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NUMERICAL ALGORITHMS AND SOFTWARE FOR  
MATRIX RICCATI EQUATIONS

FINAL REPORT

DR. ALAN J. LAUB

10/25/84

U.S. ARMY RESEARCH OFFICE

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Numerical issues related to the computational solution of the algebraic matrix Riccati equation were studied. The approach used the generalized eigenproblem formulation for the solution of general forms of algebraic Riccati equations arising in both continuous- and discrete-time applications. These general forms result from control and filtering problems for systems in generalized (or implicit or descriptor) state space form. A Newton-type iterative refinement procedure for the generalized Riccati solution was derived. The issue of numerical condition of the Riccati problem was addressed. Balancing to improve numerical condition was studied. A Fortran package called RICPACK was developed. Numerical experiments with RICPACK were performed to investigate a number of proposed condition numbers. Experience with RICPACK to date indicates that it is the most powerful software yet developed to solve general classes of Riccati equations reliably for modest-sized (say of order) a few hundred or less problems.

The special structure of models of physical systems given in linear second-order form was also examined. Exploiting that structure in solving associated Riccati equations was studied. Tests for controllability and observability were derived in terms of the original second-order-model matrices. *Originator - Joseph J. Key - no date*  
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Final Report on "Numerical Algorithms & Software for Matrix Riccati Equations"

Dr. Alan J. Laub

Department of Electrical & Computer Engineering  
University of California  
Santa Barbara, CA 93106

An interdisciplinary approach blending systems theory, numerical analysis, and mathematical software was employed to address various aspects of the numerical solution of matrix Riccati equations and some related problems. A variety of numerical issues related to the computational solution of the algebraic matrix Riccati equation were studied. Our approach concentrated on the generalized eigenproblem formulation for the solution of general forms of algebraic Riccati equations arising in both continuous- and discrete-time applications. These general forms result from control and filtering problems for systems in generalized (or implicit or descriptor) state space form.

The issue of numerical condition of the Riccati problem was addressed as was the use of various types of balancing --both numerical and system-theoretic-- to improve numerical condition. A rather sophisticated Fortran software package called RICPACK was developed for use both as a research tool and as a production vehicle. Numerical experiments with RICPACK were performed to investigate a number of proposed condition numbers. None of these numbers, both those developed under this project and those developed by others, have been found to yield completely satisfactory results. The subject remains open still. However, overall experience with RICPACK to date indicates that it is the most powerful software yet developed to solve general classes of Riccati equations reliably for modest-sized (say, of order a few hundred or less) problems.

Particular attention was focused in the latter stages of the project on the special structure of models of physical systems represented in linear second-order form. Exploiting that structure in solving associated Riccati equations was studied. As a by-product, convenient numerical tests for controllability and observability were derived directly in terms of the original second-order-model matrices.

The following papers were published in technical journals and acknowledged ARO support during this contract:

1. Laub, A.J., "Schur Techniques for Riccati Differential Equations," in Hinrichsen, D. and A. Isidori (Eds.), Feedback Control of Linear and Nonlinear Systems, Springer-Verlag, New York, 1982, 165-174.
2. Laub, A.J., "Efficient Calculation of Frequency Response Matrices from State Space Models," submitted to ACM Trans. Math. Software, June 1982.
3. Laub, A.J., L.M. Silverman, and M. Verma, "A Note on Cross-Grammians for Symmetric Realizations", Proc. IEEE, 71 (1983), 904-905.
4. Laub, A.J., and W.F. Arnold, "Controllability and Observability Criteria for Multivariable Linear Second-Order Models," IEEE Trans. Aut. Contr., AC-29(1984), 163-165.
5. Laub, A.J., Numerical Aspects of Control Design Computations, Proc. NATO AGARD Lec. Series No. 128 (Computer-Aided Design and Analysis of Digital Guidance and Control Systems), July 1983, pp. 5.1-5.16.
6. Arnold, W.F., and A.J. Laub, Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations, Proc. IEEE, Vol. 72, Dec. 1984.
7. Laub, A.J., Numerical Linear Algebra Aspects of Control Design Computations, IEEE Trans. Aut. Contr., Vol. AC-30, Feb. 1985.

The following refereed conference papers appeared in conference proceedings and acknowledged ARO support during this contract:

1. Strunce, R.R., and A.J. Laub, A Pragmatic Approach to the Development of Advanced Control Technology Software, Proc. IFAC Symposium on Computer-Aided Design of Multivariable Technological Systems, (G. Leininger, Ed.), Purdue University, West Lafayette, Indiana, September 1982, 197.
2. Laub, A.J., Schur Techniques in Invariant Imbedding Methods for Solving Two-Point Boundary Value Problems, Proc. 21st IEEE Conf. on Decision and Control, Orlando, Florida, Dec. 1982, pp. 56-61.
3. Laub, A.J., and W.F. Arnold, Controllability and Observability Criteria for Multivariable Linear Second-Order Models, Proc. Amer. Control Conf., San Francisco, CA, June 1983; pp. 925-926.
4. Laub, A.J., Numerical Aspects of Solving Algebraic Riccati Equations, Proc. 22nd IEEE Conf. on Decision and Control, San Antonio, Texas, Dec. 1983, pp. 184-186.
5. Arnold, W.F., and A.J. Laub, A Software Package for the Solution of Generalized Algebraic Riccati Equations, Proc. 22nd IEEE Conf. on Decision and Control, San Antonio, Texas, Dec. 1983, pp. 415-417.

Two Ph.D. Dissertations were supported in the Department of Electrical Engineering - Systems at the University of Southern California during this contract:

Kwae Hi Lee: Generalized Eigenproblem Structures and Solution Methods for Riccati Equations  
January 1983

Dr. Lee is now a Professor of Electrical Engineering at Sogang University, Seoul, South Korea.

William F. Arnold III: On the Numerical Solution of Algebraic Matrix Riccati Equations  
December 1983.

Dr. Arnold is now Head of Control Analysis and Simulation at the Naval Weapons Center, China Lake, CA.

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