}{\phi} \right) Q_L \tag{39}
\]

An imaginary crack length \( a' \) can be determined by fitting an elliptical equation through points C and B, as
Figure 3-39. Transitional Crack Geometry
When the back-side crack length, $c_b$, approaches that of the front side (i.e. $c_f > c'$ as shown in Figure 3-38), the factor $c_f/a'$ becomes very small and the corresponding $M_f/M$ ratio approaches unity. After that, the transitional crack becomes a through-the-thickness crack with a uniform front, i.e.

$$K_B = K_C = \sigma_0 \sqrt{\pi c} \beta_T \left( \frac{C}{R_i} \right)$$

Equations (34) and (35) were used to compute the stress-intensity factors at the intersections of the crack periphery and the hole wall and plate surfaces. The 2-D finite element method solution results from Table 3-3 were used for the basic through-the-thickness crack solution, $K(c/R_i)$. The results are shown in Figures 3-40 through 3-42 for $R_o/R_i$ ratios of 1.50, 2.25 and 3.0, respectively. In each plot, the flaw shape aspect ratio $a/c$ was assumed as constant. It should be noted that the computed part-through crack stress-intensity factors are normalized by $\sigma_{br} \sqrt{\pi c}$. These solutions correspond to corner crack solutions for $a < B$. Figure 3-43 presents the normalized stress-intensity factors at the lug surface having a constant $a/c$ ratio of 1.33 and a $R_o/R_i$ ratio ranging from 1.5 to 3.0. In this figure, the solutions are presented even after the crack breaks through the back side, i.e. $a > B$. Corresponding normalized stress-intensity factors computed using the one-parameter compounding method are also shown in the figure for comparison.

3.3 THREE-DIMENSIONAL CRACKED FINITE ELEMENT METHOD

Numerical methods such as the slicing technique [38], the conventional three-dimensional (3-D) finite element method [39], the boundary integral equation approach [40], and the 3-D alternating technique [41] have been used to analyze corner-crack problems. To accurately depict the extreme stress gradient existing in the vicinity of a crack tip, the necessary computer effort in each case is considerable. In an attempt to minimize the economic problem of the conventional approach and to extract the stress-intensity factors directly, a 3-dimensional cracked finite element was used which is capable of characterizing the crack-tip stress singularity internally. The procedure used in the development of the 3-D cracked element is described in Reference [42] and summarized as follows.
Figure 3-40. Normalized Stress Intensity Factors for Corner Cracks in a Straight Attachment Lug Having a \( R_o/R_1 \) Ratio of 1.50 and a \( B/R_1 \) Ratio of 2/3
Figure 3-41. Normalized Stress Intensity Factors for Corner Cracks in a Straight Attachment Lug Having a $R_o/R_1$ Ratio of 2.25 and a $B/R_1$ Ratio of 2/3.
Figure 3-42. Normalized Stress Intensity Factors for Corner Cracks in a Straight Attachment Lug Having a $R_0/R_1$ Ratio of 3.00 and a $B/R_1$ Ratio of 2/3
Figure 1-43. Normalized Stress Intensity Factors for Corner Cracks in Straight Attachment Lugs Having Constant Flaw Shape a/c of 1.33
Two types of 20-node 3-D cracked elements, as shown in Figure 3-44, were developed using a hybrid displacement finite element procedure. The variational principle which governs the assumed displacement hybrid finite element model is the stationary condition of a modified total potential energy functional with a relaxed requirement of interelement boundary displacement continuity, a priori. This variational principle is a three-field variable principle. The three field variables are the element interior displacements, element boundary displacements, and the Lagrange multipliers. The Lagrange multipliers are physically the interelement boundary tractions and are assumed to match the independently assumed element interior and boundary displacements at the interelement boundary. Three-dimensional asymptotic solutions for displacements ($\sqrt{r}$ - type) and stresses ($1/\sqrt{r}$ - type) near the crack front are embedded in this procedure. Apart from satisfying relevant field equations, the variational principle also enforces the conditions of displacement continuity and traction reciprocity at the interelement boundary a posteriori, assuring the convergence of the finite element procedure. The procedure is capable of analyzing mixed mode fracture problems (Modes I, II and III), and the three stress-intensity factors at various locations along the crack front are treated as unknowns along with the generalized nodal displacements of the structure. The final set of algebraic equations governing the global nodal displacements and the three stress-intensity factors at various locations along the crack front can be written as follows:

$$\begin{align*}
(K_1) \{q^*\} + (K_2)^T \{K^*\} &= \{Q_1\} \quad (42) \\
(K_2) \{q^*\} + (K_3) \{K^*\} &= \{Q_2\} \quad (43)
\end{align*}$$

Where $\{q^*\}$ are the structure's global nodal displacements, $\{K^*\}$ are the mixed mode stress-intensity factors at various locations along the crack front, $[K_1]$, $[K_2]$ and $[K_3]$ are the corresponding stiffness matrices (superscript $T$ represents the transpose), and $\{Q_1\}$ and $\{Q_2\}$ are the corresponding nodal forces. It is evident from Equations (42) and (43) that the solution for stress-intensity factors can be obtained directly from the finite element solution procedure. This eliminates additional post-processing of displacement or stress solutions to obtain the stress-intensity factors through methods such as the crack opening displacement method, nodal force method, etc.

A detailed description of the hybrid displacement procedure was documented in Reference [42].
Figure 3-44. 20-Node Three-Dimensional Cracked Elements
The accuracy and convergence of the three-dimensional hybrid displacement finite element procedure have been tested and verified through the solution of several complex fracture problems of interest in aerospace structural components, nuclear pressure vessels and components, and solid rocket motor grain applications [43-45]. Raju and Newman [46], using Tracey's wedge shaped distorted isoparametric crack elements, made a convergence study for the solution of semielliptical surface flaws in thin plates. A similar convergence study was also made using the present three-dimensional hybrid finite element procedure in Reference [44]. As discussed in Reference [44], the present solution with 4555 degrees of freedom has excellent agreement with that of Raju and Newman for their highest degrees of freedom, 6867.

The three-dimensional cracked element was used to compute the stress-intensity factors for single quarter-elliptical corner cracks at straight lugs. A total of eight problems of corner cracks in attachment lugs with various outer-to-inner radius ratios, crack aspect ratios, and crack depth-to-lug thickness ratios were considered for the present analysis. The geometry of an attachment lug with a corner crack is given in Figure 3-45(a), and the various parameters for the eight problems considered are defined in Table 3-12. The bearing pressure at the lug hole wall was assumed to be uniform and acts only on the right half of the hole as shown in Figure 3-45(a). This assumption of uniform bearing pressure over the right half of the lug hole wall was made due to the complexity involved in the generation of equivalent nodal forces corresponding to an actual pin-lug contact bearing pressure distribution—a three-dimensional case. Consequently, the solutions by the two-dimensional Green's function method generated for comparison purposes also correspond to the same bearing pressure. Also, the original Green's functions were used in the computation of two-dimensional solutions. The assumption of same bearing pressure in both cases is important, because the stress-intensity factor solutions vary significantly depending upon the bearing pressure distribution, namely, uniform, cosine and actual pin-lug pressure distributions.

A typical finite element breakdown, number of elements and total number of degrees of freedom for the present analysis are presented in Figure 3-45(b). In Figures 3-46 through 3-50, solutions of stress-intensity factor variations along the crack front by both two- and three-dimensional procedures are presented for problem numbers 1 and 2, 3, 4 and 5, 6 and 7, and 8, respectively.
Figure 1-45. Geometry and Finite Element Breakdown of an Attachment Lug with a Corner Crack for Three-Dimensional Cracked Finite Element Analysis
<table>
<thead>
<tr>
<th>PROBLEM NUMBER</th>
<th>$R_o/R_i$</th>
<th>$a/c$</th>
<th>$a/B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.50</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>1.50</td>
<td>1.5</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>1.0</td>
<td>0.1667</td>
</tr>
<tr>
<td>4</td>
<td>2.25</td>
<td>1.0</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>2.25</td>
<td>1.5</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>2.25</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>2.25</td>
<td>1.5</td>
<td>0.50</td>
</tr>
<tr>
<td>8</td>
<td>2.25</td>
<td>1.5</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Figure 1-44. Normalized Stress Intensity Factors for Corner Cracks at a Straight Attachment Lug Having a $R_o/R_i$ Ratio of 1.5 and an $a/b$ Ratio of 0.5.
Figure 3-47. Normalized Stress Intensity Factors for Corner Cracks at a Straight Attachment Lug Having a $K_o / R_i$ Ratio of 2.25 and an $a/B$ Ratio of 0.1667
Figure 3-48. Normalized Stress Intensity Factors for Corner Cracks at a Straight Attachment Lug Having a $R_o/R_i$ Ratio of 2.25 and an $a/B$ Ratio of 0.25.
Figure 7-49. Normalized Stress Intensity Factors for Corner Cracks at a Straight Attachment Lug Having a $R_o/R_i$ Ratio of 2.25 and an $a/c$ Ratio of 0.5
Figure 3-50. Normalized Stress Intensity Factors for Corner Cracks at a Straight Attachment Lug Having a $\frac{R_o}{R_i}$ Ratio of 2.25 and an $\frac{a}{b}$ Ratio of 0.75
The stress-intensity factor solutions are normalized by the exact solution of a through-the-thickness crack of length c in an infinite plate subjected to a crack pressure of $\sigma_{br}$. Elliptical angles ($\alpha$) of $0^\circ$ and $90^\circ$ refer to the intersections of the crack front with the hole wall and lug surface, respectively.

A comparison of the results in Figures 3-46 through 3-50 reveals that the two solution procedures agree well at $\alpha = 0^\circ$, but significant differences exist near $\alpha = 90^\circ$. For lugs with an outer-to-inner radius ratio of 1.5, the maximum difference at $\alpha = 90^\circ$ is about 42 percent and the difference decreases as the aspect ratio of the crack ($a/c$) increases. For lugs with outer-to-inner radius ratio of 2.25, the maximum difference at $\alpha = 90^\circ$ is about 26 percent and the difference decreases as the aspect ratio of the crack ($a/c$) and the crack depth-to-thickness ratio ($a/B$) increase. The overall behavior of the differences in the solutions is that they decrease as the outer-to-inner radius ratio, crack aspect ratio and crack depth-to-thickness ratio increase. Also, the angle at which the normalized stress-intensity factor is minimum is lower for the two-dimensional procedure in almost all of the problems. Thus, from these results, the two-dimensional procedure seems to be consistently overestimating the stress-intensity factor solutions except for elliptical angles close to $0^\circ$.

The one-parameter compounding, two-parameter weight function and the three-dimensional cracked finite element solutions are compared in terms of corner crack correction factors at the lug surface in Figure 3-51. The corner crack correction factors are obtained by normalizing $K_C$ with the stress intensity factor (by weight function method) of a through-the-thickness crack of length c and are expressed as a function of $a/B$.

4. ELASTOPLASTIC ANALYSIS

Several analytical methods of varying complexities have been developed and discussed in previous sections for through-the-thickness and corner cracks in straight attachment lugs. These analyses are basically for linear fracture mechanics problems. However, attachment lugs may undergo substantial plastic yielding around the lug hole, depending on the specific application and/or design. For example, the concepts of cold-working of the hole or installation of an interference-fit bushing prior to pin fitting have been extensively used in actual aircraft attachment lug
Figure 3-51. Comparison of Corner Crack Correction Factors at the Lug Surface by the Three Methods
design practices to improve the crack initiation life. These concepts are
basically used to introduce fatigue-improving residual stresses around the
hole of the lug. The lugs may also undergo plastic yielding due to the
application of high pin loads. It is important to develop analytical pro-
cedures for analyzing such problems.

The analytical procedure is similar to that developed and verified for
through-the-thickness cracks emanating from interference-fit and cold-worked
fastener holes in infinite plates, under a previous Air Force contract
(Reference [36]). The method involves an initial elasto-plastic stress analysis
of attachment lugs to obtain the stress distribution on the prospective crack
surface. These stresses are then used in conjunction with the Green's function
to calculate the stress intensity factors for cracks emanating from lug holes.
The Green's function developed for linear problems is also used for the
elasto-plastic case.

The elasto-plastic stress analysis has been carried out for two mate-
rinals, 4340 steel and 7075-T651 aluminum, which are being used for testing
in the present contract work. These materials have been characterized and
the nonlinear elasto-plastic stress-strain relationships were approximated
by trilinear representation and are given in Figure 3-52. The unloading
moduli have been assumed to be the same as the initial elastic moduli for
both of the materials. The analysis was performed for lugs with outer-to-
inner radius ratios, \( R_o/R_i \), of 1.50, 2.25, and 3.00 and a constant lug inner
radius of 0.75 inch. Far-field maximum stresses, \( \sigma_o \), of 35 and 15 ksi were
selected for steel and aluminum lugs, respectively. Isotropic hardening
with the von Mises yield criterion was used in the elasto-plastic analysis.
Steel pins were used in all cases, and it was also assumed that the pin did
not yield.

The finite-element method was again used for the elasto-plastic
stress analysis of attachment lugs. The finite element breakdown and
the use of spring elements with high stiffness to simulate the pin-
lug contact surface were similar to those in the linear stress analysis.
The model was loaded by a single concentrated force at the center of the pin.
In the nonlinear analysis, a loading was first applied to reach the yield
point, followed by incremental loadings to reach the maximum far-field
stresses listed above. The loadings were then reduced to give stress levels
Corresponding to far-field stress ratios, \( R \), of 0.5 and 0.1.
Figure 3-52. Material Properties Used in Elasto-Plastic Stress Analysis
In the plastic region, the following two convergence criteria were used in the analysis. A tolerance criterion of \( \Delta E_{It}/\Delta E_{L..1} \leq 0.1 \) was used in the analysis, where \( \Delta E_{It} \) and \( \Delta E_{L..1} \) are the energy changes for the iteration and the load increment, respectively. Regardless of the above tolerance criterion, the analysis was iterated at least twice for each load increment. In all the solutions presented here, the maximum value of \( \Delta E_{It} / \Delta E_{L..1} \) was less than 0.035.

The unflawed stress distribution along the prospective crack surface, 90° away from the load line, obtained from the elasto-plastic finite element analysis for the far field stress levels given before are presented in Figures 3-53 through 3-55 for steel lugs and Figures 3-56 through 3-58 for aluminum lugs. These results are given in increasing order of \( R_o/R_i \) ratios of 1.50, 2.25 and 3.00.

For a steel lug with \( R_o/R_i \) of 1.50, the stresses (\( \sigma_{\text{max}} \)) corresponding to maximum far-field stress of 35 ksi and the stresses (\( \sigma_{\text{min}} \), \( R = 0.5 \) and \( \sigma_{\text{min}} \), \( R = 0.1 \)) corresponding to minimum far-field stress for stress ratios of 0.5 and 0.1, respectively, are given in Figure 3-53. From these stresses, the difference between maximum and minimum stresses for stress ratios of 0.5 and 0.1, \( \Delta \sigma_{R=0.5} \) and \( \Delta \sigma_{R=0.1} \), respectively, can be obtained and are also included in the figure. At the maximum load, about 20 percent of the lug ligament has yielded. The effects of plastic yielding of the lug can also be seen in the distributions of \( \sigma_{\text{min}} \), \( R=0.5 \) and \( \sigma_{\text{min}} \), \( R=0.1 \). While unloading, due to plastic yielding, the stresses near the lug hole decrease at a higher rate than the rest of the ligament and become negative at the minimum stress corresponding to \( R = 0.1 \). If the loading was completely removed, significant negative residual stresses would exist near the lug hole. In calculations of crack growth any compressive minimum stresses near the lug hole are neglected due to crack closure, and \( \Delta \sigma_{R=0.1} \) is assumed not to exceed \( \sigma_{\text{max}} \). However, for illustration purposes, the actual difference in stresses near the lug hole is indicated in Figure 3-53 by a dotted line. Corresponding solutions for an aluminum lug with the same geometry are presented in Figure 3-56, and the results are qualitatively the same.

Similar analytical solutions for lugs with an \( R_o/R_i \) ratio of 2.25 are given in Figures 3-54 and 3-57 for steel and aluminum materials, respectively. In this case, for both the materials, the lugs yield only slightly;
Figure 3-53. Stress Distribution Along x-Axis for a Steel Lug with $R_o/R_i = 1.50$
4340 STEEL
$R_o/R_i = 2.25$
$R_i = 0.75$ INCH
$\sigma_o = 35$ KSI

Figure 3-54. Stress Distribution Along x-Axis for a Steel Lug with $R_o/R_i = 2.25$
4340 STEEL
$R_o/R_i = 3.00$
$R_i = 0.75$ INCH
$\sigma_o = 35$ KSI

Figure 3-55. Stress Distribution Along $x$-Axis for a Steel Lug with $R_o/R_i = 3.00$
Figure 3-56. Stress Distribution Along x-Axis for an Aluminum Lug with $R_o/R_i = 1.50$.
7075-T651 ALUMINUM
\[ \frac{R_o}{R_i} = 2.25 \]
\[ R_i = 0.75 \text{ INCH} \]
\[ \sigma_o = 15 \text{ KSI} \]

\[ \sigma_{\text{max}} \]
\[ \Delta \sigma_R = 0.1 \]
\[ \Delta \sigma_R = 0.5 \]
\[ \sigma_{\text{min}}, R = 0.5 \]
\[ \sigma_{\text{min}}, R = 0.1 \]

\( (x - R_i)/(R_o - R_i) \)

Figure 3-57. Stress Distribution Along x-Axis
for an Aluminum Lug with \( \frac{R_o}{R_i} = 2.25 \)
Figure 3-58. Stress Distribution Along x-Axis for an Aluminum Lug with $R_o/R_i = 3.00$. 

7075-T651 ALUMINUM

$R_o/R_i = 3.00$

$R_i = 0.75$ INCH

$\sigma_o = 15$ KSI
that is, about 5 percent of the lug ligament. Also, for $R=0.1$, the minimum stress near the lug hole does not become negative for this lug configuration. Thus, $\Delta \sigma_{R=0.5}$ and $\Delta \sigma_{R=0.1}$ presented are the actual differences in the stresses. Results for steel and aluminum lugs with a $R_o/R_1$ value of 3.00 are presented in Figures 3-55 and 3-58, respectively. In this case, the lugs barely yield, and very small plastic strains were calculated by the elasto-plastic analysis.

The above stress ranges, computed from the elasto-plastic analysis, are then integrated with the Green's function to compute the stress intensity factor $\gamma$ for through-the-thickness cracks in lugs that are loaded above yield.

The stress intensity factors can be calculated using the Green's function method by the following equation:

$$K(c) = \sigma_0 \sqrt{\pi c} \int_0^1 G(\epsilon, \xi) \frac{\sigma(\xi)}{d\xi} d\xi$$  \hspace{1cm} (44)

The stress intensity factor range for through-the-thickness cracks can be computed from the above equation by either inputting the maximum and minimum stresses and subtracting the values, or more directly, by using the stress range. However, in the cases where the stresses corresponding to minimum load become negative, the stress range should not be used. Rather the direct input of maximum and minimum stresses should be used and care must be exercised to neglect the negative stress intensity factors corresponding to minimum load while computing the ranges. The computed values of stress intensity factor ranges for steel and aluminum lugs are presented in Figures 3-59 and 3-60, respectively. The results are given for $R_o/R_1$ ratios of 1.50, 2.25 and 3.00 and far-field stress ratios of 0.5 and 0.1. Since this was a nonlinear analysis, the far-field stress ratio (ratio of minimum to maximum far-field stress) differs from the crack-tip stress ratio (ratio of minimum to maximum stress intensity factor). Furthermore, because of the nonlinearity, no effort was made to normalize the stress intensity factor ranges in Figures 3-59 and 3-60. However, the crack length in the presentation has been normalized by the net ligament of each lug.

For normalized crack lengths less than about 0.35, the stress intensity factor ranges are lower for an $R_o/R_1$ ratio of 1.50 and higher for increasing
4340 STEEL

$R_i = 0.75$ INCH

$\sigma_o = 35$ KSI

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>$R_o/R_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>3.00</td>
</tr>
</tbody>
</table>

Figure 3-52. Stress Intensity Factor Ranges for Steel Lugs
Figure 3-30. Stress Intensity Factor Ranges for Aluminum Lugs

7075 - T651 ALUMINUM

$R_i = 0.75$ INCH
$\sigma_0 = 15$ KSI

**SYMBOL**  
- $R_o/R_i$
- \[ \begin{align*}
- & 1.50 \\
- & 2.25 \\
- & 3.00 \\
\end{align*} \]

**STRESS INTENSITY FACTOR RANGE, \( \Delta K_i, \text{ KSI} \)**

\[ \begin{align*}
& 0 \\
& 50 \\
& 100 \\
\end{align*} \]

\[ \begin{align*}
& 0 \\
& 0.5 \\
& 1.0 \\
\end{align*} \]

\[ \begin{align*}
& c/(R_o - R_i) \\
\end{align*} \]
ratios. For normalized crack lengths above 0.35, the trend changes to higher stress intensity factor ranges for lower $R_o/R_i$ ratios. However, expressed in terms of unnormalized physical crack length, $c$, the stress intensity factor for any given crack length is always higher for a lower $R_o/R_i$ ratio.

To assess and to have a feel for the effects of plastic yielding by comparing with linear analysis, the stress intensity factor ranges are normalized by $\Delta\sigma_0\sqrt{\pi c}$, where $\Delta\sigma_0$ is the corresponding far-field stress range. These normalized values given in Table 3-13 have limited significance in the sense that these values are applicable to the specific material and loading condition. The values can be directly compared with the normalized stress intensity factor ranges given in Table 3-3 (applicable for $R=0.1$ and $R=0.5$), which correspond to linear analysis, to evaluate the effect of plasticity. Table 3-3 can be assumed to be applicable for both materials, since the differences were less than 3 percent.

Comparison of these two tables indicates that the normalized values of $\Delta K$ for the elasto-plastic analysis are lower for smaller $c/(R_o-R_i)$ ratios and become higher as the crack grows through the lug ligament. This is because of the stress redistribution in the lug due to yielding. In the yield zone and its neighborhood, the stresses will be lower when compared with the elastic stresses. Away from the plastic zone, the stresses will be higher than elastic stresses, to satisfy the equilibrium. The above behavior is true, in general; however, for an $R_o/R_i$ ratio of 1.50 and a stress ratio of 0.1, the nonlinear values are consistently lower than the linear values for almost all $c/(R_o-R_i)$ values. The reason for this behavioral change can be explained as follows. For this particular lug configuration and stress ratio, there is more plastic yielding as well as compressive residual stresses near the lug hole at the minimum load. These negative stresses are truncated in the stress intensity computation. Also, the net ligament length is the smallest. For these two reasons, i.e., truncation of negative stresses and smaller ligament size, the normalized values of stress intensity factor range never become higher than those of the linear analysis. Also, note that due to plastic yielding while loading and subsequent linear unloading, the normalized values of $K=0.5$ for lower $c/(R_o/R_i)$ ratios are greater than the corresponding values of $R=0.1$ and closer to linear elastic values given in Table 3-3.
### Table 3-13: Normalized Stress Intensity Factor Ranges from Plastic Analysis - $\Delta K / (\Delta K_c)$

<table>
<thead>
<tr>
<th>$c / (W_o R_1)$</th>
<th>4340 Steel</th>
<th>7075-T651 Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 0.1$</td>
<td>$R = 0.1$</td>
<td>$R = 0.1$</td>
</tr>
<tr>
<td>$R = 0.5$</td>
<td>$R = 0.5$</td>
<td>$R = 0.5$</td>
</tr>
<tr>
<td>$R = 1.5$</td>
<td>$R = 1.5$</td>
<td>$R = 1.5$</td>
</tr>
<tr>
<td>$R = 3.0$</td>
<td>$R = 3.0$</td>
<td>$R = 3.0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$c / (W_o R_1)$</th>
<th>4340 Steel</th>
<th>7075-T651 Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 0.1$</td>
<td>$R = 0.1$</td>
<td>$R = 0.1$</td>
</tr>
<tr>
<td>$R = 0.5$</td>
<td>$R = 0.5$</td>
<td>$R = 0.5$</td>
</tr>
<tr>
<td>$R = 1.5$</td>
<td>$R = 1.5$</td>
<td>$R = 1.5$</td>
</tr>
<tr>
<td>$R = 3.0$</td>
<td>$R = 3.0$</td>
<td>$R = 3.0$</td>
</tr>
</tbody>
</table>

115
The elasto-plastic method can also be used to calculate the stress intensity factor solutions for corner cracks emanating from attachment lugs that are subjected to loadings above yield. The above through-the-thickless solution can be modified with front-free surface, curvature correction, etc. for corner crack problems, as in Equations (34) and (35) of subsection 3.
SECTION IV

STRESS INTENSITY FACTORS FOR TAPERED ATTACHMENT LUGS

Tapered attachment lugs have been used frequently in aircraft structural fittings to provide strength against off-axis loading. To determine the critical location and direction where a fatigue crack may initiate and subsequently grow, it is necessary to determine the stress distribution of the unflawed lug and the stress intensity factors of the cracked lug. This section describes the development of stress distribution and stress intensity factors for tapered attachment lugs subjected to symmetric, off-axis and transverse loadings.

1. STRESS ANALYSIS

Because of the complexity of the lug geometry and off-axis loading, the finite element method was used to determine the tangential stress distributions along the edge of the hole in the unflawed lug. This analysis was conducted for a pin load applied at 0°, 90°, 135°, 180°, 270°, and 315° measured in the clockwise direction from the axis of the lug. Figure 4-1 depicts the geometry and typical two-dimensional finite element model used in the stress analysis of the unflawed lug. The angle between the two edge surfaces of the tapered head, \( \beta \), is 45 degrees. The finite element model shown in Figure 4-1 consists of 429 nodes, 72 triangular elements, 348 quadrilateral elements, 28 spring elements, and a total of 850 degrees of freedom. A concentrated force was applied at the center of the pin to simulate pin loading and was reacted at the base of the lug. The analysis was carried out to determine the stresses in the lug and the pin-bearing pressure distributions at the pin-lug contact area for the six loading directions mentioned above. Three outer-to-inner radius ratios of the tapered head, i.e., \( R_o/R_i = 1.50, 2.25 \) and 3.00, were evaluated. In all cases of the analysis, the rigidity of the pin was assumed to be three times the rigidity of the lug, which simulates an aluminum lug loaded by a steel pin.

For the conventional finite-element analysis using constant strain elements, the values of stress and strain obtained for a given element were assigned to the centroid location of that element. To determine the stress at the edge of the hole, the stresses at the centroids of a
Figure 4-1. Finite Element Model for a Tapered Attachment Lug Having a $R_0/R_1$ Ratio of 2.25
series of elements located in the same radial direction were used to extrapolate to the edge location. This procedure was used to determine the tangential stresses along the edge of the hole of tapered lugs having \( R_o/R_i \) ratios of 1.50, 2.25 and 3.00. The results are shown in Figures, 4-2, 4-3, and 4-4 for pin loadings applied in three principal directions, i.e., \( 0^\circ \), \(-45^\circ \) (315°), and \(-90^\circ \) (270°), respectively. In each case, the tangential stresses were normalized with the average pin bearing pressure, \( \sigma_{br} \), which is defined as \( P/(2R_iB) \), where \( B \) is the thickness of the lug.

As can be seen from these figures, for each loading direction, there are two local maximum tangential stresses located at each side of the loading direction. The locations of these maximum stresses depend upon the loading direction and \( R_o/R_i \) ratio. For a pin loading applied in the axial direction of the lug, as shown in Figure 4-2, the maximum stress locations are found at about \( \pm 85 \) to \( \pm 90^\circ \) away from the loading direction. The maximum stress locations do not change significantly with the change of the \( R_o/R_i \) ratio. When the pin loading is applied in the \(-45^\circ \) direction (see Figure 4-3), for \( R_o/R_i = 1.50 \), the absolute maximum stress occurs at about \( 65^\circ \) measured from the axis of the lug (or \( 110^\circ \) away from the load direction) with the other local maximum stress located about \( 180^\circ \) from the first. When the \( R_o/R_i \) ratio increases, the locations of the maximum stresses change only slightly. However, the absolute maximum stress location switches from the head side of the lug to the base side of the lug. When the pin loading is applied in the direction perpendicular to the axis of the lug (see Figure 4-4), for all \( R_o/R_i \) ratios, the maximum stress occurs at the location in the base of the lug at a \( \theta \) value of about \( 200^\circ \) to \( 210^\circ \). Figure 4-5 summarizes the locations of the local maximum tangential stresses at the edge of the hole for each loading direction. These are the most critical locations, where one would anticipate that a fatigue crack would initiate. For each lug geometry and loading condition, Numbers 1 and 2 shown in Figure 4-5 indicate the probable order of crack initiation. They are chosen based upon the relative magnitude of the two computed local maximum stresses. Stress distributions along the x-axis for tapered lugs with \( R_o/R_i \) ranging from 1.5 to 3.0 subjected to symmetric loading are presented in Figure 4-6 and Table 4-1.

Larsson [47, 48] conducted fatigue testing of axially and transversely loaded aluminum lugs having a \( R_o/R_i \) ratio of 2.2. He observed and tabulated the locations of fretting and crack initiation for three different lug
Figure 4-2. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in 0° Direction
Figure 4-3. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in -45° Direction
Figure 4-4. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in -90° Direction
Figure 4-5. Fatigue Critical Locations of Tapered Lugs SubJECTED TO Various Load Orientations
\[ \sigma_{br} = \frac{p}{2 R_i B} \]

**Figure 4-6. Unflawed Stress Distributions at Tapered Attachment Lugs Having Various \( R_o/R_i \) Ratios**
TABLE 4-1. NORMALIZED UNFLAWED STRESS DISTRIBUTION ALONG x-AXIS FOR TAPERED ATTACHMENT LUGS

<table>
<thead>
<tr>
<th>$(R_0 - R)$</th>
<th>$R_0 / R_1$</th>
<th>$\sigma_y / \sigma_{br}$</th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
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<td>2.25</td>
<td>2.50</td>
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<td>1.50</td>
<td>2.00</td>
<td>2.25</td>
<td>2.50</td>
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<tr>
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<tr>
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<td>1.784</td>
<td>1.580</td>
<td>1.374</td>
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<td>0.681</td>
<td>0.538</td>
<td>0.388</td>
<td>2.667</td>
<td>1.708</td>
<td>1.532</td>
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<td>0.65</td>
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<td>0.647</td>
<td>0.521</td>
<td>0.408</td>
<td>0.297</td>
<td>2.094</td>
<td>1.294</td>
<td>1.172</td>
<td>1.020</td>
<td>0.891</td>
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<tr>
<td>0.75</td>
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<td>0.564</td>
<td>0.452</td>
<td>0.355</td>
<td>0.263</td>
<td>1.869</td>
<td>1.128</td>
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<td>0.630</td>
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</table>

*Extrapolated Values
configurations. His results of crack initiation locations are summarized in Figures 4-7 and 4-8 for a pin loading applied in the 0° and 90° directions, respectively. As seen from these figures, the current predicted fatigue critical locations as well as the possible sequence of crack initiation agree well with these experimental data. These predicted critical locations will be used in the modeling of the cracked lugs in the subsequent fracture analysis.

The fatigue critical area of an attachment lug for a given loading direction is not necessarily subjected to compression when the loading direction is reversed. Thus, for some load orientations, the reversed fatigue loading might have a significant effect on crack growth behavior. Figures 4-9 through 4-11 show the tangential stress distributions along the edge of the hole for a pin loading applied in the reversed direction of the three primary load orientations presented in Figures 4-2 through 4-4, respectively. As seen from the figures, in most cases, the reversed loading stretches the critical area in tension, but the magnitude is reduced. Note that the result in Figure 4-11 is essentially the same as that in Figure 4-4 except for the definition of the angle $\Theta$. Plots of the stress concentration factors at the edge of the hole in logarithmic scales, as shown in Figure 4-12 for symmetrically loaded tapered attachment lugs, reveal that a simple empirical formula can be derived and the relationship is given by the equation

$$K_{tb} = \frac{\sigma_{\text{max}}}{\sigma_{\text{br}}} \cdot (2.75 - \frac{\beta^\circ}{135}) \left(\frac{R_0}{K_1} - 1\right) - (0.675 - \frac{\beta^\circ}{1000})$$

where $\beta^\circ$ is the taper angle of the attachment lug in degrees. Note that for straight lugs ($\beta = 0^\circ$) the above equation reduces to that given in Figure 1-5.

The stress concentration factors were obtained by normalizing the peak tangential stresses with the average pin bearing pressure, $\sigma_{\text{br}}$. The equation of the logarithmic straight lines shown in Figure 4-12 compares with the finite element solution within 0.8 percent for tapered lugs. The values computed by the above equation and the finite element method are also listed in tabular form in Figure 4-12 for comparison. This simple empirical equation may be used for interpolating for taper angles less than $45^\circ$ or extrapolating for $R_0/K_1$ values outside the range of 1.5 and 3.0. The values of
Figure 4-7. Locations of crack initiation for various attachment lugs subjected to pin loading applied in 0° direction.
Figure 4-8. Locations of Crack Initiation for Various Attachment Lugs Subjected to a Pin Loading Applied in -90° Direction
Figure 4-9. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in 180° Direction
Figure 4-10. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in 135° Direction
Figure 4-11. Normalized Tangential Stress Distributions Along the Edge of the Hole for Tapered Lugs Subjected to a Pin Loading Applied in 90° Direction
Figure 4-12. Elastic Stress Concentration Factors for Tapered Attachment Lugs

\[ K_{lb} = \left(2.75 - \frac{\beta^\circ}{135}\right) \left[\frac{R}{R_i} - 1\right]^{-(0.675 - \frac{\beta^\circ}{1000})} \]

<table>
<thead>
<tr>
<th>( \frac{R_o}{R_i} )</th>
<th>FEM</th>
<th>EQUUN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>3.768</td>
<td>3.740</td>
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<td>3.00</td>
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the stress concentration factors presented in Figure 4-12 correspond to the
\( \theta = \pm 90^0 \) locations. The actual maximum tangential stress location is be-
tween \( \pm 85^0 \) and \( \pm 90^0 \). From a knowledge of the stress distributions, it is
anticipated that there will be no appreciable difference in the solutions.
Also, in the fracture analysis of tapered lugs subjected to a pin loading
in the axial direction, the crack surface will be assumed to be in the plane
perpendicular to the loading direction. However, for off-axis loadings of
\( \pm 45^0 \) and \( \pm 90^0 \), the crack surface will be assumed to be in the critical loca-
tion as predicted by the stress analysis.

The computed pin bearing pressure distributions along the contact sur-
faces with no cracks present are presented in Figures 4-13 through 4-15 for
pin loadings applied in the three primary directions, i.e., 0\(^0\), -45\(^0\), and
90\(^0\), respectively. Similar results obtained for the reversed loadings of
180\(^0\) and 135\(^0\) are presented in Figures 4-16 and 4-17, respectively. For the
reversed loading of 90\(^0\), the contact pressure can be obtained from Figure
4-15 by redefining the angle \( \theta \) as previously discussed.

2. STRESS INTENSITY FACTOR ANALYSIS

Based on the stress analysis of unflawed tapered attachment lugs, the
most critical locations were selected for the fracture analysis to obtain
the stress intensity factors for various crack lengths. The special high-
order crack tip singularity element [8] was again used in the present analy-
sis to calculate the stress intensity factors. In the analysis, it is
assumed that, for a given tapered lug subjected to a specific direction of
pin loading, the crack will initiate from the maximum tangential stress
location at the hole and propagate radially. Figure 4-18 shows a typical
finite element model which was used for a single through-the-thickness crack
emanating from a tapered attachment lug subjected to a pin-loading applied
in the 0\(^0\) and 180\(^0\) directions. The crack surface is assumed to be in the
plane perpendicular to the loading direction. The finite element breakdown
consists of 366 nodes, 150 triangular elements, 728 quadrilateral elements,
32 spring elements, 1 crack-tip element and a total of 724 degrees of free-
dom. The computed normalized opening mode stress intensity factors, as a
function of normalized crack length, for single through-the-thickness cracks
in tapered attachment lugs having \( k_o/k_i \) ratios ranging from 1.5
to 1.0 are presented in Figure 4-19 and Table 4-2. The normalized stress
Figure 4-13. Pin-Bearing Pressure Distributions Along the Contact Surface of Tapered Lugs Subjected to a Pin Loading Applied in $0^\circ$ Direction
Figure 4-14. Pin-Bearing Pressure Distributions Along the Contact Surface of Tapered Lugs Subjected to a Pin Loading Applied in -45° Direction
Figure 4-15. Pin-Bearing Pressure Distributions Along the Contact Surface of Tapered Lugs Subjected to a Pin Loading Applied in -90° Direction.
Figure 4-16. Pin-Bearing Pressure Distributions Along the Contact Surface of Tapered Lugs Subjected to a Pin Loading Applied in 180° Direction
Figure 4-17. Pin-Bearing Pressure Distributions Along the Contact Surface of Tapered Lugs Subjected to a Pin Loading Applied in 135° Direction
Figure 4-18. Finite Element Model for a Cracked Tapered Lug Subjected to a Pin Loading Applied in 0° and 180° Directions
Figure 4-19. Normalized Stress Intensity Factors for Single Through-the-Thickness Cracks Emanating from Tapered Attachment Lugs Subjected to a Pin Loading Applied in 0° Loading Direction
TABLE 4-2. NORMALIZED STRESS INTENSITY FACTORS FOR SINGLE THROUGH-THE-THICKNESS CRACKS AT TAPERED ATTACHMENT LUGS

<table>
<thead>
<tr>
<th>( \frac{K_o}{K_i} )</th>
<th>( K/(\sigma_{br} \sqrt{\pi c}) )</th>
<th>( K/(\sigma_o \sqrt{\pi c}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.50</td>
<td>2.00</td>
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<td>0.0</td>
<td>4.220</td>
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<td>1.408</td>
</tr>
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<td>2.500</td>
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</tr>
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<td>1.224</td>
</tr>
<tr>
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<td>1.173</td>
</tr>
<tr>
<td>0.7</td>
<td>2.386</td>
<td>1.179</td>
</tr>
<tr>
<td>0.8</td>
<td>2.410</td>
<td>1.188</td>
</tr>
<tr>
<td>0.9</td>
<td>2.479</td>
<td>1.273</td>
</tr>
</tbody>
</table>
intensity factor values at the lug hole, i.e., \( c/R_1 = 0 \), were obtained by multiplying the concentration factors determined from the unflawed stress analysis by 1.12, which was derived by Gross et al [21] for a straight edge crack in a large plate subjected to remote tension. The trends of the above tapered lug solutions are very similar to those obtained for straight lugs. Similar solutions of normalized stress intensity factors as a function of normalized crack length for tapered attachment lugs subjected to pin loadings applied in the \( 180^\circ \) direction are presented in Figure 4-20.

The analysis was then extended to tapered attachment lugs subjected to off-axis loadings of \(-45^\circ, 135^\circ, -90^\circ\) and \(90^\circ\). A \( R_o/R_1 \) ratio of 2.25 was considered for the off-axis loading fracture analysis. Figure 4-21 shows the finite element model used for analyzing a single through-the-thickness crack emanating from a tapered lug subjected to a pin loading applied in \(-45^\circ\) and its reversed \((135^\circ)\) direction. The model contains 401 nodes, 224 triangular elements, 218 quadrilateral elements, 34 spring elements, 1 crack-tip element and a total of 794 degrees of freedom. Note that although there are two critical locations modeled, \(58^\circ\) and \(227^\circ\) measured from the axis of the lug, only one crack was analyzed at a time. The computed normalized stress intensity factors are shown in Figure 4-22 as a function of the normalized crack length. This figure shows that when the crack length is small, say \( c/R_1 < 0.15 \), the stress intensity factor for a crack located closer to the base of the lug is higher than the one located at the head side of the lug. When the crack length increases \( c/R_1 > 0.15 \), the stress intensity factor for a crack located at the head side becomes larger than the one located at the opposite side of the hole. The difference between the two computed K-values increases as the crack length increases. For the case when the direction of the applied pin loading is reversed, the stress intensity factors are also computed and included in the figure for a crack located at the head side of the lug. The surfaces of a crack located closer to the base of the lug are completely closed during the reversed loading. The computed K-values corresponding to the reversed loading are much smaller than the corresponding ones obtained under the primary tensile loading. In the above analysis, only one crack was assumed to exist at a time. However, in reality, both cracks may exist and grow at the same time. In such a case, the influence of one crack on the stress intensity factor of the other crack may be significant. This can be accounted for by developing a matrix of stress...
Figure 4-20. Normalized StressIntensity Factors for Single Through-the-Thickness Cracks Emanating from Tapered Attachment Lugs Subjected to a Pin Loading Applied in 180° Loading Direction
Figure 4-21. Finite Element Model for a Cracked Tapered Lug Subjected to a Pin Loading Applied in -45° and 135° Directions
Figure 4-22. Normalized Stress Intensity Factors for Single Through-the-Thickness Cracks Emanating from a Tapered Lug Subjected to a Pin Loading Applied in -45° and its Reversed Directions ($R_o/R_i = 2.25$)
intensity factor solutions for various crack lengths of both cracks. This
matrix of solutions may then be used to accurately estimate the stress in-
tensity factor for either crack. However, such an effort is not made in the
present analysis.

A similar finite element model to the one shown in Figure 4-21 was used
for a tapered lug subjected to a pin-loading applied in the +90° directions.
Two critical locations, 43° and 205° measured from the axis of the lug, were
determined from the results of the stress analysis of an unflawed tapered
lug subjected to a pin loading applied in the -90° direction. Similar to
the -45° loading case, only one crack was analyzed at a time, though two
critical locations were modeled. The computed normalized stress intensity
factors are shown in Figure 4-23 as a function of the normalized crack length.
The figure shows that when the crack length is smaller than 0.85 R, the
stress intensity factor for a crack located closer to the base of the lug
is higher than the one located at the head side of the lug. When the crack
length increases (c/R > 0.85), the stress intensity factor for a crack
located at the head side becomes larger than the one located at the opposite
side of the hole. For the case when the direction of the applied pin loading
is reversed, the stress intensity factors are also computed and included in
the figure for a crack located at the head side of the lug. The surfaces of
a crack located closer to the base of the lug are completely closed during
the reversed loading. The K-values corresponding to the reversed loading are
smaller than the corresponding ones obtained under primary tensile loading.
However, these magnitudes are significantly larger than the corresponding
ones obtained for a case where the pin loading was applied in the 135° direc-
tion.

Based on the current unflawed stress and stress intensity factor analysis,
one may conclude that, among the principal pin loading directions of 0°, -45°
and -90°, the loading of -45° is the most severe loading case from the point of
view of fatigue crack initiation and fatigue crack propagation. Also, when a
tapered lug having a R /R = 2.25 ratio is subjected to cyclic fatigue pin
loading in -45° and -90° directions, a crack will probably initiate at the
critical location closer to the base of the lug first. When this crack
propagates, a second crack will initiate at the head side of the lug. Eventu-
ally, the growth of the base-side crack will slow down while the growth rate of
the head-side crack will increase and exceed that of the base-side crack.
Figure 4-23. Normalized Stress Intensity Factors for Single Through-the-Thickness Cracks Emanating from a Tapered Lug Subjected to a Pin Loading Applied in -90° and its Reversed Directions ($R_o/R_i = 2.25$)
lug will finally fail from the head side. Failure modes experimentally studied by Larsson [46] for tapered lugs loaded in the $-90^\circ$ direction are very similar to those discussed above.
Linear elastic fracture mechanics methodology is discussed in this section to predict crack growth in damaged attachment lugs. An accurate and efficient prediction of crack growth from an initial crack length to a critical or final crack length, when subjected to a prescribed load history, depends on a damage accumulation package containing the following four basic elements:

- Applied load spectrum
- Stress intensity factor solution
- Baseline crack growth rate data or crack growth rate equation
- Spectrum load interaction model.

The applied load spectrum is the loading sequence or history the attachment lug is subjected to for which the crack growth prediction is to be made, and is therefore basically an input for the problem.

It is generally accepted that the stress intensity factor controls the rate of propagation of a fatigue crack in structural components. Any cracked structure can be expected to respond in a predictable manner to a given applied load spectrum if its associated stress intensity factor solution is available. The development of such stress intensity factor solutions was presented in the previous two sections for straight and tapered attachment lugs for various configuration, design and loading complexities.

The response of the crack is usually given in the form of constant amplitude fatigue crack propagation rate, da/dN, as a function of stress intensity factor range, ΔK. These baseline material property data are obtained experimentally and can be expressed in either tabular or equation form, i.e.,

$$\frac{da}{dN} = f(\Delta K, \ldots) \quad (46)$$

Paris' [49], Forman's [50] and Walker's [51] equations are some of the typical representative crack growth-rate equations which will be discussed in this section.
The normal crack growth rate under constant amplitude loading changes if the load cycle is preceded by a different amplitude load cycle. As examples, a tensile overload causes permanent plastic deformation at the crack tip which in turn delays the crack growth at subsequent low-load cycles, while the compressive underload may accelerate the normal crack growth.

To account for such spectrum load-interaction effects, several crack-growth retardation models have been proposed. Models such as Wheeler [52], Willenborg [53], Generalized Willenborg [54] and Hsu [55] will be discussed in this section.

1. CRACK GROWTH RATE EQUATIONS

A large and growing volume of crack growth data are available. Most of the data are expressed in terms of $\Delta K$ versus $da/dN$. In this relationship, there is a threshold value of $K$, say $K_{th}$, below which a flawed structure can be cycled without measurable crack extension. At the other extreme is $K_c$, called fracture toughness, a value of $K$ at which a flaw will propagate unstably. Therefore, fatigue crack growth must occur in the range $K_{th}$ to $K_c$. All crack growth rate data available for establishing the $da/dN$ versus $\Delta K$ relationship fall within this range. Normally, data show some experimental scatter, and the least-square mean line representation of all test data is used to establish the $\Delta K$ versus $da/dN$ relationship.

Several methods of using the mean data to describe crack growth rates have been proposed. Of these methods, three well established crack propagation equations are used most. They are proposed by Paris et al [49], Forman et al [50], and Walker [51]. Paris' equation is limited to cases where a constant stress ratio, $R$, is applied. Forman and Walker's equations are more general and applicable to cases where variable loads exist in the load spectrum.

Accuracy of the predicted crack growth life, using any crack growth equation, depends upon accurate values of $da/dN$ versus $\Delta K$ and correct stress intensity factor values for a particular geometry.

1.1 Paris' Equation

The crack growth rate equation proposed by Paris, et al., is of the following form:
\[ \frac{da}{dN} = c_p (\Delta K)^n_p \]  
\[ (47) \]

where:

\[ \frac{da}{dN} = \text{rate of crack growth (inch/cycle)} \]

\[ c_p \text{ and } n_p \text{ are constants for a particular stress ratio}\]

\[ \Delta K = \text{stress intensity factor range (K}_{\text{max}} - K_{\text{min}}). \]

Paris' equation has been an effective regression equation for crack propagation rate in unreinforced flat specimens under constant amplitude loading. However, it is not successful in pooling such data for more than one value of stress ratio \( R, \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \).

1.2 Forman's Equation

In an effort to extend Paris' equation to cover various stress ratios, and to take into account the instability of the crack growth when the stress intensity factor approaches its critical value for a given material, Forman, et al., modified Paris' equation as follows:

\[ \frac{da}{dN} = \frac{c_f (\Delta K)^{n_f}}{(1-R)K_c - \Delta K} \]  
\[ (48) \]

where:

\[ c_f \text{ and } n_f \text{ - material constants}\]

\[ K_c \text{ - critical stress intensity factor}\]

Because of crack-tip blunting which is produced by cyclic strains in the material ahead of the advancing crack, it is important to select the appropriate \( K_c \) value. The choice of a \( K_c \) value larger than the fracture toughness of the material normally will give a better result in establishing \( K \) versus \( \frac{da}{dN} \) relationships. If the available data includes more than one \( R \)-ratio, \( K_c \) should be used as a curve fitting parameter to force the best fit for all available data.
1.3 Walker's Equation

Roberts and Erdogan [56] proposed that stress ratio effects in the power law region of the crack growth rate curve could be described using

$$\frac{da}{dN} = c (K_{\text{max}}^P \cdot \Delta K^P)$$

Walker [51] reformulated the equation so that the stress ratio would be explicit and it had the form

$$\frac{da}{dN} = c [K_{\text{max}} (1 - R)]^P$$

where $c$, $m$ and $p$ are material constants.

1.4 Determination of Constants in Paris' and Forman's Equations

Taking logarithms of both sides of Paris' and Forman's equations and rearranging terms, they become, respectively,

$$\log \left( \frac{da}{dN} \right) = \log c + n \log (\Delta K)$$

and

$$\log \left\{ \left[ (1 - R) K_c - \Delta K \right] \frac{da}{dN} \right\} = \log c_f + n_f \log (\Delta K)$$

For any two given coordinate points, say $\Delta K_i$, $\frac{da}{dN}$ and $\Delta K_{i+1}$, $\frac{da}{dN}$ which represent a segment of the growth rate curve, one can solve the two simultaneous equations for the unknowns $c$ and $n$.

2. Spectrum Load Interaction Models

The majority of crack growth studies have been carried out for constant stress amplitude because of the simplicity in testing and because the data can be presented in a straightforward manner. In actual practice, engineering components are subjected to loads which are often irregular and vary in a random manner. Fluctuating loads due to atmospheric turbulence on aircraft are typical examples of random loadings. The difference between the conventional fatigue experiments and the actual loading of the components has led to the development of various crack analysis approaches. As discussed in a previous section, normal crack growth rate under constant amplitude loading changes if the load application is preceded by a loading cycle of a different
amplitude. The tensile overload causes permanent plastic deformation at the crack tip which in turn delays the crack growth at subsequent low load cycles, while the compressive overload may accelerate the normal crack growth. The importance of delay in the rate of fatigue crack growth, as produced by tensile overloads, on the accurate prediction of fatigue lives of structures has been recognized, and quite a few investigations have been stimulated in this area.

Several models have been suggested to account for the effects of delay on the prediction of fatigue crack growth. Those models use the plastic-zone size (for either plane stress or plane strain) associated with the applied load levels to characterize the load-interaction effect. These models assume that if the size of the plastic zone, $r_p$, developed due to application of the current load cycle at crack length $a$ extends to or past the extremities of a previously developed load-interaction zone, $a_p$, (i.e., $(a + r_p) \geq a_p$), there will be no load interaction and the growth rate associated with the current load cycle is the same as the one generated under constant-amplitude loading. Conversely, the crack-growth rate would be reduced (retarded) if $r_p < (a_p - a)$. Some of the most often referenced retardation models are described below.

2.1 Wheeler Model

The first retardation model proposed was that of Wheeler [52] who suggested the use of a reduction factor on the constant amplitude crack growth rate:

$$\frac{da}{dN} = c_p f(\Delta K)$$

where $c_p$ is a retardation parameter defined as

$$c_p = \left( \frac{r_p}{a_p - a} \right)^m$$

for $a_p - a > r_p$

$$c_p = 1$$

for $a_p - a \leq r_p$
and \( m \) = shaping exponent

\[ R_y = \text{extent of the current yield zone} \]

\[ a_p - a = \text{the distance from crack tip to elastic-plastic interface} \]

This model requires previous spectrum growth data to derive an empirical "shaping exponent," \( m \). Although it is a substantial improvement over the linear cumulative damage rule, this model is more of a data fitting technique than it is a predictive technique. Besides, the exponent in Wheeler's retardation parameter is dependent not only upon the material, but also upon the manner in which spectrum loads were applied. For any design spectrum different from the ones used to generate the test data, the exponent \( m \) may be very different.

### 2.2 Willenhborg Model

As mentioned earlier, the rate of propagation of a fatigue crack is controlled by the stress-intensity factor at the crack tip. Therefore, the magnitude of the stress-intensity factor is a good indicator of the extent of crack tip deformation. Since the cyclic crack tip deformation is reduced due to prior overloads, the model of Willenhborg et al [53], which uses an "effective stress" concept to reduce the applied stress, and hence the crack tip stress-intensity factor, seems more appealing. The model is described as

\[
\frac{da}{dN} = f[(\Delta K)_{\text{eff}}, (K_{\text{max}})_{\text{eff}}] \tag{54}
\]

where

\[
(\Delta K)_{\text{eff}} = (K_{\text{max}})_{\text{eff}} - (K_{\text{min}})_{\text{eff}}
\]

\[
(K_{\text{max}})_{\text{eff}} = (\sigma_{\text{max}})_{\text{eff}} \sqrt{\pi a_c / \beta_T}
\]

\[
(K_{\text{min}})_{\text{eff}} = (\sigma_{\text{min}})_{\text{eff}} \sqrt{\pi a_c / \beta_T}
\]

\[
(\sigma_{\text{max}})_{\text{eff}} = \sigma_{\text{max}} - \sigma_{\text{red}}
\]

\[
(\sigma_{\text{min}})_{\text{eff}} = \sigma_{\text{min}} - \sigma_{\text{red}}
\]

\[
\sigma_{\text{red}} = \sigma_{\text{ap}} - \sigma_{\text{max}} \text{, or zero whichever is larger}
\]
\[
\sigma_{ap} = \frac{F_{ty}}{\beta_T} \sqrt{2\left(\frac{a_p - a_c}{a_c}\right)}
\]  

(55)

and

\[
a_c = \text{the crack length at the beginning of the load cycle after the overload}
\]

\[
a_p - a_c = \text{current plastic zone caused by the overload}
\]

\[
F_{ty} = \text{tensile yield strength of the material}
\]

\[
\beta_T = \text{total geometric correction factor}
\]

This model can be used to predict the fatigue crack growth under spectrum loading without the assistance of empirical factors or data. Although the model gives fairly good predictions for crack growth under moderate spectra, it tends not to give good correlations for spectrum loading with high overloads, especially overloads exceeding 150 percent of the maximum of the low load.

2.3 Generalized Willenborg Model

According to the Willenborg et al. model, if the overload ratio, \(K_{OL}/K_{max}\), is greater than or equal to 2.0, a zero effective stress-intensity factor will be predicted and the crack will stop growing. However, experimental data contradicts such a prediction. Gallagher [54] modified the Willenborg equation for effective stress intensity factors as follows:

\[
(K_{max})_{eff} = K_{max} - \phi \left[ K_{OL}^{max} \left(1 - \frac{\Delta a}{r_{OL}}\right) ^{1/2} - K_{max} \right]
\]

and

\[
(K_{min})_{eff} = K_{min} - \phi \left[ K_{OL}^{max} \left(1 - \frac{\Delta a}{r_{OL}}\right) ^{1/2} - K_{max} \right]
\]

or zero, whichever is larger.
The factor \( \phi \) is defined as

\[
\phi = \frac{1 - \left( \frac{K_{\text{max}}}{K_{\text{max}}^{\text{Th}}} \right)}{S_{SO} - 1}
\]  

(57)

in which \( K_{\text{max}}^{\text{Th}} \) is the maximum threshold stress intensity factor, and \( S_{SO} \) is the "overload shut-off ratio." When \( \frac{K_{\text{OL}}}{K_{\text{max}}^{\text{Th}}} \geq S_{SO} \), crack arrest occurs. Note that the original Willenborg model is recovered by setting \( \phi = 1 \).

2.4 Hsu Model

The Hsu model [55] developed at the Lockheed-Georgia Company utilizes an effective stress and closure concept. It assumes that the stress singularity does not exist if the crack surface is closed and that the crack propagates only during that portion of the load cycle in which the crack surface is fully open. Let \( \sigma_o \) be the crack-opening stress, i.e., the corresponding far-field stress at the onset of crack opening. Then the effective stress range of the load cycle during crack propagation can be defined as

\[
\Delta\sigma_{\text{eff}} = \sigma_{\text{max}} - \sigma_o
\]

(58)

When the opening stress is less than the minimum stress of the applied load cycle, the effective stress range is given by

\[
\Delta\sigma_{\text{eff}} = \sigma_{\text{max}} - \sigma_{\text{min}}
\]

(59)

If the maximum stress of the applied load cycle is less than the crack opening stress due to prior loads, the crack surface will be fully closed. Hence, the fatigue crack will not propagate.

However, experimental evidence indicated that below the closure \( K \), strain concentration at the vicinity of the crack tip still exists. Since fatigue damage is normally related to the cyclic strain range, the effective \( K_{\text{min}} \) is likely to be somewhat lower than the level at the onset of closure and higher than \( K_{\text{min}} \) under steady state (constant amplitude) conditions. The effective stress range and the effective load ratio can then be re-written as
\[ \Delta \sigma_{\text{eff}} = \sigma_{\text{max}} - (\sigma_{\text{min}})_{\text{eff}} \]  
\[ R_{\text{eff}} = (\sigma_{\text{min}})_{\text{eff}} / \sigma_{\text{max}} \]  
(60)  
(61)

respectively, where \( \sigma_{\text{min}} \leq (\sigma_{\text{min}})_{\text{eff}} \leq \sigma_0 \).

For any given load cycle, the effective stress-intensity factor can then be calculated from the equation

\[ \Delta K_{\text{eff}} = \Delta \sigma_{\text{eff}} \cdot \sqrt{\pi a} \cdot \beta_T \]  
(62)

The crack growth rate associated with this applied load cycle is computed from the growth rate equation

\[ \frac{da}{dN} = f (\Delta K_{\text{eff}}, R_{\text{eff}}, \ldots) \]  
(63)

For a given loading spectrum, if one can determine the effective minimum stress corresponding to each load cycle, the fatigue crack propagation life can be predicted.

3. CRACK GROWTH ANALYSIS PROGRAM

A computer program has been developed using the state-of-the-art methodologies including the stress intensity factors developed under this program for the prediction of fatigue crack growth behaviors of single through-the-thickness cracks and single corner cracks at attachment lugs under cyclic loading. All the data presented in Sections III and IV, such as stress analysis, fracture analysis, Green's function, interference-fit bushing analysis, etc., have been embedded in the computer program to make it as automatic and simple as possible. The computer program contains the three crack growth rate equations discussed above. There are five different options to input each individual mission load profile from which a mission mix spectrum can be generated. The program predicts the crack growth using a block-by-block integration technique. Crack growth may be analyzed with or without load-interaction using any of the models described above.
For through-the-thickness cracks, either the compounding solution or the Green's function solution can be used in the prediction. In predicting the growth behavior of a single corner crack, the crack may be analyzed by either the one-parameter (i.e., constant a/c ratio) or two-parameter method. For one-parameter analysis, the prediction is straightforward and is similar to through-the-thickness crack prediction. For two-parameter analysis, it is assumed that for a given number of applied load cycles, the extension of the quarter elliptical crack border is controlled by the stress intensity factors at the intersection of the crack periphery at the hole wall and the lug surface, i.e., \( K_A \) and \( K_C \), respectively. In general, the stress intensity factors at these two locations are different, resulting in different crack growth rates. Therefore, the new flaw shape aspect ratio after each crack growth increment will be different from the preceding one. The new crack aspect ratio is computed using the new crack lengths on both the hole wall and lug surface. The process will be repeated until the crack length along the hole wall is equal to the lug thickness. At that time the transitional crack growth criteria as discussed in Section III are used until the crack has achieved a uniform format. After that, if the failure has not occurred, a one-dimensional through-the-thickness crack analysis is used to continuously predict the subsequent crack growth life. The analysis is considered to be complete when fracture occurs or when the desired final crack length or the maximum usage time is reached.

A detailed description of the computer program along with the input instructions is provided in Reference [57].
SECTION VI

SUMMARY OF RESULTS

Analytical methods have been presented to predict both fatigue crack growth and residual strength of cracked attachment lugs. Each crack growth analysis includes the following elements:

- Stress intensity factor solution
- Baseline crack growth rate relationship
- Applied load sequence
- Spectrum load interaction model.

Of these, the emphasis in this report has been upon the calculation of stress intensity factors, covered in Sections III and IV. These sections have been followed by a discussion in Section V of the alternative constant amplitude fatigue crack growth rate relationships and spectrum load interaction models.

The following summary paragraphs are intended to provide an overview and to tie together the content of this report and aid the reader in using the analytical methods and results that have been presented.

1. SUMMARY OF STRESS INTENSITY FACTORS FOR STRAIGHT LUGS (SECTION III)

Section III has covered the calculation of stress intensity factors for straight attachment lugs subjected to axial loading. Several alternative methods have been discussed, including the simple compounding, two-dimensional cracked finite element, weighting function, and three-dimensional cracked finite element method. Parameters and complexities covered in the stress intensity factor solutions presented in this section are outer-to-inner radius ratio (1.50 to 3.0), crack geometry (single corner crack, through-the-thickness crack, and the intermediate transition), crack length (measured on lug face and along bore of hole), change in distribution of pin bearing pressure due to crack length change, ratio of pin modulus to lug modulus (1.0 or 3.0), interference-fit bushings, and elastoplastic analysis when the peak stress at the hole exceeds the material tensile yield strength.
A two-dimensional finite element analysis has been used to compute the stress distribution in the uncracked lug for lugs of various $R_o/R_i$ ratios. The major useful results of this analysis are the stress distribution along the potential crack path (Table 3-1) and the stress concentration factor (Figure 3-5).

The compounding method combines known solutions to obtain an engineering approximation for the stress intensity factor, Equation (10). Stress intensity factor values calculated by this method have been listed in Table 3-2.

The two-dimensional cracked finite element method properly models the crack tip stress singularity and the distribution of pin bearing pressure, which changes drastically with crack length (Figures 3-14 through 3-16). The stress intensity factor results from this method have been shown in Figure 3-18 and listed in Table 3-3.

The weighting function method calculates the stress intensity factor as the integral of the product of the stress in the uncracked lug times the Green's function for the lug; Equation (14). The Green's functions for straight lugs, developed using two-dimensional cracked finite element analyses with point loads applied on the crack surface, have been listed in Tables 3-4 through 3-6. However, these "original" Green's functions, when used with the stress distribution in the uncracked lug from Table 3-1, obtain $K_I$ results at variance with those of the two-dimensional cracked finite element analysis. The discrepancy arises because the Green's function method fails to account for the change in the distribution of pin bearing pressure. To correct the discrepancy, the original Green's functions have been modified such that the cracked finite element results are exactly duplicated when these "modified" Green's functions (Tables 3-7 through 3-9) are used.

Figure 3-29 compares the stress intensity factors for through-the-thickness cracks computed by the various methods. Reasonable agreement among all methods is obtained at $R_o/R_i$ ratios of 2.25 and 3.0, but not at $R_o/R_i \approx 1.50$. Thus, any method could be used for larger $R_o/R_i$ ratios, but the more rigorous
two-dimensional cracked finite element method (or equivalently, the Modified Green's function method) is preferred for $R_o/R_i < 2.0$.

The weighting function method can be applied to account for residual stresses caused by a shrink-fit bushing. Assuming the lug and bushing remain in intimate contact during loading, the stress intensity factor is calculated by Equation (24) from the sum of the residual stress caused by bushing installation plus the distribution of stress caused by the applied load. The residual stress is estimated from the closed-form solution for two concentric cylinders, Equation (30). The applied stress is obtained from two-dimensional finite element analysis of an uncracked lug in intimate contact with a neat-fit bushing. If the bushing and lug are of the same material, then the applied stress distribution can be obtained from Table 3-1. Sample applied stress distributions for various bushing/lug modulus ratios are shown in Figures 3-33 and 3-34. It is to be noted here that this method can give unconservative results when separation occurs between the bushing and lug. In such instances, an improved methodology has been developed and is presented in Volume III of this report.

The solution methods for through-the-thickness cracks can be modified to analyze a corner crack, utilizing a corner crack correction factor, along with a method to account for the transitional behavior as the corner crack becomes a through-the-thickness crack. Two alternative correction factor approaches have been suggested, a one-parameter and a two-parameter method.

In the one-parameter method the flaw shape is assumed to be constant (e.g., $a/c = 1.33$) and the stress intensity factor at the lug surface point (Point C) is calculated using Equations (31) and (32). These equations apply to the crack throughout its growth from a corner crack, through transition, to a through-the-thickness crack.

In the two-parameter method, Equations (33) through (36) (in conjunction with Figures 175, 176 and 178 of Reference [61]) are used to compute stress intensity factors at the lug surface and hole wall (Points C and A). During the transition to a through-the-thickness crack, Equations (37) through (40)
are used to compute stress intensity factors at the front and back surface.

A three-dimensional cracked finite element method, although too expensive for general application, has been used to check the accuracy of the one and two-parameter corner crack solutions. Comparisons of stress intensity factors are shown in Figures 3-46 through 3-50 and comparisons of corner crack correction factors are shown in Figure 3-51.

An elastoplastic analysis has been described for use when the peak stress in the uncracked lug exceeds the material tensile yield strength. Using the stress-strain curves of Figure 3-52, an iterative finite element analysis with incremental loading and unloading is used to calculate the stress distributions in the uncracked lug for the maximum and minimum loads of the fatigue cycle. These stress distributions are used with the modified Green's function to estimate $K_{\text{max}}$ and $K_{\text{min}}$ for a lug with a through-the-thickness crack. Sample results for $\Delta K$ have been given in Figures 3-59 and 3-60 and Table 3-13. This nonlinear method is inexact and only an approximation, because strictly speaking the validity of the Green's function method requires linearity between load and stress.

2. SUMMARY OF STRESS INTENSITY FACTORS FOR TAPERED LUGS (SECTION IV)

Section IV covers the calculation of stress intensity factors for tapered attachment lugs subjected to either axial or off-axis loading. Only unbushed lugs with a 45-degree included taper angle are analyzed.

A two-dimensional finite element analysis has been used to calculate the stress distribution in an uncracked lug. For axial loading, the peak tangential stress is located at approximately 90 degrees to the lug axis (Figure 4-5), and the stresses along the 90-degree line are listed in Table 4-1 for five values of $R_0/R_1$. Equation (45) is an equation for stress concentration factor for axial loading of a tapered lug with a taper angle between 0 and 45 degrees. For off-axis as well as axial loading, the angular locations of peak stresses are given in Figure 4-5.

Stress intensity factors for tapered lugs have been calculated by two-dimensional cracked finite element analysis. Stress intensity factor results...
are listed in Table 4-2 for axial tension loading (and plotted in Figure 4-20 for axial compressive loading) of tapered lugs having $R_o/R_i$ ratios from 1.5 to 3.0. Off-axis loading solutions for stress intensity factors are plotted in Figures 4-22 and 4-23 for a tapered lug with $R_o/R_i = 2.25$, loaded in the $-45$ and $-90$ degree directions, with a crack at either of the two most critical locations.

For corner cracks in tapered lugs, the corner crack correction factors presented in Section III can be used in conjunction with the appropriate through-the-thickness stress intensity factors. For problems involving bushings or stresses above the material yield strength, the weighting function would be needed for tapered lugs for each new loading direction and crack orientation. As an approximation, the Green's function for the axially-loaded straight lug with the same $R_o/R_i$ ratio may be used although the accuracy of this approximation is questionable, particularly for off-axis loading.

3. SUMMARY OF FATIGUE CRACK GROWTH ANALYSIS METHODS (SECTION V)

Crack growth analysis methodology has been briefly summarized in Section V. Alternative constant amplitude fatigue crack growth rate equations described include those of Paris [49], Forman [50], and Walker [51]. Alternative spectrum load interaction models summarized are the Wheeler [52], Willenborg [53], Generalized Willenborg [54] and Hsu [55] models. The selection of a rate equation and retardation model is left to the discretion of the analyst who is conducting the crack growth analysis.
REFERENCES


REFERENCES (Continued)


REFERENCES (Continued)


REFERENCES (Continued)


APPENDIX A

MODE I STRESS INTENSITY FORMULAS RELATED TO CRACKED LUGS
LINE CRACKS IN DOUBLY INFINITE PLATES

CASE 11. Uniform Tensile Stress

\[ K_{11} = S \sqrt{\pi a_0} \]

Numerical Example: \( S = 1, a_0 = 3: K_{11} = 3.06998 \)

CASE 12. Central Splitting Forces on Crack Surfaces

\[ K_{12} = \frac{P}{t^{\frac{3}{2}}a_0} \]

where \( t \) = thickness

Numerical Example: \( a_0 = 3, P = 1, t = 0.6: K_{12} = 0.54289 \)

CASE 13. Arbitrary Splitting Forces on Crack Surfaces

\[ K_{13}^{(1)} = K_{12} \sqrt{\frac{a_0-x}{a_0+x}} \]
\[ K_{13}^{(2)} = K_{12} \sqrt{\frac{a_0+x}{a_0-x}} \]

**NOTE.** \( x \) is negative if force is applied closer to crack tip (1).

Numerical Example: \( a_0 = 3, x = 1, P = 1, t = 0.6: K_{13}^{(1)} = 0.78778, K_{13}^{(2)} = 0.38368 \)
LINE CRACKS IN INFINITE-LENGTH STRIPS

CASE 21. Symmetric Crack, Uniform Tension

\[ K_{21} = K_{11} \Phi_2 \left( \frac{ma_0}{2b} \right) \]

\[ \Phi_2(\lambda_0) = \sqrt{\sec(\lambda_0)} \]

Numerical Example
\[ a_0 = 3, \ b = 8, \ S = 1, \ K_{21} = 3.3668 \]

CASE 22. Symmetric Crack, Central Splitting Forces on Crack Surfaces

\[ K_{22} = K_{12} \Phi_2 \left( \frac{ma_0}{2b} \right) u_{22} \left( \frac{ma_0}{2b} \right) \]

\[ u_{22}(\lambda_0) = \frac{1}{\sin(\lambda_0)} \]

Numerical Example
\[ a_0 = 3, \ b = 8, \ P = 1, \ r = 0.6, \ K_{22} = 0.83125 \]

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CRACKS AT HOLES IN DOUBLY-INFINITE PLATES

CASE 31. Single Crack, Tension Normal to Crack

\[ K_{31} = S \sqrt{\pi a_1} u_{31} \left( \frac{a_1}{a_1 + r} \right) \]

\[ u_{31}(P_1) = \exp \left( 1.2133 - 2.205 \rho_1 + 0.6451 \rho_1^2 \right) \]

Numerical Example:
\[ a_1 = 2, \ r = 1, \ S = 1: \ K_{31} = 2.583 \]

CASE 35. Single Crack, Distributed Splitting Forces at Hole

\[ K_{35} = K_{113}^{(1)} \Psi_{31}(P_1) u_{35}(P_1) \]

\[ u_{35}(P_1) = \exp \left( 0.15 (P_1^2 - 1) \right) \]

\[ x = \frac{a_1}{2} + a_0 = r + x \]

Limitation: Total load \( P \) results from uniform radial pressure from \( \pi/8 \) to \( 7\pi/8 \) and \( -\pi/8 \) to \( -7\pi/8 \). (\( K \) drops drastically if pressure is applied between \( -\pi/8 \) and \( \pi/8 \).)
CASE 26. Unsymmetric Crack, Uniform Tension ($b_n < b$)

At (f): $K_{26}^{(f)} = K_{11}^{(f)} \Phi_{26}^{(f)} (\lambda_f, \lambda_n)$

At (n): $K_{26}^{(n)} = K_{11}^{(n)} \Phi_{26}^{(n)} (\lambda_f, \lambda_n)$

$\Phi_{26}^{(f)} = 1 + \left[ \frac{\sqrt{\sec (\lambda_n f)} - 1}{1 + 0.21 \sin \left( 8 \arctan \left( \frac{\lambda_n - \lambda_f}{\lambda_n + \lambda_f} \right) \right) \right]$

$\Phi_{26}^{(n)} = \left[ u_{26} + \left( 1 - u_{26} \right) \left( 1 + \frac{1}{4 \sec \lambda_n} \right) \right]^{2} \sqrt{\sec \lambda_n}$

$u_{26} = \sin \left[ \frac{\pi \lambda f}{\lambda_n + \lambda_f} \right]$

$\lambda_f = \frac{\pi a_0}{4b \cdot 2b_n}$ \quad $\lambda_n = \frac{\pi a_0}{2b_n}$ \quad $\lambda_{nf} = \frac{4}{7} \lambda_n + \frac{3}{7} \lambda_f$

---

CASE 27. Unsymmetric Crack, Central Splitting Forces on Crack Surfaces

$K_{27}^{(1)} = K_{13}^{(1)} \Phi_{26}^{(1)} (\lambda_f, \lambda_n) \ u_{22} (\lambda_n)$

$K_{27}^{(2)} = K_{13}^{(2)} \Phi_{26}^{(2)} (\lambda_f, \lambda_n) \ u_{22} (\lambda_{nf})$

$x = b - b_n$ \quad Combining of Cases 12, 13, 21

$u = \frac{1 + \nu}{1 - \nu}$: P. Stress

$u = \frac{1 + \nu}{1 - \nu}$: P. Strain

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CASE 41. Single Crack, Uniform Tension

\[
K_{41} = K_{31} \Phi_{26}^{(n)} (\lambda_f, \lambda_n) \Phi_{21} (\lambda_r)
\]

\[
b_n = b - \frac{a_1}{2}
\]

Combining of Cases 11, 21, 26 and 31

\[
a_0 = r + \frac{a_1}{2}
\]

\[
b_f = b + \frac{a_1}{2}
\]

CASE 44. Single Crack, Distributed Splitting Forces at Hole

\[
K_{44} = K_{27} \Phi_{31}^{(1)}(\rho_1) \Phi_{21}(\lambda_r) u_{35}(\rho_1)
\]

Limitation: Same load distribution as Case 35.

Combining of Cases 13, 21, 27, 31 and 35

Dependent Variables:

\[
x = \frac{a_1}{2}, b_n = b - x \text{ and } a_0 = r + \frac{a_1}{2}
\]
CASE 71. Quarter-Circular Corner Crack Which Becomes Through Thickness

\[
K_{71}^{(C)} = S \sqrt{\pi c} \left( \frac{c}{c + r} \right)^{1/3} \phi_{71}^{(C)}
\]

\[
\phi_{71}^{(C)} = 1.0 - \frac{0.2386}{1 + 2 \left( S \frac{t}{r} \right)^2}
\]

NOTE: For cracks beginning as quarter circular corner cracks in configurations 32 through 70, multiply the through-crack solution by \( \phi_{71}^{(C)} \).

Limitations:

- No plate bending
- \( a = c \)

Numerical Example
\( a = c = 2, r = 1, t = 3, S = 1 \): \( K_{71}^{(C)} = 2.18834 \)