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EQUATIONS AND ALGORITHMS FOR METEOROLOGICAL APPLICATIONS IN AIR WEATHER SERVICE

By

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This document lists equations or algorithms used for computing common meteorological and geophysical variables. It should serve as a handy reference for BWS computer programmers and thus promote standardization and minimize time-consuming literature researches by them. The equations and algorithms are classified into three groups: kinematic properties, thermodynamic properties, and gridded or contoured data fields.

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PREFACE

This report is largely an outgrowth of the algorithm collection effort done in preparation for the Automated Weather Dissemination System. Numerous people were involved in that project, and it is not possible to list each one here. Nevertheless, we wish to acknowledge the anonymous contributions of all these people. Without their help this report would not have been completed.

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CONVERSION FACTORS

1mb = 1000 dynes/cm^2 = .029532 in. Hg = 0.1 kilopascal (kpa)

1 inch mercury (Hg) = 33.86389mb = 33863.89 dynes/cm^2

1 dyne = 1 g cm/s^2 = 10^{-5} kg m/s^2

1 erg = 1 dyne cm = 2.3892 X 10^{-8} calorie

1 foot = 0.3048m = 30.48 cm

1 inch = 2.54 cm

1 deg lat = 111.137 km = 59.969 nautical miles

1 m/s = 3.6 km/hr = 1.9425 knots = 2.2369 statute miles/hr = 3.2808 ft/s

^{o}K = ^{o}C + 273.16

^{o}F = 9/5 (^{o}C) + 32

^{o}Rankine = 9/5 (^{o}C) + 491.69

METEOROLOGICAL CONSTANTS

mw	Molecular weight of water	18.0 g/mole
R*	Universal gas constant	8.3144 X 10^7 ergs/mole/^{o}K
R_d	Gas constant for dry air	0.068557 cal/g/^{o}K 287.04 m^2/s^2/^{o}K
g	Acceleration due to gravity	9.806 m/s^2
c_p	Specific heat of dry air	0.24 cal/g/^{o}K
p_s	Standard sea-level pressure	1013.246 mb 29.921 in Hg
T_s	Standard sea-level temperature	288.15 ^{o}K
a	Standard Tropospheric lapse rate	0.0065 ^{o}K/m
k	R_d/c_p (approximately)	0.2854
w	Earth's rotation rate	7.29X10^{-5}/s

LIST OF SYMBOLS AND ABBREVIATIONS

a	Acceleration
A	Altimeter setting
c	Wind speed
CCL	Convective condensation level
CTI	Cross totals index
D	D-value (departure of pressure height from standard atmosphere pressure height)
D(t)	Displacement distance as a function of time
DD	Dewpoint depression
d_e	Spacing (in meters) between grid points
e	Vapor pressure (of water)
e_s	Saturation vapor pressure
f	Coriolis parameter
H	Thickness of a layer of the atmosphere
H_e	Field elevation
I	Grid-point number along X-axis
J	Grid-point number along Y-axis
KI	K-index
L	Latent heat of water
LCL	Lifting condensation level
LI	Lifted index
LFC	Level of free convection
ln	Natural (base-e) logarithm (ln X = 2.3026 log X)
log	Base-10 logarithm
P	Pressure
PA	Pressure altitude
P_m	Station pressure
Q	Any continuous variable (e.g., temperature, moisture, etc.)
r	mixing ratio
r_s	saturation mixing ratio
Ri	Richardson number
RH	Relative humidity
SI	Showalter stability index
t	time

t_o	reference time
T	temperature
T_d	Dewpoint temperature
T_w	Wet bulb temperature
T_{50}	Final temperature of a parcel lifted to 500mb
TTI	Total totals index
u	eastward wind; wind in X direction in I,J grid = $\frac{dx}{dt}$
v	northward wind; wind in Y direction in I,J grid = $\frac{dy}{dt}$
VTI	Vertical totals index
VWS	Vertical wind shear
w	Upward wind
x	distance eastward
y	distance northward
Z_s	Height of pressure surface in standard atmosphere
α	Wind direction (grid-relative)
β	Wind direction (north-relative)
∂	Partial differential operator
Δ	Finite difference operator
λ	Longitude
λ_o	Reference longitude
$\emptyset$	Latitude
θ	Potential temperature
θ_d	Partial potential temperature
θ_e	Equivalent potential temperature
θ_{se}	Pseudo-equivalent potential temperature
ρ_h	Density altitude

1. INTRODUCTION

The advent of small computers in the base weather station (BWS) has brought on a new era in meteorological analysis. Among other uses, computers permit direct calculation of many meteorological variables which were previously only estimated from charts, nomograms, or tables. For some operations, e.g., computing potential temperature (θ) for given temperature (T) and pressure (p), there is only one equation in the literature; it is familiar to all meteorologists. The equations used for some operations are not as well known, or there are multiple approximations given in the literature (e.g., Tetens formula and the Goff-Gratsch equation are both used to compute saturation vapor pressure (e_s) for given temperature). This report lists equations or algorithms used to compute common meteorological and geophysical variables. It should serve as a handy reference for BWS computer programmers, promote standardization, and minimize time-consuming literature researches. Computers can be applied to every problem that involves numerical calculation, therefore, this list is not necessarily exhaustive. Meteorological algorithms which are locally developed, and not from this report, should be submitted to parent units before they are used operationally. The equations are given with little explanation or interpretation, and users are cautioned to consult the detailed references for each topic if more information is needed. In most cases, examples are given so local programmers can validate their programs. These equations and algorithms may be classified into three general groups: (1) those related to the wind fields and known as <u>kinematic</u> properties; (2) those related to temperature and pressure and known as <u>thermodynamic</u> properties of the atmosphere; and (3) those in which data are objectively analyzed to obtain gridded or contoured data fields. In this report, the meteorological algorithms are provided within each of the three groups. Following these, relations for map transformation, chart generation, and useful geophysical constants are given. Finally, when possible, the equations given here are taken from the specifications for the Automated Weather Distribution System (AWDS) so there will be minimum discontinuity in data analysis procedures when AWDS becomes operational.

2. ALGORITHMS FOR KINEMATIC PROPERTIES OF THE ATMOSPHERE

2.1 Wind. Wind observations are reported by specifying the wind direction and wind speed. For computation of kinematic properties of the atmosphere, it is necessary to resolve the wind into orthogonal components. These components are parallel to a grid system. For example, if a latitude/longitude grid is used, the component parallel to latitude lines (u) is positive when the wind is moving toward the east, and the component parallel to longitude lines (v) is positive when the wind is moving toward the north. The algorithm used to resolve the reported direction and speed to u and v components on a grid and vice versa depends upon the type of grid system and the orientation of the grid. At AFGWC, different conventions are used for the orientation of these components in the different grid systems. AFGWC/TN-79/003 describes the convention for each of these systems. Units requiring access to this technical note may request it through channels from the AWS Technical Library. The convention for wind direction is to measure it clockwise in degrees from north to the direction from which the wind is blowing.

2.1.1 Wind Components from Wind Direction and Speed. The algorithms for deriving u and v components from wind direction and speed are illustrated for several different grid systems in the following:

2.1.1.1 Wind Components on a Latitude/Longitude Grid. The u and v components are determined using:

$$u = - c \sin \beta$$

$$v = - c \cos \beta$$

where β is the wind direction and c is the wind speed.

2.1.1.2 Wind Components on the AFGWC Northern Hemispheric Whole-Mesh Reference Grid. The orientation of the u and v components and x and y axes for this grid is given in Section 3.1.2.1 and Figure 3.6, AFGWC/TN-79/003. The map is positioned with respect to the grid by specifying a reference longitude λ_0 which is parallel to the horizontal axis ($\lambda_0 = 10^\circ E$). The u component is positive for a wind blowing toward the positive I direction and the v component is positive for a wind blowing toward the negative J direction. With this convention, the u and v components of the wind vector at a point are determined from:

$$u = + c \cos [\beta - (\lambda - \lambda_0)] \qquad \text{2-2a}$$

$$v = - c \sin [\beta - (\lambda - \lambda_0)] \qquad \text{2-2b}$$

where λ is the longitude of the point where the wind was observed in degrees (0-360 degrees, positive east - see Section 2.1, AFGWC/TN-79/003).

2.1.1.3 Wind Components on the AFGWC Southern Hemispheric Whole-Mesh Reference Grid. The orientation of u and v components and x and y axes for this grid is given in Section 3.1.2.2 and Figure 3.7, AFGWC/TN-79/003. The u component is positive for a wind blowing toward the positive I direction and the v component is positive for a wind blowing toward the negative J direction. With this convention, the u and v components of the wind at a point are determined from:

$$u = - c \cos [\beta + (\lambda - \lambda_0)] \qquad 2\text{-}3a$$

$$v = + c \sin [\beta + (\lambda - \lambda_0)] \qquad 2\text{-}3b$$

where λ and λ_0 are as given in 2.1.1.2.

2.1.2 Orientation of Winds for Plotting and Isoplething. In AWS, the convention is that grid-relative wind direction is used for plotting and north-relative wind direction is used for isoplething (isogons). Wind speed is the same in both cases. Given gridded u,v wind components, wind speed c and grid-relative wind direction α are computed using the algorithm in 2.1.2.1. North-relative wind direction β is obtained by sequentially solving the algorithms in 2.1.2.1 and 2.1.2.2 for the AFGWC grid of interest. Given north-relative wind direction β, the algorithm in 2.1.2.2 can be used to solve for the grid-relative wind direction α for the grid of interest.

2.1.2.1 Computation of Wind Speed and Grid-Relative Wind Direction from Gridded Data. All of the grids discussed in Section 2.1.1 have the convention that a clockwise rotation of 90 degrees from the positive v component results in the u component. Given gridded u,v wind components, wind speed c and grid-relative direction α may be obtained from:

$$c = (u^2 + v^2)^{\frac{1}{2}} \qquad 2\text{-}4$$

$$\alpha = 180^\circ + \arctan (u/v) \qquad 2\text{-}5$$

where the sign of each wind component is taken into account so the arctangent function returns values of α between 0° and 360° as indicated in the following table:

Wind Direction and Speed	Sign	
Range ($^\circ$)	u	v
α = 180	0	+
180 < α < 270	+	+
α = 270	+	0
270 < α < 360	+	-
α = 0	0	-
0 < α < 90	-	-
α = 90	-	0
90 < α < 180	-	+

2.1.2.2 Relationship Between North-Relative and Grid-Relative Wind Directions. North-relative wind direction β and grid-relative wind direction α are related according to Equations 2-6, 2-7, and 2-8 for the three AFGWC grids described above. For computation of grid-relative wind direction α from observed north-relative direction β, λ is the longitude of the reporting station. For computation of north-relative wind direction β from the grid-relative wind direction α (obtained as specified in 2.1.2.1), λ is the longitude of the grid point. The longitude λ is obtained by solving Equations 3.7, 3.8, and 3.12, AFGWC/TN-79/003. In the following equations, the angle solved for (β or α, depending on the application) must be adjusted in multiples of 360° to keep the angle between 0° and 360°:

AFGWC Tropical Grid

$$\beta = \alpha + 180^\circ \tag{2-6}$$

AFGWC Northern Hemispheric Whole-Mesh Reference Grid

$$\beta = \alpha + (90^\circ + \lambda - \lambda_o) \tag{2-7}$$

AFGWC Southern Hemispheric Whole-Mesh Reference Grid

$$\beta = \alpha - (90^\circ + \lambda - \lambda_o) \tag{2-8}$$

2.2 Advection and Advected Field.

2.2.1 General. Advection is the process of transporting an atmospheric property solely by the velocity field of the atmosphere. It provides an estimate of the time rate of change of a parameter. The advected field is the field of values of the property after a given time interval (during which advection is applied).

2.2.2 Calculations. Mathematically, the horizontal advection of an atmospheric property Q is:

$$\partial Q/\partial t = - (U\partial Q/\partial x + V\partial Q/\partial y) \tag{2-9}$$

To calculate the advection from values of U, V and Q at grid points for specified geographical areas, finite differences are used to approximate the partial derivatives in Equation (2-9). Because of the use of more than just one grid type, it is necessary to consider the conventions for positive wind components and positive coordinate axes discussed in Section 2.1. Specifically, on the AFGWC polar stereographic (I, J) grids for the northern and southern hemispheres where I is along the X axis and J is along the Y axis, the instantaneous time rate of change of Q at a point I, J (advection) is determined by:

$$Q_t\,(I, J) = - \{ (U\,(I, J) \times [Q\,(I + 1, J) - Q\,(I - 1, J)] +$$

$$V\,(I, J) \times [Q\,(I, J + 1) - Q\,(I, J - 1)]\}/2d_e \tag{2-10}$$

On the Latitude/Longitude Grid, the instantaneous time rate of change of Q at a point I, J is determined from:

$$Q_t\,(I, J) = - \{ -U\;(I, J) \times [Q\,(I + 1, J) - Q\,(I - 1, J)] +$$

$$V\,(I, J) \times [Q\,(I, J + 1) - Q\,(I, J - 1)]\}/2d_e \tag{2-11}$$

where Q_t (I, J) = $\partial Q/\partial t$ is the time rate of change of Q at point I, J (advection); U (I, J) and V (I, J) are the horizontal components of wind at point I, J respectively; Q is the value of the atmospheric parameter at the indicated grid point (I + 1, J), (I - 1, J), (I, J +1), and (I, J - 1); and d_e is the grid spacing, a function of the grid from which data are extracted. It is variable within the grid. The following references in AFGWC/TN-79/003 allow computation of d_e. NOTE: The equations in AFGWC/TN-79/003 give d_e in km. This must be converted to the same distance units as the wind speed distance unit. In Equation 2-10, d_e is determined for the point (I, J) from Equation 3.4, AFGWC/TN-79/003. The value of the image scale σ in Equation 3.4, AFGWC/TN-79/003, is determined from Equation 3.5, AFGWC/TN-79/003. The value of latitude ϕ in Equation 3.5, AFGWC/TN-79/003, is determined from Equations 3.11, 3.7, and 3.8, AFGWC/TN-79/003. In Equation 2-11, d_e is determined for the point (I, J) from Equation 3.37, AFGWC/TN-79/003. The value of latitude ϕ used in Equation 3.37, AFGWC/TN-79/003 is given by Equation 3.43, AFGWC/TN-79/003. The value of $\Delta\lambda$ in Equation 3.37, AFGWC/TN-79/003, is 0.064775 radians and the value of $\Delta\lambda$ in Equation 3.43, AFGWC/TN-79/003, is 3.7113 degrees. Suitable approximations to Equations 2-10 and 2-11 can be used at grid boundaries (e.g., one-sided differencing). Knowing Q_t (I, J) at a given time t_o, a predicted value for Q (advected field) at a new time t_1 can be calculated by:

$$Q_1 (I, J) = Q_o (I, J) + Q_t (I, J) \times (t_1 - t_o) \qquad 2\text{-}12$$

where Q_o is the known value of Q at time t_o and Q_1 is the new value of Q at time t_1, and Q_t is from Equation 2-10 or 2-11. Equation 2-12 can be used to predict the spatial distribution of any atmospheric property at relatively near times in the future (1-12 hours). Equation 2-12 is applied in the following fashion:

a. The atmospheric property to be advected (e.g., temperature) is specified.

b. The total time interval (e.g., current time plus 12 hours) and the incremental time step (e.g., 1 hour) are specified.

c. The instantaneous time rate of change for the selected atmospheric property at grid points is determined from Equations 2-10 or 2-11, depending on the grid in use. The U and V components of the wind are assumed to remain invariant in time for this computation.

d. Predicted values for Q are calculated by Equation 2-12 for each time step until the total time interval is achieved.

2.3 Extrapolation.

2.3.1 General. For short term forecasting of various weather features such as low and high pressure systems and height and vorticity centers, extrapolation techniques are often used by meteorologists. The extrapolations are based on the information from the time sequence of analyses of the appropriate parameter.

2.3.2 Calculations. Simple synoptic extrapolation of a weather feature is defined by the formula:

$$D(t) = Vt + \tfrac{1}{2} at^2 \qquad 2\text{-}13$$

where D(t) is the displacement of the weather feature over time, t. V is the velocity of the feature, and a is the acceleration of the feature. The initial velocity of the weather feature is determined from computation of the distance traveled over a time period between the most recent known position at time t_{-1} and the current position at time t_o. In the case of a surface weather feature, 3-hour or 6-hour time intervals of are most commonly used. For rapidly moving features, shorter time intervals of 1 or 2 hours are often used for computing initial velocity. For upper air weather features, the initial velocity is determined over a 12-hour interval between the current data observation time and 12 hours earlier. Thus, the time interval for computing initial velocity is variable and should be an input parameter. To compute acceleration, the position before t_{-1} (t_{-2}) is also required. The technique for determining velocity and acceleration can use a cartesian coordinate system. The current position, X_o at time t_o, and past two positions, X_{-1} at time t_{-1} and X_{-2} at time t_{-2} are determined in x, y coordinates. If only two positions of a feature are available, the method may be applied by assuming that the acceleration is zero. To apply the extrapolation technique, compute displacements along the x and y axes separately. Equation 2-14 can be used to compute the extrapolated position along the X-axis and Equation 2-15 can be used to compute the extrapolated position for the Y-axis (X_{+1} and Y_{+1} respectively):

$$X_{+1} = X_o + [(X_o - X_{-1}/(t_o - t_{-1})] (t_{+1} - t_o) +$$

$$[(X_o - X_{-1})/(t_o - t_{-1}) - (X_{-1} - X_{-2})/(t_{-1} - t_{-2})][(t_{+1} - t_o)^2/(t_o - t_{-2})] \qquad 2\text{-}14$$

$$Y_{+1} = Y_o + [(Y_o - Y_{-1})/(t_o - t_{-1})] (t_{+1} - t_o) +$$

$$[(Y_o - Y_{-1})/(t_o - t_{-1}) - (Y_{-1} - Y_{-2})/(t_{-1} - t_{-2})][(t_{+1} - t_o)^2/(t_o - t_{-2})] \qquad 2\text{-}15$$

2.4 Vertical Wind Shear. The vertical wind shear (VWS) is the difference between wind vectors between two levels in the atmosphere. The magnitude of the VWS is usually all that is required for analysis and is computed from reported or interpolated u and v wind components using the following relationship:

$$VWS = [(u_2 - u_1)^2 + (v_2 - v_1)^2]^{\frac{1}{2}} \qquad 2\text{-}16$$

where the subscripts 1 and 2 refer to the lower and upper atmospheric levels, respectively. When reported wind speed (c) and direction (β) are available, the magnitude of the VWS is computed using the following relationship:

$$VWS = [(c_2 \sin \beta_2 - c_1 \sin \beta_1)^2 +$$

$$(C_2 \cos \beta_2 - C_1 \cos \beta_1)^2]^{\frac{1}{2}} \qquad 2\text{-}17$$

where the subscripts 1 and 2 refer to the lower and upper atmospheric levels, respectively.

2.5 Streamline Analysis. Streamline analyses are generated using wind direction and wind speed data as input within selected regions. Following a procedure described by Whittaker (1977), in Monthly Weather

Review (Volume 105, pp. 786-788), the slopes of the streamlines can be determined from observed wind directions. The display can consist of a series of lines tangent to the reported wind directions (within an instantaneous wind flow pattern). Units requiring access to Whittaker's article should forward requests to the AWS Technical Library through their parent unit.

3. THERMODYNAMIC VARIABLES AND THEIR ALGORITHMS

3.1 General. Observations of the atmosphere above the ground are taken at selected station locations over the globe four times each day at 0000, 0600, 1200, and 1800 GMT and twice each day at 0000 and 1200 GMT at other selected stations. These observations are input to the derivation of thermodynamic variables that are used as diagnostic and forecasting tools. Temperature, pressure (or height), and humidity are measured at the surface and aloft. Using the First Law of Thermodynamics, the equation of state, Dalton's Law, and other fundamental relations, a variety of thermodynamic variables can be derived. Meteorologists in the BWS have usually made these analyses by plotting the observed data on charts, such as the Skew-T, Log-P diagram, and by estimating other variables from lines on the chart. Of course, the derived quantities can be computed directly from the observations if desired. Equations and algorithms for some common thermodynamics variables are given below. Unless stated otherwise, the equations listed are those given in the AWDS specifications. For application of these thermodynamic quantities, forecasters should consult other appropriate reference documents (e.g., AWS/TR-79/006 discusses general theory and use of the Skew-T, Log-P diagram; AWS/TR-80/001 discusses aircraft icing forecasts, etc.).

3.2 Glossary of Terms. The following is a glossary of terms that appear in this section. Part A consists of terms that are derived from observed parameters. Part B lists the observed parameters and Part C contains other parameters, some of which are computed from the observed parameters that are input to the computation for the derived parameters, and others which are constants. The equations for computing the parameters in Part C are given once here, rather than being repeated several times throughout the text of this report. Part D contains useful conversion factors for the computations.

Part A. Derived Parameters:

θ,	Potential temperature ($^{\circ}$K)
$\bar{\theta}$,	Mean potential temperature of a layer ($^{\circ}$K)
H,	Thickness of a layer of the atmosphere (m)
D,	D-values (m) [departure from the standard atmospheric height of a constant pressure surface]
r,	Mixing ratio (gm/kg)
$\bar{r}$,	Mean mixing ratio of a layer (gm/kg)
r_s,	Saturation mixing ratio (gm/kg)
RH,	Relative humidity (%)
LCL,	Lifting condensation level
T_{LCL},	Temperature at the LCL ($^{\circ}$K)
P_{LCL},	Pressure of the LCL (mb)

Z_{LCL}, Height of the LCL (m)

CCL, Convective condensation level [P_{CCL} (mb)]

LFC, Level of free convection [P_{LFC} (mb)]

SI, Showalter index (^oK)

LI, Lifted index (^oK)

KI, K-index (^oK)

VTI, Vertical totals index (^oK)

CTI Cross totals index (^oK)

TTI, Total totals index (^oK)

Part B. Observed Parameters Used for Computation of Derived Parameters:

P, Atmospheric pressure (mb)

Z, Height of a constant pressure surface (m)

T, Dry bulb temperature (^oC)

DD, Dewpoint depression (^oC)

A, Altimeter setting (in, Hg.)

Part C. Constants and other Input Parameters to Required Derived Parameters (Part A) Computed from Observed Parameters:

C.1 Constants and Other Variables:

mw, Molecular weight of water vapor = 18.0 gm/mole

R*, Universal gas constant = 8.3144×10^7 ergs/mole/^oK

R_d, Gas constant for dry air, R_d = 0.068557 cal/gm/^oK
= 2.8704×10^2 $m^2/sec^2/^oK$

g, Acceleration of gravity, g = 9.806 m/sec^2

C_p, Specific heat of dry air at constant pressure = .24 cal/gm/^oK

Z_s, Height of a constant pressure surface in the standard atmosphere (m)

P_s, Standard sea level pressure - 1013.246 mb or 29.921 inches Hg. or 101.325 kilopascals (kpa)

T_s, Standard sea level temperature = 288.16°K

P_m, Station pressure (mb)

PA, Pressure altitude (m)

H' Density altitude (m)

a, The standard atmospheric lapse rate below the isothermal layer, a = 0.0065° C/m

k, Ratio of the gas constant for dry air to the specific heat capacity of dry air at constant pressure = 0.2854

H_e, Field elevation (m)

Temperatures are reported in °C, but except where otherwise indicated, temperature in the equations in the following sections use °K, where T(°K) = T(°C) + 273.16.

C.2 Other Input Parameters:

(a) T_d, Dewpoint temperature (°C), upper air

$$T_d = T - DD\ (^{\circ}C) \qquad \text{C-2(a)}$$

(b) e, Vapor pressure(†) (mb)

when $T \geq 273.16\ ^{\circ}K$, then

$$e = 10^{**}\ [23.832241 - 5.02808 \times \mathrm{Log}\ (T_d)$$

$$- 1.3816 \times (10^{**}(-7)) \times (10^{**}(11.334 - 0.0303998 \times T_d))$$

$$+ 8.1328 \times (10^{**}(-3)) \times (10^{**}(3.49149 - 1302.8844/T_d))$$

$$- 2949.076/T_d] \qquad \text{C-2(b)}$$

and when $T < 273.16\ ^{\circ}K$, then

(†) The relation given is the Goff-Gratch formula, which is correct for all values of T_d. However, as the Goff-Gratch formula is often cumbersome, approximate formulae are sometimes used (a detailed discussion is given by David Bolton, Monthly Weather Review, Volume 108, pp. 1046-1053). One formula with accuracy suitable for most purposes is Tetens' formula:

$$e = 6.11\text{mb} \times 10^{**} \frac{(aT_d)}{(T_d + b)}$$

where a = 7.5, b = 237.3 for $T_d > 0°C$, and a = 9.5, b = 265.5 for $T_d < 0°C$. C-2(b)(1)

$$e = 10^{**} [3.56654 \times \text{Log}(T_d) - 0.0032098 \times T_d - 2484.956/T_d + 2.0702294] \qquad \text{C-2(c)}$$

where T_d is dewpoint temperature in ^oK,

** means "raise to the exponent of,"

Log means "take to base-10 logarithm"

Check values for these equations are:

	Equations C-2(b) or C-2(c)	Equation C-2(b)(1)
$T_d = 50^oC$	e = 123.40mb	e = 123.39mb
$T_d = 20^oC$	23.373	23.389
$T_d = -10^oC$	2.597	2.596
$T_d = -40^oC$	0.1283	0.1262

(c) e_s, Saturation vapor pressure

To compute e_s, use Equations C-2(b) and C-2(c), C-2(b)(1), replacing the dewpoint with with the temperature.

(d) L, Latent heat of water vapor (cal/gm)

$$L = 597.3 - 0.564\,(T - 273.16) \text{ when } T \geq 273.16^oK \qquad \text{C-2(d)}$$

$$L = 597.3 - 0.574\,(T - 273.16) \text{ when } T < 273.16^oK \qquad \text{C-2(e)}$$

(e) θ_{se}, Pseudo-equivalent potential temperature (^oK)

$$\theta_{se} = \theta_d \exp(Lr/c_pT) \qquad \text{C-2(f)}$$

where "exp" means raise the natural base e to the exponent in the argument that follows, and r is the mixing ratio in grams per gram (given in g/Kg by 3-2) and θ_d is given by C-2(g).

(f) θ_d, Partial potential temperature (^oK)

$$\theta_d = T\,[(1000)/(P - e)]^k \qquad \text{C-2(g)}$$

(g) T_{50} (parcel), Final estimated temperature of a parcel lifted to 500 mb.

Part D. Conversion Factors:

1 mb = 1000 dynes/cm^2

1 in. (Hg) = 33.86389 mb

1 dyne = 1 gm cm/sec^2

1 foot = .3048 m

1 erg = 1 dyne cm = 2.3892×10^{-8} cal.

1 calorie = 4.186×10^7 ergs

= 4.186 joules

3.3 Potential Temperature, Mixing Ratio, Saturation Mixing Ratio, Relative Humidity, and Layer Average of Quantity Q.

3.3.1 Potential Temperature. The potential temperature (θ) is the temperature a parcel of air would have if brought dry adiabatically to a standard pressure of 1000 mb.

$$\theta = T \; [(1000/P)^k]$$

A check value is: if P = 850 and T = 20°C then θ = 307.1°K 3-1

3.3.2 Mixing Ratio. The mixing ratio, r is defined by:

$$r = 1000 \times .622e/(P-e) \qquad 3\text{-}2$$

where e is given by Equation C-2(b) or C-2(b)(1) or C-2(c). A check value is:

if T_d = 20°C and P = 990 mb then r = 15.04g/kg.

3.3.3 Saturation Mixing Ratio. The saturation mixing ratio, r_s is defined by:

$$r_s = 1000 \times .622e_s/(P-e_s) \qquad 3\text{-}3$$

where e_s is given by Equation C-2(b) or C-2(b)(1) or C-2(c). A check value is:

if P = 980 mb and e_s = 20 mb then r_s = 12.958.

3.3.4 Relative Humidity. The relative humidity, RH at any level is computed from:

$$RH(\%) = \frac{r}{r_s} \; (100) \qquad 3\text{-}4$$

where r and r_s are determined from Equations 3-2 and 3-3, above. A check value is:

if $T = 20^oC$ and $T_d = 10^oC$ and $P = 990$ mb then RH = 51.9%.

3.3.5 Layer Average of Quantity Q. The average of a quantity Q is calculated from Equation 3-5:

$$\bar{Q}\ (P_m, P') = [1/(\ln P_m - \ln P')] \times \{\sum_{i=m}^{n-1} [(1/2) \times (Q_i + Q_{i+1}) \times$$

$$(\ln P_i - \ln P_{i+1})] + [(1/2) \times (Q_n + Q') \times (\ln P_n - \ln P')]\} \qquad 3\text{-}5$$

where

$\bar{Q}$ = average of Q over the pressure layer from P_m to P'

P_m = station pressure (pressure at bottom of the layer)

Q_o = value of Q at P_m

P' = pressure at top of layer

Q_i, P_i = values of Q and pressure respectively at level i

P_n = highest level i such that $P_m \geq P_n > P'$

$Q' = Q_n + (Q_{n+1} - Q_n) \times (\ln P_n - \ln P')/(\ln P_n - \ln P_{n+1})$

where "ln" means "take the natural logarithm."

The first term of the two terms in curly brackets in Equation 3-5 is zero if $n = m$ (i.e., if $P_m = P_n > P'$).

3.4 Equivalent Potential Temperature. The equivalent potential temperature (θ_e) takes into account the possible gain (loss) of sensible heat that takes place during condensation (evaporation) and is defined by the dry adiabatic reduction of the equivalent temperature to a standard pressure of 1000 mb. θ_e is computed from:

$$\theta_e = T\ [(1000/P)^{k(1-0.28r)}] \times$$

$$\exp[(3376/T_{LCL} - 2.54) r (1 + 0.81r)], \qquad 3\text{-}6$$

where $K = 0.2854$, P is the pressure in mb, T_{LCL} is the temperature in ^oK at the lifting condensation level given by Equation 3-9 and r is the mixing ratio in grams per gram. A check value is:

if $T = 20^oC$, $T_d = 10^oC$, and $P = 995$ mb, then $e_s = 12.272$ mb, $r = 7.767 \times 10^{-3}$ g/g, $T_{LCL} = 280.97^oK$, $\theta_e = 316.14^oK$.

3.5 Thickness and D-Values.

3.5.1 Thickness. Thickness, H is computed from:

$$H = Z_2 - Z_1 \qquad \text{3-7}$$

Z_1 is the height of the lower (in height) constant pressure surface and Z_2 is the height of the higher constant pressure surface. Examples of constant pressure surface layers for which thickness fields are computed and their thickness in the standard atmosphere are:

Layer	Thickness in the Standard Atmosphere
a. 1000 - 500 mb	5463m
b. 1000 - 700 mb	2901
c. 1000 - 850 mb	1346
d. 850 - 500 mb	4117
e. 850 - 700 mb	1555

3.5.2 D-Values. The D-value is computed from:

$$D = Z - Z_s \qquad \text{3-8}$$

Z_s is the height of a pressure surface in the standard atmosphere and Z is the reported height of that pressure surface. Table 3-1 gives the standard height (Z_s) for each standard constant pressure surface.

Table 3-1

Standard Atmosphere Heights of Constant Pressure Surfaces

PRESSURE P (mb)	HEIGHT Z_s (m)
1000	111
950	540
900	988
850	1,457
800	1,949
750	2,466
700	3,012
650	3,591
600	4,206
550	4,865
500	5,574
450	6,344
400	7,185
350	8,117
300	9,164
250	10,363
200	11,784
150	13,608
100	16,180
70	18,442
50	20,576
30	23,849

D-values can be computed for any or all of the constant pressure surfaces.

3.6 Lifting Condensation Level. The lifting condensation level, (LCL), is the height at which a parcel of air would become saturated when lifted dry adiabatically. The temperature of the LCL, T_{LCL} is calculated from:

$$T_{LCL} = T_d - (0.212 + 0.001571 T_d - 0.000436\ T_o)(T_o - T_d) + 273.16 \qquad 3\text{-}9$$

where T_d is the surface dewpoint temperature and T_o is the surface temperature, both in ^oC. A check value is:

$$\text{if } T_o = 20^oC \text{ and } T_d = 10^oC \text{ then } T_{LCL} = 280.97^oK.$$

The pressure of the LCL is given by:

$$P_{LCL} = P_o\ (T_{LCL}/T_o)^{1/.2854} \qquad 3\text{-}10$$

where P_o is the surface pressure and T_o is the surface temperature in ^oK. A check value is:

if $t_o = 20^oC$, $P_o = 980$ mb, $T_{LCL} = 281^oK$ then $P_{LCL} = 844.8$ mb

The height of the LCL is given by:

$$Z_{LCL} = (R\bar{T}/g) \ln (P_o/P_{LCL}) \qquad 3\text{-}11$$

$\bar{T}$ is computed from Equation (3-5) with $\bar{T}$ (P_o, P_{LCL}) replacing $\bar{Q}$ (P_m, P') in Equation 3-5.

3.7 Convective Condensation Level. The convective condensation level, CCL, is defined as the level at which air that is heated near the earth's surface will rise adiabatically until it becomes saturated (condensation begins). To determine the CCL:

(a) Calculate the average mixing ratio $(\bar{r})$ in the layer from the surface to 100 mb above the surface from Equations 3-2 and 3-5 replacing $\bar{Q}$ (P_m, P') in Equation 3-5 with $\bar{r}$ $(P_o, P_o\text{-}100)$. In the following, the average mixing ratio in the lowest 100 mb will be denoted by $\bar{r}$.

(b) Calculate the saturation mixing ratio $r_s(i)$ from Equation 3-3 at each pressure level reported, until $r_s(i)$ at a given pressure level is less than or equal to $\bar{r}$.

(c) If

(1) $r_s(i) = \bar{r}$, then $P_{CCL} = P(i)$

(2) $r_s(i) < r$, compute P_{CCL} by interpolating between that level and the next lower level:

$$P_{CCL} = [(r_s(i-1) - \bar{r})(P(i) - P(i-1))/(r_s(i-1) - r_s(i))] + P(i-1) \qquad 3\text{-}12$$

3.8 Level of Free Convection. The level of free convection, (LFC), is defined as the height at which a parcel of air lifted dry adiabatically until saturated and moist adiabatically thereafter would become warmer (less dense) than its environment. The procedure for calculating the pressure P_{LFC} of the LFC includes an approximate numerical procedure which is given at the end of this section. The actual calculation of the pressure of the LFC proceeds as follows:

(a) Calculate the temperature and pressure of the LCL from the algorithms in Section 3.5.

(b) Calculate the pseudo-equivalent potential temperature Θ_{se} at the LCL from Equations C-2(f) and C-2(g), replacing T with T_{LCL} and P with P_{LCL}. NOTE: Above the LCL, the parcel is saturated; therefore, its temperature and dewpoint temperature are equal. Proceed to step (c) with P(i) taken as the lowest height level where $P(i) < P_{LCL}$.

(c) Perform the approximate numerical procedure with $\epsilon = 0.05$ to solve for $T_p(i)$.

(d) Compute the difference Δ T(i) between the reported temperature $T_s(i)$ and the computed temperature $T_p(i)$:

$$\Delta T(i) = T_s(i) - T_p(i)$$

If $\Delta T(i) = 0$, then take $T_p(i)$ as the temperature T_{LFC} of the LFC and proceed to step (h).

(e) If at step (d), $\Delta T(i) < 0$, proceed to step (h) after computing T_{LFC} with the following equation:

$$T_{LFC} = [\Delta T(i)\, T_p(i-1) - \Delta T(i-1) T_p(i)]/[\Delta T(i) - \Delta T(i-1)] \qquad 3\text{-}13$$

(f) If at step (d), $\Delta T(i) > 0$, increment the pressure level counter i (go to the next higher height level). If $i \leq n$, where n is the number of reported levels in the sounding, return to step (c).

(g) If at step (f), $i > n$, terminate the calculations, and indicate that the LFC does not exist.

(h) Perform the approximate numerical procedure with $\varepsilon = 0.1$ to solve for P_{LFC}, replacing $T_p(i)$, $P(i)$, T, T', and ΔT with T_{LFC}, P_{LFC}, P', and ΔP, respectively.

The approximate numerical procedure uses a quantity E:

$$E = T_p(i)\,[(1000)/(P(i) - e_s)]^k \exp[Lr_s/C_pT_p(i)] - \theta_{se} \qquad 3\text{-}14$$

The objective is to have E be as close to zero as possible to obtain the closest approximation for the desired parameter. The following procedure is carried out:

(a) Estimate a value of T and an arbitrary parameter value increment ΔT.

(b) Calculate E from Equation 3-14 and test for $|E| < \varepsilon$, where "$|\ |$" means "take the absolute value of." If E passes this test, then T is taken as the desired parameter value. If not,

(c) Form $T' = T + \Delta T$ and an associated E'. Again, test for $|E'| < \varepsilon$. If E' passes the test, then T' is taken as the desired parameter value. If not,

(d) Compare the signs of E and E'. If they are different (i.e., the desired zero crossing of E is between T and T'), then divide ΔT by 2 and return to step (c).

(e) If at step (d), the signs of E and E' were the same, compare the absolute values of E and E'.

(f) If $|E'| < |E|$, set T equal to the previous value of T', and E equal to the previous value of E', and return to step (c).

(g) If $|E'| > |E|$ change the sign of ΔT and return to step (c).

3.3.1 Dewpoint Temperature from Wet Bulb Temperature. In some instances the wet bulb temperature (T_w), such as measured with a sling psychrometer, may be available instead of the dewpoint temperature

(T_d). The following procedure may be used to compute T_d:

(a) Compute $\theta_o = T_o(1000/p_o)^{.2854}$ where T_o, P_o are surface temperature and pressure.

(b) As in Section 3.8, compute θ_{se} from Equations C-2(f) and C-2(g) using T_w and T_o.

(c) Follow the procedure described in Section 3.8 to compute T_i and P_i at levels above the surface. At each level i, compute θ_i using T_i and P_i and Equation 3-1. If $|\theta_i - \theta_o|$ 0.05 [illegible] step (d) below. If $\theta_i < \theta_o$, proceed [illegible] the next higher level. If $\theta_i > \theta_o$ reduce the size by which P is changed by half and recompute T_i, P_i

(d) T_d (^oC) is given by

$$T_d = 318.27 - 0.002007\ T_o - 1.212 + SQ$$

where

$$SQ = (1.212 - 0.002007\ T_o)^2 -$$

$$0.006284\ [273.16 - T_i - T_o\ (0.121 - 0.000436\ T_o)]^{1.2}$$

and T_o is ^oC and T_i is ^oK.

3.9 <u>Stability Indices</u>. Stability indices are used to provide a quantitative basis for the prediction of showers, thunderstorms, and severe local storms. Calculations for several of the stability indices are described below.

3.9.1 Showalter Stability Index. Showalter developed a stability index (SI) in 1953 that is widely used. It is the difference between the free air temperature T_{50} at the 500 mb level and the temperature $\hat{T}_{50}$ (parcel) of a parcel of air that is lifted dry adiabatically from 850 mb to its proper LCL, then moist adiabatically to 500 mb. To determine the SSI:

(a) Determine the temperature T_{LCL} and the pressure P_{LCL} of the LCL above 850 mb from Equations 3-9 and 3-10, replacing T_o and T_d with T_{85} and T_{d85} (the temperature and dewpoint temperature at pressure P_{85} at 850 mb), and setting $P_o = P_{85}$.

(b) Determine the pseudo-equivalent potential temperature θ_{se} at the LCL (above 850 mb) from Equations C-2(f) and C-2(g), replacing T with T_{LCL} and P with P_{LCL}. NOTE: Above the LCL, the parcel is saturated; therefore, its temperature and dewpoint temperature are equal.

(c) Perform the approximate numerical procedure given in Section 3.8 with = 0.05 to solve for $\hat{T}_{50}$ (parcel), replacing $T_p(i)$ and P(i) with $\hat{T}_{50}$ (parcel) and P_{50} = 500 mb, respectively.

(d) Calculate the SSI from the following relationship:

$$SSI = T_{50} - \hat{T}_{50}\ (\text{parcel}) \qquad 3\text{-}15$$

3.9.2 Lifted Index. There are times when the 850 mb parameters are not representative of the temperature and moisture conditions in the lower atmosphere, thereby causing the SSI to be unrepresentative of the stability. To overcome this problem, the Lifted Index (LI) has been developed. To determine the LI:

(a) Determine the temperature T_{LCL} and the pressure P_{LCL} at a modified LCL from Equations 3-9 and 3-10, replacing P_o with P_o - 50 mb, and replacing T_o and T_d with $\overline{T}$ and $\overline{T}_d$, the average temperature and dewpoint temperature from the surface to 100 mb above the surface. The quantities $\overline{T}$ and $\overline{T}_d$ are computed from Equation 3-5, replacing

$$\overline{Q}(P_m, P') \text{ with } \overline{T}(P_o, P - 100) \text{ and } \overline{T}_d(P_o, P_o - 100).$$

(b) Determine the pseudo-equivalent temperature θ_{se} at the modified LCL from Equations (C-2(f)) and (C-2(g)), replacing T with T_{LCL} and P with P_{LCL}. NOTE: Above the LCL, the parcel is saturated; therefore, its temperature and dewpoint temperature are equal.

(c) Perform the approximate numerical procedure given in Section 3.8 with $\varepsilon = 0.05$ to solve for $\hat{T}_{50}$ (parcel), replacing $T_p(i)$ and $P(i)$ with $\hat{T}_{50}$ (parcel) and P_{50} = 500 mb, respectively.

(d) Calculate the LI from the following relationship:

$$LI = \hat{T}_{50} - T_{50} \text{ (parcel)} \qquad \text{3-16}$$

3.9.3 K-Index. The equation for calculating the K-index is:

$$KI = T_{85} + T_{d85} - T_{70} + T_{d70} - T_{50} \qquad \text{3-17}$$

where

T_{85} = 850 mb temperature (°C).

T_{d85} = 850 mb dewpoint temperature (°C).

T_{70} = 700 mb temperature (°C).

T_{d70} = 700 mb dewpoint temperature (°C).

T_{50} = 500 mb temperature (°C).

3.9.4 Vertical Totals Index. The equation for calculating the Vertical Totals Index (VTI) is:

$$VTI = T_{85} - T_{50} \qquad \text{3-18}$$

where

T_{85} = 850 mb temperature (°C).

T_{50} = 500 mb temperature (°C).

3.9.5 Cross Totals Index. The equation for calculating the Cross Totals Index (CTI) is:

$$CTI = T_{d85} - T_{50} \qquad 3\text{-}19$$

where

T_{d85} = 850 mb dewpoint temperature (°C).

T_{50} = 500 mb temperature (°C).

3.9.6 Total Totals Index. The equation for calculating the Total Totals Index (TTI) is:

$$TTI = VTI + CTI \qquad 3\text{-}20$$

where

VTI = vertical totals index.

CTI - cross totals index.

3.9.7 Richardson Number. The Richardson number (Ri) is a nondimensional ratio of static stability and wind shear. It is usually computed across vertical layers less than about 5000 feet in depth. Values less than 0.25 indicate turbulence and values 0.25 to 1.0 indicate a chance of turbulence. The equation for Ri is:

$$Ri = g(\theta_2 - \theta_1)\ H/(VWS^2)\bar{\theta}$$

where $g = 9.806\ m/s^2$, θ_2 and θ_1 are the potential temperatures at the top and bottom of the layer respectively, H is the thickness from Equation 3-7 , $\bar{\theta} = (\theta_1 + \theta_2)/2$, and VWS is the vertical wind shear from Equations 2-16 or 2-17. A check value is: if $\theta_1 = 347°K$, $\theta_2 = 353°K$, H = 1500m, and VWS = 10 m/s, then Ri = 2.521.

3.10 Density Altitude. The altitude above sea level characterized by a known air density is called the density altitude. It is computed from Equation 3-21 and this equation is valid up to a height of 35,332 feet:

$$\rho_h = 145{,}366\ [1 - (17.326\ P_m/T_v)^{.235}] \qquad 3\text{-}21$$

where

ρ_h = density altitude in feet

$$T_v = -0.288 + (9/5)\ T\ (1 + 1.60779r)/(1 + r) \qquad 3\text{-}22$$

where

T_v is (° Rankine), T is (°K), r is (gm/gm), and

P_m = Station barometric pressure, in inches of mercury:

$$P_m = [A^{1/n} - H_e P_s{}^{1/n} a/T_s]^n \qquad \text{'3-23}$$

where A = altimeter setting (inches of mercury),

n = 5.2561,

H_e = field elevation (m),

and a, P_s and T_s are listed in Section 3.2.C.1. A check value is: if H_e = 105m, A = 30.02 im., T = 283.2°K, r = .006 g/g, then T_v = 511.32°R and P_m = 29.648 and ρ_h - 141.43 feet.

3.11 Pressure Altitude. The pressure altitude (PA) is the height above sea level in the standard atmosphere at which the altimeter setting (A) exists. It is computed from Equation 3-24:

$$PA = (T_s/a)\ [1 - (A/P_s)^{1/n}] + H_e \qquad 3\text{-}24$$

where

A = Altimeter setting (reported)

PA = Pressure altitude for the Altimeter setting

n = 5.2561

A check value is: if A = 30.02 and H_e = 105m, then PA = 77.13 m.

3.12 Sea-Level Pressure. The procedures used within AWS to reduce station pressure to sea-level pressure (SLP) are described in FMH 8(b). As the required procedures involve using a 12-hour "mean" temperature and the pressure reduction (hand-held) computer (WBAN 54-7-8 or CP-402/UM), together with a table of pressure reduction ratios, there seems to be little advantage gained at this time by using a minicomputer for this process.

4. SPATIAL ANALYSIS ALGORITHMS

4.1 General. Currently analyses of meteorological parameters at base weather stations are either done manually or they are received via the Air Force Digital Graphics System (AFDIGS) from AFGWC. Local units may desire to have the capability for automated objective analyses residing at the BWS for both those currently produced manually and those analyses received from AFGWC. Before they can perform the analyses, they must first transform observations at irregularly spaced points into data at points on a uniformly spaced grid; i.e., generate horizontal grid data. The automated analyses that may be generated at the BWS include the surface and upper air Local Area Work Chart (LAWC), and cross section analyses for various parameters. The LAWC analyses are in the horizontal (x, y) plane and the cross sections are in the vertical (Log P) plane.

4.2 Horizontal Objective Analysis. The "nearest neighbor" technique described in Section 4.2.1 (Western Region Technical Attachment No. 82-14) can be used to develop grid values from station data. Grid points are assigned values of the closest observation to the grid point.

4.2.1 The Nearest Neighbor Analysis Technique. The generation of grid point values using the nearest neighbor technique is basically a three step procedure:

(a) Step 1. Assign a value at a grid point by setting it equal to the value of the observation closest to the grid point. Each observation can be assigned to only one grid point. By using observations only once, the distance from the observation to the assigned grid point is less than one-half of the grid spacing. If there is more than one observation within one-half grid interval of any grid point, then the observation farthest from the grid point may be assigned to another grid point. Even in this case, the observation will always be moved less than one grid interval (see Figure 4-1):

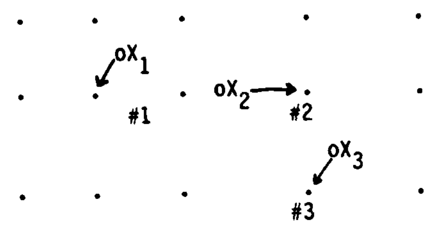

Figure 4-1. Nearest Neighbor Analysis. "X's" are observations on the indicated grid of points. X_3 is the closest to Point #2 but #2 is already assigned to a closer observation X_2 so X_3 is assigned to the next nearest point, #3.

(b) Step 2. Fill in the values at unassigned points using linear interpolation of the assigned values at surrounding points. The linear interpolation becomes a simple averaging at regularly spaced grid points. Figures 4-2(a) and (b) are examples of two point and four point averaging to fill in a midpoint.

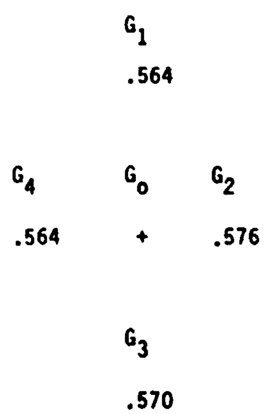

Figure 4-2(a).
Four Point Fill-In
G_o = 568.5

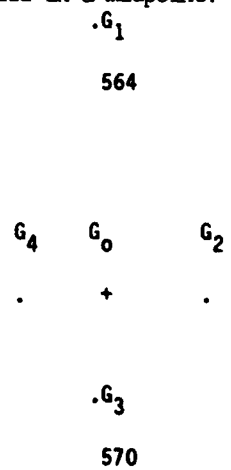

Figure 4-2(b).
Two Point Fill-In
G_o = 567

When a point has been "filled in," it can then be used as a value to assist in filling in of other adjacent unassigned points. Thus, more than one pass through the data may be required to fill in all the points. For points where interpolation is not possible, grid points can be assigned a value which is the average of grid point values over the set of grid points that have already been assigned values and are located exactly one grid interval from the grid point of interest.

(c) Step 3. Smooth the grid point values using a smoothing operator as given by Equation 4-1. Suitable smoothers can be used at grid boundary and corner points (e.g., 3 or 2 point smoothers).

$$S = (4 \times G_c + G_1 + G_2 + G_3 + G_4)/8 \qquad 4\text{-}1$$

$$\begin{array}{ccc} & +G_1 & \\ G_4+ & +G_c & +G_2 \\ & +G_3 & \end{array}$$

Figure 4-3. Grid point values used in smoothing G_c point.

The three step procedure is the same for both surface and upper air analyses except for a modification to Step 1 for upper air analyses. The value from the upper air observation is assigned to the nearest grid point as in the surface analysis. However data are then expanded to selected surrounding grid points. The expanded number of grid points with data values assigned as a result of wind information shall be based on the rules contained in Western Region Technical Attachment No. 82-14 and are as follows:

1. No wind or missing wind - no data expansion.

2. Data are expanded plus and minus two grid points in the X- and Y-directions if:

   (a) Wind direction (with respect to the grid) is 300-330°, 030-060°, 120-150°, 210-240°, regardless of speed (Figure 4-4).

   (b) Speed is less than 10 m/sec above 700 mb and less than 5 m/sec at 700 mb and less than 5 m/sec at 700 and below regardless of wind direction (Figure 4-4).

3. Data are expanded three points in the X-direction and one in the Y-direction if the wind direction is 240-300° or 060-120° and the wind speed is greater than 10 m/sec above 700 mb and greater than 5 m/sec at or below 700 mb (Figure 4-5).

4. Data are expanded three points in the Y-direction and one in the X-direction if the wind direction is 330-030° or 150-210° and the wind speed is as specified in 3, above (Figure 4-6).

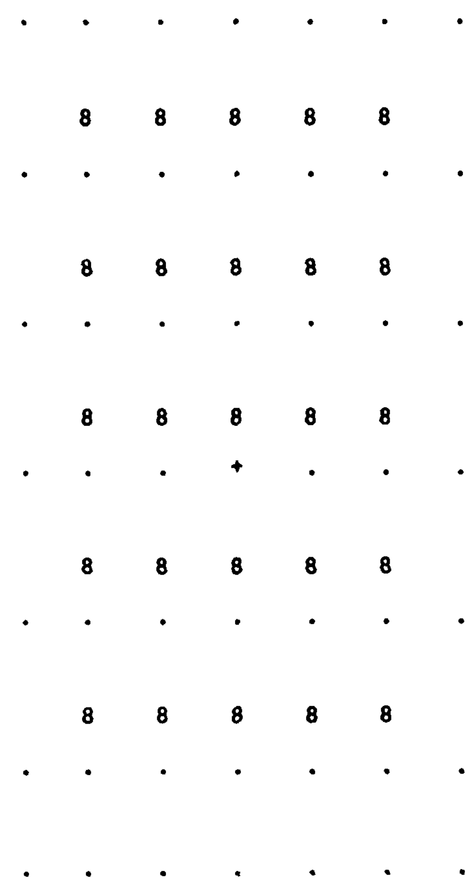

Figure 4-4. Data Expansion from + When Winds are Light or NW, SW, NE, SE.

```
.   .   .   .   .   .   .


.   .   .   .   .   .   .

8   8   8   8   8   8   8
.   .   .   .   .   .   .

8   8   8   8   8   8   8
.   .   .   +   .   .   .

8   8   8   8   8   8   8
.   .   .   .   .   .   .

.   .   .   .   .   .   .

.   .   .   .   .   .   .
```

Figure 4-5. Data Expansion from + When Winds are West or East.

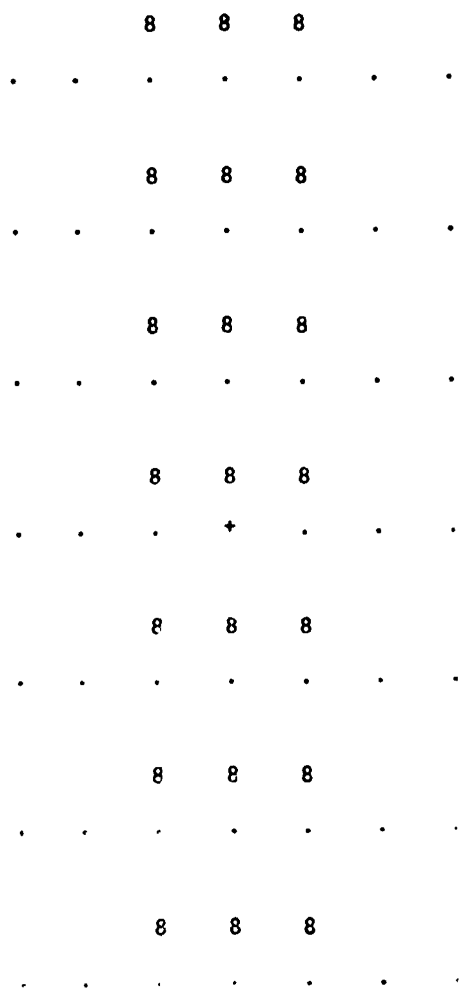

Figure 4-6. Data Expansion from + When Winds are North or South.

The value assigned to all the expanded points is the same as the central point, "+," for all fields except height of the constant pressure surface. For heights, a geostrophic gradient can be calculated from the observed wind and this height value is used to modify the values that are assigned to the surrounding points. The basic equation for computing the geostrophic gradient is

$$\partial Z/\partial n = c \times (f/g) \qquad 4\text{-}2$$

where $\partial Z/\partial n$ represents the difference in height over distance n (perpendicular to the wind direction) i.e., height gradient, c is the wind speed, f is the coriolis parameter, $f = 2\omega \sin \phi$ where $\omega = 7.29 \times 10^{-5}$ and ϕ is latitude, and g is the acceleration of gravity $g = 9.806\ m/s^2$.

To assign height values to expanded points on the grid, it is necessary to obtain u and v components of the wind (with respect to the grid) from wind direction and wind speed using the appropriate equations for the grid in use from Section 2.1.1. The value of height Z at point (I+i, J+j) where i and j are positive or negative integer grid spacing increments is then computed from the value at height Z at point (I,J) using the following equation.

$$Z(I+i,\ J+j) = Z(I,J) + \Delta Z_x + \Delta Z_y \qquad 4\text{-}3$$

where

$$\Delta Z_x = b_z\ i\ d_e\ v\ f/g \qquad 4\text{-}4a$$

$$\Delta Z_y = b_y\ j\ d_e\ u\ f/g \qquad 4\text{-}4b$$

where b_x and b_y are given in Table 4-1 for each type of grid and d_e is the grid spacing at point (I,J). The grid spacing d_e is variable and is a function of the particular grid and the location within the grid. See the discussion below Equation 2-11 in Section 2.2.2 for the method to calculate d_e.

If an expanded grid point falls on one or more other expanded grid points, the average of the two or more expanded grid point values should be used. If an expanded grid point falls on an original grid point (a grid point assigned via Step 1), the original grid point value is used.

The nearest neighbor procedure can be applied using a grid interval equivalent to that of the SGDB half-mesh polar stereographic grid and the SGDB half-mesh tropical grid.

Two smoothing passes may be made using the smoothing operator, Equation 4-1.

Table 4-1

Values of b_x and b_y

Grid Type	b_x	b_y
Northern Hemisphere Polar Stereographic Grid	+1	+1
Southern Hemisphere Polar Stereographic Grid	+1	+1
Mercator SGDB Tropical	-1	-1

4.2.2 Geostrophic Winds and Vorticity. In some cases it may be necessary to derive the wind field from the height field on a constant pressure surface. A common approximation for this purpose is the geostrophic approximation, in which it is assumed there is exact balance between the pressure gradient force and the coriolis force. The geostrophic winds are computed by inverting Equations 4-4a and 4-4b; that is

$$v_g = \Delta Z_x\ g/(f\ b_x\ i\ d_e) \qquad 4\text{-}4a'$$

$$u_g = \Delta Z_y\ g/(f\ b_y\ j\ d_e) \qquad 4\text{-}4b'$$

where the subscript g assigned to u_g and v_g indicates these are geostrophic wind components.

The geostrophic vorticity (ζ_g) is a measure of the area-integrated circulation of the atmosphere. In the Northern Hemisphere ζ_g is positive in low-pressure systems and troughs (cyclonic circulation) and is negative in high-pressure systems and ridges (anticyclonic circulation). It is computed using the equation:

$$\zeta_g = \Delta v_g/(b_x\ i\ d_e) + \Delta\ u_g/(b_y\ j\ d_e) \qquad 4\text{-}5$$

where $\Delta v_g i$ ($\Delta\ u_g$) is the difference of v_g (u_g) between the points I + 1 (J + 1) and I -1 (J -1) and ζ_g applies at the point I,J.

4.2.3 Vertical Velocity. The atmospheric vertical velocity (w) is one of the most important dynamic variables for diagnosing and predicting atmospheric processes. Unfortunately, the vertical velocity cannot be measured directly on a large scale with presently available instruments. Rather, it must be inferred or computed from other variables which are measured, such as horizontal winds or temperatures. There are basically three methods for computing the vertical velocity:

a. The adiabatic method. This method is based on the first law of thermodynamics under the assumption of isentropic flow. It requires computing time derivatives along with horizontal advection. As radiosonde data are normally available only each 12 hours, there are serious questions regarding the applications of this method. Thus, use of the adiabatic method is discouraged.

b. The omega-equation. This method is based on the full set of dynamical equations, often under the assumption of isentropic flow. It has the advantage of requiring observations only at one time; i.e., no time derivatives need to be computed. However, its solution requires inverting an operator similar to the three-dimensional Laplacian. While this can be done, (e.g., as in NOAA-TM-NWS WR-138, February 1979), it is a prodigious task requiring the proper application of boundary conditions. Comparison studies in the scientific literature find no certain evidence that the omega-equation produces more reliable estimates of vertical velocity than other methods (e.g., Smith and Lin, 1978, Monthly Weather Review, Volume 106, pp. 1687-1694). In view of these faots, use of the omega equation by local units is discouraged.

c. The Kinematic method. This method is based on the equation of continuity; i.e., mass is neither created nor destroyed, and is the most straightforward method. The success or failure of this method to give reliable results depends primarily on how well the observations of horizontal winds represent actual atmospheric conditions. Very-small-scale features of the horizontal winds must be reduced by applying a smoothing process (as in Equation 4-1) before computing vertical velocities. The Kinematic method can be applied to estimate vertical velocities at all heights once the horizontal wind fields have been gridded and smoothed as described in Section 4.2.1. The first step is to estimate the vertical velocity at the top of the planetary boundary layer (w_o, in m/s) according to:

$$w_o = (K/2f)^{\frac{1}{2}}\ \zeta_o + \{u^*(\Delta_x\ H_e) + v^*(\Delta_y\ H_e)\}/d_e \qquad 4\text{-}6$$

where K = $10m^2/s$, f is the coriolis parameter, ζ_o is the vorticity at the first level above the surface. u* and v* are composite winds which have the same speed as the surface wind but which have the same direction as the winds 2000 feet above the surface. $\Delta_x\ H_e$ and $\Delta_y\ H_e$ are the slopes of the terrain along the x and y axes of the grid system at the station. These slopes must be taken over spatial scales the same size as the spacing between points of the grid system (i.e., over distance d_e). The vertical velocity midway between layer k and layer k+1 is computed from

$$W_{k+1} = W_k - \Delta z_k \{ (\Delta_x u/b_x \, i \, d_e) + (\Delta_y v/b_y \, j \, d_e)\} \tag{4-7}$$

where $\Delta_x u$ is the difference of u between grid points on either side of the station along the x axis, and $\Delta_y v$ is the difference taken along the y axis. Δz_k is one-half the difference in height between layers k+1 and k-1. The process is illustrated in Figure 4-7:

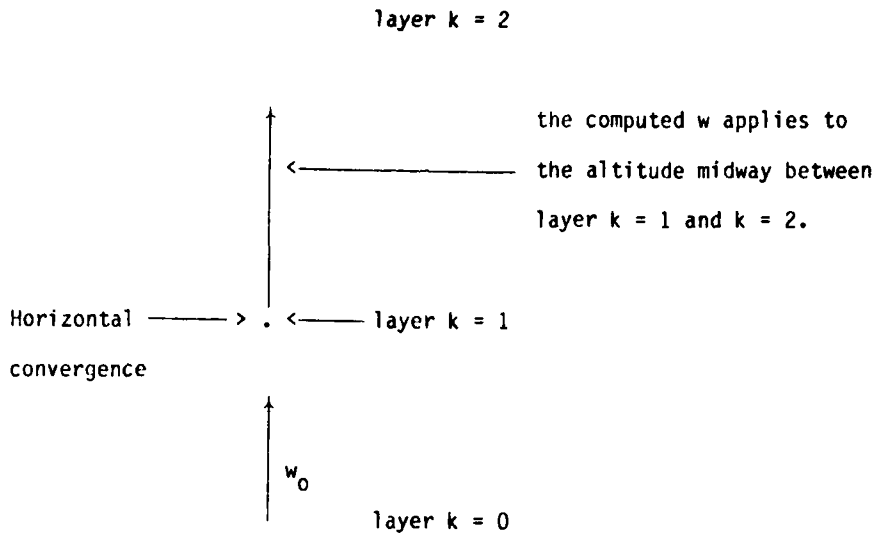

Figure 4-7. Illustration of calculation of vertical velocity.

The vertical velocity at successively higher layers is computed by resolving Equation 4-7. A note of caution must be given. The vertical velocities computed by the above algorithm are useful for qualitative analysis because the algorithm given here is cast in height rather than pressure coordinates. That is, in the lower and mid-troposphere the sign of the vertical velocity should be reliable, except for very small values, and the magnitude of the motion should permit description as "small" or "large." Finally, as with any other analysis scheme, meteorologists will appreciate the uses and shortcomings of this variable only when applied in the context of a complete meteorolgical analysis.

4.3 Cross Section Objective Analysis. Vertical cross section analysis is a tool used by meteorologists for depicting the spatial and temporal distribution of temperature, moisture, and wind velocity. Radiosonde data obtained at upper air observation stations are input. The cross section is very useful in that it shows subsynoptic scale gradients from the detailed vertical information at the upper air stations which are synoptically spaced.

4.3.1 Hermite Polynomial Interpolation for Objective Cross Section Analysis. One of the most useful types of cross sections is one that depicts the potential temperature, θ. It is called an isentropic section and it has the characteristic of showing meteorological regions of interest such as frontal zones, inversions and the tropopause in significant detail. Near these regions, there are strong gradients of θ. The one analysis method that is most widely used because it is both relatively simple and accurate is Hermitian interpolation described by Shapiro and Hastings (1973) in Journal of Applied Meteorology, Volume 12, pp. 753-762. To graph the spatial distribution of pressure along an isentropic surface, h(x), Hermite interpolation polynomials can be used to define the isentropic interpolation function, k(x). The Hermite interpolation polynomials are cubic polynomials pieced together at selected points where both function values and first derivatives are known. θ and θ_e are the two parameters to be analyzed using Hermitian polynomials. Values of θ and θ_e are determined from Equations 3-1 and 3-6, respectively. All equations used in the Hermite interpolations and the explanations for boundary conditions and differencing schemes are contained in the Shapiro and Hastings paper.

4.3.2 Objective Cross Section Analysis Using Linear Interpolation. Many fields in the vertical can, at times, be irregular in nature. Many meteorologists have found that these fields can be displayed well on a cross section using linear interpolation. Interpolation is in two dimensions, X and log P. Interpolation in the vertical may use standard and significant level data to determine parameter values at 50 mb increments over a suitable range; e.g., 1050 to 100 mb. The interpolation equation for quantity Q at pressure P = (1050 mb - n x 50 mb) where n is a integer such that $0 \leq n \leq 19$ is as follows:

$$Q = Q_i + (Q_{i+1} - Q_i)\ (\ln P - \ln P_i)/(\ln P_{i+1} - \ln P_i) \qquad 4\text{-}8$$

where P_i and P_{i+1} are reporting levels adjacent to level P such that $P_i \geq P > P_{i-1}$. Interpolation in the horizontal uses data from reporting stations or data points computed from gridded data to determine parameter values at regularly spaced grid intervals (half-mesh grid spacing) along the horizontal axis of the cross section. The interpolation equation for quantity Q at distance Z = n x d along the cross section axis from the origin, where $n \geq 0$ is an integer and d is the half-mesh grid spacing, is as follows:

$$Q = Q_i + (Q_{i+1} - Q_i)\ (X - S_i)/(X_{i+1} - X_i) \qquad 4\text{-}9$$

where X_i and X_{i+1} are distances to reporting stations or data points computed from gridded data adjacent to distance X such that $X_i \leq X < X_{i+1}$. The parameters for which bi-linear interpolations can be performed for contouring on cross sections are relative humidity, dewpoint depression, wind speed, vertical wind shear, temperature, D-values and mixing ratio. Once data are interpolated to (X,P) grid points, a graphic representation of the parameter field can be accomplished using contours that are unbroken piecewise continuous between adjacent grid boxes. One especially useful relation for the analysis of vertical profiles of wind on cross section representations is the thermal wind equation. The components of this equation show the relation of horizontal temperature gradient to vertical wind shear. In generalized X, Y coordinates, the components are given by:

$$\frac{\partial T}{\partial X} = \frac{fT}{g}\frac{\partial V}{\partial Z} \qquad 4\text{-}10a$$

$$\frac{\partial T}{\partial Y} = -\frac{fT}{g}\frac{\partial U}{\partial Z} \qquad 4\text{-}10b$$

where T is usually estimated by the temperature midway on the vertical layer across which the wind shear is computed.

5. MAP PROJECTIONS

Because reporting station locations are given in latitude and longitude, plotting of parameters at the geographic location of a station on a map requires a transformation to the coordinate system used for the map. A required first step in this process is to transform station locations from latitude and longitude ($\emptyset$, λ) to coordinates (I, J) on the appropriate AFGWC whole-mesh Satellite Global Data Base (SGDB) grid:

the Northern Hemispheric polar stereographic grid, the Southern Hemispheric stereographic grid, or the tropical (Mercator) grid. Precise location of stations on maps is required and is accomplished through the use of real-valued SGDB grid coordinates. The transformation equations for both the Northern and Southern Hemispheric SGDB whole-mesh polar stereographic grids and for the SGDB whole-mesh tropical grid are given in AFGWC/TN-79/003. Units requiring access to this TN should submit requests through their parent unit.

6. MILITARY GRID REFERENCE SYSTEM AND LATITUDE/LONGITUDE COORDINATE TRANSFORMATIONS

6.1 General. The Military Grid Reference System (MGRS) is a method for identifying geographical points on earth. This section describes the algorithms for converting latitude/longitude to MGRS points. The MGRS uses the Universal Transverse Mercator (UTM) projection. The range of latitude it covers is from 80° south to 80° north. The globe is divided into large geographical areas extending 6° in longitude and 8° in latitude, each of which is given a unique identification called the grid zone designation. The 6° by 8° areas are further subdivided into 100,000 meter squares, with each square given a unique two-letter identification. Numerical references within these 100,000 meter squares are given to 100m resolution in terms of "Easting" and "Northing" grid coordinates of the point.

6.2 Method for Calculating the Latitude/Longitude from MGRS. Necessary inputs to the computations are:

- Applicable UTM grid zone,
- Location (Easting and Northing) of the origin (lower left corner) of the pertinent 100,000 meter grid square,
- Position (Easting and Northing) of the point within the grid square for the 100m resolution.

These inputs, that define locations in the MGRS, are given in the following format:

ZZ LEN eee nnn

where ZZ is the zone number,

L is the 8° latitude band in which the point is located,

E defines the 100,000m Easting division,

N defines the 100,000m Northing division,

eee further defines the Easting location to the nearest 100m,

nnn further defines the Northing location to the nearest 100m.

A listing of variables, constants, and notations used in the conversion algorithms for the MGRS:

C1 = ZZ = Zone number
C2 = L = Eight degree latitude band (an alpha character)
C3 = E = Easting division (an alpha character)

C4 = N = Northing division (an alpha character)

C5 = eee = Easting location

C6 = nnn - Northing location

LAT = Latitude (in decimal degrees)

LON = Longitude (in decimal degrees)

R0 = 6,378,338

R1 = 0.00672267

R2 = 0.9996

R3 = 0.048481368

R4 = 3600

R5 = 500,000

R6 = 6367645.45

R7 = 16106.99

R8 = 16.976

R19 = 0.017453295

R25 = 10,000,000

R26 = 100,000

R27 = 100

** is notation for "raised to the power of."

x is notation for "multiplied by."

/ is notation for "divided by."

ABS is notation for "take the absolute value of."

SQRT is notation for "take the square root of."

INT is notation for "take the integer of" (discarding the fractional part of a number).

Latitude is determined from Equation 6-1 :

$$LAT = C2'/10000 \times INT((R10 - R12 \times R9 \times R9 - R13 \times (R9 ** 4))/R4) \times 10000 + .5) \qquad 6\text{-}1$$

R4 is a constant

Each of the terms in Equation 6-1 is determined from tables and other equations according to the following steps:

Step 1: To determine C2', find CX from Table 6-1. NOTE: The range of C4' for use in Step 3 below.

Table 6-1 - CX

C2	CX	C4' Range	C2	CX	C4' Range
C	-10	11-20	N	0	00-08
D	-9	20-28	P	1	08-17
E	-8	28-37	Q	2	17-26
F	-7	37-46	R	3	26-35
G	-6	46-55	S	4	35-44
H	-5	55-64	T	5	44-53
J	-4	64-73	U	6	53-62
K	-3	73-82	V	7	62-71
L	-2	82-91	W	8	71-79
M	-1	91-99	X	9	79-88

If CX is greater than or equal to zero, then C2' = 1. If CX is less than zero, then C2' = -1.

Step 2: Determine C3' from Table 6-2.

Table 6-2 - C3'

	A = 1	2	3	B =
	A	J	S	1
	B	K	T	2
	C	L	U	3
C3 =	D	M	V	4
	E	N	W	5
	F	P	X	6
	G	Q	Y	7
	H	R	Z	8

C3' = B (e.g., if C3 = N, then C3' = 5)

Step 3: Determine C4' from Table 6-3.

Table 6-3 = C4'

	C = 0, 2, 4, 6, 8	C = 1, 3, 5, 7, 9	D =
	A	L	0
	B	M	1
	C	N	2
	D	P	3
Use this table if C1 is odd	E	Q	4
	F	R	5
	G	S	6
C4 =	H	T	7
	J	U	8
	K	V	9

	F	R	0
Use this table if C2 is even	G	S	1
	H	T	2
	J	U	3
C4 =	K	V	4
	L	A	5
	M	B	6
	N	C	7
	P	D	8
	Q	E	9

C4' = C//D where "//" means "concatenate" and C is selected such that C//D is within the range of C4' noted in Step 1.

Step 4: Compute the R10 term in Equation 6-1 from Equation 6-2 :

R10 = ((R17/R2) + R7 x sin (2 x R17)(R6 x R2 x R19))
- R8 x sin (4 x R17/(R6 x R2 x R19)) x (1/(R6 x R19)) 6-2

R2, R7, R6, R19, and R8 are constants.

Step 5: Compute the R17 term in Equation 6-2 from Equation 6-3a :

If C2' is less than zero, then:

R17 = R25 - C6 x R27 - C4' x R26 6-3a

R25, R27, and R26 are constants, C6 is the Northing location and C4' was determined in Step 3.

If C2' is greater than or equal to zero, then:

R17 = C6 x R27 + C4' x R26 6-3b

Step 6: Compute the R12 term in Equation 6-1 from Equation 6-4 :

R12 = R16 X (10 ** 16)/ (2 X R3 X R2 X R2 X R11 X R11) 6-4

where: R2 and R3 are constants, and R16 and R11 are determined from:

R16 = TAN (R10) 6-5

R11 = R0/SQRT (1-R1 x sin (R17/(R6 x R2 x R19)) x sin
(R17/(R6 x R2 x R19))) 6-6

R0, R1, R6, R2, and R19 are constants and R17 was previously defined (Equations 6-3a or 6-3b above)

Step 7: Compute R9 in Equation 6-1 from Equation 6-7 :

R9 = (R18 - R5) x (0.000001) 6-7

where R18 is given by:

R18 = C5 x R27 + C3' x R26 6-8

C5 is the Easting location, R27 and R26 are constants and C3' was determined in Step 2.

Step 8: Compute R13 in Equation 6-1 from Equation 6-9 :

R13 = (R16 X R16 X 3 + 5) X R16 X (10 ** 28)/
(24 X (R2 ** 4) X R3 X (R11 ** 4)) 6-9

R2 and R3 are constants, R16 and R11 are defined by Equations 6-5 and 6-6, respectively.

If latitude in Equation 6-1 is negative, it is south latitude.

Longitude is determined from Equation 6-10:

$$LON = (1/10000) \times INT (((R9 \times (R14 - R15 \times R9 \times R9)/R4) + R22) \times 10000 + 0.5) \quad 6\text{-}10$$

If LON is negative, it is west longitude.

Step 1: R9 is given by Equation 6-7.

Step 2: R14 is determined from Equation 6-11:

$$R14 = (10 ** 10)/(R2 \times R3 \times R11 \times \cos (R10)) \quad 6\text{-}11$$

R2 and R3 are constants, R11 is given by Equation 6-6 and R10 is given by Equation 6-2.

Step 3: R15 in Equation 6-10 is determined from:

$$R15 = (R16 \times 2 + 1) \times (10 ** 22)/(6 \times R3 \times R2 \times R2 \times R2 \times R11 \times R11 \times R11 \times R23) \quad 6\text{-}12$$

R16 is given by Equation 6-5, R2 and R3 are constants and R11 is defined by Equation 6-6. R23 is determined from:

$$R23 = \cos (R10) \quad 6\text{-}13$$

Step 4: R22 in Equation 6-10 is determined from:

$$R22 = C1 \times 6 - 183 \quad 6\text{-}14$$

6.3 Method for Calculating the MGRS Location from Latitude/Longitude. To determine the MGRS point, calculate C1, C2, C3, C4, C5, and C6.

$$C1 = R21 \quad 6\text{-}15$$

where:

$$R21 = INT (LON/6 + 31) \quad 6\text{-}16$$

The following R's will be used in the determinations of C2 through C6. (These values of R are new sets of R's from those in Section 6.2, although the constants given in that section remain the same.)

$$R9 = -((R21 \times 6 - 183 - LON) \times 0.36 \quad 6\text{-}17$$

R10 = LAT, unless LAT is less than zero, then R10 = -LAT   6-18

R11 = RO/SQRT (1-R1 x sin (R10) x sin (R10)) 6-19

R12 = R2 x (R10 x R19 x R6 - R7 x sin (2 x R10) +
R8 x sin (4 x R10)) 6-20

R13 = (R2 x R3 x R3 x R11 x sin (2 x R10))/4 6-21

R14 = R2 x (R3 ** 4) x (1/24) x R11 x sin (R10) x cos (R10)
x cos (R10) x cos (R10) x (5 - tan (R10) x tan (R10)) 6-22

R15 = R2 x R3 x R11 x cos (R10) 6-23
R16 = R2 x (R3 ** 3) x (1/6) x R11 x cos (R10)
x (2 x cos (R10) x cos (R10) - 1) 6-24

R17 = (R14 x (R9 ** 4) + R13 x R9 x R9 + R12)
x LAT/ABS (LAT) 6-25

R29 = INT (((R9 ** 3) x R16 + R15 x R9 + R5)/R26) 6-26

R18 = (R9 ** 3) x R16 + R15 X R9 + R5 - R29 x R26 6-27

If R17 is less than zero, then:

R = R17 + R25 6-28

C2' = INT (-1 -R10/8) 6-29

If R17 is greater than or equal to zero, then:

R = R17 6-30

C2' = INT (R10/8) 6-31

To determine C3':

If (R21 + 2)/3 = INT ((R21 + 2)/3) then C3' = R29 + 10 6-32

If (R21 + 1)/3 = INT ((R21 + 1)/3) then C3' = R29 + 20 6-33

Otherwise, C3' = R29 + 30 6-34

C4' is given by:

C4' = INT (R/R26) 6-35

C5 and C6 are determined from Equations 6-36 and 6-37 , respectively:

C5 = INT (R18/R27) 6-36

$C6 = INT((R - C4' \times R26)/R27)$

At this point, C1, C2', C3', C4', C5, and C6 are known.

C2 is determined from Table 6-4:

Table 6-4 - C2 and C2'

C2	C2'	C2	C2'
C	-10	N	0
D	-9	P	1
E	-8	Q	2
F	-7	R	3
G	-6	S	4
H	-5	T	5
J	-4	U	6
K	-3	V	7
L	-2	W	8
M	-1	X	9

Determine C3 from Table 6-5 where C3' = A//B

Table 6-5 - C3

A =	1	2	3	B =
	A	J	S	1
	B	K	T	2
	C	L	U	3
C3 =	D	M	V	4
	E	N	W	5
	F	P	X	6
	G	Q	Y	7
	H	R	Z	8

C4 is determined from Table 6-6 where C4' = C//D

Table 6-6 - C4

	C = 0, 2, 4 6, 8	C = 1, 3, 5, 7, 9	D =
	A	L	0
	B	M	1
	C	N	2
Use this portion of table if C1 is odd	D	P	3
	E	Q	4
	F	R	5
	G	S	6
C4 =	H	T	7
	J	U	8
	K	V	9
	F	R	0
	G	S	1
	H	T	2
Use this portion of table if C1 is even	J	U	3
	K	V	4
	L	A	5
	M	B	6
C4 =	N	C	7
	P	D	8
	Q	E	9

Combine C1, C2, C3, C4, C5, and C6 to generate the MGRS locations.

7. SKEW-T, LOG-P BACKGROUND LINES AND TEMPERATURE AND DEWPOINT TEMPERATURE PROFILES

7.1 General. The Skew-T, Log-P diagram is discussed in AWS/TN-79/006. It is used for the graphical depiction of the vertical temperature and moisture (dewpoint temperature) structure of the atmosphere over a reporting station. It is also used for plot and display of winds at standard constant pressure surfaces and heights of these surfaces above mean sea level. Derived parameters as in Chapter 3 can also be displayed on the Skew-T, Log-P diagram. To produce the graphical depiction on a computer display terminal it is necessary to develop a version of the Skew-T, Log-P background diagram comprised of four sets of lines upon which the vertical profiles, plots, and parameter values will be displayed. The equations for the background lines and for generating profiles of temperature and dewpoint temperature are contained in the following subsections. The material in these subsections has either been taken directly (quotes) or abstracted from the document "Algorithms for Generation of a Skew-T, Log-P Diagram and Computing Selected Meteorological Quantities," by G. S. Stipanuk (1973) and published by the Army Electronics Command (ECOM-5515).

7.2 Background Lines and Vertical Profiles for Temperature and Dewpoint Temperature. Four sets of lines comprise the background Skew-T, Log-P diagram -- pressure, temperature, dry adiabat and saturation adiabat lines. The first two sets of curves, temperature and pressure, are used to locate points on the diagram. An arbitrary coordinate system is used to measure distances. The origin corresponds to the point at a temperature of 0 degrees Celsius (^{O}C) and a pressure of 1000 mb. The X-direction is parallel to the pressure lines (horizontal) with positive X to the right. The Y-direction is perpendicular to the X-direction with positive Y towards lower pressures (up). A point on the diagram specified by temperature and pressure is transformed to X, Y coordinates by Equations 7-1 and 7-2 :

$$X = .1408T - 10.53975 \log P + 31.61923 \qquad 7\text{-}1$$

$$Y = -11.5 \log P + 34.5 \qquad 7\text{-}2$$

The components in the X, Y coordinate system are given in inches. For most applications, the coordinate system for pressure ranges from 1050 to 100 mb and for temperature it ranges from -40^{O}C to +40^{O}C along the 1000 mb pressure line. Because the temperature lines are skewed, this 80^{O} Celsius range will slide to lower temperatures for decreasing pressure values. It will be necessary to scale the dimensions given by Equations 7-1 and 7-2 such that the entire background Skew-T, Log-P diagram (1050 to 100 mb and +40^{O} to -40^{O}C at 1000 mb) can be displayed within the dimensions of the graphic display device. The equations for dry adiabats and for saturation adiabats are given in Table 7-1. The temperature T at an arbitrary pressure on a saturation adiabat is determined by the bisection method. The bisection method is a numerical technique which decreases the difference between the upper and lower estimates by a factor of ½ per iteration. The temperature is assumed to lie in the range -80^{O}C to 40^{O}C. An initial guess of -20^{O}C is made and the correction, T*, computed. The correction term decreases by a factor of ½ after each correction. Terminating after 13 corrections usually gives satisfactory results. The algorithm for computing the temperature on a saturation adiabat is based on Equation 7-3 :

$$\theta = (\theta_e) \times \exp(-Lr_s/C_pT) \qquad 7\text{-}3$$

In addition to the algorithms which generate the curves for each family, it is necessary to have algorithms which determine which curve in a family passes through an arbitrary point (T, P). Algorithms to accomplish this are given in Table 7-2.

Table 7-1

SKEW-T, LOG-P Algorithms

Family	Parameter	Algorithm	
Dry Adiabat, (T_{DA})	θ, the potential temperature	$T_{DA}(\theta,P) = \theta(P/1000)^{0.2854}$	7-4
		T is in ^{o}Kelvin. $^{o}K = {}^{o}C + 273.16$	
Saturation Adiabat, (T_{SA})	θ_s, the Temperature at 1000 mb	$T_{SA}(\theta_s, P) = T_1 + \sum_{i=1}^{12} T_i^*$	7-5
		$T^* = 120/2^i \, \text{SIGN}\{a \exp[br_s(T_i,P)/T_i] - T_i(1000/P)^{0.2854}$	7-6
		$T_i = T_{i-1} + T^*_{i-1}$	7-7
		$a = \theta_s$ which is given by Equation 7-9	
		$b = -2651.8986$	
		$r_s(T, P) = 0.622e_s/(P-e_s)$	
		e_s is given by Equation C-2(b) or C-2(c)	

The SIGN function is -1 or +1 corresponding to the algebraic sign of the argument.

Table 7-2

Determining a Curve Through a Given Point

Curve Set	Parameter for Curve Passing Through (T, P)	
Dry Adiabat	$\theta = T\ (1000/P)^{0.2854}$	7-8
Saturation Adiabat	$\theta_s = T\ (1000/P)^{0.2854} /$	7-9
	$\exp\ [br_s(T,\ P)/T]$	
	$b = -2651.8986$	

8. INSOLATION AND ASTRONOMICAL ALGORITHMS

a. Hour Angles. The following formulas are useful if astronomical data, such as that given Navigational and Stellar Tables are applied to navigational purposes:

$$GHA = 15(GAST - RA)$$

$$LHA = 15(LAST - RA) = GHA - \lambda$$

$$GHA\ Aries = 15\ GAST$$

$$SHA = 360^{o} - 15\ RA$$

$$GHA = GHA\ Aries + SHA$$

where

GHA is the Greenwich hour angle in degrees;

LHA is the local hour angle in degrees;

GHA Aries is the Greenwich hour angle of the First Point of Aries (the origin of right ascension) in degrees;

SHA is the sidereal hour angle in degrees;

RA is the apparent right ascension (referred to the true equator and equinox of date) in hours;

λ is the local longitude in degrees (west is positive; east is negative);

GAST is the Greenwich apparent sidereal time in hours;

LAST is the local apparent sidereal time in hours;

W,A are constants from a tabulated list.

The GHA and declination of the sun and moon are computed from a power series of the form

$$F(X) = a_o + a_1 X + a_2 X^2 + a_3 X^3 + a_4 X^4 + a_5 X^5$$

where X is a time-like variable such that $-1 \leq X \leq 1$ and the coefficients a_o to a_5 are tabulated constants. The coefficients change from month-to-month within each year, and also change from year-to-year. Units requiring the current year's tabulated coefficients should submit requests through channels to their parent unit. To evaluate the power series for one of the navigational variables, it is first necessary to evaluate X:

(1) Compute t, where $t = N + GMT/24$ N is the day of the year at Greenwich and GMT is Greenwich Mean Time in hours. N is tabulated on many calendars, or can be computed using Equation 8-3.

(2) Compute X, where $X = (\frac{t-W}{A})-1$. W and A are constants tabulated along with the coefficients a_o to a_5 in the Almanac for Computers.

As an example, find the sun's GHA at $19^h02^m30^s$ GMT on 23 May 1983: From Equation 8-3 we find that N = 143 on 23 May. Thus, $t = 143 + \frac{19.0417}{24} = 143.7934$. Next, a page from the Almanac for Computers is shown in Figure 8-1. From this page, the appropriate values of W and A are 121 and 16, respectively. Thus,

$$X = \frac{143.7923 - 121}{16} - 1 = 0.4145875$$

Also, from Figure 8-1, the values of a_o to a_5 for GHA for 23 May 1983 are found and entered in the power series. That is,

$$GHA = 5940.9204 + X\ (5759.9128) + X^2 (-0.2935) + X^3\ (0.0195) + X^4\ (0.0046) + X^5\ (-0.0041) = 8386.45605$$

This value lies outside the range $0^o \leq GHA \leq 360^o$, so an integer multiple of 360^o is subtracted from GHA. In this case the integer is 23, so

$$GHA = 8386.45605 - 23\ (360) = 106^o.45605.$$

Days 121 thru 152 JD 2445455.5 to 2445487.5 Dates May 1 thru June 1

A = 16.0 B = -8.56250000 W = 121

Term	Aries GHA °	Sun GHA °	Sun Dec °	Sun S D °
0	5994.1267	5940.9204	19.1483	0.2642
1	5775.7719	5759.9128	3.6747	-0.0033
2	-0.0002	-0.2935	-0.6950	0.0
3	-0.0028	0.0195	-0.0522	0.0061
4	0.0001	0.0046	0.0064	0.0
5	0.0013	-0.0041	0.0010	-0.0037
Sums	11769.8970	11700.5597	22.0832	0.2633

Days 152 thru 183 JD 2445586.5 to 2445518.5 Dates June 1 thru July 2

A = 16.0 B = -10.50000000 W = 152

Term	Aries GHA °	Sun GHA °	Sun Dec °	Sun S D °
0	6024.6817	5939.8423	23.3565	0.2633
1	5775.7718	5759.1351	0.5474	0.0
2	0.0009	-0.0345	-0.8805	0.0
3	-0.0035	0.0765	-0.0072	0.0
4	-0.0010	-0.0054	0.0091	0.0
5	0.0023	-0.0074	-0.0005	0.0
Sums	11800.4522	11699.0066	23.0248	0.2633

Days 182 thru 213 JD 2445516.5 to 2445548.5 Dates July 1 thru Aug 1

A = 16.0 B = -12.37500000 W = 182

Term	Aries GHA °	Sun GHA °	Sun Dec °	Sun S D °
0	6054.2507	5938.4969	21.3321	0.2633
1	5775.7703	5759.6299	-2.6376	0.0
2	0.0027	0.2866	-0.7707	0.0
3	-0.0004	0.0377	0.0375	0.0
4	-0.0019	-0.0158	0.0033	0.0
5	0.0002	-0.0027	0.0036	0.0
Sums	11830.0216	11698.4326	17.9682	0.2633

Days 213 thru 244 JD 2445547.5 to 2445579.5 Dates Aug 1 thru Sep 1

A = 16.0 B = -14.31250000 W = 213

Term	Aries GHA °	Sun GHA °	Sun Dec °	Sun S D °
0	6084.8056	5938.9411	13.6746	0.2633
1	5775.7708	5760.8264	-5.0615	-0.0007
2	0.0026	0.2837	-0.4584	0.0004
3	-0.0014	-0.0319	0.0506	0.0031
4	-0.0017	-0.0100	-0.0033	0.0006
5	0.0008	0.0013	0.0062	-0.0015
Sums	11860.5767	11700.0106	8.2082	0.2652

Figure 8-1. Power Series Approximation of Nautical Almanac Data for Year 1983.

When using the above formulas, it may be necessary to add or subtract 360° to reduce the resulting hour angles to the range 0° -360°. Often the local hour angle values are reduced to the range -180° to $+180^{\circ}$, in which case they are called meridian angles. In all cases, positive hour angle values are measured westward from the meridian.

b. Altitude and Azimuth. Notation:

a = altitude of body above (if sin a > 0) or below (if sin a < 0) the horizon;

A = azimuth of body measured eastward from north over the range $0^{o} < A < 360^{o}$;

ϕ = latitude of observer (north is positive; south is negative);

δ = declination of body (north is positive; south is negative);

LHA = local hour angle of body;

z = zenith distance of body ($z = 90^{o} - a$).

The following formulas can be used to compute the altitude (a) and azimuth (A) of a celestial body:

$$\sin a = \cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos LHA \qquad 8\text{-}1$$

$$x = \tan A = \sin LHA/(\cos LHA \sin \phi - \tan \delta \cos \phi) \qquad 8\text{-}2$$

Since computers and calculators normally give the arctangent in the range -90^{o} to $+90^{0}$, the correct quadrant for A can be selected according to the following rules:

If $0^{o} < LHA < 180^{o}$,

$A = 180^{o} + \arctan x$, if x is positive,

$A = 360^{o} + \arctan x$, if x is negative.

If $180^{o} < LHA < 360^{o}$,

$A = \arctan x$, if x is positive,

$A = 130^{o} + \arctan x$, if x is negative.

In standard navigational notation altitude and azimuth are denoted Hc and Zn, respectively. Equations 8-1 and 8-2 are the basic formulas used in preparing sight reduction tables; they do not include the effect of refraction. EXAMPLE: Compute the altitude and azimuth of the Sun at $19^{h}02^{m}30^{s}$ GMT on 23 May 1983 at College Park, Maryland:

Latitude: $\phi = +39.0°$ $\quad$ $\sin \phi = +0.6293$ $\quad$ $\cos \phi = 0.77715$

Longitude: $\lambda = +77.0°$

Using the power series discussed earlier, the Sun's GHA and δ are found to be

$GHA = 106.457^\circ$ hence $LHA = 106.457^\circ - 77.0^\circ = 29.457^\circ$

$\sin LHA = +0.49177$ $\cos LHA = +0.87073$

$\delta = +20.579^\circ$ $\sin \delta = +0.35150$ $\cos \delta = +0.93619$ $\tan \delta = +0.37546$

$$\sin a = \cos z = (0.62932)(0.35150) + (0.77715)(0.93619)(0.87073) = +0.85471$$

$$a = 58.7^\circ$$

$$x = \tan A = 0.49177/((0.87073)(0.62932) - (0.37546)(0.77715)$$

$$= + 1.91963 \qquad \arctan x = 62.5^\circ$$

Since LHA is greater than 0° and less than 180°, and since x is positive, $A = 180^\circ + 62.5^\circ = 242.5^\circ$

c. Sunrise, Sunset, and Twilight. For locations between latitudes 65° North and 65° South, the following algorithm provides times of sunrise, sunset, and twilight to an accuracy of $\pm 2^m$, for any date in the latter half of the twentieth century. Because the phenomena depend on local meteorological conditions, attempts to attain higher accuracy are seldom justified. Although the algorithm can be used at higher latitudes, its accuracy deteriorates near dates on which the Sun remains above or below the horizon for more than twenty-four hours. Notation:

ϕ = latitude of observer (north is positive; south is negative);

λ = longitude of observer (west is positive; east is negative);

M = Sun's mean anomaly;

L = Sun's true geocentric longitude;

RA = Sun's right ascension;

δ = Sun's declination;

H = Sun's local hour angle;

z = Sun's zenith distance at rise, set, or twilight*;

t = approximate time of phenomenon in days since 31 Dec., 0^h UT;

T = local mean time of phenomenon;

UT = universal time of phenomenon.

*The proper value of z should be chosen from the following:

	z	cos z
Sunrise and Sunset	90.833°	-0.01454
Civil Twilight	96°	-0.10453
Nautical Twilight	102°	-0.20791
Astronomical Twilight	108°	-0.30902

Formulas:

(1) $M = 0.9856^{\circ}t - 3.289^{\circ}$

(2) $L = M + 1.916^{\circ} \sin M + 0.020^{\circ} \sin 2M + 282.634^{\circ}$

(3) $\tan RA = 0.91746 \tan L$

(4) $\sin \delta = 0.39782 \sin L$

(5) $x = \cos H = (\cos z - \sin \delta \sin \phi)/(\cos \delta \cos \phi)$

(6) $T = H + RA - 0.06571^{h}t - 6.622^{h}$

(7) $UT = T + \lambda$

Procedure:

(a) Step 1. With an initial value of t, compute M from Equation 1 and then L from Equation 2 . If a morning phenomenon (sunrise or the beginning of morning twilight) is being computed, construct an initial value of t from the formula

$$t = N + (6^{h} + \lambda)/24$$

where N is the day of the year* and λ is the observer's longitude expressed in hours. If an evening phenomenon is being computed, use

$$t = N + 18^{h} + \lambda)/24$$

(b) Step 2. Solve Equation (3) for RA, noting that RA is in the same quadrant as L Transform RA to hours for later use in Equation (6)

*NOTE: N is the number of days elapsed since 0 January of the current year; for example, N = 1 on 1 January, N = 32 on 1 February, and so forth. Leap year days are counted, so N = 366 on 31 December of leap years and N = 365 on 31 December other years. N is listed on most calendars. If desired, it can be computed from:

$$N = \text{Int}\left(\frac{275\ \text{Mon}}{9}\right) - \text{Int}\left(\frac{9 + \text{Mon}}{12}\right)\left(1 + \text{Int}\left(\frac{\text{Yr} - 4(\text{Int}(\text{Yr}/4) + 2)}{3}\right)\right) + \text{Day} - 30 \qquad 8\text{-}3$$

where Mon is month ($1 \leqslant \text{Mon} \leqslant 12$), Yr is the year (e.g., 1982), and Day is day of month ($1 \leqslant \text{Day} < 31$).

(c) Step 3. Solve Equation 4 for sin δ which appears in Equation 5; cos δ, which also is required in Equation 5 should be determined from sin δ. While sin δ may be positive or negative, cos δ is always positive.

(d) Step 4. Solve Equation 5 for H. Since computers and calculators normally give the arccosine in the range $0^o - 180^o$, the correct quadrant for H can be selected according to:

rising phenomena, $H = 360^o - \arccos x$;

setting phenomena, $H = \arccos x$.

In other words, for rising phenomena, H must be either in quadrant 3 or 5 (depending on the sign of cos H), whereas H must be either in quadrant 1 or 2 for setting phenomena. Convert H from degrees to hours for use in Equation (6).

(e) Step 5. Compute T from Equation 6 , recalling that H and RA must be expressed hours. If T is negative or greater than 24^h, it should be converted to the range $0^h - 24^h$ by adding or subtracting multiples of 24^h.

(f) Step 6. Compute UT from Equation 7 , where λ must be expressed in hours. UT is an approximation to the time of sunrise, sunset, or twilight, referred to the Greenwich meridian. If UT is greater than 24^h, the phenomenon occurs on the following day, Greenwich time. If UT is negative, the phenomenon occurs on the previous day, Greenwich time.

To ensure that precision is not lost during the computations, t should be carried to four decimal places. Angles should be expressed to three decimals of a degree and, upon conversion, to three decimals of an hour. Five significant digits should be carried for the trigonometric functions. Under certain conditions Equation 5 will yield a value of $|\cos H| > 1$, indicating the absence of the phenomenon on that day. At far northern latitudes, for example, there is continuous illumination during certain summer days and continuous darkness during winter days. EXAMPLE: Compute the time of sunrise on 25 June at Wayne, New Jersey:

Latitude: $40^o.9$ North   Longitude: $74^o.3$ West

$\phi = +49^o.9$ $\sin \phi = +0.654744$ $\cos \phi = +0.75585$

$\lambda = 74.3^o/15 = 4.95^h$

For sunrise: $z = 90^o50'$ $\cos z = -0.01454$

$t = 176^d + (6^h + 4.95^h)/24 = 176.456^h$

$M = 0.9856^o(176.456^d) - 3.239^o = 170.626^o$

$L = 170.626^o + 1.916^o(0.16288) + 0.020^o(-0.3214) + 282.634^o$

$= 453.566^o = 93.566^o$

d. Solar Coordinates. The true geocentric longitude of the Sun (L) can be computed to an accuracy of $\pm$ 1 minute of arc from the following formulas:

$$M = 358^{\circ}.476 + 35999^{\circ}.050T$$

$$L = 279^{\circ}.691 + 36000^{\circ}.769T \quad + (1^{\circ}.919 - 0^{\circ}.0048T) \sin M + 0^{\circ}.020 \sin 2M$$

where $T = (N - 2415010.0)/36525$ and N is from Equation 8-3.

If we consider the Sun's latitude to be identically zero, the right ascension (RA) and declination (δ) of the Sun can also be computed to $\pm$ 1 minutes of arc from

$$\tan RA = \cos \varepsilon \tan L$$

$$\sin \delta = \sin \varepsilon \sin L$$

where ε, the obliquity of the ecliptic, can be computed from $\varepsilon = 23.452^{\circ} - 0.013^{\circ}T$. The right ascension is always in the same quadrant as the true longitude. Because the obliquity varies slowly, a single value can be used for an extended period of time. During the last quarter of the twentieth century, $\varepsilon = 23.441^{\circ}$ is sufficiently accurate. Similarly the coefficient of sin M in the equation for L changes slowly; for the last half of the twentieth century a value of 1.916° can be safety used. Although there is no rigorous limit on the time span for which these formulas are valid, their accuracy gradually deteriorates for values of T greater than a couple of centuries.

e. Solar Transit. The equation of time (EqT) is the hour angle of the true Sun minus the hour angle of the mean sun. Thus, it is the difference: apparent solar (sundial) time minus meal solar (clock) time. For the current year EqT can be computed to an accuracy of $\pm$ 0.8 minutes from the formula:

$$(1) \quad EqT = -7.65^{m} \sin (0.9856^{\circ}t) + 0.53^{m} \cos (0.9856^{\circ}t)$$
$$-9.34^{m} \sin (1.9712^{\circ}t) - 2.92^{m} \cos (1.9712^{\circ}t)$$

where t is the number of days since 0 January, 0^{h} UT. If higher accuracy is required, the following formulas will give EqT to an accuracy of $\pm$ 2 seconds during the current year:

$$(2) \quad \theta = 9.094^{\circ} + 0.98561^{\circ} t + 1.916^{\circ} \sin(0.9856^{\circ} t - 3.562^{\circ})$$
$$+ 0.020^{\circ} \sin (1.9712^{\circ} t - 7.124^{\circ})$$

$$(3) \quad EqT = 36.377^{m} + 3.94244^{m} t - 4.0^{m} \arctan \ [(\tan \theta)/0.91747]$$

where t is the number of days, and fractions thereof, since 0 January, 0^{h} UT. In Equation 3 the arctangent should yield a result in degrees that is in the same quadrant as θ. Near the end of the year θ becomes greater than 360°. When this occurs, the arctangent in Equation 3 should also be greater than 360°. Equations 2 and 3 can be used to compute the time at which the Sun transits the local meridian. First use Equations 2 and 3 to compute EqT for $t = N + (12^{h} + \chi)/24$, where N is the day of the year and χ is the west longitude expressed in hours. Then the local mean time (LMT) of transit is given to an accuracy of $\pm$ 2 seconds by $LMT = 12^{h} - EqT$. The universal time of local transit is then obtained from $UT = LMT + \chi$. EXAMPLE: Compute the time of solar transit at longitude $51^{\circ}32.8^{1}$East on 31 October 1983:

$$\chi = 51.547^{\circ}/15 = 3.4365^{h} = - 3^{h}26.19^{m}$$

For solar transit: $t = 304^d + (12^h - 3^h.4365)/24 = 304.3568^d$

$$\theta = 9.094^\circ + 0.98561^\circ\ (304.3568^d) + 1.916^\circ\ (-0.8956) + 0.020^\circ\ (-0.7968) = 307.339^\circ$$

$$\begin{aligned} EqT &= 36.377^m + 3/94244^m\ (304.3568^d) - 4.0^m \arctan[-1.21084/0.91747] \\ &= 36.377^m + 1199.908^m - 4.0^m(304.989^\circ) = +16.33^m \end{aligned}$$

$$LMT = 12^h00^m - 16.33^m = 11^h43.67^m$$

$$UT = 11^h43.67^m - 3^h26.19^m = 8^h17^m29^s$$

f. Moonrise and Moonset. Time of moonrise and moonset can be computed for specified locations using the following algorithm. Between latitudes 60° North and 60° South, the phenomena can be computed to an accuracy of $\pm 5^m$. Although the algorithm can be used at higher latitudes, its accuracy deteriorates near dates on which the Moon remains above or below the horizon for more than twenty-four hours.

Notation: ϕ = latitude of observer (north is positive; south is negative);

χ = longitude of observer (west is positive; east is negative);

t_i = i-th approximation to universal time of phenomenon, expressed in days from 0 January, 0^h UT;

GHA_i = Moon's GHA at time t_i;

δ_i = Moon's declination at time t_i (north is positive; south is negative);

r_i = i-th correction to t_o, thus $t_i = t_o + r_i$;

H_i = i-th approximation to Moon's LHA at time of rise or set;

ΔH_i = i-th approximation to Moon's daily rate of change in GHA.

Formulas:

(1) $\Delta H_i = (GHA_i = GHA_o)/r_i$ for $i = 0$, let $\Delta H_o = 347.81^\circ$

(2) $x_{i+1} = \cos H_{i+1} = (.00233 - \sin\phi \sin\delta_i)/(\cos\phi \cos\delta_i)$

(3) $r_{i+1} = (H_{i+1} - H_o)/\Delta H_i$

(4) $t_{i+1} = t_o + t_{i+1}$

Procedure:

(a) Step 1. Let $t_o = N + (12^h + \chi)/24$, where N is the day of the year and χ is the observer's longitude expressed in hours. Set $i = 0$ and begin the following iterative process.

(b) Step 2. For time t_i compute the Moon's GHA and declination to navigational precision ($\pm0'.1$) using the power series method discussed earlier. Label these quantities GHA_i and δ_i, respectively, where i specifies the iteration number. For $i = 0$, compute $H_o = GHA_o - \chi$

(c) Step 3. If $i = 0$, let $\Delta H_o = 347.81^o$. Otherwise compute ΔH_i from Equation 1. If $\Delta H_i < 0$, add $360^o / r_i$ to ΔH_i.

(d) Step 4. Solve Equation 2 for H_{i+1}. Since computers and calculators normally give the arccosine in the range 0^o - 180^o, the correct quadrant for H_{i+1} can be selected according to:

moonrise computations, $H_{i+1} = 360^o - \arccos x_{i+1}$;

moonset computations, $H_{i+1} = \arccos x_{i+1}$.

In other words, near the time of moonrise H_{i+1} must be either in quadrant 3 or quadrant 1 or 2. For latitudes higher than 60^o (i.e., $|\phi| > 60^o$), the condition $\cos H_{i+1} \,|>|1$ can occur, thereby indicating the absence of the phenomenon on that day.

(e) Step 5. Compute r_{i+1} from Equation 3. If $|r_{i+1}| < 0.5^d$, proceed to Step 6. If $|r_{i+1}| > 0.5^d$, the phenomenon being computed occurs on the day before the day desired (if r_{i+1} is negative) or on the day following the day desired (if r_{i+1} is positive). Normally, the phenomenon on the desired day can be obtained by adding to r_{i+1} (if r_{i+1} is negative), or subtracting from r_{i+1} (if r_{i+1} is positive), $360^o/\Delta H_i$. If successful, this technique will produce a new value of r_{i+1} in the required range. However, two conditions may prevent the reduction to $|r_{i+1}| < 0.5^d$:

for low values of i, r_{i+1} may be a fairly crude approximation to the ultimate value, r_n;

each month there is one day (near last quarter) on which there is no moonrise, and another day (near first quarter) on which there is no moonset.

If $|r_{i+1}|\ 0.5^d$, it is probably worth attempting another iteration to see if $|r_{i+2}| < 0.5^d$.

(f) Step 6. Compute t_{i+1} from Equation 4. If $|t_{i+1} - t_i| < 0.01^d$, t_{i+1} is accurate to $\pm 5^m$. Otherwise it is necessary to iterate the solution by setting $i = i + 1$ and executing Steps 2 through 6 again.

EXAMPLE: Compute moonrise on 26 April 1983 at latitude 38°59' North and longitude 76°30' West:

$\phi = +\ 38.983^\circ$ $\quad \sin\phi = +0.62909$ $\quad \cos\phi = +0.77733$

$\chi = +\ 76.500^\circ/15 = +\ 5.100^h$

The day is found to be day 116; therefore,

$$t_o = 116^d + (12^h + 5.100^h)/24 = 116.71250^d$$

i = 0: Evaluating the power series for t_o, using constants from Figure 8-2.

$GHA_o = 262.248^\circ$ $\quad \delta_o = -7.054^\circ$

$H_o = 262.248^\circ - 76.500^\circ = 185.748^\circ$

$\Delta H_o = 347.81^\circ$

$x_1 = \cos H_o = [0.00233 - (0.62909)(-0.12280)]/[(0.77733)(0.99243)]$

$= +0.10316$ $\quad \arccos x_1 = 84.079^\circ$

Since moonrise is sought, H_1 is in quadrant 3 or 4:

$H_1 = 275.921^\circ$

$r_1 = (275.921^\circ - 185.748^\circ)/347.81^\circ = +0.25926^d$

$|r_1| < 0.5^d$ as required.

$t_1 = 116.71250^d + 0.25926^d = 116.97176^d = 26$ April 23^h19^m UT

i = 1: Evaluating the power series for t_1,

$GHA_1 = 352.576^\circ$ $\quad \delta_1 = 8.432^\circ$

$\Delta H_1 = (352.576^\circ - 262.248^\circ)0.25926^d = 348.407^\circ$

$x_2 = \cos H_2 = [0.00233 - (0.62909)(-0.14664)]/[(0.77733)(0.98919)$

$= +0.12300$ $\quad \arccos x_2 = 82.935^\circ$

Days 97 thru 102 | JD 2445431.5 to 2445437.5 | Dates Apr 7 thru Apr 12
A = 3.0 | B = -33.33333333 | W = 97

Term	Moon GHA °	Moon Dec °	Moon H P °	Moon S D °
0	1293.8731	-12.1168	0.9163	0.2497
1	1047.5575	13.1999	0.0252	0.0069
2	0.7655	2.5889	0.0122	0.0033
3	-0.6021	-0.7075	-0.0104	-0.0028
4	-0.1375	-0.0694	-0.0048	-0.0013
5	0.0302	-0.0081	0.0060	0.0016
Sums	2341.7867	2.8870	0.9445	0.2574

Days 103 thru 108 | JD 2445437.5 to 2445443.5 | Dates Apr 13 thru Apr 18
A = 3.0 | B = -35.33333333 | W = 103

Term	Moon GHA °	Moon Dec °	Moon H P °	Moon S D °
0	1226.5320	17.5393	0.9703	0.2644
1	1041.6475	12.0323	0.0227	0.0062
2	-3.5078	-4.0443	-0.0078	-0.0021
3	-0.0457	-1.4435	-0.0023	-0.0006
4	0.4431	0.0674	0.0023	0.0006
5	0.0786	0.0879	0.0	0.0
Sums	2265.1477	24.2391	0.9852	0.2685

Days 109 thru 114 | JD 2445443.5 to 2445449.5 | Dates Apr 19 thru Apr 24
A = 3.0 | B = -37.33333333 | W = 109

Term	Moon GHA °	Moon Dec °	Moon H P °	Moon S D °
0	1142.9034	17.9258	0.9882	0.2692
1	1040.0961	-12.1580	-0.0032	-0.0009
2	2.9739	-4.6920	-0.0073	-0.0020
3	0.0904	1.4035	0.0004	0.0001
4	-0.4803	0.1619	0.0002	0.0
5	0.0652	-0.0893	-0.0012	-0.0003
Sums	2185.6487	2.5519	0.9971	0.2661

Days 115 thru 120 | JD 2445449.5 to 2445455.5 | Dates Apr 25 thru Apr 30
A = 3.0 | B = -39.33333333 | W = 115

Term	Moon GHA °	Moon Dec °	Moon H P °	Moon S D °
0	1070.7377	-13.5239	0.9511	0.2592
1	1044.7045	-13.8267	-0.0301	-0.0082
2	-0.9260	3.3560	-0.0036	-0.0010
3	-0.3224	0.9767	-0.0010	-0.0003
4	0.2498	-0.1510	0.0015	0.0004
5	0.0291	-0.0206	0.0034	0.0009
Sums	2114.4727	-23.1895	0.9213	0.2510

Figure 8-2. Power Series Approximation of Nautical Almanac Data for Year 1983.

Since moonrise is sought, H_2 is in quadrant 3 or 4:

$$H_2 = 277.065^o$$

$$r_2 = (277/065^o - 185.748^o)/348.407^o = +0.26210^d$$

$$t_2 = 116.71250^d + 0.26210^d = 116.97460^d = 26 \text{ April } 23^h 26^m \text{ UT}$$

$$|t_2 - t_1| = 0.0028^d < 0.01^d$$

The extremely rapid convergence illustrated in this example occurs frequently but not invariably. Although the first approximation (t_1) will often give adequate precision for most purposes, it is recommended that the solution be iterated and that the convergence criterion ($|t_{i+1} - t_i| < 0.01^d$) be tested.

9. TOXIC CORRIDOR CALCULATIONS.

The theory and practice of toxic corridor calculations are discussed at length in AWS/TR-80/003, and a tutorial refresher on the topic is given in AWS/FM-81/007. A complete program for the TI-59 hand calculator is given in AWS/TR-80/003. One useful model is presented next, for illustration. It is a Non-Gaussian dispersion equation extracted from AWS/FM-81/007. Data from field diffusion studies have been used not only to develop and evaluate the Gaussian equation but also to develop and evaluate statistical equations which contain those meteorological parameters that "explain" or fit the field data most closely. Such empirical approaches result in statistical regression equations. Data from three field studies named Ocean Breeze, Dry Gulch, and Prairie Grass were used by the Air Force Cambridge Research Laboratories (now the Air Force Geophysics Laboratory) to develop an equation to determine downwind peak concentration of airborne contaminants from a continuous point source. This empirically derived equation was developed from data collected during extensive diffusion experiments with tracer released simulating ground-level continuous point sources. Using independent data, the normalized peak concentrations obtained from this equation have been found to be accurate within a factor of two, 65 percent of the time and within a factor of four, 94 percent of the time. The equation is

$$C_p/Q = 1.75 \cdot 10^{-4} X^{-1.95} (\Delta T + 10)^{4.92}$$

If one is concerned with the downwind distance, X, at which a predetermined concentration, C_p, will occur for a known source strength, Q, and temperature difference, delta-T (Δ T), the above equation can be easily solved for X. This has been done below. In the process, appropriate changes were made to the coefficient to convert from metric units to English units, and a factor was added to convert C_p/Q from units of seconds per cubic meter to units of PPM per lb/min. The converted equation, which was used to generate the Toxic Corridor Length Tables in AWS/TR-80/003, is

$$X = P \left[3.28 \left(\frac{29.75}{GMW} \right)^{0.153} \left(\frac{Cp}{Q} \right)^{-0.513} (\Delta T + 10)^{-2.53} \right]$$

where X = Downwind distance in feet. As used here, this distance defines a toxic corridor length. This is the downwind dimension of an area within which the forecast concentration of a toxic chemical equals or exceeds a specified exposure limit.

P = A probability factor used to determine the probability that a specified concentration is not exceeded outside the corridor. Calculations in AWS/TR-80/003 assume a 90-percent probability; therefore, P is equal to 1.63. Probability factors corresponding to other probabilities can be found in Table 35, AWS/TR-80/003.

GMW = Gram molecular weight of the toxic chemical.

C_p = Peak concentration in parts per million by volume (PPM) at a height of approximately 5 feet above the ground at a given downwind distance, X.

Q = Source strength in lb/min.

ΔT = The temperature in $^{\circ}F$ at 54 feet minus the temperature at 6 feet. NOTE: A negative ΔT means a decrease of temperature with height and a positive ΔT means an increase with height.

In this equation, the meteorological parameter that must be measured or estimated is temperature difference, delta-T, between 54 and 6 feet. Delta-T is a continuous variable that may be approximated as a discrete function in the absence of delta-T measuring equipment. Procedures are given in AWS/TR-80/003 to estimate delta-T if delta-T measuring equipment is unavailable. The delta-T input to this equation should not be confused with turbulence typing indexes such as the Pasquill-Gifford-Turner (PGT) scheme used in AWS/TR-214. The PGT index is used to estimate the stability class so that the standard deviations, σ's, of contaminant concentrations can be estimated. This estimate used with Gaussian dispersion models is different from directly in putting a value of delta-T into the Ocean Breeze/Dry Gulch equation.

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