A technique for the recognition of complex three dimensional objects is presented. The complex 3-D objects are represented in terms of their 3-D moment invariants, algebraic expressions that remain invariant independent of the 3-D objects' orientations and locations in the field of view. The technique of 3-D moment invariants has been used successfully for simple 3-D object recognition in the past. In this work we have extended this method for the representation of more complex objects. Two complex objects are represented digitally; their 3-D moment invariants have been calculated, and then the invariancy of these 3-D invariant moment expressions is verified by changing the orientation and the location of the objects in the field of view.

The results of this study have significant impact on 3-D robotic vision, 3-D target recognition, scene analysis and artificial intelligence.
Recognition of complex three dimensional objects using three dimensional moment invariants

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Abstract

A technique for the recognition of complex three dimensional objects is presented. The complex 3-D objects are represented in terms of their 3-D moment invariants, algebraic expressions that remain invariant independent of the 3-D objects' orientations and locations in the field of view. The technique of 3-D moment invariants has been used successfully for simple 3-D object recognition in the past. In this work we have extended this method for the representation of more complex objects. Two complex objects are represented digitally; their 3-D moment invariants have been calculated, and then the invariancy of these 3-D invariant moment expressions is verified by changing the orientation and the location of the objects in the field of view.

The results of this study have significant impact on 3-D robotic vision, 3-D target recognition, scene analysis and artificial intelligence.

Introduction

In order for machines or robots to recognize three-dimensional (3-D) objects, it is crucial for them to go beyond two dimensional signal processing and to exploit the three-dimensional nature of the objects. Three dimensional object recognition usually involves acquiring 3-D data about an object, extracting discriminant and invariant features from the data and then, based on these features, making decisions about the identity of the object. One way of acquiring 3-D data about an object is through the use of range information that can be obtained both passively and actively. A survey of the various range finding techniques can be found in a recent article by Jarvis.1

The next step in the recognition process after range acquisition is to extract features that are peculiar to the object and which are invariant to its position and orientation in the field of view. In this paper I explore the feasibility of the use of 3-D moment invariants as such features for complex object recognition. In 1980 Sadjadi and Hall2 introduced 3-D moment invariants and used it for recognizing a set of geometrical 3-D objects. Three dimensional moment invariants are a finite set which can represent three dimensional objects independent of any coordinate system. These moment invariants are obtained by relating 3-D statistical moments and ternary quantics, the study of which is part of the theory of invariant algebra. In 1982 Bjorklund and Los introduced 3-D moment invariants for identifying box-shaped targets in range imagery. They showed that the 3-D moment invariants of the targets are different enough to suggest their use for target classification.

In this paper two different objects of different complexities, a toy truck and a toy airplane, are expressed digitally in terms of two sets of 3-D points. Then their 3-D moment invariants are evaluated and it is shown that these features not only are invariant to position and orientation, but can be used to distinguish the two complex objects from each other.

Three-dimensional Moment Invariants

Definitions and derivations

The three dimensional moments of order $p + q + r$ of a density $\rho(x_1, x_2, x_3)$ are defined in terms of the Riemann integral as:

$$\mu_{pqr} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_1 - \overline{x}_1)^p (x_2 - \overline{x}_2)^q (x_3 - \overline{x}_3)^r \rho(x_1, x_2, x_3) \, dx_1 \, dx_2 \, dx_3$$

The central moments are defined as:

$$\mu_{pqr} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_1 - \overline{x}_1)^p (x_2 - \overline{x}_2)^q (x_3 - \overline{x}_3)^r \rho(x_1, x_2, x_3) \, dx_1 \, dx_2 \, dx_3$$

where

$$\overline{x}_1 = \frac{\mu_{100}}{\mu_{000}} ; \quad \overline{x}_2 = \frac{\mu_{010}}{\mu_{000}} ; \quad \overline{x}_3 = \frac{\mu_{001}}{\mu_{000}}$$

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The moment generating function for three dimensional moments may be defined as:

\[ M(u_1, u_2, u_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(u_1 x_1 + u_2 x_2 + u_3 x_3) \rho(x_1, x_2, x_3) \, dx_1 \, dx_2 \, dx_3 \]  

(4)

Using these definitions and applying the fundamental theorem of three dimensional moment invariants, it can be concluded that algebraic expressions involving three dimensional moments can be derived which have the property of invariancy with respect to any coordinate system. The derivations of these expressions rely on the theory of invariant algebra.

The study of ternary quadratics, which is used in the derivation of three dimensional moment invariants, leads to the study of conics, from which a set of two moment invariants is obtained.

\[ J_{1u}^2 \quad \text{and} \quad \frac{\Delta_{1u}}{J_{1u}} \]  

(5)

where

\[ J_{1u} = \mu_{200} + \mu_{020} + \mu_{002} \]  

(6)

\[ J_{2u} = \mu_{020}^2 + \mu_{002}^2 - \mu_{011}^2 + \mu_{100}^2 - \mu_{110}^2 \]  

(7)

\[ \Delta_{1u} = \text{det} \begin{pmatrix} \mu_{200} & \mu_{110} & \mu_{101} \\ \mu_{110} & \mu_{020} & \mu_{011} \\ \mu_{101} & \mu_{011} & \mu_{002} \end{pmatrix} \]  

(8)

### 3-D Moment Invariants of Complex Objects

Two objects, a toy truck and a toy airplane, were put in arbitrary positions with arbitrary orientations in the xyz space. Figure 1 displays these objects and shows their dimensions. Each object was represented by a finite though large set of discreet points that were obtained manually. A set of 3-D moment invariants for each object in three different and random positions and orientations in space was then obtained. Tables I and II show the values of the 3-D moment invariants for the two objects at different locations and orientations. The means and standard deviations of the 3-D moment invariants for the two objects are shown in Table III. As can be seen from the tables, the 3-D moment invariants remain virtually invariant regardless of the objects' positions and orientations in space. The slight variations that occur, as can be seen from the means and standard deviations in Table III, are insignificant. Furthermore, the 3-D moment invariants of the truck and the airplane are different enough to make it possible to classify these two objects by means of the values of their moment invariants.

### Summary and Conclusion

In this paper the feasibility of using 3-D moment invariants as an invariant and discriminatory attribute of three dimensional complex objects was addressed. Two different objects of different complexities were represented by sets of discreet 3-D points and the invariancy of the 3-D moments for these objects in different positions and orientations in a scene was shown. Furthermore, it was shown that the 3-D moment invariants for these objects are different enough to make their recognition based on the values of their 3-D moments possible.

These results are significant in that they demonstrate that 1) the 3-D moment invariants of these two objects, which are not composed of simple quadratic surfaces, remain invariant independent of their systems of reference, and 2) these 3-D moment invariants are different enough to make classification of the complex objects possible. These experimental results provide further impetus for using 3-D moment invariants as a discriminatory and invariant feature for machine and robotic recognition of any 3-D object.
Table I. The 3-D Moment Invariants of the Truck

<table>
<thead>
<tr>
<th>Position #</th>
<th>3-D Moment Invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.255037, 0.00899197</td>
</tr>
<tr>
<td>2</td>
<td>5.263213, 0.00352111</td>
</tr>
<tr>
<td>3</td>
<td>5.255034, 0.00959296</td>
</tr>
</tbody>
</table>

Table II. The 3-D Moment Invariants of the Plane

<table>
<thead>
<tr>
<th>Position #</th>
<th>3-D Moment Invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.236862, 0.01614955</td>
</tr>
<tr>
<td>2</td>
<td>4.257695, 0.01609848</td>
</tr>
<tr>
<td>3</td>
<td>4.237695, 0.01609848</td>
</tr>
</tbody>
</table>

Table III. The Means and Standard Deviations of the 3-D Moment Invariants of the Objects

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane</td>
<td>4.250751</td>
<td>0.009820770</td>
<td>0.016116</td>
<td>0.000024075</td>
</tr>
<tr>
<td>Truck</td>
<td>5.261821</td>
<td>0.005876207</td>
<td>0.007369</td>
<td>0.002731683</td>
</tr>
</tbody>
</table>

Figure 1. The two objects used in the experiment at three positions and orientations.
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References


