1. TITLE (Include Security Classification):
"ELECTROMAGNETIC RAIL LAUNCHER AS A CIRCUIT ELEMENT"

2. PERSONAL AUTHOR(S):
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3. ABSTRACT:
The circuit equations for the inductor driven rail launcher are integrated numerically using a fourth order Runge-Kutta method. Their solution is also obtained as a perturbation expansion for small resistance. The EMACK parameters are used to study the results. For high current shots the full numerical solution agrees with the first order approximation to 3%. For low current shots they agree to 5%. A very simple zeroth order approximation agrees with the full numerical solution to 5% in the high current case and to 3% in the low current case.
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Dr. Robert J. Barker  
AFOSR  
Attention: Physics  
Bolling AFB  
D. C. 20332  

October 25, 1984  

Reference: Grant AFOSR 83-0338, Final Scientific Report  
'The Electromagnetic Rail Launcher as a Circuit Element'  
P. I. Dr. Manuel A. Huerta  
University of Miami, Department of Physics.  

Attached please find the Final Report for the referenced Grant. The grant had a budget of $11,998 and supported about 1.5 months of work. The effort was concentrated on numerical and analytical solutions of the circuit equations for an inductor driven rail launcher. A simple analytical expression that agrees with the full numerical solution to a few percent was found.  

I would like to express my appreciation for the support I received under this grant.  

Sincerely  

Manuel A. Huerta  
Professor of Physics  

Approved for public release  
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ABSTRACT

The circuit equations for the inductor driven rail launcher are integrated numerically using a fourth order Runge-Kutta method. Their solution is also obtained as a perturbation expansion for small resistance. The EMACE parameters are used to study the results. For high current shots the full numerical solution agrees with the first order approximation to 3%. For low current shots they agree to 5%. A very simple zeroth order approximation agrees with the full numerical solution to 5% in the high current case and to 3% in the low current case.
INTRODUCTION

To avoid repetition the initial proposal for this grant is attached as an Appendix to this Final Report. The work in the grant was to study the circuit equations for the inductor driven rail launcher. The basic equations are Eq. (15) of the proposal, which comes from the circuit equation after converting from time to distance as the independent variable,

\[(L_0 + L'x) \frac{dv}{dx} + \frac{L'}{2} v^2 + 2(R_o + R_p) v + 2R' \int x \, dv = b' ,\]

and the force equation

\[m v \frac{dv}{dx} = L' i_0^2 / 2 .\]  

This is an integro-differential equation for the speed \(v\) as a function of the distance \(x\). It is in a form suitable for a regular perturbation expansion for small resistance.

We introduce dimensionless variables in terms of the characteristic parameters of the problem as follows. Let \(s\) be a dimensionless length defined as

\[x = gs ,\]

where

\[g = \frac{L_0}{L'} .\]

We introduce a velocity scale \(v_f\) by

\[m v_f^2 = L_0 i_o^2 ,\]

where \(i_o\) is the initial current in the inductor. We define a dimensionless speed \(u\) by

\[v = v_f u ,\]
and a dimensionless current $I$ by

$$I = \frac{1}{i_0}.$$  \hfill (5)

The inductance and resistance have a time constant given by

$$t_1 = \frac{L_0}{R_0 + R_p},$$  \hfill (6)

in terms of which we define an expansion parameter $\varepsilon$

$$\varepsilon = \frac{2g}{v_f+1}.$$  \hfill (7)

Let $k$ be the resistance ratio

$$k = \frac{R'g}{R_o + R_p},$$  \hfill (8)

and $q$ be the dimensionless initial speed

$$q = qv_o/v_f,$$  \hfill (9)

where $v_o$ is the initial speed of the projectile.

In terms of the above variables, Eq. (1) is rewritten as

$$(1 + s) u \frac{du}{ds} + \frac{u^2}{2} + \varepsilon u + \varepsilon k \int_q^u s \, ds = b,$$

and

$$\frac{d}{ds}(u^2) = I^2,$$  \hfill (10)

where $b$ is given by

$$b = \frac{(1 + q^2)}{2} + \varepsilon q.$$  \hfill (11)

We integrate the full Eq. (1) using a fourth order Runge-Kutta method and take the result as the standard for comparison with the approximation scheme.
developed below. We expand the solution \( u(s) \) in an asymptotic series

\[
    u = u_0(s) + \varepsilon u_1(s) + \varepsilon^2 u_2(s) + \ldots .
\]

(12)

and substitute the series into Eq. (10). The lowest order equation that results is

\[
    (1 + s) \frac{d}{ds}(u_0^2) + u_0^2 = 1 + q^2 .
\]

(13)

This equation is integrated easily to yield

\[
    u_0^2 = 1 + q^2 - \frac{1}{1 + s} ,
\]

and

\[
    I = \frac{1}{1 + s} .
\]

(14)

The lowest order approximation \( u_0 \) amounts to neglecting the resistance completely. Then the kinetic energy gained by the projectile is given by the drop in the magnetic energy stored in the system inductance \( L_0 + L'x \). Despite its simplicity, the speed of Eq. (14) agrees rather well with the Runge-Kutta solution. It overestimates the speed, but the error is not much greater than other errors that are inherent in determining what are the correct parameter values to use to simulate the experimental situation.

The error is reduced if we calculate the first order correction \( u_1 \) from the equation

\[
    \frac{d}{ds}[(1 + s)u_0 u_1] = -u_0 + q - k \int_q^{u_0(s)} s \, du_0 .
\]

(15)

After some straightforward integrations we obtain
\[ u_1 = \frac{(k - 1)(u_o - q)}{u_o} + \frac{[k + 2(1+q^2)(1-k-k(1+s))] \ln\left(\frac{(r+u_o)(r-q)}{(r-u_o)(r+q)}\right)}{2(1+q^2)^{3/2} u_o(s + 1)} + \frac{k[u_o(1+s) - q]}{2(1+q^2)^{3/2} u_o(1+s)}. \]

with the first order speed given by
\[ v = v_f (u + u_1) \] (16)

Conclusion

Below we present the numerical results of the full Runge-Kutta integration, spd R-K, compared with the first approximation spd first = \( v_f u_o \), and the second approximation spd second = \( v_f (J_o + \epsilon u_1) \). The results are presented in the tables that follow for the EMACK rail gun shots 1-5. The parameters \( L_o, L', L_o + R, R', m, j \), etc. are given above each table.

Table I contains the results for shot 1. It is the least accurate case because of the low current which leads to a relatively large value of \( \epsilon \). In this case spd first is 8% above spd R-K while spd second is 4% above spd R-K. For the other shots the current is larger and the errors are reduced. The value of spd second is off from spd R-K by only 3%. It is remarkable that spd first, which from \( u_o \) and neglects the resistance entirely is off by only 5%.
References
EMACK shot number 1
L0 = 0.500E-05H,
RO + Rp = 0.271E-04ohm,
Mass = 0.195E+00kg,
I0 = 0.500E+06A,
\( \omega \) = 0.35m
R1 = 0.450E-04ohm, \( t1 = 0.185E+00s \)
barrel length = 5.00m
\( v0 = 0.000E+00m/s, \ vF = 2531.85m/s \)

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**TABLE 1**
EMACK shot number 2
L0 = 0.500E+05H, L1 = 0.585E+06H, g = 8.55m
R0+Rp = 0.271E+04ohm, R1 = 0.450E+04ohm, t1 = 0.185E+00s
mass = 0.225E+00Kg, barrel length = 5.00m
10 = 0.800E+06A, v0 = 0.000E+00m/s, v1 = 3771.24m/s
epsilon = 0.0245674

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<th>L1</th>
<th>L2</th>
<th>R2</th>
<th>R3</th>
<th>L3</th>
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</tbody>
</table>

**Table 3**

- **L**: Length = 0.100E+00 m
- **R**: Radius = 0.100E-01 m
- **v₀**: Initial speed = 0.000E+00 m/s
- **v**: Final speed
- **r**: Radius
- **R**: Resistance
- **L**: Length
- **R**: Resistance
- **L**: Length
- **mass**: Mass
- **v₀**: Initial speed

**Constants**
- **L**: Length = 0.500E-01 m
- **R**: Radius = 0.100E-01 m
- **v₀**: Initial speed = 0.000E+00 m/s
- **v**: Final speed
- **r**: Radius
- **R**: Resistance
- **L**: Length
- **mass**: Mass
- **v₀**: Initial speed

**Calculations**
- Position: Calculated using the equations of motion for a free-falling object.
- Speed: Calculated using the formula for velocity in a vacuum.
- Resistance: Calculated using Ohm's Law.

**Notes**
- All values are in SI units.
- The calculations are based on the assumption of a frictionless environment.
<table>
<thead>
<tr>
<th>Time</th>
<th>Position</th>
<th>Spd first</th>
<th>Spd second</th>
<th>Spd R-f</th>
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**EMAC shot number 4**

EMAC: shot number 4

L0 = 0.500E-03m, RO = 0.271E-04m, R1 = 0.317E-00kg, 

barrel length = 5.0cm

v0 = 0.210E+07m/s

epsilon = 0.010Ω

Li = 0.586E-04m, 

g = 8.55m/s²

R1 = 0.163E+00s, 

v0 = 0.159E-03m/s
EMAC! shot number 5
L0 = 0.500E-05H,
R0 + Rp = 0.271E-04ohm,
mass = 0.318E+00kg,
j0 = 0.160E+07A,
\( \epsilon = 0.585E-06H, \quad g = 8.55m \)
barrel length = 5.00m
\( \nu' = 0.000E+00m/s, \quad v_f = 6344.41m/s \)
\( \epsilon = 0.0146033 \)

\begin{table}
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\textbf{time} & \textbf{position} & \textbf{spd R-f} & \textbf{spd second} & \textbf{spd first} \\
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0.40415E-02 & 0.11656E+00 & 0.11843E+00 & 0.11841E+00 & 0.11840E+00 \\
0.60623E-02 & 0.74625E+00 & 0.74778E+00 & 0.74778E+00 & 0.74778E+00 \\
0.80836E-02 & 0.37583E+00 & 0.37631E+00 & 0.37631E+00 & 0.37631E+00 \\
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0.66687E-02 & 0.41763E+00 & 0.41774E+00 & 0.41774E+00 & 0.41774E+00 \\
\hline
\end{tabular}
\end{table}
Minigrant Proposal to Air Force Office of Scientific Research

Attention: Richard W. Kopka, Major, USAF
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
Attention: XOT
Boiling AFB, D.C. 20332

Proposing Institution: University of Miami
Department of Physics
Coral Gables, FL 33124

Title: The Electromagnetic Rail Launcher as a Circuit Element

Budget: $11,998

Proposal Duration: January 1, 1983 to December 31, 1983

Principal Investigator: Dr. Manuel A. Huerta
Associate Professor of Physics
Telephone No. (305)284-2323

Manuel A. Huerta
Principal Investigator
Chairman, Department of Physics

Date 10/6/82

Ronald E. Siegwald
Authorized Official
Director, Sponsored Programs/
Fiscal Management

Date
Introduction

This proposal is a follow-up to the research the P.I. performed at the AFATL/DLDDL at Eglin AFB under the 1982 USAF-SCEEE Summer Faculty Research Program. That research identified the basic research issues involved in improving the models for the arc plasma produced in electromagnetic rail launchers with plasma driven projectiles. The results of that research are summarized in the final report entitled Basic Research Issues in Electromagnetic Rail Launchers with Plasma Driven Projectiles. A copy of that report is attached as the Appendix to this proposal.

The small scale of a migrant dictates that a problem of the appropriate scope be considered. I propose to study the behavior of rail launchers as circuit elements. The circuit properties most directly describe how the stored electrical energy is converted into kinetic energy of the projectile and determine the overall efficiency.
Previous Work by Other Authors

The current variations in rail launchers are predominantly of low frequencies. This enables us to limit our treatment to the case where the length of the rails is much shorter than the wavelength of the electromagnetic fields present. Under these conditions the current is uniform as a function of position along the rails and the distributed capacitance is negligible. We are then able to speak of the relationship between the voltage at the breech and the current through the rails and plasma.

A further simplification is possible regarding the skin depth. It is shown in the Appendix that provided that the rail thickness is much smaller than the rail separation one may take the rail resistance per unit length $R'$ and the inductance per unit length $L'$ to be constants independent of the frequency. From Eq. (56) of the Appendix we get that the breech voltage $V_b(t)$ is related to the current $i(t)$ by

$$V_b(t) = i(t) R_b + \frac{d}{dt}(L_b i)$$  \hspace{1cm} (1)

where $R_b = R_p + R' x$ and $L_b = L' x$, \hspace{1cm} (2)

with $R_p$ equal to the plasma resistance. Here $x$ is the distance from the breech to the plasma as shown in Fig. 1 of the Appendix. The i-V relation involves the variable $x$. The equation for it is

$$m \frac{d^2 x}{dt^2} = \frac{1}{2} L' i^2 - f.$$  \hspace{1cm} (3)

This equation is the statement that the magnetic force $\frac{1}{2} L' i^2$ accelerates the mass $m$ against a retarding friction $f$.

The form of Eq. (1) can be understood simply. The voltage drop at the breech is made up of the resistive drop plus the rate of change of the magnetic flux $\Phi = L_b i = L' x i$. The fact that the force on the armature is $\frac{1}{2} L' i^2$ can be understood by considering the energy into the breech. The
power into the breech is given by $iV_b$. From Eq. (1) one readily gets that

$$iV_b = i^2 R_b + \frac{d}{dt} \left( \frac{1}{2} L' x i^2 \right) + \frac{1}{2} L' i^2 \frac{dx}{dt}. \quad (4)$$

We see that the power into the breech goes into a resistive heat loss, an increase of the magnetic energy $(1/2 L_b i^2)$ stored in the rails, and a third term that gives the rate of doing work on the armature. From this term we deduce that the magnetic force on the armature is given by $1/2 L'i^2$.

The circuit behavior of the rail launcher is described by Eqs. (1)-(3). Of course the rails will be part of a complete circuit. The simplest launcher arrangement consists of a storage inductor $L_0$, with its resistance $R_0$, connected in series with the breech as shown in Fig. 1. Previous steps have energized the inductor $L_0$ so that at $t = 0$ the current has the value $i_0$.

![Fig. 1 Schematic of rail gun. $V_b$ is the breech voltage. The arc has moved to a distance $x$ from the breech. $L_0$ is a storage inductor with internal resistance $R_0$.](image-url)
The complete circuit equations are then
\[
d/dt[(L'_0 + L'x)i] + (R'_0 + R'_p + R'x)i = 0, \tag{5}
\]
and \[d^2x/dt^2 = L'i^2/2m, \tag{6}\]
where the force of friction has been neglected in eq. (6). The initial conditions are
\[
i(t = 0) = i_0, \quad x(t = 0) = 0,
\]
and \[dx/dt(t = 0) = v_0. \tag{7}\]
The initial velocity allows for a preaccelerated projectile that is injected into the launcher with a speed \(v_0\). The system of Eqs. (5) and (6) is nonlinear and has not been solved analytically in nontrivial cases. Numerical results are easy to obtain. In fact, Mr. Kenneth Cobb, of AFATL/DLDC, already has a program to integrate them.
Previous work by the P.I.

Some of the work under this proposal would go into developing further the numerical results. The main effort, however, will go into developing analytical results. Analytical results are essential to understand how the solution scales as the various parameters are varied. The P.I. has already made sufficient progress on this problem that valuable results are almost assured.

We first note that multiplying Eq. (5) by $i$ and using Eq. (6) yields a single third order equation for $x(t)$,

$$
(L_0 + L' x) \frac{d^3 x}{dt^3} + 2L' \frac{dx}{dt} \frac{d^2 x}{dt^2} + 2(R_0 + R_p + R' x) \frac{d^2 x}{dt^2} = 0.
$$

(8)

This equation is to be solved with the boundary conditions of Eq. (7) together with

$$
\frac{d^2 x}{dt^2}(t = 0) = \frac{L' i_0^2}{2m}.
$$

(9)

We note that Eq. (8) can be rewritten as

$$
\frac{d}{dt} \left[ (L_0 + L' x) \frac{d^2 x}{dt^2} \right] + \frac{L'}{2} \frac{d}{dt} \left[ \frac{dx}{dt} \right]^2 + (R_0 + R_p) \frac{d}{dt} \left[ \frac{dx}{dt} \right] + 2R' \frac{d}{dt} \int x \frac{d^2 x}{dt^2} dt = 0.
$$

(10)

Obviously Eq. (10) can be integrated once if the $R'$ term is negligible. This term would be zero in an ideal rail launcher with supercooled rails. We will carry along the $R'$ integral term and treat it as a perturbation. Integrating Eq. (10) yields
\[
\frac{d^2 x}{dt^2} + \frac{L'}{2} \left[ \frac{dx}{dt} \right]^2 + 2(R_o + R_p) \frac{dx}{dt} + 2R' \int_0^t x \frac{d^2 x}{dt^2} dt = 0.
\]

Eq. (11) follows immediately from conservation of energy, that is
\[
\frac{1}{2} L_o \dot{x}^2 + \frac{1}{2} m \dot{v}^2 = \frac{1}{2} (L_o + L' \dot{x})^2 + \frac{1}{2} m \dot{v}^2 + \int_0^t \dot{x}^2 (R_o + R_p + R_1 x) dt.
\]  

Eqs. (11) and (12) can easily be shown to be equivalent by making use of Eq. (6). Eq. (11) cannot be integrated further for \(x(t)\) even if one neglects the \(R'\) term. Progress is possible, however, if we go to the energy form. That is, we introduce the velocity \(v\) as a dependent variable,
\[
\dot{v} = \frac{dx}{dt}, \quad \frac{dv}{dt} = \frac{d^2 x}{dt^2},
\]
and furthermore use that
\[
\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx},
\]
to get
\[
(L_o + L' \dot{x}) v \frac{dv}{dx} + \frac{L'}{2} v^2 + 2(R_o + R_p) v + 2R' \int_0^x x dv = b'.
\]

Eq. (15) is in a form suitable for a nonsingular perturbation expansion of the solution where the lowest order can be found exactly. For this we introduce dimensionless variables using the principal parameters of the problem as scales. The length of the rails, \(L\), plays an important role in limiting the
final projectile velocity. We introduce a dimensionless length \( x \) by

\[ x = \frac{L'}{L_0} x. \tag{16} \]

We introduce a velocity scale \( v_f \) obtained by letting the initial acceleration

\[ a = \frac{L' i_{0}^2}{2m} \tag{17} \]

apply for the distance \( L \) to give

\[ v_f = \sqrt{2al} = \sqrt{L' i_{0}^2 / 2m}. \tag{18} \]

We then introduce a dimensionless velocity \( V \) by

\[ v = v_f V, \tag{19} \]

and a time scale \( t_f \) by

\[ t_f = \frac{L'}{v_f} \sqrt{\frac{L}{2a}} = \sqrt{\frac{m}{2L' i_{0}^2}}. \tag{20} \]

A dimensionless parameter \( h \) that gives the ratio of rail inductance to external inductance is

\[ h = \frac{L'}{L_0}. \tag{21} \]

Another dimensionless parameter \( r \) is defined by

\[ r = 2 \frac{t_f}{t_0}. \tag{22} \]

where \( t_0 \) is the time scale of the external R-L circuit,

\[ t_0 = \frac{L_0}{R_0 + R_p}. \tag{23} \]
Finally the expansion parameter is given by the ratio of rail resistance to external and plasma resistance:

\[ \varepsilon = \frac{Rf}{R_o + R_p} \]  

(24)

In terms of the dimensionless variables \( X \) and \( V \) and the dimensionless parameters defined above, Eq. (15) is rewritten as

\[ (1 + hX) V \frac{dV}{dX} + h/2 V^2 + rV + \varepsilon \int_{V_0}^{V_f} X dV = b, \]  

(25)

where \( b = \frac{b'}{L_o v_f^2} = \frac{1}{2} + h \left( \frac{\varepsilon \tau r}{\varepsilon} \right) + r \left( \frac{\varepsilon \tau r}{\varepsilon} \right) \).

A nonsingular perturbation expansion is set up by substituting the series

\[ V(X) = V_0(X) + \varepsilon V_1(X) + \varepsilon^2 V_2(X) + \ldots \]  

(26)

into Eq. (25) with the initial conditions

\[ V_0(0) = v_0/v_f, V_i(0) = 0, i = 1, 2, \ldots. \]  

(27)

The results are

\[ (1 + hX)V_0 \frac{dV_0}{dX} + h/2 V_0^2 + rV_0 = b, \]  

(28)

\[ (1 + hX)(V_1 dV_0/dX + V_0 dV_1/dX) + hV_0 V_1 + rV_1 + r \int_{V_0}^{V_f} X dV_0 = 0, \]  

(29)

\[ (1 + hX)(V_0 dV_2/dX + V_1 dV_1/dX + V_2 dV_1/dX) + h/2 (V_1^2 + 2 V_0 V_2) + rV_2 + r \int_{V_0}^{V_f} X dV_1 = 0, \]

for the terms of order \( \varepsilon^0, \varepsilon^1, \) and \( \varepsilon^2 \) respectively.
Proposed Work

The sequence of Eqs. (28) - (30) is to be solved analytically as far as practical. The analytical solution is to be compared with a numerical solution of Eq. (25).

The analytical solution is off to a solid start because the P.I. has obtained a closed form solution for Eq (28) in the form \( \text{I} = \text{I}(V_0) \). This is easily obtained by rewriting Eq. (28) as

\[
\int_{V_0}^{V} \frac{V_0 dV_0}{b - (h/2V_0^2 - rV_0)} = \int_{0}^{X} \frac{dX}{1 + hX}.
\]

(31)

A simple integration gives

\[
1 + hX = \frac{a^2 - 1}{a^2 - \beta^2 (1 + hV_0/r)^2} \left( \frac{a + 1}{a - 1} \cdot \frac{a - \beta (1 + hV_0/r)}{a + \beta (1 + hV_0/r)} \right)^{\beta/a}.
\]

(32)

where \( \beta = (1 + hV_0/rV_f)^{-1} \) and \( a = (1 + 4h\beta^2/r^2)^{1/2} \).

Eq. (32) gives us \( \text{I}(V_0) \) for an ideal launcher. Since one cannot invert for \( V_0(x) \), we simply modify Eq. (29) to continue with the asymptotic expansion.

All that is needed is to consider \( V_1, V_2, \) etc as functions of \( V_0 \).
## Budget for Minigrant Proposal

Duration: January 1, 1983 to December 31, 1983

### Salaries, Wages and Fringe Benefits

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<th>Position</th>
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<td>$35049/9 months</td>
<td>17.33% of faculty salary</td>
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<td>Graduate Student Research Assistant</td>
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### Other Direct Costs

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<td>Publication Costs</td>
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</tr>
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<td>Disc Drive (permanent equipment)</td>
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<tr>
<td>Software, and/or other equipment to upgrade microcomputer</td>
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</tr>
<tr>
<td><strong>Total Other Direct Costs</strong></td>
<td><strong>$1136</strong></td>
</tr>
</tbody>
</table>

### Total Direct Costs

- **$7705**

### Indirect Costs: 60% of MTDC(7155)

- **$4293**

### Total Budget

- **$11,998**

*The Coral Gables campus negotiated indirect cost rate for the University of Miami for fiscal year 6/1/82 to 5/31/83 is 131.0% of Modified Total Direct Costs (MTDC). The University is willing to accept an indirect cost rate that is lower than the negotiated rate. However, if the proposal is awarded, the University retains the right to exercise any cost clause or cost regulation made a part of the signed agreement.*